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ANALYTIC MODEL FOR THE NONLINEAR INTERACTION OF TEARING MODES
OF DIFFERENT PITCH IN CYLINDRICAL GEOMETRY

B. Carreras,[†] B. V. Waddell, and H. R. Hicks

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ABSTRACT

An analytic model has been developed for describing the nonlinear interaction of tearing modes of different pitch in cylindrical geometry for equilibria characterized by flat safety factor profiles. The analysis shows that the $m = 2/n = 1$ tearing mode can destabilize odd m modes, particularly the $3/2$ mode. The model compares well with our three-dimensional (3-D) code with respect to the time evolution of the $0/0$, $1/1$, $2/1$, $3/2$, and $5/3$ modes. Scaling rules are obtained for the position and location in time of the maximum or peak in the $3/2$ growth rate. The characteristic time of destabilization of the odd m modes predicted by the model correlates well with the observed time scale for the major disruption in tokamaks.

I. INTRODUCTION

In Ref. 1 it was shown in cylindrical geometry that the $m = 3/n = 2$ tearing mode can be nonlinearly destabilized by the $m = 2/n = 1$ tearing mode. Here m and n denote the toroidal and poloidal mode numbers, respectively. This result was obtained by solving numerically a reduced set of resistive MHD (magnetohydrodynamic) equations. In addition, it was found that other modes are destabilized, in particular the $m = 1/n = 1$ and $m = 5/n = 3$ modes. The growth rates of these destabilized modes exhibited maximums occurring at approximately the same time. Presently, various computer runs have been made for different initial conditions and safety factor profiles. For all of them the $m = 3/n = 2$ was initially linearly unstable. Table I summarizes the parameters of the relevant runs. Essentially the same results are obtained for all these runs.

One might argue that the destabilization of the $3/2$ and $5/3$ modes is due to the deformation of the equilibrium current by the $2/1$ mode. In order to investigate this hypothesis, we used a two-dimensional (2-D) code (see Ref. 2) to generate the $0/0$ component of the helical flux modified nonlinearly by the presence of modes with helicity or pitch $m = 2/n = 1$. The parameters of the 2-D run were the same as those for Run 1. Then, we used a linearized version of the 2-D code to determine the instantaneous growth rates of the $3/2$ and $5/3$ modes as a function of the time elapsed in the original $2/1$ run. The results of this run are compared to those of Run 1 in Fig. 2. Figure 2 clearly shows that although both the $3/2$ and $5/3$ modes are destabilized by the

Table I. The parameters and results of the computer runs

Run	$q(0)$	S	γ_{21}^0	γ_{32}^0	$ \max_{t=t_0} \psi_{21} $	$ \max_{t=t_0} \psi_{32} $	t_0	t^m	γ_{32}^m
1	1.37	1.3×10^5	340	230	1.17×10^{-4}	1.60×10^{-5}	7.3×10^{-2}	2.2×10^{-2}	810
2	1.37	2×10^4	790	440	2.30×10^{-4}	1.35×10^{-4}	4.3×10^{-3}	9.96×10^{-3}	1240
3	1.37	2×10^4	120	70	5.18×10^{-4}	2.36×10^{-5}	3.4×10^{-2}	6.65×10^{-2}	270
4	1.08	2×10^4	33.6	22.5	2.61×10^{-4}	6.85×10^{-5}	1.0×10^{-1}	2.32×10^{-1}	45.5

modification in the equilibrium, the change in the growth rates is far too small to explain the 3-D results. In addition, it is obvious that the modification of the equilibrium cannot explain the destabilization of the 1/1 mode because the maximum in its growth rate occurs before the safety factor q becomes less than one (at any value of r); according to linear theory, q must be less than one for the 1/1 mode to be unstable.

The purpose of this report is to describe a phenomenological model of the growth of the destabilized modes that explains the maximum in their growth rates, particularly the maximum in the 3/2 growth rate. The model is valid until the system becomes extremely nonlinear. Employing an approximation similar to the WKB approximation for the Fourier components of the poloidal flux function and the velocity stream function, we show that the decrease in the growth rate of the 2/1 mode is due to the modification of the equilibrium by the 2/1 mode itself. The 1/1 and 5/3 modes are essentially driven by the 2/1 and 3/2 modes; for the cases of interest, the 3/2 mode is linearly unstable. Then, in turn, the 5/3 and 1/1 modes couple to the 2/1 mode and drive the 3/2, thus producing the maximum in the 3/2 growth rate.

It should be emphasized that this model is phenomenological. Although a great deal of analysis is carried out, no rigorous perturbation scheme is employed. We do not perform the difficult task of computing coupling coefficients; instead, they are determined by fitting the results of the 3-D code. Rather than develop a rigorous perturbative analysis that would probably break down at short times, the goal is to construct an analytic model that involves only a few modes but still describes the basic physics.

Although the coupling coefficients are found by fitting the 3-D results, it is a virtue of the model that only two coefficients need to be determined for one run provided the q profile does not change drastically. Only one run is required because the scaling of the coupling coefficients with the initial parameters can be predicted. In addition, the scaling of the value and location (in time) of the maximum of the $3/2$ growth rate is predicted accurately. These scaling rules prove useful in making comparisons with experimental data because they can be used for small resistivity where the code cannot be applied because of numerical difficulties.

Although this model oversimplifies the complex nonlinear interaction of tearing modes, we believe that it illustrates a dynamical mechanism that may be responsible for some of the disruptions observed in tokamaks because the time scale derived from this model is consistent with the experimental results.

From the point of view of magnetic field line reconnection, the effect of the $2/1$ mode is different from that of other modes. The $2/1$ mode grows and produces a magnetic island with a width that is approximately the saturation width for this mode; the width at the end of a run is independent of the initial conditions. This is not the case for the other modes; the widths of their magnetic islands are a function of the initial amplitudes of the $3/2$ and $2/1$ modes.

The remainder of this report is organized as follows: in Sec. II, the equations for the Fourier components of the flux and stream functions are derived; in Sec. III, the calculation scheme is outlined; in Sec. IV, the integral equations for the growth rates are solved analytically; in

Sec. V, the results of the model are compared with those from the 3-D code; in Sec. VI, we show the compatibility of the model with the experimental results; and in Sec. VII, some concluding remarks are made.

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II. REDUCED RESISTIVE MHD EQUATIONS

We employ the reduced resistive equations derived in Refs. 1 and 3:

$$\frac{D\Psi}{Dt} \equiv \frac{\partial\Psi}{\partial t} + \underline{V}_\perp \cdot \underline{\nabla}\Psi = \eta J_\zeta - \frac{\partial\Phi}{\partial\zeta}, \quad (1)$$

$$J_\zeta = \nabla_\perp^2 \Psi, \quad (2)$$

$$\underline{V}_\perp = \underline{\nabla}\Phi \times \hat{\zeta}, \quad (3)$$

$$U = \nabla_\perp^2 \Phi, \quad (4)$$

$$\frac{DU}{Dt} = -S^2 \left[\hat{\zeta} \cdot (\underline{\nabla}\Psi \times \underline{\nabla}J_\zeta) + \frac{\partial J_\zeta}{\partial\zeta} \right]. \quad (5)$$

Here, r , θ , and ζ denote the radial, poloidal, and toroidal coordinates, respectively. The poloidal magnetic field is given in terms of the poloidal flux function Ψ by the relation $\underline{B}_p = \epsilon \underline{\nabla}\Psi \times \hat{\zeta}$, where $\epsilon \equiv a/R_0$ with a and R_0 the minor and major radii of the torus, respectively. The "1" subscript means perpendicular to $\hat{\zeta}$, the unit vector in the toroidal direction. In the preceding, $\underline{V}_\perp$ is the velocity perpendicular to ζ , η is the resistivity, J_ζ is the toroidal current density, Φ is the velocity stream function, and U is the fluid vorticity. Equations (1) and (5) are referred to as the flux and vorticity (or momentum balance) equations, respectively. Although the toroidal coordinate system is employed, all toroidal effects have been neglected in the derivation of the preceding equations. The equations are in dimensionless form; the time is normalized to the resistive skin time, $\tau_R [\equiv a^2/(\mu_0 \bar{\eta})]$, where $\bar{\eta}$ is some

typical value of the resistivity and μ_0 is the magnetic permeability. The quantity S is the ratio of τ_R and τ_{Hp} [$\equiv R_0 (\mu_0 \rho_0)^{1/2} / B_{z0}$, where ρ_0 is the (constant) mass density and B_{z0} is the (constant) toroidal magnetic field].

In order to study the nonlinear interaction of tearing modes of different helicity, it is convenient to Fourier analyze the variables as follows:

$$f = \sum_{m,n} f_{mn}(r,t) \sin(m\theta + n\zeta), \quad (6)$$

$$f = f_0 + \sum_{m,n} f_{mn}(r,t) \cos(m\theta + n\zeta), \quad (7)$$

where the sine series is employed for Φ and U , the cosine series is employed for Ψ and J_z , and f_0 denotes equilibrium quantities. It is valid to use only the sine (cosine) series for Φ (Ψ) provided the initial state has this form. For $m \geq 0$ and $n \geq 0$, the equations for the Fourier components are

$$\frac{\partial \psi_{mn}}{\partial t} = -F_{mn} \psi_{mn} + \eta_0 J_{mn} - R_1(mn), \quad (8)$$

$$\frac{\partial U_{mn}}{\partial t} = -R_2(mn) + S^2 \left[F_{mn} J_{mn} - \frac{m}{r} \frac{dJ_0}{dr} \psi_{mn} - R_3(mn) \right], \quad (9)$$

where, denoting the safety factor by q ,

$$F_{mn} \equiv \frac{m}{r} \frac{d\psi_0}{dr} + n = -\frac{1}{q} + n, \quad (10)$$

$$R_j(mn) = \sum_{\substack{m'=|m'+\sigma m''| \\ n'=|n'+\sigma n''|}} R_j(mn|m'n' + \sigma m''n'') , \quad (11)$$

$$\begin{aligned} R_1(mn|m'n' + \sigma m''n'') &= \frac{1}{2r} \left(m' \phi_{m'n'} \frac{\partial \psi_{m''n''}}{\partial r} - \sigma m'' \frac{\partial \phi_{m'n'}}{\partial r} \psi_{m''n''} \right. \\ &\quad \left. + m'' \phi_{m''n''} \frac{\partial \psi_{m'n'}}{\partial r} - \sigma m' \frac{\partial \phi_{m''n''}}{\partial r} \psi_{m'n'} \right) , \end{aligned} \quad (12)$$

$$\begin{aligned} R_2(mn|m'n' + m''n'') &= \frac{1}{2r} \left(m' \phi_{m'n'} \frac{\partial U_{m''n''}}{\partial r} - m'' \frac{\partial \phi_{m'n'}}{\partial r} U_{m''n''} \right. \\ &\quad \left. + m'' \phi_{m''n''} \frac{\partial U_{m'n'}}{\partial r} - m' \frac{\partial \phi_{m''n''}}{\partial r} U_{m'n'} \right) , \end{aligned} \quad (13)$$

$$\begin{aligned} R_2(mn|m'n' - m''n'') &= \frac{\text{sign}(m' - m'')}{2r} \left(-m' \phi_{m'n'} \frac{\partial U_{m''n''}}{\partial r} \right. \\ &\quad - m'' \frac{\partial \phi_{m'n'}}{\partial r} U_{m''n''} + m'' \phi_{m''n''} \frac{\partial U_{m'n'}}{\partial r} \\ &\quad \left. + m' \frac{\partial \phi_{m''n''}}{\partial r} U_{m'n'} \right) , \end{aligned} \quad (14)$$

$$\begin{aligned} R_3(mn|m'n' + m''n'') &= \frac{1}{2r} \left(m' \psi_{m'n'} \frac{\partial J_{m''n''}}{\partial r} - m'' \frac{\partial \psi_{m'n'}}{\partial r} J_{m''n''} \right. \\ &\quad \left. + m'' \psi_{m''n''} \frac{\partial J_{m'n'}}{\partial r} - m' \frac{\partial \psi_{m''n''}}{\partial r} J_{m'n'} \right) , \end{aligned} \quad (15)$$

$$\begin{aligned}
R_3(mn|m'n' - m''n'') &= \frac{\text{sigm}(m' - m'')}{2r} \left[m' \psi_{m'n'} \frac{\partial J_{m''n''}}{\partial r} \right. \\
&\quad + m'' \frac{\partial \psi_{m'n'}}{\partial r} J_{m''n''} - m'' \psi_{m'n'} \frac{\partial J_{m'n'}}{\partial r} \\
&\quad \left. - m' \frac{\partial \psi_{m''n''}}{\partial r} J_{m'n'} \right] . \tag{16}
\end{aligned}$$

Here, $j = 1, 2$, or 3 ; $\sigma = \pm 1$, and it is understood that in the summations the combination $m'n' + \sigma m''n''$ is the same as $m''n'' + \sigma m'n'$. In deriving Eq. (8), we have assumed that the resistivity is independent of time.

Let us construct a quadratic form. We define:

$$M_{mn} = \int_0^1 r dr \left[\left(\frac{\partial \psi_{mn}}{\partial r} \right)^2 + \frac{m^2}{r^2} \psi_{mn}^2 \right] , \tag{17}$$

$$K_{mn} = \int_0^1 r dr \left[\left(\frac{\partial \phi_{mn}}{\partial r} \right)^2 + \frac{m'^2}{r^2} \phi_{mn}^2 \right] . \tag{18}$$

Then, employing Eqs. (8) and (9) we obtain

$$\frac{d}{dt} (K_{mn} + S^2 M_{mn}) = 2L(mn) + 2N(mn) , \tag{19}$$

where

$$L(mn) \equiv S^2 \int_0^1 r dr \left(\frac{m}{r} \frac{dJ}{dr} \phi_{mn} \psi_{mn} - \eta_o J_{mn}^2 \right) , \tag{20}$$

$$N(mn) \equiv \sum_{\substack{m=|m'+\sigma m''| \\ n=|n'+\sigma n''|}} N(mn|m'n' + \sigma m''n'') , \quad (21)$$

$$\begin{aligned} N(mn|m'n' + \sigma m''n'') &\equiv S^2 \int_0^1 r dr [\phi_{mn} R_3(mn|m'n' + \sigma m''n'') \\ &\quad + S^{-2} R_2(mn|m'n' + \sigma m''n'') \phi_{mn} \\ &\quad + R_1(mn|m'n' + \sigma m''n'') J_{mn}] . \end{aligned} \quad (22)$$

Then, defining the instantaneous growth rate γ_{mn} by

$$\gamma_{mn} \equiv \frac{1}{2} \frac{d}{dt} \ln (K_{mn} + S^2 M_{mn}) , \quad (23)$$

we have

$$\gamma_{mn} = \frac{L(mn) + N(mn)}{K_{mn} + S^2 M_{mn}} . \quad (24)$$

The factor of two has been inserted in the definition of γ_{mn} so that this growth rate will coincide with that of the flux function or stream function when there is a single eigenmode.

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III. CALCULATION SCHEME

We assume that the Fourier components are given by the WKB approximation, i.e.,

$$\psi_{mn}(r, t) = \tilde{\psi}_{mn}(r) e^{\int_{t_0}^t \gamma_{mn} dt'}, \quad (25)$$

$$\phi_{mn}(r, t) = \tilde{\phi}_{mn}(r) e^{\int_{t_0}^t \gamma_{mn} dt'}, \quad (26)$$

where we take γ_{mn} to be independent of r and to be given by Eq. (23). We define the small parameters ϵ and $\delta\epsilon$ as a measure of the initial $m = 2/n = 1$ and $m = 3/n = 2$ perturbations, respectively. Then,

$$\begin{aligned} \psi_{21}(r, t) &= \epsilon \tilde{\psi}_{21}(r) e^{\int_{t_0}^t \psi_{21} dt'} = \epsilon \psi_{21}^0 e^{\gamma_{21}^0 (t-t_0)} \\ &+ \epsilon^3 \psi_{21}^1 G_{21}^1(t) + \dots, \end{aligned} \quad (27)$$

$$\begin{aligned} \psi_{32}(r, t) &= \delta\epsilon \tilde{\psi}_{32}(r) e^{\int_{t_0}^t \gamma_{32} dt'} = \delta\epsilon \psi_{32}^0 e^{\gamma_{32}^0 (t-t_0)} \\ &+ \delta\epsilon^3 \psi_{32}^1 G_{32}^1(t) + \dots, \end{aligned} \quad (28)$$

where ψ_{21}^0 and ψ_{32}^0 are the linear eigenfunctions with growth rates γ_{21}^0 and γ_{32}^0 , respectively. The time t_0 is such that at this time the modes are essentially linear eigenmodes. For the runs of interest, δ is less than one. It should be emphasized that ϵ and δ are not really expansion

parameters in the following calculation; they are merely used to delineate the order of magnitude of the various terms.

The scheme of calculation is as follows. We observe that due to the nonlinear interaction of the $m = 2/n = 1$ and $m = 3/n = 2$ modes, the $m = 1/n = 1$ and $m = 5/n = 3$ modes will be generated. In addition, all modes deform the equilibrium, i.e., generate ψ_{00} perturbations in a quasi-linear sense. First, we obtain expressions for γ_{11} and γ_{53} to lowest order. Then, we calculate ψ_{00} and γ_{00} . Next, we determine the quasi-linear correction to the growth rate of the $m = 2/n = 1$ mode due to ψ_{00} . Then we obtain the correction to the growth rate of the $m = 3/n = 2$ mode due to ψ_{00} and to the nonlinear coupling with the $m = 2/n = 1$ and $m = 5/n = 3$ modes. Finally, we compute the correction of the growth rate of the $m = 5/n = 3$ due to ψ_{00} .

In order to calculate ψ_{11} , we observe that for the cases we are considering, $q(r) > 1$ so that F_{11} does not vanish and the equations are not singular. Consequently, we can neglect inertia in the vorticity equation and use ψ_{21} and ψ_{32} to calculate the nonlinear terms. Thus, the equation for ψ_{11} is

$$F_{11}J_{11} = \frac{1}{r} \frac{dJ_0}{dr} \psi_{11} + \frac{1}{2r} \left[\psi_{32} \frac{\partial J_{21}}{\partial r} - \frac{\partial \psi_{21}}{\partial r} J_{32} + 2 \frac{\partial}{\partial r} (\psi_{32}J_{21} - \psi_{21}J_{32}) \right]. \quad (29)$$

Integration of the equation gives

$$\begin{aligned} \psi_{11} = & -rF_{11} \int_r^1 \frac{dr'}{r'^3 F_{11}^2} \left(\frac{6}{r'} \psi_{21} \psi_{32} + \frac{3}{2} r' \psi_{32} \frac{\partial^2 \psi_{21}}{\partial r'^2} \right. \\ & \left. - r' \psi_{21} \frac{\partial^2 \psi_{32}}{\partial r'^2} - \frac{r'}{2} \frac{\partial \psi_{21}}{\partial r'} \frac{\partial \psi_{32}}{\partial r'} \right). \end{aligned} \quad (30)$$

Clearly $\gamma_{11}(t) = \gamma_{21}(t) + \gamma_{32}(t)$. For $\tilde{\psi}_{53}$, we obtain

$$\begin{aligned} F_{53} \tilde{J}_{53} e^{\int_{t_0}^t \gamma_{53} dt'} &= - \frac{5}{r} \frac{dJ_0}{dr} \tilde{\psi}_{53} e^{\int_{t_0}^t \gamma_{53} dt'} \\ &+ \frac{1}{2r} \left(2\psi_{21} \frac{\partial J_{32}}{\partial r} - 3 \frac{\partial \psi_{21}}{\partial r} J_{32} \right. \\ &\left. + 3\psi_{32} \frac{\partial J_{21}}{\partial r} + 2 \frac{\partial \psi_{32}}{\partial r} J_{21} \right), \end{aligned} \quad (31)$$

outside the tearing layer because the inertia can be neglected in the vorticity equation. Since $F_{53} = 0$ at $q = 5/3$, Eq. (31) is singular and cannot be integrated without resolving the singularity at the tearing layer. Nevertheless, we observe that $\gamma_{53}(t) = \gamma_{21}(t) + \gamma_{32}(t) = \gamma_{11}(t)$. Similarly, we can argue that the growth rate of the 4/2 mode is simply given by $\gamma_{42} = 2\gamma_{21}$ because it is driven by the 2/1 mode.

Because ϕ_{00} vanishes, the flux equation can be used to calculate ψ_{00} . The resistive term may be ignored outside the tearing layer, and

$$\frac{\partial \psi_{00}}{\partial t} = - \frac{1}{2r} \frac{\partial}{\partial r} \left(\sum_{m,n} m \psi_{mn} \phi_{mn} \right). \quad (32)$$

To lowest order, only the $m = 2/n = 1$ contribution is important. Consequently,

$$\tilde{\psi}_{00}(r) = \frac{1}{r} \frac{\partial}{\partial r} (\tilde{\phi}_{21} \tilde{\psi}_{21}) , \quad (33)$$

and

$$\gamma_{00}(t) = g_0 e^{\int_{t_0}^t (2\gamma_{21} - \gamma_{00}) dt'} , \quad (34)$$

where $g_0 = \gamma_{00}(t_0)$.

Now let us evaluate the higher order corrections to the growth rates of initial modes. For the 2/1 mode, the corrections can be due to interaction with the 0/0 or the 4/2 perturbations; because the latter interaction can be shown to be smaller, we neglect it. To calculate the quasi-linear corrections, we employ the quadratic form [Eq. (24)] and obtain:

$$\gamma_{21}(t) = \gamma_{21}^0 + g_1 e^{\int_{t_0}^t \gamma_{00} dt'} , \quad (35)$$

where

$$g_1 \equiv 2\varepsilon^2 S^2 \left[\int_0^1 r dr \left(\frac{\partial J_{00}}{\partial r} \tilde{\phi}_{21} \tilde{\psi}_{21} \right) \right] (K_{21} + S^2 M_{21})^{-1} \quad (36)$$

scales as ε^2 . In obtaining Eq. (35), we have approximated the functions in the linear term by the linear eigenfunctions. There are three possible corrections to the 3/2 growth rate: the interaction of the 3/2 and 0/0,

the interaction of the 2/1 and 1/1, and the interaction of the 2/1 and 5/3. As discussed in Sec. I, the effect of the 0/0 perturbation has been studied and shown to be small. If we retain only the 2/1 and 5/3 nonlinear terms, we obtain from the quadratic form

$$\gamma_{32}(t) = \gamma_{32}^0 + g_2 e^{\int_{t_0}^t (\gamma_{53} + \gamma_{21} - \gamma_{32}) dt'}, \quad (37)$$

where

$$g_2 \equiv \epsilon^2 S^2 \left[\int_0^1 r dr N(32) \right] (K_{32} + S^2 M_{32})^{-1}, \quad (38)$$

$$N(32) \equiv \int_0^1 r dr \left\{ \tilde{\phi}_{32} [\tilde{R}_3(32|53-21) + S^{-2} \tilde{R}_2(32|53-21)] \right. \\ \left. + \tilde{J}_{32} \tilde{R}_1(32|53-21) \right\}, \quad (39)$$

and it is understood that $\tilde{R}_j(32|53-21)$ is obtained by replacing $\tilde{f}_{mn}$ with f_{mn} in Eqs. (12)-(16). Notice that g_2 scales as ϵ^2 . If we include the 2/1 and 1/1 coupling, we obtain an expression for γ_{32} with the same form as Eq. (37) but with a different g_2 . Within the context of this model, it is impossible to distinguish between the 2/1 and 5/3 driving the 3/2, and the 2/1 and 1/1 driving the 3/2.

Finally, we consider the corrections to the 1/1 and 5/3 growth rates. Because the 2/1 and 3/2 interaction has been calculated exactly, we need only consider the effect of the ψ_{00} perturbation. (We ignore the effect of coupling with the 4/2 perturbation because it is small.)

The effect of the 0/0 perturbation on the 1/1 perturbation can be neglected because of the absence of a 1/1 singular surface. Then the only effect to consider is that of the 0/0 on the 5/3, which can be computed in the same way as for the 2/1. We obtain

$$\gamma_{53}(t) = \gamma_{32}(t) + \gamma_{21}(t) + g_3 e^{\int_{t_0}^t \gamma_{00} dt'}, \quad (40)$$

$$g_3 \equiv \epsilon^2 S^2 \left(\int_0^1 r dr \ 5 \frac{\partial J_{00}}{\partial r} \tilde{\phi}_{53} \tilde{\psi}_{53} \right) (K_{53} + S^2 M_{53})^{-1}, \quad (41)$$

where g_3 scales as ϵ^2 .

IV. SOLUTION OF THE INTEGRAL EQUATIONS FOR THE GROWTH RATES

The equations derived in Sec. III are:

$$\gamma_{11}(t) = \gamma_{21}(t) + \gamma_{32}(t) , \quad (42)$$

$$\gamma_{00}(t) = g_0 e^{\int_{t_0}^t dt' [2\gamma_{21}(t') - \gamma_{00}(t')]} , \quad (43)$$

$$\gamma_{21}(t) = \gamma_{21}^0 + g_1 e^{\int_{t_0}^t dt' \gamma_{00}(t')} , \quad (44)$$

$$\gamma_{32}(t) = \gamma_{32}^0 + g_2 e^{\int_{t_0}^t dt' [\gamma_{53}(t') + \gamma_{21}(t') - \gamma_{32}(t')]} , \quad (45)$$

$$\gamma_{53}(t) = \gamma_{21}(t) + \gamma_{32}(t) + g_3 e^{\int_{t_0}^t dt' \gamma_{00}(t')} . \quad (46)$$

Recall that we have neglected the effect of ψ_{00} on γ_{32} . As discussed in Sec. I, this effect is relatively small; furthermore, if we do not include it, Eqs. (42)-(46) can be solved analytically.

First, we consider only the system of 2/1 and 0/0 modes, i.e., Eqs. (43) and (44). Defining

$$y \equiv e^{\int_{t_0}^t \gamma_{00}(t') dt'} , \quad x \equiv \int_{t_0}^t \gamma_{21}(t') dt' , \quad (47)$$

we have

$$\frac{dx}{dt} = \gamma_{21}^0 + y g_1 , \quad (48)$$

and

$$\frac{dy}{dt} = g_0 e^{2x} . \quad (49)$$

This set of equations can be reduced to

$$\frac{d^2 x}{dt^2} = g_1 g_0 e^{2x} . \quad (50)$$

The solution of Eq. (50) is

$$x = \sqrt{c} (t - t_0) + \ln \left[\frac{2 \sqrt{c} b}{b^2 - g_1 g_0 e^{2\sqrt{c}(t-t_0)}} \right] , \quad (51)$$

where

$$b = \sqrt{c} + \sqrt{c + g_1 g_0} , \quad c = (\gamma_{21}^0 + g_1)^2 - g_1 g_0 . \quad (52)$$

Then, we obtain

$$\gamma_{21}(t) = \sqrt{c} + g_1 g_0 \frac{2\sqrt{c} e^{2\sqrt{c}(t-t_0)}}{b^2 - g_1 g_0 e^{2\sqrt{c}(t-t_0)}} , \quad (53)$$

$$\gamma_{00}(t) = \frac{4cg_0be^{2\sqrt{c}(t-t_0)}}{b - g_0 + g_0(1 - g_1/b)e^{2\sqrt{c}(t-t_0)}} \times \frac{1}{b^2 - g_1g_0e^{2\sqrt{c}(t-t_0)}} \quad (54)$$

For $g_0 = 2\gamma_{21}^0 + g_1$, we have $c = \gamma_{21}^0$ and

$$\sqrt{c} + \sqrt{c + g_1g_2} - g_0 = 0 \quad (55)$$

Then, Eqs. (53) and (54) reduce to

$$\gamma_{21}(t) = \gamma_{21}^0 + g_1e^{2\gamma_{21}^0(t-t_0)}/D(t) \quad (56)$$

where

$$D(t) = 1 - \frac{g_1}{2\gamma_{21}^0} \left[e^{2\gamma_{21}^0(t-t_0)} - 1 \right] \quad (57)$$

and

$$\gamma_{00}(t) = \gamma_{21}^0 + \gamma_{21}(t) \quad (58)$$

Now, we consider the 3/2 and 5/3 modes. Substituting Eqs. (56) and (57) into Eqs. (45) and (46) yields

$$\gamma_{32}(t) = \gamma_{32}^0 + g_2e^{2\gamma_{21}^0(t-t_0)}/D^{2+g_3/g_1} \quad (59)$$

$$\gamma_{53}(t) = \gamma_{21}(t) + \gamma_{32}(t) + g_3 e^{2\gamma_{21}^0(t-t_0)/D(t)} . \quad (60)$$

We observe that if $g_1 < 0$ and $|g_1| > g_3 \geq 0$, γ_{32} has a maximum value in general agreement with the results of the 3-D code. The maximum occurs at the time t^m given by

$$t^m - t_0 \approx \frac{1}{2\gamma_{21}^0} \ln \frac{2\gamma_{21}^0}{|g_1| - g_3} , \quad (61)$$

and γ_{32}^m , the maximum value of γ_{32} , is given by

$$\gamma_{32}^m - \gamma_{32}^0 \approx g_2 \frac{2\gamma_{21}^0}{|g_1| - g_3} \frac{1}{\left(\frac{2|g_1| - g_3}{|g_1| - g_3} \right)^{(2+g_3/g_1)}} . \quad (62)$$

If $g_3 \approx 0$, Eqs. (61) and (62) reduce to

$$t^m - t_0 = \frac{1}{2\gamma_{21}^0} \ln \frac{2\gamma_{21}^0}{|g_1|} , \quad (63)$$

$$\gamma_{32}^m - \gamma_{32}^0 = \frac{g_2}{2|g_1|} \gamma_{21}^0 . \quad (64)$$

The width of the peak in γ_{32} can also be determined. We define the width Γ such that

$$\gamma_{32}(t^m \pm \Gamma) - \gamma_{32}^0 = \frac{1}{2} \Delta\gamma . \quad (65)$$

Then, if $g_3 = 0$, Eq. (60) yields

$$\Gamma = 0.88(\gamma_{21}^0)^{-1} . \quad (66)$$

It is useful to determine how $\Delta t \equiv t^m - t_0$ and $\Delta\gamma_{32} \equiv \gamma_{32}^m - \gamma_{32}^0$ scale with the initial parameters. As previously argued, all the g 's scale as ϵ^2 . Because $\tilde{\phi}_{21}$ is proportional to $\gamma_{21}^0 \tilde{\psi}_{21}$ and because K_{21} is small, Eq. (36) implies that g_1 scales as γ_{21}^0 . Similarly, if we ignore $\tilde{R}_1$ in Eq. (38) and note that $S^{-2}\tilde{R}_2$ is small, we conclude that g_2 scales as γ_{32}^0 . Thus, if

$$\tilde{\psi}_{21}' = \epsilon \tilde{\psi}_{21} , \quad \tilde{\psi}_{32}' = \delta \epsilon \tilde{\psi}_{32} , \quad (\gamma_{21}^0)' = \lambda_1 \gamma_{21}^0 , \quad (\gamma_{32}^0)' = \lambda_2 \gamma_{32}^0 , \quad (67)$$

$$g_j' = \epsilon^2 \lambda_j g_j , \quad (68)$$

where $j = 1$ or 2 , we have

$$(\Delta t)' = \frac{\gamma_{21}^0 \Delta t - \ln \epsilon}{(\gamma_{21}^0)' } , \quad (69)$$

$$(\Delta\gamma_{32})' = \frac{(\gamma_{32}^0)'}{\gamma_{32}^0} \Delta\gamma_{32} . \quad (70)$$

Because the time evolution of the 2/1 mode has been incorporated into the preceding derivation and because the analysis of its interaction with the 0/0 mode really requires a detailed evaluation of Δ' rather than the construction of a simple model, we must determine the sensitivity

of these results to the model for γ_{21} . Here, without assuming any particular form for γ_{21} , we derive some of the properties of the 3/2 mode due to the nonlinear coupling. Ignoring g_3 , which can be shown to be small, we have simply

$$\gamma_{53} = \gamma_{21} + \gamma_{32} , \quad (71)$$

$$\gamma_{32} = \gamma_{32}^0 + g_2 e^{2 \int_{t_0}^t \gamma_{21} dt'} . \quad (72)$$

Then,

$$\frac{d\gamma_{32}}{dt} = 2g_2 \gamma_{21} e^{2 \int_{t_0}^t \gamma_{21} dt'} , \quad (73)$$

$$\frac{d^2 \gamma_{32}}{dt^2} = 2g_2 \frac{d\gamma_{21}}{dt} e^{2 \int_{t_0}^t \gamma_{21} dt'} + \lambda g_2 [\gamma_{21}(t)]^2 e^{2 \int_{t_0}^t \gamma_{21} dt'} . \quad (74)$$

A maximum in γ_{32} occurs when γ_{21} vanishes if $g_2 > 0$ and $d\gamma_{21}(t_m)/dt < 0$. Consequently, the maximum in γ_{32} is independent of the particular model for γ_{21} and is a consequence of the fact that γ_{21} decreases and goes through zero. The position of the maximum is independent of the initial amplitude of the 3/2 mode.

The increase in the growth rate is given by

$$\Delta\gamma = g_2 e^{2 \int_{t_0}^{t_m} \gamma_{21} dt'} . \quad (75)$$

Then,

$$\Delta\gamma = g_2 \left[\frac{W_{21}(t^m)}{W_{21}(t_0)} \right]^4, \quad (76)$$

where W_{21} is the width of the 2/1 magnetic island. Equation (74) reflects the fact that $\Delta\gamma$ depends on the free energy associated with the 2/1 mode because $W_{21}(t^m)$ is related to the free energy. In addition, $\Delta\gamma$ depends on g_2 , which measures the overlap of the 2/1, 3/2, and 5/3 modes, i.e., g_2 measures the proximity of the three singular surfaces.

The width of the peak in γ_{32} can also be determined. We write

$$\gamma_{32} \approx \gamma_{32}^m + \frac{1}{2} (t - t^m)^2 \left. \frac{d^2\gamma_{32}}{dt^2} \right|_{t^m}. \quad (77)$$

If we define the width Γ by

$$\gamma_{32}(t^m \pm \Gamma) - \gamma_{32}^0 = \frac{1}{2} \Delta\gamma, \quad (78)$$

we obtain

$$\Gamma = \sqrt{\frac{1}{2 \left| \frac{d\gamma_{21}(t^m)}{dt} \right|}}. \quad (79)$$

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V. COMPARISON WITH THE RESULTS FROM THE THREE-DIMENSIONAL CODE

First, we consider the flux perturbation of the 1/1 mode. In Fig. 3a, the linear eigenfunctions ψ_{21} and ψ_{32} , which we obtained from a linear Δ' (see Ref. 4) code similar to the one described in Ref. 5, are plotted as a function of r . The eigenfunctions were normalized to the corresponding flux perturbations from Run 1 at $t = 0.17$, and the results are in good agreement with the 3-D code. Using these results for ψ_{21} and ψ_{32} , Eq. (30) was evaluated numerically. The result is compared in Fig. 3b with ψ_{11} obtained from Run 1. The two results for ψ_{11} agree well except for the fact that the code result is not as sharply peaked at the $q = 3/2$ surface as the theoretical result. This difference is expected because in the calculation of ψ_{21} and ψ_{32} , the tearing layer was considered to be infinitesimally thin; this approximation produces large radial gradients in ψ_{32} which are then reflected in ψ_{11} .

Next, let us compare the expressions for γ_{11} and γ_{53} [Eqs. (42) and (46) with $g_3 = 0$] with the numerical results. In Fig. 4, we compare $\gamma_{11}(t)$ and $\gamma_{21}(t) + \gamma_{32}(t)$, taken from the code for Runs 1 and 4. We see that the comparison is generally favorable and that the agreement with Run 4 is excellent.

Now, let us consider the 0/0 perturbation. From Eq. (8), we observe that outside the tearing layer where resistivity can be neglected,

$\phi_{mn} = -\gamma_{mn} \psi_{mn} / F_{mn}$. Thus, outside the tearing layer, Eq. (33) becomes

$$\psi_{00} = \frac{1}{r} \frac{\partial}{\partial r} \frac{\psi_{21}^2}{F_{21}}, \quad (80)$$

near $t = t_0$ where $\gamma_{00} \approx 2\gamma_{21}^0$. We evaluate Eq. (75) at all values of r by replacing $(nq - m)^{-1}$ by $(nq - m)/[(nq - m)^2 + b^2]$ where $b \sim \epsilon_L/q'(r_s) \ll 1$, with ϵ_L denoting the tearing layer width⁴ and the prime denoting the radial derivative; the eigenfunction in Fig. 3a is used for ψ_{21} . The result is compared with the result from Run 1 at $t = 0.17$ in Fig. 3c. The agreement is good considering the casual treatment of the tearing layer.

It is interesting to notice that the sign of the current (J_{00}) due to this ψ_{00} perturbation is opposite that of the equilibrium current near the origin. Another effect of the 0/0 perturbation is to lower the q profile near the origin, which at later times can bring the $q = 1$ surface into the plasma, destabilizing the $m = 1/n = 1$ mode even further.

In Fig. 5, the growth rates of the 0/0 and 4/2 perturbations are depicted. According to Eq. (58), $\gamma_{00} = \gamma_{21}^0 + \gamma_{21}(t)$. We compare this remarkable relation with the results from Run 4 in Fig. 5a and obtain excellent agreement. In order to illustrate that $\gamma_{21}^0 + \gamma_{21}(t)$ is distinctly different from $2\gamma_{21}(t)$, we consider the growth rate of the 4/2 mode. In Sec. III, we argued that $\gamma_{42} = 2\gamma_{21}(t)$. Consequently, in Fig. 5b we compare this result with the result from Run 4; the agreement is excellent. Furthermore, in Fig. 5c, γ_{00} as given by Eq. (58) is shown to compare favorably with the result from the same run. Rather than explicitly calculating g_1 in Eq. (58), it was determined by specifying the observed t^m in Eq. (63). Although only the position of the maximum in γ_{32} is required to determine g_1 , we obtain a fair description of γ_{00} .

In order to compute $\gamma_{32}(t)$ and $\gamma_{21}(t)$, we must determine g_1 and g_2 ; this can be accomplished by taking t^m and γ_{32}^m from the code and solving Eqs. (63) and (64). The theoretical curves [from Eqs. (56) and (60)] obtained in this fashion are compared with the results of Runs 1 and 4 in Figs. 6a and 6b, respectively. The agreement is good except for the fact that the theoretical width of the maximum in γ_{32} is larger than that obtained from the code. Notice that the code result is probably narrower in width because the 3/2 is stabilized substantially by the 0/0 perturbation that it produces itself, an effect which has not been included in the model. Although only information about γ_{32} has been used to compute g_1 and g_2 , we obtain an accurate prediction for γ_{21} .

In Table II, we test the scaling formulae [Eqs. (67) and (68)]. We take Run 1 as the base case and determine if the results of Runs 2, 3, and 4 scale properly. The table shows that $\Delta\gamma$ scales reasonably well and that Δt scales are in excellent agreement with Eq. (67).

Table II. Comparison of the numerical and model results

Run	$q(0)$	ϵ	δ	γ_{21}^0	Δt code	Δt model	$\Delta \gamma$ code	$\Delta \gamma$ model
1	1.387	1	1	340	1.5×10^{-2}	Input	400	Input
2	1.387	1.96	4.30	790	5.6×10^{-3}	5.6×10^{-3}	800	765
3	1.387	4.42	0.33	120	3.2×10^{-2}	3.01×10^{-2}	200	122
4	1.08	2.23	1.92	33.6	1.36×10^{-1}	1.28×10^{-1}	23	39

VI. COMPARISON WITH EXPERIMENT

The results presented in Ref. 1 and the results of the preceding analysis agree in a general way with the experimental observations of major disruptions.

In LT-3,⁶ it has been observed that during the disruption, modes with $n = 1$ and 2 are generated. The $n = 2$ mode is localized near the $q = 2$ mode rational surface; perhaps the localization is at the $q = 3/2$ surface. The m numbers observed are 1 , 2 , and 3 , but no serious attempt is made to correlate the m and n numbers. This combination of modes could correspond to the destabilization of the $3/2$ and $1/1$ modes by the $2/1$ mode, as reported here and in Ref. 1.

In PLT,⁷ flat q profiles similar to the ones treated here are observed preceding some of the disruptions. Furthermore, one often sees an $m = 1$ precursor oscillation while the disruption itself has an asymmetrical component. In the interior of the plasma, $m = 1$ oscillations are observed. These observations could indicate the destabilization of odd modes, particularly the $1/1$ and $3/2$, by the $2/1$ mode.

According to the preceding analysis, the time scale characteristic of the disruption should be roughly, $\Gamma \simeq (\gamma_{21}^0)^{-1}$. If we consider a PLT helium plasma with $n_e(0) = 10^{14} \text{ cm}^{-3}$, $T_e(0) = 850 \text{ eV}$, $Z_{\text{eff}}(0) = 9$, and $B_{z0} = 3.5 \text{ T}$, then $\tau_{\text{Hp}} \simeq 0.24 \text{ } \mu\text{sec}$ and $\tau_R \simeq 2 \text{ sec}$ so that $S \simeq 10^7$. Then, $\Gamma \simeq 230 \text{ } \mu\text{sec}$. If the expression for γ_{21}^0 that includes diamagnetic effects is employed, we obtain $\Gamma \simeq 440 \text{ } \mu\text{sec}$. Both these times are in general agreement with the observed disruption time of $500 \text{ } \mu\text{sec}$.

The time Γ is also in general agreement with the observations of several machines. If diamagnetic effects are ignored, Γ is proportional to $S^{2/5} \tau_R^{-1}$. Then

$$\Gamma = C (\bar{R}^2 \bar{m}_i \bar{n}_i \bar{B}_{\zeta 0} \bar{a}^6 \bar{V}^{-3})^{1/5}, \quad (81)$$

where C is a function of $q(r)$ and $q'(r_s)$, the bars indicate mks units with the ion mass m_i in terms of the proton mass and the ion density n_i in 10^{13} cm^{-3} , and $\bar{V}$ is the voltage at the limiter before the disruption. In Fig. 7, the experimental disruption time (Γ_D) versus $(\bar{R}^2 \bar{m}_i \bar{n}_i \bar{B}_{\zeta 0} \bar{a}^6 \bar{V}^{-3})^{1/5}$ is plotted for various machines. The straight line is drawn through the PLT point under the assumption that Γ_D scales with the first power of $(\bar{R}^2 \bar{m}_i \bar{n}_i \bar{B}_{\zeta 0} \bar{a}^6 \bar{V}^{-3})^{1/5}$. The results agree over two orders of magnitude with the theoretical and experimental values of C within a factor of two. One might argue that the scaling of the disruption time we have shown is a simple geometrical scaling with the size of the device; then we would also expect the disruption time to scale with $\tau_H = a/V_A = \epsilon \tau_{Hp}$, where V_A is the Alfvén velocity, or with τ_R . In Fig. 8, we have plotted Γ_D versus $\tau_R = 4\pi \bar{B}_{\zeta 0} \bar{a}^2 / \bar{V}$ sec (x points) and Γ_D versus $\tau_H = 1.4 \times 10^{-1} [\bar{a}(\bar{m}_i \bar{n}_i)^{1/2} / \bar{B}_{\zeta 0}] \mu\text{sec}$ (solid points). The lines are drawn through the PLT points under the assumption that Γ_D scales as the first power of τ_R or τ_H . The agreement between the scaling of Γ_D and the scaling of τ_R or τ_{Hp} is not as good as the agreement in Fig. 7 of the scaling of Γ_D with Γ ; the LT-3 points are particularly out of line. Furthermore, in order to even roughly predict Γ_D , one must multiply either τ_R or τ_H by an unknown factor.

VII. CONCLUSION

We have succeeded in developing an analytic model that describes the nonlinear interaction of tearing modes of different pitch in cylindrical geometry where the equilibrium is characterized by a flat safety factor profile. The analysis shows that the $m = 2/n = 1$ tearing mode can destabilize odd m modes, particularly the $3/2$ mode. The model compares well with our 3-D code with respect to the time evolution of the $0/0$, $1/1$, $2/1$, $3/2$, and $5/3$ modes. Scaling rules are obtained for the position and location (in time) of the maximum or peak in the $3/2$ growth rate. The width of the peak, as predicted by the model, correlates well with the observed time scale for the major disruption in tokamaks.

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FIGURE CAPTIONS

Fig. 1. The safety factor profiles used in the runs described in Table I.

Fig. 2. Comparison of γ_{32} and γ_{53} of the 0/0 current obtained from the 2-D code with γ_{32} and γ_{53} from the 3-D code.

Fig. 3. a) Eigenfunctions ψ_{32} and ψ_{21} as a function of r (from Δ' code). b) Comparison of ψ_{11} from the 3-D code and from Eq. (30). c) Comparison of ψ_{00} from the 3-D code and from Eq. (75).

Fig. 4. Comparison of γ_{11} and γ_{53} with $\gamma_{21} + \gamma_{32}$ for a) Run 1 and b) Run 4.

Fig. 5. a) Comparison of γ_{00} and $\gamma_{21}^0 + \gamma_{21}$ from Run 4. b) Comparison of γ_{42} and $2\gamma_{21}$ from Run 4. c) Comparison of γ_{00} from Run 4 with Eq. (58).

Fig. 6. Model growth rates γ_{21} [Eq. (56)] and γ_{32} [Eq. (60)] compared with the 3-D code results for a) Run 1 and b) Run 4.

Fig. 7. Comparison of Γ with the observed disruption time Γ_D .

Fig. 8. Comparison of the scaling of the observed disruption time Γ_D with τ_R and τ_H .

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

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Fig. 8.

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