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A MECHANISM EXPLAINING THE
INSTABILITY OF EBR-I, MARK III

by

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AN ANALYSIS OF THE STABILITY OF EBR-I, MARKS I, II AND III, CONCLUSIONS PERTINENT TO THE DESIGN OF FAST REACTORS

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A. Introduction

One of the most significant conclusions resulting from the Mark III (EBR-I) stability studies consists of the fact that there is nothing intrinsic in the mechanical or neutronic features of a fast reactor which would cause it to be unstable.^(1,2) As the result of a very comprehensive series of experiments, it has been demonstrated conclusively that the Mark III loading of EBR-I is characterized by extreme stability under all credible operating conditions. As an immediate consequence of these experiments it follows that the resonant instabilities noted in Mark II (and in Mark I) were the result of certain mechanical features of design. A detailed reinvestigation of structural Mark II feedback effects coupled with information gained in the Mark III tests has led to a credible and acceptable explanation of the Mark II instability.

The first attempt to study the dynamic behavior of the reactor was initiated in May, 1955.⁽³⁾ The results of these studies, in which the transfer function of the reactor was measured under various conditions of power and flow demonstrated that the reactor could be brought into a resonant condition at certain frequencies. Unfortunately, the measurements were relatively crude and attempts to interpret the results in terms of physical feedback processes were unsuccessful.

In November, 1955, a second more comprehensive series of tests was initiated. Again the same oscillatory phenomena were observed. A transient experiment carried out with the main coolant flow stopped demonstrated conclusively the existence of a prompt

positive power coefficient and led to an unintentional partial meltdown of the core.(4,5)

As the result of this incident a large amount of effort was devoted to an analysis of the instability and meltdown results. For the most part earlier thinking was guided by the following well established items of information. Under normal steady state operating conditions the net power coefficient was negative. Reactivity had to be added at constant flow and constant inlet temperature to initiate and to sustain a power increase. However, it seemed clear then, as it does now that the time behavior of the overall power coefficient was dominated by two major components: one positive and prompt and the other negative but larger in magnitude and much more slowly acting. Direct evidence of their existence was provided by the results of flow change tests. Immediately following a reduction in flow the power would increase, pass through a maximum and would eventually decrease to some lower equilibrium value. Following a flow increase the converse behavior was noted. Evidently the sudden increase in fuel and coolant temperature following a flow reduction was sensed by the reactor through the prompt positive component as an addition to reactivity later cancelled and over-ridden by the more slowly acting and larger negative component.

From the oscillator results it was known that the resonance peak shifted towards lower frequencies as the flow rate was reduced. This behavior suggested that the process responsible for the negative component must in some way have been associated with the physical transport time of the coolant through the reactor. The results of the excursion experiment substantiated the validity of this concept since at greatly decreased flows the normally over-riding negative component essentially disappeared and caused the resultant power coefficient to be positive.

With these facts and concepts in mind a large effort was devoted to the interpretation of the dynamic behavior of the reactor in

terms in various feedback mechanisms which, in principle, should have accounted for the existence of the prompt positive and delayed negative power coefficient components.

To explain the resonant behavior Siegel and Hurwitz⁽⁶⁾ postulated a mathematical model based on the coupling of individual power coefficient components, each with its own characteristic time dependence. As an extension of this concept Thalgott⁽⁷⁾ demonstrated that a feedback model based on prompt positive and delayed negative power coefficient components was qualitatively consistent with experimental facts.

In a similar direction Kinchin⁽⁸⁾ formulated a mathematical feedback model based on a positive component arising from a radial bowing of fuel rods and a delayed negative component resulting from a delayed expansion of the tube sheet (the perforated shield plate located immediately above the Mark I and Mark II cores). By assigning credible values for time constants and power coefficients associated with the various feedbacks, Kinchin was able to reproduce in a qualitative manner the general structure of the EBR-I resonances.

Using an analytic rather than a synthetic approach Bethe,⁽⁹⁾ in his elaboration of the Kinchin concepts, arrived at a value for the effective transport lag associated with the delayed negative component (10 sec) and a partition of the net power coefficient into prompt and delayed components. While Bethe attributed the source of the positive term to an inward bowing of fuel rods, as did Kinchin, he was unable to identify the origin of the delayed negative component with any specific structural member. It is interesting to note that Bethe commented on the substantial inconsistency existing between the analytically determined transport lag (10 sec) and the actual physical transit time of the coolant through the reactor (about one sec).

In view of the Mark III tests results it now seems clear that rod bowing was indeed the source of the postulated positive component. The origin of the delayed negative component was, however, a

matter of some mystery. One of the first attempts to explain this effect was that of Lichtenberger⁽¹⁰⁾ who postulated a mechanism based on the preheating of core inlet coolant by a transfer of heat across the flow divider from core outlet to blanket inlet. Such a mechanism was at one time considered credible since the physical transit time for coolant flowing from blanket to core was qualitatively consistent with the concept of a transport time lag. The results of intensive tests devoted to a study of such possibilities in Mark III have, however, reduced considerably the credibility of this and other pre-heating mechanisms.

A recent detailed consideration of structural feedback effects peculiar to Mark II coupled with information gained in the Mark III tests points strongly to the conclusion that the mechanism responsible for the delayed feedback of Mark II involved thermally-induced motions in the lower shield plate, a perforated plate located immediately down stream from the reactor core. It is particularly significant that a mathematical treatment of the mechanical feedback arising from these motions predicts with a high degree of accuracy the natural resonance frequency of the reactor.

B. Previous Feedback Models

The efforts of both Kinchin and Bethe have been extremely useful in the identification of the origin of the delayed structural power coefficient component and, as a basis for later discussion, it is well to consider in some detail their mathematical concepts. In common terminology Kinchin⁽⁸⁾ assumed that the fast or prompt contribution to the dynamic power coefficient was described by the following relations where X_f^0 is

$$X_f(i\omega) = \frac{X_f^0}{(1 + i\omega\tau_f)} \quad (1)$$

the zero frequency (or steady-state) power coefficient, ω is the oscillation frequency, and τ_f is the time constant describing the fast

response of the core to power changes. Kinchin further postulated that the prompt steady-state power coefficient is defined by the algebraic sum of the following components

$$X_f^0 = X_b^0 + X_c^0 + X_u^0 \quad (2)$$

where X_b^0 , X_c^0 and X_u^0 are the steady-state contributions from rod bowing, coolant expansion and uranium expansion, respectively. To explain the resultant positive coefficient at zero coolant flow Kinchin assumed that the positive rod-bowing contribution more than compensated for the various negative effects or, in other words, that X_f^0 is positive. To explain the overall negative nature of the power coefficient under normal flow conditions it was necessary to include a negative term for the effects of structural expansion described by the following relation

$$X_s(i\omega) = \frac{X_s^0}{(1 + i\omega\tau_f)(1 + i\omega\tau_s)} \quad (3)$$

where X_s^0 is the zero frequency or steady-state value of the delayed structural power coefficient component. The magnitude of X_s^0 is postulated such that the following inequality holds:

$$X_b^0 + X_c^0 + X_u^0 + X_s^0 < 0 \quad (4)$$

The sum of Eqs. 1 and 3 multiplied by the average power P defines the reactivity feedback given by

$$-H = P \left[\frac{X_f^0}{(1 + i\omega\tau_f)} + \frac{X_s^0}{(1 + i\omega\tau_f)(1 + i\omega\tau_s)} \right] \quad (5)$$

Through the substitution of credible values for the various time constants and power coefficient in Eq. 5, Kinchin was able to show that the feedback described by Eq. 5 does indeed lead to a resonant structure not unlike that experimentally observed in the Mark II oscillation studies. The physical origin of the feedback described by the delay term was attributed by Kinchin to the movement of fuel rods by thermally induced motions of the tube sheet (shield plate).

In his elaboration of the Kinchin concepts Bethe assumed that structural heating proceeds in two stages: first the convection of heat by the coolant from the fuel to the relevant portion of the structure and then diffusion of heat into the particular component. Bethe preferred to describe the convection term by the cyclic operator $e^{-i\omega\tau}$ where τ is the physical transit time of the coolant from core to structure. Accordingly, his feedback is described by the following

$$-H = P \left[\frac{X_f^0}{(1+i\omega\tau_f)} + \frac{X_s^0 e^{-i\omega\tau}}{(1+i\omega\tau_f)(1+i\omega\tau_s)} \right] \quad (6)$$

where the various power coefficients and time constants have the same significance as those in Eq. 5. Bethe's expression for the feedback is superior to Kinchin's in that Eq. 6 accounts, qualitatively at least, for the displacement of the resonance towards higher frequencies at the higher flow rates.

From the results of Mark II transfer function measurements and through a modification of Eq. 6 Bethe arrived at a value of 10 seconds for the transport lag τ , a value grossly inconsistent with the actual transit time between core and exit (about one second at one-third flow). This extremely significant discrepancy was later resolved by Storrer⁽¹¹⁾ who showed that at low frequencies a temperature signal transported by the coolant travels slower than the coolant by the factor

$$\frac{\tau_f + \tau_c}{\tau_c}$$

where τ_f is the time constant for the fuel and τ_c is the time constant for the coolant. An evaluation of this ratio for Mark II results in a value of the order of 10. Hence, a physical transit time of one second is actually sensed by the structure as a 10-second lag. The importance of Storrer's contribution cannot be overemphasized since the solution of this perplexing discrepancy prepares the way for more rigorous mathematical approaches.

Unfortunately, the use of Eq. 6 with τ modified to include Storrer's refinement predicts the existence of significant harmonics in the amplitude of the transfer function, harmonics sufficiently large in magnitude that it is inconceivable they would have been "missed" in the measurements.

The failure to observe harmonics whereas theory clearly indicates their existence seriously challenges the validity of the transport lag model. While certain items of information may be cited in support of this model other items of equal importance may be offered in contradiction. The explanation of why the transport lag model fails to explain contradictory information is essentially this: the model is much too simple to include the effects of extremely complicated and important feedbacks which are now known to exist. To explain all experimentally observed phenomena (which a proper model must) it is necessary to add a second transport lag term to Eq. 6. As a further complication a mathematical development reveals that both effective transport time constants and both power coefficients are frequency dependent.

C. Application of Mark III Results to the Mark II Problem

To appreciate the striking differences in the dynamic performances of Mark II and Mark III it is necessary to understand the nature of the changes incorporated in the Mark III loading. Following the melt-down all components enclosed within the inner tank were removed and were replaced with a core and support structure which embodied important mechanical changes.⁽¹²⁾ For the most part the nuclear and heat transfer characteristics remained unchanged. Both loadings consisted of metallic slugs of highly enriched uranium and both were designed to operate at comparable power densities. In regard to the physical relationship of fuel with respect to structure strong changes were effected. In brief, the Mark II fuel rods were loose, while those in Mark III were compacted into a rigid array through the action of longitudinal ribs and

tightening rods. A further important difference was effected by the elimination of the perforated shield plate system used for the location and orientation of fuel rods in Mark II. Other less important differences consisted of the elimination of the NaK annulus between fuel and cladding and the provisions made to permit parallel coolant flow.

Several important conclusions, each pertinent to an understanding of Mark II behavior, have resulted from the Mark III oscillator studies. The delayed negative power coefficient component, so obviously present in Mark II, was eliminated, presumably through the elimination of the perforated shield plate system. The addition of stabilizing ribs to the fuel rods was beneficial in two respects: positive feedback from rod bowing was prohibited and the much higher degree of radial coupling between fuel rods promoted a much stronger prompt negative component. It may also be argued that the inclusion of stabilizing ribs to the fuel rods reduced considerably the possibility of feedback arising from the delayed expansion of downstream structural members.

One of the most important conclusions resulting from the Mark III tests is concerned with the fact that inward rod bowing is a consequence of special conditions of rod restraint, conditions which are not easily realized in practice. In fact, the promotion of conditions conducive to a large positive rod-bowing effect proved to be one of the major difficulties encountered in the Mark III stripped-rib tests. Unless the fuel rods are rigidly fixed at upper and lower restraining points reactivity effects associated with rod deformation may be positive, negative or even zero. Since the fuel rods in Mark II were essentially loose with respect to structural bearing points it is difficult to appreciate why rod-bowing effects were so pronounced. As described below certain structural features effected conditions highly conducive to rod bowing whenever the reactor operated under high power density conditions.

D. Description of Mark II

Since it will be necessary to refer frequently to various features of the Mark II core and structure, it is essential that some effort be devoted to a sufficiently detailed description of those features particularly pertinent to the various feedback processes. A cut-away view of the reactor as it existed during the Mark I and Mark II loadings is given in Fig. 1. Surrounding the reactor tank is a massive cup composed of stacks of key-stone-shaped uranium bricks clad with stainless steel. Since there is no evidence that the cup entered into any of the feedback considerations subsequent discussion will be confined to those components included inside the reactor tank. A cross-sectional view through the reactor at core elevation is given in Fig. 2. Relevant radial and vertical dimensions are given by a sketch, Fig. 3. As an aid in later calculations pertinent features of the reactor are broken down into specific regions and are designated as such in Fig. 3.

The core region consists of a hexagonal assembly of cylindrical fuel rods each of which consists of a stainless steel tube 0.448 in. in diameter with a 0.020 in. wall thickness containing concentrically spaced fuel and blanket slugs, both of which are 0.384 in. in diameter. A 0.010 in. annulus contains NaK to serve as a heat transfer bond. The fuel rods (Fig. 4) are positioned at the bottom on 0.494 in. centers by a plate perforated with triangular holes, which in the case of Mark II, engaged conical-shaped rod tips. Dimensions and tolerances are such that a 0.005-in. diametral clearance exists between rod tips and holes. Surrounding each positioning hole are six coolant passages $\frac{3}{16}$ in. in diameter and three $\frac{1}{16}$ in. in diameter. Surrounding the hexagonal core (see Fig. 2) are 138 blanket rods nominally 0.963 in. in diameter. These are located at the bottom by 0.498 in. truncated-conical tips fitted into 0.502-in. circular holes in the bottom plate.

Approximately 16 in. above the center of the core is a four-in.-thick shield plate illustrated photographically in Fig. 5. The plate consists of two regions: an inner hexagonal section containing 217 fuel rod holes nominally 0.460 in. in diameter and an outer arrangement of 138 0.964-in. holes for blanket rods. Separating the two regions is a hexagonal annulus approximately 0.20 in. wide with grooves milled to a depth of 0.125 in. in the upper and lower surfaces. A hexagonal flow divider of stainless steel 0.088 in. in thickness fitted into grooves in the lower surface of the shield plate (and in the bottom plate) serves to separate inlet and outlet coolant. Immediately above the lower shield plate is a 3.812-in. inlet spacer. This consists essentially of a massive (hollow) ring through which the main tie rods penetrate. Holes along the periphery admit inlet coolant. Immediately above the inlet spacer is the two-in. seal plate which is identical in cross-section with the lower shield plate. A flow divider approximately four in. long fits into grooves milled in the upper surface of the lower shield plate and lower surface of the seal plate.

To permit the passage of coolant, blanket rod extensions are fluted in the vicinity of the lower shield plate while fuel rod extensions are fluted over the length extending from the bottom of the lower shield plate to the top of the seal plate. The nominal dimensions of fuel rod extensions (at maximum diameter) and shield plate perforations are 0.449 and 0.460 in., respectively, giving a nominal diametral clearance of 0.011 in. A cross section through a typical fuel rod extension and shield plate perforation at the lower surface of the shield plate is given in Fig. 6. The nominal dimensions of the fuel rod extensions in the vicinity of the seal plate and at the lower surface of the lower shield plate are slightly different. The reduction in radius from 0.224 in. at the lower surface of the lower shield plate to 0.219 in. in the vicinity of the seal plate is illustrated in exaggerated form in Fig. 10.

In series flow, the only arrangement possible, the coolant flows downward around the fluting of the blanket rods into the lower plenum where the flow reverses and passes upward through the core, around the flutings of the fuel rod extensions, and finally into the outlet spacer located immediately above the seal plate.

Above the seal plate is a series of two- and four-in. shield plates alternating with four-in. structure rings running to an elevation of 86 in. above the lower shield plate. Six 1.50-in. tie rods penetrating the outer portion of the various shield plates and structure rings tighten the entire collection.

E. Mechanisms Which Affect Fuel Rod Movement

As discussed in Appendix A, the results of attempts to detect motions induced by temperature variations in the ligaments of a spare two-in. shield plate have demonstrated conclusively that such motions do exist. For the most part the motions are of two types: a rotational or tilting mode in which the vertical axes of the rod perforations are tipped towards or away from the normal, and a translational mode in which the axes are moved radially inward or outward.

The magnitude and the time dependence of the feedback from the plate depends on the manner in which the temperature of the plate is perturbed. Two cases are of particular interest. In one the temperature of the plate is affected by a step change in power; in the other the temperature is affected by a sinusoidal variation of power.

1. Fuel Rod Movement Associated With a Step-Change in Power

The manner in which shield plate ligaments control reactivity is perhaps more readily understood in terms of elementary and greatly exaggerated illustrations. The relationship of fuel rods and their extensions with respect to pertinent structural features is illustrated in Fig. 7 for the conditions of zero power and a nominal inlet temperature of 25°C. For the most part the relationship between

rod tips and locating holes and between rod extensions and shield plate ligaments is one characterized by a complete lack of order. To illustrate this concept rod tips and rod extensions are shown at the centers of their respective positioning holes in Fig. 7. It is true, of course, that mechanical imperfections in the various components will permit a certain amount of physical contact but such physical contact will be a random phenomenon. The probability of a rod tip bearing on the inner edge of a bottom plate hole is exactly the same as the probability for its bearing on an outer edge. The same argument follows for any physical contact between rod extensions and shield plate ligaments.

At elevated inlet temperatures and at zero power a reasonably high degree of order emerges. An illustration of this change is given in Fig. 8. As a consequence of the overall unconstrained radial expansion of the bottom plate, the rod tips are preferentially gathered at the inner edges of the locating holes. After the small radial clearance ($2\frac{1}{2}$ mils) is closed the rod tips are moved farther outward as the inlet temperature increases. A similar situation exists for the rod extensions and shield plate ligaments but because rod tip clearances are closed at a lower temperature, ligamental expansions never quite "catch up" with the rod movement effected by bottom plate expansion. A small clearance between rod extensions and ligaments will, in principle, always exist regardless of how high the temperature of the inlet coolant is raised. The actual amount of the clearance is dictated by the nominal radial clearance between the rod tips and locating holes. In practice, mechanical imperfections in the system will lead to physical contact between some rod extensions and ligaments. Such contact when it does occur will preferentially be along the inner edges of the shield plate ligaments. This preferential orientation of rod tips and rod extensions, incidentally, explains why both Mark I and Mark II were characterized by a strong isothermal temperature coefficient of reactivity.

Near the center of the shield plate there will, of course, be less preferential orientation since the degree of radial expansion will be small. At greater distances from the center the degree of preferential orientation increases and attains a maximum in the outer rows. The loss of coupling in the inner region is offset insofar as feedback is concerned by the much stronger coupling in the outer rows where flux bending is the greatest.

As the reactor is brought to power the fuel portion of the rod begins to bow, Fig. 9. At this point the rod tips move outward until they eventually come to bear against the outer edges of the locating holes. From this point on the tips can no longer move outward and the fuel portion of the rods bows inward. As a result of the bowing action, rod extensions not already gathered along the inner edges of the ligaments are forced inward until the system becomes highly ordered. With few exceptions (particularly in the very important outer rows) the rod tips and shield plate bearing point situations will be as illustrated in Fig. 9. A more detailed illustration of the contact between rod extensions and ligaments is given in Fig. 10.

An interesting consequence of full power operation is a pronounced convex dishing of the seal plate, since the entire temperature differential across the core is manifested as a strong temperature differential across the outer edge of the seal plate.

The orientation of rod extensions with respect to the ligaments of the seal plate and the second and third shield plates is not clear. If, through mechanical imperfections and through a reverse bending of the extensions above the shield plate, contact occurs between the rod extensions and seal plate ligaments the bearing points act as pivots and provide a mechanism for a limited amount of mechanical amplification. Ramifications of this possibility are considered in Section I-4. For the present the development will be restricted to the simplest (non-amplifying) case and unless otherwise indicated the situation as illustrated in Fig. 9 will be assumed.

The situation existing immediately after a step increase in power is illustrated in Fig. 11. The bottom surface of the shield plate is heated relative to the upper surface. The temperature wave carried by the coolant is not "sharp" and for a limited period of time a temperature differential will exist across the plate. As a result of the preferential heating of the lower surface the plate will assume the form of a slight concave dish. (As described in Appendix A, such motions are not speculative. They have actually been observed in simulations carried out with a spare shield plate.) Since the fuel rod extensions bear at well-defined points along the inner edges of the shield plate ligaments, the action is manifested by an outward movement of the extensions at these points.

As the plate finally restores to thermal equilibrium (at a higher temperature) the temperature differential across the plate and consequently the concave dishing disappears. By this time, however, the effects of the much slower translational component of expansion are felt. Although moderately constrained, the ligaments undergo a slow overall radial expansion which effectively moves the rod extensions outward during and after a step-power increase.*

The situation existing immediately following a step-decrease in power is illustrated in Fig. 12. For a short period of time (sec) the upper surface of the plate will be at a higher temperature than the lower surface. The plate assumes the form of a slight convex dish and the rod extensions which are positioned with positive pressure along the inner edges of the ligaments move inward. Again such motions are known to exist from the simulations.

*The much slower radial expansion has been observed in the simulations. The fact that the radial mode is slower than the dishing action is attributed to the time required for the hexagonal annulus of the shield plate to expand and consequently to allow stresses built up in the ligaments to be partially relieved.

2. Fuel Rod Movement Associated With a Sinusoidal Variation in Power

When the power is varied sinusoidally the various motions and associated feedbacks are somewhat different. During the rising portion of the cycle the lower surface of the shield plate will always be at a higher temperature than the upper surface. As a consequence the plate will assume the form of a concave dish. Since the action of the shield plate ligaments is essentially the same as that for a step increase in power, Fig. 11 will suffice as an illustration of this effect. During the falling portion of the cycle the upper surface will be at a higher temperature and the plate will deform to the shape of a convex dish. This situation is illustrated in Fig. 12. During the rising portion of the cycle shield plate ligaments move outward and carry the rod extensions with them. During the falling portion of the cycle the bearing points move inward and since the rod extensions are held with positive pressure against the bearing points the rod extensions also move inward, tracing in reverse the path followed during the rising half of the cycle. Both effects, the outward movement of fuel during the rising portion of the cycle and the inward movement during the falling portion, are clearly negative.

Superficially, it is difficult to appreciate how a significantly large temperature gradient can exist across a four-in. plate immersed in a relatively fast flowing stream of coolant, the temperature of which is being oscillated at low frequencies. However, several effects not immediately obvious combine to produce a situation conducive to strong temperature gradients. First of all the larger cross-sectional flow area of the shield plate effects a sharp reduction of flow velocity in this region. Secondly, the thermal capacitance of the rod extensions and ligaments causes the temperature wave to travel much more slowly than the physical velocity of the coolant. Finally, the amplitude of the temperature wave is decreased in passage through the plate. All three effects result in a situation

in which a large fraction of the oscillating temperature amplitude of the coolant wave leaving the core can be manifested in the form of a temperature differential across the plate.

F. Oscillating Temperature Differential Across the Plate

As Storrer⁽¹¹⁾ has shown a temperature signal impressed at low frequencies may travel more slowly than the coolant itself. In the fuel region where heat is generated the delay is caused by an increase in the temperature of the coolant as it flows upward through the core. As a consequence a certain fraction of the heat produced in the fuel is expended in increasing the fuel temperature in order to maintain a required temperature gradient between fuel and coolant. The rate of temperature rise in the coolant is accordingly decreased and is manifested effectively as a lag in the velocity of the temperature wave.

As heated coolant flows past finely divided nonheat-producing materials, such as the rod handles, the coolant temperature is reduced through a loss of heat to the inert material. The effect of the loss is one of a reduction in the velocity of the temperature wave front with respect to the physical velocity of the coolant. It is apparent, then, that the delay of the temperature wave (as well as the extent of amplitude reduction) will depend on the heat capacity of the inert material and the heat transfer rate between the coolant and material. For Mark II the effect is large. In general, throughout the core, upper blanket and rod extensions, the effective transport lag of a temperature signal is slowed by a factor of approximately ten.

For the most part the temperature differential across the plate arises from the fact that the temperature of the upper surface always lags that of the lower surface. For example, if the temperature of the upper surface lags that of the lower surface by 180° (which is certainly possible at high frequencies) a large fraction of

the amplitude of the coolant temperature oscillation at the bottom of the plate will be manifested in the form of a temperature differential across the plate. The smearing of the amplitude of the temperature wave in its passage through the plate also contributes to the temperature differential although in a less important capacity. To calculate the temperature differential across the plate at any given time it is therefore necessary to establish not only the phase lag across the plate but the amplitude of the temperature wave at both lower and upper surfaces.

The phase lag across the plate is actually independent of coolant history. Its calculation relies solely on a knowledge of the flow velocity and the heat transfer properties of the materials included in the shield plate region. The amplitude of the wave at both lower and upper surfaces does, however, rely on the history of the coolant since in its passage through the core and the upper axial blanket the amplitude of a temperature signal is smeared, i.e., what might have begun as a large oscillating signal leaving the core may arrive at the lower surface of the shield plate as a mere ripple. To establish the temperature differential across the plate at any given time and for any given frequency, it is therefore necessary to evaluate the attenuation of the oscillating temperature signal in the core, blanket and rod-handle regions. Since each of these regions differs considerably in matters of heat generation and heat transfer properties it is necessary to evaluate the attenuation factors for each region separately. While the calculation of the temperature gradient across the plate does not require knowledge of the phase lag across these downstream regions an evaluation of the phase lag between the core and the shield plate essentially defines the effective transport lag.

Since the seal plate and the second shield plate may under very special conditions also contribute to the feedback, it is necessary to carry out attenuation and phase calculations over the regions

included between the lower shield plate and the second shield plate. In all, eight individual regions are involved. These are identified in Fig. 3.

With minor exceptions the treatment necessary to the evaluation of the various phase and attenuation factors is simply an application of the concepts developed by Storrer.⁽¹¹⁾ As Storrer has shown, the complex amplitude of the coolant temperature wave at an elevation z is given by

$$Y_z = Y_0 e^{-\lambda(z/v)} + \frac{e^{-\lambda(z/v)}}{vh\tau_c(1 + i\omega\tau_f)} \int_0^z a(z') e^{\lambda(z'/v)} dz' \quad (7)$$

where Y_0 is the amplitude (not complex) of the temperature oscillation at zero elevation. The parameter λ defined in Eq. 8 (below) is complex and is very strongly frequency dependent. Since v is the coolant velocity z/v defines the physical transit time of the coolant over the distance z . The time constants τ_c and τ_f are those associated with the coolant and fuel (or inert material), respectively. The evaluation of these constants for each of the eight regions is discussed in Appendix B. A summary of the various values for z/v , τ_c , and τ_f , along with other pertinent information is given in Table I. The heat transfer coefficient h appearing in the denominator of the second term is the steady-state overall heat transfer coefficient. Its evaluation for each of the various regions is also discussed in Appendix B. The increase in the temperature of the coolant in its passage through the core is considered through the inclusion of the specific heat generation term $a(z')$ which for this particular model (Storrer's axially continuous) is a function of elevation z' only.

The term λ appearing in the exponentials of Eq. 7 is defined by

$$\lambda = \frac{\omega}{1 + \omega^2\tau_f^2} \left[\frac{\tau_f^2\omega}{\tau_c} + i \left(1 + \frac{\tau_f}{\tau_c} + \tau_f^2\omega^2 \right) \right] \quad (8)$$

where ω is the oscillation frequency (rad/sec). In the absence of heat generation, for example in the upper axial blanket or in the

rod-handle region, the heat generation term $a(z')$ vanishes and Eq. 7 reduces to

$$Y_z = Y_0 e^{-\lambda(z/v)} \quad (9)$$

By breaking λ up into its real and imaginary parts Eq. 9 may be rewritten as

$$Y_z = Y_0 e^{-\alpha(z/v)} e^{-i\beta(z/v)} \quad (10)$$

where

$$\alpha = \frac{\omega^2 \tau_f^2}{\tau_c(1 + \omega^2 \tau_f^2)} \quad (11)$$

and

$$\beta = \frac{\omega}{1 + \omega^2 \tau_f^2} \left(1 + \frac{\tau_f}{\tau_c} + \tau_f^2 \omega^2 \right) \quad (12)$$

The term α determines the attenuation (or smearing) of the wave amplitude. From Eqs. 10 and 11 it is clear that as the frequency increases the amplitude of the temperature oscillation decreases. In a similar manner decreasing the flow velocity or increasing the flow path (at constant velocity) effects a decrease in the amplitude of the temperature oscillations. The term described by Eq. 12 defines the associated phase lag over a path length z and for a coolant velocity v . At low frequencies the phase lag across a region will be small: for increasingly higher frequencies the phase lag increases rapidly.

Some qualitative insight into the concept of a temperature differential across the shield plate is provided through Eq. 10. Although the temperature wave suffers little reduction in amplitude at low frequencies, the temperature differential across the plate will be unimportant since the phase lag across the plate is small. At high frequencies where the phase lag is large and conducive to strong gradients, the amplitude of the temperature oscillations is so reduced that the resulting differential is small. For intermediate frequencies the temperature differential will reach a maximum value.

The frequency at which this occurs is extremely pertinent to an understanding of the delayed negative feedback.

1. Attenuation Factor and Phase Lag Across the Core, Region I

A rigorous solution of Eq. 7 over the length of the fuel region leads to an evaluation of the complex temperature amplitude at any given elevation in the core and for any given oscillation frequency. As a major simplification in the solution of Eq. 7 it has been assumed that power production is uniform throughout the core. This simplification is not seriously compromising since the attenuation and phase lag of the temperature signal generated in the core are relatively minor compared with the corresponding cumulative effects of the other regions. The solution of Eq. 7 then becomes

$$Y_Z = \frac{e^{-\lambda(z/v)}}{vh\tau_c(1+i\omega\tau_f)} \left[\frac{av}{\lambda} (e^{\lambda z/v} - 1) \right] \quad (13)$$

where a is the specific power generation under the conditions assumed above and where the limits of integration have been taken between $z' = 0$ and $z' = z$.

A rearrangement of Eq. 13 leads to

$$Y_Z = \frac{a(1 - e^{-\lambda z/v})}{\lambda h\tau_c(1+i\omega\tau_f)} \quad (14)$$

Thus, for specific operating conditions the complex temperature amplitude Y_Z at the top of the core may be expressed in terms of the oscillation frequency. Since it becomes necessary eventually to compare the results of calculations with experimentally known facts all subsequent calculations will be based on the conditions of 470 kw and 108 gpm flow, conditions for which experimental transfer function information is available. Actually it is unnecessary to evaluate the amplitude (magnitude) of the temperature wave at the top of the core since these values may be obtained more reliably from the actual transfer function results. It is still necessary, however, to establish from Eq. 14 the attenuation factors and phase lag as a function of

frequency. The results of a solution of Eq. 14 for the conditions specified above are summarized in Table II. The attenuation factors given consider the attenuating influence of the entire 8.5 in. core. The phase lag, however, considers only that lag occurring between the middle and the top of the core. In other words, insofar as phasing is concerned, the core midplane is considered as a reference.

2. Attenuation Factor and Phase Lag Across the Upper Axial Blanket and Rod-Handles, Regions II and III

In calculating the complex temperature amplitude of the coolant wave leaving the upper axial blanket it has been assumed that the heat generation term of Eq. 7 is zero or, in other words, that heat generation in the blanket may safely be neglected. Eq. 7 then becomes

$$Y_2 = Y_1 e^{-\lambda(z/v)} \quad (15)$$

where Y_2 is the complex amplitude of the temperature wave at the top of the blanket and Y_1 is the amplitude (not complex) of the wave leaving the core. Substitution of values for z/v , τ_c and τ_f (from Table I) into Eq. 15 results in the values for the attenuation factors and phase lags summarized in Table II. Thus, for 0.001 c/sec the phase lag between the top of the core and the top of the blanket is 0.935°. The attenuation factor for this frequency is 1.000, i.e., the amplitude of the temperature oscillation at the top of the blanket is the same as that at the top of the core. For the highest frequency considered, 0.15 c/sec, the phase lag has increased to 92.5° and the amplitude of the wave leaving the blanket has been reduced to 0.344 times the input value. Since the amplitude of the wave leaving the core (at 0.15 c/sec) has already been attenuated by a factor of 0.394, the amplitude of the wave leaving the blanket is only 0.344 x 0.394 or 13.5 percent of that generated in the core.

Calculations of the attenuation and phase lag of the temperature oscillations across the rod-handle region present no

special problem since the heat generation term of Eq. 7 is clearly zero. Substitution of values for z/v , τ_c and τ_f (from Table I) into an expression similar to Eq. 15 results in the values summarized in Table II.

3. Attenuation Factor and Phase Lag Across the Shield Plate (Region IV) and Seal Plate (Region VI)

In calculating the attenuation and phase lag of the temperature wave across these regions it is necessary to extend the scope of Storrer's expressions since two media involved, ligaments and rod extensions, each with widely differing heat transfer properties, are present. Because the ligaments of the shield plate are thin, approximately 0.034 in., the time constant for response will be short. For this reason the phase lag across the plate would be expected to be small. On the other hand, the large (by comparison) time constant for the rod-handles effectively slows the propagation of the temperature signal. The effects of both (ligaments and rod-handles) cannot be found from a simple lumping of the inert material. Rather, it is necessary to revise and to solve simultaneously three transient heat balance expressions. A solution of these expressions, given in Appendix C, results in the following expression for λ :

$$\lambda = \frac{\omega^2(\tau_{f1}^2 + \tau_{f2}^2) + 2\omega^4\tau_{f1}^2\tau_{f2}^2}{\tau_c(1 + \omega^2\tau_{f1}^2)(1 + \omega^2\tau_{f2}^2)} + i\omega \left[1 + \frac{\tau_{f2}}{\tau_c(1 + \omega^2\tau_{f2}^2)} + \frac{\tau_{f1}}{\tau_c(1 + \omega^2\tau_{f1}^2)} \right] \quad (16)$$

where τ_{f1} and τ_{f2} are the time constants for the rod extensions and ligaments, respectively. A description of the method used to evaluate the time constants for these two special cases is given in Appendix B. Substitution of values for τ_{f1} , τ_{f2} , τ_c and z/v (from Table II) into Eqs. 15 and 16 results in values for the attenuation factors and phase lags summarized in Table II.

4. Attenuation Factor and Phase Lag Across the Inlet Spacer, Region V

Since there are no ligaments in this region, capacitance effects are dictated solely by the fluted rod extensions and the coolant. An evaluation of the time constants for this special case is

discussed in Appendix B. Substitution of values for τ_c , τ_f and z/v from Table I into Eqs. 8 and 15 results in the values for the attenuation factors and phase lags summarized in Table II.

5. Attenuation Factor and Phase Lag Across the Outlet Spacer, Regions VII and VIII

In regions I through VI the transit time of the coolant is exactly defined by the length of flow path and the coolant velocity. In the outlet spacer (or outlet plenum) the coolant flows outward from the interstices between the fuel rod extensions, around the blanket rod-handles and out through holes in the spacer ring into the outlet pipe. Here the concept of a physical transit time breaks down since the flow is continuously decelerating. To circumvent a detailed consideration of the velocity as a function of position, it has been assumed that the transit time may be approximated by the residence time of the coolant in this region where the residence time is defined as the time required to fill the region under consideration with coolant at the specified flow rate.

A further complication consists of the fact that the outlet spacer consists of two quite different regions: an inner, containing the fuel rod extensions; and an outer, containing the much more massive blanket rod-handles. Since the heat transfer properties of each region are quite different, each region has been considered individually, i.e., residence times and time constants have been calculated for each region. A discussion of the various calculations for the inner region (VII) and outer (VIII) is given in Appendix B. Substitution of the appropriate values into Eqs. 8 and 15 results in the values for the attenuation factors and phase lags summarized in Table II.

G. Temperature Gradient Across Lower Shield Plate as a Function of Frequency

From Table II it is clear that sizable phase lags can exist across the lower shield plate and that the phase lag increases rapidly with frequency. It is also clear that the amplitude of the input temperature wave is smeared in its passage through the plate. Both effects act to produce across the plate a temperature gradient which is a strong function of the oscillation frequency. At low frequencies, for example at 0.001 c/sec, the temperature gradient will be extremely small since the temperatures at the lower and upper surfaces of the plate at any given time will be essentially the same. This is, of course, a consequence of the small phase difference (actually 1.56°) at this frequency. At high frequencies, for example 0.098 c/sec, the phase lag is much larger (152°) and the lower and upper surfaces may, in principle, be at considerably different temperatures. On the other hand, the amplitude of the temperature wave at this frequency is considerably smeared in its passage through the core, blanket, rod-handle and plate regions. Unless the amplitude of the wave at the bottom surface of the shield plate is significantly large, the temperature gradient across the plate may be of little consequence even though strong phase lags may exist. Clearly, the maximum temperature gradient occurs at some frequency which effects a compromise between increasing phase and decreasing amplitude.

A summary of phase lag and attenuation data pertinent to an evaluation of the frequency at which the maximum temperature gradient occurs is given in Table III. All phase data are quoted relative to the center of the core. For example, the phase lag (150°) quoted at the bottom of the plate at 0.150 c/sec is simply the sum of the phase lags occurring across Regions I-III, i.e., over half of the core, the blanket, and the rod-handle region. The phase lag quoted at the top of the plate is, accordingly, the phase lags summed over region I through IV. Similarly, the attenuation factors quoted are

the products of attenuation factors over the downstream regions. For example, the attenuation factor at 0.150 c/sec is simply the product of the attenuation factors over regions I through III which from Table II amounts to $0.394 \times 0.344 \times 0.835 Y_1 = 0.113 Y_1$ where Y_1 is the amplitude (not complex) of the temperature wave leaving the core.

Since the attenuation factors and phase lags are known (for any given frequency) at both lower and upper surfaces, it is possible to express the temperature of both upper and lower surfaces as a function of time. Hence, the temperature of the coolant at the bottom of the shield plate T_3 may be expressed as a function of time t by the following relation

$$T_3 = T_1 + \mu_3 Y_1 (\sin \omega t - \theta_3) \quad (17)$$

where T_1 is the average temperature of the coolant leaving the core, μ_3 is the cumulative attenuation factor over regions I through III, Y_1 is the noncomplex amplitude of the temperature wave leaving the core, ω is the oscillation frequency and θ_3 is the cumulative phase lag over regions I through III. The temperature of the coolant at the upper surface of the shield plate is given by

$$T_4 = T_1 + \mu_4 Y_1 (\sin \omega t - \theta_4) \quad (18)$$

where μ_4 and θ_4 are the cumulative attenuation factors and phase lag, respectively, over regions I through IV.

To illustrate the concept of a temperature gradient across the plate Eqs. 17 and 18 have been evaluated as a function of time for the fixed frequency 0.0376 c/sec. The results are given in Fig. 13. To simplify the illustration the constant T_1 has arbitrarily been assumed to be zero, and the temperature of the coolant at lower and upper surfaces has been expressed in terms of Y_1 , the amplitude of the temperature wave leaving the core. From Fig. 13 it is clear that the temperature differential across the plate in the course of one cycle goes from a maximum negative differential, to zero, to a maximum positive differential and finally back through zero to a maximum negative.

The temperature differential (or gradient) across the plate at a time t and for an oscillation frequency ω is therefore defined by the difference between Eqs. 17 and 18. Hence,

$$\Delta T = T_3 - T_4 = \mu_3 Y_1 (\sin \omega t - \theta_3) - \mu_4 Y_1 (\sin \omega t - \theta_4) \quad (19)$$

or

$$\Delta T = Y_1 [\mu_3 (\sin \omega t - \theta_3) - \mu_4 (\sin \omega t - \theta_4)] \quad (20)$$

From Eq. 20 it is clear that the temperature differential across the plate is a time-dependent function of Y_1 , the oscillation frequency and the attenuation factors and phase lags accumulated over the various downstream regions.

A plot of Eq. 20 for the fixed frequency 0.0376 c/sec is given in Fig. 14. Note that the temperature gradient oscillates between maximum positive and negative values with a frequency defined by ω . Since the time constant for ligamental flexing is small (0.109 sec, Table I), the shield plate during the course of one cycle (at low frequencies) will assume the form of a concave dish for one portion of the cycle, and after straightening will form a convex dish during the second portion. The significance of these actions will be discussed in Sect. H.

For a given frequency μ_3 , μ_4 , θ_3 and θ_4 are fixed. It is therefore possible to establish the time at which the temperature differential passes through a maximum (or minimum). This value may then be substituted back into Eq. 20 to establish the value of the maximum temperature gradient. A partial differentiation of Eq. 20 results in the following

$$\left[\frac{\partial(\Delta T)}{\partial t} \right]_{\omega} = Y_1 [\omega \mu_3 \cos (\omega t - \theta_3) - \omega \mu_4 \cos (\omega t - \theta_4)] \quad (21)$$

Setting

$$\left[\frac{\partial(\Delta T)}{\partial t} \right]_{\omega} = 0$$

$$\mu_3 \cos (\omega t - \theta_3) = \mu_4 \cos (\omega t - \theta_4) \quad (22)$$

and since

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad (23)$$

$$\mu_3[\cos \omega t \cos \theta_3 + \sin \omega t \sin \theta_3] = \mu_4[\cos \omega t \cos \theta_4 + \sin \omega t \sin \theta_4] \quad (24)$$

Dividing Eq. 24 by $\cos \omega t$

$$\mu_3[\cos \theta_3 + \tan \omega t \sin \theta_3] = \mu_4[\cos \theta_4 + \tan \omega t \sin \theta_4] \quad (25)$$

A rearrangement of Eq. 25 leads to

$$\tan \omega t = \frac{\mu_4 \cos \theta_4 - \mu_3 \cos \theta_3}{\mu_3 \sin \theta_3 - \mu_4 \sin \theta_4} \quad (26)$$

Since μ_3 , μ_4 , θ_3 and θ_4 are given (Table III), it is possible through Eqs. 26 and 20 to evaluate the maximum temperature gradient for each of the discrete frequencies listed in Table III. The results of this evaluation are given in the third column of Table IV. Note that the temperature gradient expressed in terms of Y_1 (the non-complex amplitude of the wave leaving the core) is small at the low frequencies, passes through a maximum at about 0.0584 c/sec and falls off rapidly at higher frequencies.

A more realistic appraisal of the maximum temperature differential across the plate may be obtained from an evaluation of the non-complex amplitude of the core temperature wave. Fortunately, this becomes possible through an analysis of existing Mark II data for 470 kw and 108 gpm flow. The fourth column of Table IV gives the actual measured fractional change in power resulting from the oscillator input. At 0.0089 c/sec, for example, the power oscillates between the extremes of 5.85% above the average and 5.85% below the average. In the absence of attenuation effects the amplitude of the temperature oscillation is simply the product of the power variation and the temperature differential across the core which for 470 kw and 108 gpm was measured as 92°C. Hence, the non-complex amplitude of the temperature wave leaving the core at 0.0089 c/sec

amounts to $0.0585 \times 92.0^{\circ}\text{C} = 5.38^{\circ}\text{C}$. However, from Table II it is clear that the temperature wave is moderately attenuated before it leaves the core. Since the attenuation factors in the core are known, the actual non-complex amplitude of the temperature wave at the top of the core Y_1 is simply the product of Y_0 , the non-attenuated temperature amplitude, and the respective attenuation factor for that frequency. Knowledge of Y_1 , then, permits an evaluation of the actual temperature gradient across the shield plate given in the last column of Table IV. To illustrate the dependence of maximum temperature differential on frequency the results of Table IV are illustrated graphically in Fig. 15. From this it is clear that the maximum temperature differential lies in the vicinity of 0.04 c/sec.

Assuming that the amount of feedback reactivity controlled by shield plate flexing is proportional to the temperature differential, it follows that the associated power coefficient is strongly frequency dependent and reaches its maximum value at 0.043 c/sec for 470 kw and 108 gpm. It is interesting to note that the resonance frequency of the reactor under these conditions of power and flow turns out to be .040 c/sec.

H. Feedback Model

Before attempting to incorporate the plate flexing feedback into a mathematical model, it becomes necessary to scrutinize actual transfer function results for the specific conditions under which the maximum temperature differential across the plate was derived, i.e., 470 kw and 108 gpm. Unfortunately, all amplitude and phase data accumulated during the May and November (1955) tests are characterized by irregularities which are not understood. For this reason it is difficult to effect a rigorous correlation between experimental and calculated results. The extent of the irregularities in the existing transfer function data is illustrated for the specific conditions of 470 kw and 108 gpm in Fig. 16 (amplitude) and Fig. 17 (phase).

For frequencies above 0.1 c/sec the amplitude lies below the calculated curve to an extent which must be regarded as suspicious. Since all feedback processes are attenuated at the higher frequencies, the usual behavior of the transfer function amplitude is one characterized by an asymptotic approach to the calculated curve. An inspection of the phase dependence, on the other hand, does not reveal any obvious irregularities.

A more realistic appraisal of the untreated experimental data may be obtained through a study of the feedback, Fig. 18, separated from the information given in Figs. 16 and 17. A casual inspection of these results suggests a mechanism based on the existence of two power coefficient components: one prompt and positive and the other strongly negative and varying with frequency according to a transport lag dependence. Such an explanation would presumably account for the fact that the low frequency feedback is smaller in amplitude than it is at higher frequencies. The fact that the high frequency feedback is so strong in the I quadrant implies a feedback process which acts with a long transport lag. The fact that a significant transport lag is required, however, points up the fallibility of this simple explanation. If the region controlling the feedback is so far removed from the core that it must be described by a transport lag dependence it is certain from Storrer's developments that the amplitude of any temperature perturbation initiated in the core would be so small that dynamic feedback effects would be negligible. Even if strong temperature oscillations could be initiated at high frequencies the attenuation of the wave in its passage between the core and the pertinent structural region would reduce the oscillating amplitude to an ineffective ripple. From Table II it may be seen that a feedback effect originating in the vicinity of the lower shield plate for a frequency of 0.15 c/sec would be attenuated by a factor between 0.113 and 0.04. Since the behavior illustrated in Fig. 18 is characterized by little or no attenuation, it is difficult to accept the

effect as real. Rather it is more logical to question the validity of the experimental results cited in Figs. 16-18.

The first clue regarding the origin of the inconsistencies is readily apparent from a scrutiny of transfer function results obtained at 2 kw (essentially zero power), Fig. 19. At all frequencies the amplitude lies significantly and consistently below the calculated curve. Unquestionably, the origin of this discrepancy is associated with the use of an incorrect value for the absolute reactivity worth of the oscillator rod. Accordingly, the experimental amplitude results may be revised upward by a constant factor (1.25) which essentially brings experimental and calculated results into agreement. The same factor may then be applied with sufficient justification to all other amplitude results regardless of the actual power.

The experimental phase lags deviate from the calculated phase curve for frequencies greater than 0.04 c/sec, and as the frequency increases the discrepancy becomes larger. Essentially the same behavior (although to a less important degree) has been noted in the Mark III oscillator studies.⁽¹⁾ The origin of this discrepancy has been associated with the time required for neutrons originating in the core to reach the detector. As the results of the Mark III tests have shown, the phase discrepancy for a given frequency is constant with respect to power. It is therefore permissible to apply the frequency dependent discrepancies noted at zero power as corrections to the experimental phase values at other levels of power.

A normalization of the amplitude by a factor of 1.25 results in the values illustrated in Fig. 20. Again the experimental amplitude values lie significantly and consistently below the calculated curve. A separation of the feedback (with corrections applied to the phase) still results in a feedback quite similar to that illustrated in Fig. 18 for the unnormalized data. No credible explanation for the amplitude discrepancies of Fig. 20 can be offered. It is difficult to resist the conclusion that the reactivity worth of the oscillator rod decreases

strongly as the reactor power increases. On the other hand, temperature and power sensitive processes which are expected to affect the reactivity worth, i.e., neutron absorption and scattering, and thermal expansion are too small by a wide margin to account for the magnitude of the discrepancy.

To correlate the results of calculations with those obtained from experiment it becomes necessary to decide whether (a) to accept the partially normalized results of Fig. 20 as real or (b) to effect a further normalization. Since no credible explanation can be given for the partially normalized results at high frequencies (above 0.06 c/sec), the second alternative (b) is preferred. The results of a second normalization of the amplitude are given in Fig. 21. The additional normalization is based on the tacit assumption that feedback effects above 0.1 c/sec are insignificantly small. As a consequence of the developments given below, such an assumption appears to be reasonable.

A separation of the feedback associated with the amplitude given in Fig. 21 and the phase given in Fig. 17 (corrected for zero power discrepancies) results in the dependence given in Fig. 22. For the purpose of correlating calculated with experimental results it will be assumed that this particular feedback is peculiarly associated with the conditions, 470 kw and 108 gpm.

To explain the exceedingly complex feedback dependence illustrated in Fig. 21, the following mathematical model is postulated.

$$-H = P \left[\underbrace{\frac{-A_0}{1 + i\omega\tau_f}}_{\substack{\text{core} \\ \text{feed-} \\ \text{back} \\ \text{processes}}} + \underbrace{\frac{\mu_b(\omega) B_0 e^{-i\omega\tau(\omega)}}{(1 + i\omega\tau_f)(1 + i\omega\tau_p)}}_{\substack{\text{plate} \\ \text{flexing}}} + \underbrace{\frac{\mu_c(\omega) C_0 e^{-i\omega\tau(\omega)}}{(1 + i\omega\tau_f)(1 + i\omega\tau_r)}}_{\substack{\text{radial} \\ \text{plate} \\ \text{expansion}}} \right] \quad (27)$$

1. Prompt Term

The first term describes the feedback originating in the core. As a simplification it has been assumed that all core feedback processes, i.e., rod bowing, axial and radial fuel expansion, and coolant expulsion act with the same time constant, τ_f . The algebraic sum of the individual components defines the net power coefficient A_0 which in the case of Mark II is known to be positive in sign. Since the fuel rods in Mark II were not radially coupled, the normally strong radial contribution to power coefficient was greatly reduced. As a consequence the net negative component resulting from the combined effects of fuel expansion and coolant expulsion was insufficiently large to compensate for the effects of rod bowing which for Mark II were strongly positive.

2. Plate Flexing Term

The second term describes the feedback associated with shield plate flexing. The product $\mu_b(\omega) B_0$ defines the actual power coefficient for a given oscillation frequency ω where B_0 is the value for the power coefficient evaluated for the frequency at which the oscillating temperature differential across the plate reaches a maximum, Table V, and where $\mu_b(\omega)$ is the frequency dependent attenuation factor summarized in Table VI. The time constant for flexing is given by τ_p and the effective transport lag between the core and the shield plate is defined by $\tau(\omega)$. As it turns out, $\tau(\omega)$ is also frequency dependent and accordingly must be evaluated for each frequency from the accumulated phase lag between the core center and the shield plate (Table II).

3. Radial Expansion Term

The third and final term describes the delayed feedback arising from the overall radial expansion of the plate. Again the power coefficient $\mu_c(\omega) C_0$ is frequency dependent. In this case the frequency dependence enters because the amplitude of the temperature wave is strongly attenuated at the higher frequencies.

The attenuation coefficient $\mu_c(\omega)$ begins with a value of unity at low frequencies and decreases with increasing frequencies according to Eqs. 10 and 11. A summary of values for $\mu_c(\omega)$ as a function of frequency (evaluated from the information of Table II) is given in Table VI. The power coefficient C_0 is defined as the steady-state (zero frequency) value for the effects of an overall radial expansion of the plate. The transport lag $\tau(\omega)$ is identical with that appearing in the second term since both describe the effective time required to transport a temperature signal from the core to the shield plate. The time constant τ_r describes the time dependence of the overall radial expansion of the plate. Because the relatively massive hexagonal annulus (Fig. 5) is apparently involved, τ_r is considerably larger than τ_p , the time constant for simple ligamental flexing.

The frequency dependence of each term is illustrated in Fig. 23. No attempt has been made to weigh the various terms according to their actual importance in the mathematical model, i.e., each term in Eq. 27 has been assigned an arbitrary weight of unity. Various values for time constants, attenuation factors and transport lags are summarized in Table VI. With one exception, that of τ_r , values assigned to time constants and transport lags are directly the result of heat transfer considerations (see Appendix B and Table I). In this case the arbitrary value of one sec has been assigned since this value eventually proved to be more consistent with experimental results.

The attenuation factor $\mu_b(\omega)$ for the second term was evaluated from a normalization of the maximum temperature differential frequency dependence (see Fig. 15 and Table V) while the attenuation factor $\mu_c(\omega)$ for the third term is simply the cumulative attenuation factor established over the core, blanket and rod-handles, Regions I through III.

The zero frequency intercepts of the first and third term are respectively A_0 and C_0 . The zero frequency intercept for the

second term is the origin since $\mu_b(\omega)$, in this case, approaches zero at low frequencies. All three terms eventually go to zero, but at different rates, as the frequency increases indefinitely. To combine the three terms into a simulation of the experimental feedback it is necessary to assign to the individual components weighting factors which provide a satisfactory compromise between phase and amplitude values over as wide a frequency spectrum as possible.

At 0.15 c/sec nearly all feedback arises from the prompt positive effect and to a first approximation the measured feedback at this frequency may be equated to the first term. An evaluation carried out in this manner results in a value of $A_0 = 2.1 \times 10^{-6} \Delta k/k/kw$. Knowledge of A_0 considerably simplifies the evaluation of B_0 and C_0 . Substitution of A_0 , $\mu_b(\omega)$, and $\mu_c(\omega)$ into Eq. 27 for 0.0376 and 0.0484 c/sec, where $-H$ is known, results in simultaneous expressions in terms of B_0 and C_0 . Solution of these equations results in values of $B_0 = 1.7 \times 10^{-6}$ and $C_0 = 2.5 \times 10^{-6} \Delta k/k/kw$.

A test of how well the various weighting factors fit the experimental data is given in Fig. 24. It is meaningless for two reasons to force a better agreement between the calculated and experimental values by a "cut and try" shaping of the various parameters. In the first place, it has not been established unambiguously that the experimental feedback plotted in Fig. 24 is precisely defined. In the second place, the mathematical model given by Eq. 27 is somewhat over-simplified in that it fails to consider the effects of pivotal action at seal plate bearing points.

I. Credibility of Power Coefficient Values

The most important criterion to be satisfied in the quasi-analytical fit of Fig. 24 is whether thermally-induced ligamental motions are sufficiently large to explain the magnitudes of the three power coefficients, A_0 , B_0 and C_0 , as indeed they must be if the postulated mechanism is to be regarded valid.

1. Power Coefficient, A_0

As defined by Eq. 27 the power coefficient A_0 is simply the algebraic sum of the steady state power coefficients for all feedback processes limited in their action to the core. Hence,

$$A_0 = A_{rb}^0 + A_{ax}^0 + A_{rad}^0 + A_{NaK}^0 \quad (28)$$

where A_{rb}^0 , A_{ax}^0 , $A_{rad}^0 + A_{NaK}^0$ are the steady state power coefficients for rod bowing, axial expansion, radial expansion and coolant expulsion, respectively.

From flow reduction tests it is known that A_0 is positive. Since A_{rb}^0 is also positive, it follows that the sum of the negative components is insufficiently large to compensate for the positive rod-bowing term. The simplest test of whether the quasi-analytically determined value of A_0 is reasonable is to estimate the magnitudes of the various negative components and to determine whether the magnitude of the remaining rod-bowing term is consistent with the amount of rod bowing expected at 470 kw and 108 gpm flow.

A reasonably satisfactory approximation of the axial power coefficient component may be obtained from the following relation:⁽¹³⁾

$$\frac{\Delta k}{k} = -0.3 \frac{\Delta h}{h} \quad (29)$$

where $\Delta k/k$ is the reactivity change associated with a fractional change in core height, $\Delta h/h$. The power coefficient for axial expansion is then

$$A_{ax}^0 = \frac{\Delta k}{k \text{ kw}} = \frac{-0.3 \Delta h}{h \Delta P} \quad (30)$$

In terms of the linear expansion coefficient of the fuel Eq. 30 may be rewritten as

$$A_{ax}^0 = \frac{-0.3 \Delta L}{L} \times \frac{\Delta T_{eff}}{\Delta P} \quad (31)$$

where $\Delta L/L$ for the uranium-zirconium fuel alloy at $200^\circ\text{C} = 13 \times 10^{-6}/^\circ\text{C}$. To a first approximation the term $\Delta T_{\text{eff}}/\Delta P$ defines the effective temperature change of the coolant with respect to power. To account for the fact that the effects of a change in temperature differential across the core are fully felt at the top of the core and are not felt at the bottom the effective temperature differential across the core is defined as one-half that of the total differential. In particular, at 470 kw and 108 gpm the actual measured temperature differential across the core was 92°C . The effective temperature differential across the core is then 46°C and the coefficient $\Delta T_{\text{eff}}/\Delta P$ is $46^\circ\text{C}/470 \text{ kw}$ or $0.10^\circ\text{C}/\text{kw}$. Power changes of one kw result in an effective coolant temperature change of 0.10°C . Substitution of values for $\Delta L/L$ and for $\Delta T_{\text{eff}}/\Delta P$ into Eq. 31 results in a value of $-0.39 \times 10^{-6} \Delta k/k/\text{kw}$.

In estimating the magnitude of A_{rad}^0 it is necessary to consider the fact that the fuel rods in Mark II were not coupled radially. The effective change in core radius is therefore dictated by expansive effects occurring in the outer row of rods only. A measure of the power coefficient may be obtained from the relation⁽¹³⁾

$$\frac{\Delta k}{k} = -0.6 \frac{\Delta R}{R} \quad (32)$$

where $\Delta k/k$ is the reactivity change associated with the fractional change in the core radius, $\Delta R/R$. Hence,

$$A_{\text{rad}}^0 = \frac{\Delta k}{k \text{ kw}} = \frac{-0.6}{R} \frac{\Delta r}{\Delta P} \quad (33)$$

where R is the effective core radius (3.7 in.) and $\Delta r/\Delta P$ is the change in core radius effected by a power increase ΔP . (r is the radius of a fuel rod.) Since

$$\Delta r = r \frac{\Delta L}{L} \Delta T_{\text{eff}} \quad (34)$$

Eq. 33 becomes

$$A_{\text{rad}}^0 = \frac{-0.6r}{R} \times \frac{\Delta L}{L} \times \frac{\Delta T_{\text{eff}}}{\Delta P} \quad (35)$$

where $\Delta T_{\text{eff}}/\Delta P$ is defined above. Evaluation of Eq. 35 results in a value of A_{rad}^0 of $-0.048 \times 10^{-6} \Delta k/k/kw$.

The power coefficient for coolant expulsion may be estimated crudely from the relative NaK/fuel worth established from a comparison of wet and dry critical experiments in Mark I.⁽¹⁰⁾ From these experiments it was found that 1700 g of NaK reduced the critical fuel loading by approximately two kilograms. It follows, then, that one gram of NaK had an equivalent worth of 0.850 g of fuel. The change in the coolant mass with respect to effective temperature differential change may be found from the following:

$$\frac{\Delta M_c}{\Delta T_{\text{eff}}} = \left(\frac{\partial M}{\partial d} \right)_M \frac{\Delta d}{\Delta T_{\text{eff}}} + \left(\frac{\partial M}{\partial v} \right)_d \frac{\Delta V_f}{\Delta T_{\text{eff}}} \quad (36)$$

where the subscripts c and f refer to coolant and fuel respectively, $\Delta d/\Delta T_{\text{eff}}$ is the temperature coefficient of NaK density and $\Delta V_f/\Delta T_{\text{eff}}$ is the volume expansion coefficient of the fuel. Since $M = dV$

$$\frac{\Delta M_c}{\Delta T_{\text{eff}}} = V_c \frac{\Delta d}{\Delta T_{\text{eff}}} + d \frac{\Delta V_f}{\Delta T_{\text{eff}}} \quad (37)$$

In terms of fuel equivalent

$$\frac{\Delta M_f}{\Delta T_{\text{eff}}} = .850 \left(V_c \frac{\Delta d}{\Delta T_{\text{eff}}} + d \frac{\Delta V_f}{\Delta T_{\text{eff}}} \right) \quad (38)$$

Multiplying Eq. 38 by the effective change in temperature across the core per kw, $\Delta T_{\text{eff}}/\Delta P$ results in

$$\frac{\Delta M_f}{\Delta P} = .850 \frac{\Delta T_{\text{eff}}}{\Delta P} \left[\frac{V_c \Delta d}{\Delta T_{\text{eff}}} + \frac{d \Delta V_f}{\Delta T_{\text{eff}}} \right] \quad (39)$$

Using the approximation⁽¹³⁾

$$\frac{\Delta k}{k} \approx -0.27 \frac{\Delta M}{M} \quad (40)$$

the power coefficient A_{NaK}^0 may be obtained from

$$\begin{aligned} A_{NaK}^0 &= \frac{\Delta k}{k \Delta P} = \frac{-0.27 \Delta M}{M \Delta P} \\ &= \frac{-0.27 \times .85}{M} \frac{\Delta T_{eff}}{\Delta P} \left[V_c \frac{\Delta d}{\Delta T_{eff}} + d \frac{\Delta V_f}{\Delta T_{eff}} \right] \end{aligned} \quad (41)$$

The power coefficient may then be evaluated from the following information: M (wet critical mass) = 51 kg, $\Delta T_{eff}/\Delta P = 0.10^\circ\text{C}/\text{kw}$, $V_c = 2000 \text{ cm}^3$, $\Delta d/\Delta T_{eff} = 2.4 \times 10^{-4} \text{ g}/\text{cm}^3 \text{ }^\circ\text{C}$, $d = 0.847 \text{ g}/\text{cm}^3$, $\Delta V_f/\Delta T_{eff} = 0.194 \text{ cm}^3/^\circ\text{C}$. Hence, $A_{NaK}^0 = -0.29 \times 10^{-6} \Delta k/k/\text{kw}$. Since expressions 29, 32 and 40 are based on dimensional changes of a core of constant composition, the values deduced for A_{ax}^0 , A_{rad}^0 and A_{NaK}^0 are slightly in error. For the purpose of the following illustration, however, the effects of such errors are inconsequential.

A summation of the three negative power coefficients results in a value of $-0.73 \times 10^{-6} \Delta k/k/\text{kw}$. Since the net power coefficient is estimated to be $2.1 \times 10^{-6} \Delta k/k/\text{kw}$, the value of A_{rb}^0 necessary to satisfy Eq. 28 is then $2.8 \times 10^{-6} \Delta k/k/\text{kw}$. The amount of radial fuel displacement corresponding to a power coefficient of this magnitude may be found from Eq. 32. The change effected in the core radius through rod bowing in going from zero to 470 kw is

$$\Delta R = \frac{R}{0.6} \times A_{rb}^0 \times 470 \quad (42)$$

Substitution of the appropriate values into Eq. 42 results in a value of $\Delta R = 8.1 \text{ mils}$. Rod-bowing calculations based on a model in which the rod is fixed at the bottom by the grid plate and at the top by the shield plate result in the conclusion that the value of 8.1 mils is reasonable.

2. Power Coefficient, B_0

Assuming the degree of unconstrained expansion of the shield plate is 0.75 (from Appendix A), it can be shown that the angle of incline for outer row ligaments is given by

$$\alpha = \frac{.75 \delta}{4} \quad (43)$$

where δ is the differential expansion between lower and upper surfaces for a given temperature differential across the plate. For an effective plate radius of 3.7 in. and a value of $17 \times 10^{-6} \Delta L/L/^\circ\text{C}$ for the linear expansion coefficient of stainless steel $\alpha = 12 \times 10^{-6}$ radians per degree centigrade temperature differential across the plate. From Table IV it may be seen that a temperature wave with a half-amplitude of 6.30°C results in a maximum temperature differential across the plate of 4.07°C at 0.0376 c/sec (close to the frequency at which the maximum temperature differential across the plate occurs). The angle of incline per degree centigrade change in core outlet temperature is then $12 \times 10^{-6} \times 4.07/6.30$ or 7.8×10^{-6} radians/ $^\circ\text{C}$. Since the temperature differential across the core at 470 kw is 92°C , $\Delta T/\Delta P = 0.20^\circ\text{C/kw}$ and $\Delta \alpha/\Delta P = 1.56 \times 10^{-6}$ radians/kw. From simple geometric considerations it can be shown that the physical movement of outer shield plate bearing points for a one kw power change is 0.006 mils. Assuming no amplification of motion and that a one-mil ligamental displacement at plate level is sensed as a 0.5-mil displacement of fuel at core center, it follows that the power coefficient B_0 is given by

$$B_0 = \frac{-0.3 \Delta R}{R \text{ kw}} = \frac{-0.3 \times 6 \times 10^{-6}}{3.7} = -0.50 \times 10^{-6} \Delta k/k/\text{kw} \quad (43)$$

A comparison of this value with the quasi-analytically determined value of $1.7 \times 10^{-6} \Delta k/k/\text{kw}$ reveals an approximately threefold discrepancy. The treatment above, however, fails to consider the

existence of a strong amplifying mechanism which may account for the magnitude of the discrepancy. A discussion of this mechanism is given in Sect. H-4.

3. The Power Coefficient, C_0

Assuming a value of 0.75 for the degree of unconstrained expansion (Appendix A) and a 1:0.5 ratio between displacements at plate and core levels, the power coefficient C_0 expected from thermal, mechanical, and neutronic considerations is given by

$$C_0 = \frac{-.3 \times .75 \Delta R}{R \Delta P} = -.3 \times .75 \times \frac{\Delta L}{L} \times \frac{\Delta T}{\Delta P} = -.77 \times 10^{-6} \Delta k/k/kw. \quad (44)$$

Since the quasi-analytical-determined value of C_0 amounted to $2.5 \times 10^{-6} \Delta k/k/kw$, the calculated value of C_0 is too small by a factor of three to explain the experimental results. The reason for this discrepancy undoubtedly lies in the failure to consider the existence of amplifying mechanisms.

4. The Effects of Amplifying Mechanisms

In the development given thus far special emphasis has been placed on an attempt to explain the extremely complex frequency dependence of the experimental feedback. Since the postulated model satisfactorily explains the general structure of the feedback (and transfer function), there seems little doubt that the structural member responsible for the delayed feedback is the shield plate. The results of heating experiments conducted on a perforated plate of nearly identical design strongly substantiate this conclusion. The failure of the model to explain the magnitude of the feedback is believed to be the result of assumptions which are overly conservative.

To explain the discrepancy it is necessary to reconsider the assumptions upon which calculations of B_0 and C_0 were based:

- (1) that the second normalization of experimental data is justifiable,
- (2) that a 1:0.5 ratio exists between displacements occurring in the

shield plate and core, respectively, (3) that dishing of the shield plate may be described as a simple arc, (4) that ligamental expansion is somewhat constrained, and (5) that the effects of amplifying mechanisms may be neglected. For the most part the first four assumptions are reasonable. While these assumptions could be biased to some extent in one direction or another, it seems unlikely that any justifiable change in any of these would resolve the discrepancy. The assumption that the effects of amplifying mechanisms may be neglected, however, may be seriously compromising.

The most likely mechanism for amplifying ligamental motions is one which involves possible bearing points along the lower outer edges of the seal plate perforations. An illustration of this possibility is given in Fig. 25. The mechanism relies essentially on the possibility that the extremely strong temperature differential across the outer (blanket) portion of the seal plate causes the plate to assume the shape of an inverted dish. As a consequence of the dishing the lower outer edges tilt slightly upward and perhaps a little inward. In this position they effectively provide ideal pivotal bearing points for rod extensions. The credibility of the mechanism relies on whether or not a substantial fraction of the rod extensions are in actual physical contact with these bearing points.

The nominal diameters of the perforations and rod extensions in this region are 0.460 and 0.438 in., respectively, giving a nominal diametral clearance of 22 mils. From purely statistical considerations it follows that a certain unspecified fraction of the total number of rods will accidentally bear at these points.

Another mechanism tending to orient the rod extensions against these bearing points is concerned with expansive effects in the second and third shield plates. Since these and all other contiguous structural members (tank wall, tie rods, etc.) in the region above the outlet spacer are bathed in coolant at the outlet temperature their expansions will be unconstrained. Increases in

reactor power will effectively move outer bearing points outward. The seal plate bearing points are much less sensitive to power changes since the outer lower surface of the seal plate is always bathed in coolant at the inlet temperature. (Only 0.20 in. of stainless steel separates the bearing points from inlet coolant.) The effect of power increases on seal plate bearing points may actually be a slight inward movement caused by additional dishing as the temperature differential across the blanket portion of the plate increases.

Finally, rod bowing may be (and probably is) sufficient to form a slight reverse bow in the rod extensions above the shield plate bearing points (see Fig. 25). Contact between rod extensions and seal plate bearing points would require a bending of about 1.3 mils/in., an amount not at all implausible.

While the arguments outlined above apply more rigorously to rod extensions penetrating the outer row of holes, feedback effects arising from the movement of these rods are particularly important in view of the strong flux gradient at the edge of the core.

Assuming that rod extensions are oriented such that they are in physical contact with the seal plate bearing points any displacement effected in the rod extensions by shield plate flexing would be amplified at core level by a factor of roughly two. Displacements effected by the slower overall expansion of the shield plate ligaments would also be amplified by the same factor.

If the seal plate bearing points tilt upward and inward as the result of a power increase they would force the rod extensions inward. Pivotal action at the shield plate bearing points would lead to an additional two-fold amplification of motion. Unfortunately, the importance of this effect is practically impossible to assess without the benefit of extremely elaborate calculations of shield plate deformation.

Because of the time required for an increase in coolant temperature to be sensed by the outer edge of the seal plate, the effects of seal plate bearing point motion would not influence the magnitude of the plate flexing power coefficient, B_0 . Feedback effects originating as the result of such motion should more properly be considered through the inclusion of a third transport lag term in Eq. 27. Any increase in the overall value of the steady state power coefficient from this source would, of course, result in a reduction in the magnitudes of C_0 and B_0 required to explain the experimental feedback.

Admittedly, the inward motion of seal plate bearing points following a power change is highly speculative and accordingly cannot be considered seriously as a means of explaining the discrepancy between quasi-analytic and calculated power coefficient values. The amplifying mechanism based on the preferred orientation of rod extensions with respect to seal plate bearing points is, however, not only much more plausible but undoubtedly does exist to some extent. Whether the degree of orientation is complete is a question which cannot be answered with the information available. If the degree of orientation is high, and strong arguments can be cited to support such a postulate, the discrepancy between experimental and calculated results is reduced considerably. For a completely oriented system minimum values of the power coefficients B_0 and C_0 would be 1.00 and $1.60 \times 10^{-6} \Delta k/k/kw$, respectively, compared with the respective quasi-analytically determined values of 1.70 and $2.56 \times 10^{-6} \Delta k/k/kw$. Additional amplification arising from the tilting of seal plate bearing points would reduce the discrepancy even further. In view of many uncertainties involved, particularly those involving the second normalization of the experimental transfer function data and those associated with the assumptions upon which the calculated values were based, a disagreement of this magnitude is not surprising. Considerations of the fact that the postulated mechanism explains the frequency dependence of the feedback and comes within a reasonable

margin of explaining its amplitude lead to the conclusion that the lower shield plate was the structural member responsible for the Mark II delayed negative coefficient.

J. Explanation of Experimentally Observed Phenomena

To be considered valid the mechanism and its associated mathematical model must also account for all experimentally observed phenomena which have a bearing on the instability.

1. Isothermal Temperature and Power Coefficients of Reactivity

The isothermal temperature coefficient of Mark I was measured to be $31 \times 10^{-6} \Delta k/k/^{\circ}\text{C}$,⁽¹⁰⁾ a value which compares favorably with that of $28 \times 10^{-6} \Delta k/k/^{\circ}\text{C}$ measured for Mark III.⁽¹⁾ Essentially no difference exists between these values.

The effect of an isothermal temperature change in Mark II was sensed as a reactivity effect through unconstrained expansive actions in the grid plate and in the lower shield plate. After closure of nominal clearances at these points all rod tips and rod extensions were preferentially gathered along the inner edges of their respective locating holes. Under these conditions the rods were radially coupled at top and bottom and, as a consequence of the coupling, expansive actions associated with the grid plate and shield plate were transmitted to the rods. In Mark III the system of stabilizing ribs provided a very strong radial coupling and the temperature coefficient, as expected, was also strong.

Superficially it may seem that the magnitudes of the steady state power coefficients in both Mark II and Mark III would also be approximately the same. While no direct measurements of the Mark II power coefficient were ever made, a reasonable estimate may be obtained from the results of the quasi-analytical treatment of Sect. H. From the values $A_0 = 2.13$ and $C_0 = -2.56 \times 10^{-6} \Delta k/k/\text{kw}$ ($B_0 = 0$ at $w = 0$) it is clear that the steady-state power coefficient

is $-0.43 \times 10^{-6} \Delta k/k/kw$. In Mark III on the other hand, the steady-state power coefficient under comparable conditions of power and flow rate amounted to $-4.05 \times 10^{-6} \Delta k/k/kw$.⁽¹⁾ Thus, while the isothermal temperature coefficients are approximately the same, the power coefficients differ by a factor of roughly ten. The explanation of this rather large difference is concerned with rod bowing which in the case of Mark II was particularly marked and in the case of Mark III was essentially non-existent. In Mark II a considerable freedom of radial motion permitted the rods to bow in the direction of high flux. In Mark III the system of stabilizing ribs effectively prohibited radial motion and greatly minimized bowing effects.

As a result of the Mark III tests it was established conclusively that inward rod bowing is a consequence of special conditions of end restraint. From these tests it was learned that rod bowing does not necessarily result in a strong positive effect unless the fuel rods are rigidly fixed at both top and bottom. Because of the clearances afforded the rod tips and rod extensions in Mark II, it is at first difficult to appreciate why rod bowing contributed so heavily to the power coefficient. As discussed in Sect. D, rod tips and rod extensions were preferentially oriented such that any deformation resulting from an increase in power would be manifested in the form of an inward bow.

2. The Existence of Prompt Positive and Delayed Negative Components

Immediately following a reduction in flow the power would increase, pass through a maximum and would eventually decrease to some lower equilibrium level. Such behavior can be explained in terms of the postulated model. The immediate result of a flow decrease would be a sudden increase in the coolant temperature throughout the core. Since the negative effects, i.e., axial and radial fuel expansion and coolant expulsion, were insufficiently large to cancel the positive

effect from rod bowing the sudden increase in coolant temperature was sensed by the core as an addition of reactivity. The power would, accordingly, increase. Approximately 6.5 sec after the flow reduction the temperature signal would reach the shield plate. The expansive motions of the shield plate would then increase the core radius through the outward movement of rod extensions. The reactivity loss resulting from ligamental expansions would override the smaller positive effect and the power would eventually drop below the original level. The sequence of events occurring during and after a flow increase is exactly the converse of the above.

3. Excursion Results

In the last experiment conducted on Mark II the main coolant flow was reduced to zero and the reactor was placed on a positive period. As the coolant heated additional reactivity was fed back to the reactor through the influence of the prompt positive term of Eq. 27. Since the convection of heat to the shield plate was greatly reduced, the normally overriding negative effects associated with shield plate expansion essentially vanished. The dynamic behavior of the reactor under these conditions was governed almost entirely by a feedback given by the first term of Eq. 27.

It is interesting to note that at some flow rate between zero and 108 gpm the prompt positive coefficient associated with the core and the delayed negative coefficient associated with the structure would exactly cancel. The dynamic behavior of the reactor when placed on a positive period would then be governed by a zero power coefficient. The power would increase indefinitely with a period which would neither increase nor decrease.

4. The Absence of Higher Order Resonances in the Transfer Function Amplitude

The shape of a feedback governed by a constant power coefficient and a long transport lag dependence is one of a spiral which may complete several revolutions about the origin before

decreasing to zero at high frequencies. For each entry into the III quadrant a partial phase and amplitude cancellation with the inverse zero power gain $1/G_0$ always occurs. The effect would be manifested in the transfer function through the occurrence of higher order resonances lying on the high frequency side of the fundamental. The fact that such peaks were not observed whereas a simple transport lag model indicates their existence has always been perplexing. The model described by Eq. 27 resolves this ambiguity. The second term which describes the feedback associated with plate flexing approaches zero rapidly at the higher frequencies. As the frequency increases the amplitude of the temperature perturbation arriving at the shield plate is considerably reduced. As a consequence the amplitude of the oscillating temperature differential across the plate decreases strongly with frequency. The magnitude of the feedback from shield plate flexing is accordingly described by a power coefficient which is a strong function of the oscillating frequency.

The power coefficient in the third term of Eq. 27 is also affected by the amplitude attenuation of the coolant temperature wave in its passage between core and shield plate. At the higher frequencies the amplitude of the temperature oscillations in the vicinity of the shield plate are so strongly attenuated that little or no ligamental motion is effected.

Because of the strong non-linear behavior of both B_0 and C_0 the feedback as a function of frequency approaches zero much more rapidly than for the case of truly constant power coefficients.

5. The Shift of Resonant Peaks to Higher Frequencies at Higher Flow Rates

As the flow rate increases (at constant power) the physical transit time of coolant between core and shield plate decreases. The effects of the decrease are manifested through the influence of the cyclic operator $e^{-i\omega\tau(\omega)}$ (Eq. 27) as a strong increase in the phase of the feedback for all frequencies. As a consequence

constructive interference of the feedback with the input reactivity occurs at a higher frequency.

K. Conclusions

1. Because the model predicts the general shape of the experimentally measured feedback and comes within a reasonable margin of explaining the feedback magnitude, it may be concluded that the structural member responsible for the delayed negative power coefficient was the lower shield plate. There seems little doubt that any credible interpretation of the feedback must involve the concept of a delayed feedback which almost certainly must have originated at some structural member located downstream from the core. A careful scrutiny of all downstream structural members supplemented by Storrer's attenuation concepts narrows the region of suspicion to that included between the lower shield plate and the seal plate. The facts that thermally-induced ligamental motions have actually been observed in such a plate and that the maximum temperature differential across the shield plate occurs close to the natural resonant frequency of the reactor strongly support this conclusion.

2. The instability of Mark II may be directly attributed to the combination of two undesirable features both of which are easily eliminated through elementary changes in mechanical design. The lack of radial coupling between fuel rods was detrimental in two respects: rod bowing was permitted and the normally strong prompt negative feedback from radial fuel expansion was greatly weakened. The consequence was an overall prompt positive power coefficient of reactivity. The other important feature was, of course, the shield plate system which was responsible for the delayed negative power coefficient. The combination of the two effects resulted in a feedback which permitted constructive interference of periodic

reactivity insertions. It is almost certain that the addition of stabilizing ribs to the fuel rods would have eliminated the instability since radial motion of fuel from rod bowing and from shield plate ligamental motions would have been denied. For a rigidly coupled system of rods the dynamic behavior of the reactor would have been dominated by an overall prompt and strongly negative power coefficient of reactivity.

Appendix A

THE EFFECT OF TEMPERATURE TRANSIENTS ON SHIELD PLATE LIGAMENTAL MOTIONS

The results of attempts to detect motions induced by temperature variations in the ligaments of a dummy two inch shield plate have demonstrated conclusively that such motions do exist.

The first attempts to observe motions involved heating the ligaments with the exhaust from a hydrogen blast lamp. To prevent the heating of the outer portion of the shield plate (which remained at constant temperature in the reactor) a pyramidal shroud was installed beneath the plate in such a manner that the shroud effectively channeled the heated air through the central section and left unaffected areas outside the hexagonal annulus (see Fig. 5). To limit the extent of heating the blast was interrupted after approximately four seconds. Temperatures in the central portion of the plate rose rapidly to about 200°C. Measurements of expansive motions were carried out with a dial indicator butted against a quartz rod which in turn was sealed into a stainless steel insert machined to provide a flush fit with the fuel rod perforations. Asbestos shielding was installed around the measuring equipment to prevent extraneous heating effects. The responses of approximately 20 holes were tested. Although no two responded exactly alike, a general pattern of behavior was clear. With the indicator butted against the outside of the quartz rod (approximately 4 in. above the plate) heating the ligaments was followed immediately, i.e., within 2-3 sec by a movement of the rod inward. After approximately 5-6 sec the inward motion ceased and the rod began to move outward at a much slower rate coming to rest eventually at a position approximately 4-5 mils outward from its initial position. With the indicator bearing on the top of the rod, i.e., in a position measuring vertical deflections, the initial movement was 3-4 mils downward over a period of approximately 2-3 sec followed by a much slower return to a position

approximately 4-5 mils above its initial elevation. The explanation of this behavior is clear. Immediately following the 4-sec heat blast the first motion consists of a warping or a dishing of the plate. A dial indicator butting against the outside of the rod senses the motion as an inward motion. An indicator mounted on top of a rod senses the motion as a decrease in elevation. The dishing is apparently the result of preferential expansion in the lower portion of the ligaments. As the entire collection of ligaments reaches a more or less equilibrium temperature the dishing effect disappears, i.e., the axes of the perforations return to vertical. Such actions are consistent with the recoveries noted. The overshoot in elevation is the result of an overall vertical expansion which essentially raises the rod and bearing point of the indicator. The overshoot in radial displacement is a consequence of a much slower expansion of the hexagonal annulus which allows ligamental stresses to be at least partially relieved. Observations carried out on a $\frac{1}{8}$ -in. stainless steel pin positioned in the annulus substantiated this conclusion. Indicator measurements carried out as a function of elevation along the outside of the rod verified the existence of the dishing effect, i.e., significantly larger inward displacements were measured at the upper portions of the rod. Other measurements carried out on the right and left hand sides of the rod demonstrated that lateral motion also occurred.

In another series of experiments hot water was used as a heat transfer medium. In these experiments water at approximately 50°C was stored in a tank contiguous with the lower broached portion of the shield plate. An overflow chamber fitting around the periphery of the hexagonal annulus was mounted on the upper side. By gasketing mating surfaces hot water could be forced upward through the rod perforations into the overflow chamber without sensibly affecting the temperature of the outer portion of the plate. Again dial indicators were used to measure displacements by expansive motions. The results of measurements conducted on approximately 25 rod locations confirmed the results obtained in the heated air

experiments. With an indicator butting against the outside of the quartz rod at an elevation approximately 4 in. above the plate the initial motion was consistently inward. In general, the initial motion consisted of about 0.5 to 1.0 mils over a period of 2-3 sec followed by a somewhat slower recovery to the initial position. In nearly all cases an overshoot, i.e., an outward motion, of approximately one mil taking place over a period of 5-10 sec was observed. By reducing the flow of water through the plate the initial tilting could be exaggerated and the time required for tilting, recovery and overshooting could be increased. It was also observed consistently that the tilting effect was more pronounced in the outer two or three rows of rod perforations.

The results of experiments in which the bearing point of the indicator was placed on top of the rod, i.e., in a position measuring vertical movement, were consistent with the results of similar experiments conducted with heated air. The initial motion consisted of a 0.2 to 0.3-mil dip over a period of 2 to 3 sec followed by a slower recovery to initial elevation. In most cases a small overshoot of approximately 0.3 mils was observed.

The results of experiments conducted with two dial indicators both bearing against the inside of the rod but with one at 4-in. elevation above the plate and the other at 8 in. demonstrated conclusively that the initial action consisted of a tilt. In nearly all experiments of this nature the upper indicator registered significantly larger radial displacements that was the case for the lower indicator.

From these results it is clear that the immediate consequence of a sudden increase in the temperature of coolant flowing through the ligaments is a rapid dishing action which tilts the axes of the perforations towards the normal. Since the tilting is more pronounced along the periphery of the perforations, it seems likely that the thin (approximately 0.20 in.) hexagonal annulus is also involved in the dishing action. Apparently the temperature gradient

established across the plate effects a preferential expansion not only of the ligaments but of the annulus as well. The recovery and overshoot are consistent with the behavior expected for a system which eventually comes to a higher (and uniform) equilibrium temperature.

Attempts to observe the reverse effect, i.e., a convex (upward) dishing, were successful. With the ligaments at an equilibrium temperature of 50°C the hot water was drained suddenly and was replaced with water at approximately 25°C. As the cold water reached the lower surface of the ligaments an indicator bearing against the outer edge of the quartz rod registered a sudden outward motion followed by a slow return to its initial position and a somewhat slower displacement towards the center of the plate. Experiments conducted with two dial indicators, one bearing at 4 in. above the plate and the other at 8 in., demonstrated that the initial outward motion was the result of a convex dishing since the upper indicator registered displacements approximately twice those of the lower.

From the results of these experiments it is possible to estimate a crude value for the degree of unconstrained expansion. For a change of 25°C the average amount of overall radial displacement associated with holes in the outer two rows amounted to approximately 1.2 mils, in contrast with 1.57 mils expected for a completely unconstrained system. The degree of unconstrained expansion is then approximately $1.2/1.57$ or 0.75.

Appendix B

EVALUATION OF PHYSICAL PARAMETERS

A. Time Constant Calculations

In order to evaluate Eqs. 8 and 16 it is necessary to establish time constant values for each material and each region where the time constant is defined by the following

$$\tau_x = \frac{C_p}{h} \quad (B-1)$$

and where C_p is the total heat capacity per unit length of material x and h is the overall heat transfer coefficient per unit length for the same material. Physically, τ_x is analogous to the RC time constant of an electrical circuit. More specifically τ_x is the time required, after a perturbation, for the temperature of a given component to reach 63.2% of its steady-state value.

B. Film Heat Transfer Coefficients

1. Regions I-VI. In these regions coolant flows in long vertical channels. The expression used for determining the various film coefficients is⁽¹⁴⁾

$$h_f' = \frac{K}{D_e} \left[7 + 0.025 (N_{Re} \times N_{Pr})^{0.8} \right] \text{ Btu/hr-ft}^2 \text{ F}^\circ \quad (B-2)$$

$$h_f = h_f' \times S \quad \text{Btu/hr-ft F}^\circ \quad (B-3)$$

where S is the flow perimeter of all coolant channels.

2. Regions VII and VIII. In these regions the coolant flow is essentially perpendicular to both fuel and blanket rod-handles, which in this special case, are of circular cross-section. The expression used for determining the film coefficients is⁽¹⁴⁾

$$h_f' = 1.17 \frac{K}{D_e} (N_{Re} \times N_{Pr})^{0.8} N_{Pr}^{1/3} \quad (B-4)$$

The value for coolant velocity used in determining N_{Re} is the minimum value for the particular region. This procedure results in a conservative estimate of the film coefficient.

C. Method of Calculating the Equivalent Diameter (D_e) of Flow Channels in the Inlet Plenum (Region V)

In this region the fuel rod-handles are fluted. Since no provisions were made for preferential angular orientation of fuel rod tips in the bottom plate, it is assumed that the orientation of one rod-handle with respect to its neighbors is random. The equivalent diameter in this region is assumed to be equal to four times the total coolant flow area divided by the total flow perimeter of all rod-handles. From these assumptions it may be shown that $D_e = 0.44$ in. A comparison of this value with that obtained for the shield plate channels ($D_e = 0.15$ in.) shows that in the outlet plenum the equivalent diameter is larger by a factor of about three. This is to be expected in that the flow area per channel in the inlet plenum is increased by a factor slightly larger than three because of the absence of plate ligaments.

D. Heat Transfer Conduction Coefficients

1. Regions I, II, III, and VIII. In these regions the rods and rod-handles are solid and circular in cross section. The expression used for determining the conduction coefficient is⁽¹¹⁾

$$h'_c = 8\pi K \quad \text{Btu/hr-ft } F^\circ \quad (\text{B-5})$$

$$h_c = nh'_c \quad (\text{B-6})$$

where n is the total number of rods.

2. Regions IV, V, VI, and VII. The coefficients of conduction for the rod-handles and plate ligaments in these regions may be calculated from

$$h_c = \frac{KS}{d} \quad (\text{B-7})$$

where S is the flow perimeter and d is the mean distance for heat conduction. For the fluted portion of the rod-handles d is estimated to be 0.030 in. (see Fig. 6). For the shield plate ligaments d is assumed to be one-fourth of the ligamental thickness since coolant flows on both sides of the ligaments. It is also assumed that the coolant present in the 0.011 in. annular segments between the fluted rod-handles and plate ligaments is stagnant and that no heat is transferred from the coolant in these regions.

The fuel rod-handles in Region VII are circular, hollow and gas-filled. On the assumption that all heat conducted into the rod is carried by convection into the upper portion of the handles, the distance d for heat conduction is assumed to be the tube thickness.

E. Physical Properties of NaK and 347-SS at 300°C

	C_p	ρ	k	μ
	$\frac{\text{Btu}}{\text{lb m } ^\circ\text{F}}$	$\frac{\text{lb m}}{\text{ft}^3}$	$\frac{\text{Btu}}{\text{hr ft } ^\circ\text{F}}$	$\frac{\text{lb m}}{\text{ft hr}}$
NaK	0.215	49.9	14.9	0.614
347-SS	0.12	498	10.76	-

Appendix C

Calculations of the phase and amplitude reduction of a temperature wave in its passage through the shield plate (Region IV) and seal plate (Region VI) are complicated by the presence of an additional inert medium, the relatively thin 34 mil ligaments. Since Storrer's expressions⁽¹¹⁾ apply strictly to a single inert medium (rod handles in this case), it is necessary to extend the scope of his mathematical developments.

The following transient heat balance expressions may be written for each component of the shield plate region:

For the rod handles:

$$\frac{\tau_{f_1} \left[\frac{\partial T_{f_1}(z,t)}{\partial t} \right]}{\partial t} = T_C(z,t) - T_{f_1}(z,t) \quad (C-1)$$

For the shield plate ligaments:

$$\frac{\tau_{f_2} \left[\frac{\partial T_{f_2}(z,t)}{\partial t} \right]}{\partial t} = T_C(z,t) - T_{f_2}(z,t) \quad (C-2)$$

For the coolant:

$$\tau_C \left[\frac{\partial T_C(z,t)}{\partial t} + \frac{v \partial T_C(z,t)}{\partial z} \right] = T_{f_1}(z,t) + T_{f_2}(z,t) - 2 T_C(z,t) \quad (C-3)$$

where the above parameters are defined as

- τ_{f_1} = the time constant for heating or cooling the rod handles
- τ_{f_2} = the time constant for heating or cooling the shield plate ligaments
- τ_C = the time constant for heating or cooling the NaK
- $T_C(z,t)$ = the bulk coolant temperature
- $T_{f_1}(z,t)$ = the average temperature of the rod handles
- $T_{f_2}(z,t)$ = the average temperature of the ligaments
- v = the coolant velocity

Assuming that power changes in the core and temperature changes in the shield plate region vary sinusoidally, the following expressions apply

$$T_{f_1}(z,t) = X_1(z)e^{i\omega t} + T_0 \quad (C-4)$$

$$T_{f_2}(z,t) = X_2(z)e^{i\omega t} + T_0 \quad (C-5)$$

$$T_c(z,t) = Y(z)e^{i\omega t} + T_0 \quad (C-6)$$

where $X_1(z)$, $X_2(z)$ and $Y(z)$ are the non-complex amplitudes of the temperature oscillations and T_0 is the steady-state temperature. Differentiating and substituting into Eqs. C-1, C-2 and C-3 yields

$$\tau_{f_1} (i\omega X_1 e^{i\omega t}) = T_c - T_{f_1} \quad (C-7)$$

$$\tau_{f_2} (i\omega X_2 e^{i\omega t}) = T_c - T_{f_2} \quad (C-8)$$

$$T_c (i\omega Y e^{i\omega t} + v Y' e^{i\omega t}) = T_{f_1} + T_{f_2} - 2 T_c \quad (C-9)$$

Eliminating $e^{i\omega t}$ from these expressions

$$\tau_{f_1} i\omega X_1 = (T_c - T_{f_1}) e^{-i\omega t} = Y - X_1 \quad (C-10)$$

$$\tau_{f_2} i\omega X_2 = (T_c - T_{f_2}) e^{-i\omega t} = Y - X_2 \quad (C-11)$$

$$T_c (i\omega Y + v Y') = (T_{f_1} + T_{f_2} - 2T_c) e^{-i\omega t} = X_1 + X_2 - 2Y \quad (C-12)$$

Eliminating X_1 and X_2 from C-12 gives

$$\tau_c (i\omega Y + v Y') = \frac{Y(1 - i\omega\tau_{f_1})}{1 + \omega^2 \tau_{f_1}^2} + \frac{Y(1 - i\omega\tau_{f_2})}{1 + \omega^2 \tau_{f_2}^2} - 2Y \quad (C-13)$$

Solving C-13 for Y yields

$$Y_z = Y_0 e^{-\lambda \frac{z}{v}} \quad (C-14)$$

$$\lambda = \frac{1}{\tau_c} \left[i \omega \tau_c - \left\{ \frac{1 - i \omega \tau_{f_1}}{1 + \omega^2 \tau_{f_1}^2} + \frac{1 - i \omega \tau_{f_2}}{1 + \omega^2 \tau_{f_2}^2} - 2 \right\} \right] \quad (C-15)$$

Separating C-15 into real and imaginary components

$$\lambda = \frac{\omega^2 (\tau_{f_1}^2 + \tau_{f_2}^2) + 2 \omega^4 \tau_{f_1}^2 \tau_{f_2}^2}{\tau_c (1 + \omega^2 \tau_{f_1}^2)(1 + \omega^2 \tau_{f_2}^2)} + i \omega \left[1 + \frac{\tau_{f_1}}{\tau_c (1 + \omega^2 \tau_{f_1}^2)} + \frac{\tau_{f_2}}{\tau_c (1 + \omega^2 \tau_{f_2}^2)} \right] \quad (C-16)$$

where

Y_z = complex temperature amplitude of the coolant in the shield plate region

Y_0 = non-complex temperature amplitude of the coolant at the inlet of the plate

z = vertical distance at which the coolant temperature is evaluated

v = coolant velocity

ω = power oscillation frequency, rad/sec

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Figure Captions

- Fig. 1. Cutaway View of EBR-I, Mark I and Mark II Loadings
- Fig. 2. Cross-Sectional View Through Mark I and Mark II Cores
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Fig. 24. Comparison of Experimentally Measured and Calculated
Feedbacks

Fig. 25. Preferential Orientation of Rod Extensions and Seal. Plate
Bearing Points

Fig. 1

Cutaway View of EBR-I, Mark I and Mark II Loadings

Fig. 2

Cross-Sectional View Through Mark I and Mark II Cores

Fig. 3

Schematic View of EBR-I Structure

Fig. 4
EBR-I, Mark II Fuel Rod

Fig. 5

EBR-I Shield Plate

Fig. 6
Horizontal Cross Section Through Lower
Shield Plate and Rod Handle

NOTE 8
 RODS PREFERENTIALLY
 GATHERED AT INNER
 EDGE OF GRID AND
 LOWER SHIELD PLATES
 DUE TO RADIAL EXPANSION.

Fig. 8

Typical Fuel Rod Orientation at 150°C, Zero Power

Fig. 9

Typical Fuel Rod Orientation at One-Third Full Power and One-Third Flow, 150°C Inlet Temperature, 250°C Outlet Temperature

Fig. 10

Upper Structure Contact of a Bowed Fuel Rod

Fig. 11

Typical Fuel Rod Orientation During Increasing Portion of Oscillating Power Cycle or Immediately Following a Step-Power Increase

Fig. 12

Typical Fuel Rod Orientation During Decreasing Portion of Oscillating Power Cycle or Immediately Following a Step-Decrease in Power

Fig. 13

Temperature, Upper and Lower Surfaces of Shield Plate

Fig. 14
Temperature Differential Across Shield Plate

Fig. 15
Maximum Temperature Differential Across Shield Plate

Fig. 16
Unnormalized Transfer Function Amplitude, 470 kw, 108 gpm

Fig. 17

Unnormalized Transfer Function Phase, 470 kw, 108 gpm

Fig. 18

Feedback for Unnormalized Data, 470 kw, 108 gpm

Fig. 19
Zero Power Data, 2 kw, 108 gpm

Fig. 20
Transfer Function Amplitude, First Normalization

Fig. 21
Transfer Function Amplitude, Second Normalization

Fig. 22

Feedback, 470 kw, 108 gpm, Second Normalization

Fig. 23
Feedback Components

Fig. 25

Preferential Orientation of Rod Extensions
and Seal Plate Bearing Points

Table I
SUMMARY OF PHYSICAL PARAMETERS - 470 kw, 108 gpm

Region	h Btu/hr-ft °F	z in.	v in./sec	τ_c Coolant (sec)	τ_f Rod (sec)	τ_p Plate (sec)	Material	$\frac{z}{v}$ (sec)
I Upper Half of Core	4.21×10^4	4.25	28.6	0.093	0.84	-	347 SS NaK U - 2 w/o Zr	0.298
II Upper Axial Blanket	4.21×10^4	7.5	28.6	0.093	0.84	-	347 SS NaK U - 2 w/o Zr	0.260
III Rod Handles	3.46×10^4	1.69	28.6	0.112	0.77	-	347 SS NaK	0.059
IV Shield Plate	20.8×10^4 5.94×10^4 14.8×10^4	4	20.2	0.021 - -	- $0.365(\tau_{f1})$ -	- - $0.109(\tau_{f2})$	NaK 347 SS 347 SS	0.198
V Inlet Spacer	4.1×10^4	3.81	13.4	0.20	0.53	-	NaK 347 SS	0.284
VI Seal Plate	20.8×10^4 5.94×10^4 14.8×10^4	2	20.2	0.021 - -	- $0.365(\tau_{f1})$ -	- - $0.109(\tau_{f2})$	NaK 347 SS 347 SS	0.099
VII Inner Section Outlet Spacer	3.5×10^4	4	10.6 (minimum)	0.125	0.70	-	NaK 347 SS	0.160 ^a
VIII Outer Section Outlet Spacer	3.39×10^4	4	32.8 (minimum)	0.271	4.17	-	NaK 347 SS	0.580 ^a

Note: Regions IV and VI have three values for h , one each for coolant, rods and plate.

^aResidence time

Table II

PHASE LAGS AND ATTENUATION FACTORS REGIONS I-VIII

f c/sec	ω r/sec	Region I Core		Region II Blanket		Region III Rod Handles		Region IV Shield Plate	
		$\phi_{1/2}$	μ_1	ϕ_2	μ_2	ϕ_3	μ_3	ϕ_4	μ_4
001	00628	15	1 000	935	1 000	167	1 000	1 56	1 000
005	0314	2 0	1 000	4 69	998	830	1 000	7 90	999
0089	0560	4 95	993	8 28	994	1 48	999	13 9	996
0185	1162	7 80	982	16 9	974	3 06	996	29 1	982
0233	1465	9 50	970	21 6	959	3 80	993	36 6	972
0284	178	11 8	940	26 0	940	4 65	990	44 5	958
0336	212	14 0	920	30 8	915	5 50	986	52 5	941
0376	236	16 0	880	34 0	896	6 10	983	59 2	927
0484	304	18 2	858	42 8	845	7 80	973	76 0	879
0584	368	23 7	771	50 6	787	9 15	961	92 8	843
078	490	28 0	698	64 0	673	11 5	935	122	741
098	615	37 6	595	74 3	560	13 6	904	152	620
150	942	40 0	394	92 5	344	17 5	835	233	353
f c/sec	ω r/sec	Region V Inlet Spacer		Region VI Seal Plate		Region VII Core Section Outlet Spacer		Region VIII Blanket Section Outlet Spacer	
		ϕ_5	μ_5	ϕ_6	μ_6	ϕ_7	μ_7	ϕ_8	μ_8
001	00628	37	1 000	84	1 000	38	1 000	3 20	999
005	0314	1 90	1 000	4 25	1 000	1 89	1 000	15 9	983
0089	0560	3 30	999	7 48	998	3 38	998	27 2	945
0185	1162	6 90	995	15 5	991	6 96	991	48 8	790
0233	1465	8 70	992	19 6	985	8 73	987	55 7	560
0284	178	10 4	988	23 8	978	10 6	980	60 6	472
0336	212	12 5	982	28 4	969	12 5	972	64 0	392
0376	236	13 7	978	31 4	962	13 9	966	64 5	350
0484	304	17 5	964	40 3	938	17 7	945	65 7	267
0584	368	21 2	948	48 5	909	21 0	920	64 5	221
078	490	27 6	911	63 8	841	26 7	866	61 8	190
098	615	33 8	864	78 8	782	32 2	800	59 5	156
150	942	47 3	718	115	580	42 2	677	59 0	133

Table III

PHASE LAGS AND ATTENUATION FACTORS FOR SHIELD PLATE

f c/sec	ω r/sec	Bottom of Plate		Top of Plate		Phase Lag Across Plate
		Phase Lag ϕ	Amplitude $\times Y_1^a$	Phase Lag ϕ	Amplitude $\times Y_1$	
.001	.00628	1.24	1.000	2.80	1.000	1.56
.005	.0314	7.52	.998	15.4	.997	7.88
.0089	.0560	14.7	.987	28.7	.985	14.0
.0185	.1162	27.8	.953	57.9	.938	30.1
.0233	.1465	34.9	.922	71.5	.898	36.6
.0284	.178	42.5	.875	87.0	.837	44.5
.0336	.212	50.3	.832	102.8	.781	57.5
.0376	.236	56.1	.772	115.3	.716	59.2
.0484	.304	68.8	.705	144.8	.619	76.0
.0584	.368	83.5	.582	176.3	.492	92.8
.078	.490	103.5	.440	225.5	.332	122
.098	.615	124.5	.300	278	.181	154
.150	.942	150.0	.113	383	.040	233

^a Y_1 = non-complex amplitude of temperature wave leaving the core.

Table IV

TEMPERATURE GRADIENT ACROSS SHIELD PLATE

470 kw, 108 gpm, ΔT across core = 92°C

f c/sec	ω r/sec	Temperature Gradient ^a Across Plate	$\Delta n/n^b$	Attenuation Across Core	Core Temperature Amplitude Y_1	Temperature Gradient Across Plate, $^{\circ}\text{C}$
.0089	.056	.242 Y_1	.117	.993	5.35	1.29
.0185	.116	.438	.106	.982	4.80	2.10
.0233	.147	.571	.104	.970	4.65	2.55
.0284	.178	.648	.114	.940	4.93	3.19
.0336	.212	.716	.123	.920	5.20	3.71
.0376	.236	.735	.137	.880	5.55	4.07
.0484	.304	.815	.108	.858	4.73	3.85
.0584	.368	.780	.0865	.774	3.08	2.41
.078	.490	.676	.0660	.698	2.12	1.44
.098	.615	.302	.0556	.595	1.52	.46
.150	.942	.137	.0475	.086	1.17	.01

^a Y_1 = non-complex amplitude of temperature wave leaving the core^b Δn measured as peak to peak value.

Table V

POWER COEFFICIENT FROM SHIELD PLATE FLEXING

 B_0 = maximum value

f c/sec	ω r/sec	Temperature Gradient Across Plate from Table IV, °C	Power Coef- ficient for Shield Plate Flexing
.0089	.056	1.29	.280 B_0
.0185	.1162	2.10	.455
.0233	.1465	2.55	.555
.0284	.178	3.19	.692
.0336	.212	3.71	.808
.0376	.236	4.07	.882
.0484	.304	3.85	.838
.0584	.368	2.47	.522
.078	.490	1.44	.313
.0978	.615	.46	.100
.150	.942	.01	.002

Table VI

VALUES USED IN THE SOLUTION OF EQ. 27

$\tau_p = .109 \text{ sec}$ (calculated value, App. B)
 $\tau_r = 1.0 \text{ sec}$ (assumed value)

f	$\mu_b (\omega)$	$\mu_c (\omega)$	τ_ω
.0089	.280	.985	6.78 sec
.0185	.455	.940	6.35
.0233	.555	.910	6.35
.0284	.692	.875	6.35
.0336	.808	.805	6.35
.0376	.882	.742	6.35
.0484	.838	.660	5.98
.0584	.522	.535	6.15
.078	.313	.381	6.17
.098	.100	.243	5.91
.150	.002	.079	4.95