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ESTIMATING THE PERFORMANCE OF NEUTRON TIME-OF-FLIGHT SPECTROMETERS

by

George S. Stanford

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ANL-6448
Physics
(TID-4500, 16th Ed.,
Amended)
AEC Research and
Development Report

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois

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NEUTRON TIME-OF-FLIGHT SPECTROMETERS

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Reactor Engineering Division

November 1961

Operated by The University of Chicago
under
Contract W-31-109-eng-38

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ABSTRACT

Analogy with optical instruments is used to develop formulas for estimating the counting rate achieved by a neutron time-of-flight system. Input data consist of the flux at the spectrometer entrance and some of the physical characteristics of the spectrometer. Convenient relationships for the comparison of different systems are presented.

I. INTRODUCTION

There are three main types of neutron time-of-flight spectrometer: the mechanical chopper with a steady-state source of neutrons (such as a reactor), the pulsed assembly with no mechanical chopping, and the pulsed assembly with a mechanical chopper. When faced with a problem involving the use of a neutron spectrometer, it is sometimes necessary to decide which of the three methods will give the required answer in the shortest time. This paper has as its goal the establishment of criteria which will permit a rapid estimate of the counting rate from rather elementary considerations, whereby speed is gained, although at some expense in accuracy. In most cases the approximations are good enough that better than order-of-magnitude results are obtained, in an area where even this high a precision is not always achieved without considerable effort.

When comparing different systems, it will be implicit that the systems have the same energy-resolution requirements.

The symbols used are listed below; they will not all be defined in the text.

List of Symbols

a_e	Area of aperture at spectrometer entrance, if only one slit is used at a time. If n slits are open at once, then a_e is the area the entrance aperture would have if all slits but one were blocked off.
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a_x	Area of aperture at spectrometer exit, if only one slit is used at a time. If n slits are open at once, then a_x is the area the exit aperture would have if all slits but one were blocked off.
$c(\tau/\tau_c)$	Cutoff function. For a chopperless pulsed source, we use the definition $c(\tau/\tau_c) \equiv 1$.
d	Time required for flux in pulsed assembly to become insignificant (for example, to reach 1% of initial value).
D	Diameter of rotor.
E	Energy of neutron.
E_c	Energy of slowest neutron which is accepted by the spectrometer.
$J_\omega(\tau); J_\omega(\tau, t)$	Instantaneous current density per steradian per unit reciprocal speed at spectrometer entrance, in direction of axis of spectrometer.
$\bar{J}(\tau)$	<p>(a) Pulsed source with or without chopper: long-time average, before mechanical chopping, of current $J_\omega(\tau, t)$ entering spectrometer.</p> <p>(b) Chopper with steady-state source: long-time average of that portion of $J_\omega(\tau)$ which flows when chopper is open, i.e., $\bar{J}(\tau) \equiv rwJ_\omega(\tau)$.</p>
$K(\tau) \equiv \bar{J}(\tau)/Q$	The attenuation factor relating $\bar{J}(\tau)$ and Q . $K(\tau)$ is characteristic of the particular test assembly.
ℓ	Distance from entrance aperture to exit aperture.
ℓ_f	Length of flight path.
n	Number of slits used simultaneously by the spectrometer.
N	Number of spectrometer runs required to cover specified energy region.
Q	<p>(a) Pulsed source with or without chopper: average number of neutrons per second entering test assembly.</p> <p>(b) Chopper with steady-state source: that portion of the number of neutrons per second entering test assembly which enter while chopper is open, i.e., $Q \equiv rw$ times total rate of entry of neutrons into test assembly.</p>

r	Number of bursts per unit time. For a given chopper, the product rw is a constant.
$R(\tau)$	Counting rate per unit reciprocal speed at reciprocal speed τ .
$R_c(\tau)$	Counting rate per channel at reciprocal speed τ .
s	Width of narrowest part of chopper slit.
S	"Specific area" of chopper, defined by Eq. (4).
t	Time variable.
T	Total running time for measuring a complete spectrum.
U_i	Total number of counts required for adequate statistics in time channel centered at τ_i .
w	Duration (full width at half-maximum) of burst of neutrons which enter drift path of spectrometer.
$\beta(\tau)$	Spectrometer constant, independent of neutron source and test assembly, defined by Eq. (15).
δ	Duration of channel in time analyzer.
$\epsilon(\tau)$	Efficiency of detector at energy corresponding to reciprocal speed τ .
ν	Number of bursts per revolution of chopper rotor.
τ	Reciprocal speed of neutron.
τ_c	Reciprocal speed of cutoff neutron, i.e., of neutron with energy E_c .
τ_i	Value of τ for the i th spectrometer run for the time channel in which the statistics are least adequate.
$\phi(E)$	Neutron flux per unit energy.

II. GENERAL OPTICAL CONSIDERATIONS

In a neutron time-of-flight spectrometer, the beam must be defined in space, as well as in time. It is usually possible to identify, exactly or approximately, two apertures which determine the spatial extent of the beam, and we will refer to these as the entrance aperture and the exit aperture, respectively. They correspond to the aperture stop and field stop of geometrical optics, and are defined operationally as follows: If a neutron passes through both apertures, it will strike the detector (neglecting any scattering along the way), and any neutron which strikes the detector has passed through both apertures. (The preceding definition is for the ideal, and often realized, case. It sometimes happens, however, that there is an obstacle along the way which removes some of the neutrons which otherwise would have passed through the apertures and reached the detector; this type of obstruction is often referred to as "vignetting.")

Figure 1 shows some aspects which are common to all neutron spectrometers. (For cross-section measurements, the sample is typically placed at or near the entrance aperture.) The relationships to be derived presuppose a knowledge of the areas and locations of the entrance and exit apertures. If they are not immediately obvious, they can generally be discovered with a little ray-tracing.

FIG. 1
GENERALIZED SPATIAL ASPECTS OF A NEUTRON SPECTROMETER

In any case, the actual or approximate values of the areas a_e and a_x of the entrance and exit apertures can probably be determined, along with the distance l between them. If $R(E)$ is the counting rate per unit energy at the detector, it follows from the geometry of Fig. 1 that

$$R(E) \propto n a_e a_x l^{-2} J_\omega(E) \quad , \quad (1a)$$

where n is the number of slits which are open at once, and $J_\omega(E)$ is the current density per unit energy per steradian at the spectrometer source in the direction of the spectrometer axis. In the special case of an isotropic neutron flux $\phi(E)$ at the spectrometer source,

$$J_\omega(E) = \frac{1}{4\pi} \phi(E) \quad . \quad (2)$$

In discussing spectra as measured by time of flight, it is more convenient to think in terms of the reciprocal speed τ (time of flight per unit distance: proportional to the deBroglie wavelength λ) than in terms of the neutron energy E . Consequently, the expression (1a) will not be used directly, but rather

$$R(\tau) \propto n a_e a_x \ell^{-2} J_\omega(\tau) \quad . \quad (1b)$$

III. CHOPPERS WITH STEADY-STATE SOURCES

We will specify that where a chopper is involved, the entrance and exit apertures are to be calculated for the case of a stationary rotor with one set of slits parallel to the axis of the spectrometer; the effect of rotation of the rotor will be taken into account later by means of the cutoff function. When the rotor slits are narrower at one end than at the other, it is likely that the narrowest part of the slit constitutes the entrance aperture, and the detector the exit aperture. The general case of the slit which is the same width at both ends is less predictable. However, if the detector is narrow with respect to the neutron beam,⁽¹⁾ it is probably a good approximation to assume that the detector constitutes the exit aperture, and that the entrance aperture has the dimensions of the ends of the slit, with correction if necessary for neutron leakage through the slit edges, and that it is located midway between the ends of the slit. If the detector and source are wider than the neutron beam, the ends of the slit are the defining apertures, and $\ell = D$.

A mechanical chopper passes bursts of neutrons at a rate r and with half-width (full width at half-maximum) w . For any specific chopper, w is inversely proportional to r , so that rw is a constant for the chopper: it is the fraction of time that the spectrometer accepts neutrons from the source. In general, $rw = \theta \nu s / \pi D$: if the rotor slit is of the diverging, "one-ended" variety, then $\theta = 1$. If the slit is the same size at both ends, the value of θ usually depends on the width of the detector (or of the source, if the latter has a smaller angular width). For a narrow detector,⁽¹⁾ $\theta = \frac{1}{2}$.

Every chopper has a cutoff function $c(\tau/\tau_c)$ (sometimes called the "transmission function"). All cutoff functions approach zero more or less gradually as the energy approaches the cutoff energy E_c , and equal zero for $E \leq E_c$ (i.e., for $\tau \geq \tau_c$). The exact form of $c(\tau/\tau_c)$ depends on the slit design.

Making use of Eq. (1b), it can be seen that the counting rate per unit reciprocal speed is given by

$$R(\tau) = [n a_e a_x \ell^{-2} r w] [c(\tau/\tau_c) \epsilon(\tau) J_\omega(\tau)] \quad . \quad (3a)$$

The geometrical factors in the first bracketed term in Eq. (3a) are conveniently combined into a quantity S , which we will call the "specific area" of the chopper,* so that we have

$$\frac{R(\tau)}{J_{\omega}(\tau)} = Sc \left(\frac{\tau}{\tau_c} \right) \epsilon(\tau) \quad , \quad (3b)$$

where

$$S \equiv na_e a_x \ell^{-2} rw \quad . \quad (4)$$

For an order-of-magnitude comparison of two choppers, it is often sufficient to assume the same cutoff function and detector efficiency, so that the relative counting rates are directly indicated by the relative specific areas.

To correspond to equations derived below for the pulsed-source case, we can write

$$rwJ_{\omega}(\tau) = \bar{J}(\tau) = K(\tau)Q \quad , \quad (5)$$

so that Eq. (3a) becomes

$$R(\tau) = na_e a_x \ell^{-2} c(\tau/\tau_c) \epsilon(\tau) K(\tau) Q \quad . \quad (6)$$

It is sometimes necessary to apply corrections to Q , as mentioned in the next section. The form of $K(\tau)$ depends on the test assembly.

Equation (3) gives the counting rate per unit reciprocal speed at reciprocal speed τ . In practice, however, it is usually not this that determines the running time, but rather the counting rate per time channel; the duration δ of the time channel is typically adjusted from run to run to be the same fraction of the burst duration. Since the reciprocal-speed interval represented by a channel of duration δ is δ/ℓ_f , the counting rate in the channel at reciprocal speed τ is given by

$$R_c(\tau) = R(\tau) \delta / \ell_f \quad . \quad (7)$$

Equations (3), (6), and (7) give the counting rate at one energy. When the spectrum over an extended energy interval is to be determined, it is usually necessary to make several runs at different rotor speeds, the actual number of runs per energy decade being dependent upon the resolution requirements and the cutoff function. Therefore, in using any of those equations

*The quantity $nrwc(\tau/\tau_c)$ corresponds closely to the factor F of Ref. (2). The Russian name assigned to the quantity by the authors was rendered as "luminosité" by the translator responsible for the French version. The term "luminosité" has since been used in a similar sense by S. Hautecler.(3)

to estimate the total running time required, it is necessary to apply it separately to each energy interval. The time required for each run will be determined in general by the counting rate in the channel with the poorest statistics. Let τ_i be the reciprocal speed corresponding to the channel which determines the duration of the i th run. If N runs are required to cover the energy region, and since the time for each run is inversely proportional to the counting rate, the total running time T for a chopper measurement is given by

$$T = \sum_{i=1}^N \frac{U_i}{R_c(\tau_i)} = \ell_f \sum_{i=1}^N \frac{U_i}{\delta_i R(\tau_i)} \quad , \quad (8)$$

where U_i is the total number of counts required in the channel at τ_i . It will be seen below that this equation is applicable to all three types of time-of-flight neutron spectrometer.

IV. PULSED ASSEMBLIES WITH NO MECHANICAL CHOPPING

In the case of a pulsed assembly, the neutron flux is not constant in time, but diminishes after each pulse with a die-away time characteristic of the test assembly. It is possible to apply an equation of the form of Eq. (3) [with $c(\tau/\tau_c) \equiv 1$], but w is a rather nebulous quantity, and $J_\omega(\tau)$ is a function of time, better written as $J_\omega(\tau, t)$. If we let $\bar{J}(\tau)$ be the average of $J_\omega(\tau, t)$ over a long time [i.e., $\bar{J}(\tau) \equiv r \int_0^{1/r} J_\omega(\tau, t) dt$], then [remembering Eq. (1b)],

$$R(\tau) = na_e a_x \ell^{-2} \epsilon(\tau) \bar{J}(\tau) \quad . \quad (9)$$

Furthermore, we can see that, for a fixed spectral and spatial distribution of neutrons entering the test assembly,

$$\bar{J}_1(\tau) = K(\tau) Q_1 \quad (9a)$$

and

$$\bar{J}_2(\tau) = K(\tau) Q_2 \quad , \quad (9b)$$

where the subscripts refer to two different spectrometers. For any τ , the factor $K(\tau)$ is a constant characteristic of the test assembly. If Q_1 and Q_2 are not distributed identically in space or energy, it is sometimes necessary to apply a correction to one or the other to reduce them, at least approximately, to the same basis. Then, from Eq. (9), we can write

$$R(\tau) = na_e a_x \ell^{-2} \epsilon(\tau) K(\tau) Q \quad , \quad (11)$$

which is Eq. (6) without the cutoff function. Equation (6) can therefore be used for direct comparison of a chopper spectrometer with a pulsed-source spectrometer, for a measurement at any specified energy.

In a pulsed-source experiment it is usually the assembly rather than the length of the accelerator pulse which determines the duration of the neutron burst. In this case there is no point in using more than one run to cover the entire energy region (unless the number of time channels is limited). If advantageous, however, it is possible to combine the counts from adjacent channels in those energy regions where the resolution requirements permit it. The total running time T is determined by the channel or group of channels with the least-adequate statistics. If this occurs at τ_i , then T is given by Eq. (8), with $N = 1$.

V. PULSED ASSEMBLIES WITH MECHANICAL CHOPPING

Mechanical choppers are used in conjunction with pulsed assemblies when a burst width is required which is shorter than the die-away time in the test assembly. Sometimes it is desired to know the spectrum as a function of time after the injection of neutrons into the test assembly. This is a case which we will not consider further here, since this type of measurement can be made only with the chopper-pulsed-source system, and comparison of different setups reduces to comparison of choppers, as discussed in Section III.

When it is the steady-state neutron spectrum that is desired, it is necessary to vary the chopper timing with respect to the injected neutron pulse, so as to sample the spectrum evenly as the pulse decays. Thus the number of pulses required to give the number of counts at energy E which one pulse would give in the absence of the chopper is approximately d/w .

For the chopper itself, the entrance and exit apertures are determined as in Section III, and Eq. (4) of course applies. For the chopper-pulsed-source combination, the relationship corresponding to Eqs. (6) and (11) is

$$R(\tau) = na_e a_x \ell^{-2} c(\tau/\tau_c) \epsilon(\tau) K(\tau) Q[w/d] \quad , \quad (12)$$

or, in terms of the specific area [Eq. (4)],

$$R(\tau) = S c(\tau/\tau_c) \epsilon(\tau) K(\tau) Q/r d \quad . \quad (13)$$

The counting rate per channel and the total running time are given by Eqs. (7) and (8), respectively.

VI. COMPARISON OF NEUTRON TIME-OF-FLIGHT SPECTROMETERS

Equation (12) applies to all three types of time-of-flight systems, provided that we make the following two definitions:

- (1) $w/d \equiv 1$ for a chopper with a steady-state source and for a chopperless pulsed source;
- (2) $c(\tau/\tau_c) \equiv 1$ for a chopperless pulsed source.

If we collect in the function $\beta(\tau)$ those factors on the right side of Eq. (12) which refer to the spectrometer itself rather than to the neutron source or test assembly, Eq. (12) becomes

$$R(\tau) = \beta(\tau)K(\tau)Q[w/d] \quad , \quad (14)$$

where

$$\beta(\tau) \equiv na_e a_x \ell^{-2} c(\tau/\tau_c) \epsilon(\tau) \quad . \quad (15)$$

Thus for any two spectrometer systems, the ratio of counting rates at any specific energy is given by

$$\frac{R_1(\tau)}{R_2(\tau)} = \frac{\beta_1(\tau)}{\beta_2(\tau)} \frac{Q_1}{Q_2} \frac{K_1(\tau)}{K_2(\tau)} \frac{(w/d)_1}{(w/d)_2} \quad . \quad (16)$$

We have also seen that the total running time is given by Eq. (8), in all cases.

When two choppers for use with the same primary source (pulsed or otherwise) and test assembly are to be compared, Eq. (3b) gives

$$\frac{R_1(\tau)}{R_2(\tau)} = \frac{S_1 c_1(\tau/\tau_{c1}) \epsilon_1(\tau)}{S_2 c_2(\tau/\tau_{c2}) \epsilon_2(\tau)} \quad . \quad (17)$$

To the extent that the cutoff functions and detector efficiencies are similar, this reduces to

$$\frac{R_1(\tau)}{R_2(\tau)} = \frac{S_1}{S_2} \quad . \quad (18)$$

If, moreover, the resolutions are similar, so that the same energy interval per run can be covered in each case, the relative total running times are given by

$$\frac{T_1}{T_2} = \frac{S_2}{S_1} \quad . \quad (19)$$

SUMMARY

Some of the aspects of the three principal types of neutron time-of-flight spectrometer have been considered, insofar as they influence the counting rate achieved at the detector. On the basis of certain characteristics of the system, it is possible to estimate the counting rate, and hence the running time required, for any given problem for which such a spectrometer is at all suitable.

For estimating the counting rate at any energy E (corresponding to reciprocal speed τ), Eq. (12) can be used in all cases. In addition, Eq. (3) is convenient when the system at hand is a chopper-reactor combination, and Eq. (13) is suitable for the chopper-pulsed-source system. The total running time is given in all three cases by Eq. (8), which in the case of a pulsed source with no chopper is proportional to the reciprocal of Eq. (12). In general, it is necessary to have some idea of the energy spectrum at the spectrometer source, as well as the flux, in order to make these estimates.

For comparing the performance of time-of-flight spectrometers, the most convenient equation depends on the pair being compared. Suitable relations for each purpose are listed in Table I. The first half of each column indicates equations for comparison at a single energy, and the second half designates the ones for comparing total running times for extended energy intervals. It will be observed that when pairs of the same type of spectrometer are being compared, the same equation often serves for comparing both counting rate and total running time. When there are similarities in the experimental arrangements, for instance when similar neutron sources or test assemblies are to be used, quantities such as those related to the flux or to the moderating properties of the test assembly will often cancel in the course of the comparison.

Table I

EQUATION NUMBERS FOR COMPARING
TIME-OF-FLIGHT SPECTROMETERS

	Chopper		Pulsed Source		Chopper plus Pulsed Source	
	$R(\tau)^a$	T	$R(\tau)^a$	T	$R(\tau)^a$	T
Chopper	3 16 18 ^b	8 16 ^b 19 ^b	16	8	16	8
Pulsed Source	-	-	16	8 16	16	8
Chopper plus Pulsed Source	-	-	-	-	16 18	8 16 ^b 19 ^b

^a $R(\tau)$ is the counting rate per unit reciprocal speed. The counting rate per channel is given by $R_c(\tau) = R(\tau)\delta/\ell_f$.

^b Accurate only under certain conditions.

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