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CONTROL OF XENON INSTABILITIES
IN LARGE PWR'S
QUARTERLY PROGRESS REPORT
FOR THE PERIOD ENDING
SEPTEMBER 30, 1968

M. J. O'Boyle
Project Engineer

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WESTINGHOUSE ELECTRIC CORPORATION
Atomic Power Divisions
PWR Plant Division
Pittsburgh, Pennsylvania 15230
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SECTION 1
PROGRAM DESCRIPTION

This program investigates the characteristics and control of spatial instabilities in large pressurized water reactors (large PWR's), with particular emphasis on azimuthal xenon instabilities (x-y plane). The program consists of the following technical tasks:

1. Task EUXE-200 - Effect of Core Parameters on Spatial Oscillations

   The aim of this task is to analyze the effect of variations in core design and operating parameters on the propensity for spatial oscillations, with emphasis on those resulting from xenon redistribution. Parameters to be analyzed include, but are not limited to, core dimensions, fuel and moderator temperature feedbacks, and power distributions.

2. Task EUXE-300 - Remedial Control Procedures

   Under this task, two-dimensional x-y calculations will be performed to establish control methods and detector locations which will prevent divergent oscillations. Calculated results will be used to develop criteria for the application of remedial measures to large pressurized water reactors.

3. Task EUXE-400 - Three-Dimensional Analysis

   Under this task, either direct or synthesized three-dimensional calculations will be performed to further study spatial instabilities in large PWR cores with conditions conducive to oscillations. If a three-dimensional oscillation develops after perturbations have been introduced, selected remedial methods (as developed under Task EUXE-300) will be applied and evaluated. The unique characteristics of three-dimensional oscillations (if observed) will be identified and the criteria for the application of remedial measures will be developed.
SECTION 2

PROGRESS SUMMARY

Major progress for the three-month period ending September 30, 1968 is summarized below:

Technical efforts under Task EUXE-300 were completed. A procedure for controlling xenon-induced spatial oscillations in large PWR's was developed and demonstrated effective in all cases studied. A topical report describing this work in detail is in preparation.

The time-dependent flux expansion technique formulated last quarter for the purpose of eliminating time-step errors in digital xenon-redistribution calculations was further developed for numerical solution. The theoretical basis for the treatment of associated temperature feedback effects was more fully investigated and appropriate feedback equations derived. The one-dimensional, two-group diffusion code ZEST is being altered to facilitate testing the adequacy of the proposed formulation. The changes to ZEST involve the numerical integration of the xenon and iodine equations and computation of the higher flux, xenon, and iodine derivatives.
SECTION 3
EUXE-100

PROGRAM MANAGEMENT

M. J. O'Boyle, Project Engineer
Development Projects
PWR Plant Division

A. P. Suda

This is the eighth in a series of technical progress reports on the U.S.-Euratom program titled "Control of Xenon Instabilities in Large PWR's." The preceding reports in this series are:

EUROEC-1721 WCAP-3680-1
"Control of Xenon Instabilities in Large PWR's,
Technical Progress Report for the Period Ending
September 30, 1966"

EUROEC-1781 WCAP-3680-2
"Control of Xenon Instabilities in Large PWR's,
Quarterly Progress Report for the Period Ending
December 31, 1966"

EUROEC-1830 WCAP-3680-3
"Control of Xenon Instabilities in Large PWR's,
Quarterly Progress Report for the Period Ending
March 31, 1967"

EUROEC-1880 WCAP-3680-4
"Control of Xenon Instabilities in Large PWR's,
Quarterly Progress Report for the Period Ending
June 30, 1967"

EUROEC-1925 WCAP-3680-5
"Control of Xenon Instabilities in Large PWR's,
Quarterly Progress Report for the Period Ending
September 30, 1967"

EUROEC-2008 WCAP-3680-6
"Control of Xenon Instabilities in Large PWR's,
Semi-Annual Progress Report for the Period Ending
March 31, 1968"
"Control of Xenon Instabilities in Large PWR's, Quarterly Progress Report for the Period Ending June 30, 1968"

The following topical report has also been prepared under this program:

"Xenon-Induced Spatial Instabilities in Large Pressurized Water Reactors"

Program management is not discussed in this technical progress report, since separate reports emphasizing the administrative aspects of the program are published monthly for limited distribution.
The work performed under this task has been completed. The work is described in detail in the following topical report, the abstract of which is also presented:


Abstract:

The characteristics of free-running xenon-induced spatial oscillations in large pressurized water reactors (large PWR's) are investigated both from a phenomenological point of view and from the standpoint of the theoretical and calculational methods employed in their analysis. The digital simulation of spatial xenon instabilities with multidimensional, multi-group diffusion theory programs is investigated in detail. An extensive study of the effect of a finite, non-zero time-step length on calculated stability characteristics is presented, including a set of correlations which permits the extrapolation of digital calculations to effectively zero time-step length. The sensitivity of the digital results to the spatial mesh length, the energy mesh, and the treatment of temperature feedback effects is investigated. The effect of a non-
zero flux convergence criterion on spatial xenon oscillations is investigated. A comparison is made of modal theory results and digital simulation calculations. The space-dependent transfer function formalism is employed in a number of analyses, including the effect of delayed neutrons and the effect of finite temperature feedback time lags.

The space-time characteristics of xenon-induced instabilities in the two-dimensional plane perpendicular to the direction of coolant flow are studied for a large variety of core sizes, power distributions and temperature feedback effects. The effect of perturbation size and location is evaluated. Extensive parametric and sensitivity analyses are performed, based on a modal expansion method. Parameters investigated are core size, power distribution, power level, power coefficient, fuel enrichment, Xe-135 absorption cross section, I-135 fission yield and direct Xe-135 fission yield. Parametric calculations with one- and two-dimensional diffusion theory programs are performed to corroborate the modal theory results.
SECTION 5

EURO-300

REMEDIAL CONTROL PROCEDURES

A. F. McFarlane, Manager

Nuclear Design

4 Loop Plants

F. B. Skogen

1. SUMMARY

The technical aspects of the EURO-300 program have been completed. A procedure for controlling xenon-induced spatial oscillations in large PWR's has been developed and demonstrated to be successful for all cases studied. A topical report covering the work is being prepared.

2. INTRODUCTION

A detailed digital analysis of xenon-induced spatial oscillations in typical, large PWR cores, 11 feet in diameter and with a power density of 105 kw/ft, has shown these cores to be stable in the x-y plane provided Doppler coefficient and moderator temperature coefficient feedbacks are present. Consequently, for the purposes of studying x-y remedial control procedures an unstable system was artificially created by eliminating the moderator density feedback in the calculations. The resulting divergent xenon oscillation was shown in the previous quarterly report[1] Figure 4.

Two modes of oscillation were studied in the analysis; namely, an oscillation about a principal axis of symmetry (0° or 90°) and an oscillation diagonal axis of symmetry (+45°). Emphasis was placed on the diagonal mode of oscillation because it proved to be the more unstable condition.

3. CONTROL OF X-Y OSCILLATIONS

In large PWR's the means available for the control of xenon-induced spatial oscillations in a plane at right angles to the flow are limited to the insertion of one or more control rods. An important part of the study was concerned with identifying which rods were most suitable for the control of xenon during the oscillation.

Any effective control procedure must be capable of limiting the power density in any part of the core to within a specified percentage of its value before the commencement of the oscillation. In this study the criterion for the initiation of remedial action was chosen to be a quadrant tilt of ten percent. Quadrant tilt is defined as the ratio of the power in one quadrant of the core to the average power as indicated by the nuclear detectors. When the tilt reaches ten percent, a xenon control rod is inserted by the operator in the region where the local power density is rising. The control rods which have proved to be most effective for xenon control in the core studied are shown in Figure 1.

The effectiveness of the procedure is illustrated by applying control to the unstable cases discussed in the last quarterly. For the case where the oscillation is about a diagonal, the xenon control rod located on the diagonal in the quadrant where the power was increasing, was inserted. After allowing the xenon to redistribute for four hours, the rod is withdrawn. The subsequent behavior of the system is illustrated in Figure 2 which shows the power peaking in a horizontal plane as a function of time throughout the operation. The oscillation immediately after withdrawal of the rod is seen to be damped and the stability index is changed from +.014 hr\(^{-1}\) to -.0035 hr\(^{-1}\).
FIGURE 1.
X-Y XENON CONTROL ROD ARRANGEMENT

0°

270°

90°

180°

X - XENON CONTROL RODS
FIGURE 2

X-Y POWER PEAKING VS. TIME AFTER A PERTURBATION

Notes: Two-Dimensional (X-Y) Calculation
Half Core with Diagonal Symmetry
No Moderator Temperature Feedback
Power Density = 104.5 kW/ft²
An oscillation may also be set up about the other axes of symmetry (0° or 90°) and previously reported work has shown this mode to have a greater degree of stability. By following a similar control procedure, using the appropriate rod, as shown in Figure 1, the desired control has been achieved for this type of oscillation. The power peaking in this case is plotted in Figure 3 and the stability index is changed from +.0039 hr\(^{-1}\) to -.009 hr\(^{-1}\).

Various other alternatives involving the use of one and two control rods for controlling xenon-induced spatial oscillations were found unacceptable. It was found that any outer rod located on the diagonal depressed the quadrant power to such an extent that the quadrant power tilt became intolerable after a relatively short period of time. In addition, a divergent oscillation resumed following the withdrawal of these rods.

Another procedure that was tried, was to insert a rod in the quadrant were the power density was decreasing, the object being to burn out the xenon peak more rapidly. This procedure failed because unacceptable power distribution quickly occurred. An attempt was also made to use non-symmetrical rods, i.e. rods located off either symmetry axes, and although this proved unsuccessful, it is interesting to note that no precession of the oscillation resulted following this action.

Even though the four-hour duration of rod insertion proved to be very effective in the analysis of remedial control procedures, a study was made to determine the effect on subsequent oscillations resulting from variations in insertion times. This work revealed that any insertion times longer than four hours produced unacceptably large power tilts across the core while the rod was inserted, and, in addition, they produced a more divergent oscillation following withdrawal. A two-hour insertion was tried which caused a subsequent oscillation having a
FIGURE 3
X-Y POWER PEAKING VS. TIME AFTER A PERTURBATION

Notes: Two-Dimensional (X-Y) Calculation
Half Core Symmetric about Principal Axis
No Moderator Temperature Feedback
Power Density = 104.5 kW/t
stability index of approximately zero. Consequently, for this procedure to be successful, the corrective action would have to be repeated in order to return the system to a steady-state condition. This demonstrates that successful control can be achieved with insertion times ranging from two to four hours, the longer time being more preferable. It also illustrates that within this range successful control is not critically dependent upon time duration of rod insertion.
SECTION 6
EUXE-400
THREE-DIMENSIONAL ANALYSIS

G. H. Minton, Manager

J. E. Olhoeft, Manager

C. G. Poncelet

A. M. Christie

1. INTRODUCTION

In studies of xenon oscillations, the effect of the finite time-step length over which the xenon is depleted is of considerable importance. This effect was investigated in the EUXE-200 study[1] and is being continued in the present program. The basic equations and theory were discussed in the previous quarterly[2] and only a brief outline of these will be given here. Some further theory will then be described in more detail and some practical aspects of the investigation discussed.

As a brief summary, the basic equations are shown below. These are the linearized one-group diffusion equations with xenon feedback and the coupled linearized xenon and iodine equations.

\[ M \delta \phi = -\sigma^X_o \delta X \]  \hspace{1cm} (1)

\[ \delta X = a \delta X + b \delta I + c \delta \phi \]  \hspace{1cm} (2)

\[ \delta I = d \delta I + e \delta \phi \]  \hspace{1cm} (3)

A description of the variables in equations (1), (2) and (3) is given in Reference 2.


If equation (1) is differentiated \( n \) times and equations (2) and (3), \((n-1)\) times, then the \(n\)th derivative of the flux can be determined from the \(n\)th xenon derivative. This, in turn, can be calculated from the \((n-1)\)th derivatives of the flux, xenon and iodine through equation (2). In this manner, a recursive relation for the \(n\)th derivatives can be established which is based on the base values of the variables in question. Using these derivatives, a time dependent expansion of the flux can easily be derived.

2. NUMERICAL SOLUTION TO THE PROBLEM

It will be noticed that the operator \( M \) in equation (1) is the operator for the steady-state diffusion equation and as such is singular. For the general case consider the homogeneous equation

\[
MX = 0
\]  

(4)

This is equivalent to the steady-state diffusion equation and only has a non-trivial solution when \( M \) is singular. Equation (1) can be written as

\[
MY = S
\]  

(5)

This has a solution if and only if the inhomogeneous term \( S \) is orthogonal to the fundamental solution, i.e.,

\[
\int X \cdot S \, dr = 0
\]  

(6)

In our case we must satisfy the equivalent condition

\[
\int_{\text{reactor}} \phi \cdot \phi \delta X^{(n)} \, dr = 0
\]  

(7)

where the superscript represents the \(n\)-th derivative.

Due to the boundary conditions on the problem, this condition is always met. The general solution to equation (5) is

\[
Z = AX + Y
\]  

(8)
where \( A \) is an arbitrary constant. In the numerical solution of equation (1), we run into the problem that, if we do not impose another restriction on the system, we shall obtain a solution of the form

\[
\delta \phi (n) = A \phi + \delta \phi_p (n) \tag{9}
\]

where \( A \) is arbitrary and \( \delta \phi_p (n) \) is the solution to the inhomogeneous equation and represents the true \( n \)th derivative without any contamination from the fundamental. Obviously we wish to eliminate any contamination by setting \( A \) to zero. We do this through the condition that the total power level must remain constant over an infinitesimal period of time around the time step being considered. Hence

\[
P(t) = P(t + \delta t) \tag{10}
\]

Therefore

\[
\int \nu \Sigma_f \phi dr = \int \nu \Sigma_f (\phi + \phi \delta t) dr \tag{11}
\]

and

\[
\int \nu \Sigma_f \phi \delta t dr = \delta t \int \nu \Sigma_f \phi dr = 0
\]

In equation (1) the operator, as discussed is singular. Numerically, this is equivalent to stating that in the matrix \( M \) of equation (4), one row must be a linear combination of some of the other rows. In particular, for the case of a one-dimensional reactor, \( M \) is tri-diagonal and this imposes the restriction that only the first row or the last row can be linear combination of the other rows. There is no reason why one should be preferred over the other and hence either the first row can be a linear combination of some of the others, including the first. Therefore, we must eliminate either the first or the last row and replace it by the condition given in equation (11). This procedure can be generalized for a two or three-dimensional core where the matrix \( M \) will become 5 or 7 band.
To give some insight into the picture, consider the 3-band form of the matrix \( M \) as shown below.

![Exhibit A]

It can be seen that for the internal rows, no row can be a linear combination of any other row due to the fact that rearranging any group of rows will result in a matrix which is no longer tri-diagonal. Only when the first or last row is included can a rearrangement of rows result in the matrix still being tri-diagonal. However, if we remove either the first or last row, the system becomes under-determined. This can be remedied by replacing the last row by the numerical equivalent of equation (11). The matrix \( M \) now becomes of the form shown in

![Exhibit B]
With the extra condition imposed on the last row, we have effectively suppressed the contamination due to the fundamental and will see, in the solution of the equivalent set of linear equations, only that part of the solution for the derivative in question. In exactly the same manner, we could have replaced the first row by the numerical equivalent of equation (11) but the resulting set of equations would have been considerably more inconvenient to solve numerically.

3.0 TREATMENT OF FEEDBACK EFFECTS

In the foregoing discussion, temperature feedback effects have been ignored. If we assume that some arbitrary cross section $\Sigma$ (absorption, removal etc.) is a function of fuel temperature and moderator density we can write

$$\Sigma = \Sigma(T_f, \rho_m)$$

If we linearize the one-group diffusion equation and include feedback through the absorption cross section, we get

$$-\nabla \cdot \nabla \delta \phi + \Sigma_{a_o} \delta \phi + X_o \sigma \delta \phi - \frac{\nu \Sigma_f}{\lambda} \delta \phi = -\sigma \phi \delta X - \phi_0 \delta \Sigma$$

(13)

It should be noted that this expression is for feedback in the plane at right angles to coolant flow. The more general expression also including feedback parallel to coolant flow will be considered later in this section. In a multigroup scheme, feedback through removal cross sections could also be included in a similar manner. Differentiating equation (12) with respect to time now gives us a term $\partial \delta \Sigma_a / \partial t$. This could be calculated analytically through the expression

$$\frac{\partial}{\partial t} \delta \Sigma_a (T_f, \rho_m) = \frac{\partial \phi}{\partial t} \cdot \frac{\partial P}{\partial \phi} \left[ \frac{\partial T_f}{\partial P} \cdot \frac{\partial}{\partial T_f} + \frac{\partial H}{\partial P} \frac{\partial \rho_m}{\partial \rho_m} \right] \delta \Sigma_a$$

(14)
where
\[ H \] is the enthalpy
\[ P \] is the power density
\[ T_f \] is the fuel temperature
\[ \rho_m \] is the moderator density

However evaluating equation (14) would be extremely difficult in practice and for higher derivatives, the situation would become almost intractable.

One alternative and simpler approach is to fit the cross sections to a polynomial in the fundamental flux distribution for each fueled region. This will uniquely determine the cross sections for small deviations away from the equilibrium values of the flux. Thus we can write

\[ \Sigma(\phi) = \sum_{n=0}^{N} a_{n}^{(i)} \phi^n \]  

where the superscript \( i \) denotes the region dependence of the fit. Since a general fit is still awkward, and the theory is basically linear, we carry the expansion only up to \( N=2 \); i.e.,

\[ \Sigma(\phi) = a_0^{(i)} + a_1^{(i)} \phi + a_2^{(i)} \phi^2 \]  

Hence, in linear theory,

\[ \delta \Sigma(\phi) = (a_1^{(i)} + 2a_2^{(i)} \phi_0) \delta \phi = b(x) \delta \phi \]  

The coefficient \( b(x) \) is point-wise spatially dependent due to \( \phi \) varying point-wise. Hence

\[ \frac{\partial (\delta \Sigma)}{\partial t} = \frac{\partial}{\partial \phi} (\delta \Sigma) \frac{\partial \phi}{\partial t} \]  

Since for linear feedback, \( b \) is not a time-varying quantity.

\[ \frac{\partial^n (\delta \Sigma)}{\partial t^n} = b \delta \phi(n) \]
Equations (14) to (19) are valid only for a system in which the flow of moderator is at right angles to the direction of the calculation (i.e., the cross section is a function of the point-wise flux). This allows us to assume a fit of the form in equation (15). However, if the flow is parallel to one axis (e.g., z axis) of the calculation, then the cross section will also be proportional to the integrated power (in the direction of flow). Thus we may write

\[ \Sigma(r) = c_o + c_1 \phi o + c_2 \phi^2 + \int o \left( d_o + d_1 \phi + d_2 \phi^2 \right) dz \]  

(20)

If there is no axial variation in fuel composition then the c's and d's will be constant up the core (e.g., at the beginning of life). However due to burnup through the life of the core, the c's and d's will subsequently become space-dependent. The change in the cross sections will therefore be of the form

\[ \delta \Sigma(r,t) = (c_1 + 2c_2 \phi) \delta \phi + \int o \left( \delta d_o + \delta d_1 \phi + \delta d_2 \phi^2 \right) + (d_1 + 2d_2 \phi) \delta \phi \right) dz \]  

(21)

Therefore

\[ \frac{\partial (\delta \Sigma(r,t))}{\partial t} = c \frac{\partial \delta \phi}{\partial t} + \int o \left( \frac{\partial}{\partial t} \delta \phi \right) dz \]  

(22)

In general

\[ \frac{\partial^n (\delta \Sigma(r,t))}{\partial t^n} = c \delta \phi(n) + \int o \left( \delta \phi(n) \right) dz \]  

(23)
It has been assumed that the functions under consideration in equation (21) and (22) are well behaved in respect to the fact that differentiation can be taken inside the integral sign. It should also be noted that \( \phi \) refers, not to the flux at the time-step being considered, but to the fundamental flux where the coefficients were derived.

The general form of equation (1) with feedback now becomes

\[
M \delta \phi(n) = -\sigma x \delta X(n) - \phi \delta \Sigma_a(n) \tag{24}
\]

Therefore

\[
M \delta \phi(n) = -\phi_o (\sigma x \delta X(n) + c \delta \phi(n) + \int_0^z g \delta \phi(n) dz) \tag{25}
\]

Both terms on the right hand side involving \( \delta \phi(n) \) can be incorporated into the left hand side. Numerically, the integral term will produce a matrix with zeros in the upper triangular half in a one-group scheme. Thus, the general solution can be written as

\[
M_L \delta \phi(n) = -\phi_o \sigma_x \delta X(n) \tag{26}
\]

For the particular case with flow at right angles to the calculation equation (24) simply becomes

\[
(M+b) \delta \phi(n) = -\phi_o \sigma_x \delta X(n) \tag{27}
\]

4.0 DEPLETION OF XENON

Having derived a time-varying flux at a particular time step, we could then solve the xenon and iodine equations using a numerical scheme such as Runge-Kutta integration. As was found by Pearce and Roth\[1\], this approach leads

to numerical instabilities for a combination of high flux and long time step. The numerical scheme investigated here therefore approaches the problem differently. The proposed method involves theta differencing as discussed by Henry and Vota\textsuperscript{[1]} in the description of the WIGL-2 program. This finite difference scheme essentially weights forward and backward differencing schemes so that the numerical solution is a more accurate approximation than the standard forward, central or backward differencing schemes.

For a detailed description of the theory, reference \textsuperscript{1} should be examined. Only the derivation of the basic equation and final numerical scheme based on theta differencing will be stated here.

The xenon and iodine equations in a one-group scheme can be written as

\[
\frac{\partial X}{\partial t} = -\lambda_X X + \lambda_I I + (y_X \Sigma_f - \sigma_X) \phi \tag{28}
\]

\[
\frac{\partial I}{\partial t} = -\lambda_I I + y_I \Sigma_f \phi \tag{29}
\]

In matrix notation these can be written as

\[
\frac{\partial}{\partial t} \begin{bmatrix} X \\ I \end{bmatrix} = \begin{bmatrix} -\lambda_X + \sigma_X \phi & \lambda_I \\ 0 & -\lambda_I \end{bmatrix} \begin{bmatrix} X \\ I \end{bmatrix} + \begin{bmatrix} y_X \Sigma_f \phi \\ y_I \Sigma_f \phi \end{bmatrix} \tag{30}
\]

or

\[
V = AV + S \tag{31}
\]

The resulting finite difference scheme is given by

\[
V^{j+1} = [I-\Delta M]^{-1} [I+\Delta (A-M)] V^j + [I-\Delta M]^{-1} S \Delta \tag{32}
\]

where $V^j$ is the value of the vector $V$ at time step $j$, $\Delta$ is the subinterval of time over which the equations are being integrated and the matrix $M$ is defined by

$$M = \begin{bmatrix} -(\lambda_x + \sigma_x \phi) \theta_{11} & \lambda_I \theta_{12} \\ 0 & -\lambda_I \theta_{22} \end{bmatrix}$$  \hspace{1cm} (33)$$

It can be seen that when all the $\theta$'s are zero, the scheme becomes of backward difference form while when all the $\theta$'s are unity, $M$ is identical with $A$ and the scheme is of the forward type. The $\theta$'s are defined by

$$\theta_{11} = (2 - (\lambda_x + \sigma_x \phi) \Delta)^{-1}$$  \hspace{1cm} (34)$$

$$\theta_{22} = (2 - \lambda_I \Delta)^{-1}$$  \hspace{1cm} (35)$$

$$\theta_{12} = \frac{\lambda_x + \sigma_x \phi}{\lambda_x - \lambda_I + \sigma_x \phi} \theta_{11} - \frac{\lambda_I}{\lambda_x - \lambda_I + \sigma_x \phi} \theta_{22}$$  \hspace{1cm} (36)$$

The time between two diffusion calculations is broken down into subintervals over which equation (32) is solved. For these subintervals an appropriately averaged flux between the limits of the subinterval being considered can be derived from the time dependent form of the flux.

5.0 PROGRAMMING ASPECTS

Using the one-dimensional two-group diffusion code ZEST\textsuperscript{[1]}, the preceding equations, except those for feedback, have been programmed and are at present being debugged. The changes in ZEST involve the numerical integration of xenon

and iodine equations using equation (32) and the computation of the higher flux, xenon and iodine derivatives. Modifications must also be made to the subroutine which solves the diffusion equation in order that it can solve a system of equations whose coefficient matrix is of the form shown in Figure 2.

The above formulation is applicable to any number of dimensions or groups.