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ISPO TASK C.35

**CALCULATION OF PARAMETERS FOR INSPECTION PLANNING
AND EVALUATION—MIXED-OXIDE FUEL FABRICATION FACILITIES**

AUGUST 1982

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**PROGRAM FOR
TECHNICAL ASSISTANCE
TO IAEA SAFEGUARDS**

Department of Energy
Safeguards & Security

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PACIFIC NORTHWEST LABORATORY
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CALCULATION OF PARAMETERS FOR
INSPECTION PLANNING AND EVALUATION -
MIXED-OXIDE FUEL FABRICATION FACILITIES

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SUMMARY

As part of Task C.35 (Calculation of Parameters for Inspection Planning and Evaluation) of the U.S. Program of Technical Assistance to IAEA Safeguards, Pacific Northwest Laboratory has performed some quantitative analyses of IAEA inspection activities for mixed-oxide fuel fabrication facilities.

There were four distinct efforts involved in this task. These were as follows:

- show the effect on a material balance verification of using two variables measurement methods in some strata
- perform additional calculations for the reference facility described in STR-89
- modify the INSPECT⁽²⁾ computer programs to be used as an after-inspection analysis tool, as well as a preinspection planning tool
- provide written comments and explanations of text and graphs of the first draft of STR-89, Safeguards Considerations for Mixed-Oxide Fuel Element Fabrication Facilities, by W. Bahm, T. Shea, and D. Tolchenkov, System Studies Section, IAEA.

Section 2.0 analyzes the effect on a material balance verification of using two variables measurement methods in some strata. It describes the measurement models used by the inspector and operator, shows how to find the variances of the difference statistic for the different measurement methods, and provides numerical examples to illustrate the effect of using two methods. The analysis showed there is little to be gained by combining information from different measurement methods for the purpose of detecting bias as can be the case when destructive and non-destructive techniques are used in some strata. Non-destructive techniques still play a key role in gross defect detection, however.

Section 3.0 deals with additional calculations for the reference MOX facility described in STR-89. Material strata which contribute most heavily to material variances (hence non-detection probability) were identified. These were PuO_2 Powder in receipts and MOX Fuel Rods in Shipments. The measurement capability was varied in these strata, and the values of key inspection parameters noted. If state-of-the-art measurement techniques are employed, inspection effectiveness is greatly enhanced. The better measurement techniques do not reduce the total effort of the inspection substantially, as only about 20% of the total effort is the sampling plan.

Section 4.0 describes modifications made to the INSPECT computer programs so that it can be used as an after inspection analysis tool. For this purpose, the program POST was written. POST calculates an achieved probability of detection using input sample sizes based on the methodology described in Part F of the IAEA Safeguards Technical Manual. POST was used in Section 3.0 for studying the effects of limiting the number of variables samples taken during an inspection. The complete program listing is included in Appendix C.

Another program was written to help make the INSPECT package easier to use. This is the PLANT program which provides a listing of the stratum description information, and the measurement method relative standard deviations. PLANT provides the user of the INSPECT package a method of checking the necessary inputs to the remainder of the package for completeness and accuracy. The program listing for PLANT is included in Appendix B.

Section 4.0 also includes descriptions of further modifications which would increase the versatility of the INSPECT package. These additions and modifications are primarily in the areas of statistical methodology, calculation procedural changes, and input-output clarity.

Appendix A contains the written comments and explanations of text and graphs presented in STR-89. The INSPECT package was extensively used in preparing the report. Often there were what appeared to be surprising results. Further analysis showed the results to be completely consistent with the methodology on which the programs are based.

Appendix D contains a discussion of current measurement methodology utilized in mixed-oxide facilities. This information is based on recent developments in measurement technology.

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1.0 INTRODUCTION

The United States Program of Technical Assistance to IAEA Safeguards, managed by the International Safeguards Project Office (ISPO), includes several tasks in the general area of inspection effort modeling. The first of these was Task C.5, Estimation of Inspection Effort for Chosen Inspection Procedures. The objective of Task C.5 was to develop and demonstrate a method which provides a systematic way of determining inspection effort and safeguards effectiveness as a function of safeguards goals, facility models, and safeguards approach. This task produced two documents which describe the method used, and computer program package, INSPECT, which is useful in determining safeguards effort and effectiveness.^(2,3)

The next task was C.19, Model for Analysis of the Impact of Safeguards Criteria.⁽⁴⁾ The objective of Task C.19 was to apply the method developed in C.5 to analyze the effect on inspection effort and safeguards effectiveness of varying safeguards goals and procedures. A set of input parameters to INSPECT was systematically varied using an experimental design (Box-Behnken design). Results were analyzed using graphs, response surfaces and regression analysis.

The IAEA found the INSPECT programs to be very useful for studies of material accountancy, and is in the process of applying them to comprehensive calculations related to mixed-oxide fuel element fabrication, and low-enriched uranium conversion and fuel element fabrication. A part of this activity is to be accomplished under ISPO Task C.35, Calculation of Parameters for Inspection Planning and Evaluation.

Already completed under Task C.35 was a study of low-enriched uranium conversion and fuel-element fabrication facilities.⁽⁵⁾ The two primary objectives of that study were as follows: 1) to quantify certain aspects of the effectiveness of possible IAEA inspection approaches at LEU conversion and fuel fabrication facilities; and 2) investigate the effect of large plant inventories (up to 80% of throughput) on the detection probability and the corresponding inspection effort. The inspection approaches included

verification of fuel assemblies, verification of fuel rods, verification of pellets at the rod loading station, or some combination of the three approaches.

The portion of Task C.35 which deals primarily with mixed-oxide (MOX) fuel fabrication facilities evolved over time to four distinct categories. These are as follows:

- show the effect on a material balance verification of using two variables measurement methods in some strata
- perform additional calculations for the reference facility described in STR-89
- modify the INSPECT⁽²⁾ computer programs to be used as an after-inspection analysis tool, as well as a preinspection planning tool
- provide written comments and explanations of text and graphs of the first draft of STR-89, Safeguards Considerations for Mixed-Oxide Fuel Element Fabrication Facilities, by W. Bahm, T. Shea, and D. Tolchenkov, System Studies Section, IAEA.

The structure of this report is to treat each category in a separate section. Section 2.0 contains the analysis of using two variables measurement methods in some strata. This is of interest because in practice inspectors often perform quantitative non-destructive assay (NDA) measurements and also take some samples for destructive analysis. Intuitively, it seems reasonable that an evaluation method incorporating information from both methods may be superior to a method based on only one of the two. Section 3.0 describes the additional calculations performed on the reference MOX facility used in STR-89. The section starts with a complete facility description, then identifies material strata which cause the largest measurement variances, and examines ways of improving inspection effectiveness. Section 4.0 describes changes that have been made to the INSPECT computer package, and outlines further developmental changes that could make the package even more powerful. The commentary provided on STR-89 is included in Appendix A.

2.0 THE EFFECT ON A MATERIAL BALANCE VERIFICATION OF USING TWO VARIABLES MEASUREMENT METHODS IN SOME STRATA

The methods described for determining the variance of the difference statistic ($\hat{D}$) in the IAEA Safeguards Technical Manual, Part F: Statistical Concepts and Techniques⁽⁶⁾ only allow for one variables measurement method by the inspector per material stratum. Often the situation occurs where an inspector will perform a quantitative NDA measurement as well as take some samples for destructive analysis in some strata. Hence, there are often two types of variables measurement data available for analysis for certain strata. It would be reasonable to expect that an evaluation method incorporating data from both measurement techniques may be superior to a method based on only one technique.

The objectives of this section can be listed as follows.

- Consider an extension of Part F to consider a more general inspector variables measurement plan;
- Show how to analyze data from an inspection when two variables verification measurement methods are used in some strata; and
- Appraise the incremental benefit of using information from both measurement methods rather than only one.

To explore these objectives, the section is broken into different parts. The first portion lays out the measurement models for the operator and inspector. The second portion shows how the variance is calculated, with a final portion which is a numerical example.

2.1 MEASUREMENT MODELS FOR OPERATOR AND INSPECTOR

The measurement models for a single measurement are presented here. This model is then generalized for a complete stratum. It is assumed that the sample sizes are given, and the problem is to evaluate the effectiveness of the inspection. It is also assumed that the operator uses one weighing method, one sampling method, and one analytical method. The inspector will have two measurement methods; one will typically be NDA, and the other will involve weighing, sampling, and chemical analysis.

The model for the operator's measurements will be as follows:

$$X_i = \mu_i(1 + \delta_0 + \epsilon_{0i}) \quad (1)$$

where X_i = operator's recorded value (i.e., element weight) for item i
 μ_i = true value for item i
 δ_0 = operator's relative systematic error variable
 ϵ_{0i} = operator's relative random error variable for item i

The recorded value for an item, X_i , is a function of the true value for the item, μ_i , and the systematic and random error variables involved.

The model for the inspector is similar except there is an equation for each measurement method. As before, the recorded value is a function of the true value and the random and systematic error variables.

$$Y_{1i} = \mu_i(1 + \delta_1 + \epsilon_{1i}) \quad (2)$$

$$Y_{2i} = \mu_i(1 + \delta_2 + \epsilon_{2i}) \quad (3)$$

where Y_{1i} = inspector's value for item i based on measurement method 1
 Y_{2i} = inspector's value for item i based on measurement method 2
 δ_1 = inspector's relative systematic error variable for method 1
 δ_2 = inspector's relative systematic error variable for method 2
 ϵ_{1i} = inspector's relative random error variable for method 1 on item i
 ϵ_{2i} = inspector's relative random error variable for method 2 on item i

The next step in the analysis is to determine the differences between operator's values and inspector's values. For the two measurement methods, these differences are as follows:

$$d_{1i} = X_i - Y_{1i} \quad (4)$$

$$d_{2i} = X_i - Y_{2i} \quad (5)$$

To find the mean difference for each measurement method, one merely sums the individual differences, and divides by the number of measurements made by the inspector using each method. This is shown as

$$\overline{d_1} = \frac{1}{n_1} \sum_i d_{1i} \quad (6)$$

$$\overline{d_2} = \frac{1}{n_2} \sum_i d_{2i} \quad (7)$$

where $\overline{d_1}$ = mean difference found with method 1

$\overline{d_2}$ = mean difference found with method 2

n_1 = number of measurements made by the
inspector using method 1

n_2 = number of measurements made by the
inspector using method 2

This result can be extrapolated across the entire stratum by multiplying the mean difference by the number of items in the stratum as follows:

$$D_1 = N \overline{d_1} \quad (8)$$

$$D_2 = N \overline{d_2} \quad (9)$$

where N = total number of items in the stratum.

This completes the measurement models for the operator and inspector.

2.2 MEASUREMENT VARIANCES

The next step in the analysis of two measurement methods is to determine the variances of D_1 and D_2 . Using equations (1), (2), (4), (6), and (8) it is possible to work backwards to consider the individual components of D_1 and D_2 . For D_1 , this is as follows:

$$D_1 = \frac{N}{n_1} \sum_i d_{1i} \quad (10)$$

$$D_1 = \frac{N}{n_1} \sum_i (X_i - Y_{1i})$$

$$D_1 = \frac{N}{n_1} \sum_i (\mu_i + \delta_0 \mu_i + \epsilon_{0i} \mu_i - (\mu_i + \delta_1 \mu_i + \epsilon_{1i} \mu_i))$$

$$D_1 = \frac{N}{n_1} \sum_i \mu_i [(\delta_0 - \delta_1) + (\epsilon_{0i} - \epsilon_{1i})]$$

The variance of D_1 is then the variance of the above expression.

$$\text{VAR}(D_1) = \text{VAR} \left[\frac{N}{n_1} \sum_i \mu_i [(\delta_0 - \delta_1) + (\epsilon_{0i} - \epsilon_{1i})] \right]$$

There is no variance associated with the sample sizes, or the number of items in the stratum, and these values may be pulled from the above equation. This gives the following:

$$\text{VAR}(D_1) = \frac{N^2}{n_1^2} \text{VAR} \sum_i \mu_i [(\delta_0 - \delta_1) + (\epsilon_{0i} - \epsilon_{1i})]$$

This can be approximated by breaking the summation into two parts.

$$\text{VAR}(D_1) \approx \frac{N^2}{n_1^2} \text{VAR} \left[(\delta_0 - \delta_1) \sum \mu_i \right] + \frac{N^2}{n_1^2} \text{VAR} \left[\sum \mu_i (\epsilon_{0i} - \epsilon_{1i}) \right]$$

At this point, it is necessary to make substitutions and an assumption. The assumption is that the μ_i are all approximately equal for the items in the stratum. The substitution is the relative and systematic error variances for their associated systematic and random variables; i.e., σ^2 for δ and ϵ . The above equation can then be rewritten as follows:

$$\text{VAR}(D_1) \approx \frac{N^2 n_1^2 \mu^2}{n_1^2} \text{VAR}(\delta_0 - \delta_1) + \frac{N^2}{n_1^2} \sum \mu^2 (\sigma_{r0}^2 + \sigma_{r1}^2)$$

Simplifying gives

$$\text{VAR}(D_1) = N^2 \mu^2 \left(\sigma_{s0}^2 + \sigma_{s1}^2 \right) + \frac{N^2 \mu^2}{n_1} \left(\sigma_{r0}^2 + \sigma_{r1}^2 \right) \quad (11)$$

Similarly,

$$\text{VAR}(D_2) = N^2 \mu^2 \left(\sigma_{s0}^2 + \sigma_{s2}^2 \right) + \frac{N^2 \mu^2}{n_2} \left(\sigma_{r0}^2 + \sigma_{r2}^2 \right) \quad (12)$$

In equations (11) and (12), the following notation was introduced:

σ_{s0}^2 = operator's relative systematic error variance

σ_{s1}^2 = inspector's relative systematic error variance associated
with method 1

σ_{s2}^2 = inspector's relative systematic error variance associated
with method 2

σ_{r0}^2 = operator's relative random error variance

σ_{r1}^2 = inspector's relative random error variance associated with method 1

σ_{r2}^2 = inspector's relative random error variance associated with method 2.

Equations (11) and (12) give the variance equations for the difference statistic needed in evaluating the effects of using two different measurement methods for bias detection. The next section will illustrate the relative size of each variance in a typical case prior to actually combining the information to get a single variance.

2.3 EXAMPLES

It will be fruitful at this point to consider a couple examples which illustrate the relative magnitudes of $\text{VAR}(D_1)$ and $\text{VAR}(D_2)$.

Example 1

For this example, assume the following conditions. The random and systematic portions of the variance are equal, the operator's measurement method is comparable in precision and accuracy to the inspector's method 1. Also assume that method 1 has a standard deviation an order of magnitude more precise and accurate than method 2. This corresponds to method 1 being chemical analysis and method 2 being NDA. This gives the following:

$$\sigma_{s0}^2 = \sigma_{r0}^2 = \sigma_{s1}^2 = \sigma_{r1}^2 = \sigma^2$$

$$\sigma_{s2}^2 = \sigma_{r2}^2 = 100\sigma^2$$

From Equations (11) and (12) the following relations are true:

$$\text{VAR}(D_1) = N^2 u^2 \left[(2\sigma^2) + \frac{1}{n_1} (2\sigma^2) \right]$$

$$\text{VAR}(D_2) = \left[N^2 \mu^2 (101\sigma^2) + \frac{1}{n_2} (101\sigma^2) \right]$$

If the case is considered where one chemical analysis sample is taken, and an infinite number of NDA samples are taken, i.e., $n_1 = 1$ and $n_2 \rightarrow \infty$, then

$$\text{VAR}(D_1) = N^2 \mu^2 (4\sigma^2)$$

$$\text{VAR}(D_2) = N^2 \mu^2 (101\sigma^2).$$

Clearly, only one chemical analysis is better than an infinite number of NDA measurements when one method is an order of magnitude better than the other. This conclusion will fail to hold when the following occurs:

- the systematic errors are negligible relative to the random errors (which is probably not very realistic)
- the inspector's measurement errors are negligible relative to the operator's (in this instance, it may be better to ignore the operator's data altogether).

Example 2

Now consider a numerical example. For this example a stratum consisting of 200 buckets of plutonium oxide powder with each bucket containing 2.2 kilograms of plutonium has the following measurement standard deviations:

	<u>Random</u>	<u>Systematic</u>
Operator	0.0025	0.0025
Inspector (Method 1)	0.0025	0.0025
Inspector (Method 2)	0.02	0.02

$$\begin{aligned}
\text{VAR}(D_1) &= (200 \times 2.2 \text{ kg})^2 (0.0025^2 + 0.0025^2) + \\
&\quad + \frac{1}{n_1} (200 \times 2.2 \text{ kg}) (0.0025^2 + 0.0025^2) \\
\text{VAR}(D_1) &= (440 \text{ kg})^2 (0.0025^2 + 0.0025^2) \left[1 + \frac{1}{n_1} \right] \\
\text{VAR}(D_1) &= 2.42 \left[1 + \frac{1}{n_1} \right] \text{ kg}^2
\end{aligned}$$

From equation (12)

$$\begin{aligned}
\text{VAR}(D_2) &= (440 \text{ kg})^2 (0.02^2 + 0.0025^2) \left[1 + \frac{1}{n_2} \right] \\
\text{VAR}(D_2) &= 78.65 \left[1 + \frac{1}{n_2} \right] \text{ kg}^2
\end{aligned}$$

For the 200 buckets of PuO_2 , if there is one chemical analysis ($n_1=1$) and 199 NDA measurements ($n_2=199$), then the following results are obtained:

$$\text{VAR}(D_1) = 4.84$$

$$\text{VAR}(D_2) = 79.05$$

This calculation shows that a single chemical analysis is far more valuable than 199 NDA measurements for bias detections. It should be remembered that the NDA measurements are still very useful for detecting gross defects because the fraction of the population sampled by this method is much greater.

2.4 COMBINING MEASUREMENT INFORMATION

It is now possible to combine the information of the two measurement methods. A combination of D_1 and D_2 is considered as follows:

$$D = a D_1 + (1-a)D_2$$

where a and $(1-a)$ are multipliers used to ensure D will be unbiased. Using standard calculus techniques (taking derivatives and setting them to zero) it can be shown that the variance of D is minimized when

$$a = \frac{\text{VAR}(D_2) - \text{COV}(D_1, D_2)}{\text{VAR}(D_1) + \text{VAR}(D_2) - 2\text{COV}(D_1, D_2)}$$

The variance of D then takes on the value

$$\text{VAR}(D) = \frac{\text{VAR}(D_1) \times \text{VAR}(D_2) - \text{COV}^2(D_1, D_2)}{\text{VAR}(D_1) + \text{VAR}(D_2) - 2\text{COV}(D_1, D_2)}$$

It is now necessary to find the covariance between D_1 and D_2 . The standard definition of covariance gives

$$\begin{aligned}\text{COV}(D_1, D_2) &= E \left[(D_1 - E(D_1)) (D_2 - E(D_2)) \right] \\ &= E (D_1 \times D_2)\end{aligned}$$

where E is notation for expected value.

In Section 2.2, the following was shown to be true:

$$D_1 = \frac{N}{n_1} \sum_i \mu_i (\delta_{0i} - \delta_{1i}) + (\epsilon_{0i} - \epsilon_{1i})$$

$$D_2 = \frac{N}{n_2} \sum_i \mu_i (\delta_{0i} - \delta_{2i}) + (\epsilon_{0i} - \epsilon_{2i})$$

The expected value of these expressions will be centered on systematic terms. This gives

$$\begin{aligned}
E(D_1 \times D_2) &= E(N\mu(\delta_0 - \delta_1) \times N(\delta_0 - \delta_2)) \\
&= N^2 \mu^2 E(\delta_0^2 + \delta_1 \delta_2 - \delta_0 \delta_1 - \delta_0 \delta_2) \\
&= N^2 \mu^2 \sigma_{\delta_0}^2 \\
\therefore \text{COV}(D_1, D_2) &= N^2 \mu^2 \sigma_{\delta_0}^2
\end{aligned}$$

From the numerical example in Section 2.3

$$\text{VAR}(D) = \frac{4.84 \times 79.04 - [(440 \text{ kg})^2 (0.0025)^2]}{4.84 + 79.04 - 2((440 \text{ kg})^2 (0.0025)^2)}$$

$$\text{VAR}(D) = \frac{4.84 \times 79.04 - (1.21)^2}{4.84 + 79.04 - 2.42} \text{ kg}^2$$

$$\text{VAR}(D) = 4.68 \text{ kg}^2$$

$$\text{Sigma}(D) = \sqrt{\text{VAR}(D)} = 2.16 \text{ kg}$$

The variance and standard deviation obtained from method 1 alone was found to be

$$\text{VAR}(D_1) = 4.84 \text{ kg}^2$$

$$\text{SIGMA}(D_1) = 2.20 \text{ kg}$$

2.5 CONCLUSION

There is little to be gained by combining the information from inspector methods 1 and 2 for the purpose of detecting bias. A reduction of approximately 2% in the standard deviation was achieved when 200 samples were taken (199 NDA and 1 chemical analysis) and the methods have one order of magnitude measurement standard deviations difference. It is to be remembered that the NDA measurements play a key note in gross defect detection, and by no means should be eliminated.

3.0 MIXED-OXIDE FACILITY CALCULATIONS

The mixed-oxide fuel fabrication reference facility is defined by the collection of many parameters used in the INSPECT codes. Among the important physical parameters are plant throughput, material types (strata), batch and item sizes and weights, and inventories. Also defined are operator and inspector measurement capabilities (as relative error standard deviations) necessary to calculate the operator's variance of MUF (material unaccounted for), and the inspector's difference statistic ($\hat{D}$). These statistics play a key role in determining the probability of detection of diversion of a significant quantity by material accounting techniques.

This section is organized by first describing the strata which comprise the reference MOX facility. This includes material amounts, and measurement method error standard deviations. The strata which produce the largest material variances are then isolated. Once these strata are identified, measurement parameters are varied to simulate improved measurement capability in the trouble strata. A sample was run to show state of the art measurement techniques as a threshold case.

3.1 STRATA DESCRIPTIONS

The strata descriptions are summarized using the computer program PLANT, and appear in Tables 3.1 and 3.2. Table 3.1 gives the index notation used in the descriptions for Table 3.2. Table 3.3 summarizes the relative measurement error standard deviations used in STR-89. Sampling measurement error standard deviations were assumed to be included in the analytical measurement method, and between-laboratories components were assumed to be zero.

The reference facility has five flow strata, two as receipts, and three as shipments. The receipt strata are containers of PuO_2 powder with 2.205 kilograms of Pu per container, and recycled fabrication scrap fed into the blending vessel containing 2.646 kilograms of Pu. Shipment strata consist of finished fuel rods, fabrication scrap not recycled, and waste. Each fuel rod is an item and contains 0.053 kilograms Pu, scrap contains 2.646 kilograms of Pu, and waste drums contains 0.025 kilograms Pu.

TABLE 3.1 Notation for STR-89 Mixed-Oxide Reference Facility Description

NOTATION:

X(IJKQPT)	= AVERAGE ITEM WEIGHT
QOP	= OPERATOR BULK MEASUREMENT METHOD INDEX
QIN	= INSPECTOR BULK MEASUREMENT METHOD INDEX
POP	= OPERATOR MATERIAL SAMPLING METHOD INDEX
PIN	= INSPECTOR MATERIAL SAMPLING METHOD INDEX
TOP	= OPERATOR ANALYTICAL MEASUREMENT METHOD INDEX
TIN	= INSPECTOR ANALYTICAL MEASUREMENT METHOD INDEX
M(K)	= NUMBER OF BATCHES
N(K)	= NUMBER OF ITEMS PER BATCH
R(K)	= NUMBER OF SAMPLES PER BATCH (OPERATOR)
C(K)	= NUMBER OF ANALYSES PER SAMPLE (OPERATOR)
U(K)	= NUMBER OF BATCHES MEASURED BY INSPECTOR
W(K)	= NUMBER OF SAMPLES PER BATCH FOR WHICH INSPECTOR MAKES BULK MEASUREMENTS
V(K)	= NUMBER OF SAMPLES PER BATCH FOR WHICH INSPECTOR MAKES ELEMENT DETERMINATION
A(K)	= NUMBER OF ANALYSES PER SAMPLED ITEM (INSPECTOR)
B(K)	= NUMBER OF LABS USED BY INSPECTOR IN THE STRATUM

The reference facility also contains ten inventory strata, five beginning and five ending. To simulate a facility in steady state operation, beginning and ending inventory sizes are the same. The strata consist of PuO_2 powder from the calcination process step, mixed-oxide (MOX) powder from blending operations, fuel pellet stacks after sintering, grinding, and drying, fuel rods at the pellet loading station, and scrap awaiting recovery. The amount of material in each of these strata is shown in Table 3.2.

3.2 IDENTIFYING KEY MATERIAL STRATA

To identify key material strata, those which contribute most heavily to the MUF and $\hat{D}$ material variances, each stratum was given a separate measurement index. This simulates completely independent measurement techniques for each strata. STR-89 lumped all inspector's measurements for the 15 strata into two indices, thus losing the ability to identify individual strata contributions. Having the common indices also leads to non-zero covariances between strata, and smaller total variances.⁽⁶⁾ The smaller variances will yield larger detection probabilities. The independent measurements achieved by assigning separate indices to each strata, will yield the more conservative results of somewhat higher total variances and smaller detection probabilities. It will allow isolation of the key material strata.

TABLE 3.2. STR-89 Mixed-Oxide Reference Facility Description

BEGINNING INVENTORY																
STRATUM NAME AND INSPECTOR LAB INDEX	X(IJKQPT)	QOP	QIN	POP	PIN	TOP	TIN	M(K)	N(K)	R(K)	C(K)	U(K)	W(K)	V(K)	A(K)	B(K)
PUO2 POWDER BI (1 0 0 0 0 0)	13.230	1	1	1	1	6	11	1	1	3	3	1	1	1	1	1
MOX POWDER BI (1 0 0 0 0 0)	13.230	1	1	1	1	7	12	1	1	3	3	1	1	1	1	1
PELLETS BI (1 0 0 0 0 0)	0.132	2	2	1	1	8	12	30	4	3	3	1	1	1	1	1
FUEL RODS BI (1 0 0 0 0 0)	0.053	3	3	1	1	9	12	11	10	3	3	1	1	1	1	1
SCRAP BI (1 0 0 0 0 0)	3.700	1	1	1	1	10	12	2	1	3	3	1	1	1	1	1
RECEIPTS																
PUO2 POWDER REC (1 0 0 0 0 0)	2.205	1	1	1	1	1	11	100	2	3	3	1	1	1	1	1
SCRAP REC (1 0 0 0 0 0)	2.646	1	1	1	1	2	12	16	1	3	3	1	1	1	1	1
SHIPMENTS																
FUEL RODS SH (1 0 0 0 0 0)	0.053	3	3	1	1	3	12	817	10	3	3	1	1	1	1	1
SCRAP SH (1 0 0 0 0 0)	2.646	1	1	1	1	4	12	16	1	3	3	1	1	1	1	1
WASTE SH (1 0 0 0 0 0)	0.025	3	3	1	1	5	5	16	20	1	3	1	1	1	1	1
ENDING INVENTORY																
PUO2 POWDER EI (1 0 0 0 0 0)	13.230	1	1	1	1	6	11	1	1	3	3	1	1	1	1	1
MOX POWDER EI (1 0 0 0 0 0)	13.230	1	1	1	1	7	12	1	1	3	3	1	1	1	1	1
PELLETS EI (1 0 0 0 0 0)	0.132	2	2	1	1	8	12	30	4	3	3	1	1	1	1	1
FUEL RODS EI (1 0 0 0 0 0)	0.053	3	3	1	1	9	12	11	10	3	3	1	1	1	1	1
SCRAP EI (1 0 0 0 0 0)	3.700	1	1	1	1	10	12	2	1	3	3	1	1	1	1	1

TABLE 3.3. STR-89 Measurement Error Relative Standard Deviations for the Reference Facility Description

BULK MEASUREMENT ERROR DATA

BULK MEASUREMENT METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.000800	0.000300
2	0.000200	0.000100
3	0.000000	0.000000

ANALYTICAL MEASUREMENT ERROR DATA

ANALYTICAL METHOD INDEX	RANDOM ERROR RELATIVE STANDARD DEVIATION	SYSTEMATIC ERROR RELATIVE STANDARD DEVIATION
1	0.002000	0.002000
2	0.002000	0.002000
3	0.002000	0.002000
4	0.002000	0.002000
5	0.100000	0.100000
6	0.002000	0.002000
7	0.002000	0.002000
8	0.002000	0.002000
9	0.002000	0.002000
10	0.002000	0.002000
11	0.010000	0.010000
12	0.020000	0.020000

For the reference MOX facility with independent indices, all material strata which contained PuO_2 had random and systematic error variances set at 1%. Material strata which contained MOX had random and systematic error variances set at 2%. This conforms to the values used in STR-89. Table 3.4 summarizes the results. Two material strata dominate variances, PuO_2 Powder from receipts, and MOX Fuel Rods from shipments. The reason for this is that these strata contain about 88% of the total plutonium materials in the facility. Improved detection probabilities will primarily depend on measurement improvements for these strata.

The measurement scheme shown in Table 3.4 when run through the INSPECT package led to the values shown in Table 3.5 for key inspection parameters. A comparison to STR-89 is also included to illustrate the effects of independent measurement indices for this case.

TABLE 3.4. Material Variances for Independent Inspector Measurements for the Reference MOX Facility

Strata	Relative Error Standard Deviations		Material Type	Material Variance (kg ²)
	Random	Systematic		
<u>Inventories</u> (a)				
PuO ₂ Powder	.01	.01	PuO ₂	0.035
MOX Powder	.02	.02	MOX	0.140
Fuel Pellets	.02	.02	MOX	0.201
Fuel Rods	.02	.02	MOX	0.027
Scrap	.02	.02	MOX	0.044
<u>Receipts</u>				
PuO ₂ Powder	.01	.01	PuO ₂	19.488
Scrap	.02	.02	MOX	0.717
<u>Shipments</u>				
Fuel rods	.02	.02	MOX	74.999
Scrap	.02	.02	MOX	0.717
Waste	.02	.02	MOX	0.026

(a) The material variance for inventories represents the sum of beginning and ending inventories which were identical.

Examination of Table 3.5 shows the slightly conservative effect of using independent measurement indices for each material strata by producing slightly higher variances and a somewhat lower probability of detection. The advantage gained is to isolate those material strata which contribute most heavily to the material variances.

3.3 EFFECTS OF VARYING MEASUREMENT CAPABILITY IN SOME STRATA

The measurement capability in PuO₂ Powder (Receipts) and MOX Fuel Rods (Shipments) was altered to investigate the effect this would have on key inspection parameters. Cases were run using INSPECT where the systematic error standard deviation for PuO₂ Powder (Receipts) was varied from 0.2% to 0.3%, and the systematic measurement error standard deviation for MOX Fuel

TABLE 3.5. Values of Key Inspection Parameters

	Independent Measurement Indices	STR-89
Variance of MUF (kg^2)	2.202	2.194
Variance of $\hat{D}$ (kg^2)	98.573	97.295
Probability of Detection (%)	21.2	21.4
Attribute Sample Size	318	308
Variable Sample Size	79	75

Rods (Shipments) was varied from 0.3% to 0.7%. For all other strata, measurement capability was held constant at 2% and 0.2%, random and systematic, respectively, for PuO_2 materials, and 3% and 0.3%, random and systematic, respectively, for MOX materials. These strata will not effect the key inspection parameters significantly as they do not contain large amounts of plutonium.

Table 3.6 summarizes the results from these INSPECT cases for key inspection parameters. Improved measurement capability produced low variances of $\hat{D}$, and resulted in higher detection probabilities. This is shown in Figure 3.1 for the case where PuO_2 Powder has a systematic error variance of 0.2%, and the goal quantity is 8.0 kgs. Similar curves would exist for the other case where PuO_2 Powder has a systematic error variance of 0.3% and a goal quantity of 9.3 kgs. Another relationship to examine is that between measurement capability with probability of detection and sample sizes. As shown in Figure 3.2, the higher detection probabilities require substantial inspection sampling effort. The curve of variables sample size has a similar slope to that of probability of detection, hence they decrease at about the same rate. The tradeoff between these factors is manpower related, both by what is made available at the IAEA, and what is allowed by the facility.

3.4 IMPROVED MEASUREMENT CAPABILITY

A recent study⁽⁷⁾ performed by Battelle Pacific Northwest Laboratories examined currently available measurement techniques used in mixed-oxidized facilities in the United States. Frequently, these measurement techniques had

TABLE 3.6. Summary of Key Inspection Parameter Values for Changing Measurement Capability

Inspector Measurement Capability (RSD)				Goal Quantity (kg)	Amount Taken As Large Defects (kg)	Variance of MUF (kg ²)	Variance of $\hat{D}$ (Systematic) (kg ²)	Probability of Detection (%)	Attribute Sample Size	Variable Sample Size
PuO ₂ Powder (Receipts)		MOX Fuel Rod (Shipments)								
Random	Systematic	Random	Systematic							
0.02	0.002	0.03	0.003	8.0	3.9	2.202	4.728	91.5	318	685
				9.3	4.7			92.0	282	446
0.02	0.002	0.03	0.004	8.0	2.9	2.202	6.041	89.2	318	659
				9.3	4.6			91.3	282	596
0.02	0.002	0.03	0.005	8.0	1.3	2.202	7.728	84.7	318	546
				9.3	3.3			88.9	282	546
0.02	0.002	0.03	0.006	8.0	0.0	2.202	9.790	76.8	318	429
				9.3	1.4			84.6	282	429
0.02	0.002	0.03	0.007	8.0	0.0	2.202	12.228	67.0	318	265
				9.3	0.0			76.1	282	265
0.02	0.003	0.03	0.003	8.0	3.2	2.202	5.700	89.9	318	686
				9.3	4.7			91.5	282	559
0.02	0.003	0.03	0.004	8.0	1.9	2.202	7.013	86.7	318	586
				9.3	3.9			90.1	282	586
0.02	0.003	0.03	0.005	8.0	0.3	2.202	8.700	81.4	318	498
				9.3	2.5			87.1	282	498
0.02	0.003	0.03	0.006	8.0	0.0	2.202	10.763	71.5	318	298
				9.3	0.0			80.4	282	298
0.02	0.003	0.03	0.007	8.0	0.0	2.202	13.389	63.7	318	237
				9.3	0.0			72.8	282	237

FIGURE 3.1. Effect of Measurement Capability on the Probability of Detection and Variance of $\hat{D}$ for Goal Quantity = 8.0 kg

much better measurement error standard deviations than those used in STR-89. Table 3.7 summarizes these results. Appendix D gives a description of the measurement methods in more detail.

INSPECT cases were run utilizing the values in Table 3.7 where appropriate. It was assumed that the inspector will never measure better than the facility. If the values in Table 3.7 were less than those used in STR-89 for the operator, then the operator's analytical measurement error values were used for the inspector. Two other cases were also run. One doubled the operator analytical measurement errors from STR-89 in all strata for the inspector (except waste which was left at 10%), the other increased the measurement errors by a factor of five. These numbers are shown in Table 3.8. The measurement indices refer to the reference MOX facility with the first 10 corresponding to those in Table 3.2. Indices 11 through 25 correspond to a separate inspector measurement index for each stratum. The key strata are PuO_2 Powder (receipts), index number 16, and MOX Fuel Rods (shipments), index number 18. Waste shipments, index number 20, also effected material variances.

Values of key inspection parameters for these cases are summarized in Table 3.9. As expected, as the variance of $\hat{D}$ got larger, the probability of detection was reduced. The number of variables samples required also dropped as the measurement capability improved. These two results imply that inspectors performing fewer, high-quality measurements can be more effective.

The computer program POST was utilized to investigate how few variables samples need to be performed by the inspector with STR-89 operator times two measurement capability before effectiveness as measured by the probability of detection less than STR-89 operator times five measurement capability is reached. Table 3.10 summarizes the results. The probability of detection drops sharply from the full plan at first, but levels off, before dropping sharply again. This result occurs as the random variance of $\hat{D}$ grows. The systematic variance of $\hat{D}$ is fixed by the measurement capability. Figure 3.3 shows a plot of the results. From the figure, it is seen that about 70 samples of times two measurement capability gives equal effectiveness to the 433 samples of times five measurement capability. Substantial inspection effort at the facility can be saved with improved measurement capability.

FIGURE 3.2. Effect of Measurement Capability on Probability of Detection and Sample Sizes for Goal Quantity = 8.0 kg

TABLE 3.7. Current MOX Measurement Capability

Method	Application	Analytical Measurement Error Relative Standard Deviations	
		Random	Systematic
Amperometric Titrations	PuO ₂ receipts	0.003	0.005
	Pu in MOX powder	0.002	0.001
	Pu in MOX Pellets	0.002	0.001
	Pu in recycled MOX	0.0025	0.001
Coulometric Titrations	Pu scarp	0.0025	0.001 → 0.002
	Pu in MOX	0.002	0.001 → 0.002
NOA γ-Scan	Pu in MOX samples	0.01	0.01
	Pu in scrap	0.02 → 0.05	0.02 → 0.04
	HEPA filters	0.1	0.05
	Waste cartons	0.1	0.05
NOA Neutron Coincidence	PuO ₂ powder	0.03	0.01 → 0.02
	Pu in MOX samples	0.04	0.01 → 0.02
	FFTF fuel pino	0.01	0.003

3.5 INSPECTION EFFORT

The Agency performs different types of inspections. These inspections include routine inspections, ad hoc inspections, and special inspections. This study concern itself only with the routine inspections.

Inspection activities can be grouped into five major categories. These are as follows:

- planning and preparation
- audit
- check and service containment and surveillance devices
- post-inspection and evaluation
- verification of measurement data.

The maximum routine inspection effort (MRIE) is defined as the maximum number of man-days or man-years of inspection per annum allowable for a

TABLE 3.8. Analytical Measurement Error Relative Standard Deviation Data

Analytical Method Index	Material Stratum(a)	Current Technology		Operator Data Times Two		Operator Data Times Five	
		Random	Systematic	Random	Systematic	Random	Systematic
1		0.002	0.002	0.002	0.002	0.002	0.002
2		0.002	0.002	0.002	0.002	0.002	0.002
3		0.002	0.002	0.002	0.002	0.002	0.002
4		0.002	0.002	0.002	0.002	0.002	0.002
5		0.100	0.100	0.100	0.100	0.100	0.100
6		0.002	0.002	0.002	0.002	0.002	0.002
7		0.002	0.002	0.002	0.002	0.002	0.002
8		0.002	0.002	0.002	0.002	0.002	0.002
9		0.002	0.002	0.002	0.002	0.002	0.002
10		0.002	0.002	0.002	0.002	0.002	0.002
11	PuO ₂ Powder, B.I.	0.002	0.002	0.004	0.004	0.010	0.010
12	MOX Powder, B.I.	0.002	0.002	0.004	0.004	0.010	0.010
13	Pellets, B.I.	0.002	0.002	0.004	0.004	0.010	0.010
14	Fuel Rods, B.I.	0.002	0.002	0.020	0.004	0.050	0.010
15	Scrap, B.I.	0.005	0.002	0.010	0.004	0.025	0.010
16	PuO ₂ Powder, R.	0.003	0.002	0.006	0.004	0.015	0.010
17	Scrap, R.	0.005	0.002	0.010	0.004	0.025	0.010
18	Fuel Rods, S.	0.010	0.002	0.020	0.004	0.050	0.010
19	Scrap, S.	0.005	0.002	0.010	0.004	0.025	0.010
20	Waste, S.	0.100	0.100	0.100	0.100	0.100	0.100
21	PuO ₂ Powder, E.I.	0.002	0.002	0.004	0.004	0.010	0.010
22	MOX Powder, E.I.	0.002	0.002	0.004	0.004	0.010	0.010
23	Pellets, E.I.	0.002	0.002	0.004	0.004	0.010	0.010
24	Fuel Rods, E.I.	0.010	0.002	0.020	0.004	0.050	0.019
25	Scrap, E.I.	0.005	0.002	0.010	0.004	0.025	0.010

(a) Indices 1 through 10 are defined in Table 3.2 for the operator. B.I. = beginning inventory, R = receipts, S = shipments, and E.I. = ending inventory.

TABLE 3.9. Values of Key Inspection Parameters for Varying Inspector Measurement Capability

	<u>STR-89</u>	<u>Current Technology</u>	<u>Operator Values Times 2</u>	<u>Operator Values Times 5</u>
Variance of MUF (kg ²)	2.202	2.202	2.202	2.202
Variance of $\hat{D}$ (kg ²)	97.295	4.406	9.048	41.552
Probability of Detection (%)	21.4	92.5	85.5	35.3
Attribute Sample Size	308	318	318	318
Variable Sample Size	75	82	364	433

TABLE 3.10. Effect of Limiting Variables Sample Size on the Probability of Detection (1-Qmax)

<u>Total Variables Sample Size</u>	<u>Systematic Variance of $\hat{D}$ (kg²)</u>	<u>Random Variance of $\hat{D}$ (kg²)</u>	<u>Probability of Detection (%)</u>
364	9.048	0.545	86.2
272	9.048	0.721	56.1
188	9.048	1.052	55.7
102	9.048	2.070	54.2
75	9.048	2.707	41.8
50	9.048	4.500	9.7

facility as provided for in paragraph 80 of INFCIRC/153.⁽⁸⁾ For mixed-oxide fuel fabrication facilities, the value of MRIE in man-days is given as follows:

$$\text{MRIE} = 30 \times \sqrt{E}$$

where

E = inventory or annual throughput (whichever is greater) in effective kilograms.

The IAEA safeguards glossary defines an effective kilogram as a special unit used in safeguarding nuclear material which reflects its strategic value. For

FIGURE 3.3. Inspection Effectiveness for Restricted Sample Sizes Using the POST Computer Program

plutonium, it is just the weight of the plutonium in kilograms. For the reference facility, this is 500 kilograms, hence the MRIE is 671 man-days.

Of the five categories of effort, three involve time spent at facilities. These are audit, check and service containment and surveillance equipment, and verification of measurement data (sampling plan effort). Table 3.11 summarizes the total inspection effort for a mixed-oxide facility for the example where the inspector's measurement capability is twice that of the operator. This led to 364 variables samples (342 for flow verification), and 318 attribute samples (296 for flow verification). In Table 3.11, the "In Field" column is the sum of "At Facility" and "Travel." The "Total" is the sum of "In Field" and "At HQ." The total time spent at the facility in a year is 130 man-days, or 19% of the MRIE, and total effort is 346 man-days.

Planning and preparation includes such activities as equipment checkouts, facility stratification, sample plan calculations, trip arrangements, and personnel packing. Post inspection activities include editing of data, data analysis, and report writing.

3.6 CONCLUSION

Key material strata for the reference facility which contribute most heavily to the MUF and $\hat{D}$ material variances were identified. The two material strata which dominated the variances were PuO_2 Powder from receipts, and MOX Fuel Rods from shipments. The effects of improving measurement capability in these key strata, hence overall detection probability, were studied. The results show that improvements in these two strata will lead to much improved inspection effectiveness. Current state-of-the-art measurement technology as applied to the reference MOX facility was also examined. The improved measurement capability lead to improved effectiveness, and reduced variables sample sizes. The reduced sample sizes will not greatly effect the total effort spent for inspections. Table 3.11 showed that the sampling effort consisted of only 64 man-days from a total of 346 days of effort. The primary advantage to improving measurement capability is the improvement in inspection effectiveness, not effort.

TABLE 3.11. Inspection Effort for the Reference Mixed-Oxide Facility (Man-Days)

Activity	At Facility	Travel	In Field	At HQ	Total
Planning and Preparation	0	30 ^(a)	30	76 ^(b)	106
Audit	38 ^(c)	0	38	0	38
C/S Devices	28 ^(f)	0	28	0	28
Sampling Effort	64 ^(d)	0	64	0	64
Post Inspection	0	30	30	80 ^(e)	110

(a) One inspector, one day of travel, for each of 26 flow verifications, plus two inspectors, each one day of travel, for two inventory verifications.

$$(1 \times 26) + (2 \times 2) = 30 \text{ man-days}$$

(b) One inspector, 2 days, for each of 26 flow verifications, plus two inspectors, 6 man-days each, for two inventory verifications.

$$(2 \times 26) + (2 \times 6 \times 2) = 76 \text{ man-days}$$

(c) One inspector, 1 day, for each of 26 flow verifications, plus two inspectors, 3 days each, for two inventory verifications.

$$(1 \times 26) + (2 \times 3 \times 2) = 38 \text{ man-days}$$

(d) One inspector, 2 days of sampling, for each of 26 flow verifications, plus two inspectors, 3 days of sampling, for two inventory verifications.

$$(2 \times 26) + (2 \times 3 \times 2) = 64 \text{ man-days}$$

(e) One inspector, 2 days of analysis, for each of 26 flow verifications, plus two inspectors, 7 days of analysis, for two inventory verifications.

$$(2 \times 26) + (2 \times 7 \times 2) = 80 \text{ man-days}$$

(f) One inspector, 1 day, for each of 26 flow verifications and two inventory verifications.

$$(1 \times 26) + (1 \times 2) = 28 \text{ man-days}$$

4.D INSPECT COMPUTER CODE DEVELOPMENT

The INSPECT computer program package was developed as a tool to aid in inspection effort modeling for Task C.5, Estimation of Inspection Effort for Chosen Inspection Procedures. The INSPECT programs were found to be useful for studies of material accountancy by the IAEA. The programs were not originally designed to be used by inspectors as an inspection data analysis tool. Some work has been done to improve the usefulness of the INSPECT package as an inspection tool.

Two computer programs have been added to the INSPECT package. One program, PLANT, provides a listing of the stratum description information, and the stratum measurement method relative error standard deviations. The purpose of PLANT is to provide the user of the INSPECT package a method of checking the necessary inputs to the remainder of the package for completeness and accuracy. The other computer program, POST, calculates the achieved probability of detection using input sample sizes. The previous version of the INSPECT package calculated inspection sample sizes as a function of various parameters such as goal quantity, false alarm probability, and goal probability of detection, and can still be used as a planning tool. POST can be used as an a posteriori evaluation tool. Program listings are shown in Appendix B for PLANT and Appendix C for POST.

POST calculates an achieved probability of detection using input sample sizes based on methodology described in Part F of the IAEA Safeguards Technical Manual.⁽⁶⁾ To use the program requires many of the same steps described in ISPO-58, PNL-2559 which first documented the INSPECT package. As in the previous INSPECT package, the following procedure is followed.

- Input data required must be accumulated and organized.
- Data files must be set up on the computer.
- Direct access files must be created from the data files utilizing the programs SAMSIZ and RELVAR from the previous INSPECT package.
- The variance of MUF is calculated using the program PROPER from the previous INSPECT package.

The procedure varies at this point when calculating the random and systematic error variances of $\hat{D}$. As previously, the program DERR is used. However, during the interactive session when the question "Do you want to compute the random component of the variance of $\hat{D}$ using user-specified sample sizes?" is asked, the user should respond with "1 (yes)." The interactive session then continues with the user supplying variables sample sizes for each stratum. The variances of $\hat{D}$ are then calculated. The final step in the procedure is to use POST to determine the a posteriori effectiveness of the inspection. The program interactively prompts the user for necessary input data. This data consists of the following:

- title for the output
- name of the stratum description data file (from SAMSIZ)
- number of the last record for plant components (see ISPO-58)
- goal quantity
- false alarm probability
- variance inflation factor
- variance of MUF (from PROPER)
- systematic and random error variance of $\hat{D}$ (from DERR)
- attribute and variables sample sizes for each stratum
- crossover factor for each stratum.

The output from POST is stored in a file called POST.PRT which was created during the execution of POST. A sample of the output from POST is shown in Table 4.1.

PLANT is also an interactive program where the user must supply information. This information is as follows:

- choose an output option
- enter the number of strata in beginning inventory, receipts, shipments, and ending inventory
- supply stratum description data file name (from SAMSIZ)
- supply measurement error data file names (from RELVAR)
- enter a file name for the internally created output file

TABLE 4.1. Sample Output from the POST Program

Test 75 Samples

Results for the Attribute Tests

NA = Attribute Sample Size
 NV = Variables Sample Size
 Beta = Probability of Non-Detection

The Goal Quantity is 8.000

<u>Stratum Description</u>	<u>Item Weight</u>	<u>Number of Items</u>	<u>Crossover Factor</u>	<u>NA</u>	<u>Beta(NA)</u>	<u>NV</u>	<u>Beta(NV)</u>
<u>Beginning Inventory</u>							
PuO ₂ Powder BI	6.620	2	0.20	1	0.4327	1	0.0152
MOX Powder BI	7.620	2	0.20	1	0.4327	1	0.0152
Pellets BI	0.132	120	0.20	1	0.6022	1	0.0792
Fuel Rods BI	0.053	110	0.20	1	0.2520	1	0.0010
Scrap BI	3.700	2	0.20	1	0.2234	1	0.0006
<u>Additions to Plant</u>							
PuO ₂ Powder REC	2.205	200	0.20	112	0.0509	13	0.2955
Scrap REC	2.646	16	0.20	10	0.0515	2	0.1328
<u>Removals from Plant</u>							
Fuel Rods SH	0.053	8170	0.20	161	0.0496	41	0.0224
Scrap SH	2.646	16	0.20	10	0.0515	2	0.1328
Waste SH	0.025	320	0.20	3	0.0491	7	0.0000
<u>Ending Inventory</u>							
PuO ₂ Powder EI	6.620	2	0.20	1	0.4327	1	0.0152
MOX Powder EI	6.620	2	0.20	1	0.4327	1	0.0152
Pellets EI	0.132	120	0.20	1	0.6022	1	0.0792
Fuel Rods EI	0.053	110	0.10	1	0.2520	1	0.0000
Scrap EI	3.700	2	0.20	1	0.2234	1	0.0006

Summary:

The maximum value of Beta is 0.602.
 The total attribute sample size is 306.
 The total variables sample size is 75.

Test 75 Samples

QMAX Calculation

<u>Goal Quantity</u>	<u>Variance Inflation Factor</u>	<u>Amount Taken via Large Defects</u>	<u>Non-Detection Probability for Large Defects</u>	<u>Non-Detection Probability for Muf-D Strategy</u>	<u>QMAX</u>
8.00	1.00	6.87	0.6471	0.8994	0.5820
8.00	4.00	7.36	0.6270	0.8550	0.5361

<u>Goal Quantity</u>	<u>VAR D(Random)/HO</u>	<u>VAR D/HO</u>	<u>VAR D(Random)/H1</u>	<u>VAR D/H1</u>	<u>VAR MUF-D/HO</u>	<u>VAR MUF-D/H1</u>
8.00	2.707	11.755	10.828	19.876	9.553	17.674

VAR MUF = 2.2020
 VAR D(Systematic) = 9.0480
 The False Alarm Probability = 0.050

- supply the number of entries in each of the four measurement error files corresponding to the number of measurement methods
- enter a title for the output.

A sample PLANT output is shown in Section 3.0 for the mixed-oxide base case from STR-89.

The INSPECT package could be made more versatile with further additions and modifications. The most significant additions would be in the areas of statistical methodology, calculation procedural changes, and input-output clarity. Brief descriptions of these changes follow.

A primary change in the input data structure would have the data be more consistent with inspection working papers and reports. This would require accepting stratum weights, number of items in a stratum, and the number of batches in a stratum as the input elements. Another change in the input structure would be to convert certain input parameters to REAL data type from INTEGER data type. This allows the use of decimal fractions instead of just whole numbers. This is necessary in cases where average numbers (such as the average number of items per batch) are all that is known. This change would also overcome some limitations minicomputers have in representing large integers. A method of streamlining the programs would be to eliminate much of the repetitive interactive inputs by storing certain parameters (such as false alarm probability, probability of non-detection) on the stratum description and measurement error files. The output of the program can also be improved to make key results more readable and understandable. One important aid would be to add graphical outputs. There are many circumstances, a plot of detection probability versus sample sizes or inspection effort, for example, where graphical outputs are useful. Other inspection parameters currently printed out should be better defined to maximize the usefulness to persons with limited expertise in statistics and evaluation methodology.

Several calculational procedural changes could be made to the INSPECT package. Currently, computer software is used to round fractional sample sizes to integer values. Sample sizes should almost always be rounded up to the next integer (software sometimes rounds down), and minimum sample sizes

should be defined (at least one sample per stratum). Once sample sizes have been determined, a routine to generate random numbers for use in choosing samples would be a welcome addition. Such routines are plentiful, and an appropriate one could be incorporated into INSPECT. Proposals have been made to extend existing evaluation techniques. Volume 3 of Part F, for example, contains extensions which are not included in the INSPECT programs. Each extension should be studied to determine its utility in the context of current IAEA inspection planning and evaluation, and useful features should be incorporated into the programs. Such extensions include a calculation of material unaccounted for (MUF) and the difference statistic ($\hat{D}$). The programs currently calculate the error variances of MUF and $\hat{D}$, but not the values themselves. The programs currently calculate systematic error variances for each measurement method when perhaps the evaluation should be done stratum-by-stratum. When the same measurement method is used in several strata, the program does not separate out systematic error variance by stratum, hence the ability to perform statistical tests involving individual strata is lost.

Versatility can be added to INSPECT by incorporating various statistical methodology options. One such option would be to include an absolute measurement error model in addition to the current relative measurement error model. There are applications, particularly for weight measurements, where an absolute error model is preferable. To implement this option it will be necessary to modify the error propagation programs (PROPER, DERR) for both systematic and random errors. For systematic errors it is necessary to determine the net number of items weighed on a particular scale, square this number, and multiply by the associated absolute measurement error variance. For random errors, one determines the net number of items weighed on a particular scale, and multiply by the associated absolute measurement error variance. Volume 3 of Part F also includes procedures for incorporating short-term systematic errors into MUF and $\hat{D}$ statistic calculations. Currently, there is no distinction between short-term and long-term systematic errors. The programs also assume independence of operator and inspector measurement methods, that is, there are no covariances between operator and

inspector measurements. The assumption of independence may not always be true if an inspector utilizes an operator system, such as the same sample lines from an accountability tank. Methods of incorporating covariances are described in Volume 3 of Part F. Another feature (particularly for enrichment plants) which would be of interest would be to propagate errors for isotopic quantities. The programs calculate error variances for element quantities only, such as total uranium or total plutonium. There may be applications where total uranium-235 is needed to be known.

The INSPECT package has undergone significant changes since its inception for Task C.5. Two new programs, PLANT and POST, have been added which add clarity and completeness to necessary inputs, and provides an a posteriori evaluation tool. Other changes described in this section would increase the usefulness of the INSPECT package even more.

APPENDIX A

DISCUSSION OF GRAPHS PRESENTED IN STR-89
"SAFEGUARDS CONSIDERATIONS FOR MIXED-OXIDE
FUEL ELEMENT FABRICATION FACILITIES"

APPENDIX A

DISCUSSION OF GRAPHS PRESENTED IN STR-89 SAFEGUARDS CONSIDERATIONS FOR MIXED-OXIDE FUEL ELEMENT FABRICATION FACILITIES

The International Atomic Energy Agency's System Studies Section issued in draft form a Safeguards Technical Report (STR-89)⁽¹⁾ on the application of safeguards at mixed-oxide fuel fabrication facilities. The INSPECT package of computer programs was extensively used in preparing the report. There were often what appeared to be surprising results that needed further explanation.

This appendix includes an analysis of the more involved graphs presented in STR-89. A copy of the graph and a discussion of the results is provided.

A.1 DISCUSSION OF STR-89 FIGURE 2: THE EFFECT OF THE INSPECTOR'S RANDOM AND SYSTEMATIC MEASUREMENT ERRORS

INTRODUCTION

Figure 2 shows the overall probability of detection and the total variables sample size for four levels of inspector measurement errors. Two goal quantities are considered - 8 kg Pu and 9.26 kg Pu.

The assumed measurement standard deviations are in the range of 1% to 5%, indicating the measurement methods in question are nondestructive (NDA).

Probability of Detection

The material balance period on which Figure 2 is based is one year and a throughput of plutonium of 440 kg. The goal quantities (8 kg and 9.26 kg) are about 2% of the throughput. Intuitively, one would expect that with measurement errors in the percent range, it should be difficult to detect diversions in the range of a few percent of throughput. The basic diversion paths are reduced to two types, and the mathematics underlying these paths are explained in Part F of the Safeguards Technical Manual. Gross defects and partial defects are lumped together into a diversion path called "large defects," i.e., defects which can be detected by a single measurement, either of the attributes or variables type. The paths involving bias defects and diversion into MUF are combined into a path called "diversion into MUF- $\hat{D}$." Diversions into the MUF- $\hat{D}$ path can be detected with high probability only when the measurement error standard deviation of MUF- $\hat{D}$ is small compared to the amount diverted. The probabilities of detection shown in Figure 2 confirm this. They are all in the 10% to 30% range. The optimal diversion strategy in this case is clear: the preferred diversion path is via MUF- $\hat{D}$. Diversions via large defects will be relatively less attractive since they can still be detected even if the inspector's measurement errors are in the percent range.

Two other features of the probability curve can be noted. First, the probability of detection decreases as the inspector's measurement errors increase. Second, the smaller the goal quantity, the smaller the probability of detection.

Variables Sample Size

The behavior of variables sample size as a function of the inspector's measurement capability is striking: the sample size is essentially constant. At first glance it appears that the variables sample size is independent of the inspector's measurement capability. This is only true in certain special cases. To understand why, it is useful to review briefly the procedure used in determining variables sample sizes.

From the point of view of IAEA inspections, variables samples can serve several purposes. (See, for example, INFCIRC/153, Paragraphs 70-75.) In the mathematical model on which the INSPECT programs are based, two particular uses of variables samples are defined:

- detection of partial defects
- detection of bias.

To determine variables sample sizes, the following procedure is used:

1. For each stratum, find the sample size needed to detect partial defects. A binomial approximation is used.
2. For the complete material balance, find the total sample size required to detect bias.
3. Allocate this sample size to the various strata.
4. Define a minimum sample size of one sample per stratum.
5. For each stratum, define the actual variables sample size as the largest of a) the sample size for detecting partial defects, b) the sample size for detecting bias, or c) the minimum sample size of one per stratum.

It sometimes happens that in all strata the sample size for detecting partial defects is larger than that for detecting bias. In some cases, the sample size for detecting bias dominates. In still other cases, none of the three sample sizes is dominant in all strata so the actual variables sample size is determined in some strata by the need to detect partial defects, in some strata by the requirement for detecting bias, and in some strata by the requirement that at least one sample should be taken from each stratum.

In Figure 2, it is primarily the need to detect partial defects that governs the behavior of the sample size curve. The reason the curve in Figure 2 is flat is that the sampling plan for detecting partial defects is an attributes sampling plan; it does not depend on the inspector's variables measurement capability.

A look at the formula for calculating these sample sizes will demonstrate this point. In each stratum, the sample size for detecting partial defects is chosen according to the following rule:

$$n_{v1} = N(1 - \beta^{\gamma\bar{x}/M})$$

where

n_{v1} = sample size for detecting partial defects

N = number of items in the stratum

β = goal probability of nondetection

$\bar{x}$ = average weight of an item

M = goal quantity

γ = the crossover point (The crossover point defines the size of a partial defect. The quantity $\gamma\bar{x}$ is the largest falsification that could not be detected by the attributes (gross defects) test instrument. The fraction γ depends on the measurement capability of the attributes tester, but not on the variables measurement capability.)

If one graphed in Figure 2 the variables sample size for detecting bias, one would find that the curve is not flat. The actual behavior of the curve as a function of the inspector's measurement errors is quite complex, for reasons that will be explained in the discussions of Figures 3 and 4.

FIGURE 2, Set 1. Detection Probability for MUF-D Strategy and Number of Variables Samples for Increasing Inspector's Random and Systematic Error

A.2 DISCUSSION OF STR-89 FIGURE 3: THE EFFECT OF THE INSPECTOR'S RANDOM ERRORS

INTRODUCTION

Figure 3 shows the overall probability of detection and the variables sample size as a function of the inspector's random error. The goal quantities are the same as in Figure 2 and the material balance period is one year. The operator's measurement errors are the same as in Figure 2, but the inspector's systematic error standard deviation is now 0.2%. The measurement method in this case is some kind of destructive (chemical) analysis.

Probability of Detection

The probability of detection in Figure 3 is consistently above 90%, whereas in Figure 2 it was always less than 30%. The reason for the difference is that the inspector's systematic error standard deviation is 0.2% instead of 2% to 5% as in Figure 2. Improvements in inspector measurement capability permit improvements in detection probability.

The probability of detection is essentially constant and is fairly close to the goal detection probability of 95%. Over the range of measurement errors considered, the technical objectives for protracted diversion are achievable. The tradeoff is that the variables sample sizes are extremely large in some instances. The goals become increasingly difficult to achieve as the inspector's random errors increase.

Variables Sample Size

In Figure 2, the total variables sample size was in the neighborhood of 70 or 80. This sample size was determined primarily by the requirement for detecting partial defects. In Figure 3, the sample sizes are quite a bit larger. Since the sample size for detecting partial defects is not dependent on the inspector's measurement errors, the increased variables sample sizes in Figure 3 must be attributable to the requirement for detecting bias.

As the random error increases, a larger number of samples is needed to achieve a fixed detection goal. A more precise explanation of the behavior of the curve requires some discussion of how variables sample sizes for detecting bias are determined.

Although the mathematical details are somewhat complicated, the basic idea is fairly simple. To detect bias, the difference statistic, $\hat{D}$, which measures the difference between the operator's and the inspector's variables measurements is used. The ability to detect bias of a given magnitude depends on the measurement error variance of the $\hat{D}$ statistic. If the variance is large the probability of detection will be low; if the variance is small the probability of detection will be large. Generally speaking, there exists a value of the variance of $\hat{D}$ at which the actual probability of detection will be equal to the goal probability of detection. This value of the variance can be referred to as the target value for the variance of $\hat{D}$.

To determine a variables sample size for detecting bias, the first step is to determine the target value for the variance of $\hat{D}$. Once this target variance is known, one attempts to find an inspection sampling plan that is consistent with it. The variance of $\hat{D}$ consists of a random component, which is a decreasing function of the inspector's sample size, and a systematic component, which is independent of the inspector's sample size. As the inspector's sample size becomes very large, the random component of the variance of $\hat{D}$ approaches zero. If the systematic component is not too large, then it is possible to achieve the target value for the variance of $\hat{D}$ by appropriately choosing the sample size. If the systematic component of the variance is too large, then the goal probability of detection will be unachievable no matter how large the sample size. In this case, an arbitrary "point of diminishing returns" is invoked to determine the sample size. Any further increase of the sample size beyond this point is regarded as not cost-effective.

Figure 3 shows that as the inspector's random errors increase, more and more measurements are needed to achieve a fixed goal. On the right side of the graph, the sample sizes become extremely large - as many as 3000 samples - and it is questionable whether it is cost-effective to take so many samples. At this point, the inspection planner should consider in more detail the tradeoff between probability of detection and sample size and select the actual variables sample sizes on the basis of cost-effectiveness considerations. More details on the sample selection procedure are contained in Chapter 8, Part F, IAEA Safeguards Technical Manual.

FIGURE 3, Set 2: Detection Probability for MUF-D Strategy and Number of Variables Samples for Increasing Inspector's Random Error

A.3 DISCUSSION OF FIGURE 4: THE EFFECT OF THE INSPECTOR'S SYSTEMATIC MEASUREMENT ERRORS

INTRODUCTION

Figure 4 shows the overall probability of detection and the total variables sample size as a function of the inspector's systematic measurement errors. The goal quantities and material balance period are the same as in Figure 2 and 3. The operator's measurement errors are also unchanged. The inspector's systematic errors range from 0.2% to 1.5% indicating the measurement methods in question could be NDA in some cases, and chemical analysis in others.

Probability of Detection

The general trend of the probability of detection as a function of inspector's systematic errors is easily seen and understood: as the systematic error standard deviation increases, the probability of detection tends to decrease. This behavior was also noticed in Figure 2. However, it should be noted that there are really two regions in the graph, even though there are not enough points to show them very clearly. In one region, the design goals for the inspection are achievable, and the probability of detection is essentially flat. In the other region, the design goals for the inspection are not achievable and the overall probability of detection begins to decline as the systematic errors get larger. The precise transition point between the two regions cannot be determined from examining the graph, but clearly it lies somewhere between the first and the second set of points. The curve should look something like this:

Variables Sample Size

The behavior of the variables sample size as a function of the inspector's systematic error requires explanation. The curve has two distinct regions, as did the curve showing probability of detection. In one region, the design goals of the inspection are achievable and the variables sample size increases as the inspector's systematic errors increase. In the other region, the design goals are not attainable and the sample size decreases as the inspector's systematic errors increase.

The reason for the increasing sample sizes in the first region, is not hard to find. In order to attain the design goals for the inspection, a certain target value for the variance of $\hat{D}$ must be achieved. As stated in the previous section, the variance of $\hat{D}$ consists of a systematic component, which is independent of the inspector's sample size but which does depend strongly on the inspector's systematic measurement errors, and a random component, which depends strongly on sample size but is independent of systematic measurement error. If the sum of the two components is to be held equal to the target value, then any increase in the systematic component must be compensated by a decrease in the random component. Conversely, any decrease in the systematic component can be counterbalanced by an increase in the random component. Because of the inverse relationship between the random error variance of $\hat{D}$ and the sample size, an increase in the inspector's systematic measurement errors is translated into an increase in the sample size, so long as the target value of the variance is achievable (i.e., the design goals for the inspection are achievable). This accounts for the shape of the sample size curve in the left hand region of the graph.

The behavior of the curve in the right hand region is more difficult to explain. It involves the "point of diminishing returns" mentioned in the previous section. When the systematic error standard deviation of the $\hat{D}$ statistic is large in relation to the goal quantity, then the design goals for the inspection will be unattainable no matter how large the sample size. In this situation the sample size selection procedure uses a cut-off value - a point of diminishing returns - to keep the sample size from increasing without bound. This cut-off point is expressed in terms of the ratio of the random

variance component of $\hat{D}$ to the systematic variance component. Sample sizes are chosen so that this ratio is no smaller than a specified value. When the design goals are unattainable then the ratio of the random variance of $\hat{D}$ to the systematic variance of $\hat{D}$ is held constant.

If the systematic measurement error variance of $\hat{D}$ increases and the ratio is to remain constant, the random variance of $\hat{D}$ must also increase. Because of the inverse relationship between sample size and random error variance, this increase in the random variance translates into a decrease in sample size. Thus, increasing systematic measurement errors lead to a decrease in variables sample sizes for detecting bias.

FIGURE 4, Set 3: Detection Probability for MUF-D Strategy and Number of Variables Samples for Increasing Inspector's Systematic Error

A.4 DISCUSSION OF STR-89 FIGURE 5: THE EFFECT OF INCREASING GOAL QUANTITIES

INTRODUCTION

Figure 5 shows the overall probability of detection and the total variables sample size as a function of goal quantity. The operator's measurement errors and the material balance period are the same as in the preceding figures. The inspector's measurement errors are in the percent range indicating the measurement methods are nondestructive (NDA). Two levels of inspector measurement capability are considered. The goal quantities range from 8 kg Pu to 38 kg.

The behavior of the probability curves is not surprising. As the goal quantity increases, so does the probability of detection. Also, the better the inspector's measurement capability (i.e., the smaller the measurement error standard deviations), the higher the detection probability.

Both of the probability curves are good examples of what statisticians call power curves (power refers to power of detection).

Variables Sample Size

The behavior of the sample size curve is complicated. There are four distinct regions. In order to characterize these regions in a simple way, the following nomenclature is useful:

n_{v1} = variables sample size for detection of partial defects

n_{v2} = variables sample size for detection of bias

n_{vmin} = variables sample size based on a minimum sample size of one per stratum

n_v = actual variables sample size.

The first region of the curve on the far left is downward sloping. In this region, n_{v1} is the dominant influence on n_v . The shape of the curve depends primarily on the requirement for detection of partial defects. To see why the curve is downward sloping, consider the binomial sampling formula used to calculate n_{v1} :

$$n_{v1} = N(1 - \beta^{\gamma \bar{x}/M}),$$

where

n_{v1} = sample size for detection of partial defects

N = number of items in the stratum

β = goal probability of nondetection

M = goal quantity

$\bar{x}$ = average item weight

γ = crossover point, i.e., point above which a defect would be detectable with a simple attribute check; typically, $\gamma = 0.2$ to 0.3 (20% to 30% of the item weight). In effect, $\gamma \bar{x}$ defines the size of a partial defect.

From this formula, when M increases, n_{v1} decreases. In other words, the larger the goal quantity, the larger the number of defects required to accomplish the diversion and thus the smaller the sample size needed to detect at least one defect.

The next region in the curve of variables sample size is flat. Here n_{v2} is the dominant influence; the n_{v1} value has dropped below that of n_{v2} . The reason the curve is flat is that the design goals of the inspection are not attainable and the point of diminishing returns determines the sample size. Since this point does not depend on goal quantity, n_{v2} is independent of changes in goal quantity over this region.

The third region of the curve is once again downward sloping, though this behavior only appears in the curve that corresponds to small inspector measurement errors. For the case of high inspector measurement errors, the flat region extends beyond the range of goal quantities considered in this graph. In this region, n_{v2} is still dominating, but now the design goals for the inspection are attainable (or nearly so). The point of diminishing returns no longer plays a role and sample sizes once again vary as a function

of goal quantity. The slope of the curve is what one would expect - larger goal quantities do not require as large a sample size as small goal quantities.

The last region is on the right hand side of the graph. (Again, it only appears in the curve that corresponds to small inspector measurement errors.) The variables sample size begins to level off once again. The reason is that n_{vmin} is beginning to play the dominant role. Recall that the sample size selection procedure included the constraint that at least one variables sample would be taken in each stratum, irrespective of any other consideration. Since the reference facility included fifteen strata, the variables sample size must begin to level off as it drops down toward fifteen. No matter how large the goal quantity, at least 15 samples will still be taken.

FIGURE 5, Set 4: Detection Probability and Number of Variables Samples for Increasing Goal Quantities (one year)

A.5 DISCUSSION OF STR-89 FIGURE 6: THE EFFECT OF GOAL QUANTITY (TWO WEEK MATERIAL BALANCE)

INTRODUCTION

Figure 6 shows the total detection probability, the detection probability for the MUF- $\hat{D}$ strategy (this quantity will be explained below) and the variables sample size as a function of goal quantity for a two-week material balance. The inspector's measurement errors are in the percent range indicating NDA techniques are being used.

Probability of Detection

The behavior of the probability of detection as a function of goal quantity is easy to explain. On the left side of the graph, the goal quantities are too small to be detected with high probability. On the right side, the goal quantities are large enough so that the goal detection probability of 95% is attainable.

Detection Probability for the MUF- $\hat{D}$ Strategy

This curve by itself is not too meaningful, but in conjunction with the total probability of detection, it does reveal something about the diverter's optimal strategy.

Recall that the diverter has two basic diversion paths, the large defects path and the MUF- $\hat{D}$ path. In many cases, the diverter's best strategy is to use both paths together, taking a fraction of the goal quantity through large defects and the remainder through MUF- $\hat{D}$.

When the diverter employs such a mixed strategy, the probability of detection is calculated in the following way:

$$\begin{aligned}\text{Probability of detection} &= 1 - \text{probability of nondetection} \\ &= 1 - Q_{LD}(G) \times Q_{\text{MUF-}\hat{D}}(M-G)\end{aligned}$$

where

$Q_{LD}(G)$ = Probability of failing to raise an alarm on the large defects tests, given that the amount G has been diverted via large defects

$Q_{MUF-\hat{D}}(M-G)$ = probability of failing to raise an alarm on the MUF- $\hat{D}$ test, given that an amount $M-G$ has been diverted via the MUF- $\hat{D}$ path.

The total amount of material diverted is M , the goal quantity. This diversion has been partitioned across the two basic paths, the amount G being diverted via large defects, the remainder, $M-G$, via the MUF- $\hat{D}$ path. The total nondetection probability is the product of the nondetection probabilities for the two basic diversion paths. This implies that detection fails to occur only when neither of the two basic diversion tests raise an alarm.

Since G is a variable that can range from 0 to M , the probability of detection is indeterminate unless G is somehow fixed. The approach used in the INSPECT programs is to use the value of G that minimizes the total probability of detection.

The behavior of the curve showing the detection probability for the MUF- $\hat{D}$ strategy reflects the fact that the optimal strategy for the diverter changes as goal quantity increases. As the goal quantity increases, the MUF- $\hat{D}$ diversion path becomes relatively less attractive while the large defects path becomes relatively more attractive. The decreasing attractiveness of the MUF- $\hat{D}$ path is responsible for the downward slope of the detection probability for the MUF- $\hat{D}$ strategy.

Number of Variables Samples

The variables sample size shows the same behavior here as in the previous section. As the goal quantity increases, the variables sample size decreases until the lower bound of one sample per stratum is approached.

A.19

FIGURE 6, Set 5: Detection Probability for Different Goal Quantities (two weeks)

Fig. 1. Dependence of the rate of polymerization on the concentration of the initiator.

R_p — rate of polymerization;
 $[I]$ — concentration of the initiator.

1 — rate of polymerization;
 2 — concentration of the initiator.

APPENDIX B

LISTING OF THE PLANT
COMPUTER PROGRAM

```

PROGRAM PLANT
  DIMENSION LAB(6),STNM(5),DQ(2),DP(2),DT(2),TITLE(18)
  1,TITLE1(5,4) ,NUM(8),FILNM1(9),FILNM2(9),FILNM3(9)
  2,FILNM4(9),FILNM7(9),FILNM8(9)
  INTEGER*4 IND(21)
  DATA TITLE1/4HBEGI,4HNNIN,4HG IN,4HVENT,4HORY ,4H ,4H RE,
  14HCEIP,4HTS ,4H ,4H ,4H SH,4HIPME,4HNTS ,4H ,4HENDI,
  24HNG I,4HNVEN,4HTORY,4H /
C
C   ENTER THE NECESSARY DATA
C
  TYPE 119
119  FORMAT(' ENTER OUTPUT REQUIREMENTS',/10X,'1=STRATA TABLES ONLY',
1/10X,'2=ERROR DATA TABLES ONLY',/10X,'3=STRATA AND ERROR TABLES')
  ACCEPT*, ITABLE
  IF(ITABLE.EQ.2) GO TO 500
  TYPE 100
100  FORMAT(' ENTER THE NUMBER OF BEGINNING INVENTORY STRATA')
  ACCEPT*, NBIS
  TYPE 101
101  FORMAT(' ENTER NUMBER OF RECEIPTS STRATA')
  ACCEPT*,NRCT
  TYPE 102
102  FORMAT(' ENTER NUMBER OF SHIPMENTS STRATA')
  ACCEPT*,NSHIP
  TYPE 103
103  FORMAT(' ENTER NUMBER OF ENDING INVENTORY STRATA')
  ACCEPT*, NEIS
  TYPE 112
112  FORMAT(' ENTER DIRECT ACCESS DATA FILES AS REQUESTED')
  TYPE 104
104  FORMAT(' ENTER FILE NAME CONTAINING STRATUM DATA'/ ' >'$)
  READ(5,200) FILNM1
  IF(ITABLE.EQ.1) GO TO 510
  500 TYPE 105
105  FORMAT(' ENTER FILE NAME CONTAINING BULK MEASUREMENT ERROR'/ ' >'$)
  READ(5,200) FILNM2
  TYPE 106
106  FORMAT(' ENTER FILE NAME OF MATERIAL SAMPLING ERROR DATA'/ ' >'$)
  READ(5,200) FILNM3
  TYPE 107
107  FORMAT(' ENTER FILE NAME OF ANALYTICAL MEASUREMENT ERROR DATA'
1/' >'$)
  READ(5,200) FILNM4
  TYPE 108
108  FORMAT(' ENTER FILE NAME OF BETWEEN LABS ERROR DATA'/ ' >'$)
  READ(5,200) FILNM7
  200  FORMAT(9A4)
  510 TYPE 109
C

```

ENTER THE NECESSARY DATA

```
109 FORMAT(' ENTER FILE NAME OUTPUT IS TO BE PRINTED ON'/' >'$)
    READ(5,200) FILNM8
    IF(ITABLE.EQ.1) GO TO 511
    TYPE 113
113 FORMAT(' ENTER NUMBER OF ENTRIES IN EACH DATA FILE IN THE FOL
LOWING ORDER: BULK MEASUREMENT,SAMPLING,ANALYTICAL,LAB')
    ACCEPT*,NBM,NMS,NAS,NLB
511 TYPE 110
110 FORMAT(' ENTER TITLE FOR THE OUTPUT. USE TWO LINES.'/' STAY
1 BETWEEN THE MARKERS.'/' *,36X,*'/' >'$)
    ACCEPT 300,(TITLE(IC),IC=1,9)
    TYPE 111
111 FORMAT(' >'$)
    ACCEPT 300,(TITLE(IC),IC=10,18)
300 FORMAT(9A4)
    FILNM1(9)=0.
    FILNM2(9)=0.
    FILNM3(9)=0.
    FILNM4(9)=0.
    FILNM7(9)=0.
    FILNM8(9)=0.
    IF(ITABLE.EQ.2) GO TO 501
```

C
C
C

```
    OPEN THE NECESSARY FILES

    OPEN(UNIT=1,NAME=FILNM1,TYPE='OLD',FORM='UNFORMATTED',READONLY,
1ACCESS='DIRECT',RECORDSIZE=30)
    IF(ITABLE.EQ.1) GO TO 512
501 OPEN(UNIT=2,NAME=FILNM2,TYPE='OLD',FORM='UNFORMATTED',READONLY,
1ACCESS='DIRECT',RECORDSIZE=2)
    OPEN(UNIT=3,NAME=FILNM3,TYPE='OLD',FORM='UNFORMATTED',READONLY,
1ACCESS='DIRECT',RECORDSIZE=2)
    OPEN(UNIT=4,NAME=FILNM4,TYPE='OLD',FORM='UNFORMATTED',READONLY,
1ACCESS='DIRECT',RECORDSIZE=2)
    OPEN(UNIT=7,NAME=FILNM7,TYPE='OLD',FORM='UNFORMATTED',READONLY,
1ACCESS='DIRECT',RECORDSIZE=1)
512 OPEN(UNIT=8,NAME=FILNM8,TYPE='NEW',FORM='FORMATTED')
    IF(ITABLE.EQ.2) GO TO 502
```

C
C
C

SET UP LOOP PARAMETERS

```
NUM(1)= 1
NUM(2)= NBIS
NUM(3)= NUM(2)+1
NUM(4)= NUM(2)+NRCT
NUM(5)= NUM(4)+1
NUM(6)= NUM(4)+NSHIP
NUM(7)= NUM(6)+1
NUM(8)= NUM(6)+NEIS
```

C


```

C      PRINT OUT NOTATION TO BE USED
C
      WRITE(8,800)(TITLE(I),I=1,18)
800  FORMAT('1',35X,18A4)
      WRITE(8,801)
801  FORMAT('///' NOTATION: '/')
      WRITE(8,802)
802  FORMAT(' X(IJKQPT)=AVERAGE ITEM WEIGHT'/' QOP      = OPERATOR BULK
1 MEASUREMENT METHOD INDEX'/' QIN      = INSPECTOR BULK MEASURE
2MENT METHOD INDEX'/' POP      = OPERATOR MATERIAL SAMPLING
3 METHOD INDEX'/' PIN      = INSPECTOR MATERIAL SAMPLING METHOD
4 INDEX'/' TOP      = OPERATOR ANALYTICAL MEASUREMENT METHOD
5 INDEX'/' TIN      = INSPECTOR ANALYTICAL MEASUREMENT METHOD
6 INDEX'/' M(K)      = NUMBER OF BATCHES'/' N(K)      = NUMBER OF
7 ITEMS PER BATCH'/' R(K)      = NUMBER OF SAMPLES PER BATCH (OP
8ERATOR)'/' C(K)      = NUMBER OF ANALYSES PER SAMPLE (OPERATOR)'/'
9' U(K)      = NUMBER OF BATCHES MEASURED BY INSPECTOR'/' W(K)
1      = NUMBER OF SAMPLES PER BATCH FOR WHICH INSPECTOR MAKES
2 BULK MEASUREMENTS'/' V(K)      = NUMBER OF SAMPLES PER BATCH FOR
3 WHICH INSPECTOR MAKES ELEMENT DETERMINATION'/' A(K)      = NUMBER
4 OF ANALYSES PER SAMPLED ITEM (INSPECTOR)'/' B(K)      = NUMBER OF
5 LABS USED BY INSPECTOR IN THE STRATUM')
      WRITE(8,47)
47  FORMAT('///' ERROR TABLE DATA IS DECIMAL FRACTION, NOT PERCENT')
C
C      START READING IN DATA
C
      DO 50 K=1,4
      WRITE(8,803) (TITLE(I),I=1,18)
803  FORMAT('1',35X,18A4'///')
      WRITE(8,820) (TITLE1(J,K),J=1,5)
      M=2*K-1
      N1= NUM(M)
      N2= NUM(M+1)
820  FORMAT(40X,5A4,/)
      IF(N2.LT.N1) GO TO 51
      WRITE(8,804)
804  FORMAT(' STRATUM NAME AND X(IJKQPT) QOP QIN POP PIN
1 TOP TIN M(K) N(K) R(K) C(K) U(K) W(K)
2 V(K) A(K) B(K)'/' INSPECTOR LAB INDEX')
      DO 10 II=N1,N2
      READ(1'II) STNM,WT,IND,LAB
      WRITE(8,805) STNM,WT,(IND(J),J=1,11,2),(IND(J),J=13,21),LAB
805  FORMAT(1X,5A4,T23,F10.3,3(2X,I3),3(3X,I3),2(3X,I6),
13X,I4,2X,I6,3X,I6,4(3X,I3)'/' (' ,6I3,''))
10  CONTINUE
      GO TO 50
51  WRITE(8,806)
806  FORMAT(35X,' THIS COMPONENT CONTAINS NO STRATA')
50  CONTINUE
      IF(ITABLE.EQ.1) GO TO 515

```

```

C      SET UP ERROR DATA TABLES
C
502 WRITE(8,825) (TITLE(I),I=1,18)
    WRITE(8,47)
825  FORMAT('1',35X,18A4)
    WRITE(8,830)
830  FORMAT('/',37X,'BULK MEASUREMENT ERROR DATA')
    WRITE(8,831)
831  FORMAT(T16,'BULK MEASUREMENT',T46,'RANDOM ERROR',
1T74,'SYSTEMATIC ERROR')
    WRITE(8,832)
832  FORMAT(T21,'METHOD',T48,'RELATIVE',T78,
1'RELATIVE')
    WRITE(8,833)
833  FORMAT(T21,'INDEX',T44,'STANDARD DEVIATION',
1T74,'STANDARD DEVIATION')
    DO 60 I=1,NBM
    READ(2'I) DQ
    WRITE(8,834) I,DQ
834  FORMAT(T22,I3,T47,F10.6,T79,F10.6)
60  CONTINUE
    WRITE(8,825) (TITLE(I),I=1,18)
    WRITE(8,835)
835  FORMAT('/',42X,'SAMPLING ERROR DATA')
    WRITE(8,836)
836  FORMAT(T46,'RANDOM ERROR',T74,'SYSTEM
1ATIC ERROR')
    WRITE(8,837)
837  FORMAT(T18,'SAMPLING',T48,'RELATIVE',T78,
1'RELATIVE')
    WRITE(8,838)
838  FORMAT(T16,'METHOD INDEX',T44,'STANDARD
1 DEVIATION',T74,'STANDARD DEVIATION')
    DO 70 I=1,NMS
    READ(3'I) DP
    WRITE(8,839) I,DP
839  FORMAT(T20,I3,T47,F10.6,T79,F10.6)
70  CONTINUE
    WRITE(8,825) (TITLE(I),I=1,18)
    WRITE(8,840)
840  FORMAT('/',35X,'ANALYTICAL MEASUREMENT ERROR DATA')
    WRITE(8,841)
841  FORMAT(T46,'RANDOM ERROR',T74,'SYSTEM
1ATIC ERROR')
    WRITE(8,842)
842  FORMAT(T17,'ANALYTICAL',T48,'RELATIVE',
1T78,'RELATIVE')
    WRITE(8,843)
843  FORMAT(T16,'METHOD INDEX',T44,'STANDARD
1 DEVIATION',T74,'STANDARD DEVIATION')

```

SET UP ERROR DATA TABLES

```
DO 80 I=1,NAS
  READ(4'I) DT
  WRITE(8,839) I,DT
80  CONTINUE
    WRITE(8,825) (TITLE(I),I=1,18)
    WRITE(8,850)
850  FORMAT(//,32X,'BETWEEN-LABORATORIES ERROR COMPONENTS'/)
    WRITE(8,851)
851  FORMAT(T25,'BETWEEN LABORATORY',T57,'BETWEEN-
1LABORATORIES')
    WRITE(8,852)
852  FORMAT(T27,'COMPONENT INDEX',T60,'ERROR COMPON
1ENT')
    DO 90 I=1,NLB
      READ(7'I) Z
      WRITE(8,853)I,Z
853  FORMAT(T31,I3,T62,F10.6)
    90  CONTINUE
515  STOP
      END
```

 

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APPENDIX C

LISTING OF THE POST
COMPUTER PROGRAM

```

PROGRAM POST
DIMENSION TITLE(18),ICT(4),CMPNM(5,4),STNM(5),W(13),FILNM1(9)
INTEGER*4 IND(21),NA,NV,N,NAT,NVT
DATA CMPNM/4HBEGI,4HNNIN,4HG IN,4HVENT,4HORY ,4HADDI,4HTION,
14HS TO,4H PLA,4HNT ,4HREMO,4HVALS,4H FRO,4HM PL,4HANT ,4HENDI,
24HNG I,4HNVEN,4HTORY,4H /
C
C PART 1: INPUTS
C
      TYPE 630
630  FORMAT('OENTER A TITLE FOR THE OUTPUT.  USE TWO LINES.'/
1' STAY BETWEEN THE ASTERISKS.'/ *',36X,'*'/' >'$)
      ACCEPT 199, (TITLE(IC),IC=1,9)
      TYPE 633
633  FORMAT(' >'$)
      ACCEPT 199, (TITLE(IC),IC=10,18)
C
      TYPE 101
101  FORMAT('OENTER THE NAME OF THE STRATUM DESCRIPTION FILE.'/ >'$)
      READ(5,199) FILNM1
199  FORMAT(9A4)
      FILNM1(9)=0.
C
      DO 149 I=1,4
      TYPE 102, I, (CMPNM(IA,I),IA=1,5)
102  FORMAT('OENTER THE NUMBER OF THE LAST RECORD FOR COMPONENT',
1I2,/16X,5A4/' IF THIS COMPONENT IS NOT PRESENT, ENTER 0/' >'$)
149  ACCEPT *, ICT(I)
C
      TYPE 103
103  FORMAT('OENTER THE GOAL QUANTITY.'/ >'$)
      ACCEPT *, GQ
C
      TYPE 104
104  FORMAT('OENTER THE FALSE ALARM PROBABILITY.'/ >'$)
      ACCEPT *, ALPHA
C
      TYPE 105
105  FORMAT('OENTER THE VARIANCE INFLATION FACTOR.'/ >'$)
      ACCEPT *, F
C
      TYPE 106
106  FORMAT('OENTER THE VARIANCE OF MUF.'/ >'$)
      ACCEPT *, VMUF
C
      TYPE 107
107  FORMAT('OENTER THE SYSTEMATIC ERROR VARIANCE OF D.'/ >'$)
      ACCEPT *, VDST
C
      TYPE 108
108  FORMAT('OENTER THE RANDOM ERROR VARIANCE OF D.'/ >'$)
      ACCEPT *, VDR0
C

```

C PART 1: INPUTS

```

OPEN(UNIT=1,NAME=FILNM1,TYPE='OLD',ACCESS='DIRECT',READONLY,
1FORM='UNFORMATTED',RECORDSIZE=30)
OPEN(UNIT=3,NAME='POST.PRT',TYPE='NEW')

```

C

C

C PART 2: ATTRIBUTE TEST EVALUATION

C

C

```

WRITE(3,200) TITLE !PRINT
WRITE(3,201) !ATTRIBUTES
WRITE(3,202) GQ !HEADING
WRITE(3,203)

```

C

```

BETAMX=0.
NAT=0
NVT=0
KK=0
DO 250 IA=1,4
IF(ICT(IA).EQ.0) GO TO 250
K1=KK+1
K2=ICT(IA)
WRITE(3,204) (CMPNM(IC,IA),IC=1,5) ! WRITE COMPONENT NAME
DO 251 IB=K1,K2
READ(1'IB) STNM,WT,IND
WRITE(5,220) IB, STNM
220 FORMAT('OSTRATUM # ',I3,' : ',5A4)
WRITE(5,221)
221 FORMAT('OENTER THE ATTRIBUTE SAMPLE SIZE.'/ >'$)
ACCEPT *, NA
WRITE(5,222)
222 FORMAT('OENTER THE VARIABLES SAMPLE SIZE.'/ >'$)
ACCEPT *, NV
WRITE(5,223)
223 FORMAT('OENTER THE CROSSOVER FACTOR.'/ >'$)
ACCEPT *, C

```

C

```

R=GQ/WT
N=IND(13)*IND(14)
BETAG=(1.-FLOAT(NA)/FLOAT(N))*R ! BETA FOR GROSS DEFECTS
R=R/C
BETAP=(1.-FLOAT(NV)/FLOAT(N))*R ! BETA FOR PARTIAL DEFECTS
BETAMX=AMAX1(BETAMX,BETAG,BETAP)
NAT=NAT+NA !ACCUMULATE ATTRIBUTE SAMPLE SIZE
NVT=NVT+NV !ACCUMULATE VARIABLES SAMPLE SIZE
WRITE(3,210) STNM,WT,N,C,NA,BETAG,NV,BETAP
251 CONTINUE
KK=ICT(IA)
250 CONTINUE
WRITE(3,211) BETAMX,NAT,NVT

```

C

C FORMAT STATEMENTS FOR PART 2

C

```

200 FORMAT(1H1,35X,18A4/1H0)
201 FORMAT(1H , 'RESULTS FOR THE ATTRIBUTE TESTS' /
11H0, 'NA=ATTRIBUTE SAMPLE SIZE' /
21H , 'NV=VARIABLES SAMPLE SIZE' /
31H , 'BETA=PROBABILITY OF NON-DETECTION')
202 FORMAT(1H0, 'THE GOAL QUANTITY IS ', F12.3)
203 FORMAT(1H0, 'STRATUM DESCRIPTION ', 5X, 'ITEM', 4X, 'NUMBER', 2X,
1 'CROSSOVER', 8X, 'NA', 2X, 'BETA(NA)', 8X, 'NV', 2X, 'BETA(NV)' / 1H ,
224X, 'WEIGHT OF ITEMS FACTOR')
204 FORMAT(1H0, T21, 5A4)
210 FORMAT(1H , 5A4, F10.3, I10, F10.2, 2(I10, F10.4))
211 FORMAT(/ 1H0, 'SUMMARY: ' / 1H , 'THE MAXIMUM VALUE OF BETA IS ', F7.3 /
11H , 'THE TOTAL ATTRIBUTE SAMPLE SIZE IS ', I7 /
21H , 'THE TOTAL VARIABLES SAMPLE SIZE IS ', I7)

```

C

C

C PART 3: QMAX CALCULATION

C

C

```

WRITE(3,300) TITLE !PRINT
WRITE(3,301)      !HEADING
WRITE(3,302)

```

C

```

ZA=GAUPER(ALPHA) !PERCENTAGE POINT OF GAUSSIAN DISTRIBUTION
W(2)=F*VDRO !VAR D(RANDOM)/H1
W(3)=VDRO+VDST!VAR D/H0
W(4)=W(2)+VDST!VAR D/H1
W(5)=W(3)-VMUF!VAR MUF-D/H0

```

C

```

DO 349 I=3,4 !LOOP:I=3-->VARIANCE NOT INFLATED
W(6)=W(I)-VMUF ! I=4-->VARIANCE INFLATED

```

C CALCULATE OPERATOR'S OPTIMAL ALLOCATION OF THE GOAL QUANTITY

```

W(8)=-1.*ALOG(BETAMX)*SQRT(W(6))/GQ !Y
W(9)=ZA*SQRT(W(5)/W(6)) !THETA(MAX)
W(10)=W(9)-GQ/SQRT(W(6)) !THETA(MIN)
IF(W(10).GE.-5.) GO TO 341
W(11)=W(8)*(2.867E-7)-1.486719515E-6
IF(W(11).LE.0.) W(10)=-5.
IF(W(11).LE.0.) GO TO 341
W(7)=-7. !ARBITRARILY CHOSEN THETA VALUE
WRITE(3,351) !ALLOCATION IS NOT OPTIMAL
GO TO 340

```

C AT THIS POINT W(10), I.E., THETA(MIN), IS .GE. -5.

```

341 W(11)=.3989423*EXP(-.5*W(10)**2) !F THETA(MIN)
W(12)=W(11)-W(8)*CDF(W(10)) !F THETA(MIN) - Y*CDF THETA(MIN)
IF(W(12).LE.0.) W(7)=W(10) !THETA(MIN)
IF(W(12).LE.0.) GO TO 340
W(13)=.3989423*EXP(-.5*W(9)**2) !F THETA(MAX)

```


C PART 3: QMAX CALCULATION

```

W(12)=W(13)-W(8)*CDF(W(9))      !F THETA(MAX) - Y*CDF THETA(MAX)
IF(W(12).GT.0.) W(7)=W(9)      !THETA(MAX)
IF(W(12).GT.0.) GO TO 340
CALL THAYTA(W(8),W(10),W(9),W(7))
340 G=MAX(0.,MIN(GQ,GQ+W(7)*SQRT(W(6))-ZA*SQRT(W(5))))
W(8)=BETAMX*(G/GQ)              !Q1:NON-DETECTION PROB FOR LARGE DEFECTS
W(9)=ZA*SQRT(W(5))+G-GQ
W(11)=CDF(W(9)/SQRT(W(6)))      !Q2:NON-DETECTION PROB FOR MUF-D
W(12)=W(8)*W(11)               !QMAX
IF(I.EQ.3) WRITE(3,303) GQ,1.0,G,W(8),W(11),W(12)
IF(I.EQ.4) WRITE(3,303) GQ,F,G,W(8),W(11),W(12)
349 CONTINUE
WRITE(3,304)
WRITE(3,305) GQ,VDR0,W(3),W(2),W(4),W(5),W(6)
WRITE(3,306) VMUF,VDST,ALPHA

C
CLOSE(UNIT=3,DISPOSE='SAVE')

C
C FORMAT STATEMENTS FOR PART 3
C
300 FORMAT(1H1,35X,18A4/1H0)
301 FORMAT('QMAX CALCULATION')
302 FORMAT(1H0,3X,'GOAL',4X,'VARIANCE',3X,'AMOUNT TAKEN NON-DETECTION'
1,'NON-DETECTION',5X,'QMAX'/1H ,
2' QUANTITY INFLATION',6X,'VIA',7X,'PROBABILITY',4X,'PROBABILITY'/
31H ,12X,'FACTOR',3X,'LARGE DEFECTS',6X,'FOR',12X,'FOR',/1H ,
435X,'LARGE DEFECTS MUF-D STRATEGY'//)
303 FORMAT(1H ,F9.2,F10.2,F11.2,F15.4,F15.4,F12.4)
304 FORMAT(///1H0,4X,4HGOAL/12H QUANTITY ,17HVAR D(RANDOM)/H0 ,
19X,9HVAR D/H0 ,18H VAR D(RANDOM)/H1 ,9X,9HVAR D/H1,5X,
213HVAR MUF-D/H0 ,5X,13HVAR MUF-D/H1 )
305 FORMAT(1H ,F10.2,6F18.3)
306 FORMAT(//1H0,'VAR MUF = ',G12.5/1H , 'VAR D(SYSTEMATIC) = ',G12.5/
11H , 'THE FALSE ALARM PROBABILITY = ',F6.3)
351 FORMAT(' THE ALLOCATION OF THE GOAL QUANTITY IS NOT OPTIMAL')

C
C
STOP
END
FUNCTION CDF(X)
C CALCULATION OF CUMULATIVE PROBABILITY FOR THE GAUSSIAN DISTRIBUTION
C REFERENCE: NBS HANDBOOK OF MATHEMATICAL FUNCTIONS P. 932 (26.2.17)
W1=1.0
IF(X.LT.0.) W1=0.0
W2=ABS(X)
T=1./(1+.2316419*W2)
ZX=.3989423*EXP(-.5*W2**2)
W3=T*(T*(T*(T*(T*1.330274429-1.821255978)+1.781477937)-.356563782)

```

C FORMAT STATEMENTS FOR PART 3

```

1 +.319381530)
  CDF=ABS(W1-ZX*W3)
D   PRINT 100, T,ZX,W3,X,CDF
D 100 FORMAT(1X,13(2H<>)/1X,3(F12.4,1X),`CDF(`,F12.4,`= ` ,F12.4)
      RETURN
      END
      FUNCTION GAUPER(BETA)
C   CALCULATION OF THE PERCENTAGE POINT OF 1.-BETA
C   THE INVERSE GAUSSIAN DISTRIBUTION FUNCTION FOR THE UPPER TAIL CONTAINING
C   A PROBABILITY OF BETA
C   REFERENCE NBS HANDBOOK OF MATHEMATICAL FUNCTIONS P. 933 (26.2.23)
      W1=0.
      IF(BETA.GT..5) W1=1.
      W2=ABS(W1-BETA)
      W3=SQRT(-2.*ALOG(W2))
      W4=W3*(W3*.010328+.802853)+2.515517
      W5=W3*(W3*(W3*.001308+.189269)+1.432788)+1.
      GAUPER=W3-W4/W5
      IF(W1.EQ.1.) GAUPER=-1.*GAUPER
      RETURN
      END
      SUBROUTINE THAYTA(Y,X1,X2,T)
100  FORMAT(` THETA DID NOT CONVERGE IN 21 ITERATIONS. `)
C   AT THIS POINT X1 IS A LOWER BOUND AND X2 IS AN UPPER BOUND.
C   USING THE METHOD OF BISECTION TO OBTAIN X
      N=0
2    N=N+1
      X=.5*(X1+X2)
      Y3=.3989423*EXP(-.5*X**2)
      DD=CDF(X)
D    PRINT 700, N,Y,X1,X,X2,DD
D 700 FORMAT(1H ,25(1H+)/` THAYTA *** `,I3,1X,5F12.4)
      IF((Y3.GT.(Y+.0001)*DD.OR.Y3.LT.(Y-.0001)*DD).AND.N.GT.20)
1WRITE(6,100)
      IF((Y3.LT.(Y+.0001)*DD.AND.Y3.GT.(Y-.0001)*DD).OR.N.GT.20)
1GO TO 3
      IF(Y3.GT.Y*DD) X1=X
      IF(Y3.LT.Y*DD) X2=X
      GO TO 2
3    T=X
      RETURN
      END

```

CONFIDENTIAL - SECURITY INFORMATION

1. The purpose of this document is to provide information regarding the activities of the [redacted] and the [redacted] in the [redacted] area. This information is being provided to you for your information only and is not to be distributed outside of your organization.

2. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

3. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

4. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

5. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

6. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

7. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

8. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

9. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

10. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

11. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

12. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area. The [redacted] and the [redacted] are both active in the [redacted] area and are both active in the [redacted] area.

APPENDIX D

CURRENT MIXED-OXIDE MEASUREMENT TECHNOLOGY

APPENDIX D

CURRENT MIXED-OXIDE MEASUREMENT TECHNOLOGY

A recent study⁽⁷⁾ examined current measurement technology in MOX fuel fabrication facilities. Appendix D summarizes the study's findings.

MASS MEASUREMENTS

Good quality commercial scales can be used for all mass measurements of SNM in a process. The capacity of the scale at each weighing station should be just sufficient for weighing the largest mass expected at that station. The load limit recommended by the manufacturer would not be exceeded.

Several readout options are available in commercial scales, digital readout and printing, binary digital readout for direct input to a computer, and manual reading of either digital or analog displays. As an alternative to an all-automatic system, manual digital reading could be used for production control weighings and a printing scale for the pellet stack scale in pin loading.

The gross weighing of a can or tray of process material and the tare weighing, if required, are normally made on the same scale (before and after filling or emptying). Although that entails the use of an over-capacity scale for the tare weighings, resulting in less than optimum weighing precision, it is generally impractical to provide more than one scale per station. Cleanup powder and scrap produced in a unit operation are also weighed on the scale provided for the main flow material, i.e., feed or product. Tare weights of reuseable containers and pellet trays may be premeasured and used instead of independent tare weights being determined at each weighing station. The tare values are written or stamped on the container.

Normally, scales are checked daily with reference standards and control charts are maintained. Each calibration period, the quality control statistician calculates the average bias for the period. If the bias is statistically significant, the scale is readjusted to a zero bias, if such an adjustment can be made, or bias corrections are applied to weighing results.

CHEMICAL AND ISOTOPIC ANALYSES OF PROCESS MATERIALS FOR PLUTONIUM AND URANIUM

Methods of analysis that are widely used and accepted in the industry are discussed here. The amperometric and coulometric titration methods are options for plutonium assay in feed and product compounds and high-grade intermediate products. They have about the same capabilities in accuracy and precision. Two basic NDA methods for plutonium are commonly used, a gamma spectrometric method and a neutron coincidence counting method. The first method is applied to the plutonium assay of bulky waste in barrels and cartons and to isotopic analysis of scrap and waste. The second method is applied to all PuO_2 and MOX scrap and intermediate materials and laboratory wastes.

Uranium assays of the UO_2 powder are done by the standard gravimetric method and uranium in MOX is determined by the Davies-Gray titration method.

The surface ionization mass spectrometric method is used to determine the ratios of the plutonium isotopes with the exception of ^{238}Pu , which is determined by alpha spectrometry. The combined data provides a total isotopic analysis. Alpha spectrometry is also used for determining the plutonium concentration of liquid wastes.

The assay precision and accuracy assessment for each of the measurement methods in each principal application or material type was given in Table 3.7. Random error standard deviations indicate the repeatability (precision) of assays and calibration standard deviations indicate the limits of the calibrations and establish bounds to the bias of the method. The assay methods are checked for bias by analyzing two reference standard materials per day and new calibration curves or factors are calculated weekly if the change from the reference value is statistically significant at the 90% level.

NDA DEVICES AND TECHNIQUES

Nondestructive assays (NDA) are used wherever their sensitivity, accuracy and precision are acceptable. The widest usages are in determining the contents of fuel pins and containers of scrap and waste.

Gamma Ray Spectroscopy and Calorimetry

The plutonium isotopic composition and ^{241}Am concentration must be known in order to calculate the total plutonium from calorimetric heat measurement data. The ^{238}Pu , ^{239}Pu , ^{240}Pu , and ^{241}Am can be measured by gamma-ray spectrometry. Since ^{242}Pu decay emits no gamma rays, it must be estimated by isotopic correlation techniques. The plutonium isotopic abundance, ^{241}Am concentration range and typical precision of a four-hour gamma-ray assay of mixed oxide are shown in Table D.1. The typical precision of calorimetric heat measurements is about 0.5%. These measurement uncertainties yield a total uncertainty in the plutonium assay of about 1%.

TABLE D.1. Plutonium Isotopic Abundance and Precision of a 4-Hour Gamma-Ray Assay of Mixed Oxide Material⁽⁹⁾

	^{238}Pu	^{239}Pu	^{240}Pu	^{241}Pu	^{242}Pu	^{241}Pu
Abundance	0.061%	86.58%	11.79%	1.35%	0.20% ^(a)	595 ppm
Precision (% RSD)	3.5%	0.38%	2.8%	0.57%	--	6.3%

(a) ^{242}Pu not measured by gamma-ray spectrometry. Abundance must be estimated by isotopic correlation.

Neutron Coincidence Assay as NDA Technique

The spontaneous-fission neutrons from plutonium are measured by neutron coincidence counters. The fission neutrons come primarily from the even-mass nuclei (^{238}Pu , ^{240}Pu , ^{242}Pu) of plutonium. Table D.2 lists the number of spontaneous-fission neutrons per gram per second for each plutonium isotope. The neutron coincidence count rate can be measured with a standard deviation of about 0.5%. Combining the plutonium isotopic and neutron counting variances yields a total standard deviation in the plutonium assay of about 1%.

Californium-252 Rod Scanning

The assembly MOX fuel rod is measured by a ^{252}Cf rod scanner. Each fuel rod can be assayed individually in less than five minutes by an automated

TABLE D.2. Spontaneous-Fission Rate for Plutonium(10)

	^{238}Pu	^{239}Pu	^{240}Pu	^{241}Pu	^{242}Pu
No. Spontaneous-Fissions (Neutrons/Grams/Second)	2,490	0.022	1,020	0.024	1,730

scanner. The relative standard deviation of the scanner is less than 1% when used to measure fuel rods containing between 30 and 40 grams fissile plutonium.

The ^{252}Cf source and NaI(Tl) detectors are located in a large cylindrical shield. The fuel rod being measured is loaded on a horizontal tray and automatically inserted into the analysis chamber. The results of the assay can be printed on the terminal by the computer control unit.

The Segmented Gamma Scanner as an NDA Device

The segmented gamma scanner is used to measure low density process waste such as paper, plastic, glass or other potentially contaminated items. The items to be assayed are placed in cans. The gamma-ray measurement could also verify plutonium isotopic composition of selected waste items. The uncertainty of the segmented gamma-scanner is a function of the waste matrix, chemical form, particle size of the plutonium, and position of the plutonium in the waste can. The measurement uncertainty can be as large as 50% for "worst case" conditions but relative standard deviations should range from 10% to 20% on waste items that are homogeneous and are similar to the calibration standards used to calibrate the segmented gamma scanner.

The item being measured is placed on a turntable between a collimated transmission gamma-ray source and a Ge detector. Under computer control the gamma-rays from the item and transmission source are measured as the item is rotated and vertically moved past the detector. The results of the assay can be computed by computer and printed.

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