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STUDY OF A NATIONAL 2 GeV CONTINUOUS
BEAM ELECTRON ACCELERATOR

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ABSTRACT

Current trends in research in medium energy physics with electromagnetic probes are reviewed briefly and design objectives are proposed for a continuous beam 2 GeV electron accelerator. Various types of accelerator systems are discussed and exploratory designs developed for two concepts, the linac-stretcher ring and a double-sided microtron system. Preliminary cost estimates indicate that a linac-ring system which meets all the design objectives with the exception of beam quality and uses state-of-the-art technology can be built for approximately \$29 million. However, the "double-sided" microtron shows promise for development into a substantially less expensive facility meeting all design objectives. Its technical feasibility remains to be established. Specific areas requiring additional engineering studies are discussed, and current efforts at Argonne and elsewhere are identified.

I. INTRODUCTION

A strong consensus has developed recently among the nuclear physics community that research with electromagnetic probes in the 1-2 GeV range generated by a high current 100% duty factor electron accelerator represents an exciting frontier. Traditionally the small duty factor and relatively poor beam quality which characterize available electron accelerators have been a continuing limitation on research capability. However, recent advances in accelerator technology present the possibility of realizing an accelerator design of acceptable capital and operating cost which could furnish the long sought c.w. (100% duty factor) electron beams with high currents and with good properties. It is probable that new developments will occur in this area with increasing rapidity and that construction of a facility with the requisite properties will receive the broad support of the physics community.

Because of the rapidly growing interest in such a high-current c.w. electron accelerator, a design group of five physicists and accelerator specialists has been formed at ANL to review recent developments in accelerator technology, and to develop conceptual designs for technical evaluation and subsequent cost analysis. The design objective is a multiple beam facility in the 1-2 GeV range with a capital cost of \$30 - 40M.

Various accelerator concepts have been examined. Two possible microtron designs and a linac-stretcher ring have been studied in considerable detail. One of the conclusions of the study is that the technology exists for construction of a linac-stretcher ring system which could provide 100 μ amp beams between 1 and 2 GeV. Such a system would have the advantage that it can readily be scaled upward in energy. However, the capital and operating costs are projected to be higher than those of a microtron, and the expected beam quality relatively poor. Although the scientific feasibility of microtron operation in this energy range remains to be established, the potential cost savings and excellent beam characteristics warrant continued research and development on microtron type designs. Specific areas requiring additional conceptual or engineering studies are identified, and programs in progress are discussed.

II. ELECTROMAGNETIC PROBES AT MEDIUM ENERGIES — TRENDS AND OPPORTUNITIES

The electron and photon have proved to be two of the most valuable projectiles available to the nuclear physicist for probing the structure of nuclei and the nature of nuclear forces. The basic electromagnetic interactions can be calculated precisely, and the interactions are sufficiently weak that the probe does not distort the structure of the target nuclei under study. Although the properties of available electron and photon beams have not always met all the expectations of experimental physicists, progress in the area of electron accelerators has in general kept pace with that in other areas, and the study of electromagnetic interactions of electron and photon-beams has proved to be a prolific source of information of great detail and precision on the structure of nuclei.

Beginning with the pioneering work of Robert Hofstadter¹ in the 1950's, most of our information on nuclear size and surface thickness has come from measurements of electron scattering. With recent refinement, measurements² of elastic scattering over a large-range of momentum transfer, have provided values of the charge density in ^{40}Ca and ^{208}Pb with a spacial resolution of 0.5 fermi and a precision in the density of 1%. Such measurements are basic to our understanding of nuclear structure. The results provide stringent tests of Hartree-Fock calculations of nuclear density distributions and convincing indications of the presence of meson-exchange currents.

Our understanding of collective modes of excitation of nuclei, e.g. the giant multipole resonances, has in large measure come from studies of photon absorption and electron scattering. The giant electric dipole resonance is one of the most basic features of nuclear matter. Measurements of photon absorption and scattering cross sections have provided us with a detailed data base describing the properties of the giant dipole state and their variation throughout the periodic table. Almost all our information on magnetic dipole resonances and their systematics have come from electromagnetic probes. Much of the quantitative information on higher order multipole excitations has come from measurements of inelastic

electron scattering with high resolution. Electron scattering is the most promising probe for isolating electric monopole transition strength which is of fundamental importance because it provides a direct measure of the compressibility of nuclear matter.

In inelastic electron scattering to discrete final states, a transition density is measured which is analogous to the charge or magnetization density for the ground state. This density, in effect, probes directly the shape of the wave function which describes the excited nuclear state. It can provide a stringent test of theoretical descriptions of the excited states, whether they be collective in character or simple shell-model configurations. The recent advances in high energy spectrometer design have been dramatic³ and the experimenter can now make measurements of electron scattering with sufficient resolution to study structure in even the heaviest deformed nuclei such as ^{238}U . Currently, such electron-scattering spectroscopy is one of the most active and successful areas of photonuclear physics.

The shell model and its concepts play a vital role in our current understanding of nuclear structure. However, until now the properties of shell model levels have been experimentally accessible only for those levels near the Fermi surface. Comparatively little experimental data is available for deeply bound states. Quasi-elastic electron-scattering at high momentum transfer measures directly the single particle structure of deeply-bound levels. Inclusive measurements of quasi-elastic electron scattering have provided our best estimates of the fermi-momentum and the average interaction energy describing particle-hole excitations. Coincidence electron scattering experiments in which the recoil proton is observed,⁴ (e,e'p) have given direct measurements of the spectral function which describes the momentum distribution and separation energies for deeply bound shell model levels in nuclei.

And more recently, two landmark experiments have provided the most dramatic evidence to date for the presence of subnuclear constituents in nuclei. In the first, the electrodisintegration of the deuteron was studied under kinematic conditions corresponding to high momentum transfer

but excitation energies near the electrodisintegration threshold.⁵ The cross sections measured are as much as a factor of ten larger than predictions of the simplest impulse approximation. The observed enhancement is the strongest evidence for meson exchange currents in nuclei. In the second experiment,⁶ elastic electron scattering on light nuclei such as the ^2H , ^3He and ^4He were studied at very high energies of 0.8 - 2.5 GeV. Anomalies in the behavior of the form factors at very high momentum transfer can be attributed to the properties of the underlying quark-constituents of the nuclei. The evidence suggests that asymptotic form factors for these light nuclei obey scaling laws consistent with their predicted quark structure.

The implications of these research trends for future major facilities and the characteristics of future accelerators which will guarantee the continued vitality of the field have been widely discussed.⁷ There is a clear need for measurements of nuclear form factors and transition densities with much better spacial resolution. The much higher momentum transfer necessary for such measurements can be accomplished by measurements through large scattering angles. However, at moderate incident electron energies, typical counting rates are very low. The scattering cross section for a given momentum transfer goes as the electron energy squared. For a given momentum transfer increasing the incident energy from 400 MeV to 1.3 GeV increases the counting rate by an order of magnitude.

Coincidence measurements, because of their selectivity and complete characterization of the reaction detected, will become a major component of future experimental programs. Simple arguments based on the systematics of quasifree-proton scattering ($e, e'p$) indicate that in such experiments data acquisition rates will increase by an order of magnitude when the incident electron energy is increased from 0.9 to 1.8 GeV.⁸ The study of mesonic effects in nuclei through photo and electroproduction of pions and kaons will undoubtedly become a major thrust of research programs, again necessitating beams in the 0.5 to 2.0 GeV. Poor duty factor has always been a major limitation on photonuclear experiments. Even now, high energy beams are available only at a duty factor of <5%. Under such

conditions, coincidence measurements are limited by accidental rates, and can be made only with the greatest efforts, and by use of prodigious amounts of beam time. The availability of full duty factor beams in the energy range of 0.5 — 2.0 GeV make this class of measurement a realistic option which can be used on an extensive scale.

The conclusion of the nuclear-physics community as reported by the Livingston panel⁹ is that "An electron accelerator with peak energy of 1 to 2 GeV with 100% duty factor, and with beam currents on the order of 100 μ A, promises to open a completely new and exciting range of electron nuclear research which would address many of the most interesting questions at the frontiers of nuclear physics."

III. DESIGN OBJECTIVES

In order to assess possible accelerator concepts for a 1-2 GeV facility, we have reviewed recent discussions of future needs. The design objectives listed in Table I result from the consideration of prospective experiments which place the most stringent requirements on the pertinent beam properties. They are design goals and may or may not be simultaneously achievable, but should serve as the standards against which to measure the various alternatives. The criteria leading to their choice are given below.

Energy — Based on the considerations of the Livingston panel,⁹ a momentum transfer in electron scattering experiments of ~ 400 MeV/c is typical of the proposed program. Because cross sections go as the square of the incident energy, high electron energy is desired in the range of $\sim 0.5 - 2.0$ GeV. We have chosen a maximum electron beam energy of 2 GeV. However, there is considerable sentiment toward even higher energies if the cost and design compromises are not prohibitive.

Energy Variability — Suitable variation of the beam energy, E_e , over the range 0.5 - 2 GeV is essential for electron scattering studies. In addition a major component of the research program will probably involve the use of a "tagged photon" monochromator. Such an instrument can be realized easily with a capability of magnetically analyzing electrons in the range $\sim 10 - 150$ MeV corresponding to photon energies of $\sim E_e$ to $E_e - 150$ MeV. Variability in 100 MeV steps above 500 MeV would guarantee that the full photon energy range can be scanned.

Average Current, I_{av} — No clear upper limit of current can be given since one can always take advantage of higher intensities in experimental designs which are more selective or in searches for rare processes. The beam current of 100 μ A was proposed by the Livingston panel as adequate for a broad class of low cross section single-arm spectrometer experiments and for the requirements of many proposed coincidence measurements.

Duty Factor — "In pulsed operation, the accidental to true signal

Table I. Accelerator system design objectives

E_{max}	$\geq 2 \text{ GeV}$
Variability	in steps of $\sim 100 \text{ MeV}$ from $\sim 500 \text{ MeV}$
I_{av} (per beam)	$\geq 100 \mu\text{A}$
Number of beams	> 1
Duty factor	70 - 100%
ΔE	$\leq \pm 200 \text{ keV}$
Emittance ¹	$0.2 \pi \text{ mm mrad}$
E reproducibility	$\sim 100 \text{ keV}$
E stability	$\sim 300 \text{ keV}$
I_{av} stability	1 - 5%

¹For beam energies above 500 MeV.

ratio, dead-time losses and pulse pile up effects are all proportional to the peak current in the macropulse. For a given peak current, the average current and hence the gain in overall count rate is therefore proportional to the macroscopic duty factor, hence the highest possible duty factor is required. The microscopic duty factor is unimportant provided the frequency f obeys $f^{-1} < 2\tau$ where 2τ is the coincidence resolving time. Since 2τ is typically 10^{-8} sec, we require $f > 100 \text{ MHz}$." - CRNL-550.¹⁰

Energy Spread - It is difficult at this time to give a precise objective for energy spread. Resolution of 10^{-4} to 10^{-5} are required to observe details of nuclear structure in scattering of 2 GeV electrons. Single arm "dispersed beam" spectrometers can achieve this precision with incident energy spread on target of only $\Delta E/E = \pm 0.5\%$. However, in coincidence measurements where simultaneous dispersion matching of two spectrometers is not possible, in at least one of the arms resolution will be limited by the beam parameters. Should it be possible to achieve

$\Delta E/E \approx 10^{-4}$ in the electron beam, one could avoid dispersion matching techniques, and develop spectrometers of simple less-expensive design. With this in mind we have established a design objective of $\Delta E/E = \pm 10^{-4}$ for energy spread at 2 GeV and $\Delta E = \pm 200$ keV everywhere.

Emittance - Beam emittance will be limited by the requirements of "dispersed beam" high resolution spectrometers which are capable of measurements with $\Delta p/p < 10^{-4}$. These instruments require only modest energy spread in the incident beam, $\sim 0.5\%$ but they do require excellent beam emittance. For a spectrometer with a maximum field of 1.6 T and $\rho = 4$ meters at 2 GeV, state-of-the-art resolution is $\Delta p/p \sim 10^{-4}$ leading to a restriction on spot size of 0.4 mm. In typical design (see S. Kowalski et al.¹¹) 2nd order aberrations lead to a restriction in beam divergencies consistent with 10^{-4} resolution of $\theta_{\text{div}} \approx 0.5$ mr. We estimate the required emittance to be 0.2π mmmr.

Energy Stability and Reproducibility - These limits are imposed on the beam to be consistent with the energy resolution goal of 500 keV.

Current Stability - It is important in a machine of this type to avoid sizable variations in the beam current which would have the effect of making the effective duty factor appreciably less than the design objective.

IV. POSSIBLE ACCELERATOR CONCEPTS

The primary constraint on an accelerator design which meets the design objectives of Section III is low capital and operating cost. In the past, high duty factor electron accelerators have not been feasible because of their prohibitive power costs. High energy electron beams are normally achieved by accelerating electrons in intense electric fields generated in appropriate wave guides excited by means of intense pulses of microwave power. Economical operation is achieved by restricting the accelerating cycle to very short pulses. Operation of such systems in a continuous beam mode is not feasible. For example, consider a 1 km long 2 GeV linear accelerator using a SLAC type of wave guide structure. Assuming a reasonable shunt impedance of $ZT^2 \approx 60 \text{ M}\Omega/\text{m}$ for the r.f. wave guide, 67 megawatts of power would be required for acceleration to 2 GeV. Such operation would be prohibitively expensive. Several proposals to overcome this problem have been widely discussed.

A. Microtrons

One alternative is an outgrowth of an idea used in the classic microtron design proposed by Veksler.¹² By inserting an r.f. accelerating section in a magnetic field whose value is chosen so as to guarantee correct beam phasing, one can recirculate the electron beam through the same cavity many times. The power required for a given beam energy is reduced relative to one pass acceleration by $\frac{1}{n}$ where n is the number of passes. The evolution of the microtron concept to the most recent proposal is shown schematically in Fig. 1. In the race-track microtron, the magnetic field is split into two half circular sectors separated by sufficient drift space to allow insertion of a linac section of substantial accelerating power. In order to avoid the prohibitive costs of the mass of the return magnets required for high energy operation (e.g. $\geq 500 \text{ MeV}$), the double-sided microtron has been proposed.¹³ This system required about 1/5 the volume of steel of the conventional microtron design. The advantage of the microtron concept is the moderate r.f. power required. Disadvantages include a possible limit on the maximum current accelerated imposed by microwave beam blowup. Because of multiple turn recirculation

Fig. 1. Evolution of Microtron Accelerator Designs

such a limit is more serious than it would otherwise have been. A second problem is the small beam emittance and large number of focusing elements required for beam transport through the full accelerating cycle.

B. Superconducting Recyclotrons

In a second approach, the large power dissipated in the accelerating structures is decreased by using high Q superconducting cavities. Workers hope eventually to achieve voltage gradients of ~ 6 MV/m. A 50 m structure would achieve 0.5 - 2.0 GeV operation with only 5 turns of recirculation. The recirculation is accomplished by means of a so-called "unconventional" magnet system. This concept is attractive because a guide field of discrete elements could replace the massive end magnets characteristic of the conventional microtron. The r.f. power requirements are least for this design. A major difficulty with such high Q systems is severe limit on beam current resulting from cumulative beam blowup. Experience to date indicates that the maximum current achievable is in the 100 μ A range.¹⁴ With recirculation, the external beam limit is $\sim 100/n$ μ A.

C. Linac-Stretcher Rings

The third type of accelerator draws heavily on the electron-storage ring technology developed in high energy physics. A typical system is shown schematically in Fig. 2. A conventional pulsed linac, such as the SLAC design, is used to inject up to 4 or 5 turns of electrons into a conventional stretcher ring. The r.f. power requirements are competitive with those of other alternatives. Beam extraction is accomplished by controlled translation of the circulating beam into a sextupole field which induces a third order resonance moving the beam into one or more extraction septa. Such a system is attractive because it is based on existing technologies¹⁵ and because the systems scale linearly with energy so that expansion to higher energy presents no fundamental problem. Also multiple beam extraction has been demonstrated.¹⁶ Disadvantages include higher operating costs and a beam quality significantly poorer than that of other systems. A 500-MeV stretcher ring system currently under consideration at Frascati, would be the first such accelerator.¹⁷

Fig. 2. Proposed Linac-Stretcher Ring Accelerator Design

In the following sections, we describe conceptual designs for microtron and linac-stretcher ring systems. These designs have been developed to permit comparative evaluation of capital costs and to determine areas of the technology requiring additional research development.

V. THE MICROTRON APPROACH - A CONCEPTUAL DESIGN

A. Design Criteria

The basic feature of the microtron accelerator is the coherence condition which requires the path difference in the successive orbits of the beam to be an integral multiple of the wave length of the accelerating r.f. field. This condition is met by appropriate choice of r.f. accelerating voltage and strength of the magnetic field in the guide magnets. It insures that r.f. phase stability can be maintained and that the energy gain per revolution of the beam is constant. A variety of magnet configurations have been proposed for microtrons,¹⁸ and the specific coherence condition is somewhat different in each case. In order to illustrate the basic properties of each choice we derive the appropriate conditions for beam coherence and phase stability. The coherence condition requires that the time taken by an electron to travel along a trajectory from linac exit to linac entrance is an integral multiple of the r.f. field period. This requirement can be written in the form

$$\frac{L_1}{\beta_1} = \mu_1 \lambda , \quad (1)$$

$$\frac{L_{n+1}}{\beta_{n+1}} - \frac{L_n}{\beta_n} = \nu \lambda ,$$

where λ is the r.f. field wavelength, μ_1 and ν are integers, and β_n the velocity of the electrons in units of the light velocity c . From Fig. 3 we see that

$$L_n = 2\pi r_n + S \quad (2a)$$

for the conventional race track microtron (Fig. 3a)

$$L_{n,i} = (\pi - 2)r_{n,i} + S_i \quad i = 1, 2 \quad (2b)$$

Fig. 3. Possible Microtron Accelerator Designs

for the double-sided microtron (Fig. 3b), and

$$L_n = 2(\pi - 1) r_n + S \quad (2c)$$

for the hybrid magnet structure (Fig. 3c). In Eq. (2) S and S_i are constant distances, independent of the turn number n . The radius of the circular orbits in the bending magnets is given by

$$r_n = \frac{W_n \beta_n}{eBc} , \quad (3)$$

where W_n is the total energy of the electron. The magnetic field B is assumed to be uniform. Substituting Eq. (3) in Eqs. (2a, b, c) and using Eq. (1) we obtain the coherence conditions

$$\frac{2\pi W_1}{eBc} + \frac{S}{\beta_1} = \mu_1 \lambda , \quad (4a)$$

$$\frac{2\pi \Delta W}{eBc} + S \left(\frac{1}{\beta_{n+1}} - \frac{1}{\beta_n} \right) = \nu \lambda ,$$

$$\frac{(\pi - 2)W_{1,i}}{eBc} + S_i / \beta_{1,i} = \mu_{1,i} \lambda \quad (4b)$$

$$\frac{(\pi - 2)\Delta W}{eBc} + S_i \left(\frac{1}{\beta_{n+1,i}} - \frac{1}{\beta_{n,i}} \right) = \nu \lambda ,$$

$$i = 1, 2$$

$$\frac{2(\pi - 1)W_1}{eBc} + \frac{S_1}{\beta_1} = \mu_1 \lambda , \quad (4c)$$

$$\frac{2(\pi - 1)\Delta W}{eBc} + S \left(\frac{1}{\beta_{n+1}} - \frac{1}{\beta_n} \right) = \nu \lambda$$

where ΔW is the energy gain per turn. In general $\left(\frac{1}{\beta_{n+1}} - \frac{1}{\beta_n} \right)$ is small

enough that it can be set equal to zero and we have

$$\frac{2\pi\Delta W}{eBc} = \nu\lambda \quad (5a)$$

$$\frac{(\pi - 2)\Delta W}{eBc} = \nu\lambda \quad (5b)$$

$$\frac{2(\pi - 1)\Delta W}{eBc} = \nu\lambda \quad (5c)$$

By using this approximation, the usual coherence condition, one introduces a phase error on the n th orbit of the form

$$\Delta\phi_{ni} = \frac{2\pi S_i}{\lambda} \left(\frac{1}{\beta_{n,i}} - \frac{1}{\beta_{n-1,i}} \right)$$

As shown below for stable longitudinal motion, there is a maximum permissible phase error relative to the synchronous phase $(\phi_s)_{\max}$. For no beam loss $\sum_n \Delta\phi_{n,i}$ must be smaller than $(\phi_s)_{\max}$.

The range of phase acceptance and the energy spread of the beam are related to the phase slip per turn, $2\pi\nu$, and differ for the three different designs. Consider an electron that leaves a linac with a phase error $\delta\phi_k$ and an energy error δW_k . Here k enumerates half turns for the double-sided microtron and whole turns for the other two magnetic structures. After the next passage through a linac the phase and energy errors are respectively

$$\delta\phi_{k+1} = \delta\phi_k + \frac{\delta L_k}{\beta_k \lambda} 2\pi \quad (6a)$$

$$\delta W_{k+1} = \delta W_k + eV_o [\cos(\phi_s + \delta_{k+1}) - \cos \phi_s] \quad (6b)$$

where V_o is the peak linac voltage and ϕ_s is the synchronous phase. Note that the energy gain per turn is given by

$$\Delta W = 2eV_0 \cos\phi_s \quad (7)$$

for the double-sided microtron and

$$\Delta W = eV_0 \cos\phi_s \quad (8)$$

for the other two magnet structures. Substituting

$$\begin{aligned} \delta L_k &= \frac{2\pi}{eBc} \left(\beta_k \delta W_k + \delta \beta_k W_k \right) \approx \frac{2\pi \beta_k \delta W_k}{eBc} \\ \text{or } \delta L_k &= \frac{\pi-2}{eBc} \left(\beta_k \delta W_k + \delta \beta_k W_k \right) \approx \frac{(\pi-2) \beta_k \delta W_k}{eBc} \\ \text{or } \delta L_k &= \frac{2(\pi-1)}{eBc} \left(\beta_k \delta W_k + \delta \beta_k W_k \right) \approx \frac{2(\pi-1) \beta_k \delta W_k}{eBc} \end{aligned}$$

in Eq. (6a), neglecting higher order terms of $\delta\phi_{k+1}$ in Eq. (6b) and using Eqs. (5a,b,c), we obtain, after some manipulation

$$\delta\phi_{k+1} = \delta\phi_k + \frac{2\pi v}{\Delta W} \delta W_k$$

$$\delta W_{k+1} = \delta W_k - eV_0 \sin\phi_s \left(\delta\phi_k + \frac{2\pi v}{\Delta W} \delta W_k \right)$$

or in matrix form

$$\begin{pmatrix} \delta\phi_{k+1} \\ \delta W_{k+1} \end{pmatrix} = M \begin{pmatrix} \delta\phi_k \\ \delta W_k \end{pmatrix} \quad (9)$$

where

$$M = \begin{pmatrix} 1 & \frac{2\pi\nu}{\Delta W} \\ -eV_o \sin\phi_s & 1 - \frac{2\pi\nu}{\Delta W} eV_o \sin\phi_s \end{pmatrix}$$

Since the determinant of M is equal to one, the motion is stable if the trace of M satisfies the condition

$$-2 < 2 - \frac{2\pi\nu}{\Delta W} eV_o \sin\phi_s < 2$$

or more fully using Eqs. (7) or (8)

$$0 < \tan\phi_s < \frac{2m}{\pi\nu} \quad (10)$$

where $m = 2$ for the double sided microtron and $m = 1$ for the other two magnet structures. From a comparison of Eqs. (5a,b,c) and Eq. (10) it is evident, that for a given energy gain per turn, ΔW , and magnetic field B the double-sided microtron has the largest phase acceptance.

Since $\Delta W = m e V_o \cos\phi_s$ the matrix M can be written

$$M = \begin{pmatrix} 1 & \frac{2\pi\nu}{W} \\ -\frac{\Delta W}{m} \tan\phi_s & 1 - \frac{2\pi\nu}{m} \tan\phi_s \end{pmatrix}$$

Assuming that condition (10) is satisfied we can define quantities θ, a, b and c by

$$\begin{aligned}
\cos\theta &= 1 - \frac{\pi v}{m} \tan\phi_s \\
a \sin\theta &= \frac{\pi v}{m} \tan\phi_s \\
b \sin\theta &= \frac{2\pi v}{\Delta W} \\
c \sin\theta &= \frac{\Delta W}{m} \tan\phi_s
\end{aligned} \tag{11}$$

The condition that $\det M = 1$ becomes

$$bc - a^2 = 1 \tag{12}$$

The matrix M can be rewritten in the form

$$M = \begin{pmatrix} \cos\theta + a \sin\theta & b \sin\theta \\ -c \sin\theta & \cos\theta - a \sin\theta \end{pmatrix} = I \cos\theta + J \sin\theta$$

where I is the unit matrix, and

$$J = \begin{pmatrix} a & b \\ -c & -a \end{pmatrix}, \quad J^2 = -I$$

is a matrix with zero trace and unit determinant (see Eq. 12). It is not difficult to show that [see E. D. Courant and H. S. Snyder, *Annals of Physics*: 3, 1-48 (1958)]

$$M^k = I \cos k\theta + J \sin k\theta$$

The usefulness of introducing the quantities a , b and c arises mainly from the feature that in the $\delta\phi\delta W$ space the quadratic form

$$c \delta\phi_k^2 + 2a \delta\phi_k \delta W_k + b \delta W_k^2 = \frac{A}{\pi}, \quad (13)$$

is constant independent of k . The quantity A is the area of the $\delta\phi - \delta W_k$ space occupied by the beam. If the electrons are injected in such a way that

$$c \delta\phi_1^2 + 2a \delta\phi_1 \delta W_1 + b \delta W_1^2 = \frac{A}{\pi}, \quad (14)$$

Then for any k we have

$$\begin{aligned} (\delta\phi)_{\max} &= \sqrt{bA/\pi} \\ (\delta W)_{\max} &= \sqrt{cA/\pi} \\ (\delta W)_{\max} &= \sqrt{c/b} (\delta\phi)_{\max} \end{aligned} \quad (15)$$

using Equation (11) to evaluate the above we find that

$$\delta W_{\max} = \sqrt{\frac{\tan\phi_s}{2\pi m\nu}} \Delta W \delta\phi_{\max} \quad (16)$$

Hence, from Equation (10)

$$(\delta W)_{\max} < \frac{\Delta W}{\pi\nu} (\delta\phi)_{\max}$$

For typical designs the energy gain per turn is less than 50 MeV. Assuming that the spread in phase $(\delta\phi)_{\max}$ can be limited to less than 1° it follows for each of the designs that

$$(\delta W)_{\max} < 0.28 \text{ MeV}$$

thus on the basis of these preliminary considerations it appears that regardless of configuration microtrons will furnish beams with the requisite energy definition.

B. Conceptual Design

1. Introduction

The conventional microtron has the disadvantages of smallest phase acceptance $(\phi_s)_{\max}$ for given values of ΔW , B , and λ and also is the most difficult configuration in which to achieve good transverse phase space matching with the acceptance of the linac. The double-sided microtron has the largest $(\phi_s)_{\max}$ and better transverse phase space matching but has the disadvantage of considerable vertical defocusing at the magnet edges. The hybrid system shown in Fig. 3c is a possible compromise. In this case of four 90° bending magnets and one linac, the majority of focusing quadrupoles can be accommodated in the short straight sections. The path length corrections and extraction can be done in the long straight section. The longitudinal "acceptance" $(\phi_s)_{\max}$ is intermediate between the race-track microtron and the double-sided microtron. Pole-face windings in the two large volume bending magnets where the orbits are concentric could ease the path length corrections.

For both the double-sided and the hybrid designs a significant problem of beam optics arises because of the vertical defocusing which occurs at the entrance and exit faces of the two sector magnets. For these elements the beam enters and exits at 45° to the normal to the pole face. There is equal and opposite edge focusing in the x and y directions which may be characterized by the matrix equations

$$\begin{pmatrix} x \\ x \end{pmatrix}_{\text{out}} = \begin{pmatrix} 1 & 0 \\ -\frac{\tan\gamma}{r_n} & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{\text{in}},$$

$$\begin{pmatrix} y \\ y' \end{pmatrix}_{\text{out}} = \begin{pmatrix} 1 & 0 \\ +\frac{\tan\gamma}{r_n} & 1 \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix}_{\text{in}},$$

where γ is the orbit angle relative to the normal of the magnet edge. In this geometry $\tan \gamma = 1$ and $r_n = W_n \beta_n / eBc$. For small values of r_n the effects are too strong to be corrected with discrete elements. Therefore in the region traversed by low energy orbits, the pole edges will be shaped (see Fig. 4) so that the edge angle, γ will be very small, both for entrance and exit. The residual effects of the edge will be compensated for by a quadrupole singlet to form a very weak double focusing lens. Focusing and phase-space matching is done on individual orbits by quadrupole magnets located in each of the straight sections. The complete magnet configuration for a hybrid system is shown in Fig. 4. The quadrupole magnets in the long straight sections are all identical with the same field strengths. The magnets in the short straight sections increase in strength in proportion to the orbit momentum.

It is apparent that if we can solve the dominant problem of vertical defocusing, the choice between the double-sided microtron and hybrid microtron must be based on other factors. There are several factors strongly favoring the double-sided design. As the discussion above shows, the phase acceptance in the double-sided system is much greater than other designs. In addition, considerable saving in magnet costs accrue from the less massive sector magnets in the double-sided design. Finally the availability of both long straight sections for locating linac sections allows a much more compact design for a given r.f. accelerating gradient. Our preliminary considerations focused on a hybrid microtron design but in light of the above factors we have concluded that the double-sided system is the technical optimum. Therefore, our conceptual design for the microtron is based on the double-sided geometry.

2. Longitudinal motion

Substituting the energy gain per turn $\Delta W = 50$ MeV, the mode number $\nu = 1$ and the r.f. wavelength $\lambda = 0.125$ m in the coherence condition (Eq. 5b)

$$(\pi - 2) \frac{\Delta W}{eBc} = \nu \lambda$$

Fig. 4. Complete Magnet Configuration for Hybrid Microtron Design

we obtain

$$B = 1.523T$$

Subtracting in Eqs. (4b) L_{11} from L_{12} we find

$$(\mu_{12} - \mu_{11}) \lambda = \frac{(\pi - 2)\Delta W}{2eBc} + \frac{S_2}{\beta_{12}} - \frac{S_1}{\beta_{11}}$$

For the microtron under consideration we have $\beta_{12} \approx \beta_{11} \approx 1$ so that

$$\mu_{12} - \mu_{11} = 0.5 + \frac{S_2 - S_1}{\lambda}$$

The rotation period must be an integral multiple of the r.f. field period. To satisfy this condition, μ_{11} and μ_{12} in Eq. (4b) must be integers. In this case the linac are in phase. If the two linacs are 180° out of phase we have μ_{11} and μ_{12} half integers. There is no reason to choose the latter.

The first equation of (4b) can be written in the form

$$S_1 = \left(\mu_{11} \lambda - \frac{(\pi - 2)W_{11}}{eBc} \right) \beta_{11}$$

$$S_2 = \left(\mu_{12} \lambda - \frac{(\pi - 2)W_{12}}{eBc} \right) \beta_{12}$$

Choosing the injection energy $W_i = 5$ MeV we find (see Fig. 5)

$$W_{11} \approx W_i + \frac{\Delta W}{2} = 30 \text{ MeV}, \quad \beta_{11} \approx 1$$

$$W_{12} \approx W_i + \Delta W = 55 \text{ MeV}, \quad \beta_{12} \approx 1$$

Choosing $\mu_{11} = \mu_{12} = 115$ and recalling that $\lambda = 0.125$ m and $B = 1.523T$ we obtain

Fig. 5. Plan View of Double-Sided Microtron

$$S_1 \approx 14.30 \text{ m and } S_2 \approx 14.24 \text{ m}$$

From Fig. 5 we see that

$$S_1 = S + 2s_1 \text{ and } S_2 = S + 2s_2$$

With $s_1 \approx 1.50 \text{ m}$ we find $S = 11.3 \text{ m}$ and $s_2 \approx 1.47 \text{ m}$. The choice of ϕ_s is determined by two conflicting requirements. Equation (10) shows that for a large stable region we should have

$$\phi_s \approx \frac{1}{2} \tan^{-1} \frac{2m}{\pi v}$$

On the other hand, from Eq. (7) we see that for an effective use of the linac ϕ_s should be as small as possible. We choose $\phi_s = 9^\circ$ which is sufficiently small that $\cos\phi_s$ is close to one, but large enough that linac input power fluctuations and magnetic field errors do not put ϕ_s outside the stable region. Substitution of $\Delta W = 50 \text{ MeV}$, $\phi_s = 9^\circ$ and $m = 2$ in Eqs. (7) and (11) give $eV_0 = 25.3 \text{ MeV}$, $\theta = 41.3^\circ$, $a = 0.377$, $b = 0.19 \text{ MeV}^{-1}$ and $c = 6 \text{ MeV}$. Setting in Eq. (15) $\delta\phi_{\max} = 1^\circ = 0.0175$ radian we obtain $\delta W_{\max} = 0.1 \text{ MeV}$. Assuming an accelerating field $E \approx 1.4 \text{ MV/m}$ and 0.125 m space between 3 m sections the linac length is 18.75 m . Including the drift distances S_1 and S_2 we have the total long straight section length $L_\ell = 21.719 \text{ m}$.

3. Transverse motion

As pointed out in the introduction, the main problem of the double-sided microtron is the strong vertical defocusing effect due to the 45° angle the beam makes with the normal to the magnet boundary. The beam crosses the fringing fields bordering the linac straight sections at the same location. From Fig. 5, we see that for these edges it is not difficult to have the beam normal to the boundary of the magnetic field. We will also attempt to reduce the short straight sections edge-angles for the first six turns. The linac is divided into 3 m long sections with quadrupole magnets between the sections. The gradient of these quadru-

pole magnets is constant and chosen such that the phase advance of the betatron motion would be 90° per focusing period if the energy of the electrons remains 5 MeV. In other words, in the transformation matrix per focusing period

$$M = \begin{pmatrix} 1 - L^2/2f^2 & 2L \pm L^2/f \\ -\frac{L}{2f^2} \mp \frac{L^2}{4f^3} & 1 - L^2/2f^2 \end{pmatrix}$$

we have $1 - \frac{L^2}{2f_i^2} = 0$ with $\frac{1}{f_i} = \frac{eB'l_q c}{W_i \beta_i}$

$$W_i = 5 \text{ MeV } (\beta_i \approx 1), \quad L = 3 \text{ m.}$$

Solving for the quadrupole gradient we find $B'l_q \approx 0.008 \text{ T/m} - \text{m}$.

Because of the quadrupole focusing the beam is not circular but elliptical. From Fig. 5 we see that at the entrance of the linac we have a y-focusing quadrupole, so that the maximum beam size is given by

$$a_y = \left[2L \left(1 + \frac{L}{2f_i} \right) \epsilon \right]^{1/2} = \left[6 \left(1 + \frac{1}{2}\sqrt{2} \right) \epsilon \right]^{1/2}$$

For $\epsilon = 1 \text{ mm} - \text{mrad}$ we obtain $a_y = 3.2 \text{ mm}$. The effects of the quadrupole between the linac sections decreases as the energy of the electrons increase. For the second and subsequent turns we can neglect the effects of these quadrupoles. The focusing is mainly achieved by the quadrupoles in the short straight sections.

The x- and y-directions transformation matrices of the 90° bending magnets are respectively

$$M_x = \begin{pmatrix} 0 & r & r \\ -\frac{1}{r} & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}, M_y = \begin{pmatrix} 1 & \frac{\pi}{2}r \\ 0 & 1 \end{pmatrix}$$

$$\text{Let } \begin{pmatrix} a_x & b_x & 0 \\ c_x & d_x & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} a_y & b_y \\ c_y & d_y \end{pmatrix}$$

be respectively the x- and y-directions transformation matrices from magnet edge to the center of the straight section. The effects of the short straight sections edge angles are included in these matrices. We assume the system has mirror symmetry about the mid point of the straight section. The following transformation matrices from point P to point Q are then obtained, for the x- and y- directions respectively

$$M_x = \begin{pmatrix} -(a_x d_x + b_x c_x) & 2a_x c_x r^2 & 2a_x r(c_x r + d_x) \\ 2b_x d_x / r^2 & -(a_x d_x + b_x c_x) & -\frac{2b_x}{r}(c_x r + d_x) \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_y = \begin{pmatrix} a_y d_y + b_y c_y + \pi a_y c_y r & 2\left(\frac{\pi}{2} a_y r + b_y\right)\left(\frac{\pi}{2} c_y r + d_y\right) \\ 2 a_y c_y & a_y d_y + b_y c_y + \pi a_y c_y r \end{pmatrix}$$

Where P is the point of entry into the sector magnet preceding a short straight section and Q is the point of exit from the sector magnet following a short straight section. Setting in M_x

$$c_x r + d_x = 0$$

we have a dispersion free trajectory in the linac straight section. To minimize the beam size in the linac we require that the beam envelope have waists at the midpoints of the straight sections. Let β_0 be the β -function at the midpoint of the linac. At a distance ℓ from this midpoint we have $\beta(\ell) = \beta_0 + \ell^2/\beta_0$ differentiation of β with respect to β_0 shows that $\beta(\ell)$ has its minimum value $\beta(\ell)_{\min} = 2\ell$ for $\beta_0 = \ell$. The value of ℓ is: $\ell = \frac{1}{2}L_\ell$ where L_ℓ is the length of the linac. The mirror symmetry and the requirement that at the midpoint of the short straight section we have a waist give Twiss parameters at the points P and Q that satisfy the relations

$$\alpha_Q = -\alpha_P, \beta_Q = \beta_P \text{ and } \gamma_Q = \gamma_P$$

4. Extraction

A movable septum magnet SM_1 in the short straight section gives the beam an outward deflection

$$X'_1 = \frac{eB\ell_m c}{W}$$

where B is the magnet field strength and ℓ_m its effective length. The resulting trajectory is shown in Fig. 6. After passing through the 90° bending magnet the displacement from the normal trajectory is given by

$$X_2 = \left(r - \ell - \frac{\ell\ell_1}{r} \right) X'_1$$

This displacement is outward when

$$r > \frac{\ell}{2} \left(1 + \sqrt{1 + \frac{4\ell_1}{\ell}} \right)$$

Fig. 6. Schematic of beam extraction system using septum magnets (SM_1 and SM_2).

and inward when

$$r < \frac{\ell}{2} \left(1 + \sqrt{1 + \frac{4\ell_1}{\ell}} \right)$$

Choosing $\ell = \ell_1 = 0.5$ m we find $X_2 = 0$ when $r = 0.809$ m or $W = 370$ MeV.

Table II gives the value of $B\ell_m$ and X_1' as a function of W for $X_2 = 0.05$ m.

TABLE II. Beam extraction condition

W(Gev):	0.5	0.75	1.00	1.25	1.50	1.75	2.00
$B\ell_m$ (T-m)	0.2276	0.1263	0.1058	0.0971	0.0923	0.0893	0.0872
X_1' rad	0.0623	0.0154	0.0072	0.0063	0.0028	0.0020	0.0015

Another septum magnet of fixed location SM_2 deflects the beam to a bending magnet for further extraction.

5. Beam blowup

At this time no coherent transverse instability has been observed in room temperature microtrons. However, in high energy c.w. microtrons we should anticipate beam blowup associated with the excitation of HEM deflecting modes in the linac cavities. The deflecting modes could be excited by all bunches but the effect of the HEM field will be large only for the first turn bunches.

To elevate the blowup threshold a feedback damping system will be installed in the short straight sections. The electrodes for observing the coherent displacement are placed downstream of the first quadrupole. At this location we have large displacement and small deflection. The electrodes that can be driven to damp the coherent transverse motion will be placed in a region where the deflection is large and the displacement small, such as the midpoint of the short straight sections. Damping systems will be installed along the orbits of the first three turns. Ideally the deflection in the second and subsequent turns can be made negligibly small. If this is the case, the threshold current is then determined by the maximum displacement in the first linac. Assuming that three e-folds can be tolerated a blowup threshold current of approximately 300 μ A can be obtained (see Appendix A).

6. Accelerator parameters

Focusing and phase-space matching is done on individual orbits by quadrupole magnets located in the straight sections. The complete magnet configuration is shown in Fig. 5. The quadrupoles in the short straight section increase in strength in proportion to the particle momentum. In the complete system there are ~ 500 quadrupoles. The maximum total power required is < 50 kW. The properties of the magnet elements is given in Table III. For a beam emittance of less than 1π mm-mr one finds a maximum beam size less than 15 mm. The gap of the sector magnets is 35 mm, thus allowing 10 mm for vacuum chamber and orbit distortion, and an additional 10 mm for pole-face windings.

Table III. Properties of Magnet Elements

Type	Dimensions	Operating Characteristics
quad. magnet	aperture - 3 cm length ~15 cm	$B' l_q \lesssim 3\text{T/m-m}$
sector magnet	gap = 3.5 cm radius = 4.38 m wt = 1.5×10^5 kg(Fe) 1.2×10^3 kg(cu)	$B_{\text{max}} = 1.53\text{T}$ power = 225 kW

From Eqs. 5 and 10 it is evident that the operating tolerances increase with the wavelength S-band. The choices commonly considered are S-band, $\lambda = 12.5$ cm and L-band, $\lambda = 22.5$ cm. L-band has the advantage of more favorable operating tolerances, but the disadvantages that high power L-band klystrons are not commercially available and r.f. losses are somewhat greater for L-band compared to S-band. High-power klystrons are readily available at 2400 MHz, and r.f. beam splitting of extracted beams will have less effect on duty factor at the higher frequency. Consequently in Table IV we have tabulated the beam conditions for the conceptual design based on S-band operation. The energy gain per turn is 50 MeV and the synchronous phase, $\phi_s = 0.16$ radians. Ideally the phase between the center of the bunch and the peak of the r.f. field is equal to the chosen ϕ_s . However, fluctuations of the magnetic field and/or the linac peak voltage will change the instantaneous value of ϕ_s . From Eqs. (2b) and (3) we find

$$\Delta\phi_{s1} = (\pi - 2) \frac{r}{\lambda} \frac{\Delta B}{B} 2\pi \text{ radian}$$

Eq. (7) gives

$$\Delta\phi_{s2} = \frac{\Delta V_o}{V_o} \cot \phi_s \text{ radian}$$

$\Delta\phi_{s1}$ and $\Delta\phi_{s2}$ are uncorrelated and we have

$$\Delta\phi_s = \sqrt{\left[(\pi - 2) \frac{r}{\lambda} \frac{\Delta B}{B} 2\pi \right]^2 + \left[\frac{\Delta V_o}{V_o} \cot \phi_s \right]^2} \text{ radian}$$

To estimate the effect of $\Delta\phi_s$ on the final energy of the electrons we assume that, because of the stable phase motion, the energy deviation is mainly determined in the last turn. Assuming that the reproductability of the magnetic field and the linac voltage are given by $\frac{\Delta B}{B} \leq 10^{-3}$, and $\frac{\Delta V_o}{V_o} \leq 2 \times 10^{-3}$ we find for $r = 4.38$ m and $\phi_s = 0.16$ radians $\Delta\phi_s = 1.94^\circ$ and $\Delta E = (\Delta W \sin \phi_s \Delta\phi_s) = 0.265$ MeV. Similarly for magnetic field and linac voltage stability of $\frac{\Delta B}{B} = 2 \times 10^{-5}$, and $\frac{\Delta V_o}{V_o} = 10^{-3}$ we find $\Delta\phi_s = 0.364^\circ$, $\Delta E = 50$ keV.

Table IV. Microtron Operating Conditions

Wavelength λ	12.5 cm
Mode number ν	1
First turn harmonic number	530
Magnetic field	1.523 T
Stability limit $(\phi_s)_{\max}$	0.905 radian
E reproducibility	265 keV
E stability	50 keV

The linac in a 2 GeV single stage microtron will be long enough that the cumulative multisection type beam break-up may be the limiting factor for the total current in the linac. The choice of the number of recirculations is then a compromise between r.f. losses and beam break-up problems. A low energy linac increases the number of recirculations and consequently enhances the beam break-up problem. The starting current for beam break-up is not known for a steady state operation. In this preliminary conceptual design, the number of recirculations is tentatively set at 40. In Table V the basic parameters of the conceptual design are tabulated.

The properties of the microtron beam are expected to be excellent. The energy spread can be calculated from (16) if one assumes that the injection phase is stable relative to the synchronous phase ϕ_s . It is reasonable to expect this phase uncertainty to be no more than about 1° , $\delta\phi_1 \lesssim 1^\circ$ in which case the calculated energy spread at 2 GeV will be less than 200 keV. The beam emittance can be estimated by using state-of-the-art values for the emittance from a 5 MeV linac injector, 21π mm mr. Thus emittances, $\epsilon < .1 \pi$ mm mr should be achievable at 500 MeV.

Table V. Parameters for Microtron Conceptual Design

Basic parameters	
Maximum energy (GeV)	2
Current (μA)	300
Energy spread, $\Delta E/E$	$\approx 10^{-4}$
Beam emittance	$\leq 0.2\pi \text{ mm mr}$
Injection energy (MeV)	5
Max energy gain per turn (MeV)	50
Synchronous phase ϕ_s (radian)	0.16
Accelerating field $E(\text{MV/m})$	1.4
Number of recirculations	40
Length long straight section (m)	21.72
Maximum length, short straight section (m)	≈ 11.17
Wavelength λ (cm)	12.5
Mode number	1
Maximum B (Tesla)	1.523
Linac length (m)	18.75 m
Number of linacs	2
$P_{\text{r.f.}}$ (r.f. losses)	0.94 MW
ZT^2 (shunt impedance)	75 $\text{M}\Omega/\text{m}$
R_{max} (m)	4.38
R_{min} (m)	0.0657
ΔR (m)	0.1095

Appendix A

Transverse Beam Blow-Up

The electromagnetic fields in a wave guide can be written in the form (see Slater, Microwave Electronics, p. 7)

$$\begin{aligned}\vec{E} &= \left(\vec{E}_t + \vec{a}_z E_z \right) e^{i(\omega t - \beta z)} , \\ \vec{H} &= \left(\vec{H}_t + \vec{a}_z H_z \right) e^{i(\omega t - \beta z)} ,\end{aligned}\tag{A.1}$$

where $\vec{a}_z$ is the unit vector in the z-direction $\beta = \frac{\omega}{v_p}$ and v_p is the phase velocity. Substitution of Eq. (A.1) in Maxwell's equations:

$$\text{curl } \vec{E} = - \frac{\partial \vec{B}}{\partial t} = - i\omega\mu\vec{H}\tag{A.2}$$

$$\text{curl } \vec{H} = \frac{\partial \vec{D}}{\partial t} = i\omega\epsilon\vec{E}$$

$$\text{give } i\omega\vec{B}_t = i\omega\mu\vec{H}_t = \vec{a}_z \times (\text{grad}_t E_z + i\beta\vec{E}_t)\tag{A.3}$$

With the aid of Eq.(A.3) the transverse deflecting force $\vec{F}_t = e[\vec{E}_t + v\vec{a}_z \times \vec{B}_t]$ becomes:

$$\vec{F}_t = e \left[\left(1 - \frac{v}{v_p} \right) \vec{E}_t + i \frac{v}{\omega} \text{grad}_t E_z \right]\tag{A.4a}$$

The deflecting field is effective only when its phase velocity is approximately the same as the bunch velocity. Thus $v \approx v_p$ and Eq.(A.2) reduces to

$$F_t = \frac{iev}{\omega} \text{grad}_t E_z\tag{A.4b}$$

In the central region of the linac structure the deflecting field can be written in the form

$$E_z = \sum_{m=-\infty}^{\infty} E_m J_1(k_m r) e^{-j\beta_m z} \cos\phi\tag{A.5a}$$

$$k_m^2 + \beta_m^2 = \left(\frac{\omega}{c} \right)^2 = k^2$$

In general the higher harmonics can be neglected and we have, setting $k_1 = k$

$$E_z \approx E_1 J_1(kr) \cos \phi \approx E_1 \frac{1}{2} kx \quad (\text{A.5b})$$

Substituting (A.5b) in Eq. (A.4b) we obtain

$$m \frac{d}{dt} \gamma \frac{dx}{dt} = \frac{iev}{\omega} E_1 \frac{1}{2} k$$

or since $\frac{dz}{dt} = v$ is approximately equal to the speed of light

$$\frac{d}{dz} \gamma \frac{dx}{dz} = \frac{ie}{\omega mc} E_1 \frac{1}{2} k \quad (\text{A.6})$$

We now make the following assumptions:

- 1) The frequency of the deflecting field and the electron rotation frequency are not commensurable
- 2) Each section of the linac is replaced by a single short cavity oscillating in the TM_{11} mode
- 3) The phase between the recirculated bunches and the relevant deflecting field can have any value between $-\pi$ and $+\pi$.

The net effect of the recirculated bunches on the first turn bunches can be neglected because of the feedback system and the transverse focusing. For the same deflecting field the first turn bunches will have the largest displacement from the axis. In what follows only these bunches will be considered. The energy exchange of a field given by Eq. (A.5b) with a short bunch moving at a distance x from the cavity axis is given by

$$\Delta U = \int_0^L N_b e E_z dz \approx L N_b e E_1 \frac{1}{2} kx ,$$

where L is the length of the cavity and N_b is the number of electrons in the bunch. Noting that the average current $I = -f N_b e$ (f = number of bunches per second) we find the rate of energy exchange

$$\frac{\partial U_b}{\partial t} = f \Delta U = -IL E_1 \frac{1}{2} kx$$

The rate of the energy loss in the cavity walls is given by

$$\frac{\partial U_{\omega}}{\partial t} = - \frac{\omega}{Q} U$$

Equilibrium will be reached when the rate of the wall losses is equal to the rate of the energy exchange with the beams. Thus

$$\frac{\omega}{Q} U = 1LE_1 \frac{1}{2} kx \quad (A.7)$$

to estimate U we assume that the field is given by Eq. (A5b):

$$E_z = E_1 J_1(kr) \cos \phi$$

over the whole volume of the cavity (assumption 2). Substitution in

$$U = \frac{1}{2} \epsilon \int_0^L \int_0^{2\pi} \int_0^a E_z^2 dr r d\phi dz \text{ gives}$$

$$U = (0.4028)^2 \frac{\pi}{4} \epsilon_o E_1^2 a^2 L$$

Substituting this in Eq. (A.7) we obtain, after some manipulation

$$E_1 = \frac{\frac{1}{2} kx I Q}{(0.4028)^2 \frac{\pi}{4} \epsilon_o a^2 \omega} \quad (A.8)$$

Eqs. (A.6) and (A.8) yield

$$\frac{d}{dz} \gamma \frac{dx}{dz} = \frac{ie}{\omega mc} \frac{k^2 Q I x}{(0.4028)^2 \epsilon_o a^2} \quad (A.9a)$$

Using the relations $ka = 3.8319$ and $\frac{1}{\epsilon_o} = c Z_o$ ($Z_o = 120\pi \text{ ohm}$ and $\omega = 2\pi c/\lambda$) we obtain

$$\frac{d}{dz} \gamma \frac{dx}{dz} = \frac{i 90.5 \lambda^2 Q Z_o I}{U_e 4\pi^3 a^4} x \quad (A.9b)$$

where $U_e = 5.11 \times 10^5$ Volt

Setting $x = \frac{1}{\sqrt{\Gamma}} \xi$ Eq. (A.9b) becomes

$$\frac{d^2 \xi}{dz^2} + \left[\left(\frac{1}{2\gamma} \frac{d\gamma}{dz} \right)^2 - \frac{1}{2} \frac{1}{\gamma} \frac{d^2 \gamma}{dz^2} - \frac{i}{U_e} \frac{90.5 \lambda^2 Z_o QI}{4\pi^3 a^4 \gamma} \right] \xi = 0 \quad (\text{A.10a})$$

We assume a constant energy gain per unit length thus

$$\frac{d\gamma}{dz} = \frac{eE}{mc^2}, \quad \frac{d^2 \gamma}{dz^2} = 0 \text{ and Eq. (A.10a) becomes}$$

$$\frac{d^2 \xi}{dz^2} + \Gamma^2 \xi = 0 \quad (\text{A.10b})$$

$$\text{where } \Gamma^2 = \left(\frac{eE}{2\gamma mc^2} \right)^2 - \frac{i 90.5 \lambda^2 Z_o QI}{4\pi^3 a^4 U_e \gamma} \quad (\text{A.11})$$

Introducing the new variables

$$n = \sqrt{\Gamma} \xi \text{ and } \phi = \int_0^Z \Gamma dz$$

we obtain from Eq. (A.10b)

$$\frac{d^2 \eta}{d\phi^2} + \left[1 + \frac{3}{4} \left(\frac{1}{\Gamma^2} \frac{d\Gamma}{dz} \right)^2 - \frac{1}{2} \frac{1}{\Gamma^3} \frac{d^2 \Gamma}{dz^2} \right] \eta = 0 \quad (\text{A.12})$$

$$\text{For } \frac{3}{4} \left(\frac{1}{\Gamma^2} \frac{d\Gamma}{dz} \right)^2 - \frac{1}{2} \frac{1}{\Gamma^3} \frac{d^2 \Gamma}{dz^2} \ll 1$$

The solution of Eq. (A.12) will have the form $\eta = e^F$. From the definitions of η and ξ it follows that the solution of the equation of motion for the electrons, Eq. (A.9b) can be written in the form

$$x = \frac{\text{constant}}{\sqrt{\gamma \Gamma}} e^F,$$

$$\text{where } F = \int_0^{z_1} i \Gamma dz = \int_{\gamma_i}^{\gamma_1} \frac{imc^2}{eE} \Gamma d\gamma = \left[2i\gamma \frac{mc^2}{eE} \Gamma - \tan^{-1} \left(\frac{2i\gamma mc^2}{eE} \Gamma \right) \right]_{\gamma = \gamma_i}^{\gamma = \gamma_1}$$

Substituting (A.11) in $2i\gamma \frac{mc^2}{eE} \Gamma = p + iq$ we find

$$p(\gamma) = \frac{1}{2} \sqrt{2} \left(\sqrt{1 + \alpha^2 \gamma^2} - 1 \right)^{1/2} \text{ and } q(\gamma) = \frac{1}{2} \sqrt{2} \left(\sqrt{1 + \alpha^2 \gamma^2} + 1 \right)^{1/2}, \quad (\text{A.13})$$

$$\text{when } \alpha = \frac{90.5 \lambda^2 Z_o Q I U_e}{\pi^3 a^4 E^2}$$

The real value of F can thus be written in the form

$$\text{Re}F = p(\gamma_1) - p(\gamma_i) + \frac{1}{2} \left[\cotan^{-1} p(\gamma_1) - \cotan^{-1} p(\gamma_i) \right] \quad (\text{A.14})$$

For $E = 1.40 \text{ MV/m}$, $\gamma_i = 10.78$, $\gamma_1 = 59.71$, $\lambda = 0.0833 \text{ m}$, $Q = 10^4$,
 $a = -.05 \text{ m}$, $Z_o = 120\pi \text{ ohm}$ and $U_e = 5.11 \times 10^5 \text{ Volt}$. We obtain for
 $I = 300 \mu\text{A}$, $\text{Re}F = 3$.

VI. LINAC - STRETCHER RING - A CONCEPTUAL DESIGN

A. Introduction

The use of a pulse stretcher storage ring to convert a pulsed beam to continuous beam is an attractive option because it draws heavily on the existing technology developed in high energy physics for colliding beam experiments using storage rings and because it utilizes the short pulse linac injectors of well-established design. The linac-stretcher-ring design presented here consists of a 2 GeV SLAC type linac which injects into a storage ring consisting of a lattice computer-controlled-separated-function element. The possibilities of using a standing-wave type linac in place of the SLAC traveling-wave structure are discussed in the Appendix following this Section. The experience in their operation in the transient mode is limited, and the cost advantages which they offer appear to be marginal. In view of these uncertainties we feel a conceptual design based on their use would be premature.

The design developed here differs from those discussed recently¹⁹ in several important respects. The storage ring includes an r.f. cavity system whose purpose is to control the beam orbit and the rate of extraction from the ring. A constant rate of extraction can be maintained for all energies. Such controlled extraction would be difficult in a storage ring system which relies on synchrotron radiation to translate the beam orbit. With an r.f. system in the ring, beam injection must be employed which has $\approx 100\%$ capture efficiency. Clearly, injection of a linac beam of the power anticipated precludes substantial injection losses. In order to achieve $\approx 100\%$ capture, synchronous transfer of beam between the linac and storage ring would be used. Consideration of the synchrotron frequency in the storage ring preclude the use of the same frequencies in the linac and ring r.f. systems. However, synchronous injection into the ring r.f. is possible when the frequency of the storage ring is a sub-harmonic of the linac r.f., and the micropulses of the linac are chopped so that the spacing of r.f. bunches is the same as the ring system. To achieve this a chopped system used elsewhere²⁰ or a

sub-harmonic buncher at the linac injector would be used. The storage ring r.f. system can then be similar to that of existing $e^+ - e^-$ accelerators. Electron beams of 900 mA have been stored in the DORIS storage rings²¹ at DESY. In the DORIS system there are separate e^+ and e^- rings. With that performance established, our design objective, storage of 500 ma of electrons by synchronous injection appears sound. Time dependent orbit distortion will be adjusted to avoid the stored beam hitting the injection septum magnet.

Beam extraction would be accomplished by inducing a third integer resonance in the horizontal phase space. Extraction sextupole magnets would be placed at appropriate places to induce the third integer resonance separatrices at the extraction septum magnets. In order to facilitate smooth efficient extraction during the entire extraction period a "hollow" phase space technique would be used. The technique has been employed extensively in the ZGS extraction system²² at ANL and can be done either by injecting into off-equilibrium orbits or by using a tickler system. The third resonance extraction also offers the possibility of extracting up to three beams simultaneously. The use of horizontal extraction would enhance the quality of the extracted beam, particularly the energy spread.

B. Linac Design

The characteristics of the linac injector will be determined by the radius of the stretcher ring, the magnitude of the circulating current, and the peak current in the linac, itself. Operation at the highest acceptable peak current is desirable because the cost of linac r.f. components is directly proportional to the r.f. duty factor. High peak currents permit operation at smaller duty factors. Limits on peak currents of ~100 mamps have been established on the basis of operational experience with SLAC structures of much longer lengths than contemplated in this study. Earlier conceptual designs of 2 GeV systems based on such limits have resulted in design values for Klystron duty factors which are typically twice that of SLAC design and operational experience.

Upgrading the existing SLAC klystrons to the performance required would be a formidable task necessitating the redesign of the klystron collector and cooling as well as development of r.f. windows of new materials. Even if the redesign is partially successful the operational problems and failure rate will probably make use of the upgraded version prohibitively expensive. All of these problems can be avoided if the peak current in the linac injector can be raised to 400 mamp and injection into the stretcher ring accomplished in a single turn. The resulting klystron duty factor would be within present operating range.

Because beam breakup thresholds are a strong function of linac length and focusing lattice, it is important to study breakup thresholds in systems appropriate to the Pulse-Stretcher Injector Linac. R. Helm (private communication) has carried out a series of calculations of current thresholds for beam breakup for a SLAC type linac with the specifications given in Table VI. The linac in-line system used, consists of 3 m sections grouped into four sectors, each containing 14 sections, with a 3 meter drift for extraction at the end of each sector. Nine quadrupoles are distributed within each sector. The strengths of the quadrupoles are scaled with energy to maintain a phase advance of about 90 degrees per cell. The four quadrupoles in matching cells at the end of each sector are adjusted to match the beam envelope from one sector to the next. The cumulative beam breakup arises from a few resonant modes in the HEM_{11} passband, which occur in the first few cells of each 3 m section. Values for the frequencies and transverse shunt impedances were chosen on the basis of experience with SLAC systems. The beam breakup was assumed to be driven by an initial betatron oscillation of about 1 mm, and the threshold was defined arbitrarily as the current at which the beam displacement grows to 1 cm.

Calculations were made for 3 cases of linac structure design. Case A consisted of a linac of identical sections; Case B consisted of a structure in which the frequency of the blowup mode in the first 11 sections was 2 MHz below that of the remaining sections; In Case C the frequency of sections 8 through 21 were 2 MHz above the initial sections and the remainder 4 MHz above the initial sections. These frequency shifts for

Table VI. Linac parameters used in beam breakup calculations

Structure	SLAC $2\pi/3$
Section length	3m
Filling time	0.44 μsec
Gradient	12 MeV/m
Number of sections	56
Extraction points	0.5, 1.0, 1.5, 2.0 GeV
Beam pulse length	2.5 μsec
Bunching frequency	476 MHz

transverse blowup modes can be realized in realistic designs with negligible effect on the accelerating properties of the structure. The computed breakup thresholds for these three cases were as follows:

Case A	320 mA
Case B	680 mA
Case C	1060 mA

Breakup occurs 2.5 microsec after the current is switched on. While these calculations were not exhaustive they did establish the feasibility of design objectives for peak linac currents in the range of 500 mA.

Using the operating experience in the DORIS storage ring as a guide we have chosen a maximum circulating current of 400 mA in order to minimize problems of beam instabilities in the ring. Injection in the ring can be accomplished in a single turn with a peak linac current of 400 mA. The design value of the external beam will fix the number of turns in each extraction cycle; i.e. $n_{\text{ext}} = i_{\text{cir}}/i_{\text{ext}}$, and accordingly the repetition rate will be

$$n_{\text{pps}} = \frac{1}{n_{\text{ext}} t}$$

where t is the revolution period.

C. Design Optimization

Using these parameters one can optimize the design in terms of the stretcher ring radius using overall construction cost as the criterion. We have used SLAC wave guide structures modified to decrease the effect of beam loading by increasing the group velocity and reducing the attenuation. The operating properties of a typical section are given in Table VII. The energy gain per section, E_1 , is given by ²³

$$E_1 = (1 - e^{-2\tau})^{1/2} (P \ell r)^{1/2} - i_{inj} \frac{r \ell}{2} \left(1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}} \right)$$

Table VII. Wave-guide parameters

Wave-guide type	constant gradient disc loaded
Section length, ℓ	3m
Operating mode	$2\pi/3$
Attenuation/section, τ	0.3 nep.
Shunt impedance, r	53 M Ω /m
Q	13,000
Filling time, t_f	0.44 μ sec
R.f. power input/section, P	40 MW

where the first term is the no-load energy and the second term is the steady state beam loading. Assuming two inactive klystrons and linac sections, for full 2 GeV operation

$$N = \frac{E + 2 i_{inj} \frac{r_l}{2} \left(1 - \frac{2\tau e^{-2\tau}}{1 - 2^{-2\tau}} \right)}{E_1}$$

where E is the beam energy giving a total number of active sections, $N = 46$. The overall linac length is

$$L = 1.2 (N + 2) 3m \approx 175m.$$

The r.f. duty factor for single turn injection will be

$$D_{r.f.} = \left(t_f + \frac{2\pi pR}{c} \right) n_{pps}$$

These values for L and $D_{r.f.}$ were used together with component cost estimates in ref. 15 to obtain the overall accelerator cost versus radius given in Fig. 7. A total external beam current corresponding to 3 external 100 μ A beams is assumed. Although the minimum cost occurs at a radius of about 8m we have chosen a somewhat larger radius, $R = 15$ meters, in order to permit operation at higher energies should such a need develop. The linac parameters are fixed at the values given in Table VIII. Assuming an emittance of $2 \pi \text{ mm mr}$ for the injector, at all energies above 500 MeV the linac beam should have an emittance of less than the design objective of $.2 \pi \text{ mm mr}$. The energy spread will be determined primarily by the transient beam loading. By proper phasing of several linac sections it should be possible to minimize this effect. The design objective will be an energy spread of 0.5% in the beam transported to the stretcher ring. Our design provides for a 10% loss in energy analysis of this beam.

To avoid beam loading from the unexcited wave guides during low energy operation a transport system will take the beam at the appropriate energy, 500 MeV, 1000 MeV, or 1500 MeV and bypass the remaining linac sections. The beam transport is shown schematically in Fig. 8.

Fig. 7. Estimated cost of Linac-Stretcher Ring as a Function of Ring Radius

Table VIII. Linac parameters *

Maximum energy	2 GeV
Beam loading	415 MeV
Linac peak current	440 mA
Linac total length	175 m
Number of sections	46
Linac frequency	2856 MHz
Linac repetition rate	894 pps
Linac duty factor	1.1×10^{-3}
Linac AC power	4.0 MW
Pulse length	1.27 μ sec

* For operation with 3 external beams of 100 μ A each.

Fig. 8. Beam Transport System for Injector in Linac Ring Accelerator

D. Stretcher Ring Design

The design of the stretcher ring lattice structure has to fulfill the following requirements:

1. The straight sections must be long enough to accommodate injection, extraction (2 or 3) and r.f. systems.
2. At the locations of the injection and extraction septum magnets and r.f. straight sections the dispersion-function must be zero.
3. At the location of the resonance extraction magnet the dispersion function must not be too small.
4. For 2 or 3 simultaneous extractions the betatron phase advance between the extraction straight sections (locations of extraction septum magnets) must be a multiple of 120° .

The above requirements may be satisfied by a FODO lattice with a betatron phase advance per cell of approximately 60° . Fig. 9 shows the lattice layout of one of the 4 straight sections. The dispersion function is made zero by leaving out bending magnets.

The bending radius of the magnets is chosen to be 15 m. This corresponds to a field of 0.445 T at 2 GeV and 0.0446 T at 200 MeV. This field is high enough that the effects of coercive force and permeability of the magnet iron will be negligible. There are 64 magnets each 1.473 m long. The maximum strength of the 88 quadrupole magnets is 2.34 T/m-m. Between the bending magnets and the quadrupole there is enough space to accommodate sextupole magnets for chromaticity correction. To inject the linac beam in the stretcher ring the equilibrium orbit is distorted by four fast bumper magnets (see Fig. 10). Two septum magnets put the linac beam in the ring in such a way that the radial phase space is hollow. At the end of the injection period the 4 bumper magnets are turned off simultaneously.

The extraction makes use of the third order resonance $\nu_x = 22/3$. The resonance sextupole magnet will be located in a region with non-zero

Fig. 10. Linac-Stretcher Ring Injection System

dispersion function. The magnetic field of this magnet seen by the particle is then a function of the betatron amplitude and the particle energy (location of its equilibrium orbit). A particle with its equilibrium orbit going through the center of the resonance magnet will see only a sextupole field. Particles with higher energy (larger equilibrium orbit) will also see a positive quadrupole field and its ν_x value moves closer to the resonance value $22/3$. (It is assumed that the unperturbed ν_x values in 7.25 and outward extraction). Particles with lower energies are farther away from the stopband. By slowly decreasing the R.F. frequency (i.e. increasing the electron energy) the particles will be squeezed out of the stable region along the three arms of the separatrix (see Fig. 11). Particles with lower energy need larger betatron amplitude to be extracted. The electrons jump from one arm to the next each turn with increasing displacement. When the increase per turn of this displacement is large enough the electrons will enter an extraction septum. By having 2 or 3 septa separated by multiple of 120° in betatron phase one can extract 2 or 3 beams simultaneously. It turns out that in practice the extraction efficiency decreases with the area inside the separatrix. By having a hollow radial betatron phase space area (see Fig. 11, shaded area) the extraction efficiency can be kept high. The energy spread of the extracted beam is also improved and is approximately given by

$$\frac{\Delta p}{p} \approx \sqrt{\frac{\beta_{xR} \epsilon_x}{\eta_{xR}}}$$

where β_{xR} = value of β_x at the location of the resonance magnet, ϵ = linac beam emittance, and η_{xR} = dispersion function value at the location of the resonance magnet. Using values typical of the lattice of the ring design of Fig. 9, i.e. $\beta_{xR} = 8$ m and $\eta_{xR} = 1$ m, and taking $\epsilon_x = 2.5 \times 10^{-7}$ m-rad we find

$$\frac{\Delta p}{p} = 1.4 \times 10^{-3}$$

The main parameters of the stretcher ring are listed in Table IX.

Fig. 11. Extraction Separatrix in Betatron Phase Space

TABLE IX.

Main parameters of stretcher ring	
Maximum energy	2 GeV
Current	400 mA
Machine circumference ($2\pi R$)	251.3 m
Bending radius	15 m
Maximum bending field	0.445 T
Length of bending magnet	1.473 m
Bending magnet gap	0.031 m
Powerloss/bending magnet	5 kW
Number of bending magnets	64
Maximum quadrupole gradient	4.68 T/m
Length of quadrupole magnet	0.5 m
Diameter of quadrupole magnet	0.3 m
Powerloss/quadrupole magnet	1kW
Number of quadrupole magnets	88
Radial betatron frequency (ν_x)	7.25
Vertical betatron frequency (ν_y)	6.25
Number of cells (total)	44
Number of normal cells	32
Super period	4
Synchrotron Rad. loss/turn	94 kV
R. F. voltage per turn	800 kV
Harmonic number	399
R. F. frequency	476 MHz
Rotation frequency	1.193 MHz
Synchrotron frequency/turn (ν_s)	0.02
β_x max	9.93 m
β_x min	3.35 m
β_y max	10.50 m
β_y min	4.17 m
Normal cell $\eta_{\max}$	1.25 m
Normal cell $\eta_{\min}$	0.75 m

Appendix B
Use of Standing Wave Structures
in a 2 GeV Short-pulse Linac-Injector

The development of biperiodic r.f. structures with high shunt impedances operating in the $\pi/2$ mode offers an attractive solution for the acceleration of electrons in short pulses. In the case of c.w. operation the biperiodic structure is the clear choice. Such systems are also interesting for pulsed acceleration of electrons because their high Q makes more energy available for heavily beam-loaded applications and their high shunt impedance means a more efficient accelerating structure. However, when acceleration of short pulses (≈ 2 μ sec) is the objective—as in our case—the transient response of the standing-wave guide must be established in order to determine the energy spread of the accelerated beam. For a complete understanding of the transient response, a detailed study of the behavior of the excitation and subsequent decay of all the important cavity modes is necessary. Although it remains to be demonstrated, it is expected that the shock excitation of all but the r.f. accelerating mode will decay rapidly and not interact with a significant portion of the electron pulse. With this qualification we can develop a simple heuristic derivation of the proper injection time for beam acceleration with minimum energy spread. The derivation is based on the energy balance and as we would expect indicates that proper injection time is a function of the beam current and should take place at that instant in the excitation of the r.f. cavity when the required beam power per section is equal to the rate at which energy is being fed into the structure to reach the final no-load voltage V_0 .

Assume a periodic wave guide operating in the $\pi/2$ mode with an effective shunt impedance per unit length, ZT^2 given by

$$ZT^2 = (V/L)^2 / (dP/dZ) \quad (B.1a)$$

or equivalently

$$V^2 = ZT^2 PL \quad (B.1b)$$

where V is the peak energy gain of the accelerator, L the accelerator

length, and P the total power. For a square pulse of electrons injected with an average current in the pulse I_b and a well bunched structure at the frequency of the cavity accelerating mode the beam energy will be

$$V^2 = ZT^2(P - I_b V)L \quad (B.2)$$

or

$$V^2 + V(I_b ZT^2 L) - ZT^2 PL = 0 \quad (B.3)$$

The positive solution is

$$V = \sqrt{ZT^2 PL + \left(\frac{I_b ZT^2 L}{2}\right)^2} - \frac{I_b ZT^2 L}{2} \quad (B.4)$$

Written in terms of the incident power and the coupling coefficient this becomes

$$V = \sqrt{\frac{4\beta}{(1+\beta)^2} ZT^2 P_i L + \left(\frac{I_b ZT^2 L}{1+\beta}\right)^2} - \frac{I_b ZT^2 L}{(1+\beta)} \quad (B.5)$$

It is evident that the last term in Eq(B.5) is the steady state beam loading.

The time dependence of the field in a coupled cavity section is shown in Fig. B-1. For no beam loading the field is given by:

$$V(t) = V_0 [1 - \exp(-\omega t/2Q)] \quad (B.6)$$

For a matched system the power into the structure is given by

$$P_{in}(t) = P_0 [1 - \exp(-\omega t/Q)] \quad (B.7)$$

and the reflected power by

$$P_{ref}(t) = P_0 \exp(-\omega t/Q) \quad (B.8)$$

We assume that a square edge current pulse enters the cavity at time t_0 and that the cavity coupling is matched to the beam loaded wave guide.

Fig. B-1. Accelerating potential as a function of time in a simplified standing-wave linac section. The heavy lines indicate the time dependence expected for injection of a square edge current at time t_1 before and at time t_2 later than the correct time t_0 for which no transient is expected in the voltage form.

The field in the structure will be given by

$$V(t) = V_0[1 - \exp(-\omega t_0/2Q)] + \{V_1 - V_0[1 - \exp(-\omega t_0/2Q)]\} \left[1 - \exp\left(-\frac{(t - t_0)}{2Q_1}\right) \right] \quad (B.9)$$

where

$$Q = Q_0/1 + \beta \quad Q_1 = Q_0/1 + \beta'$$

and β and β' are the no-load and loaded couplings. V_1 is the asymptotic beam loaded voltage. From and inspection of Eq.(B.9) it is evident beam injection when the cavity voltage is equal to the asymptotic value, V_1 results in transient-free acceleration of the beam, (see Fig. B-1). The corresponding injection time is given by the condition

$$V_1 - V_0[1 - \exp(-\omega t_0/2Q)] = 0 \quad (B.10)$$

or equivalently

$$t_0 = -Q_0 \ln(1 - V_1/V_0) / \pi f(1 + \beta) \quad (B.11)$$

Noting from Eq. (B.5) that

$$V_0 = \sqrt{\frac{4\beta}{(1 + \beta)^2} ZT^2_{PL}} \quad (B.12)$$

we find

$$\frac{V_1}{V_0} = \sqrt{1 + \frac{I_b^2 ZT^2_L}{4\beta P}} - \sqrt{\frac{I_b^2 ZT^2_L}{4\beta P}} \quad (B.13)$$

We can use Eqs. (B.11) and (B.13) to determine the correction injection time in a cavity structure with parameters appropriate to 2 GeV design under consideration.

A conceptual design for a standing wave linac has been studied. Although operational parameters for such a system are not established, we assumed a peak power of 20 megawatts per 3 meter linac section whose effective shunt impedance was 100 megohms/meter. The beam current was assumed to be 440 ma and a β of 1.4 corresponding to 80% loading was assumed. Fifty-four sections are required to accelerate the beam to 2 GeV. The gradient is 13 MeV/m. Using the parameters $Q_0 = 20,000$, $f = 2800$ megahertz, we can solve Eqs. (B.11) and (B.13) to find the optimum injection time t_0 . The result

$$t_0 = 0.68 \text{ } \mu\text{sec}$$

for 440 mA. The repetition rate for the ring design described in this section is 898 pulses/sec and the required current pulse .939 μsec long. The resulting r.f. duty is

$$\text{Duty factor}_{\text{r.f.}} = 1.36 \times 10^{-3}$$

Assuming 50% r.f. efficiency (the same value used in evaluating the traveling wave design), this duty factor leads to a power requirement for the standing wave structure of

$$\text{Power}_{\text{r.f.}} = 2.94 \text{ MW}$$

From a comparison of these results with the linac parameters of Table VIII which describe the SLAC type linac uses in our conceptual design, it is evident that use of standing wave structure in the future may result in significant saving in operating cost. However there does not appear at this time to be a major saving in capital cost or operational efficiency. The klystron duty factor is somewhat above the current performance of available models. A larger number of sections are required in a standing wave system, and their fabricating costs remain to be established. Clearly, research and development of these systems for possible use in a 2 GeV design should continue. However, their use in this particular situation where the required beam pulse is so short offers no dramatic advantage. For the purpose of our evaluation of possible conceptual design use of a SLAC traveling-wave structure seems adequate.

VII. PRELIMINARY COST ESTIMATES

The cost estimates for the linac-stretcher ring and double-sided microtron accelerator facilities are given in Tables X and XI respectively. The estimates given in 1980 dollars include a 20% contingency factor. The stretcher ring system is estimated to cost $\$28.6 \times 10^6$ as compared to $\$17.4 \times 10^6$ for the microtron, a substantial difference. To a large extent, the estimates are based on numbers obtained from recent proposals for research facilities of a similar character. Where necessary we have made allowances for subsequent inflation.

The recent experience of the staff of the Stanford Linear Accelerator Center was of particular value in determining costs of the stretcher-ring facility. Where we have made independent judgements generally our assessments agree with estimates made by the SLAC staff. A major part of the microtron cost will be magnet fabrication. Here, we have used as a guide inflation escalated estimates based on the cost of the cyclotron magnets originally proposed as part of the ANL Midwest Tandem Cyclotron (MTC) proposal.²⁴ The results are in reasonable accord with costs recently estimated for the cyclotron magnets in the Colorado University cyclotron proposal²⁵ submitted to DOE recently. Informal consultations with klystron manufacturers were the basis for estimates for R. F. components in the microtron design.

TABLE X. 2 GeV linac-stretcher ring, cost estimate

A. S-band pulsed linac		
1. Linac injector	\$ 440K	
2. Beam line components (guides, support, alignment vacuum & cooling)	3700K	
3. Klystrons and modulators	8800K	
4. Linac transport system	440K	
5. Linac tunnel and klystron gallery	1960K	
6. Instrumentation and control	1100K	
7. Substation 5 MW	330K	
8. Water cooling tower	220K	
		<hr/>
		\$16990K
B. Stretcher-Ring		
1. Ring tunnel	\$1375K	
2. Ring lattice magnets	2200K	
3. Ring alignment, support cooling, correction magnets	1650K	
4. Power supplies	330K	
5. Ring vacuum system	330K	
6. Computer, control, diagnostics	440K	
7. Extraction system	220K	
8. R.f. system	220K	
		<hr/>
		\$ 6765K
		<hr/>
	TOTAL	\$23755K

Estimated cost $\$23.8\text{M} \times 1.2 = \28.6M

TABLE XI. 2 GeV double-sided microtron, cost estimate

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A. S-band C.W. linac		
1. Injector with chopping	\$ 275K	
2. Wave guides	2640K	
3. Klystrons (3)	594K	
4. Power supplies	660K	
5. Cooling system	165K	
6. Miscellaneous r.f. comp.	275K	
7. Instrumentation and control	451K	
8. 5 Mw substation	330K	
	<hr/>	\$5390K
B. Microtron elements (excluding r.f.)		
1. Sector magnets (4)	1320K	
2. Support, alignment, and cooling	880K	
3. Quadrupoles and power supplies	380K	
4. Vacuum system and beam tube	1210K	
5. Magnet controls	1210K	
	<hr/>	\$5000K
C. Building (25m x 60m) @ (\$3300/sq.m)		<hr/> \$4070K
	TOTAL	\$14460K
Estimated cost \$14.5M x 1.2 = \$17.4M		
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VIII. COMPARISON OF CONCEPTUAL DESIGNS

A major attraction of the linac-stretcher ring system lies in its use of state-of-the-art technology developed in the design of storage ring systems for research in elementary particle physics. It is evident from the conceptual design data of Section VI that no research and development is needed to prepare a proposal for a system capable of furnishing a 100 μ A single external beam. Such a system can be built with designs taken from existing high-energy physics facilities. With a limited development effort directed at refining the extraction technique this design can be exploited to give stretcher ring currents suitable for multiple beam operation. The stretcher ring has the added appeal of great flexibility for extension to operation at higher beam energies. No change in ring design would be required and a simple linear increase in linac-accelerator structure is all that is required.

However, the linac-ring system does not meet our design objective for beam quality. In this respect the microtron design is a much better option. This may be a very important factor in the design selection because of the implications of beam quality for the design of the high-resolution spectrometers used in electron scattering measurements. At the present time we estimate that the capital cost of experimental facilities will be approximately \$20 million. Spectrometer design could be considerably simplified if a high quality electron beam ($\Delta E/E \approx 10^{-4}$) is available, with attendant savings in capital costs. In addition certain classes of coincidence nuclear structure measurements may not be possible with the beam quality characteristic of the linac-ring system.

Of course such performance for the microtron designs is based strictly on design expectations. It remains to explore these questions in careful computer simulations and ultimately to confirm the predictions in experimental measurements. To date, such performance has never been realized at high electron energy with a functioning microtron system. The same reservation must be made with regard to beam current blowup limit. The stretcher-ring limit has been established empirically in storage ring operation at DORIS. The limit for the microtron systems has not been

established but is believed to be in the $\sim 100-1000$ μamp range. Multiple beam operation appears to be assured with both designs. At Cornell extraction of two beams from the 12 GeV electron synchrotron has been accomplished with 97% efficiency²⁶ and the beam quality of all three designs is such that beam splitting with an r.f. subharmonic splitter should be possible. Operation with beams of more than one energy is not possible with the linac ring system. However beam extraction at several points in a microtron system is a question that should be explored.

The power consumption of the microtron systems is substantially less than that of the linac ring system, and major differences in the systems occur in their capital cost and the manner in which that cost scales with beam energy. The estimated cost of the double-sided microtron is only 60% of that of linac-ring system. Such a substantial prospective saving dictates that in the absence of a major technical limitation the double-sided microtron is the preferred design and that research and development efforts should focus on its utilization. Table XII also indicates the manner in which the alternatives scale with energy by giving the power of the leading term in their cost. The actual costs are shown as a function of electron energy in Fig. 12. There is substantial sentiment in the research community for reserving the possibility of running at higher energies. The microtron could be designed so as to permit operation as high as 3.0 GeV and still be cost competitive with the linac-ring system, if the segment magnets are appropriately designed.

TABLE XII. Comparison of accelerator designs

Characteristic	Linac-Ring	Double-Sided Microtron
Beam quality, $\Delta E/E$	$\sim 10^{-3}$	$\sim 10^{-4}$
Emittance	$\sim .2$ mm-mr	$\sim .1\pi$ mm-mr
Beam current blowup limit	~ 600 μamp ^b	$\sim 100\text{--}500$ μamp (computer study)
Multiple beam capability	yes	with external beam splitter
Multiple beam energies	no	to be studied
Capital cost	\$28.6M	\$17.4M
Power, AC (2 GeV operation)	5.0 MW	3.9 MW
Scaling law, capital cost ^a (leading term)	$\sim E$	$\sim E^3$
Flexibility of design for increased energy	excellent	underdesign of magnet and linac required

^aSee Fig. 12 for comparison of capital costs versus energy^bRef. 21.

Fig. 12. Cost comparison for possible accelerator designs. The curve for the hybrid microtron refers to the design discussed briefly in Section V.

IX. SUMMARY

From the discussion presented in this study we have reached the following conclusions:

- A GeV continuous beam accelerator consisting of a SLAC type Linac and a conventional stretcher-ring can be built using existing technology. Such a system can be upgraded to 300 μ amp operation.
- Multiple beam extraction at high efficiency from electron storage rings of similar design has been demonstrated. However the energy spread in the extracted beam is expected to be an order of magnitude larger than the design objective.
- The double-sided microtron is a promising alternate option which may meet all the design objectives established in this report.
- The savings in capital cost which could be realized by development of a microtron accelerator could be as large as \$11M.
- Operating costs for the microtron systems would be substantially less than those expected for a linac-stretcher ring system.

It is evident that two major issues exist which must be resolved before a sound decision can be made on the proper option to be pursued in building a national 2 GeV electron accelerator facility. The first is whether the threshold for beam breakup in a microtron system with reasonable energy gain per turn is high enough to permit acceleration of the equivalent of 300 μ amps of external beam at 2 GeV. The second is whether magnetic guide fields of the requisite stability and precision are attainable in a 1-2 GeV accelerator. These problems are under study in projects at several major laboratories. An 820 MeV electron accelerator consisting of three cascaded c.w. microtrons is under construction at the University of Mainz.²⁷ The first of these, a 17 MeV system is already operational. The design objective of this system is 100 μ amps of beam. We anticipate that the performance of the first two stages of this system should provide

much-needed experimental data on beam breakup and the character of blowup modes. A National Bureau of Standards—Los Alamos collaboration is engaged in an accelerator research project directed at establishing operational limits for high-current, continuous beam electron accelerators by constructing a 185 MeV race track microtron. A major goal of this group is to establish that 300 μ amps of beam can be accelerated without beam breakup. They will explore possible r.f. accelerating structures for use in the microtron, and will attempt to employ a disc and washer structure²⁸ which appears to be ideal for a recirculating machine.

A microtron research and development project in progress at Argonne is concentrating on an engineering study and design of the basic sector magnet which would be used to generate the magnetic guide field for a 2 GeV double-sided c.w. microtron. This project should resolve the question of the feasibility of sector magnets of the requisite field precision. A detailed design of an appropriate sector magnet is planned. This will be followed by construction of a prototype of the portion of the sector magnet corresponding to the fields traversed in the first six turns of beam operation in a 2 GeV accelerator, i.e. 50–300 MeV. Field measurements will be made and compared with assumptions used in the preliminary design studied. Pending confirmation of orbit containment, a complete prototype will be constructed.

By 1982 the necessary data should be available to make a decision on the optimum design for a 2 GeV accelerator. Magnet studies at ANL will have been completed. Studies of possible r.f. accelerating structures at LASL will have reached a definitive stage. Data on electron beam breakup should be available from the microtron systems under construction. We anticipate that at that time a firm decision will be reached on accelerator concept to be employed.

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