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PHENOMENOLOGICAL ANALYSIS OF CURRENT LIMITS IN STORAGE RINGS*

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ABSTRACT

We study the limitations on the current which can be stored in a storage ring. A simple model is used for the impedance of the beam environment. We assume that feedback systems are used to their full capacity to provide beam stability and that in the frequency range where feedback can no longer be used Landau damping will determine how much current can be stored in the ring. Estimates are made for proton storage rings and for electron storage rings of interest as synchrotron radiation sources and as colliding beam systems for high energy physics.

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I. INTRODUCTION

In this paper we discuss the limitations on the amount of current which can be stored in a ring. The effects limiting the current are assumed to be the collective instabilities due to the interaction of the beam with the vacuum chamber walls, rf cavities, pick-up electrodes and other devices constituting the environment in which the beam moves. We neglect other effects which, for very high currents, might introduce difficult technical problems, like, for instance, heating of the walls or radiation effects, since we want to study what is the maximum current obtainable under idealized conditions. We will consider mainly the limitations due to high frequency instability, assuming that all low frequency effects can be cured by feedback. We will only use a simplified and approximate description of the instability to obtain an order of magnitude estimate of the beam current and to see how it scales with different machine parameters.

In the next section we describe the model we assume for the beam environment, i.e. the impedance that we will use for the subsequent calculations. In section III we define our criterion for beam stability and in sections III and IV we apply it to the cases of coasting and bunched beams.

II. THE CHAMBER IMPEDANCE

To discuss the impedance seen by a beam in a storage ring, let us define first what are the different regions of interest in the frequency domain. Let ω_0 be the revolution frequency $\omega_B = c/L$ the frequency associated with the bunch length L and $\omega_{co} \approx c/d$ the cut-off frequency of

the vacuum pipe, whose transverse dimension is d . The beam environment is assumed to be a pipe with nearly uniform cross section, plus the rf cavities and beam measuring devices.

We assume that the total chamber impedance is the sum of three contributions:

- 1) the high-Q resonant contributions at low frequency ($\omega \ll \omega_B$);
- 2) a nonresonant broadband impedance extending to frequencies of the order of ω_{co} .
- 3) the resistive wall impedance.

The resonant part of the impedance gives rise to many bunch instabilities or to single bunch multiturn effects. We assume that all these effects can be cured with a feedback system, and in the following we will neglect this part of the impedance.

The broadband, nonresonant impedance is responsible for the high frequency instabilities involving the internal bunch modes, like the head-tail effect or the microwave instability. This is the part of the impedance that will be considered here.

The resistive wall impedance is usually small compared to the resonant impedance at low frequency and to the broadband impedance at high frequency and will also be neglected.

The broadband impedance is produced by discontinuities in the vacuum chamber, bellows, pick-up electrodes and other possible devices present in the beam environment. The characteristics of the longitudinal broadband impedance have been obtained at SPEAR from bunch length measurements.¹ The shape of this longitudinal impedance, is given in

Fig. 1, where we have plotted the real and imaginary parts of $(Z_{\parallel}/\omega)$ versus ω .

This impedance can be approximately described as that of a parallel R, C, L, resonant circuit, having the resonant frequency equal to ω_{co} and a Q of the order of 1. To completely define $Z_{\parallel}$ we only need to know ω_{co} and the shunt impedance, R_o . We then have

$$\frac{Z_{\parallel}(\omega)}{\omega} = \left(\frac{R_o}{\omega_{co}} \right) \frac{(\omega/\omega_{co}) + i Q \left[1 - \left(\frac{\omega}{\omega_{co}} \right)^2 \right]}{\left(\frac{\omega}{\omega_{co}} \right)^2 + Q^2 \left[1 - \left(\frac{\omega}{\omega_{co}} \right)^2 \right]^2} . \quad (2.1)$$

For our instability calculation we want to know the value of $Z_{\parallel}/\omega$ at $\omega = m\omega_o$ and in particular the peak value. For the new generation of storage rings one can assume that $(Z_{\parallel}/\omega)_{MAX} \approx 1\Omega$ and in the following we will use this value.

The chamber irregularities that give rise to $Z_{\parallel}$ will also generate fields which give the beam particles a transverse kick, and can cause unstable, coherent transverse oscillations. The transverse effect of the wall fields is most conveniently described in terms of a "Transverse impedance" Z_T --see W. Schnell²--which can, quite generally, be written in terms of $Z_{\parallel}$ by

$$Z_T(m\omega_o) \approx \frac{2R}{d^2} \frac{Z_{\parallel}(m\omega_o)}{m} , \quad (2.2)$$

where R is the ring radius.

Space-charge effects can be taken into account by adding an additional term to Z_T , namely,³

$$Z_T(\text{space charge}) = R \frac{Z_0}{\gamma} \left(\frac{1}{a^2} - \frac{1}{d^2} \right) \quad (2.3)$$

where γ is the beam energy factor, a is the beam radius and Z_0 is the impedance of free space. We will take for Z_T the sum of the two contributions in (2.2) and (2.3).

III. CURRENT LIMITS FOR UNIFORM BEAMS

To determine the maximum current in a stable beam we use the following assumptions:

- 1) for $\omega < c/d$ we can use feedback systems to stabilize all transverse and longitudinal modes;
- 2) the effect of resonant impedances is also stabilized by feedback;
- 3) frequencies $\omega \geq \omega_{co}$ are important, although $Z_{||}$ starts to decrease at $\omega \approx \omega_{co}$;
- 4) for $\tau \geq \omega_{co}$ the only cure for unstable modes is Landau damping;
- 5) we only consider unstable mode with a rise time, τ , longer than the revolution period, or

$$\frac{1}{\tau} < \frac{\omega_0}{2\pi}$$

thus excluding phenomena like the beam break-up in linear accelerators.

We now write down the conditions for beam stability considering the two cases of a coasting beam (that is, an unbunched beam) or of a bunched beam. For both the coasting and bunched beam case we consider separately the effect of longitudinal and transverse instabilities. For the

longitudinal case we consider essentially the effect of the negative mass instability generalized to an arbitrary wall impedance⁵⁻⁷ and its extension to bunched beams, i.e. the microwave instability.^{8,9} For the transverse case the main effect comes from high frequency transverse coherent modes¹⁰ and the head-tail instability for bunched beams.^{11,12} In all cases stability is provided by Landau damping.

For a coasting beam we write the condition for longitudinal stability as⁷

$$e I_0 \left(\frac{Z_{||}}{w} \right) \leq \alpha E \left(\frac{\Delta E}{E} \right)^2 \quad (3.1)$$

where I_0 is the beam current, α the momentum compaction factor, E the beam energy and $(\Delta E/E)$ the beam energy spread.

The transverse stability condition is written as^{10,13}

$$\delta \nu \leq \Delta \nu \quad (3.2)$$

where $\delta \nu$ is the shift in the betatron tune, ν_β due to the collective interaction with the environment

$$\delta \nu = \frac{e I_0 Z_T R}{2\pi \nu_\beta E} \quad (3.3)$$

and $\Delta \nu$ is the tune spread producing the Landau damping. We assume Z_T to be given by (2.2), (2.3), or

$$Z_T = \frac{2R}{d^2} \frac{Z_{||}}{n} + R \frac{Z_o}{2} \left(\frac{1}{a^2} - \frac{1}{d^2} \right) . \quad (3.4)$$

The amount of tune spread, $\Delta \nu$, which can be allowed in a beam is limited by the distance between nonlinear resonances. In the following we will assume that $\Delta \nu \approx 10^{-2}$. If the machine chromaticity is of the order of one we also have $\Delta E/E \approx 10^{-2}$.

With these values of Δv and $\Delta E/E$ and assuming $\alpha = 10^{-3}$, $E = 30$ GeV, $v = 30$, $Z_{\parallel}/n = 1\Omega$, we obtain from (3.1)

$$I_0 \leq 3 \times 10^3 \text{ A} .$$

It is also interesting to rewrite (3.1) as a condition on the current density per unit energy spread, which is the quantity of interest during injection in a proton storage ring. For $\Delta E/E = 10^{-3}$ this gives

$$I_0 / \frac{\Delta E}{E} \leq 3 \times 10^4 \text{ A} .$$

Assuming $E \sim RB$, $\alpha \approx v_{\beta}^{-2} \approx R^{-2}$ we also see that, for given $Z_{\parallel}$, I_0 scales as B/R so that, for a given energy, it is convenient to use high magnetic fields B .

To evaluate the transverse instability, we assume again $E = 30$ GeV, $\alpha = 10^{-3}$, $Z_{\parallel}/n = 1\Omega$ and $(r/d) = 10^4$, $v_{\beta} = 30$. The first and second term in (3.4) are of the same order for $(d/a) \approx 3$ and we assume $Z_T R = 5 \times 10^8 \Omega$. With these values we have from (3.3) (3.2)

$$I_0 \leq 10^4 \Delta v$$

or $I_0 \leq 10^2 \text{ A}$ for $\Delta v = 10^{-2}$. Notice that I_0 scales as $v_{\beta} E/R^2$ or as $(E/R) = B$ so that; as in the longitudinal case, it is convenient to work at high magnetic fields.

IV. BUNCHED BEAMS

We consider now the bunched beam case--of interest primarily for electron rings. The microwave stability condition for a bunched beam can be written as^{8,9}

$$e I_p (Z_{\parallel}/n)_{\text{eff}} < \alpha \bar{z} \left(\frac{\Delta E}{E} \right)^2 \quad (4.1)$$

where I_p is the peak current, $\Delta E/E$ is the beam energy spread, E the beam energy, α the momentum compaction. The quantity $(Z_{\parallel}/n)_{\text{eff}}$ takes into account the fact that the frequency spectrum of a bunched beam contains many harmonics of the revolution frequency. It is defined as⁹

$$\left(\frac{Z_{\parallel}}{n\omega_0}\right)_{\text{eff}} = \sum_{p=-\infty}^{+\infty} \frac{Z_{\parallel}(p\omega_0)}{p\omega_0} \rho_m(p)$$

$\rho_m(\omega)$ being the bunch form factor for m -th mode of oscillation ($m = 1$ dipole, $m = 2$ quadrupole, etc.).

The condition for transverse stability can again be written in the form (3-2), with δv given by¹³

$$\delta v = \frac{e I_{\text{peak}} Z_{\text{Teff}} R}{2\pi v \beta E} \quad (4.2)$$

where the effective transverse impedance is

$$Z_{\text{Teff}} = \frac{2R}{d^2} \left(\frac{Z_{\parallel}}{n\omega_0}\right)_{\text{eff}} + Z_0 \frac{R}{2} \left\{ \frac{1}{a^2} - \frac{1}{d^2} \right\} \quad (4.3)$$

Both (4.1) and (4.2) depend on the peak current. For a given peak current we can increase the average current, I_0 , by increasing the bunch length L . What limits the bunch length is the requirement of maintaining a long lifetime, determined mainly by quantum fluctuations for electrons and intrabeam scattering or other diffusion effects for proton beams. To keep down these effects we can use the condition that the bunch should fill less than one tenth of the rf period or

$$\frac{L}{2\pi R} \leq \frac{1}{10h} \quad (4.4)$$

where h is the harmonic number. Assuming this maximum possible bunch length we then have for the average current per bunch

$$I_o = I_{\text{peak}} \frac{L}{2\pi R} = \frac{I_{\text{peak}}}{10h} \quad (4.5)$$

We can now evaluate the limits on I_o for two different cases: a low energy, $E = 1$ GeV, and a high energy, $E = 15$ GeV, electron storage ring. For the low energy ring we use the following parameters:

$$\begin{aligned} E &= 1 \text{ GeV} & (Z_{\parallel}/\omega)_{\text{eff}} &= 1 \, \Omega \\ R &= 15 \text{ m} & Z_{\text{Teff}}^R &= 2 \times 10^4 \, \Omega \\ \alpha &= 10^{-3} & \Delta v_{\beta} &= 10^{-3} \\ v_{\beta} &= 5 \\ v_s &= 2 \times 10^{-2} \end{aligned}$$

The synchrotron tune, v_s is needed to relate $\Delta E/E$ and L , namely $(\Delta E/E) = (v_s/\alpha)(L/R)$. Using (4.1), (4.5) we obtain a threshold condition for the microwave instability

$$I_o \leq \frac{v_s^2 E}{8\pi \alpha h^3}$$

or

$$I_o \leq 10^7/h^3 \text{ Ampere} .$$

If we fill all the bunches we have a total current, $I_T = hI_o$, given by

$$I_T = \frac{10^7}{h^2} \text{ Ampere} .$$

Notice the very strong dependence of I_o on h , which makes it convenient to choose a low value of the harmonic number. For h of the order of 20, corresponding to an rf frequency of the order of 60 MHz, the limit on I_o is still very high, of the order of 10^3 A/bunch.

In the transverse case we obtain from (4.2), (4.5)

$$I_o \leq \frac{10^2}{h} \text{ Ampere}$$

or

$$I_T < 10^2 \text{ Ampere} .$$

In this case there is no dependence on h , although it is still convenient to use a small h if one wants a large current per bunch.

For the high energy ring we use the parameters

$$\begin{aligned} E &= 15 \text{ GeV} & (Z_{||}/\omega)_{\text{eff}} &= 1 \Omega \\ R &= 300 \text{ m} & A_{T\text{eff}} \times R &= 7 \times 10^7 \Omega \\ \alpha &= 10^{-3} & \Delta v_{\beta} &= 10^{-3} \\ v_{\beta} &= 20 \\ v_s &= 5 \times 10^{-2} \end{aligned}$$

For the microwave instability we have

$$I_o \leq 2 \times 10^9 / h^3 \text{ Ampere} .$$

Choosing $h = 3 \times 10^3$, or a rf frequency $f_{\text{rf}} = 475 \text{ MHz}$, we have

$$I_o \leq 7 \times 10^{-2} \text{ Ampere} .$$

For the transverse case we obtain

$$I_o \leq \frac{1}{2h}$$

and for $h = 3 \times 10^3$

$$I_o \leq 9.5 \times 10^{-4} \text{ A} .$$

The corresponding value of the peak current is rather large

$$I_p = 28 \text{ A}$$

and the bunch length L is about 7 cm.

Although the value of δv is rather large $\delta v = 10^{-3}$, causing the loss of Landau damping, it is still possible to use the radiation damping to cure instability if the rise time, τ , is longer than the radiation damping time. We can estimate this rise time for the case of a head-tail instability using the expression

$$\frac{1}{\tau} = \omega_0 \delta v \, 2\pi \frac{\xi v}{v_s} \left(\frac{\Delta E}{E} \right) \quad (4.6)$$

where ξ is the machine chromaticity, and the term $\xi v/v_s (\Delta E/E)$ measures the phase shift due to synchrotron oscillations. Ideally one can have $\xi = 0$ and thus an infinite rise time. However this condition can only be approximately satisfied because of machine nonlinearities making ξ a function of momentum. Assuming $\xi v/v_s \Delta E/E \approx 10^{-4}$ we obtain a rise time, for the current corresponding to $\delta v = 10^{-3}$,

$$\frac{1}{\tau} \approx 10 \, \text{s}^{-1}.$$

The order of magnitude of the radiation damping time is

$$\frac{1}{\tau_{\text{rad}}} = \frac{2}{3} \left(\frac{r_e}{\rho} \right)^3 \omega_0$$

and assuming the radius of curvature ρ to be 100 m, $\omega_0 = 10^6 \, \text{s}^{-1}$ we have $1/\tau_{\text{rad}} = 20 \, \text{s}^{-1}$ at 5 GeV and $1/\tau_{\text{rad}} \approx 5.4 \times 10^2 \, \text{s}^{-1}$ at 15 GeV. Hence, at 5 GeV one can store twice the current corresponding to $\delta v = 10^{-3}$, or $0.66 \times 10^{-3} \, \text{A}$, and at 15 GeV one can store about 50 mA.

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Fig. 1.

Broadband longitudinal impedance.