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MUON CAPTURE IN LEAD

by

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October 1968

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MUON CAPTURE IN LEAD

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MUON CAPTURE IN LEAD^{*}

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^{*} Work supported in part by the United States Atomic Energy Commission.

[†] Submitted to the Department of Physics, Carnegie-Mellon University, March 8, 1968, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

ABSTRACT

The disappearance rate, decay rate and decay electron energy spectrum were measured for negative muons stopping in a thin lead sheet sandwiched between two optical spark chambers. A carbon reference target was used to eliminate the necessity of knowing the precise value of the detection efficiency of the apparatus, and the muon concentration of the beam of particles from the cyclotron. The decay electron energy spectrum was also found for muons stopping in the carbon target. Scanning the muon and electron tracks in the spark chambers enabled the elimination of most of the background of muons stopping in material other than the target. The decay electron energy was measured in a large sodium iodide crystal.

The decay and disappearance rates for negative muons in lead were found to be $.46 \pm .03$ ($10^6/\text{sec}$) and $13.1 \pm .9$ ($10^6/\text{sec}$), respectively.

A theoretical spectrum previously calculated by Huff and corrected for experimental resolution was compared to the measured decay electron energy spectrum for negative muons in lead. A best value for $\rho = .72 \pm .05$ was obtained with a χ^2 of 23.8 compared to an expected χ^2 of 19 ± 6.2 . In a similar way the data for negative muons stopping in carbon were analyzed to give a best value of $\rho = .72 \pm .06$ with a χ^2 of 15.6 compared to an expected χ^2 of 19 ± 6.2 .

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I. INTRODUCTION

When negative mu mesons stop in matter, they enter an atomic orbit and cascade¹ to the 1s shell in less than 10^{-10} seconds. The mu meson can then either decay ($\mu^- \rightarrow e + \nu_\mu + \bar{\nu}_e$) or be captured by the nucleus ($\mu^- + p^+ \rightarrow n + \nu_\mu$). The disappearance rate is the sum of the decay rate and the capture rate. Positive muons stopping in matter behave differently. Since they are repelled by the positively charged nucleus, they do not go sufficiently close to it to be affected by its weak forces and be captured. Therefore, the positive muon disappearance rate is equal to the free muon decay rate of $4.55 \times 10^5 \text{ sec}^{-1}$ (mean life 2.2 μsec). The experimental results for the disappearance rate of negative muons stopping in matter approach this value as Z approaches zero. Therefore, the free particle decay rates for negative and positive muons are equal.

In lead, the capture rate is found to be about 30 times the free particle decay rate; whereas in carbon it is about 10% of the free decay rate. The disappearance rates for muons are about $13.4 \times 10^6 \text{ sec}^{-1}$ in lead² and $.493 \times 10^6 \text{ sec}^{-1}$ in carbon.³ These correspond to mean lives of 74.87 nsec and 2.0026 μsec in lead and carbon respectively. In contrast to the large differences in capture rates and total disappearance rates between these two elements, their decay rates are much less disparate and are approximately equal to the free particle decay rate.

The muon decay rate in the muon rest frame is proportional to the phase space available to the decay products and to the matrix element of the decay interaction between initial and final states.

$$\frac{dn}{dt} = \int (M_{if})^2 \frac{dn}{dE} d\vec{P}_e$$

where $\frac{dn}{dt}$ is the decay rate in the muon rest frame,

M_{if} is the matrix element,

$\frac{dn}{dE}$ is the available phase space,

and $\vec{P}_e$ is the decay electron's momentum in the muon rest frame.

In addition, to obtain the decay rate in the laboratory frame, a factor due to the Lorentz transformation between the muon reference system and the laboratory system is needed. Thus, there are three reasons causing the decay rate for bound muons in the laboratory reference frame to differ from the free muon decay rate:

1. Phase Space Effect. The phase space is proportional to the energy available for the decay products. Since the bound muon has less energy than the free muon due to the binding potential of the nucleus, the phase space available to the bound muon decay products is less. Since the decay rate is proportional to the available phase space, the decay rate for bound muons is reduced. This effect increases as Z increases, due to the increased binding.

2. Time Dilatation Effect. Since the free muon decays at rest, the laboratory and the rest frames are identical. Therefore, the decay rate for free muons is the same in the rest frame and the laboratory frame. The bound muon is moving about the nucleus when it decays, and therefore the muon rest frame differs from the laboratory frame. This will cause the laboratory decay time to increase by virtue of the time dilatation effect inherent in the Lorentz transformation between the two frames. Thus, the number of decays per unit time will decrease in going between these two frames and the laboratory decay rate will be less for bound muons than for free muons. This effect also increases as Z increases.

3. The Electron Coulomb Effect. When bound muons decay, the electrons are attracted by the nucleus. This attraction causes the electron wave function to have an amplitude near the nucleus greater than if there were no attraction, as in the free muon decay. Since the bound muon wave function is peaked near the nucleus, this causes a greater overlap of muon and electron wave functions, and therefore, a greater matrix element. Since the decay rate is proportional to the matrix element, it will be increased for bound muons. This effect increases as Z increases due to greater attraction of the electron wave function.

The following is an illustration of these effects in lead as calculated by Huff.⁴ The bound muon decay rate divided by the free decay rate is called R . The phase space effect gives $R_1 = .37$, the time dilation effect gives $R_2 = .92$, and the electron coulomb effect gives $R_3 = 2.47$. The product of all these effects⁴ is $R = R_1 R_2 R_3 = 0.84$. In carbon, R for each effect is very close to 1. Fig. 1 shows R as a function of Z .

As can be seen in Fig. 1, the effect of Z on the decay rate⁴ goes from 1 at low Z to about .84 at high Z . In contrast to this, there is an enormous change in the disappearance rates and the capture rates as Z runs through the elements.

The decay electron energy spectrum from bound muons differs from the energy spectrum from free muons for two reasons:

1. The decay electron will lose some of its kinetic energy in escaping due to the fact that it must overcome the attraction of the nucleus. Therefore, the bound muon decay electron spectrum compared to the free decay spectrum is shifted downward in energy. This effect is small for light and medium elements but is quite pronounced for heavy elements.
2. The bound muon decays while moving in the laboratory reference frame. This motion gives a Doppler broadening to the decay electron energy spectrum. Since the motion is greater for heavy elements, this effect is greater for heavy elements.

FIG. 1

Several independent theoretical calculations of the decay rate λ_D as a function of Z were made by Uberall,⁵ Terent'ev,⁶ Gilinski and Matthews,⁷ and Kruger and Rothleitner.⁸ The approximations used for the muon and electron wave functions were not valid for high Z . Huff⁴ calculated the decay rate λ_D and the decay electron energy spectrum for stopping muons as a function of Z using approximations which were valid for all Z . He used the data on nuclear charge distributions obtained from electron scattering experiments⁹ to calculate the electron and muon wave functions. The calculations were made by numerical integration. Fig. 2 shows the results of Huff's calculation for the case of lead compared to the free decay spectrum. The large shift in the peak toward lower energy is evident. The spectrum in the case of carbon is almost the same as the free spectrum because of the weak binding of the muon to the carbon nucleus.

The capture rates for negative muons in matter were calculated by Primakoff¹⁰ and go as the effective nuclear charge to the fourth power.

This experiment was undertaken to investigate the weak interaction of muons stopping in lead since all of the previously discussed effects are more pronounced for a high Z target. Optical spark chambers were used to enable a visual selection of muons which stopped in the proper regions of the lead target. In the determination of the muon decay rate in lead, a comparison is made of the muon decay spectrum in lead to the spectrum in carbon to eliminate the need to know the electron detection solid angle and other absolute quantities which are difficult to measure accurately. This reduces some of the sources of error.

DECAY ELECTRON ENERGY SPECTRUM

FIG. 2

The measurement of the decay electron energy spectrum is necessary for two reasons:

1. To understand muon decay, it is necessary to know whether the decay interaction is a vector interaction or some combination of tensor and scalar interaction.
2. Previous experimental measurements of the muon decay rate used a correction factor due to the loss of electrons in the lead target. The correction factor was calculated from the theoretical spectrum. It is necessary to measure the decay electron energy spectrum in order to ascertain if the corrections are valid in these previous experiments.

The following quantities were measured:

1. The muon disappearance rate in lead.
2. The muon decay rate in lead.
3. The decay electron energy spectrum from muons stopping in lead.
4. The decay electron energy spectrum from muons stopping in carbon.

II. METHOD

A. Decay Rate

The muons were stopped in a thin target which was sandwiched between two spark chambers. Carbon and lead were alternately used as stopping materials. The amount of material was adjusted so that both targets had the same stopping power. By examining the decay electron time distribution in the case of lead and comparing it to the time distribution in the case of carbon, the decay rate of muons in lead can be calculated. This method eliminated the need to know the percentage of the beam that was muons and the percentage of muons that stopped in the target.

The measured time distribution of decay electrons from negative muons stopping in a target is of the form $N_e = N_o e^{-\lambda_T t}$; where N_e is the number of electrons detected per unit time at time t , and λ_T is the muon disappearance rate for the particular target.

N_o can be written as:

$$N_o = (\lambda_D) (M) (E_M) (E_a) (E_{SN}) (E_{SC}) (E_S) \quad (1)$$

where:

λ_D = The decay rate for negative muons stopping in the target. This is the quantity that is being determined.

M = The number of particles that "stop" in the apparatus as read on an appropriate scaler.

E_M = The fraction of M that are muons that stop in the usable fiducial region of the target.

(See Appendix F).

E_a = The fractional solid angle of acceptance for decay electrons. This quantity is the same for both targets since the geometry is independent of target.

E_{SN} = The efficiency of scanning a good event. This quantity is discussed in Appendix D.

E_{SC} = The spark chamber efficiency. This quantity is the probability of a good event being a single track in the spark chamber and is the same for both targets.

E_S = The fraction of decay electrons whose energies are above the minimum detectable energy (see Appendix C).

The value of N_0 can be evaluated by counting the number of electrons detected at time t in the interval dt and extrapolating back to the number at time $t=0$. In order to solve for λ_D^{Pb} , the equation for N_0^{Pb} was divided by the equation for N_0^C .

$$\frac{N_o^{Pb}}{N_o^C} = \frac{(\lambda_D^{Pb})}{(\lambda_D^C)} \cdot \frac{(M E_M E_a E_{SN} E_{SC} E_S)^{Pb}}{(M E_M E_a E_{SN} E_{SC} E_S)^C} \quad (2)$$

Since E_a and E_{SC} are the same for both targets, they cancel.

Thus,

$$\frac{N_o^{Pb}}{N_o^C} = \frac{(\lambda_D^{Pb})}{(\lambda_D^C)} \cdot \frac{(M^{Pb})}{(M^C)} \cdot \frac{(E_M^{Pb})}{(E_M^C)} \cdot \frac{(E_{SN}^{Pb})}{(E_{SN}^C)} \cdot \frac{(E_S^{Pb})}{(E_S^C)} \quad (3)$$

For carbon, λ_D^C equals the free decay rate; therefore, solving for λ_D^{Pb} expressed in units of the free decay rate (called R) gives:

$$R = \frac{(N_o^{Pb})}{(N_o^C)} \cdot \frac{(M^C)}{(M^{Pb})} \cdot \frac{(E_M^C)}{(E_M^{Pb})} \cdot \frac{(E_{SN}^C)}{(E_{SN}^{Pb})} \cdot \frac{(E_S^C)}{(E_S^{Pb})} \quad (4)$$

E_S for carbon is unity for two reasons: (1) the carbon decay electron spectrum has a negligible number of electrons that are below the energy cutoff of the apparatus; (2) the entire spectrum was used in the analysis (i.e., no artificial cutoff was imposed).

E_S for lead was calculated from the energy spectrum data (see Appendix C). E_{SN}^C and E_{SN}^{Pb} were determined by examining the scanning data (see Appendix D). The fraction $\frac{E_M^C}{E_M^{Pb}}$ was calculated and is discussed in Appendix F. Therefore, since the other factors on the right of equation (4) are known, λ_D^{Pb} can be evaluated.

B. Energy Spectrum

1. Carbon

The theoretical decay electron energy spectrum for muons in carbon can be written in the form $N(E) = R_S(E) + \rho \{R_T(E) - R_S(E)\}$. R_S is the scalar coupling term and R_T is the tensor term. ρ is the familiar Michel¹¹ parameter. $N(E)$ is the number of electrons detected with an energy E in the interval dE . Theoretical calculations using a two component neutrino theory¹² give the following values:⁴

$$R_S = 3E^2 (6-12 E)$$

$$R_T = E^2 (6-4 E)$$

$$\rho = .75$$

where E is the electron energy in units of the free muon mass.

The crystal resolution, (see Appendix A), and the decay electron ionization energy loss were folded into the theoretical expressions for R_S and R_T and the energy spectrums were calculated for various values of ρ (see Fig. 3). The folded theoretical curve was compared to the experimental data to evaluate ρ . Since the value of ρ had been determined by other more precise experiments, this served as an additional check of the crystal calibration.

2. Lead.

The theoretical decay electron energy spectrum from muons in lead has been calculated by Huff.⁴ As in carbon, the spectrum has the form $N(E) = R_S(E) + \rho \{R_T(E) - R_S(E)\}$. The crystal resolution and decay

DECAY ELECTRON ENERGY SPECTRUM
FOR
MUONS IN CARBON

- A $\rho = 0$ SCALAR INTERACTION
B $\rho = .75$ VECTOR INTERACTION
C $\rho = 1.0$ TENSOR INTERACTION

FIG. 3

electron ionization loss were folded into the theoretical expression for R_S and R_T and the resulting spectrum for various values of ρ are shown in Fig. 4. The data were compared to the folded theoretical expression in order to evaluate ρ .

DECAY ELECTRON ENERGY SPECTRUM
FOR
MUONS IN LEAD

FIG. 4

III. EXPERIMENT

A. Beam

The beam of negative muons used in this experiment was obtained from the 440 MeV proton synchrocyclotron at Carnegie-Mellon University. The beam (which was produced every 5 milliseconds in pulses of about 100 μ sec duration) was spread uniformly in time through the use of an electronic stretcher.¹³ The stretching greatly reduced accidental events which are more probable when the beam is very intense for a short time. Negative muons of approximately 70 MeV were selected by the cyclotron's fringe magnetic field and by a deflection magnet placed outside the cyclotron shielding, (see Fig. 5), and were transported to the experimental apparatus.

The mean energy and the energy spread of the muon beam were measured by taking a range curve. The scintillation counters used for the range curve had the following dimensions:

- 1 4" x 4" x 1/4"
- 2 4" x 4" x 1/4"
- 3 5" x 5" x 1/4"
- 4 6" x 8" x 1/2"

Fig. 6 is a side view of the range telescope. Counter 4 was placed against counter 3 to insure that counter 4 would detect all charged particles that passed through counter 3. Copper absorber was placed upstream and against counter 3. $C(1234)$ was counted for a fixed

FIG. 5

SIDE VIEW OF APPARATUS FOR RANGE CURVE

FIG. 6

number of C(12) for different thicknesses of copper. By using C(12) as a monitor, beam intensity fluctuation did not affect C(1234) (see Fig. 7 for the range curve). The thickness of copper that would cause a maximum number of muons to stop in the target was determined from the range curve.

A solid block of copper of this thickness was used to degrade the energy of the beam for the duration of the experiment. The apparatus was then transformed to the principal arrangement used in the experiment using the following scintillation counters and crystal (see Fig. 8):

- 1 4" x 4" x 1/4"
- 2 4" x 4" x 1/4"
- 3 4" x 4" x 1/4"
- 4 4" x 4" x 1/16"
- 5 1-3/4" x 1-3/4" x 1/16"
- 6 4" x 4" x 1/4"
- 7 8" x 8" x 1/2"

CRYSTAL Sodium Iodide crystal 8" x 8" diameter (see Appendix A)

Counter 3 was covered with 2 mill aluminum foil to reduce the number of muons stopping in the covering rather than in the target. The copper block was placed upstream and against counter 3. Counters 4 and 5 were made thin and were covered with aluminum foil to minimize the energy that the decay electrons would lose when passing through them. Counter 5 was placed at the center of the face of the sodium

FIG. 7

SIDE VIEW OF APPARATUS

COUNTER 6 IS AGAINST THE LIGHT PIPE OF COUNTER 5

FIG. 8

iodide crystal. This, plus certain scanning rules, made certain that the decay electron lost its entire energy inside the crystal.

B. Targets

Two targets were used: lead and carbon. The thickness of the lead target was chosen to stop a number of muons sufficient to give a statistically significant number of detectable decay electrons in a reasonable time and to minimize the decay electron ionization loss within the target.

The thickness of the carbon was chosen to make the number and distribution of muons stopping in it equal to the number and distribution of muons stopping in the lead target (see Appendix F for the discussion of the two targets).

C. Spark Chambers

The target was placed between two spark chambers and the entire arrangement was held tightly together in a lucite frame. The plates of the spark chambers were made of aluminum foil .35 mills thick. The plates were constructed by gluing the aluminum foil to 1/4" thick lucite frames at 45°F. After the glue had dried, the frames were brought to room temperature which caused the foil to stretch tightly since the thermal expansion of lucite is greater than that of aluminum. The plates with an active area of 12" x 12" were kept thin to reduce the number of muons that would stop in them and to minimize the energy loss of decay electrons which passed through them. There were

seven gaps upstream and six downstream of the target. The target was spaced 1/2" from the nearest aluminum plate to prevent confusion from arising when determining whether the muon stopped in the aluminum or in the target. Since the thickness of each aluminum foil plate normal to the beam direction was 3.47 mg/cm^2 , which was much smaller than the 1223 mg/cm^2 of the lead target, very few muons stopped in the aluminum foil plates and even fewer were confused as a muon stop in the target.

Since the spark chambers were made separate, it was possible to change targets in about one hour plus one more hour to refill the chambers with gas. The gas mixture of 50% Ne and 50% He flowed continuously at a very slow rate. The clearing field on the spark chamber was 20 volts DC all the time. The high voltage was 7000 volts (see Fig. 9 for trigger circuit).

D. Optics

Mirrors and lens were placed in a proper position to enable both 90° stereo and direct views of the spark chamber to be photographed on a single frame of film. This frame also contained the muon decay time and decay electron pulse height in digital form.

A reticule was placed in front of the spark chamber to serve as a reference system in which measurements were made. The position of the various portions of the apparatus relative to the reticule was measured with the use of reference wires stretched in known positions and photographed. A lens was placed in front of the reticule to make

the entire volume of the spark chamber visible to the camera. The apparatus was carefully adjusted by using transits and mirrors to keep the total distortion in the system, including the scanning machine, below 1/2%.

E. Logic

1. Event.

A stopping particle was defined by a coincidence of counters 1, 2 and 3 in anticoincidence with counters 4 and 7; $\mu_s = C(123\overline{47})$ (see Fig. 10). An electron was defined by a coincidence of counters 4 and 5 in anticoincidence with counter 3 and 6, $e_s = C(45\overline{63})$. Counter 3 prevented beam particles from giving an electron event. Counter 6 prevented a particle that passed through the light pipe of counter 5 and gave a detectable Cerenkov pulse from giving an electron event. The pulse, μ_s , triggered a gate, μ_g , which was set for 360 nsec for most of the experiment. The e_s pulse was placed in coincidence with the μ_g gate and this coincidence was an event, $C(\mu_g, e_s) = E_v$.

2. Decay Time.

The decay time was measured by using a time-to-amplitude converter (TAC). To obtain a sharp "start" pulse, fast pulses (3 nsec) from counter 3 were placed in coincidence with the slower μ_s pulses (20 nsec). The time of the resulting coincidence $C(3, \mu_s) = \mu_T$ was determined by the time at which the counter 3 pulse occurred. In a similar manner a sharp "stop" pulse was obtained from a coincidence between a fast pulse from

FIG. 10

counter 4 (3 nsec) and the E_V ($5, E_V$) = e_T . If no e_T followed within 1.2 μ sec of a μ_T pulse, the TAC recycled giving no output. The normal TAC output represented the decay time and was analyzed by an analog to digital converter (ADC1).

3. Electron Energy.

The sodium iodide crystal output pulse went through an amplifier to an analog to digital converter ADC2. ADC2 was gated by a pulse derived from E_V , so that it only analyzed pulses that occurred in time with an event. This gate was 4 μ sec.

The ADC1 and ADC2 outputs each went to TSI scalers and to pulse height analyzers. Nixie lights located at the spark chamber and connected to the TSI scalers were flashed every event, displaying the corresponding decay time and pulse height in digital form on the same film frame with the spark chamber tracks.

4. Dead Time.

Every time the cyclotron went through a cycle (200/sec), there was a small amount of beam which was bunched together for a short time (~ 100 μ sec). The apparatus was gated off during this time to reduce accidental events. The apparatus was also gated off for 2.2 sec after every event to allow time for the spark chamber to fire and for the event to be photographed. Suitable gating was provided to prevent a second trigger of the μ_S circuit from occurring while the μ_g gate was on due to a first μ_S pulse.

F. Data Analysis

1. Scanning the Film.

The location of the scintillation counters with respect to the sparks on each film frame can be seen in Fig. 11. The film was scanned in three steps. The first step was the selection of events that could be used in the energy spectrum and decay time calculations. In the second and third steps, the scanner classified the events according to where in the target the tracks intersected. This was done for stereo and direct views.

In step 1, the events were selected using Template A (see Fig. 12) using the following criteria:

1. There is only one μ track and only one e track in each frame (to eliminate accidental events).
2. The backward extension of the μ track must pass through counter 3 (to eliminate events that passed through the light pipe of 3).
3. The forward extension of the e track must pass through counter 5 (to eliminate events that passed through the light pipe of 5).
4. The μ track must intersect the e track at the target in the usable region (to eliminate events that might be stops in the gas, the aluminum, or the plastic).

APPARATUS AND DATA FILM

FIG. 11

TEMPLATE A

FIG. 12

Fig. 13 shows examples of both acceptable and unacceptable events.

The second step was to group the events selected in step 1 according to where they passed through the target. The target was divided into three regions in each view -- 1, 2 and 3 using Template 1 (see Fig. 14). After the scanner selected the regions, one region in each view, he punched a two-digit number on tape. The right digit was the region in the stereo view and the left digit was the region in the direct view. Region 22 was the only region in which it was possible for a path to go from counter 3 through counter 5 without bending. All events for which the intersection was not in region 22 had to bend at the intersection point and were classified according to the direction of the bend in each view (see Fig. 14). By making an analysis of the decay time data for each pair of regions (stereo and direct) in the case of lead, it was possible to identify the regions that had low background. The regions with low background were then used for the decay rate determination. It was important for the regions used to have low background with the lead target since in the case of the carbon target, which has to have the same regions, it was difficult to separate the background from the real events.

The third step was to group the events selected in step 1 according to the angles the electron paths had when entering the sodium iodide crystal. To accomplish this, the target was again divided into three regions in both the stereo and direct view using Template 2 (see Fig. 15).

GOOD AND BAD EVENTS

Good event above.

Pulse height was channel 208
and TAC was channel 730.

Bad event above.

No electron track.

FIG. 13

TEMPLATE 1

FIG. 14

TEMPLATE 2

FIG. 15

A two-digit number was put on tape in the same way as for step 2.

All the electrons whose path intersected the target in region 1 entered the crystal with an angle of less than .25 radians to the normal and therefore lost all their energy in the crystal. All the electrons whose path intersected the target in region 2 entered the crystal with an angle of less than .47 radians and had a high probability of losing all their energy in the crystal. Region 3 contained all the remaining events. The three regions were mutually exclusive.

For each event the following data were punched on tape by the scanner.

1. pulse height
2. decay time
3. regions in Template 1
4. regions in Template 2
5. film frame register number

2. Calculations and Results.

a. Disappearance Rate for Lead

For the calculation of the disappearance rate in lead, a histogram was made of the decay time of events that intersected the lead target in all regions. To reduce background from carbon decay which shows a peak at 43 MeV, the following requirements were made:

1. Events with intersection in all regions except 22, Template 1, had to have an electron with energy below 35 MeV.

2. Events with intersection in region 22, Template 1, had to have an electron with energy below 19 MeV.

This requirement was necessary because region 22 had a much larger number of background particles than the other regions.

A theoretical curve, $y = A e^{-\lambda_T t} + B$ was fit to the histogram using a least squares technique. The quantities A, B, and λ_T were determined. λ_T is the disappearance rate of muons stopping in lead.

$$A = 45.7 \pm 1.3 \quad \text{counts per time interval}$$

$$B = 4.2 \pm .7 \quad \text{counts per time interval}$$

$$\lambda_T = (1.31 \pm .09) \times 10^7 \text{ (sec}^{-1}\text{)}$$

$$\text{time interval is } .703 \text{ nsec}$$

$$\chi^2 = 55.2$$

$$\text{expected } \chi^2 = 52 \pm 10.1$$

number of
analyzed events $N = 5768$

Fig. 16 shows the data with the curve that gave the best fit to the data.

b. Calculation of N_o^{Pb}

A histogram was made of the decay time of events that intersected the lead target in regions 11, 12, and 13, Template 1. These were the regions in which the background was found to be low. The histogram included only events with the electron energy greater than 11 MeV.

DECAY CURVE IN CALCULATION OF λ_T FOR LEAD

FIG. 16

Since many more muons were captured than decayed, there was a probability that the nuclear gamma rays and neutrons emitted from excited nuclei could knock out electrons from atomic levels and these would have the same time distribution as decay electrons. These electrons, however, would all have had energies less than 11 MeV. The energy requirement thus insured that only electrons from muon decay were accepted. There was no upper energy cutoff since the background was already very low. Any upper energy cutoff would have involved losing good events with very little reduction in the background.

The function $N_e = N_o^{Pb} e^{-\lambda_T t} + B$ was fitted to the histogram using a least squares technique. λ_T was put equal to $13.4 \times 10^6 \text{ sec}^{-1}$ * and the quantities N_o^{Pb} and B were evaluated. The background B came from three processes:

1. Muons could decay in the edge of counter 3 and thus not give an electron anti pulse and the decay electrons could scatter in the lead and enter the electron telescope.
2. Muons could scatter in the lead and stop and decay in the edge of counter 4. (and thus not give a beam anti).
3. An accidental event could occur where one particle goes through the muon telescope and a different particle goes through the electron telescope in such a way that their tracks intersect at the target. Because the time spectrum is random for these events, data for events at negative times can be used to subtract away these accidental events.

* This is the weighted average of many experiments (see Ref. 14).

The results of the fit were:

λ_T was put equal to $13.4 \times 10^6 \text{ (sec}^{-1}\text{)}$

$N_O^{\text{Pb}} = 15.4 \pm .6$ counts per time interval

$B = 1.1 \pm .02$ counts per time interval

time interval is .703 nsec

$\chi^2 = 41.3$

expected $\chi^2 = 52 \pm 10.2$

number of
analyzed events $N = 1846$

See Fig. 17 for a comparison of data and fitted curve. The function $N_e = N_O^{\text{Pb}} e^{-\lambda_T t} + B$ was fit to the data for several different values of λ_T to find the dependence of N_O^{Pb} on λ_T . N_O^{Pb} was found to vary very slowly with any change in λ_T (see Fig. 18).

c. Calculation of N_O^{C}

A histogram was made of the decay time for the carbon data using the same fiducial regions as in the case of N_O^{Pb} . However, there was no energy cutoff since there are very few captures and, therefore, negligible background due to this effect. Because the lifetime in carbon was long compared to the times investigated, the decay curve $N_e = N_O^{\text{C}} e^{-\lambda_T t}$ reduced to the form $N_e = A t + N_O^{\text{C}}$. This curve was fit to the data. N_e is the number of electrons detected at time t in interval dt . N_O^{C} is the number detected at time $t = 0$. The background in the carbon analysis was negligible due to the following reasons:

DECAY CURVE IN CALCULATION OF N_0^{Pb}

FIG. 17

FIG. 18

1. Decay electrons from muons that stopped in counter 3 would not scatter sufficiently in the carbon target to pass through counter 5 and give a background event, since the rms scattering angle was much lower for electrons in carbon than in the lead and only a total of 7.2% background was detected in the case of lead from all causes.

2. Muons which could scatter in the lead and stop and decay in the edge of counter 4 would do so at a very reduced ratio when carbon was the target because of the reduction in the rms scattering angle for muons in carbon.

3. Accidental events which occurred when one particle went through the μ telescope and another went through the e telescope could be measured by looking at data for negative times. This was found to be negligible in the case of carbon as a target.

The results were:

N_e = the number of electrons at time t per time interval

$N_o^C = 7.17 \pm .31$ counts per time interval

$A = -1.32 \pm .42$ counts per time interval per 200 nsec

time interval was .703 nsec

$\chi^2 = 45.6$

expected $\chi^2 = 52 \pm 10.2$

number of
analyzed events $N = 2101$

See Fig. 19 for data and fit curve.

FIG. 19

d. Energy Spectrum for Carbon and Lead Data.

1) Carbon

Because the energy spectrum for muons decaying in carbon has a peak of 52.5 MeV (see Fig. 3), only electrons that decayed from region 11, Template 2, were used for the pulse height analysis. These electrons intersecting the carbon in region 11, Template 2, entered the crystal at a small angle and went through the center portion of the crystal. The resulting shower of electrons and gamma rays were absorbed in the crystal giving a pulse to the photomultiplier that was proportional to the initial electron energy. A histogram was made of all the pulse height data from that region and a theoretical folded curve of the form $N(E) = R_S(E) + \rho\{R_T(E) - R_S(E)\}$, normalized to the number of events in the region between 6 and 60 MeV, was fitted to the data using a least square technique. ρ was set equal to .75 for the normalization because the integral was not sensitive to ρ . The values of R_S and R_T were obtained from the theory and ρ was the variable fit. The following values were found:

$$\rho = .72 \pm .06$$

$$\chi^2 = 15.6$$

$$\text{expected } \chi^2 = 19 \pm 6.2$$

number of
analyzed events $N = 745$

See Fig. 20 for a comparison of the theoretical curve and the data.

DECAY ELECTRON ENERGY SPECTRUM
FOR
CARBON TARGET

FIG. 20

2) Lead

A histogram was made of the pulse heights of events that intersected the lead target in regions 11, 12, and 13, Template 1. These regions have a small number of background events (see calculation of N_0^{Pb} , page 36). Only events having decay electrons in less than 130 nsec after the muon stopped were included in the histogram in order to further reduce the ratio of background to good events. This is because the background events have a flat time distribution while the muon decay in lead events have an exponential time distribution with a lifetime of 74.6 nsec.¹⁴ No additional requirement on where the electrons intersected the lead target was made because the decay electrons from muons in lead (having a pulse height distribution with a peak at about 24 MeV, see Fig. 4) from all the regions would lose their entire energy in the crystal since the resulting gamma ray and electron shower has a low energy. A theoretical folded curve in the form $N(E) = R_S(E) + \rho\{R_T(E) - R_S(E)\}$ normalized to the number of events between 11 and 70 MeV was fitted to the pulse height histogram using a least square technique. As in carbon for the normalization ρ was set equal to .75 since the normalization was not sensitive to ρ . Data for energies below 11 MeV were not fitted for the reasons explained in section b, page 36. The values of R_T and R_S were obtained from theory and the value of ρ was determined. From the energy spectrum it was possible to calculate the fraction of decay electrons that were above

11 MeV from muon decay in lead. This fraction was used in the calculation of the decay rate (see Appendix C). The following values were found:

$$\rho = .72 \pm .05$$

$$\chi^2 = 23.8$$

$$\text{expected } \chi^2 = 19 \pm 6.2$$

number of
analyzed events $N = 1653$

See Fig. 21 for a comparison of the theoretical curve and the data.

e. Calculation of the Decay Rate of Lead

The decay rate λ_D^{Pb} of muons stopping in lead in terms of the free decay rate called (R) can be calculated since all the terms in the following expression were determined:

$$R = \frac{N_o^{\text{Pb}}}{N_o^{\text{C}}} \cdot \frac{M^{\text{C}}}{M^{\text{Pb}}} \cdot \frac{E_M^{\text{C}}}{E_M^{\text{Pb}}} \cdot \frac{E_{\text{SN}}^{\text{C}}}{E_{\text{SN}}^{\text{Pb}}} \cdot \frac{E_S^{\text{C}}}{E_S^{\text{Pb}}}$$

$$\frac{N_o^{\text{Pb}}}{N_o^{\text{C}}} = \frac{15.4 \pm .6}{7.17 \pm .31}$$

$$\frac{M^{\text{C}}}{M^{\text{Pb}}} = \frac{.640 \times 10^9}{1.56 \times 10^9}$$

$$\frac{E_M^{\text{C}}}{E_M^{\text{Pb}}} = 1.00 \pm .01$$

(see Appendix F)

$$\frac{E_{\text{SN}}^{\text{C}}}{E_{\text{SN}}^{\text{Pb}}} = 1.01 \pm .03$$

(see Appendix D)

$$\frac{E_S^{\text{C}}}{E_S^{\text{Pb}}} = \frac{1}{.88 \pm .02}$$

(see Appendix C)

The final result is: $R = 1.01 \pm .07$

DECAY ELECTRON ENERGY SPECTRUM
FOR
LEAD TARGET

FIG. 21

IV. DISCUSSION OF RESULTS

A. Disappearance Rate for Lead

This measurement of the muon disappearance rate with spark chambers to visually discriminate against background yields a result that is consistent with the results of other experiments and with theoretical predictions. This result also served as a check of the performance of the apparatus (see Table I)..

B. Negative Muon Decay Rate in Lead

The measurement of the muon decay rate in lead determined a value about 10% higher than previous experimental results (see Table II). The decay rate is also about 15% higher than the value predicted by theory.⁴ Two of the previous results were consistent with this value^{20,21} but one result¹⁹ is about two standard deviations from this result. One result is much less than the others¹⁸ and is unexplained. The theoretical prediction is slightly more than two standard deviations from this result. The difference could be resolved by an experiment with much higher statistics.

One of the sources of error in the previous experiments might be found in the correction made for electrons lost in the target. By measuring the energy spectrum at the same time as the decay spectrum, this source of error was minimized for this experiment.

C. Energy Spectrum

1. Carbon

The carbon pulse height data fit the theoretical spectrum for a value of close to .75 which is both the theoretical prediction and the

TABLE I

Theoretical and experimental results in the determination of the
negative muon disappearance rate λ_T in lead

$$\lambda_T (10^6 \text{ sec.}^{-1})$$

Theory	Experiment	Reference
	$12.2 \pm .7$	15
	$15.0 \pm .7$	16°
	$13.43 \pm .10$	2
	$13.72 \pm .22$	14
	$13.40 \pm .11$	14*
	$13.1 \pm .9$	**
12.4		17

* weighted average of previous experimental results

** this experiment

TABLE II

Theoretical and experimental results in the determination of the muon decay rate (λ_D^{Pb}) for muons stopping in lead expressed in units of the free muon decay rate (called R)

R		
Theory	Experiment	Reference
	$.4 \pm .1$	
	$.28 \pm .1$	18
	$.86 \pm .04$	19
	$.88 \pm .12$	20
	$.88 \pm .13$	21
	$1.01 \pm .07$	*
.884		8
.483		7
.684		
.844		4

* The results of this experiment.

results of other experiments.^{22,23*} This served as a check of the performance and of the calibration of the apparatus in the experiment.

2. Lead

The lead pulse height data fit the theoretical spectrum for a value of ρ close to .75 which is a vector interaction. A vector interaction is predicted. In other experiments to determine the decay rate in lead, the decay electron spectrum was used to correct for the electron loss in the lead. This result gives assurance that those corrections were valid when the vector spectrum was used.

* These references are for the positive muon spectrum which is the same as the negative spectrum in carbon after folding, since the crystal folding masks the slight spectrum differences before folding.

Appendix A

Sodium Iodide Crystal Calibration

I. Abstract

Electrons of momenta 25.2 MeV/c through 63.3 MeV/c, produced at the Carnegie-Mellon University synchrocyclotron, were used to calibrate a large sodium iodide crystal (8" thick x 8" diameter). Optical spark chambers, located upstream and downstream of a bending magnet, enabled a measurement of the angle the electrons bent when passing through the magnet. To reduce the momentum spread of the electrons due to multiple scattering, the material in the beam path was kept at a minimum. Small scintillation counters triggered the spark chambers when the electrons passed through the proper position in space and entered the center of the crystal. The magnet was calibrated using a floating wire technique. Using electrons with a momentum spread of less than $\pm 2\%$ at all of the momenta used, the total resolution of the crystal for electrons of momenta 25.2 and 40.8 MeV/c was measured. The full width at half height of spectrum was 10.5% of the peak position for both the 25.2 and 40.8 MeV/c electrons. The calibration determined the values for A' and B' in the following equation:

$$P = A' \text{ TSI} + B'$$

P = electron momentum in MeV/c

TSI = peak channel position of the spectrum from electrons
of momentum P

$$A' = .076 \pm .000 \text{ MeV/c per channel}$$

$$B' = 1.25 \pm .16 \text{ MeV/c}$$

Using gamma sources the crystal resolution was measured as follows:

Source	Energy (MeV)	Resolution (full width at half height over energy)
Cs ¹³⁷	.661	9.6%
Co ⁶⁰	1.17	7.5%
	1.33	6.8%
PuBe	4.43	6.8%

II. Experiment

A. Beam

The beam of negative electrons used in the calibration was obtained from the 440 MeV proton synchrocyclotron at Carnegie-Mellon University. The beam was spread uniformly in time through the use of an electronics stretcher.¹³ The stretching greatly reduced double tracks in the spark chamber which are more probable when the beam is very intense for a short time. The electrons traveled through an opening in the cyclotron shielding and entered the apparatus (see Fig. 22).

The scintillation counters used in the experiment were:

- 1 $1\text{-}3/4'' \times 1\text{-}3/4'' \times 1/32''$
- 2 $4'' \times 4'' \times 1/4''$
- 3 $1\text{-}3/4'' \times 1\text{-}3/4'' \times 1/32''$
- 4 $4'' \times 4'' \times 1/4''$
- 5 $4'' \times 4'' \times 1/4''$

Counters 1 and 3 were made very thin and were covered with thin aluminum foil to reduce the ionization energy loss and the multiple scattering the electrons had when they traveled through them.

B. Spark Chambers

Four spark chambers were used for the measurement of the bending angle of the electron in the magnet. Each chamber was made of two

COUNTER 2 IS AGAINST THE LIGHT PIPE OF COUNTER 1.
COUNTER 4 IS AGAINST THE LIGHT PIPE OF COUNTER 3

FIG. 22

aluminum foil plates, .35 mills thick, and separated by a 1/4" plastic frame. The chambers were used in pairs. One pair defined the entry angle and the other defined the exit angle of an electron in the magnet. The separation of the chambers in the pairs was by a 1-1/2" lucite spacer. A thin Aclar cover, .5 mills thick, was placed on either side of each pair to protect the pairs from the effect of the differential pressure of the gas inside the chamber and the air outside of it. The whole assembly was held together by a plastic frame. Electrons of 21 MeV/c had a rms scattering angle of .026 radians (1.6%) in the four chambers. The clearing field on each chamber was 75 volts DC all the time. The pulsed high voltage on the chambers was 7 KV. A gas mixture of 50% Ne and 50% He flowed continuously through the chambers at a very low rate.

C. Optics

Each pair of spark chambers was held in place by a frame which projected a 90° stereo view into the same direction as the direct view, to enable both views to be photographed on the same film frame. A reticule in front of the frame enabled the scanner to determine the laboratory coordinate of the sparks. The reticule position in space was determined by passing wires through the spark chamber frame and photographing them. The film then was analyzed to determine the relative position of the wire to the reticule. This enabled a correlation between the experimental apparatus and the reticule. Then by measuring the relative position of the sparks to the reticule, the actual position of the sparks in the laboratory system could be calculated.

A lens of 16' focal length placed in front of each reticule made the whole volume of the spark chambers visible. A system of four mirrors bent the images from both lenses into a Bettie camera with a lens of 8-1/4" focal length. The film used was TRIX.

D. Helium Bag

A polyethylene bag containing Helium was placed inside the magnet to reduce the scattering of electrons passing through the magnet. The bag had windows of .5 mill Aclar and a total length of 6 feet. Electrons of 21 MeV/c would scatter an rms angle in the Helium bag of .02 radians (1.2%).

E. Magnet

The magnet, placed at an angle of .810 radians to the input electron beam, was positioned so that an acceptable electron was bent an average angle of 1.620 radians. The magnet current was monitored by a 100 amp 50 millivolt shunt connected to a Leeds & Northrup potentiometer that was accurate to more than four decimal places. The magnetic field was regulated by a servo system that kept the current in the magnet constant to better than $\pm 1\%$.

The magnet was calibrated by wire measurements. One end of a wire was placed in the center of the opening in the cyclotron shielding and the other end was placed over a pulley placed at the crystal center. The current through the wire I_w and the tension on the wire T were adjusted

for each magnet current setting I_m to position the wire in an orbit that corresponded to the mean orbit of an electron that was an acceptable event. The momentum P of the electron is related to the wire current and tension:

$$P = \frac{2.94T}{I_w}$$

P units are MeV/c.

T units are grams.

I_w units are amps.

By fitting the following equation to the data, using a least squares technique, the parameters A and B were evaluated (see Fig. 23):

$$P = A I_m + B \quad (1a)$$

$$A = 1.02 \pm .02 \quad \text{MeV/c per amp}$$

$$B = 2.11 \pm .87 \quad \text{MeV/c}$$

F. The Sodium Iodide Crystal

The crystal was a cylinder 8" thick and 8" diameter with four photomultiplier tubes looking into the base. These tubes were carefully balanced with gamma ray sources. The pulse height distribution of the crystal with these sources are given in Fig. 24 and 25.

FIG. 23

FIG. 24

FIG. 25

The resolution of the crystal with each of these sources is the following:

Source	Energy (MeV)	Resolution (full width at half height divided by peak position)
Cs ¹³⁷	.661	9.6%
	1.17	7.5%
Co ⁶⁰	1.33	6.8%
PuBe	4.43	6.8%

G. Logic

1. Event.

An electron event was defined by a coincidence of counters 1 and 3, in anticoincidence with counters 2, 4, and 5, $C(13\overline{2}45) = E_v$ (see Fig. 26). Counter 2 prevented electrons that passed through the light pipes of counter 1 and gave a detectable Cerenkov pulse from giving an event. Counter 4 acted the same way as counter 2 with regard to counter 3.

A lead wall was placed between the opening in the cyclotron shielding and the sodium iodide crystal to reduce the background in the crystal. Counter 5 was against the lead wall to prevent electrons scattered from the wall from triggering an electron event.

FIG. 26

2. Electron Energy.

The sodium iodide crystal output went to an amplifier, whose output went to an analog to digital converter ADC and to an oscilloscope. The oscilloscope trace was photographed each event. The ADC was gated by a pulse derived from E_v so that it only analyzed pulses that occurred in time with an event. The gate width was 4 μ sec.

The ADC output went to a TSI scaler and to a pulse height analyzer (PHA). Nixie lights located at the spark chamber and connected to the TSI scaler were flashed every event displaying the pulse height in digital form on the same film frame with the spark chamber track. Two registers, one located at the spark chamber and the other at the oscilloscope were photographed each event to enable a correlation of the spark chamber and scope pictures. The PHA accumulated the pulse height data in its memory whose information was punched on tape at frequent intervals.

3. Dead Time.

Every time the cyclotron went through a cycle (200/sec), there was a small amount of beam which was bunched together for a short time ($\sim 100 \mu$ sec). The apparatus was gated off during this time to reduce double tracks in the spark chambers. The apparatus was also gated off for 2.2 sec after every event to allow time for the logic to function and the event to be photographed.

H. Data Analysis

The scanning of the spark chamber film was done on line to a computer to enable an accurate measurement of the electron bending angle in the magnet. However, before the measurement was started the scanner checked that only one event appeared in each spark chamber and that this event went through the center portions of the chambers. When these conditions were satisfied, the scanner measured four points on the reticule (stereo view upstream of the magnet) to establish a coordinate system for the angle measurement. Since only three points are needed in a coordinate transformation, the fourth point served as a check point. If the error on the fourth point's position was large, the computer instructed the scanner to remeasure the reticule. Because of the scanners dislike of remeasuring the reticule, the scanner became more careful in his scanning resulting in a reduction of the errors in the angle measurement. The scanner measured two angles for each event (the entrance and exit to the magnet). The scanner also input the pulse height and the register number to the computer. After the measurements were completed, the computer punched out the input angle, the exit angle, the total bending angle, the pulse height, and the register number on tape for analysis later. The on-line scanning added an error of less than 1% in the measurement of the electron bending angle in the magnet. See Fig. 27 for a typical good event.

1. Scanning the Oscilloscope Film.

The oscilloscope film was analyzed to determine whether there were any double pulses from the crystal in some events. Double pulses, which would have spread the crystal pulse height spectrum were found in only a small percentage of events, thus having a negligible effect on the crystal calibration (see Fig. 28 for a typical oscilloscope picture).

GOOD EVENT

Typical spark chamber picture from crystal calibration. Upstream chambers are to the right and downstream chambers are to the left. Stereo views are the middle pictures and the outside pictures are the direct views.

FIG. 27

OSCILLOSCOPE PICTURE

0.3 2-5 2 6

7800

Sodium Iodide anode pulse as displayed on oscilloscope.

Notice the lack of spark chamber noise on the pulse.

FIG. 28

V. RESULTS

For magnet currents of 25 and 40.5 amps, data were accumulated using the spark chambers to define the electron's bending angle through the magnet and, hence, the electron's momentum. For magnet currents of 52.5 and 62.5 amps, data were accumulated using only the pulse height analyzer.

A histogram was made of the electron pulse height data for each of the four magnet settings used in order to determine the channel position of the peak. The electron's momenta were calculated from equation 1a (see page 59) for the four cases. The ionization losses due to the material that the electron passed through before entering the crystal were subtracted from the momenta. The following equation was then fitted to the data.

$$P = A' \text{ TSI} + B' \quad (2a)$$

$$A' = .076 \pm .000 \text{ MeV/c per channel}$$

$$B' = 1.25 \pm .16 \text{ MeV/c}$$

(see Fig. 29)

FIG. 29

The crystal resolution was measured for the two magnet settings in which the spark chambers were used. A histogram was made of the electron bending angle in the magnet (θ) for each of those cases (see Fig. 30 and 31). Histograms of the crystal pulse height for both cases (see Fig. 32 and 33) were made for the following three conditions:

1. All data that entered the PHA were used.
2. Only events that passed through the center part of the spark chamber were used.
3. Only events that passed through the center part of the spark chamber and had a bending angle ± 20 mrad to the mean bending angle were used.

From the histogram in condition 3, the resolution of the crystal for each of the two momenta used was measured (see Table III).

TABLE III

Crystal Resolution

P (MeV/c)	Left Resolution	Right Resolution	Total Resolution
25.2	12.5%	8.5%	10.5%
40.8	12.5%	8.5%	10.5%

Left Resolution = The spread in channels between peak position and half height left of peak times 2 divided by peak position.

Right Resolution = The spread in channels between peak position and half height right of peak times 2 divided by peak position.

Total Resolution = The spread in channels between right half height and left half height of the peak divided by peak position.

FIG. 30

BENDING ANGLE HIST.

FIG. 31

PULSE HEIGHT SPECTRUM

X DATA FROM PHA

O DATA FROM SCANNING $\Delta\theta = 400\text{mrad}$ $\bar{\theta} = 1619\text{ mrad}$

• DATA FROM SCANNING $\Delta\theta = 40\text{mrad}$ $\bar{\theta} = 1619\text{ mrad}$

FIG. 32

PULSE HEIGHT SPECTRUM

x DATA FROM PHA

○ DATA FROM SCANNING $\Delta\theta = 400\text{ mrad}$ $\bar{\theta} = 1619\text{ mrad}$

• DATA FROM SCANNING $\Delta\theta = 40\text{ mrad}$ $\bar{\theta} = 1619\text{ mrad}$

FIG. 33

Appendix B

Calibration of the TAC

The differential and integral linearity of the TAC was measured using a pulse generator output for a start pulse and counter 5 for a stop pulse. A Co^{60} source placed at an appropriate distance from counter 5 gave a sufficient number of stops. The TAC was connected to the ADC1 which, in turn, was connected to a pulse height analyzer (PHA). The data were stored in the memory of the PHA and punched out at the end of the test. This test was done at frequent intervals during the experiment to make certain the apparatus was functioning properly. The slope of a straight line fit to the data is the integral linearity. The differential linearity was determined by examining the CHI square of the fit to the data. The analysis showed a differential linearity of better than .2% and an integral linearity of $(.68 \pm .08)\%$ per 200 nsec (see Fig. 34).

The calibration of the TAC was performed with delay cables. The output of the pulse generator was split into two pulses. One went directly to the start of the TAC and the other went through a delay cable to the stop of the TAC. The output from the TSI seen on the Nixie lights was determined for each delay cable used. A least squares fit was made for the line $D = A (\text{TSI}) + B$ to the data. D is the delay time and TSI is the reading of the lights. The value of A calculated is $.703 \pm .005$ nsec per channel.

FIG. 34

Appendix C

Energy Cutoff

The fractional number of decay electrons from muons in lead above the energy cutoff was calculated from the theoretical folded spectrum which best fit the data.

The integral of the spectrum from the cutoff energy to infinity was divided by the integral from minus infinity to infinity. This gave the efficiency E_S^{Pb} . The value for a cutoff of 11 MeV is .876.

To check the calculation of E_S , λ_D was calculated for various cutoff energies (see Fig. 35). The conclusion drawn from these calculations is that the value of λ_D measured is independent of the cutoff energy used.

DECAY RATE VERSUS CUTOFF ENERGY

$R = \lambda_D^{\text{Pb}}$ DIVIDED BY FREE DECAY RATE

FIG. 35

Appendix D

Scanning Efficiency

After all the rolls had been scanned once using the scanning rules, the good events were rescanned to eliminate any events that had double tracks and were wrongly accepted on the first scanning. The events that remained after the rescanning were then used in all of the analysis.

In order to calculate the scanning efficiency, a sample number of rolls were scanned a second time. The number of good events found in the sample rolls after the first scanning was calculated and is called N_1^α where α is either Pb or C. The number of good events in the sample found after the second scanning was calculated and is called N_2^α . The number of events accepted in both the first and second scanning was called N_3^α . All of the events that were accepted in either the first or second scanning of the sample, but not in both, were rescanned to eliminate bad events. The number of bad events found in the sample in the first scanning was subtracted from N_1^α and the new number was called M_1^α . In the same way the new number in the second scanning was calculated and called M_2^α .

The following efficiencies were calculated:

$$E_1^\alpha = \frac{M_1^\alpha}{N_1^\alpha} = \text{the fraction of accepted events in the first scanning that were really good.}$$

$E_2^\alpha = \frac{N_3^\alpha}{E_2^\alpha} =$ the fraction of total good events that were accepted in the first scanning.

The total efficiency $E_{SN}^\alpha = \frac{E_2^\alpha}{E_1^\alpha}$

The results are presented in Table IV.

TABLE IV

Scanning Efficiency Results

	E_1	E_2	E_{SN}	E_{SN}^C / E_{SN}^{Pb}
Lead	$.919 \pm .014$	$.841 \pm .014$	$.915 \pm .02$	
				$1.01 \pm .03$
Carbon	$.964 \pm .013$	$.888 \pm .013$	$.921 \pm .02$	

Appendix E

Zero Time Calculation

Data were taken at the end of every roll of film of events (E_V) which had the $\bar{4}$ in μ_s and the $\bar{3}$ in e_s removed. These events, called "prompt events," were due to beam particles going directly through counters, 1, 2, 3, 4, and 5. By visually observing the channel numbers on the Nixie lights after each roll of film, a check was made on the performance of the apparatus. The data on the film were scanned using the rules for good events. A histogram was made of the decay time of events in region 22, Template 1. Region 22 is the only region of the target in which unscattered particles would pass. The peak of the histogram was channel 648.4 (see Fig. 36). This corresponds to zero decay time for fast particles in region 22.

The muons which stopped in the target were slow particles with an average energy of only 4.5 MeV. The muons which were used in the calculation of N_O^{Pb} also traveled further than the particles which gave prompt events. A calculation of these two effects increased the average zero channel to 653.1 for the muons that stopped in region 11, 12, 13, Template 1, which were used in the calculation of N_O^{Pb} and N_O^C .

FIG. 36

Appendix F

Target Calculation

A Monte Carlo calculation was made to determine the relative stopping effectiveness of carbon and lead targets to negative muons. The probability of an incident muon of energy E stopping in each target was calculated. The muon entered the target, traveled a small distance r , losing some of its energy* that was a function of the muon energy, the target material and the distance r , and scattered angles θ and ϕ randomly. The angle ϕ was completely random while the angle θ had a probability that was a function of the distance traveled r , the muon energy, and the target material. Random numbers were used to obtain the angles. The new position of the muon was then calculated. The muon again traveled a distance r and the scattering angles and energy loss were found. This process was continued until either the muon left the target or lost its entire energy in the target. The calculations were repeated many times at each energy but of course the random numbers used were always different. The probability of an incident muon stopping was found by dividing the number of stops by the total number of calculations for each case of incident muons energy E . The probability distributions (see Fig. 37) were integrated from energies 0 through 10.5 MeV for each target. Each integration yields an energy E_x . If all the muons less than E_x were stopped and all above were not then the integration would also yield E_x . Since the number of muons at each energy below 10.5 MeV was the same, the ratio of the value of E_x for the two

* The energy loss as a function of distance traveled was found from the range energy tables (ref. 24).

targets yields the relative stopping power. The calculation shows that both targets stopped the same percentage of muons in the beam (see Table V for a comparison of the targets).

A check was made on the relative stopping power of the lead target to the carbon target. $T = C(12347)/C(123)$ was determined for both the cases of the lead and carbon targets sandwiched between the spark chambers. T is the relative number of particles stopped in the apparatus to the number of particles entering the apparatus. Therefore, the value of T is sensitive to the stopping power of the target. The value of T was found to be the same for both targets verifying the conclusion of the theoretical calculations on the relative stopping power of the two targets.

F = FRACTION OF MUONS AT ENERGY E THAT STOP IN THE TARGET

FIG. 37

TABLE V

Comparison of Targets*

target	density gm/cm ³	thickness normal to plate in inches	thickness normal ₂ to beam in gm/cm ²	effective*** maximum energy of muons that will stop in target in MeV	purity
Carbon	1.68	.108	.651	9.05	99.9%
Lead***	11.35	.030	1.223	9.06	99.989%

* both targets were 12" x 12"

** composition of lead was

A	Fraction
208	.520
207	.220
206	.240
204	.015

$$\frac{E_M^C}{E_M^{Pb}} = 1.00 \pm .01$$

*** If all the muons below that energy were stopped in the target, the number of muons stopped would equal the actual number that stopped assuming the muons having energies below 10.5 MeV are equally probable. This was the case in this experiment.

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