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stress analysis of the SM-1A reactor vessel

Contract No. AT[30 1] 2639
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ALCO PRODUCTS, INC.
NUCLEAR POWER ENGINEERING DEPARTMENT

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AEC Research and
Development Report
UC-81, Reactors Power
Special Distribution

STRESS ANALYSIS OF THE SM-1A
REACTOR VESSEL

By:

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M. H. Dixon, Project Engineer

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New York Operations Office

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ABSTRACT

This report presents the stress analysis performed on the SM-1A reactor vessel and cover as part of the Program for Engineering Support and Development of Army Pressurized Water Reactor Power Plants. The maximum combined stress (51,360 psi in compression) occurs in the vessel cover during a 50°F/hr transient. A fatigue analysis of these stresses indicated that they could be applied safely at least 2500 times, and since the vessel is expected to receive less than 900 cycles, it should not suffer any fatigue damage.

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1.0 SUMMARY AND CONCLUSIONS

The following primary and secondary stresses were calculated at various points in the reactor vessel using methods specified in the Navy Code,(1) and well known theory of shells of revolution:

(1) Membrane stresses due to pressure, including stress concentration factors where applicable.

(2) Secondary bending stresses due to the discontinuity in geometry at the middle transition head and the bottom ellipsoidal head.

(3) External load stresses at the support ring and main coolant nozzles.

(4) Cover, flange and stud stresses for loading conditions of:

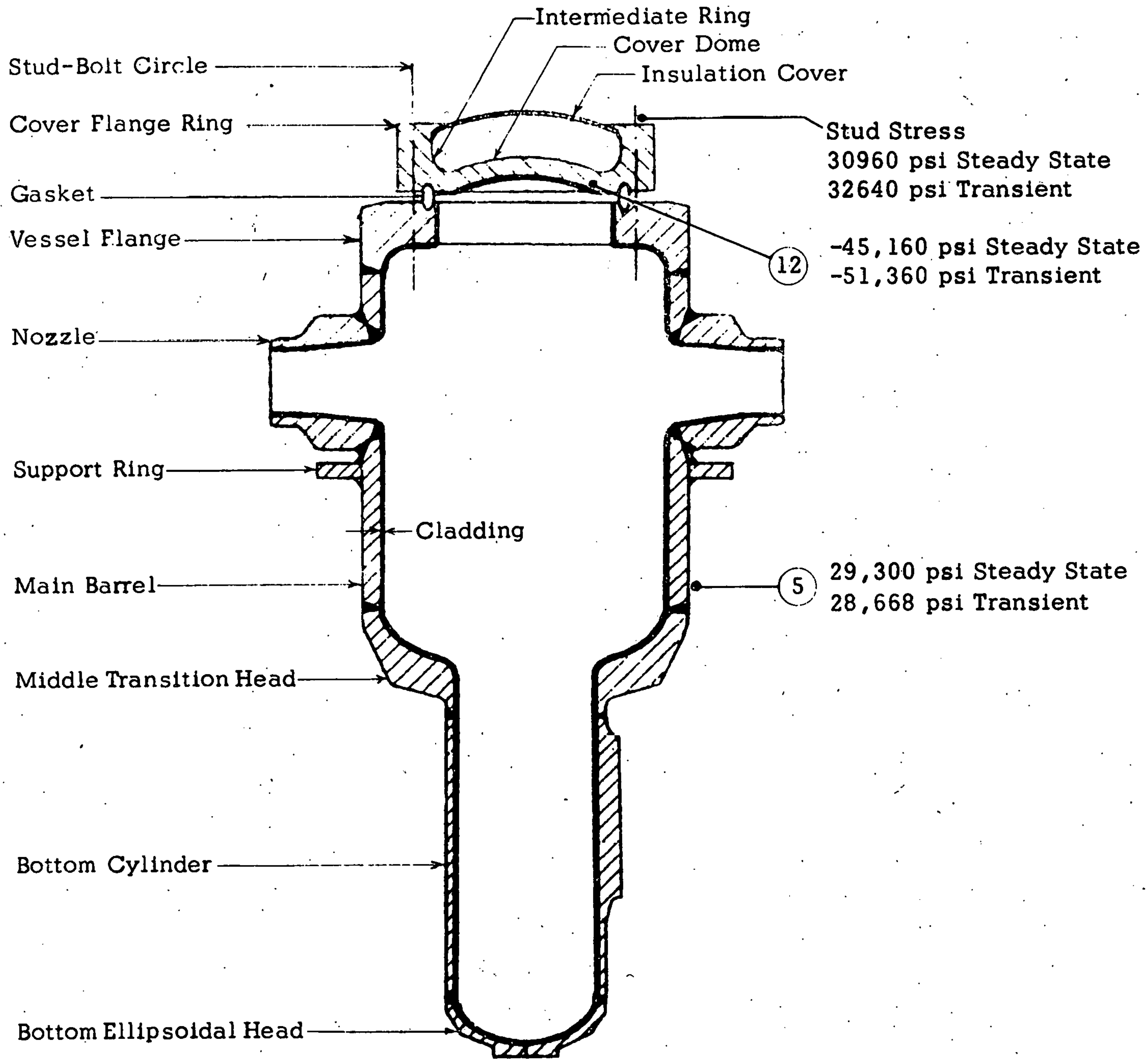
   a. Initial stud tension
   b. Stud load plus operating pressure
   c. Steady-state temperatures
   d. 50°F/hr. heatup

(5) Stress in the vessel wall due to cladding.

(6) Thermal stresses in the vessel wall for steady state and transient conditions.

Figure 1 shows the location and magnitude of the maximum combined stresses obtained in the vessel, cover and studs. The maximum combined stress in the vessel wall occurs at the junction of the main barrel and middle transition head (point 5) where the axial stress reaches 29,300 psi after reaching steady state conditions. During a 50°F/hr transient, the axial stress at this point is 28,668 psi.

The maximum combined stress in the cover, which is the highest stress anywhere in the vessel, occurs during a 50°F/hr transient, at the junction of the cover dome and the intermediate ring (point 12), where the axial stress reaches 51,360 psi in compression at the outside surface. After reaching steady state conditions, the stress at this point drops to 45,160 psi in compression. A fatigue analysis of these stresses indicated that they could be applied safely at least 2500 times. Since the vessel is expected to receive less than 900 cycles of stress,



SM-1A SHELL AND COVER OUTLINE

Figure 1. Maximum Combined Stresses

it should not suffer any fatigue damage.

The total stud stress during the transient condition is 32,640 psi of which 28,950 psi is tension, and 3690 psi bending. After reaching steady state conditions, the total stud stress is 30,960 psi of which 27,590 psi is in tension and 3370 psi is bending. The stud stresses are somewhat conservative, since they do not take into account any local yielding that takes place under the nut during vessel and cover flange rotation.

Detailed stress calculations are presented in Appendix B of this report. Table B.1 of Appendix B presents a summary of individual and combined stresses which were calculated for each point in the vessel and cover.

The thermal analysis, from which temperature distributions were obtained for the thermal steady state and transient conditions, is presented in Appendix A.

2.0 INTRODUCTION

This report presents a stress analysis performed on the SM-1A reactor vessel as part of the Program Plan For Engineering Support and Development of Army Pressurized Water Reactor Power Plants.*

The SM-1A reactor vessel is fabricated of SA-350 Gr. LF-1 Modified, with a Type 304 stainless steel cladding. Drawings F-41201-1-2 and D-41201-1-9 show the reactor vessel and cover configuration.

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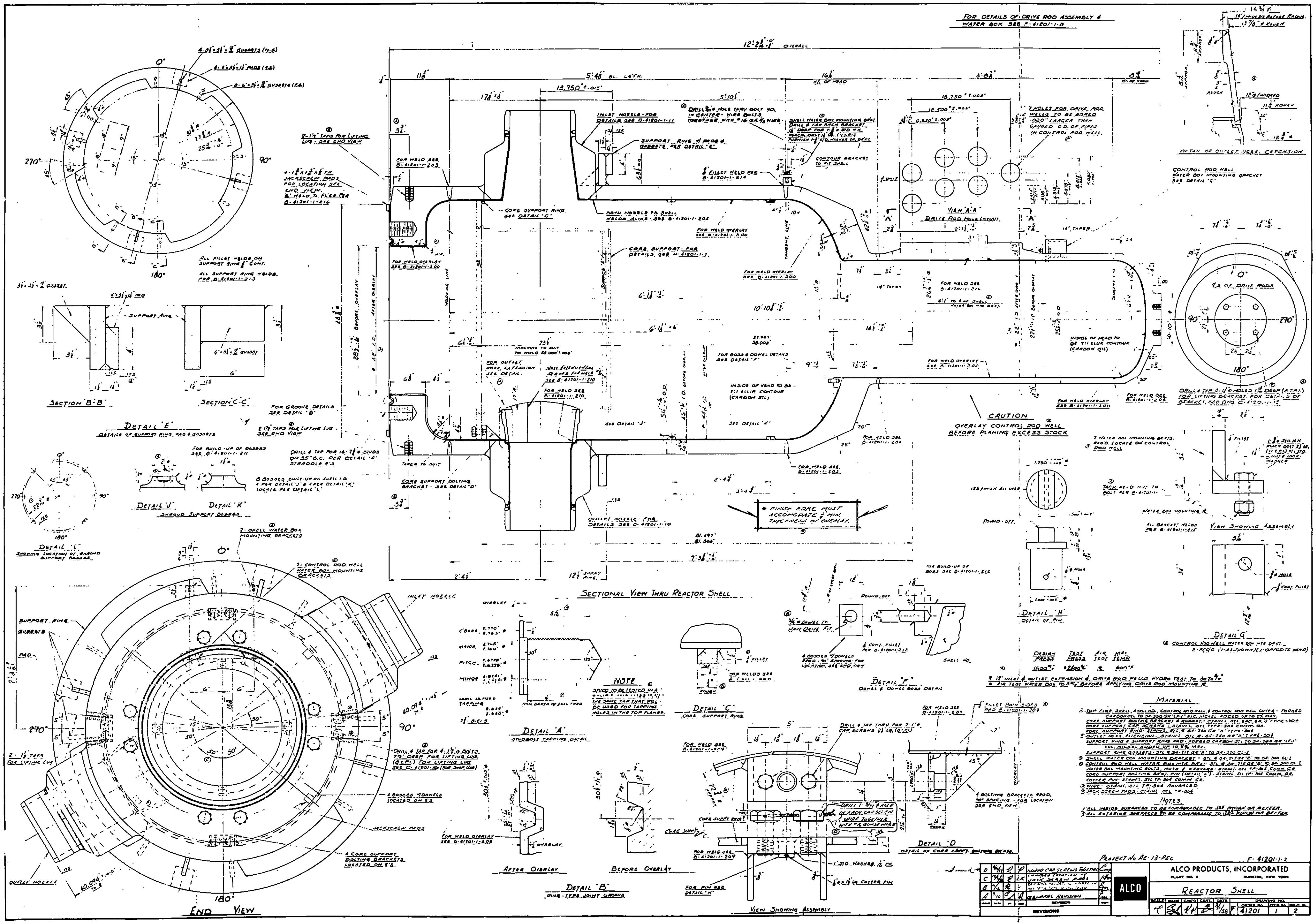
* AP Note 286, Addendum 1 Revision 1 (Item 6.9), May 1, 1961.



DESIGN PRESS	TEST PRESS	AIR TEST	MAX. TEMP.
1600#	*2400#	*	600°F

* 12" INLET & OUTLET EXTENSION & DRIVE ROD WELLS HYDRO TEST TO 3020#
* AIR TEST WATER BOX TO 5# BEFORE APPLYING DRIVE ROD MOUNTING R

MATERIAL

Ⓐ TOP FLGE, SHELL, SHELL HD., CONTROL ROD WELL & CONTROL ROD WELL COVER - FORGED CARBON STL. TO SA-350 GR "LF-1" EXC. NICKEL ADDED UP TO 2% MAX.
CORE SUPPORT BOLTING BRACKET & GUSSET - STAIN'L. STL. 240 GR. "S" TYPE 304
CORE SUPPORT CAP SCREWS - STAIN'L. STL. TYPE 304 COMM. GR.
CORE SUPPORT RING - STAIN'L. STL. R SA-240 GR "S" TYPE 304
Ⓑ OUTLET NOZZ. EXTENSION - STAIN'L. STL. R SA-240 GR "S" TYPE 304
SUPPORT RING & SUPPORT RING PAD - FORGED CARBON STL. TO SA-350 GR "LF-1" EXC. NICKEL ADDED UP TO 2% MAX.
SUPPORT RING GUSSETS - STL. R SA-212 GR "B" TO SA-300 CL-1
Ⓒ SHELL WATER BOX MOUNTING BRACKET - STL. R SA-212 GR "B" TO SA-300 CL-1
Ⓓ CONTROL ROD WELL WATER BOX MTG. BRKT. - STL. R SA-212 GR "B" TO SA-300 CL-1
WATER BOX MOUNTING BOLTS, NUTS & WASHERS - STAIN'L. STL. TP-304 COMM. GR.
CORE SUPPORT BOLTING BRKT. PIN (DETAIL "H") - STAIN'L. STL. TP-304 COMM. GR.
COTTER PIN - STAIN'L. STL. TP-304 COMM. GR.
Ⓔ WIRE - STAIN'L. STL. TP-304 ANNEALED
Ⓕ JACK SCREW PADS - STAIN'L. STL. TP-304

NOTES

1. ALL INSIDE SURFACES TO BE COMPARABLE TO 125 FINISH OR BETTER.
2. ALL EXTERIOR SURFACES TO BE COMPARABLE TO 250 FINISH OR BETTER.

PROJECT No AE-13-PEC

F-41201-1-2

D	[illegible]			WIRED CAP SCREWS TOGETHER	
C	[illegible]		LK	CHANGED LOCATION OF JACK SCREW PADS	
B	[illegible]			[illegible]	
A	[illegible]			GENERAL REVISION	
	DATE	BY		REVISION	

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REACTOR SHELL

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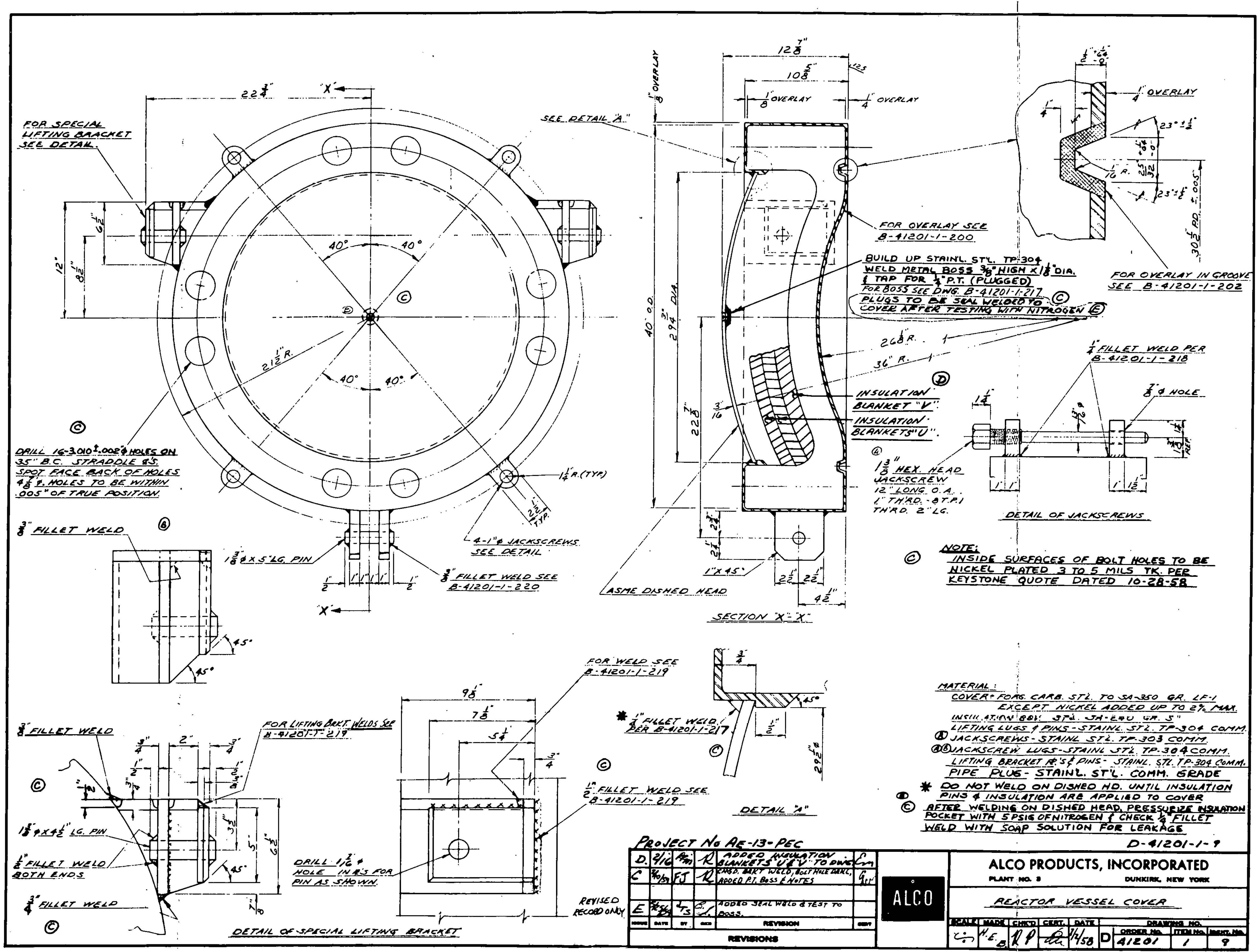

Ⓒ NOTE: INSIDE SURFACES OF BOLT HOLES TO BE NICKEL PLATED 3 TO 5 MILS TK. PER KEYSTONE QUOTE DATED 10-28-58

MATERIAL:

COVER - FORG. CARB. ST'L. TO SA-350 GR. LF-1 EXCEPT NICKEL ADDED UP TO 2% MAX.

INSULATION BOX ST'L. SA-240 GR. S"

LIFTING LUGS & PINS - STAINL. ST'L. TP-304 COMM.

Ⓑ JACKSCREWS - STAINL. ST'L. TP-303 COMM.

ⒶⒷ JACKSCREW LUGS - STAINL. ST'L. TP-304 COMM.

LIFTING BRACKET PL'S & PINS - STAINL. ST'L. TP-304 COMM.

PIPE PLUG - STAINL. ST'L. COMM. GRADE

Ⓓ * DO NOT WELD ON DISHED HD. UNTIL INSULATION PINS & INSULATION ARE APPLIED TO COVER

Ⓒ AFTER WELDING ON DISHED HEAD, PRESSURIZE INSULATION POCKET WITH 5 PSIG OF NITROGEN & CHECK 1/4" FILLET WELD WITH SOAP SOLUTION FOR LEAKAGE

D-41201-1-9

PROJECT No AE-13-PEC

ISSUE	DATE	BY	CHK'D	REVISION	CERT
D.	2/16	Am	R	ADDED INSULATION BLANKETS "U & V" TO DWG.	Em
C	3/10/59	FJ	R	CHGD. BRKT. WELD, BOLT HOLE DEML., ADDED P.T. BOSS & NOTES	G
E	[illegible]	TS	B.L.	ADDED SEAL WELD & TEST TO BOSS.	

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REACTOR VESSEL COVER

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	H.E.B.	R.P.		1/4/58	D	41201	1	9

3.0 ANALYSIS OF VESSEL EXCLUDING COVER AND FLANGE

3.1 MEMBRANE STRESSES DUE TO 1200 PSI INTERNAL PRESSURE

Principal stress components at the inner and outer surfaces of the cylindrical sections of the vessel were calculated as outlined in the Navy Code.(1) The maximum pressure stress obtained was 11,200 psi circumferentially and is the same for the main barrel and bottom cylinder. Figure 2 illustrates the location and magnitude of the circumferential membrane stresses calculated.

Stress indices at the nozzles were obtained from the Navy Code in order to determine the increase in pressure stress at the nozzle. The maximum stress occurs at the inside corner of the longitudinal section of the nozzle (points 8 and 10). It is a circumferential stress of 25,600 psi.

The maximum membrane stress in the bottom ellipsoidal head (point 1) is 10,900 psi in compression. The ellipsoidal middle transition section (point 4) membrane stress was calculated to be 11,100 psi in compression.

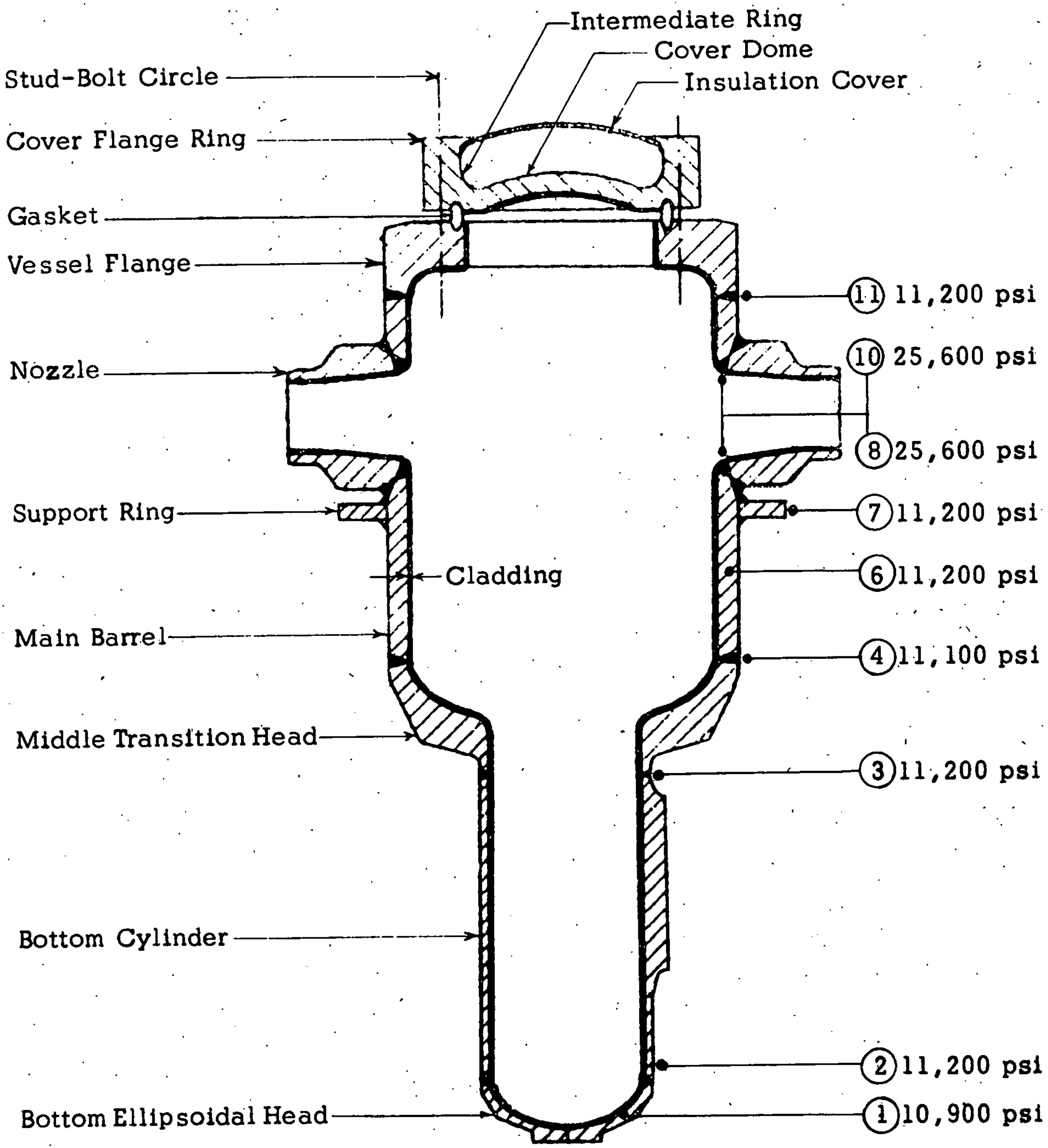
3.2 SECONDARY BENDING STRESSES AT TRANSITION AND LOWER HEADS

A discontinuity analysis was made at the region where the bottom ellipsoidal head joins the bottom cylinder of the vessel; at the junction of the bottom cylinder with the transition head; and at the junction of the main barrel with the transition head.

Figure 3 shows the magnitude and location of the secondary bending stresses.

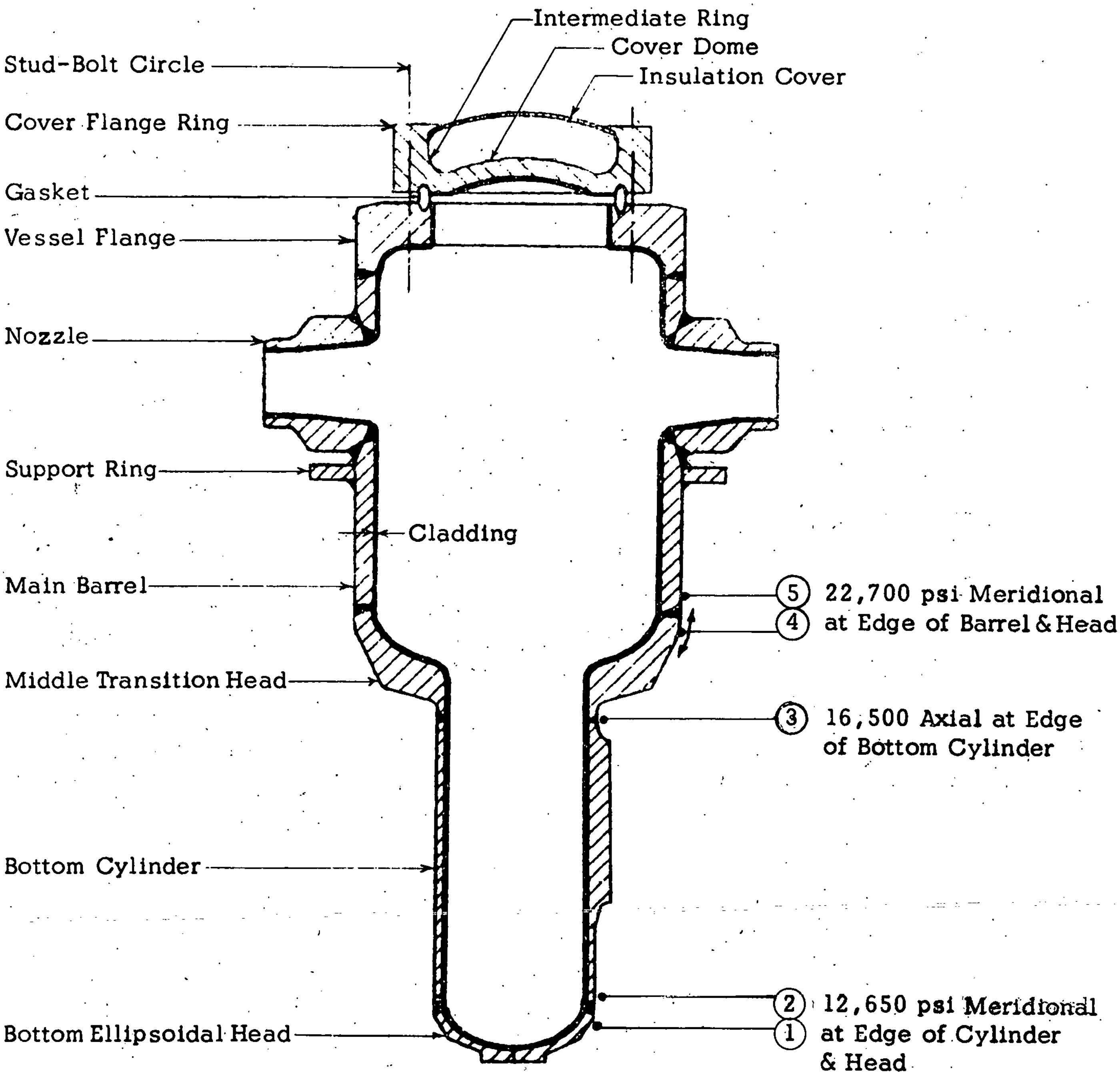
In each case, membrane deflections were calculated for the head and shell. The discontinuity deflection and edge rotation for the head and shell were obtained in terms of the discontinuity bending moments and shear forces. Boundary conditions required that the edge rotations for the head and cylinder be equal and that the algebraic sum of the membrane and discontinuity deflections be the same for both the head and shell.

From the discontinuity analysis at the bottom ellipsoidal head, it was found that the maximum stress due to discontinuity forces occurs at the edges of the bottom cylinder and the ellipsoidal head (point 1 and 2). The axial stress at each edge is 12,650 psi.



SM-1A SHELL AND COVER OUTLINE

Figure 2. Circumferential Membrane Stresses Due to 1200 Psi Pressure



SM-1A SHELL AND COVER OUTLINE

Figure 3. Secondary Bending Stresses at Transition and Lower Heads

In order to determine the discontinuity stresses in the bottom cylinder where it joins the middle transition head, it was conservatively assumed that the massive middle transition head completely restrained the edges of the relatively thin bottom cylinder. The boundary conditions for the bottom cylinder were that there was no rotation of the cylinder edge and the sum of the membrane and discontinuity edge deflections were equal to zero. The maximum discontinuity stress, 16,500 psi axially, occurred at the inside edge surface of the bottom cylinder (point 3) where it joins the middle transition head.

The discontinuity analysis at the middle transition head where it joins the main barrel resulted in a maximum stress of 22,700 psi axially, this occurred at the edge of the transition head and the main barrel (points 4 and 5)

3.3 STRESSES DUE TO EXTERNAL LOADS

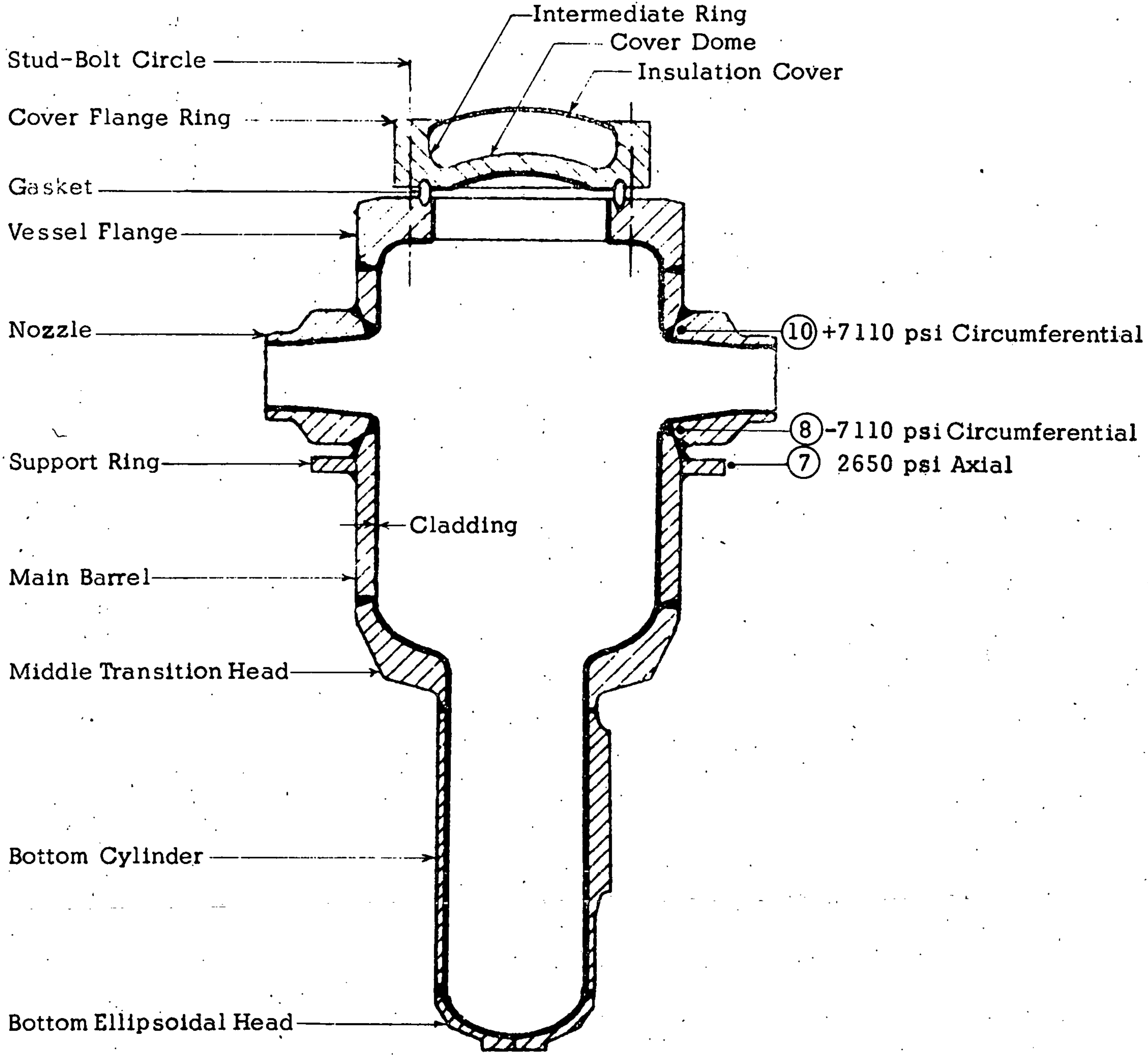
External loads are applied to the reactor vessel at the support ring and the nozzles. The locations of the stresses are shown in Fig. 4.

At the support ring, the external load stresses come from two sources. One is the total dead weight of the vessel including its internals and the water it contains, plus the weight of shield tank water acting on the top of the vessel. The other source is the restraining force of the support ring on the reactor shell due to the slower response of the support ring to primary fluid thermal transients.

The external load stresses at the nozzles are due to the external moments applied at the nozzles by the primary loop piping. External piping loads were previously calculated in the SM-1A piping design analysis.(2)

In order to calculate these external load stresses, the method outlined by the Navy Code for obtaining stresses from radial loads and external moments acting on cylindrical shells was used. The maximum external load stress due to total weight acting on the support ring (point 7) was 2650 psi axially. The maximum stress due to the restraint of the support ring on the vessel during thermal transients of 50°F/hr was 1990 psi in the circumferential direction.

The maximum stress in the vessel wall due to external primary piping loads on the nozzles (points 8 and 10) was 7110 psi



SM-1A SHELL AND COVER OUTLINE

Figure 4. Stresses Due to External Loads

acting circumferentially. The stress was relatively low because the steam generator is mounted on lubrite pads, allowing relatively free expansion of the primary piping.

3.4 THERMAL STRESSES IN THE VESSEL WALL

Thermal stresses for a temperature transient of 50°F/hr were calculated using the method outlined in the Navy Code. The thermal stress at the inner wall surface was found to be 632 psi compression while at the outer surface it was 281 psi tension.

The thermal stress in the vessel wall due to gamma heating was previously calculated(3) as 1900 psi.

3.5 CLADDING STRESS

An analysis was made to determine the stresses induced in the vessel cladding and wall as a result of each of the following operations:

(1) Application of the welded overlay.
(2) Stress relieving.
(3) Hydrostatic testing.
(4) Application of normal operating temperature and pressure.

The analysis was based on the assumption that the cladding was an elastic, perfectly plastic material and that interaction forces between the cladding and base metal caused stresses which were uniformly distributed over the thickness of each.

After the cladding has been applied at some elevated temperature and cools to room temperature, it is assumed to be at its yield strength of 30,000 psi in tension at 80°F. The stress which is induced in the vessel wall by the cladding is 2860 psi in compression and is due to the difference in thermal coefficients of expansion between the cladding and the base material.

Stress relieving the vessel at a temperature of 1150°F causes compression in the cladding, equal to its yield strength of 12,500 psi in compression at 1150°F. This results in a total stress range of 45,000 psi. Assuming the stress to be uniformly distributed over the thickness, the stress induced in the vessel wall, when stress relieved at 1150°F, will be increased by 4050 psi. Thus, it will go from its prior value of 2860 psi in compression to 1190 psi in tension. When the vessel returns to room temperature, the stresses in the cladding and wall return to their initial values before stress relieving, of 30,000 psi tension and 2860 psi compression respectively.

Based on the test pressure of 2400 psi, the average increase in the vessel wall stress during hydrostatic test was calculated as 21,200 psi in tension. The cladding carried none of the load since it was already at its yield stress of 30,000 psi. It simply yielded further at test pressure. When test pressure was relieved, the cladding stress dropped to 8850 psi in tension. This resulted in reduction of compression stress in the vessel wall from 2860 psi to 841 psi.

Upon heating the vessel to operating temperature of 433°F, the change in cladding stress was calculated to be 25,700 psi in compression. Thus the resultant stress in the cladding was 16,850 psi in compression since it was 8850 psi in tension following stress relieving and hydrostatic testing. The stress induced in the vessel wall by the cladding stress of 16,850 psi at operating temperature, was 1605 psi tension.

The stresses resulting from the operating pressure of 1200 psi were also calculated. As a result of applying operating pressure to the vessel, the calculated average tensile stress in the cladding was 9808 psi. The resultant average stress in the cladding was thus reduced from 16,850 psi in compression to 7042 psi in compression. The average tensile stress in the vessel wall due to 1200 psi pressure was 9208 psi, which, added to the residual stress in the vessel wall of 1605 psi tension, resulted in a total stress of 10,813 psi tension.

3.6 SUMMARY

The combined stress is the sum of the various stresses at each point which have been calculated for the vessel. The calculated stresses are tabulated and summarized in Table 8.1 of Appendix B.1

The maximum combined stress in the vessel shell, excluding the cover and flange region, occurred at the junction of the main barrel with the middle transition head where it reached 29,300 psi axially at steady state conditions. This was made up of 5,000 psi membrane stress, 22,700 psi discontinuity stress and 1600 psi cladding stress. During a 50°F/hr heatup, the thermal stress at this point was 632 psi compression and the combined stress became 28,668 psi axially.

4.0 REACTOR VESSEL COVER AND FLANGE ANALYSIS

The SM-1A reactor vessel cover consists of a cover dome 3 in. thick formed to a spherical radius of 26-1/8 in. The cover dome is connected by a transition fillet to a heavy outer flange ring which is 10.625 in. high and 4.12 in. thick and has 16 (3.01 in. diam) stud holes. The upper face of the cover dome is covered with thermal insulation, which is covered and protected from water by a 3/16 in. thick Type 304 stainless steel dished head, welded to the outer flange ring.

The reactor vessel main barrel has an outside radius of 25-5/8 in. with a 2-7/8 in. wall thickness, including 1/4 in. of cladding. The flange, which is 7.12 in. high and 11-5/8 in. thick, is joined to the main barrel through a fillet of 4-1/4 in. radius.

The cover is fastened to the vessel by 16 studs. A fully annealed Type 304 stainless steel gasket is used to seal the closure. Drawings F-41201-1-2 and D-41201-1-9 show the reactor vessel cover and flange dimensions.

In order to calculate stresses, the cover was divided into four pieces consisting of the cover dome, intermediate ring, outer flange ring, and the insulation cover. The vessel-flange section was divided into two pieces consisting of the flange and cylindrical vessel wall. An analysis was made to determine the forces and moments required at the boundaries of each piece to establish continuity of the assembly. It was assumed that the gasket acted as a hinge connecting the cover and vessel, transmitting only radial shear loads between the two.

Deflections, rotations and corresponding stresses for the various pieces were calculated for the following cases:

Case A - Stud load only
Case B - Stud load plus 1200 psi internal pressure
Case C - Steady state thermal conditions
Case D - Transient thermal condition for a heatup rate of 50°F/hr.

The transient analyzed was a heatup at 50°F/hr because the heatup produces the most severe temperature distributions. This can be deduced from Fig. A.1 and A.2 of Appendix A, which show that the outer boundaries of the flanges are close to the surrounding air and water temperatures even under steady-state conditions. The result is a large temperature difference between inner and outer surfaces. The cooldown transient tends to collapse this temperature difference so that the magnitude of thermal stresses is reduced.

4.1 CASE A - STUD LOAD ONLY; NO INTERNAL PRESSURE

The studs are assumed to be tightened to the ASME Code allowable stress of 20,000 psi. With the gasket acting as a hinge point, the outer flange ring rotates causing it to move downward along the stud centerline a distance of 0.0011 in. The vessel flange also rotates causing it to move upward along the stud centerline a distance of 0.0009 in. Thus the total distance the cover ring and vessel flange move toward each other is the sum of the two, or 0.002 in. Figure 5 shows the maximum stresses and their location for the case of stud load only.

The maximum stress in the cover is a circumferential stress of 6570 psi which occurs at the top inside corner of the outer flange ring (point 14). It is caused by the rotation and displacement of the ring.

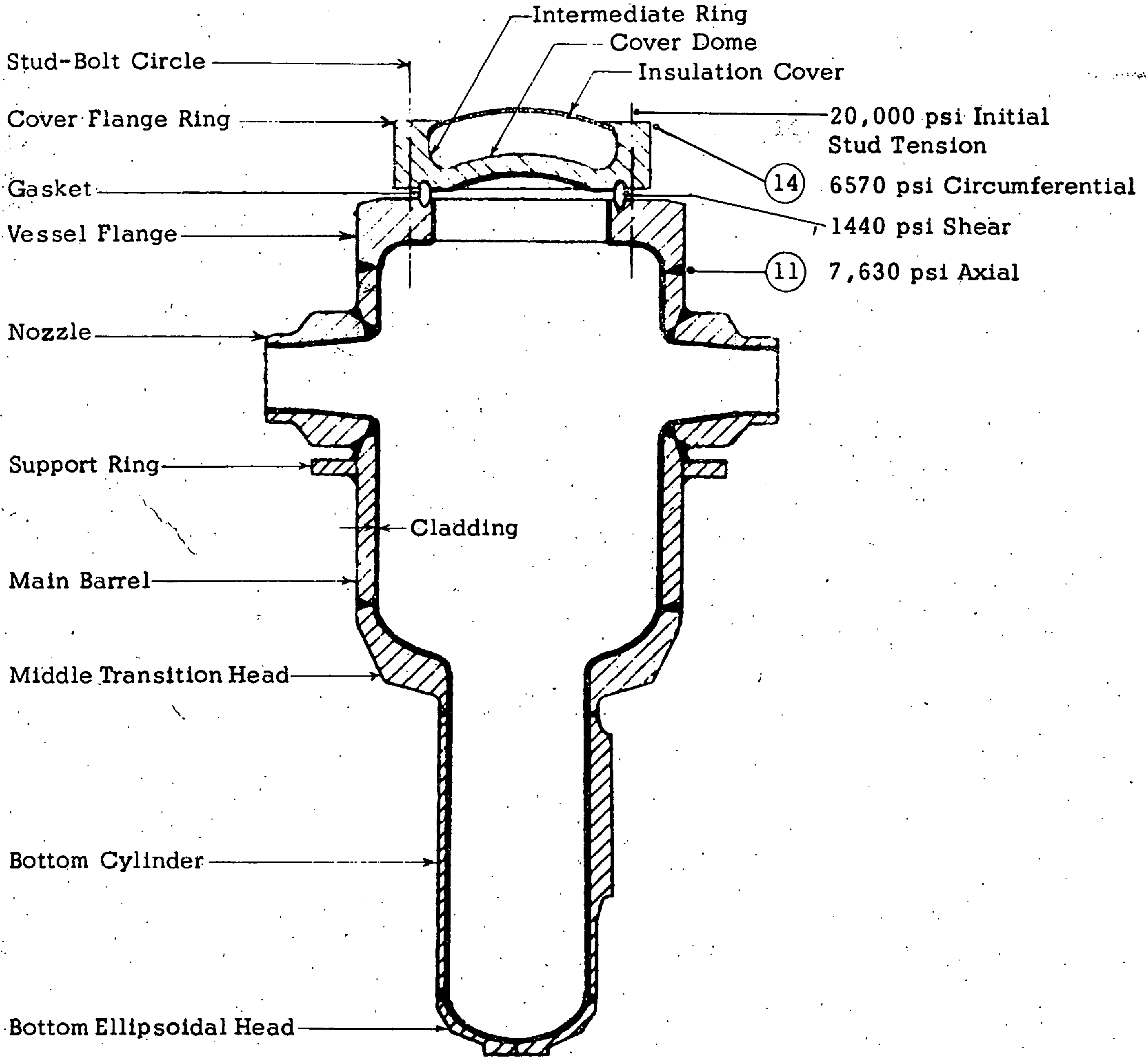
The maximum stress developed in the vessel as a result of the stud load occurs at the junction of the flange and main barrel (point 11). This is an axial stress of 7630 psi caused by the discontinuity bending moment at this point.

The shear stress developed in the gasket due to initial stud tightening is 1440 psi.

Due to the absence of internal pressure, the stresses are all relatively low. A summary of the stresses, deflections and rotations for the cover and flange region is presented in the stress analysis calculations for Case A, in Appendix B.

4.2 CASE B - STUD LOAD PLUS 1200 PSI INTERNAL PRESSURE

Applying an internal pressure of 1200 psi, after the studs have been tightened to a stress of 20,000 psi, causes the cover ring and vessel flange to move apart, again rotating about the gasket as a hinge point. Measuring along the vertical centerline of the studs, the cover ring moves 0.0006 in. upward, while the vessel flange moves downward 0.0009 in. The total distance that they have moved apart, which corresponds to the distance that the stud has been stretched as a result of applying pressure, is the sum of the movements for the cover ring and vessel flange, or 0.0015 in. Due to internal pressure, then, the stud tension has increased by 3800 psi to 23,800 psi.



SM-1A SHELL AND COVER OUTLINE

Figure 5. Case A - Stud Load Only

Figure 6 shows the location and magnitude of the maximum stresses in the cover and flange for Case B. The maximum discontinuity vessel stress still occurs at the junction of the vessel flange and cylinder (point 11) and has increased to 12,700 psi axially, due to the increase in discontinuity bending moment. The maximum discontinuity stress in the cover occurs at the junction of the intermediate ring and the outer flange ring. It is a meridional compressive stress of 9950 psi at the outer surface. The rotation of the flanges has increased the shear stress in the gasket to 18,150 psi.

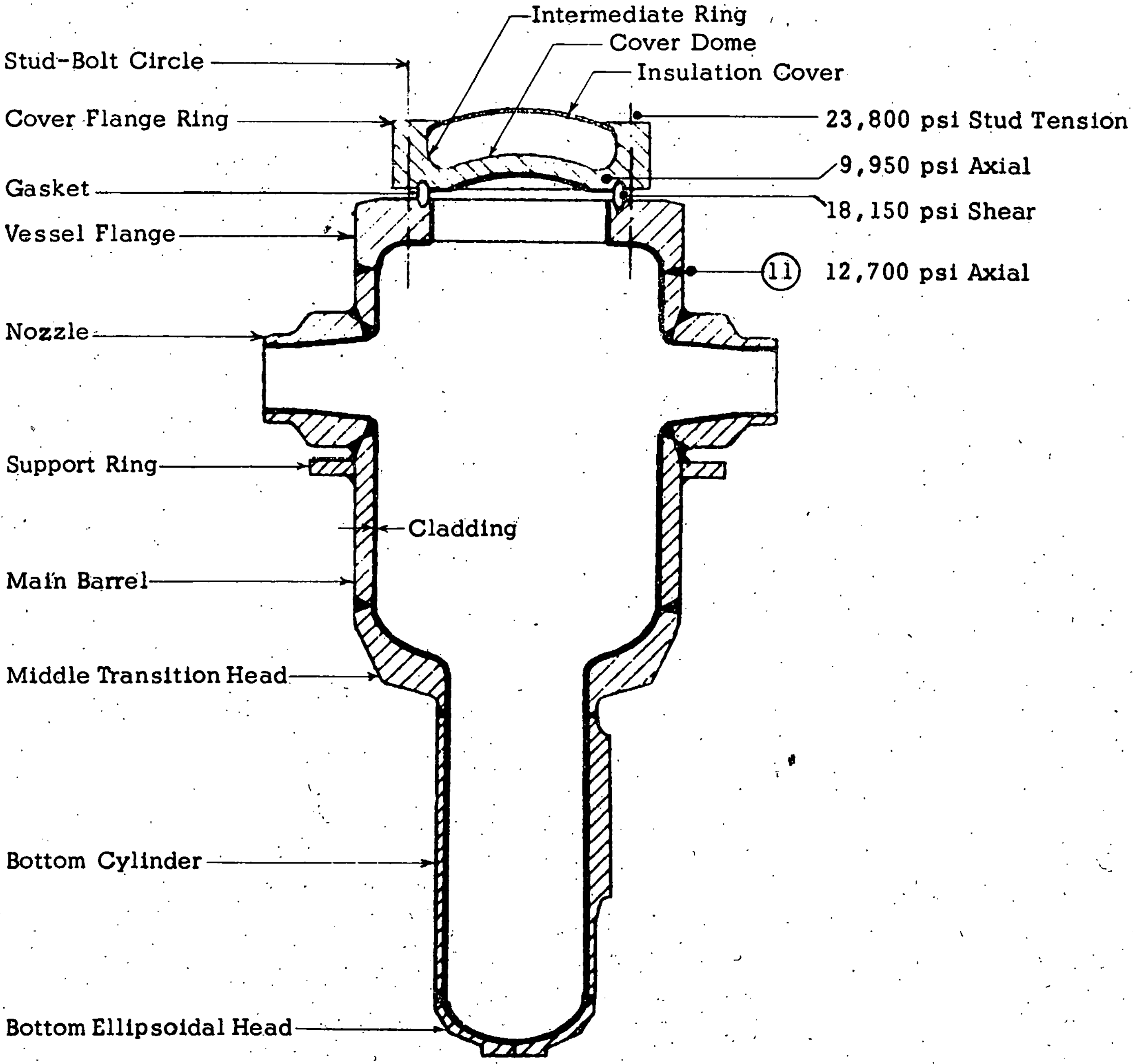
A summary of stresses, deflections and rotations for the cover and flange region is presented in the stress analysis calculations for Case B, in Appendix B.

4.3 CASE C - STEADY STATE THERMAL STRESSES

The steady state temperature distributions through the SM-1A reactor vessel cover and flange presented in Appendix A were determined by using the HOC Code. Hoc is a steady state heat conduction program which determines temperature distribution in two dimensions by the relaxation technique. The code can handle solids of irregular shape, that can be represented as a solid of revolution, or solids with non-uniform heat generation.

The bulk water temperature inside the reactor was assumed to be at an average temperature of 433°F and the mode of heat transfer was assumed to be natural convection. The water surrounding the vessel cover region was assumed to be at a bulk temperature of 150°F. The effect of gamma heat generation on temperature was investigated and found to be negligible. Temperature increases with and without gamma heating differed by less than 1°F.

A discontinuity analysis was performed for the steady state condition, based on free body thermal displacements and rotations calculated from the average temperature of each piece of the cover and flange. For this case, the cover ring and the vessel flange both rotate upward about the gasket. Due to this rotation, the cover ring moves upward 0.0017 in. while the vessel flange moves upward 0.0018 in., reducing the stud stretch by 0.0001 in. All distances are measured along the vertical stud centerline. However, the difference in coefficients of thermal expansion between the cover ring and stud results in an increase in stud stretch of 0.0016 in. Thus, the net change in stud length for the steady state thermal condition is an increase of 0.0015 in.



SM-1A SHELL AND COVER OUTLINE

Figure 6. Case B-Stud Load and 1200 Psi Internal Pressure

The maximum stresses produced by the combined effects of stud load, pressure, and steady-state temperatures (Cases B and C) were as follows:

1. The stress in the stud for the steady state temperature condition has increased by 3810 psi and the bending stress in the stud due to flange rotations is 3370 psi giving a combined stud stress of 30,960 psi.

2. The maximum steady state thermal stress developed in the cover occurs at the junction of the cover dome with the intermediate ring (point 12). The thermal stress at this point is a meridional stress of 45,000 psi in compression. The steady state combined stress at this point is 45,160 psi.

3. The maximum steady state thermal stress in the vessel flange occurs at the top inside corner of the flange (point 15) where the circumferential thermal stress is 17,700 psi. The total combined stress for the steady state condition is 19,482 psi at this point.

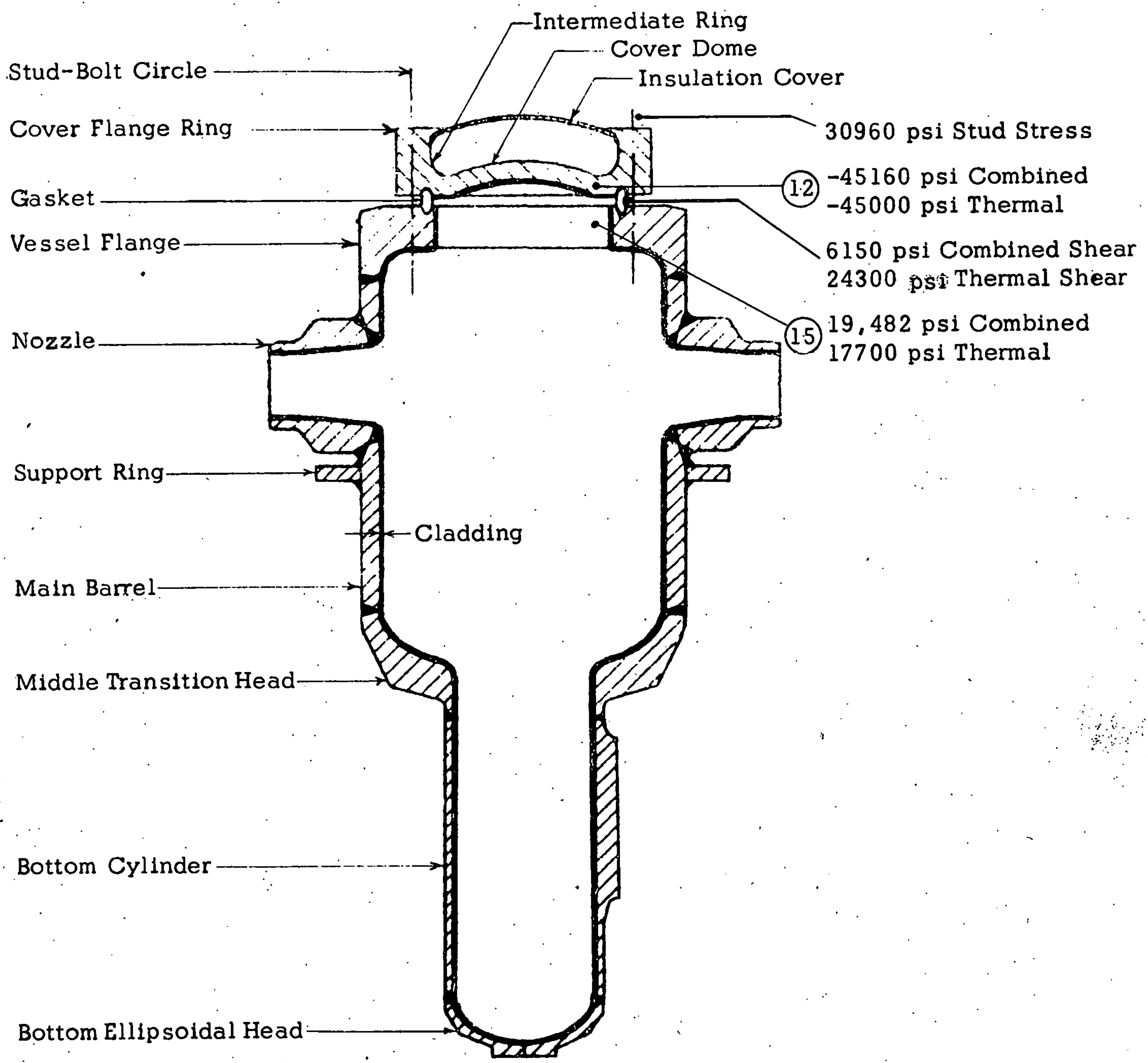
4. The shear load in the gasket has increased in magnitude for the thermal case but has reversed direction so the gasket shear stress is decreased to 6150 psi for the combined cases B and C.

Figure 7 shows the location of the above stresses. Deflections, rotations and stresses for the cover and flange region are tabulated on a summary sheet in the stress analysis calculations for Case C, in Appendix B. Combined stresses are tabulated in Table B.1 of Appendix B.

4.4 CASE D - 50^oF/hr HEATUP STRESSES

The transient temperature distributions through the SM-1A reactor vessel cover and flange, described in Appendix A, were determined by using the TIGER I Code. This is a transient state heat conduction program which determines temperature distribution in two dimensions by matrix solutions.

The initial temperature of all nodes was 100^oF. A 50^oF/hr heat-up was used, holding the environment surrounding the cover and flange to 100^oF throughout the heat-up. The heat-up was stopped as the bulk water temperature reached 400^oF. A preliminary investigation showed the worst case to be at a bulk water temperature of 433^oF. Therefore, all nodal points were extrap-



SM-1A SHELL AND COVER OUTLINE

Figure 7. Case C - Combined Effect of Stud Load, Pressure & Steady-State Temperature

olated from the bulk water temperature of 400°F to 433°F.

A discontinuity analysis was performed for the end-of-transient condition, based on free body thermal displacements and rotations calculated from the average temperature of each piece of the cover and vessel flange. The outer flange ring and vessel flange both rotate upward about the gasket. Due to this rotation, the outer flange ring moves upward 0.0025 in. while the vessel flange moves upward 0.0011 in. Both these distances are measured along the vertical stud centerline. Due to the difference in coefficients of expansion between the outer flange ring and stud, the stud is stretched 0.0005 in. Thus, the net increase in stud length is 0.0019 in.

The maximum stresses produced by the combined effects of stud load, pressure and transient temperatures (Cases B and D) were as follows:

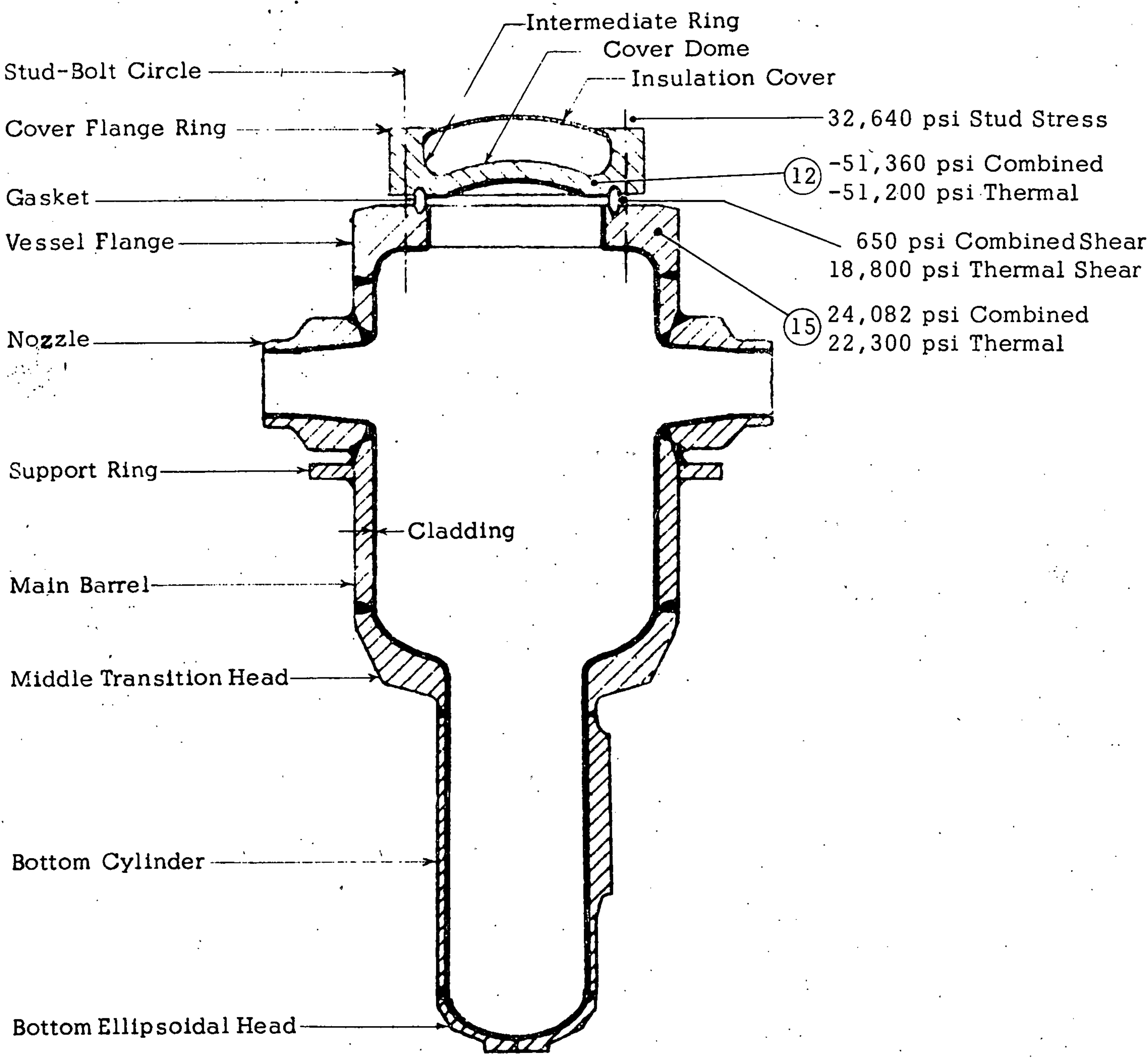
1. The tensile stress in the stud for the transient state condition has increased by 5170 psi, and the bending stress in the stud due to flange rotations by 3690 psi, giving a total stud stress of 32,640 psi.

2. The maximum transient thermal stress developed in the cover occurs at point 12, the junction of the cover dome and intermediate ring. It is a meridional stress of 51,200 psi in compression which gives a combined stress of 51,360 psi in compression.

3. The maximum transient thermal stress in the vessel occurs at point 15, the top inside corner of the flange where the circumferential stress is 22,300 psi. The total combined transient stress at this point is 24,082 psi.

4. The shear load in the gasket for case D is about the same magnitude as for case B but opposite in direction. The net effect for the combined cases is a shear stress of 650 psi.

Figure 8 shows the location of the above thermal stresses. A stress summary sheet which tabulates deflections, rotations and stress at various points in the cover and flange region is presented in the stress calculations for case D the thermal transient condition, Appendix B. Combined stresses are tabulated in Table 8.1 of Appendix B.



SM-1A SHELL AND COVER OUTLINE

Figure 8. Case D - Combined Effect of Stud Load, Pressure & Transient Temperature

4.5 SUMMARY

After stud tightening, the maximum discontinuity stress in the cover occurs at the inside corner of the outer flange ring where the stress is 6570 psi. The maximum discontinuity stress in the vessel is 7630 psi at the junction of the flange and main barrel. Stud stresses are assumed to be at the ASME Code allowable of 20,000 psi.

When an internal pressure of 1200 psi is applied, a maximum discontinuity stress of 9950 psi occurs in the cover at the junction of the cover dome and intermediate ring. The vessel stress increases from 7630 psi to 12,700 psi. At the same time, the stud stress is increased to 23,800 psi.

At the end of a 50°F/hr heat-up, the thermal stress in the cover is maximum at the junction of the cover dome and intermediate ring where it reaches 51,200 psi in compression. The vessel flange thermal stress reaches a maximum of 22,300 psi at the inside corner. The maximum combined stress for the cover and flange region at these points is then, 51,360 psi in compression and 24,082 psi tension respectively. The combined stud stress is 32,640 psi.

After reaching steady state conditions, the maximum thermal stress in the cover has dropped to 45,000 psi compression at the junction of the cover dome and outer ring. The maximum combined stress at this point is 45,160 psi in compression. The thermal stress at the inside corner of the flange is reduced to 17,700 psi for a combined stress of 19,482 psi. The stud stress is reduced slightly to 30,960 psi.

5.0 EVALUATION OF MAXIMUM STRESS

Referring to Appendix B, Table B.1, Stress Summary Chart, the highest combined stress occurs at point 12, in the vessel cover, where the cover dome joins the intermediate ring. The combined stress at this point is 51,360 psi in compression and occurs at the end of a 50°F/hr heatup.

The principal stresses for the outer surface at point 12 are listed in Table 5.1. Each principal stress and stress intensity is tabulated versus time for a complete cycle. The stress intensity is the algebraic difference between principal stresses at any given point.

The worst stress intensity is S_{31} which alternates between -110 psi and 51,360 psi resulting in an alternating stress intensity of

$$S_{alt} = \frac{110 + 51,360}{2} = 25,740 \text{ psi.}$$

The basic value of the calculated mean stress intensity is

$$S'_{mean} = 25,740 - 110 = 25,630 \text{ psi.}$$

From the Navy Code, when $S_{alt} + S'_{mean} > S_b$ and $S_{alt} < S_b$, the adjusted value of the mean stress intensity is given by

$$S_{mean} = S_b - S_{alt}$$

$$S_{mean} = 42,000 - 25,740 = 16,260 \text{ psi.}$$

Plotting this point on a fatigue diagram for the vessel material, Fig.9, shows that the worst condition is safe for at least 2500 cycles.

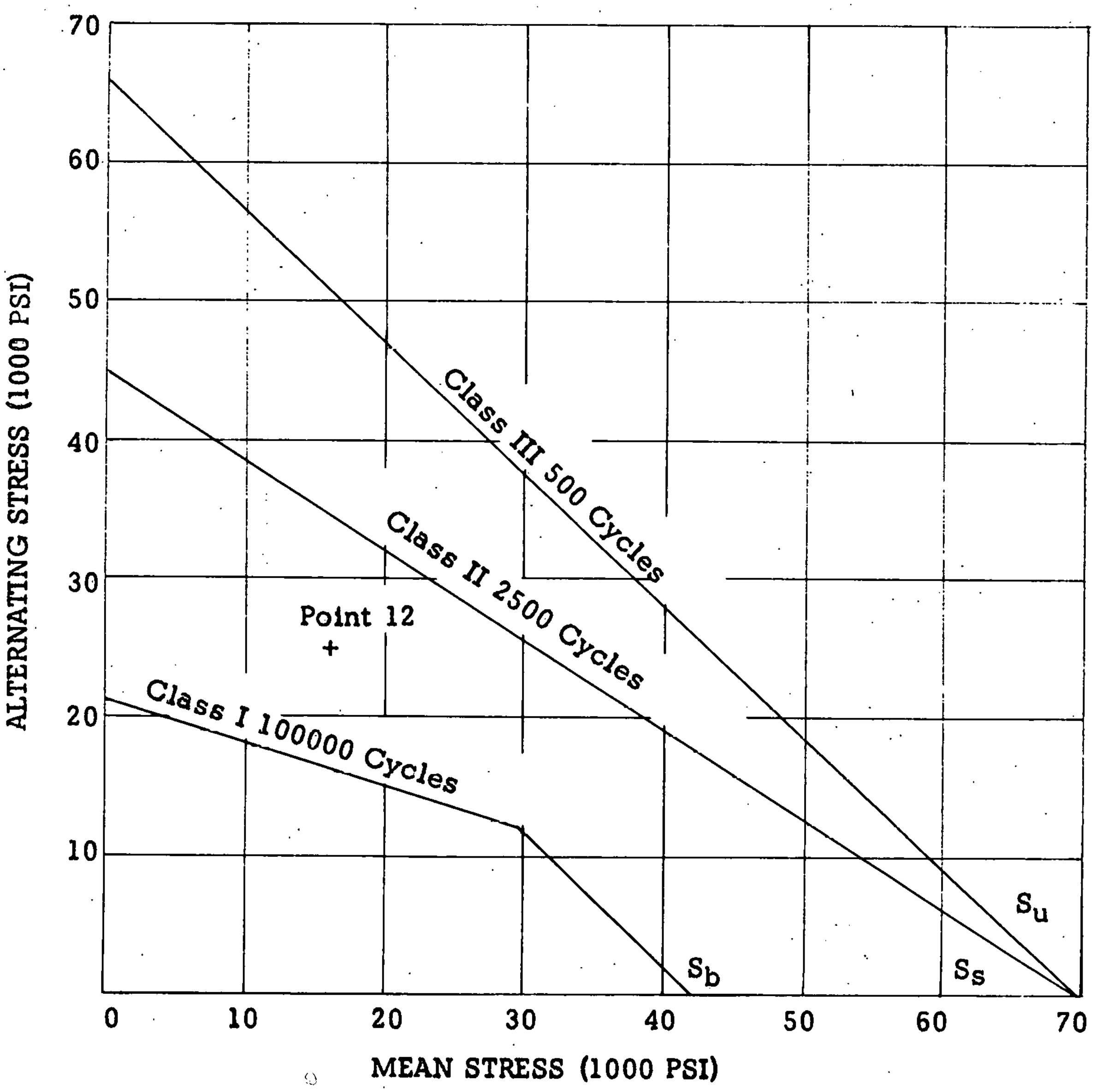
The Navy Code does not list allowable stress intensities for A-350 Gr. LF-1. Therefore, as a close approximation the allowable values for A-212 Gr. B were used.

The number of startup and shutdown cycles which will occur during the lifetime of the SM-1 has been estimated to be less than 900.(4) Since the SM-1A will be utilized primarily as a power reactor rather than a training facility, the number of startup and shutdown cycles will be considerably less than those of the SM-1. Therefore, the stresses which have been calculated will be safe for the SM-1A.

TABLE 5.1

SUMMARY OF PRINCIPAL STRESSES AND STRESS INTENSITIES (POINT 12)

Time	Principal Stresses, psi			Stress Intensities		
	σ_1	σ_2	σ_3	S_{12}	S_{23}	S_{31}
After Stud Tightening	110	2,056	0	- 1,946	2,056	- 110
After Pressurizing to 1200 psi	- 160	4,100	0	- 4,260	4,100	160
At End of 50°/hr Heatup To Operating Temperature	-51,360	-28,000	0	-23,360	-28,000	51,360
At Steady-State Operating conditions	-45,160	-26,180	0	-18,980	-26,180	45,160



MAT'L. - SA 350 GR LF-3 MOD.

Figure 9. Fatigure Diagram SM-1A Reactor Vessel Cover Point 12

6.0 MATERIALS DATA

The SM-1A vessel is fabricated from A-350 Grade LF-1 steel modified by the addition of up to 2% nickel. The base material is clad with a Type 304 stainless steel overlay.

Figures 10, 11, 12 and 13 present the tensile and physical properties for Type 304 stainless steel and A-350 Grade LF-1 (modified). The tensile properties of the base material are based on actual test data obtained from an SM-1A test slab at testing temperatures of 1100°F, 600°F and room temperature. Intermediate values were obtained by interpolation.

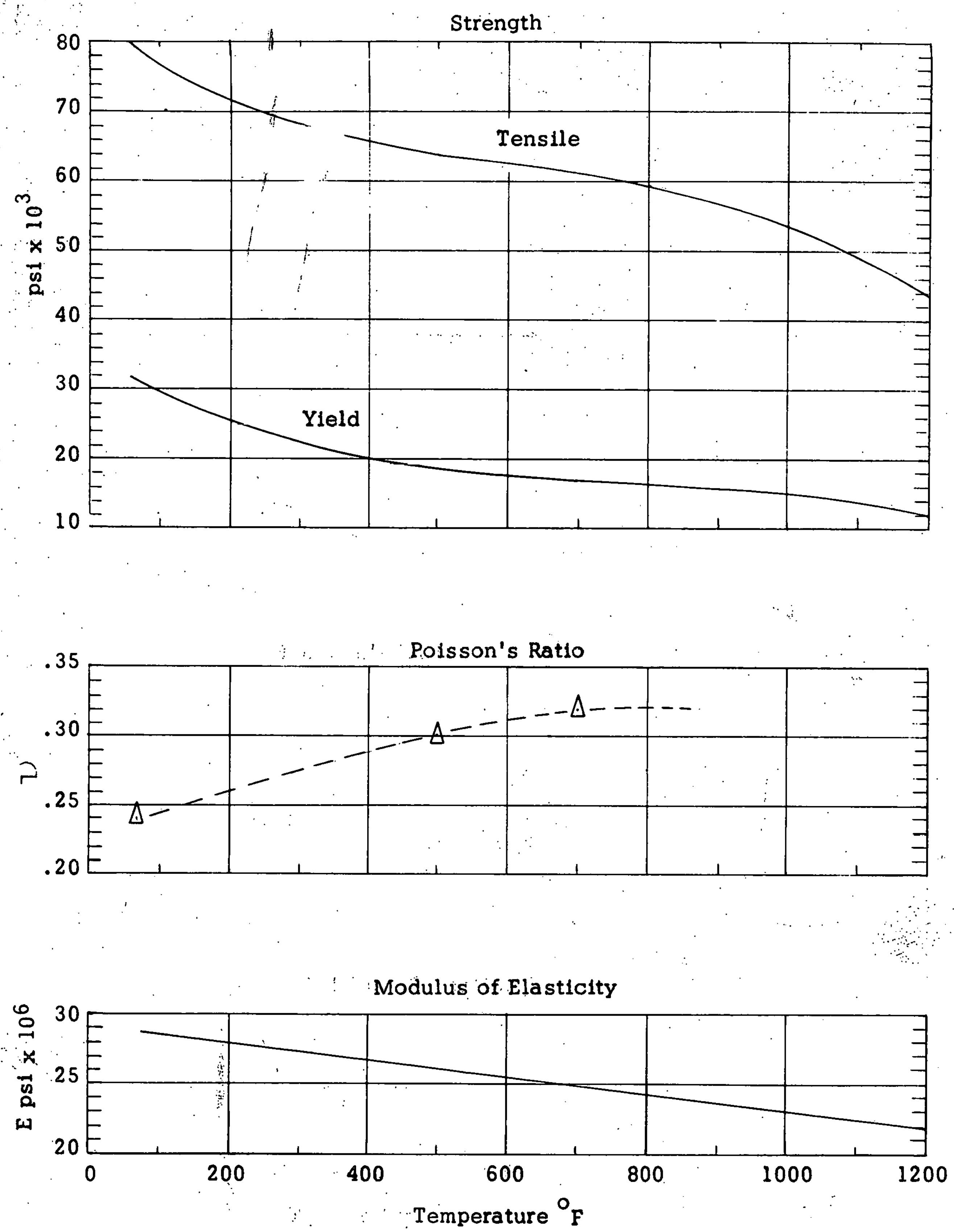


Figure 10. Tensile Properties Type 304 Stainless Steel

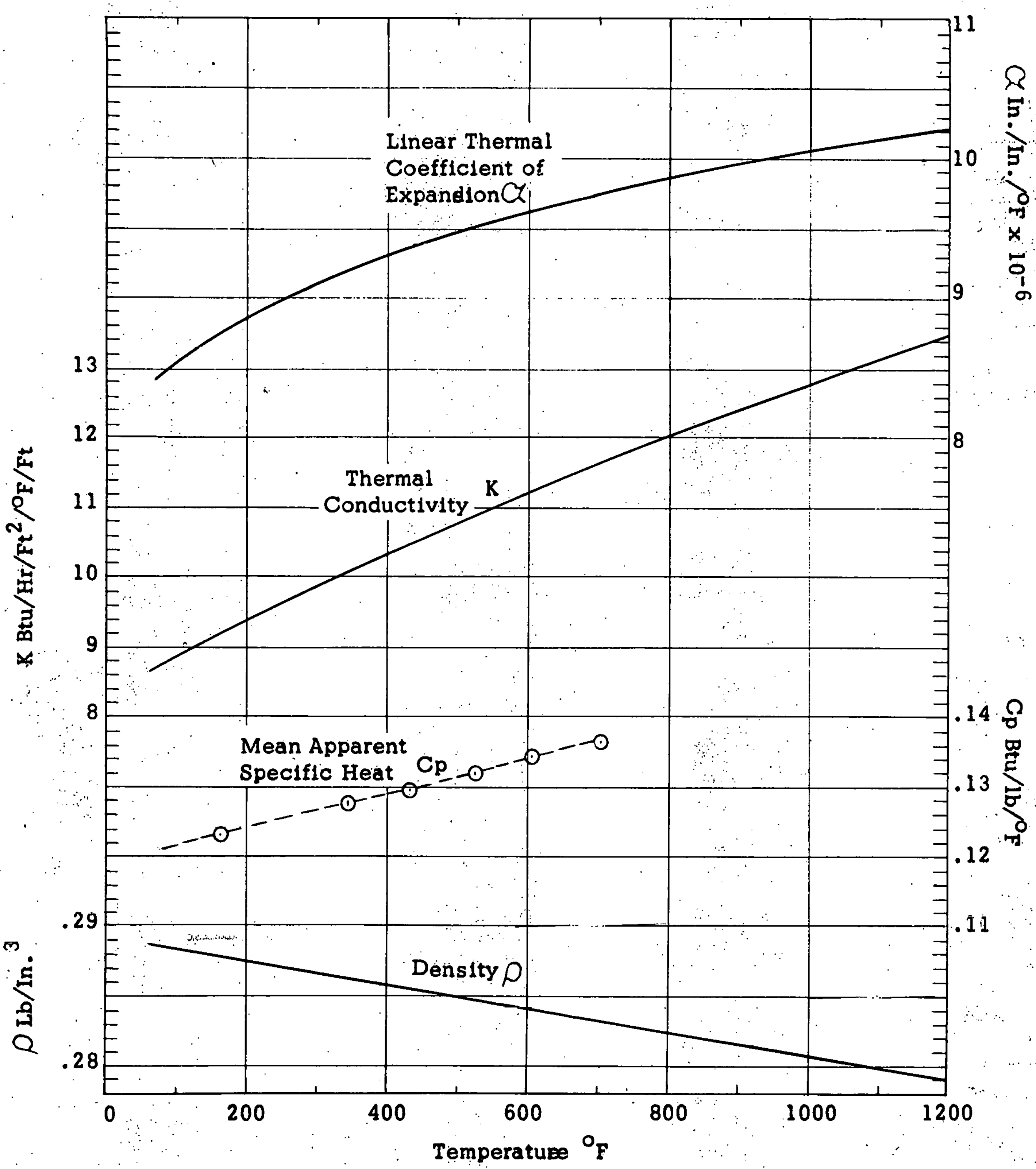


Figure 11. Physical Properties Type 304 Stainless Steel

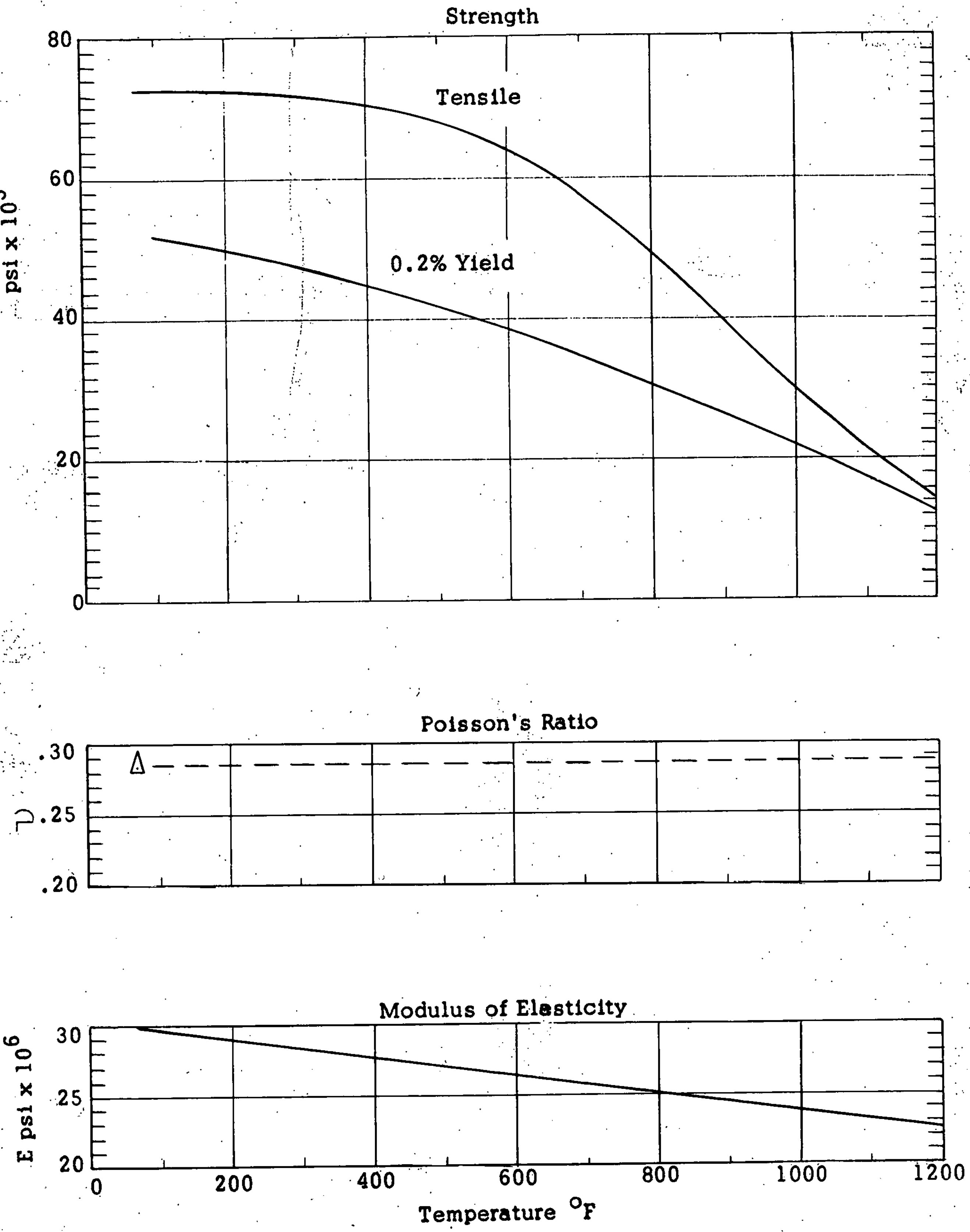


Figure 12. Tensile Properties A-350 GR LF-1 (Modified)

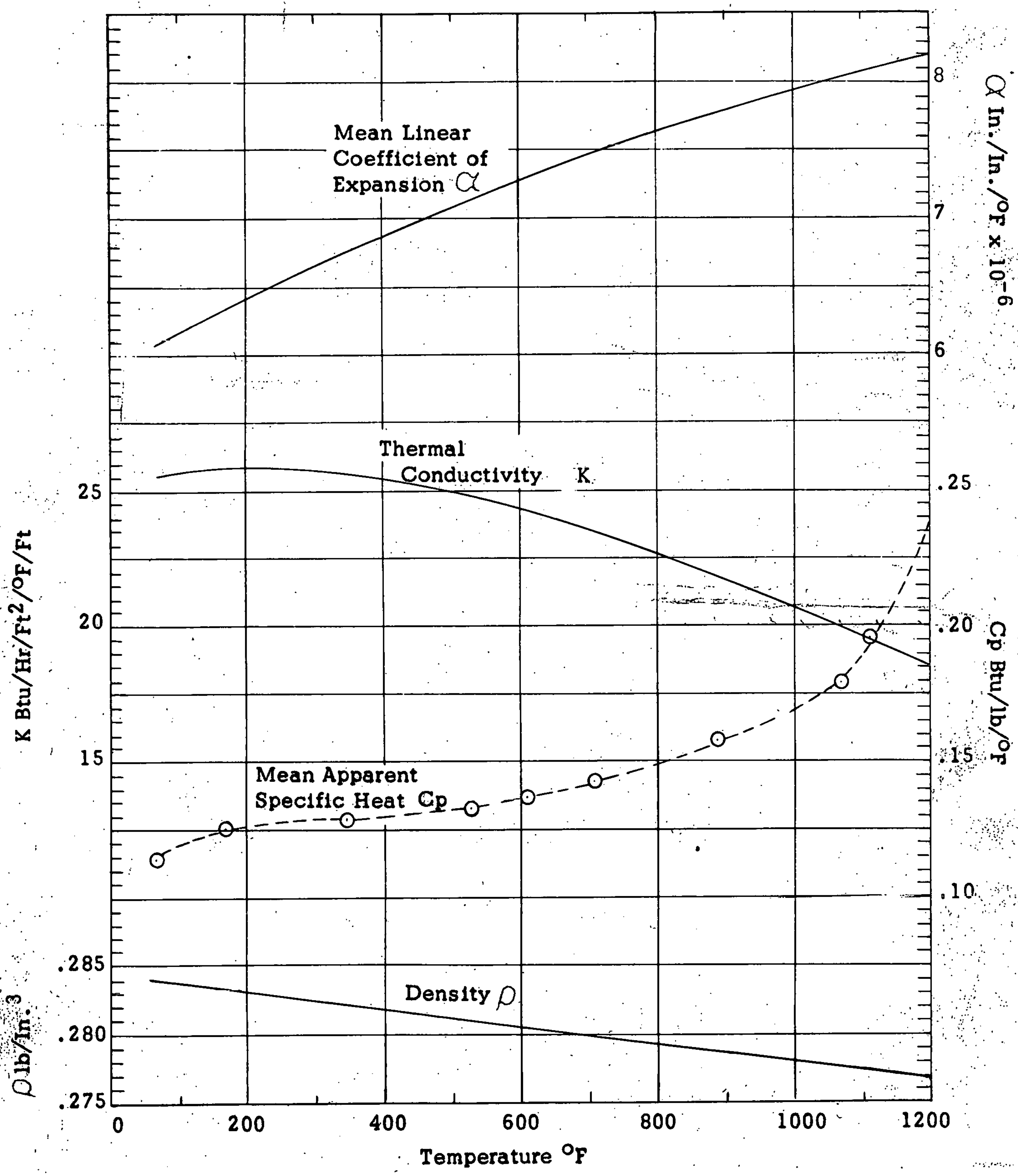


Figure 13. Physical Properties A-350 GR LF-1 (Modified)

7.0 REFERENCES

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APPENDIX A - THERMAL ANALYSIS

A.1 STEADY STATE TEMPERATURE DISTRIBUTION ANALYSIS

A.1.1 Method of Analysis

To provide data for the thermal stress analysis of the SM-1A vessel and cover, steady state temperature distributions were calculated using the HOC code.

The physical geometry of the vessel and cover was reduced to rectangular regions, each region having uniform properties. After estimates of boundary temperatures and convective film heat transfer coefficients were made, the problem was run in r,z geometry.

For conservatism, the cross section analyzed passed through the center of the cover studs. In r, z geometry, this cross section is rotated to determine region volumes and integrated region temperatures. The heat flow paths between the studs will reduce the severity of the temperature gradients somewhat.

A.1.2 Boundary Conditions

The bulk water temperature inside the vessel is at an average temperature of 433°F and natural convection is employed as the mode of heat transfer to determine the heat transfer film coefficient. The water surrounding the vessel cover is assumed to be at a bulk temperature of 150°F. The air outside of the vessel wall is also assumed at 150°F. Two further boundary conditions were employed for the water surrounding the nuts on the flange stud circle (sections 36 and 37) and the water between the cover vessel wall (section 50). In these sections, the bulk water temperature was assumed to be at 175°F. A final boundary condition was chosen for the water on section 24 which was assumed to be at 350°F. Natural convection was assumed to be the governing mode of heat transfer in all the determinations of film coefficient.

The water in the gap between the stud and cover was assumed to be at 200°F and for the steam condition at 250°F - 300°F. Conduction was assumed to be the mode of heat transfer in the gap. This gap was divided into four section as seen in Fig. A.1 - sections 45-48.

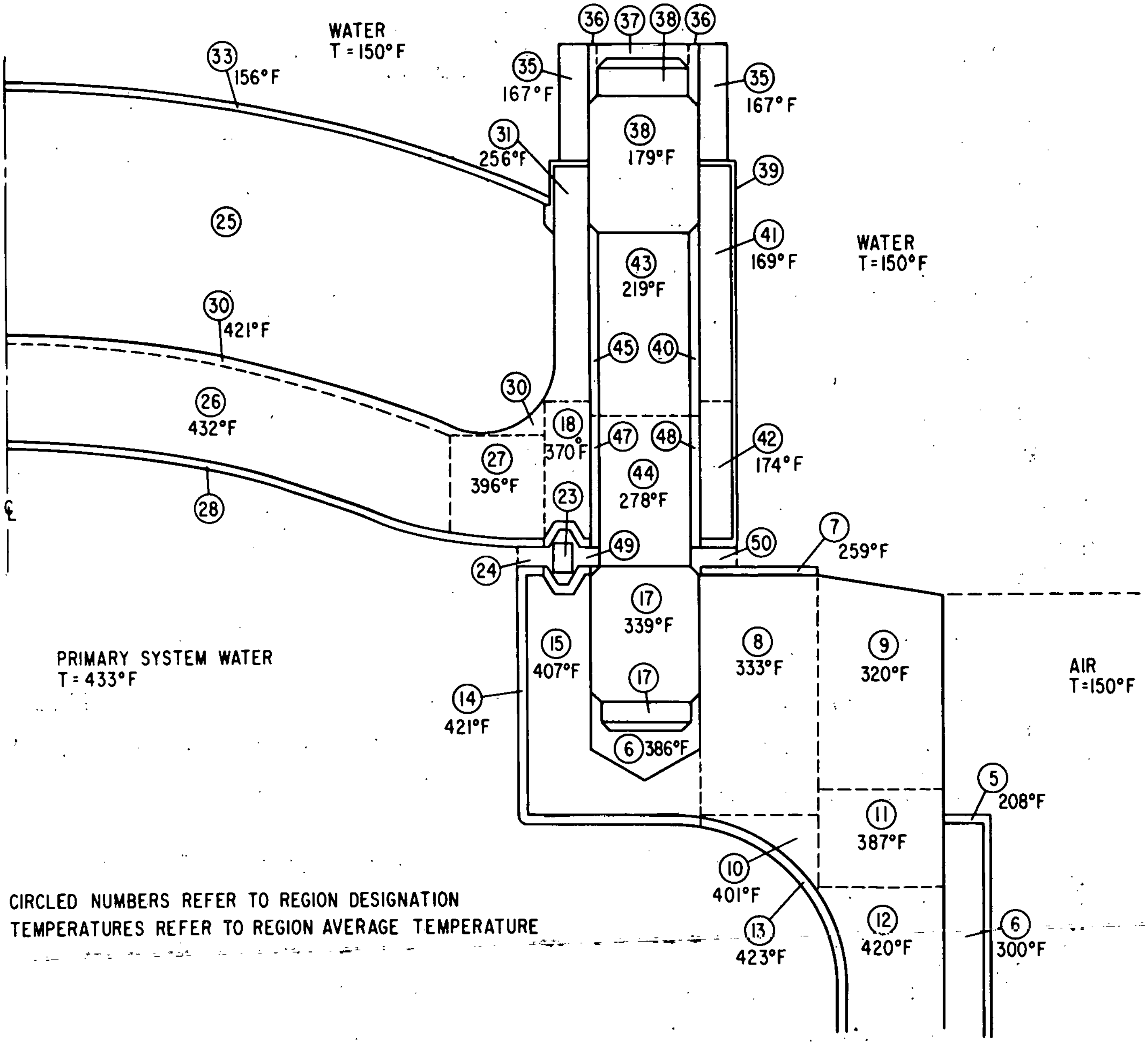
The effect of gamma heat generation on temperature was investigated and found to be negligible; the greatest difference being only 0.6°F. A further variable in this analysis was the effect of steam or water on the space between the stud and vessel cover. (sections 45 and 46 in Fig. A.1).

Table A.1 summarizes the boundary conditions for this problem.

TABLE A.1

SUMMARY OF BOUNDARY VALUES

Region Number	Location	Bulk Temp.	Heat Transfer Coefficient Btu/hr-ft^{2}F
2	Water above reactor vessel cover	150°F	64.0
3	Air alongside of reactor vessel	150°F	1.0
4	Water inside reactor vessel	433°F	340
24	Water between cover and vessel, rear gasket (inside)	350°F	230
36	Water above stud - vertical	175°F	80
37	Water above stud - horizontal	175°F	60
50	Water between cover and cover near gasket (outside)	175°F	75



CIRCLED NUMBERS REFER TO REGION DESIGNATION
TEMPERATURES REFER TO REGION AVERAGE TEMPERATURE

Figure A.1. SM-1A Vessel and Cover, Steady State Temperature Distribution

Table A.2 lists the materials and properties used in this analysis.

TABLE A.2

STEADY STATE ANALYSIS MATERIALS AND PROPERTIES

Part	Material	Btu/hr-ft^o-F
Vessel body	A-350 LF-1	26.0
Vessel clad	A-304	10.3
Vessel insulation	Fiberglas	.04
Cover	A-350-LF-1	25.5
Cover clad	A-304	10.3
Cover insulation	Fiberglas	.04
Nut	A-403	9.2
Bolt	A-403	14.5
Gasket	A-304	10.00

A.1.3 Results

Figure A.1 shows a sketch of the vessel and cover, the outline of the various regions and the region average temperature as calculated by the HOC Code.

The solution indicates severe temperature gradients across the vessel flange and across the bolt hole in the cover ring. It should be remembered that they are somewhat more severe than actual vessel temperature because the geometry and boundary conditions used in the analysis tended to be conservative.

A.2 TRANSIENT TEMPERATURE DISTRIBUTION ANALYSIS

A.2.1 Method of Analysis

The most severe temperature differences and thermal stress will occur during a heatup of the primary system. Accordingly, the transient thermal stress analysis was based on a heatup of the reactor vessel starting at a uniform metal and water temperature of 100°F. A fifty degree per hour transient was imposed on the reactor system. The transient TIGER II code was programmed to provide temperature distributions at intermediate times between start of heatup and full system operating conditions in order to determine the point of worst temperature distribution.

The physical geometry of the vessel flange and cover was broken down into rectangular regions to facilitate code input. The cross section of the cover and vessel was again taken through the bolt. The TIGER code calculates the problem as a slab rather than as a body of revolution. The results therefore will tend to be unconservative; i.e., temperatures at outer boundaries, especially in radially thick members like the flanges, will be higher than is the case in the actual vessel. For the same reason, radial gradients will be less severe in the calculated case. However, this weakness in the code was counteracted by selection of conservative boundary conditions which tended to lower the exterior surface temperatures.

A.2.2 Boundary Conditions

In order to determine the time-temperature characteristics of the boundary fluids for this reactor, the problem analyzed would have to include all physical regions out to an absolute heat sink. To do this within the limit of 25 surface nodes, the detail picture of the vessel and cover would have been lost. Therefore, it was decided to hold all fluid boundaries to a constant 100°F and use the nodal capacity of the code to describe the vessel and cover.

This approach also introduces some conservatism into the results by creating heat sinks adjacent to the reactor. It forces reactor vessel surface temperatures down and counteracts the effects of the slab approximation discussed earlier.

Table A.3 indicates the boundary values for this problem. The code calculates film coefficients as a function of temperature.

TABLE A.3

BOUNDARY VALUES AT WORST TEMPERATURE

Area	Fluid	Fluid Temp.	Direction of Heat Removal	Distribution Heat Transfer Coefficient Btu/hr-ft^2-^{o}F
Inside Vessel	Water	433^{o}F	Horiz. & Vert.	340
Above Cover insulation	Water	100	Horizontal	24
Along side of Cover Flange	Water	100	Vertical	126
Along top of Vessel Flange	Air	100	Horizontal	1.5
Along side of Vessel	Air	100	Vertical	1.2
Along Vessel wall insulation	Air	100	Vertical	0.6

Table A.4 lists the materials and the properties used in this analysis.

TABLE A.4

TRANSIENT ANALYSIS MATERIALS AND PROPERTIES

Part	Material	Heat Capacity Btu/lb-^{o}F	Density lb/in^3	Productivity Btu/hr-ft^{o}F
Vessel & Cover	A-350 LF-1	.128	.282	22.50
Insulation	Fiberglas	.190	.089	0.40
Nut & Bolt	A-403	.128	.283	14.50
Boundary	Water	1.000	.0360	.385

Analysis of the data indicated that the most severe temperature gradients occurred just at the point where the primary system water reached its operating temperature of 433°F. This occurred 400 min after the beginning of a 50 $^{\circ}$F/hr heatup.

A.2.3 Results

Figure A.2 shows a sketch of the SM-1A vessel and cover, the outline of the regions used for this analysis and the average temperatures for each region at the point of the worst temperature distribution from a thermal stress standpoint.

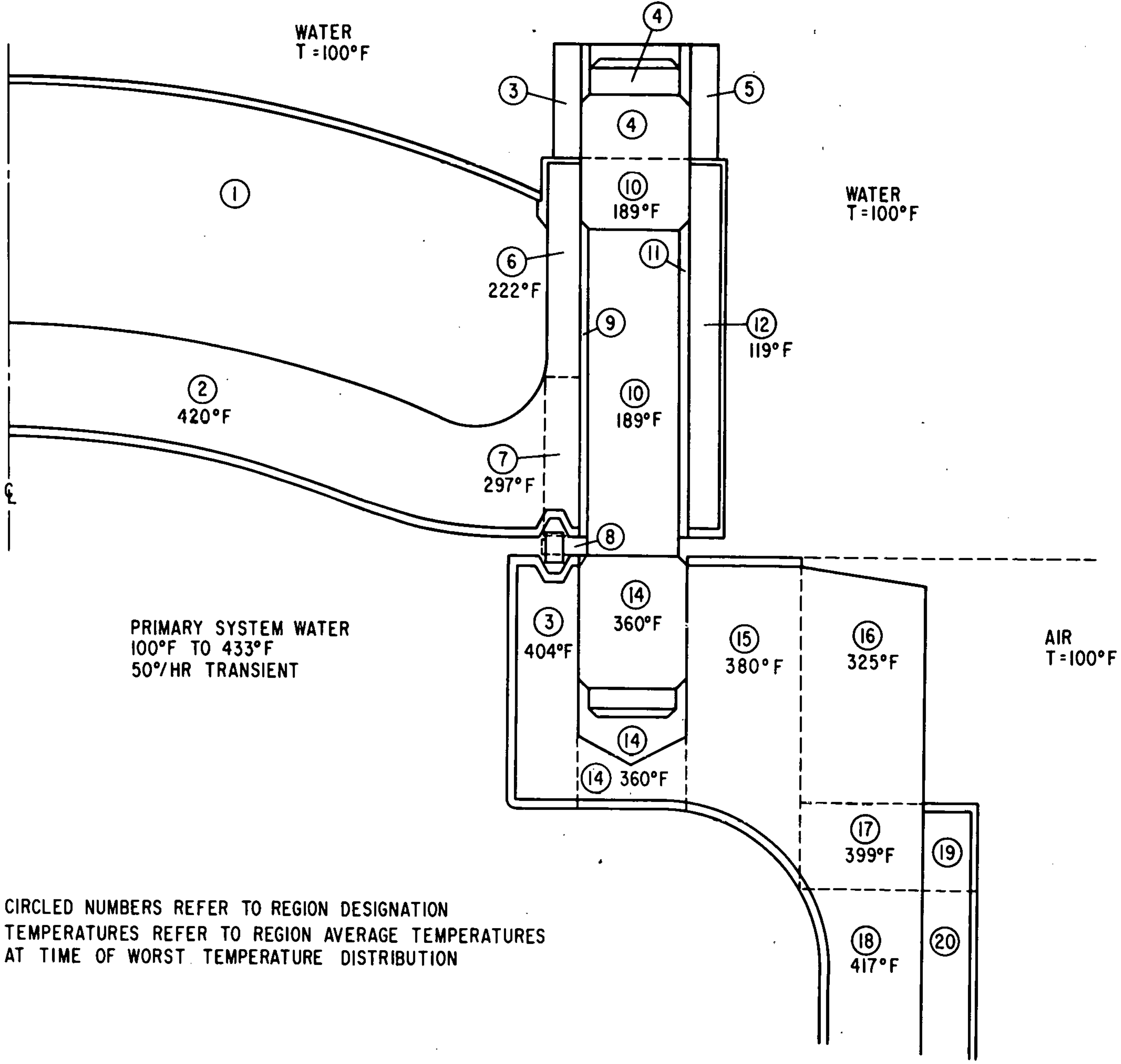


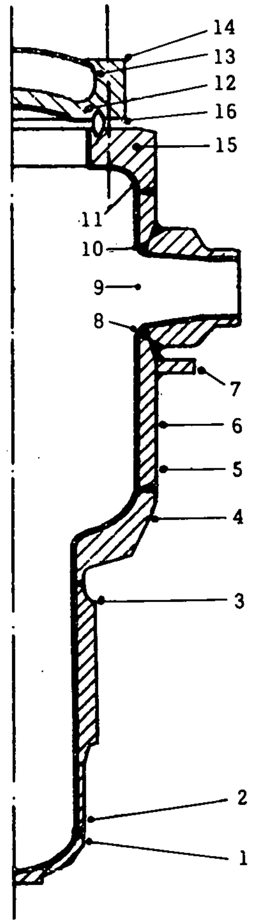
Figure A.2. SM-1A Vessel and Cover, Transient Temperature Distribution

APPENDIX B - STRESS CALCULATIONS

This section presents the detailed stress calculations for the various vessel locations. The results are summarized in Table B.1

INDEX TO DATA SHEETS

TABLE B-1
STRESS SUMMARY SHEETS



Point	Location		Membrane Stress	Thermal Stresses			Ext. Load Stress	Discont. Stress	Cladding Stress	Total Stress	
				γ Heating	Steady State	Transient				Trans.	Steady State
1	Inside	σ_1	5450	-	-	-	0	12650	-	-	18100
		σ_2	-10900	-	-	-	0	10110	-	-	- 790
		σ_3	- 1200	-	-	-	0	0	-	-	- 1200
	Outside	σ_1	5450	-	-	-	0	-12650	-	-	- 7200
		σ_2	-10900	-	-	-	0	2500	-	-	- 8400
		σ_3	0	-	-	-	0	0	-	-	0
2	Inside	σ_1	5000	-	-	-	0	12650	-	-	17650
		σ_2	11200	-	-	-	0	- 5600	-	-	5600
		σ_3	- 1200	-	-	-	0	0	-	-	- 1200
	Outside	σ_1	5000	-	-	-	0	-12650	-	-	- 7650
		σ_2	10000	-	-	-	0	-13200	-	-	- 3200
		σ_3	0	-	-	-	0	0	-	-	0
3	Inside	σ_1	5000	-	-	-	0	16500	-	-	21500
		σ_2	11200	-	-	-	0	-14010	-	-	- 2810
		σ_3	- 1200	-	-	-	0	0	-	-	- 1200
	Outside	σ_1	5000	-	-	-	0	-16500	-	-	-11500
		σ_2	10000	-	-	-	0	- 4110	-	-	5890
		σ_3	0	-	-	-	0	0	-	-	0
4	Inside	σ_1	5550	-	-	-	0	22700	-	-	28250
		σ_2	-11100	-	-	-	0	14380	-	-	1280
		σ_3	- 1200	-	-	-	0	0	-	-	- 1200
	Outside	σ_1	5550	-	-	-	0	-22700	-	-	-17150
		σ_2	-11100	-	-	-	0	740	-	-	-10360
		σ_3	0	-	-	-	0	0	-	-	0
5	Inside	σ_1	5000	-	-	- 632	0	22700	1600*	28668	29300
		σ_2	11200	-	-	- 632	0	- 7480	1600*	+ 4328	- 4960
		σ_3	- 1200	-	-	- 632	0	0	-	- 1200	- 1200
	Outside	σ_1	5000	-	-	281	0	-22700	1600*	-15819	-16100
		σ_2	10000	-	-	281	0	-21460	1600*	9579	3760
		σ_3	0	-	-	-	0	0	-	-	-
6 (Mid-plane)	Inside	σ_1	5000	1900	-	- 632	0	0	1600*	7868	8500
		σ_2	11200	1900	-	- 632	0	0	1600*	14068	14700
		σ_3	- 1200	0	-	-	0	0	-	- 1200	- 1200
	Outside	σ_1	5000	1900	-	281	0	0	1600*	8781	8500
		σ_2	10000	1900	-	281	0	0	1600*	13781	13500
		σ_3	0	0	-	-	0	0	-	0	0
7	Inside	σ_1	5000	-	-	-	- 2420	0	1600*	-	4180
		σ_2	11200	-	-	1990	- 1160	0	1600*	13630	11640
		σ_3	- 1200	-	-	-	0	0	-	-	- 1200
	Outside	σ_1	5000	-	-	-	2650	0	1600*		9550
		σ_2	10000	-	-	- 1990	1840	0	1600*	11450	13440
		σ_3	0	-	-	-	0	0	-	0	0
8	Inside	σ_1	- 1665	-	-	-	4780	-	-	-	3115
		σ_2	25600	-	-	-	1360	-	-	-	26960
		σ_3	- 1665	-	-	-	-	-	-	-	- 1665
	Outside	σ_1	2780	-	-	-	- 6870	-	-	-	- 4090
		σ_2	2780	-	-	-	- 7110	-	-	-	- 4330
		σ_3	0	-	-	-	-	-	-	-	0
9	Inside	σ_1	9450	-	-	-	-	0	-	-	9450
		σ_2	- 1665	-	-	-	-	0	-	-	- 1665
		σ_3	- 1665	-	-	-	-	0	-	-	- 1665
	Outside	σ_1	12730	-	-	-	-	0	-	-	12730
		σ_2	12730	-	-	-	-	0	-	-	12730
		σ_3	0	-	-	-	-	0	-	-	0
10	Inside	σ_1	- 1665	-	-	-	-4780	0	-	-	- 6445
		σ_2	25600	-	-	-	-1360	0	-	-	23240
		σ_3	- 1665	-	-	-	-	0	-	-	- 1665
	Outside	σ_1	2780	-	-	-	6870	0	-	-	9650
		σ_2	2780	-	-	-	7110	0	-	-	9890
		σ_3	0	-	-	-	-	0	-	-	0
11	Inside	σ_1	5000	-	53	-13850	-	12700	-	3850	17753
		σ_2	11200	-	- 9434	-14350	-	- 3540	-	- 6690	- 1774
		σ_3	- 1200	-	0	0	-	0	-	- 1200	- 1200
	Outside	σ_1	5000	-	- 53	13850	-	-12700	-	6150	- 7753
		σ_2	10000	-	- 9466	- 6050	-	-11160	-	- 7210	-10626
		σ_3	0	-	0	0	-	0	-	0	0
12	Inside	σ_1	5260	-	20200	-24000	0	- 420	-	28840	25040
		σ_2	5260	-	-10720	- 9500	0	743	-	- 3497	- 4717
		σ_3	- 1200	-	0	0	0	0	-	- 1200	- 1200
	Outside	σ_1	4680	-	-45000	-51200	0	- 4840	-	-51360	-45160
		σ_2	4680	-	-30280	-32100	0	- 583	-	-28003	-26183
		σ_3	0	-	0	0	0	0	-	0	0
13	Inside	σ_1	0	-	2705	2130	0	292	-	2422	2997
		σ_2	0	-	39000	30700	0	4190	-	34890	43190
		σ_3	0	-	0	0	0	0	-	0	0
	Outside	σ_1	0	-	0	0	0	0	-	-	-
		σ_2	0	-	0	0	0	0	-	-	-
		σ_3	0	-	0	0	0	0	-	-	-
14	Inside	σ_1	0	-	-	-	0	-	-	-	-
		σ_2	0	-	10800	8500	0	4370	-	12870	15170
		σ_3	0	-	-	-	0	-	-	-	-
	Outside	σ_1	0	-	-	-	0	-	-	-	-
		σ_2	0	-	5480	3040	0	2800	-	5840	8280
		σ_3	0	-	-	-	0	-	-	-	-
15	Inside	σ_1	0	-	-	-	0	0	-	0	0
		σ_2	2220	-	17700	22300	0	- 438	-	24082	19482
		σ_3	- 1200	-	-	-	0	0	-	- 1200	- 1200
	Outside	σ_1	0	-	-	-	0	0	-	0	0
		σ_2	1020	-	8810	11500	0	352	-	12872	10182
		σ_3	0	-	-	-	0	0	-	0	0
16	Inside	σ_1	0	-	-	-	0	0	-	-	-
		σ_2	0	-	-	-	0	0	-	-	-
		σ_3	0	-	-	-	0	0	-	-	-
	Outside	σ_1	0	-	-	-	0	-	-	0	0
		σ_2	0	-	6280	12400	0	- 688	-	11712	5592
		σ_3	0	-	-	-	0	-	-	0	0

ALCO PRODUCTS INC.

BY B/R DATE 12/2/60 SUBJECT SM-1A STRESS ANALYSIS AE-90 TASK 6.9 SHEET NO. 1 OF 7
CHKD. BY ______ DATE ______ JOB NO. CONTR. NO. AT(30-1)-2639

TABULATION OF PRINCIPAL STRESS COMPONENTS (VESSEL WALL, & HEADS)

LOCATION	σ_3	σ_2	σ_1
VESSEL SHELL			
INSIDE	-1200	11200	5000
OUTSIDE	0	10000	5000
BOTTOM CYLINDER			
INSIDE	-1200	11200	5000
OUTSIDE	0	10000	5000.
MIDDLE TRANSITION HEAD	-1200	-11100	5500
BOTTOM HEAD	-1200	-10900	5450
NOZZLES			
LONGITUDINAL SECTION			
INSIDE	-1665	25600	-1665
OUTSIDE	0	2780	2780
TRANSVERSE SECTION			
INSIDE	-1665	-1665	9450
OUTSIDE	0	12730	12730

σ_1 = AXIAL STRESS

σ_2 = CIRCUMFERENTIAL STRESS

σ_3 = RADIAL STRESS

PRINCIPAL STRESS COMPONENTS AT WALL SURFACES DUE TO INTERNAL PRESSURE —

I. VESSEL SHELL —

INSIDE SURFACE

$$\sigma_r = -P$$

$$\sigma_h = P\left[\frac{Y^2+1}{Y^2-1}\right]$$

$$\sigma_a = \frac{P}{Y^2-1}$$

OUTSIDE SURFACE

$$\sigma_r = 0$$

$$\sigma_h = \frac{2P}{Y^2-1}$$

$$\sigma_a = \frac{P}{Y^2-1}$$

where P = internal pressure = 1200 psi
Y = Radius Ratio $= \frac{R_i + t}{R_i}$, R_i = inside rad. t = wall thick.

INSIDE SURFACE

Vessel SHELL (PTS. 4,5,9,&8 ON STRESS CHART)

$D = 46''$
$R_i = 23$
$t = 2\,5/8$

$$Y = \frac{R_i + t}{R_i} = \frac{23+2.65}{23}$$

$Y = 1.113$
$Y^2 = 1.24$

1. $\sigma_r = -P = \underline{\underline{-1200 \text{ psi}}}$

2. $\sigma_h = P\left[\frac{Y^2+1}{Y^2-1}\right] = 1200\left[\frac{1.24+1}{1.24-1}\right]$

$\sigma_h = \underline{\underline{11200 \text{ psi}}}$

3. $\sigma_a = \frac{P}{Y^2-1} = \frac{1200}{1.24-1}$

$\sigma_a = \underline{\underline{5000 \text{ PSI}}}$

BOTTOM PORTION (PTS. 1,2&3 ON STRESS CHART)

$D = 22\,1/2''$
$R_i = 11.25$
$t = 1\,5/16 = 1.3125$

$$Y = \frac{R_i + t}{R_i} = \frac{11.25+1.3125}{11.25}$$

$Y = 1.114$
$Y^2 = 1.24$

1. $\sigma_r = -\underline{\underline{1200 \text{ psi}}}$

2. $\sigma_h = P\left[\frac{Y^2+1}{Y^2-1}\right]$

$\sigma_h = 1200\left[\frac{1.24+1}{1.24-1}\right]$

$\sigma_h = \underline{\underline{11200 \text{ psi}}}$

3. $\sigma_a = \frac{P}{Y^2-1} = \frac{1200}{1.24-1}$

$\sigma_a = \underline{\underline{5000 \text{ psi}}}$

OUTSIDE SURFACE

VESSEL SHELL

1. $\sigma_r = 0$

2. $\sigma_h = \dfrac{2P}{Y^2-1} = \dfrac{2(1200)}{1.24-1}$

$\sigma_h = 10000$ PSI

3. $\sigma_a = \dfrac{P}{Y^2-1} = \dfrac{1200}{1.24-1}$

$\sigma_a = 5000$ PSI

BOTTOM PORTION

1. $\sigma_r = 0$

2. $\sigma_h = \dfrac{2P}{Y^2-1} = \dfrac{2(1200)}{1.24-1}$

$\sigma_h = 10000$ PSI

3. $\sigma_a = \dfrac{P}{Y^2-1} = \dfrac{1200}{1.24-1}$

$\sigma_a = 5000$ PSI

II ELLIPSOIDAL HEADS

INSIDE SURFACE

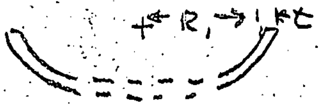
$$\sigma_r = -P$$
$$\sigma_h = -\frac{P}{t}\left(R_1 + \frac{t}{2}\right)$$
$$\sigma_a = \frac{P}{2t}\left(R_1 + \frac{t}{2}\right)$$

OUTSIDE SURFACE

$$\sigma_r = 0$$
$$\sigma_h = -\frac{P}{t}\left(R_1 + \frac{t}{2}\right)$$
$$\sigma_a = \frac{P}{2t}\left(R_1 + \frac{t}{2}\right)$$

INSIDE SURFACE

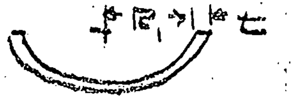
TRANSITION HEAD



$R_1 = 23''$, $t = 2\,5/8'' = 2.625''$

1. $\sigma_r = -P = -1200$ PSI

LOWER HEAD



$R_1 = 11.25''$; $t = 1\,5/16 = 1.3125''$

1. $\sigma_r = -P = -1200$ PSI

BY________ DATE________ SUBJECT SM-1A STRESS ANALYSIS SHEET NO. 4 OF 7

CHKD. BY________ DATE________ JOB NO.________

2. $$\sigma_h = -\frac{P}{t}\left(R_1 + \frac{t}{2}\right)$$

$$= -\frac{1200}{2.625}\left(23 + \frac{2.625}{2}\right)$$

$$\sigma_h = -111.00 \text{ psi}$$

3. $$\sigma_a = \frac{P}{2t}\left(R_1 + \frac{t}{2}\right)$$

$$\sigma_a = \frac{11100}{2} = 5550 \text{ psi}$$

2. $$\sigma_h = -\frac{P}{t}\left(R_1 + \frac{t}{2}\right)$$

$$= \frac{-1200}{1.3125}\left(11.25 + \frac{1.3125}{2}\right)$$

$$\sigma_h = -10900 \text{ psi}$$

3. $$\sigma_a = \frac{P}{2t}\left(R_1 + \frac{t}{2}\right)$$

$$\sigma_a = 5450 \text{ psi}$$

III STRESS INDICES AT NOZZLES —

Ref Navy Code Sect C.1.7 pg 133

$$\text{Stress Indices} = \frac{\text{STRESS IN QUESTION}}{\text{MEMBRANE STRESS in Shell}}$$

$$\text{MEMBRANE STRESS} = \frac{P(2R_i + t)}{2t}$$

$$S_M = \frac{1200\left[2(23) + 2.625\right]}{2(2.625)}$$

$$S_M = 11100 \text{ P.S.I.}$$

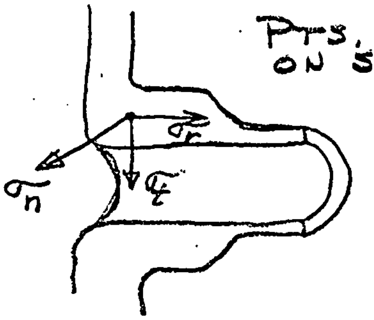
ALCO PRODUCTS INC.

BY________ DATE________ SUBJECT SM-1A STRESS ANALYSIS SHEET NO. 5 OF 7
CHKD. BY________ DATE________ JOB NO.________

REF: NAVY CODE FIG C, 3-2

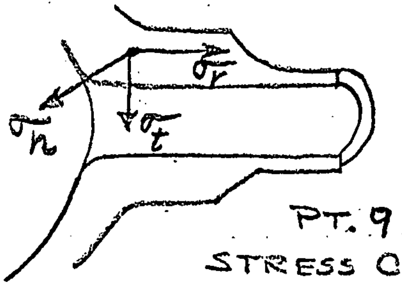
LONGITUDINAL SECTION

$\sigma_n = \sigma_2$ CIRCUMFERENTIAL
$\sigma_r = \sigma_3$ RADIAL
$\sigma_t = \sigma_1$ AXIAL



PTS. 8 & 10 ON STRESS CHART

INSIDE CORNER	OUTSIDE CORNER
$\sigma_n = \sigma_2 = 2.30(11100) = 25600$ psi	$\sigma_n = \sigma_2 = .25(11100) = 2780$ psi
$\sigma_r = \sigma_3 = -.15(11100) = -1665$ psi	$\sigma_r = \sigma_3 = 0(11100) = 0$ psi
$\sigma_t = \sigma_1 = -.15(11100) = -1665$ psi	$\sigma_t = \sigma_1 = .25(11100) = 2780$ psi



PT. 9 ON STRESS CHART

TRANSVERSE SECTION

$\sigma_n = \sigma_1$ AXIAL
$\sigma_r = \sigma_3$ RADIAL
$\sigma_t = \sigma_2$ CIRCUMF.

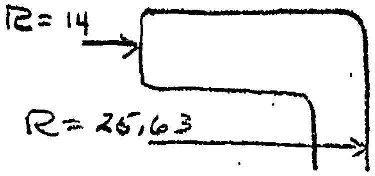
INSIDE CORNER	OUTSIDE CORNER
$\sigma_n = \sigma_1 = .85(11100) = 9450$ psi	$\sigma_n = \sigma_1 = 1.15(11100) = 12730$ psi
$\sigma_r = \sigma_3 = -.15(11100) = -1665$ psi	$\sigma_r = \sigma_3 = 0(11100) = 0$ psi
$\sigma_t = \sigma_2 = -.15(11100) = -1665$ psi	$\sigma_t = \sigma_2 = 1.15(11100) = 12730$ psi

ALCO PRODUCTS INC.

BY .... DATE .... SUBJECT SM-1A STRESS ANALYSIS SHEET NO. 6 OF 7

CHKD. BY .... DATE .... JOB NO. ....

PRESSURE STRESS AT PT. 15 ON STRESS CHART



INSIDE

$\sigma_3 = -P = -1200$ psi

$\sigma_2 = P\left[\frac{y^2+1}{y^2-1}\right]$

where $y^2 = \left[\frac{R_i + t}{R_i}\right]^2$

$y^2 = \left[\frac{25,63}{14}\right]^2 = 3,35$

$\sigma_2 = 1200\left[\frac{3,35+1}{3,35-1}\right]$

$\sigma_2 = 2220$ psi

$\sigma_1 = 0$

OUTSIDE

$\sigma_3 = 0$

$\sigma_2 = \frac{2P}{y^2-1}$

$\sigma_2 = \frac{2(1200)}{3,35-1}$

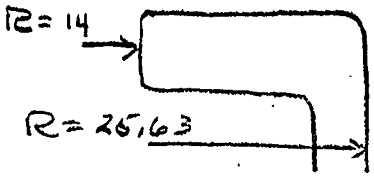
$\sigma_2 = 1020$ psi

$\sigma_1 = 0$

BY ______ DATE ______ SUBJECT SM-1A STRESS ANALYSIS SHEET NO. 6 OF 7

CHKD. BY ______ DATE ______ JOB NO. ______

PRESSURE STRESS AT PT. 15 ON STRESS CHART



INSIDE

$\sigma_3 = -P = -1200$ psi

$\sigma_2 = P\left[\frac{y^2+1}{y^2-1}\right]$

where $y^2 = \left[\frac{R_i + t}{R_i}\right]^2$

$y^2 = \left[\frac{25.63}{14}\right]^2 = 3.35$

$\sigma_2 = 1200\left[\frac{3.35+1}{3.35-1}\right]$

$\sigma_2 = 2220$ psi

$\sigma_1 = 0$

OUTSIDE

$\sigma_3 = 0$

$\sigma_2 = \frac{2P}{y^2-1}$

$\sigma_2 = \frac{2(1200)}{3.35-1}$

$\sigma_2 = 1020$ psi

$\sigma_1 = 0$

BY ______ DATE ______ SUBJECT SM-1A Stress Analysis SHEET NO. 7 OF 7
CHKD. BY ______ DATE ______ JOB NO. ______

Pressure Stress Pt 12 on Stress Chart

Ref. Navy Code Pg. 52 Par. A.3.3 —

Inside Surface

$\sigma_r = \sigma_3 = -P = -1200\,psi$

$$\sigma_h = \sigma_a = \sigma_{1\&2} = \frac{P}{2}\left[\frac{Y^3+2}{Y^3-1}\right] \qquad \text{where } Y = \frac{R_i+t}{R_i}$$

$$= \frac{26.125+3}{26.125} = \frac{29.125}{26.125}$$

$$Y = 1.115$$

$$= \frac{1200}{2}\left[\frac{\overset{1.386}{1.115^3}+2}{1.115^3-1}\right]$$

$$= 600\left[\frac{3.386}{.386}\right]$$

$$\sigma_{1\&2} = \underline{\underline{5260\,psi}}$$

Outside Surface

$\sigma_r = \sigma_3 = 0$

$$\sigma_R = \sigma_a = \sigma_{1\&2} = \frac{3P}{2}\left[\frac{1}{Y^3-1}\right]$$

$$= \frac{3(1200)}{2}\left[\frac{1}{.386}\right]$$

$$= \underline{\underline{4680\,psi}}$$

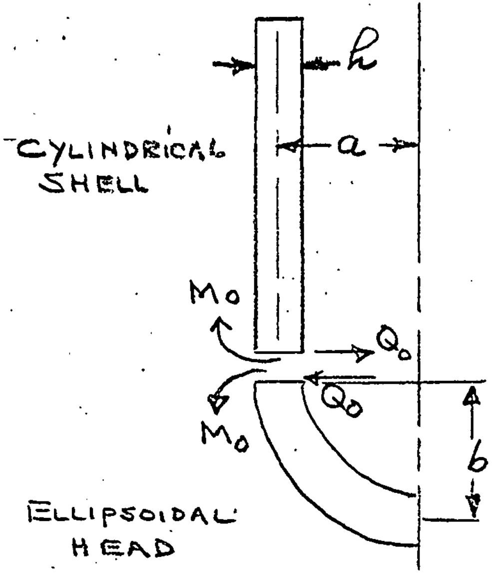
BY CJR DATE 1/14/61 SUBJECT SM-1A STRESS ANALYSIS AE-90 TASK 6.9 SHEET NO. 1 OF 9
CHKD. BY [illegible] DATE 4/25/61 JOB NO. CONTR. NO. AT(30-1)-2439

DISCONTINUITY STRESSES AT BOTTOM HEAD

w_1 = + INWARD
θ_1 = + CLOCKWISE

δ = + INWARD

w_2 = + INWARD
θ_2 = + COUNTER C.W.



$a = 11.656''$

$h = 1.312''$

$b = 6.781''$

(6)

① MEMBRANE DEFLECTION — *

* REF. P. 407 & 409 "THEORY OF PLATES & SHELLS."

SHELL — $$\delta_1 = -\frac{Pa^2}{Eh}\left(1 - \frac{\nu}{2}\right)$$

HEAD — $$\delta_2 = -\frac{Pa^2}{Eh}\left(1 - \frac{a^2}{2b^2} - \frac{\nu}{2}\right)$$

② DISCONTINUITY DEFLECTION — *

SHELL — $$w_1 = -\frac{2\beta a^2}{Eh}\left[\beta M_0 - Q_0\right]$$

HEAD — $$w_2 = -\frac{2\beta}{Eh} b^2 \left[\beta \frac{b^2}{a^2} M_0 + Q_0\right]$$

③ EDGE ROTATION *

SHELL — $$\theta_1 = \frac{2\beta^2 a^2}{Eh}\left[2\beta M_0 - Q_0\right]$$

HEAD — $$\theta_2 = \frac{2\beta^2 b^2}{Eh} \frac{b^2}{a^2}\left[2\beta \frac{b^2}{a^2} M_0 + Q_0\right]$$

① CALCULATION of MEMBRANE DEFLECTION —

$$b^2 = (6.781)^2 = 46 \quad , \quad \frac{b^2}{a^2} = .339$$

$$a^2 = (11.656)^2 = 135.7$$

SHELL

$$\delta_1 = -\frac{Pa^2}{Eh}\left(1-\frac{v}{2}\right)$$

$$\delta_1 = -\frac{1200(135.7)}{(30\times10^6)(1.312)}\left(1-\frac{.3}{2}\right)$$

$$\delta_1 = -.00413(.85)$$

$$\checkmark \; \underline{\underline{\delta_1 = -.00352''}}$$

HEAD

$$\delta_2 = -\frac{Pa^2}{Eh}\left(1-\frac{a^2}{2b^2}-\frac{v}{2}\right)$$

$$\delta_2 = -.00413\left(1-\frac{2.95}{2}-.15\right)$$

$$\delta_2 = -.00413(-.625)$$

$$\checkmark \; \underline{\underline{\delta_2 = +.00258''}}$$

② DISCONTINUITY DEFLECTIONS —

a) SHELL

$$w_1 = -\frac{2\beta}{Eh}a^2\left[\beta M_0 - Q_0\right]$$

$$\text{where} \quad \beta = \frac{1.285}{\sqrt{a\times h}} = \frac{1.285}{\sqrt{\underset{15.3}{11.656\times1.312}}} = \frac{1.285}{\sqrt{15.3}} = .328 \checkmark$$

$$w_1 = -\frac{2(.328)}{30\times10^6(1.312)}a^2\left[.328M_0 - Q_0\right]$$

$$= -.01665\times10^{-6}a^2\left[.328M_0 - Q_0\right] \qquad a^2 = 135.7$$

$$= -.741\times10^{-6}M_0 + 2.26\times10^{-6}Q_0$$

$$\boxed{w_1\times10^6 = -.741\,M_0 + 2.26\,Q_0}$$

b) ELLIPSOIDAL HEAD

$$\omega_2 = -\frac{2\beta}{Eh}\, b^2\left[\beta\frac{b^2}{a^2}M_0 + Q_0\right]$$

$$\omega_2 = -.01665\times10^{-6}(46)\left[.328(.339)M_0 + Q_0\right]$$

$$\omega_2 = -.085\times10^{-6}M_0 - .765\times10^{-6}Q_0$$

✓ $$\omega_2\times10^{6} = -.085\,M_0 - .765\,Q_0$$

③ EDGE ROTATION

a) SHELL

$$\theta_1 = \frac{2\beta^2}{Eh}\,a^2\left[2\beta M_0 - Q_0\right]$$

$$= \frac{2\beta}{Eh}\,\beta a^2\left[2\beta M_0 - Q_0\right]$$

$$= .01665\times10^{-6}(.328)(135.7)\left[2(.328)M_0 - Q_0\right]$$

$$= .487\times10^{-6}M_0 - .74\times10^{-6}Q_0$$

✓ $$\theta_1\times10^{6} = .487\,M_0 - .74\,Q_0$$

b) HEAD

$$\theta_2 = \frac{2\beta^2}{Eh}\,b^2\frac{b^2}{a^2}\left[2\beta\frac{b^2}{a^2}M_0 + Q_0\right]$$

$$\theta_2 = .01665(.328)(46)(.339)\left[2(.328)(.339)M_0 + Q_0\right]$$

$$\theta_2 = .0189\times10^{-6}M_0 + .085\times10^{-6}Q_0$$

✓ $$\theta_2\times10^{6} = .0189\,M_0 + .085\,Q_0$$

④ BOUNDARY CONDITIONS – $\left\{ \begin{matrix} \theta_1 = -\theta_2 \\ \omega_1 + \delta_1 = \omega_2 + \delta_2 \end{matrix} \right\}$

a) $\theta_1 = -\theta_2$

$$.487\,M_o - .74\,Q_o = -\left[.0189\,M_o \overset{-}{\textcircled{+}} .085\,Q_o\right]$$

$$\begin{array}{l} .019\,M_o \\ \hline .506\,M_o \end{array} \qquad \begin{array}{l} +.740\,Q_o \\ \hline .655\,Q_o \end{array}$$

$$.506\,M_o = .655\,Q_o$$

$$M_o = 1.295\,Q_o$$

b) $\omega_1 + \delta_1 = \omega_2 + \delta_2$

$$-.741\,M_o + 2.26\,Q_o - .00352 \times 10^6 = -.085\,M_o - .765\,Q_o + .00258 \times 10^6$$

$$\begin{array}{l} +.085\,M_o \\ \hline -.656 \end{array} \qquad \begin{array}{l} .765\,Q_o \\ \hline 3.025 \end{array}$$

$$-.656\,M_o + 3.025\,Q_o = .0061 \times 10^6$$

Subst above value of M_o

$$-.656\,(1.295\,Q_o) + 3.025\,Q_o = .0061 \times 10^6$$

$$-.85$$

$$2.175\,Q_o = .0061 \times 10^6$$

$$Q_o = .0028 \times 10^6$$

$$M_o = 1.295\,(.0028 \times 10^6) = .00363 \times 10^6$$

✓ $Q_o = 2800$ #/in

✓ $M_o = 3630$ in#/in

Check $\theta_1 = -\theta_2$

$$.487 M_0 - .74 Q_0 = -[.0189 M_0 + .085 Q_0]$$

$.487(3630) = 1765$	$.0189(3630) = 68.5$
$-.74(2800) = -2070$	$.085(2800) = 238$
-305	$= -306.5$

OK

⑤ DEFLECTION -

SHELL	HEAD
$w_1 \times 10^6 = -.741 M_0 + 2.26 Q_0$	$w_2 \times 10^6 = -.085 M_0 - .765 Q_0$
$= -.741(3630) + 2.26(2800)$	$= -.085(3630) - .765(2800)$
$= -2690 + 6330$	$= -309 - 2140$
$= 3640$	$= -2449$
✓ $w_1 = .00364''$	✓ $w_2 = -.00245''$

Check $w_1 + \delta_1 = w_2 + \delta_2$

$.00364$	$-.00245$
$-.00352$	$+.00258$
✓ $.00012$	✓ $.00013$

OK

BY B.J.R. DATE 1/14/61 SUBJECT SM-1A STRESS ANALYSIS SHEET NO. 6 OF 9
CHKD. BY ______ DATE ______ JOB NO. ______

⑥ DETERMINATION M_x & Q_x FOR SHELL —

$$* \; M_x = -D \times \frac{d^2w}{dx^2} = -D_x - \frac{1}{\beta D}\left[\beta M_o \varphi(\beta x) \overset{*}{-} Q_o \zeta(\beta x)\right]$$

$$M_x = M_o \varphi(\beta x) - \frac{Q_o}{\beta} \zeta(\beta x)$$

$$M_x = 3630 \; \varphi(\beta x) - \frac{2800}{.328} \zeta(\beta x)$$

$$\boxed{M_x = 3630 \; \varphi(\beta x) - 8530 \; \zeta(\beta x)}$$

$$Q_x = -D \frac{d^3w}{dx^3} = -D \times \frac{1}{D}\left[2\beta M_o \zeta(\beta x) - Q_o \psi(\beta x)\right]$$

$$Q_x = -2\beta M_o \zeta(\beta x) + Q_o \psi(\beta x)$$

$$Q_x = -2(.328)(3630)\zeta(\beta x) + 2800 \psi(\beta x)$$

$$\boxed{Q_x = -2380 \; \zeta(\beta x) + 2800 \psi(\beta x)}$$

Substitute values from Table 45 Pg. 394 "Theory of Plates & Shells" into above expressions to obtain M_x & Q_x

* Note the above expressions have different sign for Q_o term than those equations given in "Theory of Plates & Shells". The equations in Timoshenko are for the case of moment & shear acting in the same direction, where as here the moment is in the same direction but the shear is in the opposite direction. Therefore it is given an opposite sign from Timoshenko equation.

βx	$\varphi(\beta x)$	$\zeta(\beta x)$	$3630\varphi(\beta x)$	$-8530\zeta(\beta x)$	M_x	Q_x
0	1.0	0	3630	0	3630	2800
.3	.9267	.2189	3360	−1870	1490	
.5	.8231	.2908	2985	−2480	505	
.6	.7628	.3099	2770	−2640	130	
.7	.6997	.3199	2535	−2730	−195	
.9	.5712	.3185	2075	−2720	−645	
1.2	.3899	.2807	1415	−2400		
1.5	.2384	.2226	866	−1900		

The maximum bending moment occurs at the edge of the shell, where the shear force is also a maximum.

From sheet 5 The deflection w for $M_o = 3630$

$Q_o = 2800$

$$w_1 \times 10^6 = -.741(3630) + 2.26(2800)$$

$$= -2690 + 6330$$

$$= 3640$$

$w_1 = .00364$ Deflection at edge of shell where Moment & Shear are a maximum

⑦ STRESSES IN SHELL — $\left\{ \begin{array}{l} \text{at edge} \\ M_o = 3630 \\ Q_o = 2800 \end{array} \right.$

a) Axial —

$$\sigma_1 = \frac{ap}{2h} \pm \frac{6M_x}{h^2}$$

$$= \frac{11.656(1200)}{2(1.312)} \pm \frac{6(3630)}{(1.312)^2}$$

$$= 5330 + \begin{bmatrix} +12650 \text{ inside} \\ -12650 \text{ outside} \end{bmatrix}$$

Discontinuity Stress @ Pt 2 on Stress Chart
+ inside
− outside

$$\sigma_1 = \left\{ \begin{array}{ll} 18000 \text{ psi} & \text{inside} \\ -7330 \text{ psi} & \text{outside} \end{array} \right.$$

b) Circumferential

$$\sigma_2 = \frac{ap}{h} - \frac{Ew_o}{a} \pm \frac{6\nu M_o}{h^2}$$

$$= 10660 - \frac{(30\times10^6)(3640\times10^{-6})}{11.656} \pm .3(12660)$$

$$= 10660 - 9400 \pm 3800$$

$$= 10660 + \begin{bmatrix} -5600 \\ -13200 \end{bmatrix}$$

Disc. Stress @ Pt (2)
inside
outside

$$\sigma_2 = \left\{ \begin{array}{ll} +5060 \text{ psi} & \text{inside} \\ -2540 \text{ psi} & \text{outside} \end{array} \right.$$

⑧ STRESSES IN HEAD — AT EDGE where

$M_0 = 3630$

$Q_0 = 2800$

a) MERIDIONAL — (OR AXIAL)

$$\sigma_1 = \frac{ap}{2h} \pm \frac{6M}{h^2} = 5330 \pm \frac{6(3630)}{(1.312)^2}$$

$$\sigma_1 = 5330 + \left[\begin{array}{ll} -12650 \text{ psi} & \text{OUTSIDE} \\ +12650 \text{ psi} & \text{INSIDE} \end{array}\right]$$ Discont. Stress at POINT 2

$\sigma_1 =$ 17980 psi INSIDE

−7320 psi OUTSIDE

b) Circumferential —

$$\sigma_2 = \frac{ap}{h} - \frac{E\omega_x}{a} \pm \frac{6\nu M_x}{h^2}$$

$$= -10660 - \frac{(30\times10^6)(-2449\times10^{-6})}{11.656} \pm .3(12650)$$

$$= -10660 + 6300 \pm 3800$$

$$= -10660 + \left[\begin{array}{ll} 2500 & \text{OUTSIDE} \\ 10100 & \text{INSIDE} \end{array}\right]$$ Discont. Stress at Pt. 2

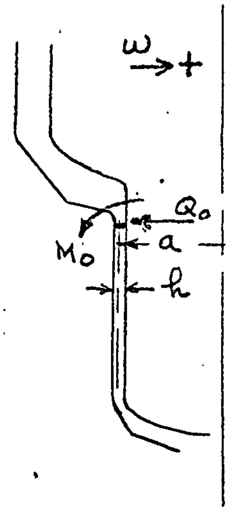
$\sigma_2 =$ −8160 psi OUTSIDE

−560 psi INSIDE

BY B J R. DATE 1/14/61 SUBJECT SM-1A STRESS ANALYSIS AE 90 TASK 6.9 SHEET NO. ① OF 4
CHKD. BY [illegible] DATE 1/25/61 JOB NO. CONTR. NO. AT(30-1)-2639

DISCONTINUITY STRESSES IN SHELL AT MIDDLE HEAD –

THE LOWER CYLINDRICAL SECTION OF THE VESSEL IS CONSIDERED TO HAVE FIXED EDGES WHICH CAN NOT MOVE SO THAT LOCAL BENDING OCCURS AT THE EDGES. The deflection due to bending plus the deflection due to pressure is equal to zero.



$a = 11.656''$

$h = 1.312''$

$$\omega + \delta = 0$$

$$M_o = \frac{P}{2\beta^2}\left(1 - \frac{\nu}{2}\right)$$

$$Q_o = -\frac{P}{\beta}\left(1 - \frac{\nu}{2}\right)$$

Ref: Pg. 399 THEORY of PLATES & SHELLS TIMOSHENKO

$$\beta = \frac{1.285}{\sqrt{a h}} = \frac{1.285}{\sqrt{11.656 \times 1.312}} = .328$$

$$\beta^2 = .1075$$

$$M_o = \frac{1200}{2(.1075)}\left(1 - \frac{.3}{2}\right) \qquad Q_o = -\frac{1200}{.328}\left(1 - \frac{.3}{2}\right)$$

✓ $M_o = 4750$ in #/in ✓ $Q_o = -3110$ #/in

BY BJR DATE 1/14/61 SUBJECT SM-1A STRESS ANALYSIS SHEET NO. ③ OF 4
CHKD. BY ✓ DATE JOB NO.

② DETERMINATION of M_x & Q_x –

$$M_x = -D\frac{d^2w}{dx^2} = -D \times -\frac{1}{\beta D}\left[\beta M_0\,\varphi(\beta x) + Q_0\,\zeta(\beta x)\right]$$

$$M_x = M_0\,\varphi(\beta x) + \frac{Q_0}{\beta}\,\zeta(\beta x)$$

$$M_x = 4750\,\varphi(\beta x) - \frac{3110}{.328}\,\zeta(\beta x)$$

$$\underline{M_x = 4750\,\varphi(\beta x) - 9450\,\zeta(\beta x)}$$

$$Q_x = -D\frac{d^3w}{dx^3} = -D \times \frac{1}{D}\left[2\beta M_0\,\zeta(\beta x) - Q_0\,\psi(\beta x)\right]$$

$$Q_x = -2\,\beta M_0\,\zeta(\beta x) + Q_0\,\psi(\beta x)$$

$$Q_x = -2(.328)(4750)\,\zeta(\beta x) - 3110\,\psi(\beta x)$$

$$\underline{Q_x = -3120\,\zeta(\beta x) - 3110\,\psi(\beta x)}$$

BY B/R DATE 1/14/61 SUBJECT SM-1A STRESS ANALYSIS SHEET NO. ③ OF 4

CHKD. BY ______ DATE ______ JOB NO. ______

βx	$\varphi(\beta x)$	$\zeta(\beta x)$	$4750\varphi(\beta x)$	$-9450\zeta(\beta x)$	M_x
0	1,0	0	4750	0	4750
,3	,9267	,2189	4400	−2170	2230
,6	,7628	,3099	3620	−2920	700
,9	,5712	,3185	2710	−3010	−300
1,2	,3899	,2807	1840	−2660	−820
1,5	,2384	,2226	1130	−2110	−980
1,8	,1234	,1610	586	−1510	−924

THE MAX MOMENT ~ $M_x = 4750$ #/IN. } OCCUR AT

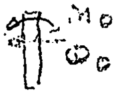
SHEAR FORCE ~ $Q_x = -3110$ #/IN } THE EDGE

③ STRESSES IN CYLINDER –

a) AXIAL –

$$\sigma_1 = \frac{ap}{2h} \pm \frac{6M_x}{h^2} = \frac{11,656(1200)}{2(1,312)} \pm \frac{6(4750)}{(1,312)^2}$$

$$\sigma_1 = 5330 \pm 16500 \begin{Bmatrix} +\text{ INSIDE} \\ -\text{ OUTSIDE} \end{Bmatrix} \text{ PT. 3}$$



$$\sigma_1 = \begin{cases} 21,830 \text{ PSI} & \text{INSIDE} \\ -11,170 \text{ PSI} & \text{OUTSIDE} \end{cases}$$

b) CIRCUMFERENTIAL —

$$\sigma_2 = \frac{ap}{h} - \frac{Ew_x}{a} \pm \frac{6\nu M_x}{h^2}$$

FROM the boundary condition
$w + \delta = 0$ at the edge,

$$w = -\delta$$

$$w = \frac{Pa^2}{Eh}\left(1 - \frac{\nu}{2}\right)$$

$$\sigma_2 = \frac{ap}{h} - \frac{\cancel{E}}{\cancel{a}}\left[\frac{Pa^{\cancel{2}}}{\cancel{E}h}\left(1 - \frac{\nu}{2}\right)\right] \pm 6\frac{\nu M_x}{h^2}$$

$$\sigma_2 = \frac{ap}{h} - \frac{Pa}{h}\left(1 - \frac{\nu}{2}\right) \pm 6\frac{\nu M_x}{h^2}$$

$$\sigma_2 = 10660 - 10660\overset{.85}{(.85)} \pm \frac{6(.3)(4750)}{(1.312)^2}$$

$$\sigma_2 = 10660 - 9060 \pm 4950$$

$$\sigma_2 = 10660 \begin{matrix} -4110 & \text{OUTSIDE} \\ -14010 & \text{INSIDE} \end{matrix}$$

$$\sigma_2 = \begin{cases} 6550 \text{ PSI} & \text{INSIDE} \\ -3350 \text{ PSI} & \text{OUTSIDE} \end{cases}$$

BY B.F.R. DATE 2/9/61 SUBJECT SM 1A STRESS ANALYSIS AE-90 TASK 6.9 SHEET NO. 1 OF 10
CHKD. BY [illegible] DATE 1/25/62 JOB NO. CONTR. NO. AT(30-1)-2639

DISCONTINUITY STRESSES AT SHELL & MIDDLE TRANSITION HEAD

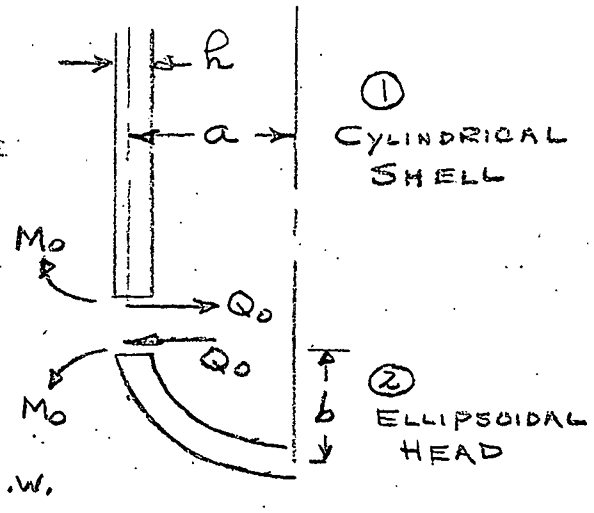
w_1 = + INWARD

θ_1 = + CLOCKWISE

δ = + INWARD

w_2 = + INWARD

θ_2 = + COUNTER C.W.



$a = 24.312$

$h = 2.625''$

$b = 12.156''$

① MEMBRANE DEFLECTION —

SHELL

$$\delta_1 = -\frac{Pa^2}{Eh}\left(1 - \frac{\nu}{2}\right)$$

HEAD

$$\delta_2 = -\frac{Pa^2}{Eh}\left(1 - \frac{a^2}{2b^2} - \frac{\nu}{2}\right)$$

② DISCONTINUITY DEFLECTION —

SHELL

$$w_1 = -\frac{2\beta a^2}{Eh}\left[\beta M_0 - Q_0\right]$$

HEAD

$$w_2 = -\frac{2\beta}{Eh} b^2\left[\beta \frac{b^2}{a^2} M_0 + Q_0\right]$$

③ EDGE ROTATION —

SHELL

$$\theta_1 = \frac{2\beta^2 a^2}{Eh}\left[2\beta M_0 - Q_0\right]$$

HEAD

$$\theta_2 = \frac{2\beta^2}{Eh} b^2 \frac{b^2}{a^2}\left[2\beta \frac{b^2}{a^2} M_0 + Q_0\right]$$

BY______DATE______ SUBJECT______ SHEET NO. 2 OF 10

CHKD. BY______DATE______ JOB NO.______

$a^2 = 24{,}312^2 = 590$

$b^2 = 12{,}156^2 = 147.5$

① MEMBRANE DEFLECTION –

SHELL	HEAD
$\delta_1 = -\frac{Pa^2}{Eh}\left(1-\frac{\nu}{2}\right)$	$\delta_2 = -\frac{Pa^2}{Eh}\left(1-\frac{a^2}{2b^2}-\frac{\nu}{2}\right)$
$\delta_1 = -\frac{1200\overset{590}{(24{,}312)^2}}{30\times10^6(2.625)}\overset{.85}{\left(1-.\frac{3}{2}\right)}$	$= -.00899\left(1-\frac{590}{2(147.5)}-.15\right)$
$\delta_1 = -.00899(.85)$	$= -.00899(1-2.15)$
$\delta_1 = -.00765''$	$\delta_2 = +.01032''$

② DISCONTINUITY DEFLECTIONS –

a) SHELL

$$w_1 = -\frac{2\beta a^2}{Eh}\left[\beta M_o - Q_o\right]$$

where $\beta = \frac{1.285}{\sqrt{a\times h}} = \frac{1.285}{\sqrt{24.312\times2.625}} = \frac{1.285}{\sqrt{63.8}} = .161$

$$w_1 = -\frac{2(.161)}{(30\times10^6)(2.625)}\, a^2 \left[.161\, M_o - Q_o\right]$$

$$= -.00408\times10^{-6}(590)\left[.161M_o - Q_o\right]$$

$$w_1 = -.388\times10^{-6} M_o + 2.41\times10^{-6} Q_o$$

$$w_1\times10^6 = -.388\ M_o + 2.41\ Q_o$$

BY________DATE________ SUBJECT________ SHEET NO. 3 OF 10

CHKD. BY________DATE________ JOB NO.________

b) ELLIPSOIDAL HEAD

$$\frac{b^2}{a^2} = \frac{147.5}{590} = .25$$

$$\omega_2 = -\frac{2\beta}{Eh} b^2 \left[\beta \frac{b^2}{a^2} M_0 + Q_0\right]$$

$$= -.00408 \times 10^{-6} (147.5) \left[.161(.25) M_0 + Q_0\right]$$

$$= -.0242 \times 10^{-6} M_0 - .602 \times 10^{-6} Q_0$$

$$\omega_2 \times 10^6 = -.0242 M_0 - .602 Q_0$$

③ EDGE ROTATION —

a) SHELL

$$\theta_1 = \frac{2\beta^2}{Eh} a^2 \left[2\beta M_0 - Q_0\right]$$

$$= \frac{2\beta}{Eh} \beta a^2 \left[2\beta M_0 - Q_0\right]$$

$$= \overset{.388 \times 10^{-6}}{.00408(.161)(590) \times 10^{-6}} \left[\overset{.322}{2(.161)} M_0 - Q_0\right]$$

$$= .125 \times 10^{-6} M_0 - .388 \times 10^{-6} Q_0$$

$$\theta_1 \times 10^6 = .125 M_0 - .388 Q_0$$

b) HEAD

$$\Theta_2 = \frac{2\beta^2}{Ek} b^2 \frac{b^2}{a^2}\left[2\beta\frac{b^2}{a^2}M_0 + Q_0\right]$$

$$= .000658(147.5)(.25)\left[.322(.25)M_0 + Q_0\right]\times 10^{-6}$$

$.0242$

$$= .00195\times 10^{-6} M_0 + .0242\times 10^{-6} Q_0$$

$$\Theta_2 \times 10^6 = .00195 M_0 + .0242 Q_0$$ ✓

④ BOUNDARY CONDITIONS — $\Theta_1 = -\Theta_2$

$\omega_1 + \delta_1 = \omega_2 + \delta_2$

a) $\Theta_1 = -\Theta_2$

$$.125 M_0 - .388 Q_0 = -\left[.00195 M_0 + .0242 Q_0\right]$$

$.125$
$\underline{.002}$
$.127 M_0$

$$.127 M_0 = .3638 Q_0$$

$$M_0 = 2.86 Q_0$$

b) $\omega_1 + \delta_1 = \omega_2 + \delta_2$

$-.388 M_o + 2.41 Q_o - .00765 = -.0242 M_o - .602 Q_o + .01032$

$+ \underline{.024} \qquad + \underline{.160} Q_o \qquad\qquad + \underline{.00765}$

$-.364 M_o \qquad 3.01 Q_o \qquad\qquad .01797$

$-.364 M_o + 3.01 Q_o = .01797$

Subst. value for M_o

$-.364(2.86 Q_o) + 3.01 Q_o = .01797 \times 10^6$

$-1.04 Q_o + 3.01 Q_o = .01797 \times 10^6$

$1.97 Q_o = .01797 \times 10^6$

$Q_o = .00912 \times 10^6$

$M_o = 2.86 Q_o$

$M_o = 2.86 (-.00912) \times 10^6$

$M_o = -.0261 \times 10^6$

$M_o = 26100$ in#/in

$Q_o = -9120$ #/in

⑤ Check $\Theta_1 = -\Theta_2$

$$.125 M_o - .38 Q_o = -\left[.00195 M_o + .0242 Q_o\right]$$

$.125(26100) = 3260$ $.00195(26100) = -51$

$-.388(9120) = -3540$ $.0242(9120) = -221$

-280 -272

$-280 = -272$ OK

$$\omega_1 + \delta_1 = \omega_2 + \delta_2$$

$$-.388 M_o + 2.41 Q_o - .00765 = -.0242 M_o - .602 Q_o + .01032$$

$$-.388(26100) + 2.41(9120) - .00765 = -.0242(26100) - .602(9120) + .01032$$

$$\underbrace{-10120 + 21950}_{\omega_1 \times 10^6} - .00765 = \underbrace{-633 - 5480}_{\omega_2 \times 10^6} + .01032$$

21950
-10120
11830

5480
633
6113

$.011830 = \omega_1$ $-.006113 = \omega_2$

$-.00765$ $+.01032$

$.00418 = .00421$ OK

$\omega_1 = .01183$ for SHELL

$\omega_2 = -.00611$ for HEAD

⑥ DETERMINATION M_x & Q_x FOR SHELL –

$$M_x = M_o\,\varphi(\beta x) - \frac{Q_o}{\beta}\,\zeta(\beta x)$$

$$M_x = 26100\,\varphi(\beta x) - \frac{9120}{.161}\,\zeta(\beta x)$$

$$M_x = 26100\,\varphi(\beta x) - 56600\,\zeta(\beta x) \qquad Ⓐ$$

$$Q_x = -2\beta M_o\,\zeta(\beta x) + Q_o\,\psi(\beta x)$$

$$= -2(.161)(26100)\,\zeta(\beta x) + 9120\,\psi(\beta x)$$

$$Q_x = -8410\,\zeta(\beta x) + 9120\,\psi(\beta x) \qquad Ⓑ$$

SUBSTITUTE VALUES FROM TABLE 45 Pg 394 "Theory of PLATES & SHELLS" TO obtain VALUE of MAXIMUM MOMENT M_x AND MAXIMUM SHEAR Q_x

BY ________ DATE ________ SUBJECT ________________ SHEET NO. 8 OF 10
CHKD. BY ________ DATE ________ JOB NO. ________

βx	$\varphi(\beta x)$	$\zeta(\beta x)$	$26100\,\varphi(\beta x)$	$56600\,\zeta(\beta x)$	M_x
0	1.00	0	26100	0	26100

M_x is max, at edge of shell = 26100

Q_x " " " " " " = 9120

7. The deflection of the shell at the edge where the moment is a maximum, is

$$w \times 10^6 = -.388\,M + 2.41\,Q$$

$$= -.388(26100) + 2.41(9120)$$

$$= -10130 + 22000$$

$$w \times 10^6 = 11870$$

8. STRESSES IN SHELL AT THE EDGE where M = 26100
Q = 9120

a) AXIAL —

$$\sigma_1 = \frac{ap}{2h} \pm \frac{6M}{h^2} = \frac{24.312(1200)}{2(2.625)} \pm \frac{6(26100)}{(2.625)^2}$$

$$\sigma_1 = 5550 + \left[\begin{array}{ll} +22700 & \text{INSIDE} \\ -22700 & \text{OUTSIDE} \end{array}\right] \text{DISCONT STRESS}$$

$$\sigma_1 = \begin{array}{ll} 28250 & \text{INSIDE} \\ -17150 & \text{OUTSIDE} \end{array}$$

b) Circumferential –

$$\sigma_2 = \frac{ap}{h} - \frac{Ew}{a} \pm \frac{6\nu M}{h^2}$$

$$= 11100 - \overset{14650}{\frac{(30\times10^{6})(11870\times10^{-6})}{24.312}} \pm \overset{6810}{.3(22700)}$$

$$= 11100 - 14650 \pm 6810$$

$$= 11100 + \begin{bmatrix} -21{,}460\text{ PSI} & \text{OUTSIDE} \\ -7840\text{ PSI} & \text{INSIDE} \end{bmatrix}$$

9. DETERMINATION of M_x & Q_x in HEAD —

AS EXPLAINED IN the DISCONTINUITY CALCULATIONS for THE BOTTOM HEAD, ONLY THE STRESSES AT THE EDGE OF THE HEAD WILL BE DETERMINED.

$$M_x = 26100\,\varphi(\beta x) + 56600\,\zeta(\beta x)$$

$$Q_x = -8410\,\zeta(\beta x) + 9120\,\varphi(\beta x)$$

AT THE EDGE WHERE $x = 0$, THE SHEAR & MOMENT IS

$$M_o = 26100$$
$$Q_o = 9120$$

THE DEFLECTION AT THE EDGE IS

$$w = -6122 \times 10^{-6}$$

10. STRESSES IN HEAD

a) AXIAL or MERIDIONAL —

$$\sigma_1 = \frac{ap}{h} \pm \frac{6M}{h^2} = \frac{24.312(1200)}{2(2.625)} \pm \frac{6(26100)}{(2.625)^2}$$

$$\sigma_1 = 5550 + \begin{bmatrix} +22{,}700\,\text{psi} & \text{INSIDE} \\ -22{,}700\,\text{psi} & \text{OUTSIDE} \end{bmatrix} \begin{array}{l}\text{DISCONT} \\ \text{STRESS} \\ \text{AT PT}\end{array}$$

$$\sigma_1 = \begin{array}{ll} 28250\,\text{psi} & \text{INSIDE} \\ -17150\,\text{psi} & \text{OUTSIDE} \end{array}$$

b) CIRCUMFERENTIAL

$$\sigma_2 = -\frac{ap}{h} - \frac{E\omega}{a} \pm \frac{6\nu M}{h^2}$$

$$= -11100 - \frac{(30\times10^6)(-6122\times10^{-6})}{24.312} \pm .3(22700)$$

$$= -11100 + 7560 \pm 6820$$

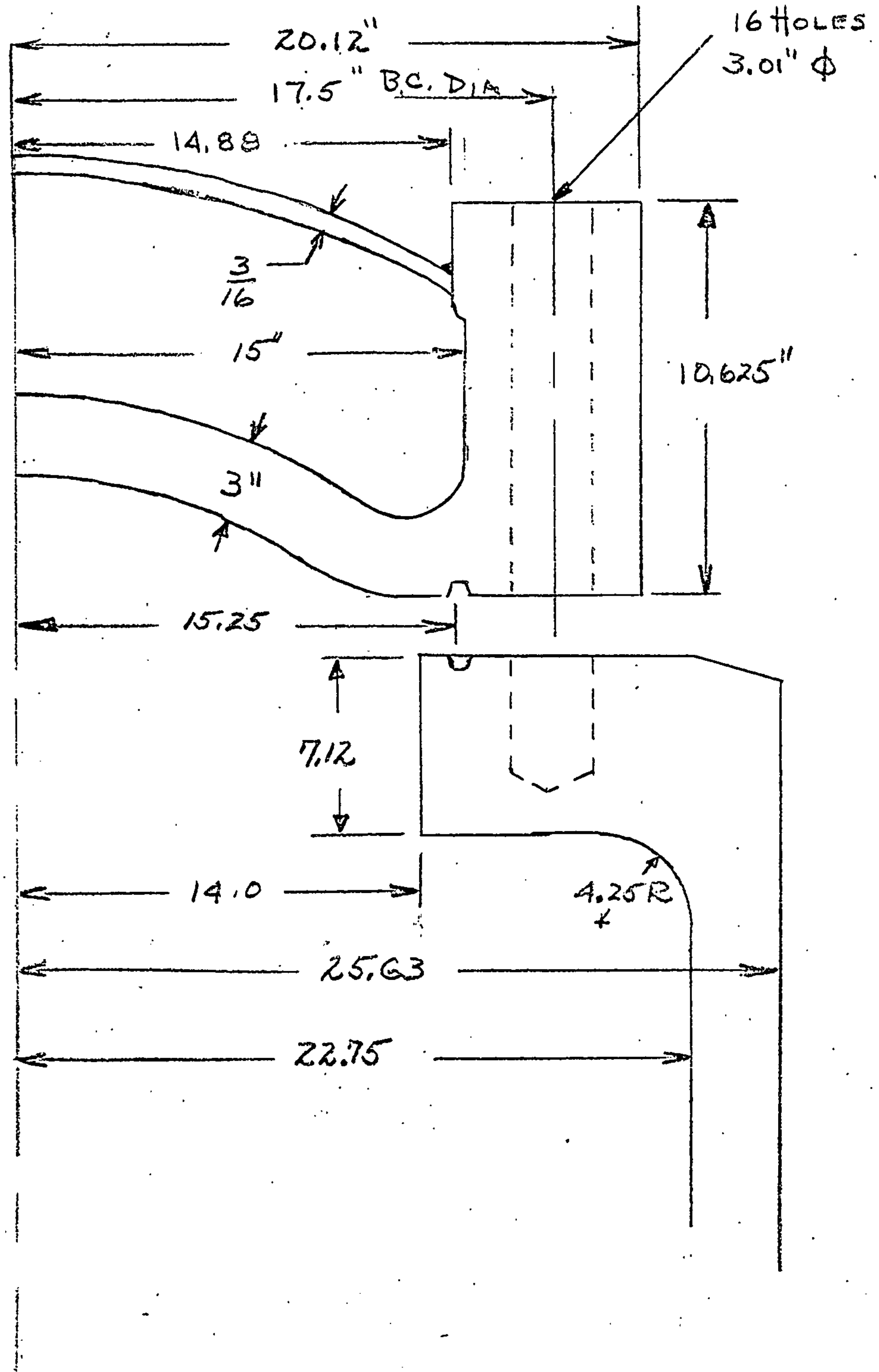
$$= -11100 + \begin{bmatrix} 740\,\text{psi} & \text{OUTSIDE} \\ 14380\,\text{psi} & \text{INSIDE} \end{bmatrix} \begin{array}{l}\text{DISCONT} \\ \text{STRESS} \\ \text{AT PT.①}\end{array}$$

$$= \begin{array}{ll} 3280\,\text{psi} & \text{outside} \\ -10360\,\text{psi} & \text{inside} \end{array}$$

BY BJR DATE
CHKD. BY DATE
SUBJECT SM-1A VESSEL AND COVER STRESS ANALYSIS AE-90 TASK 6.9
SHEET NO. 1 OF 31
JOB NO. CONTR NO AT (30-1)-2639

SM-1A COVER & FLANGE ANALYSIS

REF. DUNKIRK DWG. 41201-1-1
REACTOR VESSEL
OUTLINE & ASS'Y.



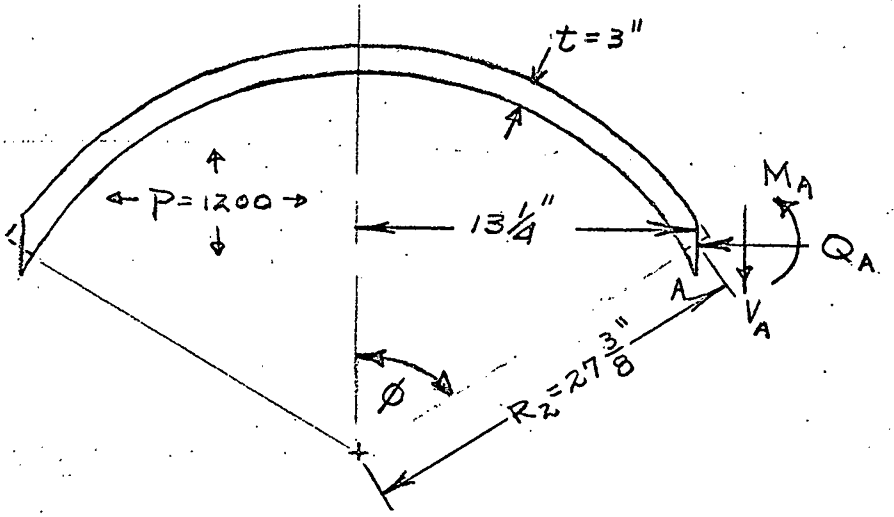
ALL DIMENSIONS INCLUDE CLADDING

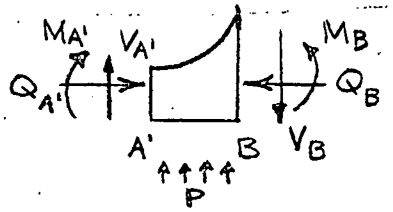
BY BJR DATE SUBJECT SM-1A VESSEL AND COVER ANALYSIS SHEET NO. 2 OF 21

CHKD. BY DATE JOB NO.

CASE "A" – STUD LOAD ONLY

I COVER DOME –





SIGN CONVENTION ΔR + →
θ + ↶ ccw

$$\phi = \sin^{-1}\frac{13.25}{27.375} = \sin^{-1} .484 = 29^\circ$$

1. REACTION TO EDGE LOADS (Ref. Roark$^{(5)}$ P.272 Cases 14 & 15)

$$\beta = \sqrt[4]{3(1-\nu^2)\left(\frac{R_2}{t}\right)^2} = 1.283\sqrt{\frac{27.375}{3}} = 3.883$$

$$K_1 = 1 - \frac{1-.6}{2\times3.88}\cot 29^\circ = 1 - .0951 = .907$$

$$K_2 = 1 - \frac{1+.6}{2\times3.88}\cot 29^\circ = 1 - .372 = .628$$

$$\Delta R_A = -\frac{Q_A}{(28.5\times10^6)(3)}\left(3.88\times27.375\times.484^2\right)\left(.628 + \frac{1}{.907}\right) + \frac{M_A}{(28.5\times10^6)(3)}\left(\frac{2\times3.88^2\times.484}{.907}\right)$$

$$\Delta R_A = -.505\times10^{-6}\,Q_A + .187\times10^{-6}\,M_A \qquad (1)$$

BY DATE
CHKD. BY DATE
SUBJECT SM-1A VESSEL AND COVER ANALYSIS
SHEET NO. 3 OF 31
JOB NO.

$$\theta_A = -0.187 \times 10^{-6} Q_A + \frac{M_A}{(28.5 \times 10^6)(3)} \left(\frac{4 \times 3.88^3}{27.375 \times .907} \right)$$

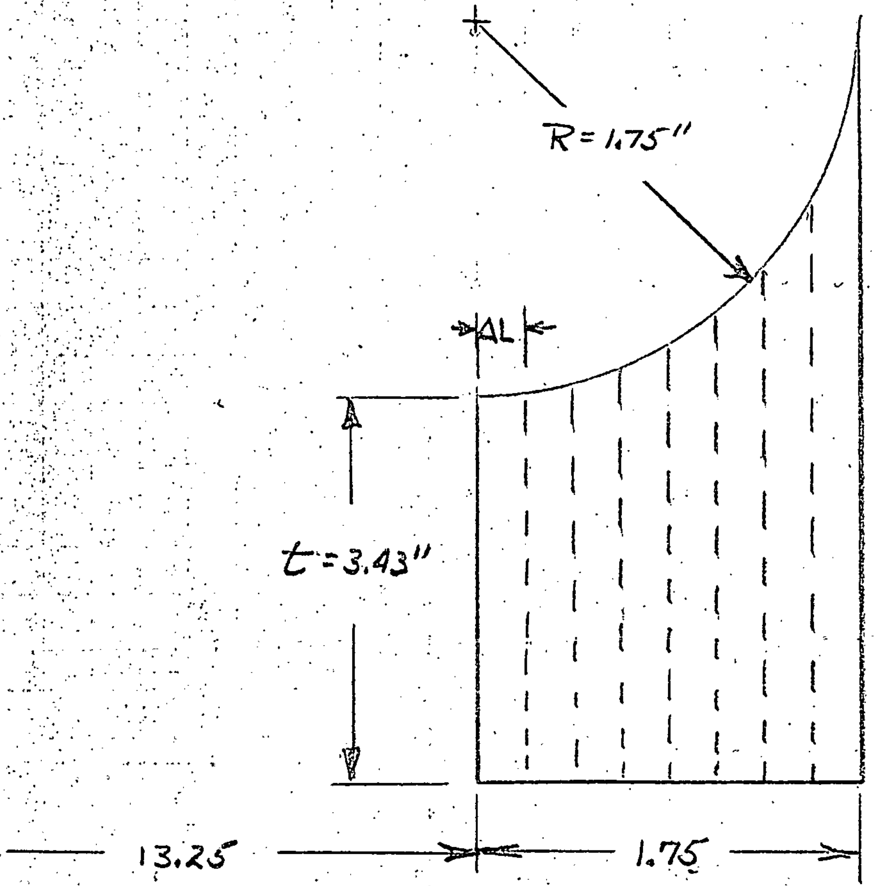
$$\theta_A = -0.187 \times 10^{-6} Q_A + 0.1158 \times 10^{-6} M_A \qquad (2)$$

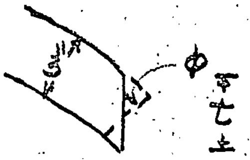
II INTERMEDIATE RING —

The intermediate ring is treated as a beam using numerical integration and moment area method. Correct approximately for taper.

$$\Delta\theta = \Sigma \frac{M}{EI} \Delta L$$

for $\frac{1}{EI}$ use $\frac{1}{D}$ to account for plate behavior





$$\text{thickness } t = \frac{3}{\cos\phi} = \frac{3}{.875} = 3.43''$$

BENDING OF INTERMEDIATE RING

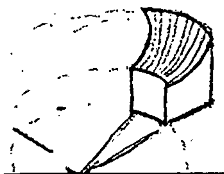
$$\frac{1}{EI} = \frac{1}{D} = \frac{12(1-\nu^2)}{Et^3} = \frac{10.92}{28.5 \times 10^6 \times t^3}$$

Point x	Thickness t	$\frac{1}{EI} \times 10^6$	$\frac{x}{EI} \times 10^6$
.125	3.43392	.0094625	.0011828
.375	3.46547	.0092064	.0034524
.625	3.53016	.0087095	.0054434
.875	3.63156	.0080001	.0070001
1.125	3.77642	.0071143	.0080036
1.375	3.97764	.0060883	.0083714
1.625	4.26408	.0049419	.0080306
1.75	Σ =	.051463	.038770

$$\text{Change in } \theta = \int \frac{M\,dL}{EI} = 0.25 \times .0515 \times 10^{-6} M_A' + 0.25 \times .0385 \times 10^{-6} V_A'$$

$$= 0.01288 \times 10^{-6} M_A' + 0.0097 \times 10^{-6} V_A'$$

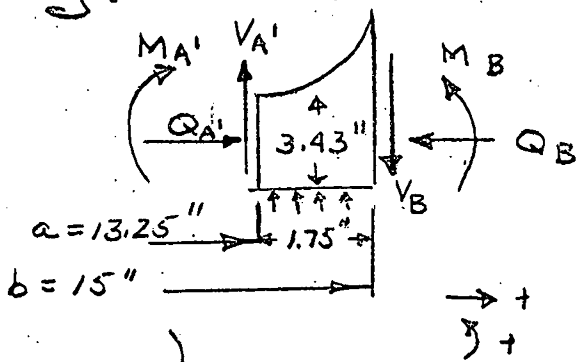
Correction for taper (beam increases in width with increasing radius)



$$\Delta\theta_B = \frac{13.25}{14.125}\int\frac{M\,dL}{EI} = 0.01205\times10^{-6}M_A' + 0.0091\times10^{-6}V_A'$$

$$\theta_B = \theta_A + \Delta\theta_B = 0.1279\times10^{-6}M_A' + 0.0091\times10^{-6}V_A' - 0.187\times10^{-6}Q_A \qquad (3)$$

The intermediate ring is treated as a thick walled cylinder 3.43 inches long.



REF. ROARK

$$\Delta R_{A'} = \frac{Q_A'}{3.43}\times\frac{13.25}{28.5\times10^6}\left(\frac{15^2+13.25^2}{15^2-13.25^2}+.03\right) - \frac{Q_B}{3.43}\times\frac{13.25}{28.5\times10^6}\left(\frac{2\times15^2}{15^2-13.25^2}\right)$$

$$\Delta R_{A'} = 1.14\times10^{-6}Q_A' - 1.235\times10^{-6}Q_B \qquad (4)$$

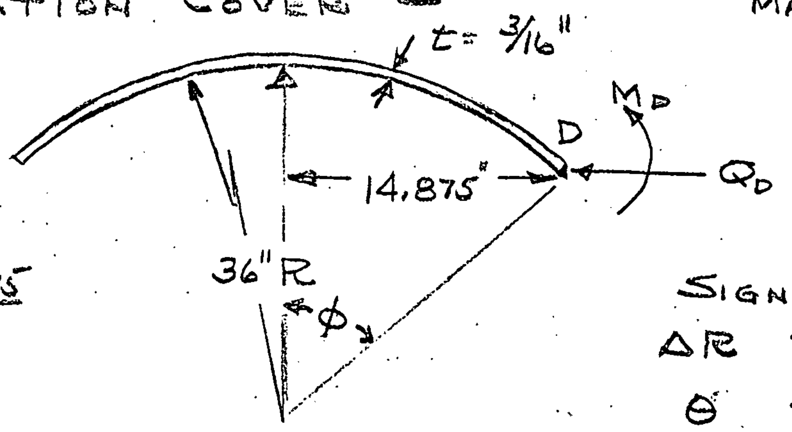
$$\Delta R_B = \frac{Q_A'}{3.43}\times\frac{15}{28.5\times10^6}\left(\frac{2\times13.25^2}{49.4}\right) - \frac{Q_B}{3.43}\times\frac{15}{28.5\times10^6}(8.1-0.3)$$

$$\Delta R_B = 1.09\times10^{-6}Q_A' - 1.197\times10^{-6}Q_B \qquad (5)$$

III INSULATION COVER —

MAT'L. SA 240 GR S

$E = 28 \times 10^6$



$$\phi = \sin^{-1} \frac{14.875}{36}$$

$$\phi = 24.4°$$

SIGN CONVENTION

ΔR + →

θ + ↺ CCW

1. REACTION TO EDGE LOADS

$$\beta = 1.283 \sqrt{\frac{36}{.1875}} = 17.8$$

$$K_1 = 1 - \frac{1-.6}{2 \times 17.8} \cot 24.4° = 1 - .0248 = .9752$$

$$K_2 = 1 - \frac{1+.6}{2 \times 17.8} \cot 24.4° = 1 - .0993 = .9007$$

$$\Delta R_D = \frac{-Q_D}{28.5 \times 10^6 \times .1875} (17.8 \times 36 \times .413^2)\left(.9008 + \frac{1}{.9752}\right)$$

$$+ \frac{M_D}{28.5 \times 10^6 \times .1875} \left(\frac{2 \times 17.8^2 \times 0.413}{0.9752} \right)$$

$$\Delta R_D = -40.1 \times 10^{-6} Q_D + 51.2 \times 10^{-6} M_D$$

$$\theta_D = -51.2 \times 10^{-6} Q_D + \frac{M_D}{28 \times 10^6 \times .1875} \left(\frac{4 \times 17.8^3}{36 \times .9752} \right)$$

$$\theta_D = -51.2 \times 10^{-6} Q_D + 122.4 \times 10^{-6} M_D$$

ASSUME THE INSULATION COVER TO HAVE A HINGED EDGE WHERE IT IS WELDED TO THE COVER RING AT POINT D.

$$\therefore \quad M_D = 0$$

$$\Delta R_D = -40.1 \times 10^{-6} Q_D \text{ OR } Q_D = -0.025\times10^{-6}\Delta R_D \qquad (6)$$

$$\theta_D = -51.2\times10^{-6}\ Q_D \qquad (7)$$

$10^6 \times \Delta R_D = \Delta R_B - 8.41\ \theta_B$ Subst. expression for ΔR_B & θ_B

$$10^6\times \Delta R_D = 1.09\ Q_A - 1.197\ Q_B - 8.41\,(0.1279\ M_A - 0.187\ Q_A + 0.0091\ V_A)$$

FOR CASE "A" STUD LOAD ONLY $V_A = 0$ and using expression $10^6 \Delta R_D = -40.1\ Q_D$ from above

$$Q_D = -0.0665\ Q_A + 0.0299\ Q_B + 0.0269\ M_A$$

FOR CASE "A" STUD LOAD ONLY $\Delta P = 0$ and $P =$ LOAD WHICH PRODUCES ASME CODE ALLOWABLE STRESS IN STUDS (REF C9-16-2005)

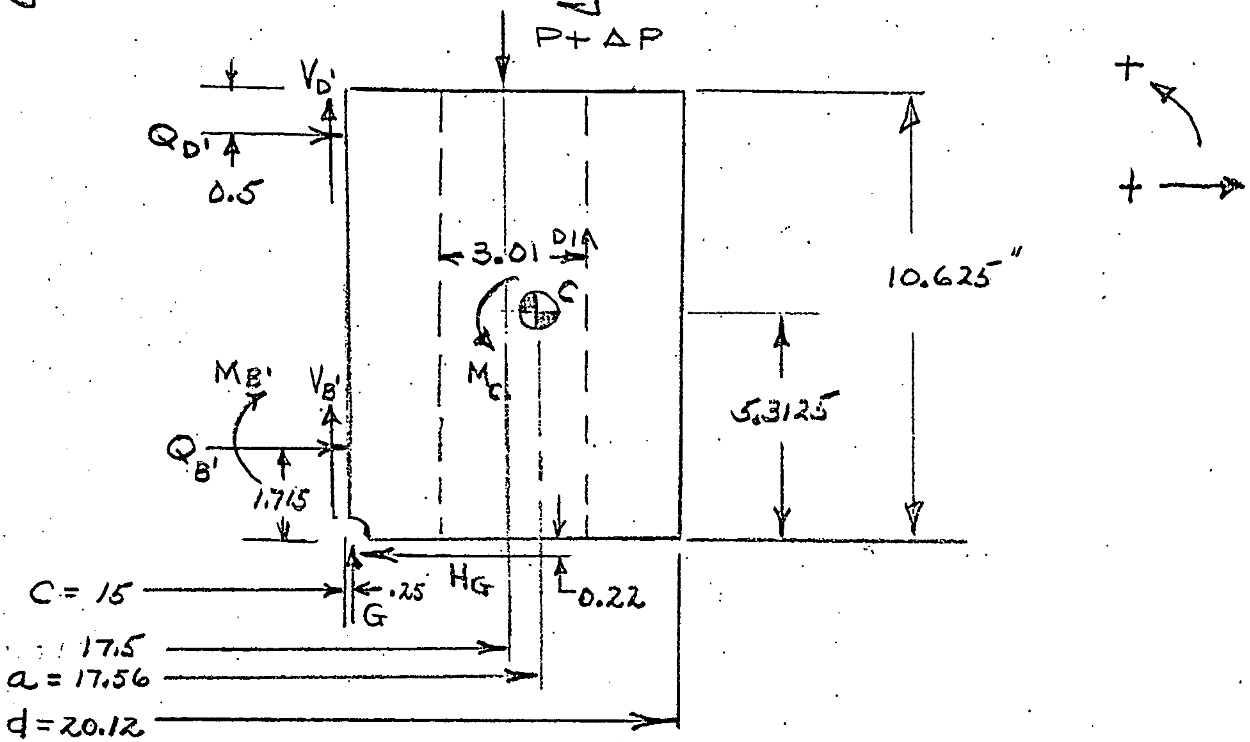
$$\text{STUD AREA} = \frac{\pi}{4}\times 2.531^2 \times 16 = 80.5 \text{ in}^2$$

ALLOWABLE STRESS (TYPE 403) = 20,000 PS.I.

$$P = \frac{80.5 \times 20000}{\pi \times 35} = 14{,}640\ \#/\text{in}$$

IV OUTER RING —

The outer ring is treated as a thick walled cylinder 10.625" long.



1. DEFLECTION DUE TO RADIAL LOADS —

$$\Delta R_c = \frac{(Q_{B'} + Q_{D'} - H_G)}{10.625} \times \frac{15}{28.5\times10^6}\left(\frac{20.12^2 + 15^2}{20.12^2 - 15^2} + 0.3\right)$$

$$= 0.188\times10^{-6}\,(Q_B' + Q_D' - H_G)$$

The loss of area due to bolt holes = $\frac{\pi}{4}(3.01)^2 16 = 114\ in^2$

Total area of ring = $\pi(20.12^2 - 15^2)$ = $565\ in^2$

Assume stiffness is proportional to area

$$\Delta R_c \text{ CORRECTED} = 0.188\times10^{-6}(Q_{B'} + Q_{D'} - H_G)\,\frac{565}{565-114}$$

$$\Delta R_c = 0.236\times10^{-6}(Q_{B'} + Q_{D'} - H_G) \qquad (8)$$

2. ROTATION — TREAT AS A RING OF RIGID CROSS SECTION.

$$\theta_c = 12 \frac{M_c a}{E h^3 \ln \frac{d}{c}} = \frac{12 \times 17.56 \times M_c}{28.5 \times 10^6 (10.625)^3 \ln\left(\frac{20.12}{15}\right)}$$

$$\theta_c = 0.021 \times 10^{-6} M_c \qquad (9)$$

Correcting for bolt holes in the cover ring

$$\theta_c = 0.021 \times 10^{-6} M_c \times \frac{565}{451}$$

$$\theta_c = 0.0263 \times 10^{-6} M_c$$

Taking sum of moments about C

$$17.56\, M_c = 15 Q_{B'} \times 3.6 - 15 M_{B'} - 15.25 H_G \times 5.533 - 15.25 (1.15 P) \times 2.31 \quad (a)$$

$$+ 17.5 P \times .06 - 15 Q_{D'} \times 4.813 - 15 (V_{B'} + V_{D'}) \times 2.56$$

for CASE "A" STUD LOAD ONLY

$$\left.\begin{array}{l} V_{B'} = V_{D'} = 0 \\ \text{and from pg 8} \\ Q_D = -0.0665\, Q_A + 0.0299\, Q_B + 0.0269\, M_A \\ \text{Also } P = 14{,}640 \text{ \#/in. from pg. 8} \\ M_B = \frac{13.25}{15} M_A = 0.883\, M_A \end{array}\right\} (b)$$

Substituting eqns (b) in (a) gives

$$M_c = 0.273\, Q_A + 2.975\, Q_B - 0.8655 - 4.8 H_G - 33000 \quad (c)$$

Then by substituting eqn. (c) in (9)

$$10^6 \theta_c = 0.0263 \, M_c$$

$$10^6 \theta_c = 0.00719 \, Q_A + 0.0777 \, Q_B - 0.0228 \, M_A - 0.126 \, H_G - 869 \qquad (10)$$

Compatibility Equations –

$$\Delta R_A = \Delta R_A' \qquad (11)$$

$$\Delta R_B = \Delta R_B' = \Delta R_c + 3.6 \, \theta_B \qquad (12)$$

$$\theta_B = \theta_B' \text{ or } \theta_B = \theta_c \qquad (13)$$

Using eqn. (13) and substituting eqns (10) & (3) gives

$$0.1507 \, M_A - 0.1942 \, Q_A - 0.0777 \, Q_B + 0.126 \, H_G = -869 \; * (14)$$

Using eqn. (12) and substituting eqns (5) & (8) gives

$$1.09 \, Q_A - 1.197 \, Q_B - 0.236 \, Q_B + 0.0157 \, Q_A - 0.007 \, Q_B - 0.0064 \, M_A - 0.46 \, M_A + 0.236 \, H_G + 0.673 \, Q_A = 0.0328 \, V_A$$

FOR CASE "A" BOLT LOAD ONLY $V_A = 0$

$$1.779 \, Q_A - 1.440 \, Q_B - 0.4664 \, M_A + 0.236 \, H_G = 0 \quad * \quad (15)$$

Using eqn.(11) and substituting eqns (1) & 4 gives

$$-1.645\,Q_A + 1.235\,Q_B + 0.187\,M_A = 0$$ * (16)

Using eqn (12) and substituting eqn.(8) gives

$$\Delta R_B = 0.236\,(Q_B + Q_D - H_G) + 3.6\,\theta_B$$

From eqn.(7) $Q_D = -0.0195\,\theta_D$ & $\theta_D = \theta_B$

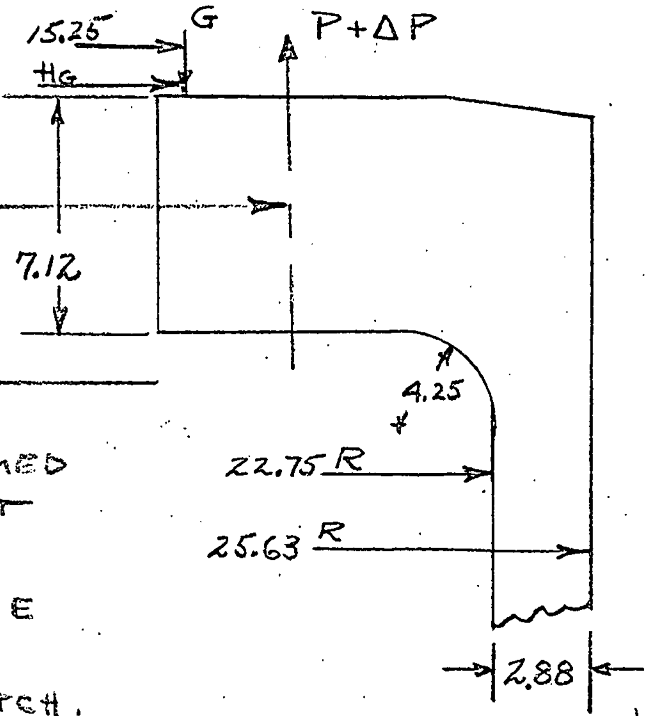
$\therefore .236\,Q_D = -.0046\,\theta_B$ can be neglected compared to $3.6\,\theta_B$

$$1.764\,Q_A - 1.433\,Q_B + 0.236\,H_G - 0.46\,M_A - 0.0328\,V_A = 0$$

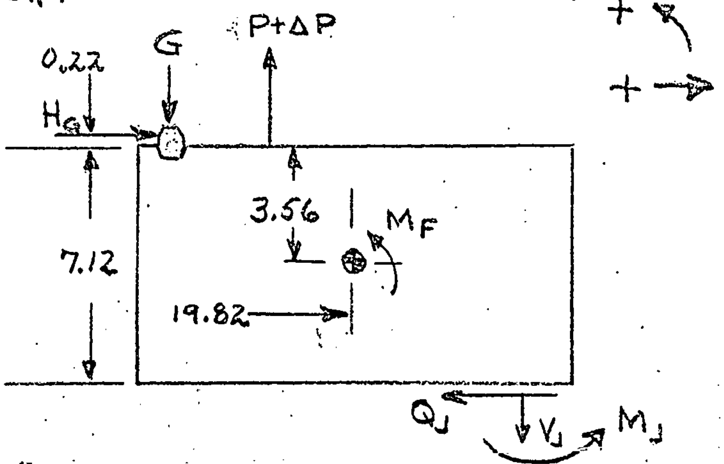
for Case "A" - Bolt load $P = 0$ & $V_A = 0$

$$1.764\,Q_A - 1.433\,Q_B + 0.236\,H_G - 0.46\,M_A = 0$$ (17)

V VESSEL FLANGE —



THE LOWER FLANGE IS ASSUMED TO BE RIGID SUCH THAT IT ROTATES BUT DOES NOT BEND. FOR SIMPLICITY, THE VESSEL FLANGE IS TREATED AS SHOWN IN LOWER SKETCH.



THE FLANGE IS ASSUMED TO ROTATE ABOUT POINT G, THE MID POINT OF THE GASKET

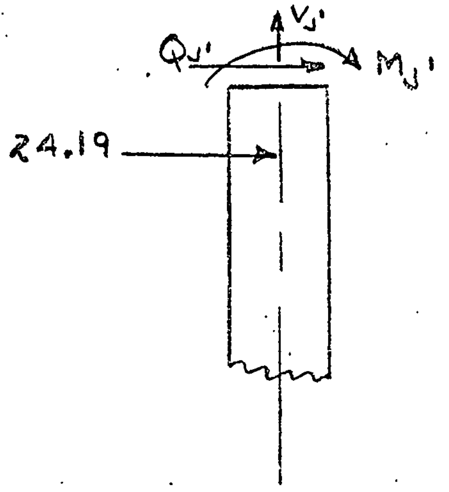
$$\Delta R_G = \Delta R_B + \left(\frac{3.43}{2} + .22\right)\theta_B$$

$$\Delta R_G = \Delta R_B + 1.94\,\theta_B \qquad (18)$$

$$\Delta R_G = \Delta R_{F_G} - 3.78\,\theta_F \qquad (19)$$

$$\Delta R_{J'} = \Delta R_J = \Delta R_{F_J} + 3.56\,\theta_J \qquad (20)$$

$$\theta_F = \theta_J \qquad (21)$$



DEFLECTION of LOWER FLANGE AT F −

$$\Delta R_{F_G} = \frac{H_G}{7.12} \times \frac{14}{28.5\times10^6}\left(\frac{25.63^2+14^2}{25.63-14^2}+0.3\right) -$$

$$\frac{24.19}{25.63}\left(\frac{Q_J}{7.12}\right)\frac{14}{28.5\times10^6}\left(\frac{2\times25.63^2}{25.63^2-14^2}\right)$$

$$\Delta R_{F_G} = 0.1483\times10^{-6} H_G - 0.186\times10^{-6} Q_J \qquad (22)$$

ROTATION $\theta_F = \frac{12\ M_F\, a}{E\, h^3 \ln\frac{d}{c}} = \frac{12\, M_F\times19.82}{28.5\times10^6\times7.12^3 \ln\frac{25.63}{14.0}}$

$$\theta_F = 0.0382\times10^{-6} M_F \qquad (23)$$

Sum of Moments about F

$$19.82\, M_F = 15.25\, G \times 4.57 - 15.25\, H_G \times 3.78 - 17.5\, P\times 2.32 - 24.19\, Q_J \times 3{,}56$$

$$+ 24.19\, M_J$$

where $G = 1.15\, P$ & $P = 14{,}640$ #/in

$$\therefore\ M_F = -2.9\, H_G - 4.34\, Q_J + 1.22\, M_J + 29200 \qquad (24)$$

Substituting eqn. 24 in eqn. 23

$$10^6\times\theta_F = -0.111\, H_G - 0.166\, Q_J + 0.0466\, M_J + 1115 \qquad (25)$$

DEFLECTION of LOWER FLANGE AT J −

$$\Delta R_{F_J} = \frac{H_G(24.19)}{7.12(28.5\times10^6)}\left(\frac{2\times(14)^2}{25.63^2-14^2}\right) - \frac{Q_J(24.19)(24.19)}{7.12(25.63)(28.5\times10^6)}\left(\frac{25.63^2+14^2}{25.63^2-14^2}-\nu\right)$$

$$\Delta R_{F_J} = .1013\times10^{-6} H_G - .174\times10^{-6} Q_J \qquad (25a)$$

DEFLECTION of LOWER FLANGE AT J

$$\beta = \frac{1.285}{\sqrt{24.19\times 2.88}} = 0.154$$

$$D = \frac{28.5\times 10^{6}\times 2.88^{3}}{12(1-0.09)} = 62.4\times 10^{6}$$

$$\Delta R_{j'} = \frac{1}{2(0.154)^{3}\,62.4\times 10^{6}}\left(0.154\,M_j + Q_j\right)$$

$$\Delta R_{j'} = 0.337\times 10^{-6} M_j + 2.19\times 10^{-6} Q_j \qquad (26)$$

Rotation of Lower Flange at J

$$\theta_J = -\frac{1}{2\times(0.154)^{2}\times 62.4\times 10^{6}}\left(2\times 0.154\,M_J + Q_j\right)$$

$$\theta_J = -0.104\times 10^{-6} M_j - 0.337\times 10^{-6} Q_j \qquad (27)$$

FOR THE LOWER FLANGE CALCULATIONS, no correction WAS MADE FOR THE VOLUME of metal removed FOR THE STUD HOLES. This is balanced by the volume of metal in the fillet, between the Lower flange and the cylinder, which was not taken into account

Vol. fillet	Vol. of Holes
$V = \left[4.25^{2} - \frac{1}{4}\pi(4.25^{2})\right]2\pi\times 21.75$	$V = 16\,\frac{\pi}{4}(2.76)^{2}\times 5.75$
$V_{fillet} = 530\ in^{3}$ $\approx$	$V_{STUD\ HOLES} = 550\ in^{3}$

Using eqn.(19) and substituting eqns.(22), (18), & .(25)

gives –

$$.1483\, H_G - .0586\, Q_J - .727\, Q_A + 1.197\, Q_B - .2479\, M_A + .393\, M_J = 0 \quad * \ (28)$$

Using eqn(21) and substituting eqns. (25) & (27)
gives –

$$.111\, H_G - .171\, Q_J - .1506\, M_J = 1115 \quad * \ (29)$$

Using eqn.(20) and substituting eqns(25a) & (27)

gives:

$$.707\, H_G + 3.564\, Q_J - .1013\, H_G = 0 \quad * \ (30)$$

Thus with 6 equations* the 6 unknown FORCES & MOMENTS CAN BE FOUND

(14) $-1.645\, Q_A + 1.235\, Q_B + 0.187\, M_A = 0$
(15) $1.779\, Q_A - 1.440\, Q_B - 0.4669\, M_A + 0.236\, H_G = 0$
(16) $-0.1942\, Q_A - 0.0777\, Q_B + 0.1507\, M_A + 0.126\, H_G = -869$
(28) $-0.727\, Q_A + 1.197\, Q_B - 0.2479\, M_A + 0.1483\, H_G + 1.089\, Q_J + 0.393\, M_J = 0$
(29) $0.111\, H_G - 0.171\, Q_J - .1506\, M_J = 1115$
(30) $-0.1013\, H_G + 3.564\, Q_J + 0.707\, M_J = 0$

SOLUTION –

$Q_A = 1461$ #/in $\qquad H_G = -1074$ #/in

$Q_B = 2225$ #/in $\qquad Q_J = 2059$ #/in

$M_A = -1838$ in-#/in $\qquad M_J = -10533$ in-#/in.

BY________DATE________ SUBJECT SM-1A VESSEL AND COVERS ANALYSIS SHEET NO. 17 OF 31

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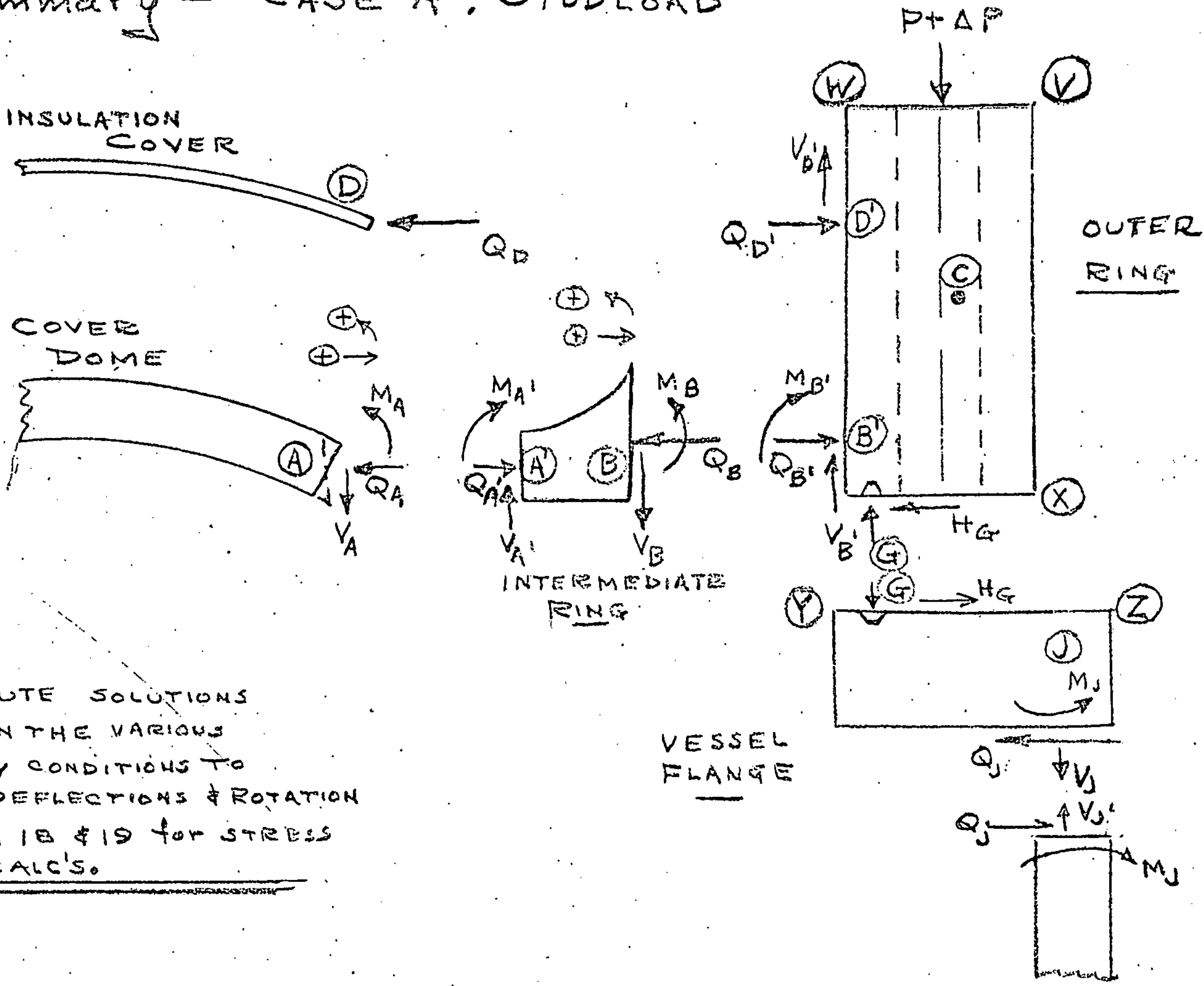
Summary - CASE "A"; STUD LOAD



SUBSTITUTE SOLUTIONS FOUND IN THE VARIOUS BOUNDARY CONDITIONS TO OBTAIN DEFLECTIONS & ROTATION

SEE pgs 18 & 19 for STRESS CALC'S.

LOCATION	DEFLECTION $(\Delta R)\ 10^{-6}$ in	ROTATION $(\theta)\ 10^{-6}$ RAD.	STRESSES			
			CIRCUMF (σ_2) PSI	AXIAL σ_1 PSI	RADIAL (σ_3) PSI	SHEAR PSI
A	-1085	-486	-2624 INNER -2056 OUTER	-1780 INNER +110 OUTER	0	
B	-1070	-508	-2288 inner -1792 outer	-1261 INNER 391 outer	0	
D	3210	-508	6160	-428 INNER -428 OUTER	0	
G	-2054	—	—	—	—	-1435
J	958	400	3410 outer -1170 inner	-7630 INNER +7630 outer	0	
V	3235	-508	4580	—	—	
W	3460	-508	6570	—	—	
X	2165	-508	3080	—	—	
Y	-2059	400	-4200	—	—	
Z	-1890	400	-2100	—	—	

STRESS CALCULATIONS ~ CASE A

A) AXIAL STRESS:

$$\sigma_{1_{THRUST}} = -\frac{Q_A}{t_A} = -\frac{1461}{1.75} = -835 \text{ psi}$$

$$\sigma_{1_{BENDING}} = \pm \frac{6M_A}{h^2} = \frac{6(1838)}{(3.43)^2} = \begin{matrix} +945 \text{ psi OUTSIDE} \\ -945 \text{ psi INSIDE} \end{matrix}$$

$$\sigma_1 = -835 \pm 945 = \begin{matrix} +110 \text{ psi OUTSIDE} \\ -1780 \text{ psi INSIDE} \end{matrix}$$

CIRCUMFERENTIAL:

$$\sigma_2 = \frac{\Delta R_A}{R_A} \times E \pm \frac{\nu 6M_A}{h^2} = \overset{-2340}{\frac{-1085(28.5)}{13.25}} \pm \overset{\pm 284}{.3(945)}$$

$$\sigma_2 = \begin{matrix} -2624 \text{ psi INSIDE} \\ +2056 \text{ psi OUTSIDE} \end{matrix}$$

RADIAL: $\sigma_3 = 0$

B) AXIAL:

$$\sigma_{1_{THRUST}} = -\frac{Q_B}{t_B} = \frac{2224}{5.12} = -435 \text{ psi}$$

$$\sigma_{1_{BENDING}} = \frac{6M_B}{h^2} = \frac{6(.833 \times -1838)}{(3.43)^2} = \begin{matrix} -826 \text{ psi INSIDE} \\ +826 \text{ psi OUTSIDE} \end{matrix}$$

$$\sigma_1 = 435 \pm 826 = \begin{matrix} -391 \text{ psi OUTSIDE} \\ -1261 \text{ psi INSIDE} \end{matrix}$$

CIRCUMFERENTIAL:

$$\sigma_2 = \frac{\Delta R_B}{R_B} \times E \pm \frac{\nu 6M_B}{h^2} = \overset{-2040}{\frac{-1074(28.5)}{15}} \pm \overset{\pm 248}{.3(826)}$$

$$\sigma_2 = \begin{matrix} -2288 \text{ psi INSIDE} \\ -1792 \text{ psi OUTSIDE} \end{matrix}$$

RADIAL: $\sigma_3 = 0$

D) AXIAL:

$$\sigma_1 = \sigma_{1_{THRUST}} \pm \sigma_{1_{BENDING}}$$

$$\sigma_1 = -\frac{80,2}{.1875} \pm \left[\sigma_{1_{Bending}} = 0 \text{ since } M_D = 0\right]$$

$$\sigma_1 = -428 \text{ PSI}$$

Circumferential:

$$\sigma_2 = \frac{\Delta R_D}{R_D} \times E = \frac{3210}{14.88} \times 28,5 = 6160 \text{ psi}$$

RADIAL: $\sigma_3 = 0$

G) SHEAR STRESS IN GASKET $= -\frac{1074}{.75} = 1435$ p.s.i.

J) AXIAL:

$$\sigma_1 = \frac{6 M_J}{h^2} = \frac{6(10583)}{(2.88)^2} = \begin{matrix} -7630 \text{ PSI INSIDE} \\ +7630 \text{ psi OUTSIDE} \end{matrix}$$

CIRCUMFERENTIAL:

$$\sigma_2 = \frac{\Delta R_J}{R_J} \times E \pm \frac{\nu 6 M_J}{h^2}$$

$$= \frac{958}{24.19} \times 28,5 \pm .3(7630)$$

$$= 1120 \pm 2290$$

$$\sigma_2 = \begin{cases} 3410 \text{ PSI OUTSIDE} \\ -1170 \text{ PSI INSIDE} \end{cases}$$

RADIAL:

$$\sigma_3 = 0$$

V) CIRCUMF.

$$\sigma_2 = \frac{\Delta R_V}{V} \times E$$

$$\sigma_2 = \frac{3235}{20,12} \times 28.5 = 4580 \text{ psi}$$

W)

$$\sigma_2 = \frac{\Delta R_W}{R_W} \times E$$

$$= \frac{3460}{15} \times 285 = 6570 \text{ psi}$$

X)

$$\sigma_2 = \frac{\Delta R_X}{R_X} \times E$$

$$= \frac{2165}{20,12} \times 28,5 = 3080 \text{ psi}$$

Y)

$$\sigma_2 = \frac{\Delta R_Y}{R_x} \times E$$

$$= \frac{-2059}{14} \times 28,5 = -4200 \text{psi}$$

Z)

$$\sigma_2 = \frac{\Delta R_Z}{R_Z} \times E$$

$$= \frac{-1890}{25,63} \times 28,5 = -2100 \text{ psi}$$

CASE "B" - STUD LOAD PLUS PRESSURE

1. GASKET LOAD WITH INTERNAL PRESSURE

a) WITH NO PRESSURE THE STUD LOAD AT 20,000 psi per BOLT = 14,640 #/in (ref pg. 8)

$$\text{Gasket Load} = \frac{17.5}{15.25}(14640) = 16800\ \#/\text{in.}$$

Relative stiffness of gasket & studs

$$\delta_s = \frac{P\,l_s}{A_s E_s} \quad ; \quad \delta_G = \frac{G\,l_g}{A_g E_g}$$

$$\text{but } G = \frac{17.5}{15.25} P = 1.15 P$$

$$\therefore \quad P = \frac{\delta_s A_s E_s}{l_s} = \frac{\delta_g A_g E_g}{1.15\, l_g}$$

$$\frac{\delta_s}{\delta_g} = \frac{E_g}{E_s} \times \frac{A_G}{A_s} \times \frac{l_s}{l_g} \times \frac{1}{1.15} = \frac{\pi \times 15.25 \times .75}{80.5 \times 1.15} \times \frac{11.125}{0.5} \times \frac{26 \times 10^6}{28 \times 10^6}$$

$$\frac{\delta_s}{\delta_g} = 8.0$$

Thus for a given load the studs deflect 8 times as far as the gasket.

b) WITH PRESSURE - ref. PM-2A calcs. of 12/16/60

LOAD ON STUD = $P + \Delta P$ where ΔP = change in stud tension due to added pressure load and unloading of gasket.

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LOAD ON STUD = B + G where B = blow off load
G = load on gasket with load applied.

$$B = \frac{1200\,\pi\,(15.25)^2}{2\,\pi\,(15.25)} = 9{,}150 \text{ \#/in.}$$

CHANGE IN STUD LENGTH & GASKET HEIGHT

$$\Delta\delta_s = \frac{(B+G-P)\,l_s}{A_s E_s} \qquad \Delta\delta_g = \frac{(1.15P-G)\,l_g}{A_g E_g}$$

but $\Delta\delta_s = \Delta\delta_g$ when pressure load is applied

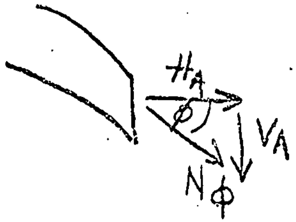
$$B+G-P = \left(\frac{A_s}{A_g} \times \frac{E_s}{E_g} \times \frac{l_g}{l_s}\right)(1.15P-G)$$

SOLVING for G GASKET LOAD WITH PRESSURE

$$G = 6{,}770 \text{ \#/in}$$

$$P + \Delta P = B + G = 9150 + 6770 = 15920 \text{ \#/in.}$$

2. COVER DOME — Ref. Sketch pg. 2



$$V_A = 1200 \times \frac{13.25}{2} = 7{,}950 \text{ \#/in.}$$

$$H_A = V_A \cot\phi = 14{,}360 \text{ \#/in.}$$

$$\text{Tensile Stress} = \frac{N_\phi}{A} = \frac{1200 \times 25.875^2 \times \pi}{(28.875^2 - 25.875^2)\,\pi}$$

$$= 4845 \text{ psi}$$

$$\Delta R_A = \frac{\sigma}{E}(1-\nu)R = .001577''$$

DEFLECTION COVER DOME –

FOR CASE A –

$$10^6 \Delta R_A = -.505 Q_A + .187 M_A \quad \text{(ref. pg. 2) eqn(1)}$$

$$10^6 \Delta R_A' = 1.14 Q_A' - 1.235 Q_B \quad \text{(ref. pg. 6) eqn(4)}$$

FOR CASE B –

$$\Delta R_A' = \Delta R_A + 0.001577$$

$$Q_A' = Q_A - H_A = Q_A - 14350 \qquad (a)$$

Eqn (1) Becomes $10^6 \Delta R_A = -.505 Q_A + .187 M_A + 1577$ (1B)

Eqn (4) Becomes $10^6 \Delta R_A' = 1.14 Q_A - 1.235 Q_B - 16370$ (4B)

EQUATING THE TWO EQUATIONS GIVES

$$-1.645 Q_A + 1.235 Q_B + 0.187 M_A = -17950 \qquad *$$

3. ROTATION OF INTERMEDIATE RING –

From Pg 6 Eqn (3)

$$10^6 \Theta_B = 0.1279 M_A + 0.0091 \times 10^{-6} V_A' - 0.187 Q_A$$

where $V_A' = 7950$ #/in for Case B

$$\therefore 10^6 \Theta_B = 0.1279 M_A - 0.187 Q_A + 72.3 \qquad (3B)$$

4. DEFLECTION OF INTERMEDIATE RING –

Using eqn(5) Pg.6 $10^6 \Delta R_B = 1.09 Q_A' - 1.197 Q_B$ and Substituting $Q_A' = Q_A - 14350$

Becomes

$$10^6 \Delta R_B = 1.09 Q_A - 1.197 Q_B - 15650 \qquad (5B)$$

5. OUTER RING —

Using eqn (8) Pg 9 $10^6 \Delta R_c = 0.236 (Q_B + Q_D - H_G)$

but $Q_D = -.025 \times 10^6 \Delta R_D = -0.0665 Q_A + 0.0299 Q_B + 0.0269 M_A + 406$ (a)

$10^6 \Delta R_D = \Delta R_B - 8.41 \theta_B = 2.66 Q_A - 1.197 Q_B - 1.075 M_A - 16200$

$10^6 \Delta R_c = -0.0157 Q_A + 0.243 Q_B + .0064 M_A - 0.236 H_G + 96$ (8B)

From eqn. (12) pg. 11 $\Delta R_B = \Delta R_c + 3.6 \theta_B$, substituting eqns (5B), (8B) & 3B gives

$1.779 Q_A - 1.44 Q_B - 0.464 M_A + 0.236 H_G = 16000$ * (9B)

From Pg. 11

$10^6 \theta_c = 0.0263 M_c$ (b)

$$M_c = 3.08 Q_B - 0.855 M_{B'} - 4.8 H_G - \frac{15.25}{17.56} (6770) \times 2.31 + \frac{17.5}{17.56} \times 15920 \times 0.06$$
$$- \frac{15}{17.56} Q_D \times 4.813 - \frac{15}{17.56} \times 9150 \times 2.56$$

$M_c = 3.08 Q_B - 0.855 M_{B'} - 4.8 H_G - 4.11 Q_D - 32650$

Subst. expression for $M_B = \frac{13.25}{15} (M_A + 1.75 V_A)$ $V_A = 7950$ and Q_D from (a) above to obtain M_c & in turn substitute in (b) gives

$$10^6 \theta_c = 0.00719 Q_A + 0.0777 Q_B - 0.0228 M_A - 0.126 H_G - 1180$$ (10B)

From eqn. (13) Pg 11

$$\theta_B = \theta_C \quad \text{or} \quad \theta_B - \theta_C = 0$$, substituting

eqns. (3B) & (10B) gives

$$-0.1942\, Q_A - 0.0777\, Q_B + 0.1507\, M_A + 0.126\, H_G = -1252$$

$* (15B)$

6. LOWER FLANGE —

PRESSURE DEFLECTION AT J

For Flange $\delta R_{F_J} = 1200 \times \frac{24.19}{28.5 \times 10^6} \left(\frac{2 \times 14^2}{25.63^2 - 14^2} \right)$

$$\delta R_{F_J} = 866 \times 10^{-6}$$

CYLINDER

$$\delta R_J = \frac{1200 \times 22.75 \times 24.19}{28.5 \times 10^6 \times 2.88} \left(1 - \frac{.3}{2} \right)$$

$$\delta R_J = 6830 \times 10^{-6}$$

PRESSURE DEFLECTION — FLANGE AT GASKET

$$\delta R_{F_G} = 1200 \times \frac{15.25\,(14)^2}{461 \times 28.5 \times 10^6} \left[1.3 + (0.7) \frac{(25.63)^2}{(15.25)^2} \right.$$

$$\delta R_{F_G} = 895 \times 10^6$$

Eqn (22) pg 14 becomes

$$\Delta R_{F_G} = 0.1483 \times 10^{-6} H_G - 0.186 \times 10^{-6} Q_J + 895 \times 10^{-6} \quad (14B)$$

eqn.(25a) pg 14 becomes

$$\Delta R_{F_J} = 0.1013 \times 10^{-6} H_G - .174 \times 10^{-6} Q_J + 866 \times 10^{-6} \quad (25aB)$$

eqn (26) pg 15 becomes

$$\Delta R_J = 0.337 \times 10^{-6} M_J + 2.19 \times 10^{-6} Q_J + 6830 \times 10^{-6} \quad (26B)$$

eqn (24) pg 15 becomes

$$M_F = \frac{15.25}{19.82} \overset{(G)}{(6770)} \times 4.57 - 2.05 \overset{P+\Delta P}{(15920)}$$

$$- \frac{24.19}{19.82} \times 1200 \overset{(V_J)\quad 10.7}{\left(\frac{\pi (22.75)^2}{2\pi\, 24.19} \right)} \times 4.37$$

$$- 1200\pi \left(\frac{22.75^2 - 15.25^2}{2\pi \times 19} \right) \frac{19}{19.82} \times 0.82 - 2.9 H_G$$

$$- 4.34 Q_J + 1.22 M_J$$

$$M_F = 2.9 H_G - 4.34 Q_J + 1.22 M_J - 84370 \quad (24B)$$

eqn(25) pg 14 becomes $\theta_F = 0.0382 \times 10^{-6} M_F$

$$10^6 \theta_F = -0.111 H_G - 0.166 Q_J + 0.0466 M_J - 3220 \quad (25B)$$

eqn(28) pg. 16 becomes $(\Delta R_G = \Delta R_G)$

$$\Delta R_B + 1.94\, \theta_B = \Delta R_{F_G} - 3.78\, \theta_F = \Delta R_{F_G} - 3.78\, \theta_J$$

Substituting eqns (5B), (3B), (14B) & 27 gives

$$.727 Q_A - 1.197 Q_B + 0.248 M_A - 0.1483 H_G - 1.085 Q_J - 0.393 M_J =$$

$$16{,}400 \quad (26B)^{*}$$

eqn. (29) pg. 16 $(\theta_F = \theta_J)$

Substituting eqn. (25-B) & (27) gives

$$-0.111 H_G + 0.171 Q_J + 0.1506 M_J = 3220 \quad *(29B)$$

eqn. (30) pg. 16 $\Delta R_J = \Delta R_{F_J} + 3.56\,\theta_J$

Substituting (26B) (25-aB) & 27 gives

$$.707 M_J + 3.564 Q_J - 0.1013 H_G = -5964 \quad {}^{*}(30B)$$

THUS, WITH 6 EQUATIONS* AND 6 UNKOWNS, THE UNKNOWN FORCES & moments may be found

$$-1.645 Q_A + 1.235 Q_B + 0.187 M_A = -17950$$

$$1.779 Q_A - 1.44 Q_B - 0.462 M_A + 0.236 H_G = 16000$$

$$-0.1942 Q_A - 0.0777 Q_B + 0.1507 M_A + 0.126 H_G = -1252$$

$$.727 Q_A - 1.197 Q_B + 0.298 M_A - 0.1483 H_G - 1.088 Q_J - 0.393 M_J = 16400$$

$$.707 M_J + 3.564 Q_J - 0.1013 H_G = 5964$$

SOLUTION

Q_A = 4600 #/in

H_G = −13620 #/in

Q_B = −9065 #/in

Q_J = −5560 #/in

M_A = 4330 in#/in

M_J = 17660 in#/in

BY______________DATE________
CHKD. BY_______DATE________

SUBJECT SM-1A VESSEL
AND FLANGE ANALYSIS

SHEET NO. 27 OF 31
JOB NO.________________

SUMMARY CASE "B" STUD LOAD PLUS PRESSURE

REF. SKETCH PG. 17

LOCATION SK. PG 17	LOCATION STRESS CHARTS	DEFLECTION (ΔR) 10^{-6} IN.	ROTATION (θ) 10^{-6} RAD	STRESSES CIRCUMF σ_2 (PSI)	AXIAL σ_1 (PSI)	RADIAL σ_3 (PSI)	SHEAR (PSI)
A	(12)	37		-583 outer 743 inner	-4840 outer - 420 inner	0	
B		231	-235	2899 outer -2021 inner	-9950 outer +6450 inner	0	
C		1072	-235	1740	—	—	
D	(13)	2190	-235	4190	292	0	
J	(11)	- 588	33	-11160 outer -3540 inner	-12700 +12700	0	
G		- 215	—	—	—	—	18150
W	(14) INSIDE	2300	-235	4370	—	—	
V	(14) OUTSIDE	1976	-235	2800	—	—	
X	(16)	- 486	-235	-688	—	—	
Y	(15) INSIDE	-215	38	-438	—	—	
Z	(15) OUTSIDE	316	38	352	—	—	

— STRESSES —

A) AXIAL:

$$\sigma_1 = \sigma_{THRUST} + \sigma_{BENDING}$$

$$= -\frac{Q_A}{t_A} \pm \frac{6M_A}{t_A} = -\frac{4600}{1.75} \pm \frac{6(4330)}{(3.43)^2}$$

$$\sigma_1 = -2630 \pm 2210 = \begin{matrix} -4840 \text{ PSI OUTSIDE} \\ -420 \text{ PSI INSIDE} \end{matrix}$$

CIRCUMFERENTIAL:

$$\sigma_2 = \frac{\Delta R_A}{R_A} \times E \pm \frac{\nu 6 M_A}{h^2} = \frac{37}{13.25}(28.5) \pm .3(2210)$$

$$\sigma_2 = \begin{cases} -583 \text{ PSI OUTSIDE} \\ 743 \text{ PSI INSIDE} \end{cases}$$

B) AXIAL:

$$\sigma_1 = \sigma_{THRUST} + \sigma_{BENDING}$$

$$= -\frac{Q_B}{t_B} \pm \frac{6M_B}{h^2} = \overset{-1750}{-\frac{9065}{5.12}} \pm \overset{\pm 8200}{6\frac{(.883 \times 4320 + 12300)}{(3.43)^2}}$$

$$\sigma_1 = \begin{cases} +6450 \text{ PSI INSIDE} \\ -9950 \text{ PSI OUTSIDE} \end{cases}$$

CIRCUMFERENTIAL:

$$\sigma_2 = \frac{\Delta R_B}{R_B} \times E \pm \frac{\nu 6 M_B}{h^2} = \overset{439}{\frac{231}{15}(28.5)} \pm \overset{2460}{.3(8200)}$$

$$\sigma_2 = \begin{cases} +2899 \text{ PSI OUTSIDE} \\ -2021 \text{ PSI INSIDE} \end{cases}$$

RADIAL: $\sigma_3 = 0$

C) CIRCUMFERENTIAL:

$$\sigma_2 = \frac{\Delta R_C}{R_C} \times E = \frac{1072}{17.56} \times 28.5 = 4190 \text{ psi}$$

D) AXIAL:

$$\sigma_1 = \sigma_{Thrust} + (\sigma_{BENDING} = 0) \text{ SINCE } M_D = 0$$

$$\sigma_1 = -\frac{Q_D}{t} \pm 0 \qquad \text{See pg 23 for } Q_D$$

$$\sigma_1 = -\left(\frac{-54.8}{.1875}\right) = 292 \text{ psi}$$

CIRCUMFERENTIAL:

$$\sigma_2 = \frac{\Delta R_D}{R_D} \times E = \frac{2190}{14.88}(28.5) = 4190 \text{ psi}$$

RADIAL: $\sigma_3 = 0$

G) SHEAR STRESS IN GASKET $= \frac{H_G}{t} = \frac{13620}{.75} = 18160$ psi

J) AXIAL:

$$\sigma_1 = \frac{6M_J}{h^2} = \frac{6(17660)}{(2.88)^2} = \begin{matrix} +12700 \text{ psi INSIDE} \\ -12700 \text{ psi outside} \end{matrix}$$

CIRCUMFERENTIAL:

$$\sigma_2 = \frac{Pa}{h} + \frac{Ew}{a} \pm \frac{\nu 6 M_J}{h^2}$$

where w is the deflection due to discontinuity only.

$$w = .337 M_J + 2.19 Q_J$$
$$w = -6242$$

$$\sigma_2 = \frac{1750(24.19)}{2.88} + \frac{(28.5)(-6242)}{24.19} \pm .3(12700)$$

$$\sigma_2 = 14700 - 7350 \pm 3810$$

$$\sigma_2 = 14700 + \begin{bmatrix} -3540 \text{ psi INSIDE} \\ -11160 \text{ psi outside} \end{bmatrix}$$

V) CIRCUMF: $\sigma_2 = \frac{\Delta R_V}{R_V} \times E$

$= \frac{1976}{20,12} \times 28,5 = 2800$ PSI

W) CIRCUMF: $\sigma_2 = \frac{\Delta R_W}{R_W} \times E$

$= \frac{2300}{15} (28,5) = 4370$ psi

X) CIRCUMF. $\sigma_2 = \frac{\Delta R_X}{R_X} (E)$

$= \frac{-486}{20,12} (28,5) = -688$ psi

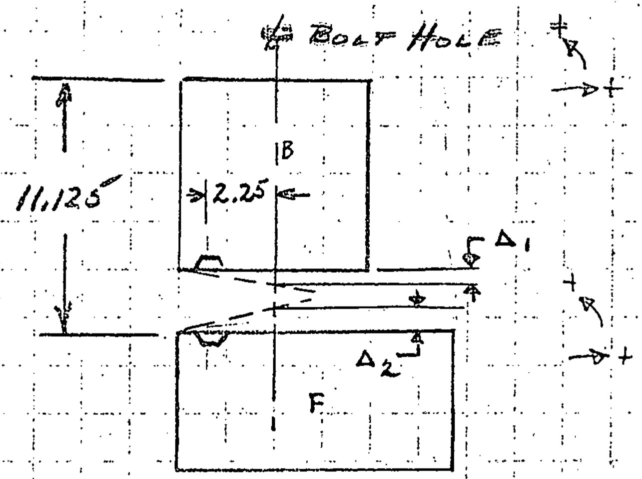
Y) Circumf. $\sigma_2 = \frac{\Delta R_Y \simeq \Delta R_G}{R_Y} (28,5)$

$= \frac{-215}{14} (28,5) = -438$ psi

Z) Circumf $\sigma_2 = \frac{\Delta R_Z}{R_Z} (E)$

$= \frac{316}{25,63} (28,5) = 352$ PSI

DEFLECTIONS OF TOP & BOTTOM FLANGES ALONG CENTERLINE OF BOLT HOLE



CASE "A" – BOLT LOAD ONLY

$\Delta_1 = 2.25\,\theta_B$
$= 2.25(-508 \times 10^{-6})$
$\Delta_1 = -1142 \times 10^{-6}$ in.

$\Delta_2 = 2.25\,\theta_F$
$= 2.25\,(400 \times 10^{-6})$
$\Delta_2 = 900 \times 10^{-6}$ in

$\Delta_{TOTAL\,CASE\,"A"} = 2042 \times 10^{-6}$ in

CASE "B" – BOLT LOAD + PRESSURE

$\Delta_1 = 2.25\,\theta_B$
$= 2.25\,(-235 \times 10^{-6})$
$\Delta_1 = -528 \times 10^{-6}$ in.

$\Delta_2 = 2.25\,\theta_F$
$\Delta_2 = 2.25\,(38 \times 10^{-6})$
$\Delta_2 = 86 \times 10^{-6}$ in.

$\Delta_{TOTAL\,CASE\,"B"} = 614 \times 10^{-6}$

FOR CASE A — STUD LOAD ONLY -

THE COVER RING MOVES DOWNWARD ALONG THE STUD CENTERLINE 1142×10^{-6} INCHES, & THE VESSEL FLANGE MOVES UPWARD 900×10^{-6} INCHES.

THEREFORE, FOR AN INITIAL STRESS IN THE STUDS OF 20,000 PSI THE FLANGE SURFACES AT THE STUD CIRCLE MUST MOVE A TOTAL OF 2042×10^{-6} INCHES.

FOR CASE B - STUD LOAD PLUS 1200 P.S.I. PRESSURE

THE COVER RING has BEEN MOVED DOWNWARD 528×10^{-6} INCHES INSTEAD OF 1142×10^{-6} AND THE VESSEL FLANGE MOVES UPWARD 86×10^{-6} INSTEAD OF 900×10^{-6} INCHES.

THEREFORE, THE SURFACES OF BOTH FLANGES MOVE APART DUE TO APPLICATION OF PRESSURE A DISTANCE

$$10^{6}(1142 - 528) + 10^{6}(900 - 86) = \underline{\underline{1428 \times 10^{-6}\ \text{IN.}}}$$

STUD STRESS FOR CASE "B" —

$$\sigma_{STUD} = 20000 + \frac{1428 \times 10^{-6}}{11.125} \times 29.5 \times 10^{6}$$

$$= 20000 + 3780$$

$$\sigma_{STUD} = \underline{23780\ \text{PSI}}$$

BY R.L. DATE 7/10/61 SUBJECT SM-1A STRESS ANALYSIS AE 90 TASK 6.9 SHEET NO. 1 OF 28
CHKD. BY CH DATE 9/13/61 JOB NO. CONTR. NO. AT(30-1)-2639

STEADY STATE CASE "C"

THERMAL STRESS ANALYSIS — COVER & FLANGE

REF. (M) — NPED MEMO No. 77 "STEADY STATE TEMPERATURE DISTRIBUTION THROUGH SM-1A VESSEL COVER AND FLANGE"

DETERMINATION of AVERAGE TEMPERATURES of COVER & FLANGE SECTIONS based on REGION AVERAGE TEMPERATURES from NPED MEMO 77.

COVER

REGION NO	VOL (ft^3)	TEMP VAP.	TEMP LIQUID	VOL × TEMP VAP	VOL × TEMP LIQUID
30 ↗ $\frac{3}{4} \times .2521$	.189	421.3	417.3	79.6	78.9
26	.9363	425.7	432.3	398.6	404.8
	1.1253			478.2	483.7

$$\text{Av. Temp. of Cover} = \frac{478.2}{1.125} = 425^\circ \quad \text{VAP. Cond.}$$

$$\text{Av. Temp. of Cover} = \frac{483.7}{1.125} = 430^\circ \quad \text{LIQUID Cond.}$$

COVER DOME

35 Read Av. Temp Directly from Region Av. Temp Listed in Memo. 77

for Vapor Cond Av Temp = 425°
" Liquid " " " = 430°

BY ________ DATE ________ SUBJECT ________ SHEET NO. 2 OF 28
CHKD. BY ________ DATE ________ JOB NO. ________

COVER RING (INTERMEDIATE RING SHOWN IN SKETCH FOR COVER & FLANGE PRESSURE ANALYSIS)

REGION NO.	VOL (ft³)	TEMP VAP	TEMP LIQ	VOL × TEMP VAP	VOL × TEMP LIQ
30 → $\frac{1}{4} \times .2521$	.0630	421.3	417.3	27	26.3
27	.4982	396.1	384.4	197.5	191.5
	.5612			224.5	217.8

AV TEMP VAP COND = $\frac{224.5}{.561}$ = 399°

" " LIQ. COND = $\frac{217.8}{.561}$ = 388°

COVER FLANGE — (SPLIT INTO TWO SECTIONS BOTTOM HALF & TOP HALF)

BOTTOM HALF

REGION NO					
18	.2865	370.3	347.2	106.1	99.5
42	.3054	156.5	174.2	47.7	53.1
	.5919			153.8	152.6

Av. Temp. Liquid = $\frac{153.8}{.5919}$ = 260°

Av. Temp. Vap. = $\frac{152.6}{.5919}$ = 258°

TOP HALF

31	.4098	255.7	236.4	104.8	96.9
41	.4932	161.6	169.2	79.7	83.4
	.9030			184.5	180.3

Av. Temp Liquid = 204°
" " Vapor = 199.7°

BY______ DATE______ SUBJECT______ SHEET NO. 3 OF 28

CHKD. BY______ DATE______ JOB NO.______

VESSEL FLANGE

REGION	VOL (ft³)	TEMP. VAP	TEMP. LIQ.	TEMP × VOL VAP	TEMP × VOL LIQ
10	1.026	402.9	407	413.4	417
8	1.974	311.2	333	614.3	657
9	2.190	267.0	320	584.7	700
	5.190			1612.4	1774

Av Temp Vap. $\frac{1612.4}{5.19} = 311°$

Liq. $\frac{1774}{5.19} = 342°$

VESSEL CYLINDER

10	.0862	382.3	401	33	34.5
11	.4709	345.5	327	162.5	182.0
12	2.339	408.1	420	954.5	981.0
	2.8961			1150.0	1197.5

Av Temp $= \frac{1197.5}{2.8961} = 414°F$ { LIQ & VAP

STUD

44	.01357	271	266.5	3.68	3.52
43	.01357	219.1	213.9	2.97	2.90
38 & 17	.00537	179	177	.96	.95
	.03251			7.61	7.37 4

Av. Temp. $= \frac{7.61}{.0325} = 234°$ Vapor

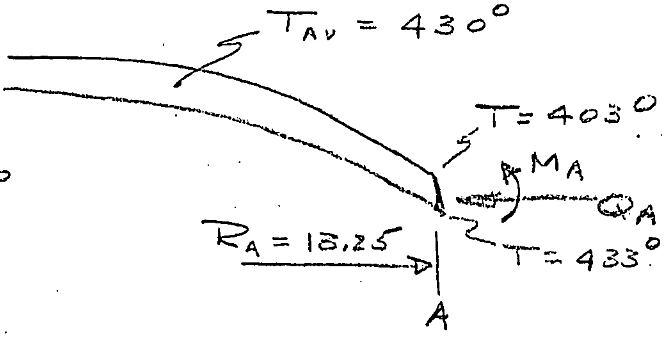
$= \frac{7.37}{.0325} = 227°$ Liquid

The Average Temp for the liquid Condition is used for following calc's.

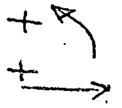
BY______________DATE__________ SUBJECT________________________________ SHEET NO. 9 OF 28

CHKD. BY________DATE__________ JOB NO.__________________

COVER DOME —



SIGN CONVENTION



USE SAME SIGN CONV AS FOR COVER & FLANGE PRESSURE ANALYSIS

① DEFLECTION AT A DUE TO TEMP DISTRIBUTION –

$$\Delta R_{A_{Th}} = \alpha_{430^\circ} \Delta T R_A$$

$$= (6.9\times10^{-6})(430-70)(13.25)$$

$$\Delta R_{A_{Th}} = +32900$$

REFERRING TO COVER & FLANGE STRESS ANALYSIS

$$10^6 \Delta R_A = -.505\, Q_A + .187\, M_A$$

but for Temp. Distribution

$$10^6 \Delta R_A = -.505\, Q_A + .187\, M_A + 32900 \text{ ——— } (1)$$

② ROTATION —

The equivalent BENDING MOMENT AT EDGE A which would produce THE SAME EFFECT AS THE TEMP DIFFERENCE AT THE EDGE, IS

$$M = \frac{\Delta T}{2}\, \alpha\, \frac{E}{1-\nu}\, \frac{t^2}{6}\ ^*$$

ΔT = Temp diff ACROSS Thickness at edge

$$M_{Th} = \left(\frac{433-403}{2}\right)(6.9\times10^{-6})\frac{(27.5\times10^6)}{(1-.3)}\frac{(2.59)^2}{6}$$

E & α are taken at Av. Temp of the EDGE

$$M_{Th} = +4550$$

REFERRING TO COVER & FLANGE STRESS ANALYSIS THE ROTATION AT A WAS —

$$10^6\Theta_A = -.187\,Q_A + .1158\,M_A$$

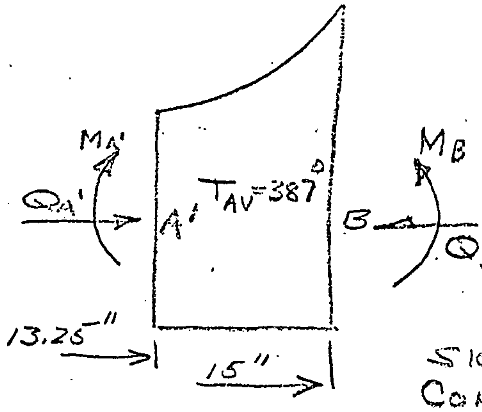
but due to temp distribution there is an additional EQUIVALENT MOMENT of +4550

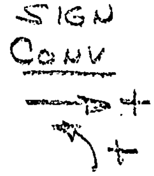
$$\therefore 10^6\Theta_A = -.187\,Q_A + .1158\,(M_A + 4550)$$

$$10^6\Theta_A = -.187\,Q_A + .1158\,M_A + 527 \text{ ———— } (2)$$

INTERMEDIATE RING —

DEFLECTION AT A' —





$$\Delta R_{A'_{Th}} = \alpha_{387}\,\Delta T\,R_{A'}$$

$$= (6.85\times10^{-6})(387-70)(13.25)$$

$$\Delta R_{A'_{Th}} = 28770$$

REFERRING TO COVER & FLANGE ANALYSIS

$$10^6\Delta R_{A'} = 1.14\,Q_{A'} - 1.235\,Q_B \quad \text{becomes}$$

$$10^6\Delta R_{A'} = 1.14\,Q_{A'} - 1.235\,Q_B + 28770 \text{ ———— } (3)$$

DEFLECTION AT B —

$$\Delta R_{B_{Th}} = (6.85\times10^{-6})(387-70)(15) = 32570$$

$$10^6\Delta R_B = 1.09\,Q_A - 1.197\,Q_B + 32570 \text{ ———— } (4)$$

ROTATION AT B —

From previous Cover & Flange Analysis

$10^6 \theta_B = \theta_A + \Delta\theta_B$

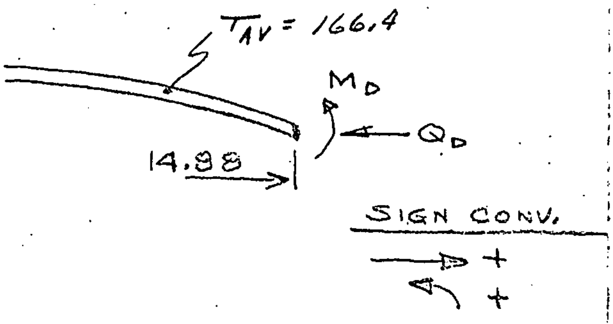
$10^6 \theta_B = -.187 Q_A + .1158 M_A + .01208 M_A'$

Then for Temperature distribution

$10^6 \theta_B = -.187 Q_A + .1158(M_A + 4550) + .01208 M_A'$

$10^6 \theta_B = -.187 Q_A + .1279 M_A + 527$ ——— (5)

INSULATION COVER —



DEFLECTION AT D —

$\Delta R_D = \alpha_{166.4} \Delta T R_D$

$= (8.72\times10^{-6})(166.4-70)(14.88)$

$10^6 \Delta R_{D_{TH}} = 12500$

From previous Cover & Flange Analysis

$10^6 \Delta R_D = -40 Q_D$ becomes

$10^6 \Delta R_D = -40 Q_0 + 12500$

Also

$10^6 Q_D = -.0665 Q_A + .0299 Q_B + .0269 M_A$ becomes

$= -.0665 Q_A + .0299 Q_B + .0269 M_A - .025(325-70)$
$+ .025(8.41)(527)$

$10^6 Q_0 = -.0665 Q_A + .0299 Q_B + .0269 M_A - 703$ (6

BY__________ DATE__________ SUBJECT__________ SHEET NO. 7 OF 28

CHKD. BY__________ DATE__________ JOB NO.__________

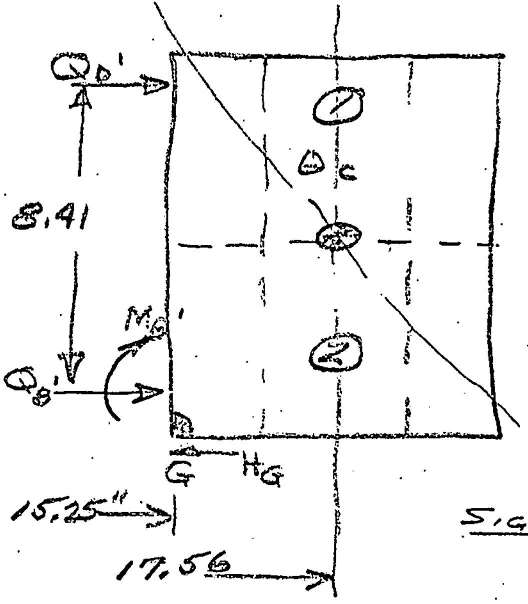
COVER RING —

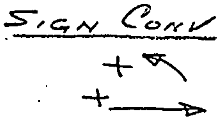
$\overline{T}_{AV②} = 258°$

$\overline{T}_{AV①} = 199.5$

457.5

T_{AV} for Complete pc = 229°





DEFLECTION —

$$\Delta R_{D_{Th}} = \alpha \Delta T_1 R_D$$

$$= (6.4\times10^{-6})(199.5-70)(14.88)$$

$$10^6 \Delta R_{D_{Th}} = 12400$$

$$\Delta R_{B_{Th}} = \alpha \Delta T_2 R_B$$

$$= (6.55\times10^{-6})(258-70)(15)$$

$$10^6 \Delta R_{B_{Th}} = 18300$$

$$\Delta R_{C_{Th}} = \alpha_{229} \Delta T R_C$$

$$= (6.5\times10^{-6})(229-70)(17.56)$$

$$10^6 \Delta R_{C_{Th}} = 18150$$

ROTATION —

$$\theta_c = \frac{\Delta R_{B_{Th}} - \Delta R_{D_{Th}}}{L} = \frac{18300-12400}{8.41}$$

$$\theta_{C_{Th}} = 702$$

Eqn. for θ_c

$$10^6 \theta_c = .0263\, M_c \quad \text{becomes}$$

$$10^6 \theta_c = .0263\, M_c + 702$$

TAKING SUM OF MOMENTS ABOUT C

$$17.56\, M_C = 15\, Q_B (3.6) - 15\, M_B - 15.25\, H_G (5.533)$$
$$- 15\, Q_D (4.813)$$

$$M_C = 3.08\, Q_B - .855\, M_B - 4.8\, H_G - 4.11\, Q_D$$

Subst. IN θ_C ABOVE

$$10^6 \theta_C = .081\, Q_B - .0225\, M_B - .126\, H_G - .108\, Q_D + 702$$

but $M_B = .883\, M_A$

$$10^6 \theta_C = .081\, Q_B - .0195\, M_A - .126\, H_G - .108\, Q_D + 702 \qquad (7)$$

DEFLECTION OF COVER RING AT C

$$10^6 \Delta R_c = .236 (Q_B + Q_D - H_G) \quad \text{becomes}$$

$$10^6 \Delta R_c = .236 (Q_B + Q_D - H_G) + 18150 \quad \text{———} \quad (8)$$

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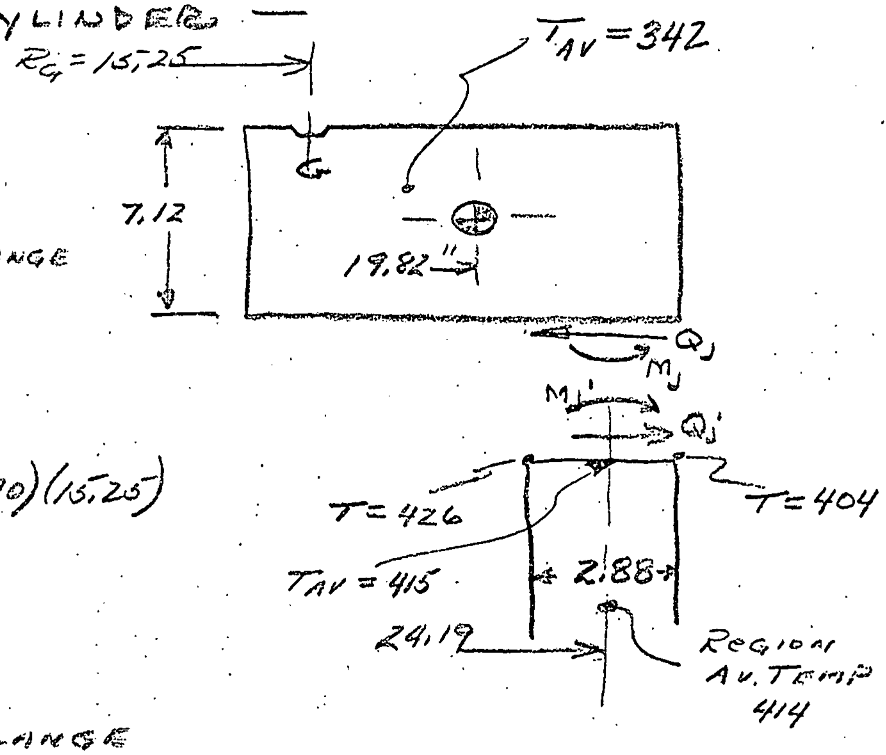
VESSEL FLANGE & CYLINDER —



DEFLECTIONS —

ΔR_{FG} – deflection of FLANGE AT GASKET

$$\Delta R_{FG} = \alpha_{342} \Delta T R_G$$

$$= (6.74 \times 10^{-6})(342-70)(15.25)$$

$$10^6 \Delta R_{FG} = \underline{\underline{27,750}}$$

ΔR_{FJ} – deflection of FLANGE AT J

$$\Delta R_{FJ} = \alpha_{342} (342-70)(24.19)$$

$$10^6 \Delta R_{FJ} = \underline{\underline{43950}}$$

SIGN CONV.

↖ +

→ +

ΔR_J – deflection of cylinder AT J

$$\Delta R_J = \alpha_{415} \Delta T R_J$$

$$= (6.9 \times 10^{-6})(415-70)(24.19)$$

$$10^6 \Delta R_J = \underline{\underline{57300}}$$

FROM ORIG. EQUATIONS

$10^6 \Delta R_{FG} = .1483 H_G - .186 Q_J$ becomes

$$10^6 \Delta R_{FG} = .1483 H_G - .186 Q_J + 27750 \text{ ——————— } (9)$$

$10^6 \Delta R_{E_j} = .1013 H_G - .174 Q_j$ becomes

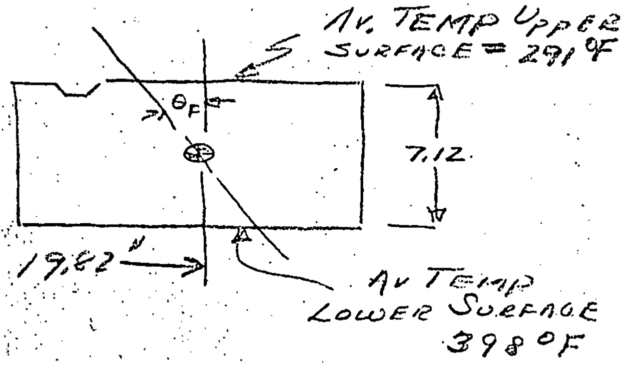
$10^6 \Delta R_{E_j} = .1013 H_G - .174 Q_j + 43950$ ———— (10)

$10^6 \Delta R_j = .337 M_j + 2.19 Q_j$ becomes

$10^6 \Delta R_j = .337 M_j + 2.19 Q_j + 57300$ ———— (11)

ROTATION —

Rotation of Flange due to Temperature Distribution



$\Delta R_{upper} = \alpha_{291} \Delta T R$

$= (6.62\times10^{-6})(291-70)(19.82)$

$10^6 \Delta R_{upper} = \underline{\underline{29000}}$

$\Delta R_{lower} = \alpha_{398} \Delta T R$

$= (6.87\times10^{-6})(398-70)(19.82)$

$\Delta R_{lower} = \underline{\underline{44700}}$

$$10^6 \theta_F = \frac{\Delta R_{lower} - \Delta R_{upper}}{7.12}$$

$$= \frac{44700 - 29000}{7.12}$$

$10^6 \theta_F = \underline{\underline{2208}}$

ORIG. EQUATI.

$$10^6 \theta_F = .0382\, M_F \quad \text{becomes}$$

$$10^6 \theta_F = .0382\, M_F + 2208$$

Obtain expression for M_F by taking moments about F

$$19.82\, M_F = -15.25\, H_G \times 3.78 - 24.19\, Q_j \times 3.56 + 24.19\, M_j$$

$$M_F = -2.9\, H_G - 4.34\, Q_j + 1.22\, M_j$$

Subst. in θ_F above

$$10^6 \theta_F = -.111\, H_G - .166\, Q_j + .0466\, M_j + 2208 \qquad (12)$$

ROTATION of CYLINDER —

EQUIVALENT BENDING MOMENT AT EDGE of CYLINDER which would produce the SAME EFFECT AS THE TEMP DIFFERENCE AT THE EDGE.

$$M = \frac{\Delta T}{2}\, \alpha\, \frac{E}{1-\nu}\, \frac{t^2}{6}$$ where ΔT = Temp diff across thickness at the edge

$$M = \frac{(426-404)}{2}(6.9 \times 10^{-6})\frac{27.8 \times 10^6}{.7}\frac{(2.64)^2}{6}$$ E & α are taken at av. Temp at the Edge.

$M = -4170$

ORIG. SIGN CONVENTION ↻ +
But due to Temp Diff. the shell rotates ↷
∴ M IS NEGATIVE

The orig. equation was

$$10^6 \theta_j = -.104 M_j - .337 Q_j$$ ✓

But due to Temp distribution there is an additional equivalent moment of $-4170 \Delta T$ ✓ which gives

$$10^6 \theta_j = .104(-M_j - 4170) - .337 Q_j$$ ✓

$$10^6 \theta_j = -.104 M_j - .337 Q_j - 434 \quad\text{———} \quad (13)$$

SUMMARY OF EQUATIONS —

COVER DOME —

$$10^6\,\theta_A = -.187\,Q_A + .1158\,M_A + 527 \quad (2)$$

$$10^6\,\Delta R_A = -.505\,Q_A + .187\,M_A + 32900 \quad (1)$$

INTERMEDIATE RING

$$10^6\,\Delta R_{A'} = 1.14\,Q_{A'} - 1.235\,Q_B + 28770 \quad (3)$$

$$10^6\,\theta_B = -.187\,Q_A + .1279\,M_A + 527 \quad (5)$$

$$10^6\,\Delta R_B = 1.09\,Q_A - 1.197\,Q_B + 32570 \quad (4)$$

INSULATION COVER

$$10^6\,\Delta R_D = -40\,Q_D + 12500 \quad (5A)$$

$$10^6\,Q_D = -.0665\,Q_A + .0299\,Q_B + .0269\,M_A - 703 \quad (6)$$

COVER RING

$$10^6\,\theta_C = .081\,Q_B - .0195\,M_A - .126\,H_G - .108\,Q_D + 702 \quad (7)$$

$$10^6\,\Delta R_C = .236\,(Q_B + Q_D - H_G) + 16150 \quad (8)$$

FLANGE

$$10^6\,\theta_F = -.111\,H_G - .166\,Q_J + .0466\,M_J + 2208 \quad (12)$$

$$10^6\,\Delta R_{F_G} = .1483\,H_G - .186\,Q_J + 27750 \quad (9)$$

$$10^6\,\Delta R_{F_J} = .1013\,H_G - .174\,Q_J + 45950 \quad (10)$$

CYLINDER

$$10^6\,\Delta R_J = .337\,M_J + 2.19\,Q_J + 57300 \quad (11)$$

$$10^6\,\theta_J = -.104\,M_J - .337\,Q_J - 434 \quad (13)$$

BOUNDARY CONDITIONS —

$\Delta R_A = \Delta R_{A'}$ (14)

$\Delta R_B = \Delta R_{B'} = \Delta R_C + 3.6\,\theta_B$ (15)

$\theta_B = \theta_C$ (16)

$\Delta R_G = \Delta R_B + 1.94\,\theta_B = \Delta R_{FG} - 3.78\,\theta_F$ (17)

$\Delta R_{J'} = \Delta R_J = \Delta R_{FJ} + 3.56\,\theta_J$ (18)

$\theta_F = \theta_J$ (19)

$Q_D = f(Q_B, Q_A, M_A)$ (19A)

Subst. equat's. 1 & 3 in 14 gives

$-1.646Q_A + .187\,M_A + 1.235\,Q_B = -4130$ ———— (20)

Subst. equat's, 5, 8 & 4 in 15 gives

$1.763\,Q_A - 1.433\,Q_B - .236\,Q_D + .236\,H_G - .46\,M_A = -12520$ (21)

Subst. equat's, 5 & 7 in 16 gives

$-.187\,Q_A + .1474\,M_A - .081\,Q_B + .126\,H_G + .108\,Q_D = 175$ (22)

Subst. equat's 4, 5, 9 & 12 in 17 gives

$.727\,Q_A - 1.197\,Q_B + .248\,M_A - .568\,H_G - .441\,Q_J + .176\,M_J$

$= -14184$ — (23)

Subst equat's 10, 11, & 13 in 18 gives

$.709\,M_J + 3.564\,Q_J - .1013\,H_G = 14898$ — — (24)

Subst equat's 12 & 13 in 19 gives

$-.111\,H_G + .171\,Q_J + .1506\,M_J = -2642$ --- (25)

Also from (19A)

$$Q_D + .0665 Q_A - .0299 Q_B - .0269 M_A = -703 \quad (26)$$

Solving above 7 equations (20) to (26) yields —

Q_A = 42,545 #/in

M_A = 63,928 in#/in

Q_B = 43,645 #/in

Q_D = − 507 #/in

H_G = 18,237 #/in

Q_J = −3676 #/in

M_J = 73 in#/in

Also from pg 8 $M_B = .833 (\overset{63928}{M_A}) = 53,300$ in#/in.

Substituting above values into boundary condition equations to check the solution the values for the deflections & rotation at the various points are obtained & listed on the following page —

CHECK OF BOUNDARY CONDITIONS —

① $\Delta R_A = \Delta R_A'$

$$-.505\underset{Q_A}{\overset{-21500}{(42545)}} + .187\underset{M_A}{\overset{+11950}{(63928)}} + 32900 = 1.14\underset{48500}{\overset{Q_A'}{(42545)}} - 1.235\underset{-52900}{\overset{Q_B}{(43645)}} + 27770$$

$$23350 = 23370$$

ΔR_A due to discont. forces only $= -21500 + 11950 = -9550$

② $\Delta R_{B'} = \Delta R_C + 3.6\,\Theta_B$

$$\Delta R_C = .236\underbrace{\overset{43645 - 507 - 18237}{(Q_B + Q_D - H_G)}}_{5880}$$

$$1.09\underset{Q_A}{\overset{46374}{(42545)}} - 1.197\underset{Q_B}{\overset{-52243}{(43645)}} + 32570 = .236(24901) + 18150$$

$$+\,3.6\Big[\underbrace{-.187\overset{-7960}{(42545)} + .1279\overset{8180}{(63928)} + 527}_{\theta_B = 747}\Big]$$

$$26700 = 26680$$

③ $\Theta_B = \Theta_C$

$$747 = .081\overset{Q_B}{(43645)} - .0195\overset{M_A}{(63928)} - .126\overset{H_G}{(18237)} - .108\overset{Q_D}{(-507)} + 702 = 747$$

④ $$\Delta R_G = \Delta R_B + 1.94\theta_B = \Delta R_{FG} - 3.78\Big[\overbrace{\theta_F = .111\underset{H_G}{(18237)} - .166\underset{663}{(-3676)} + .0466(75) + 2208}^{799.4}\Big]$$

$$\rightarrow .1483\underset{H_G}{\overset{2700}{(18237)}} - .186\underset{Q_F}{(-3676)} + 27750 = 31143$$

$$\rightarrow 1.09\underset{Q_A}{(42545)} - 1.197\underset{Q_B}{(43645)} + 32570 = 26770$$

$$\Delta R_G = 26770 + 1.94(737) = 31143 - 3.78(799.4)$$

$$28200 = 28123$$

⑤ $\Delta R_j = \Delta R_J \approx \Delta R_{F_j} + 3.56\,\theta_j$

$$\overset{25}{.337(\underset{M_j}{73})} + \overset{-8050}{2.19(\underset{Q_j}{-3676})} + 57300 = \overset{1850}{.1013(\underset{H_G}{18237})} - \overset{+640}{.174(\underset{Q_j}{-3676})} + 43950$$

$$+3.56\left[\theta_j = \overset{-7}{-.104(73)} - \overset{+1229}{.377(-3676)} - 434 = 798\right]$$

$+2840$

$49274.6 = 49280$

⑥ $\theta_F = \theta_J$

$799.4 = 798$

⑦ $Q_D = f(Q_A\ Q_B\ M_A)$

$$Q_D = -.0665(\underset{Q_A}{42545}) + .0299(\underset{Q_B}{43645}) + .0269(\underset{M_A}{63928}) - 703$$

$-507 = -507$

TOTAL ROTATIONS AT CORNERS OF FLANGES

PT X

$\Delta R_{x_{TOT}} = \Delta R_{x_{TH}} + \Delta R_{x_{DISC}} + 5.312\,\theta_{C_{DISC}}$

where $\Delta R_{x_{TH}} = \alpha\,\Delta T\,R_x = 6.55(258-70)(20.12) = 24800$

$\Delta R_{x_{DISC}} = 4200$

$\Delta R_{x_{TOT}} = 24800 + 4200 + 5.312(45)$

$\Delta R_{x_{TOT}} = 29239$

<u>Pt. W</u>

$$\Delta R_W = \Delta R_B - 8.91\ \theta_C$$

$$\Delta R_{W_{TOT}} = 26670 - 8.91(747) = 20400$$

<u>Pt. V</u>

$$\Delta R_{V_{TOT}} = \Delta R_{V_{TH}} + \Delta R_{V_{DISC}} - 5.312\ \theta_{C_{DISC}}$$

$$= \Delta R_{V_{TH}} = \alpha_{199.5}\ \Delta T\ R_V = 6.4(199.5-70)20.12 = 16650$$

$$\Delta R_{V_{DISC}} = 4200 \quad (pg\ 21)$$

$$\Delta R_{V_{TOT}} = 16650 + 4200 \overset{-239}{- 5.312(45)}$$

$$\Delta R_{V_{TOT}} = 20611$$

<u>Pt Z</u>

$$\Delta R_{Z_{TOT}} = \Delta R_{Z_{TH}} + \Delta R_{Z_{DISC}} + 3.56\ \theta_{F_{DISC}}$$

where $\Delta R_{Z_{TH}} = \alpha_{291}\ \Delta T\ R_Z$

$$10^6 \Delta R_{Z_{TH}} = 6.62\overset{221}{(291-70)}25.63 = 38900$$

and $\Delta R_{Z_{DISC}} \approx \Delta R_{F_J} = 2490$

$$\theta_{F_{DISC}} = \overset{-2020}{-.111\underset{H_G}{(18237)}} \overset{610}{-.166\underset{Q_J}{(-3676)}} \overset{3.4}{+.0466\underset{M_J}{(\ 73\ )}}$$

$$\theta_{F_{DISC}} = -1407$$

$$10^6 \Delta R_{Z_{TOT}} = 38900 + 2490 + \overset{5000}{3.56(1407)} = 46390$$

<u>Pt Y</u>

$$\Delta R_{Y_{TOT}} \approx \Delta R_{G_{TOT}} = 28200$$

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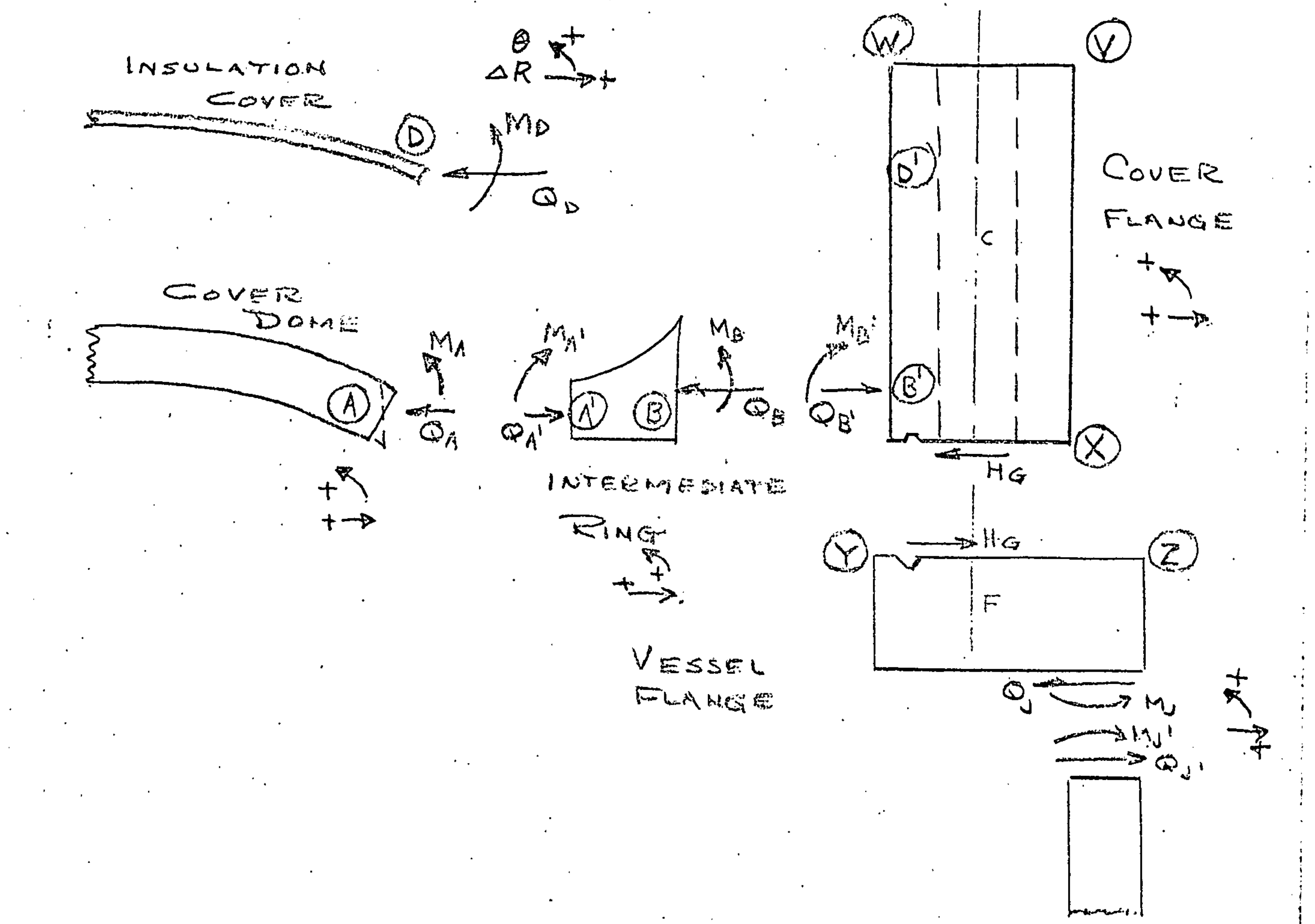
CHKD. BY ________ DATE ________ JOB NO. ________

SUMMARY — CASE C: STEADY STATE THERMAL STRESSES; Coolant at 433°



LOCATION ABOVE SK.	STRESS CHARTS	TOTAL DEFLECTION ΔR (10^{-6} IN)	TOTAL ROTATION θ (10^{-6} RAD)	STRESSES (PSI) σ_2 CIRCUMF	 σ_1 AXIAL	 σ_3 RADIAL	 SHEAR
A	12	23350		−10720 INNER −30280 OUTER	−45000 OUTER +20200 INNER	0	—
B		26790	747	− 2660 INNER −18980 OUTER	−35720 OUTER +18680 INNER	0	—
D	13	20100	747	39000	+ 2705	0	—
G		28200	—			—	24300
J	11	49275	798	−9466 OUTER −9434 INNER	−53 +53	0	—
V	14 OUTSIDE	20611	747	5480	—	—	—
W	14 INSIDE	20200	747	10800	—	—	—
X	16	29239	747	6280	—	—	—
Y	15 INSIDE	28200	798	17700	—	—	—
Z	15 OUTSIDE	46390	798	8810	—	—	—

STRESSES – THE DEFLECTION DUE TO DISCONTINUITY FORCES IS USED TO find CIRCUMFERENTIAL STRESSES.

A) AXIAL: $\sigma_1 = \sigma_{THRUST} + \sigma_{BENDING}$

$$\sigma_1 = -\frac{Q_A}{t_A} \pm \frac{6M_A}{h^2} = \overset{-12400}{-\frac{42545}{3,43}} \pm \overset{32600}{\frac{6(63928)}{(3,43)^2}}$$

$$\sigma_1 = \begin{matrix} 20,200 \text{ psi INSIDE} \\ -45,000 \text{ psi OUTSIDE} \end{matrix}$$

CIRCUMF: $\sigma_2 = \frac{\Delta R_A}{R_A} \times E \pm \frac{6\nu M_A}{h^2}$

$$= \overset{-20500}{-\frac{2050}{13,25}(28,5)} \pm \overset{9780}{.3(32600)}$$

$$= \begin{cases} -10720 \text{ psi inner surface} \\ -30280 \text{ psi outer surface} \end{cases}$$

RADIAL: $\sigma_3 = 0$

B) AXIAL:

$$\sigma_1 = \sigma_{THRUST} + \sigma_{BENDING}$$

$$\sigma_1 = -\frac{Q_B}{t_B} \pm \frac{6M_B}{h^2} = \overset{-8520}{-\frac{43645}{5,12}} \pm \overset{\pm 27200}{\frac{6(53300)}{(3.43)^2}}$$

$$\sigma_1 = \begin{cases} -35720 \text{ psi. OUTSIDE} \\ 18680 \text{ psi. INSIDE} \end{cases}$$

CIRCUMF:

$$\sigma_2 = \frac{\Delta R_B}{R_B} \times E \pm \frac{\nu 6 M_B}{h^2}$$

$$= \frac{-5700}{.15}(28,5) \pm .3(27200)$$

$$= -10820 \pm 8160$$

$$\sigma_2 = \begin{cases} -2660 \text{ psi INSIDE} \\ -18980 \text{ psi OUTSIDE} \end{cases}$$

RADIAL: $\sigma_3 = 0$

D) AXIAL:

$$\sigma_1 = \sigma_{THRUST} + (\sigma_{BENDING} = 0) \quad \text{SINCE } M_D = 0$$

$$\sigma_1 = -\frac{Q_D}{t_D} = \frac{+507}{.1875} = +2705 \text{ PSI}$$

CIRCUMF:

$$\sigma_2 = \frac{\Delta R_D}{R_D} \times E$$

where $\Delta R_D = -40 Q_D$

$= -40(-507)$

$\Delta R_D = 20280$

$$= \frac{20280}{14,88}(28,5)$$

$$\sigma_2 = 39000 \text{ PSI}$$

RADIAL: $\sigma_3 = 0$

G) SHEAR STRESS IN GASKET $= \frac{H_G}{.75} = \frac{18237}{.75} = 24300$ PSI

J) AXIAL:

$$\sigma_1 = \frac{6M_J}{h^2} = \frac{6(73)}{(2,88)^2} = \begin{matrix} -52.9 \text{ PSI OUTSIDE} \\ +52.9 \text{ PSI INSIDE} \end{matrix}$$

CIRCUMF:

$$\sigma_2 = \frac{Pa}{h} + \frac{Ew}{a} \pm \nu\frac{6M_J}{h^2}$$

where w is the discontinuity deflection at J.

$$\sigma_2 = \frac{1750(24.19)}{2.88} + \frac{28.5(-8025)}{24.19} \pm .3(52.9)$$

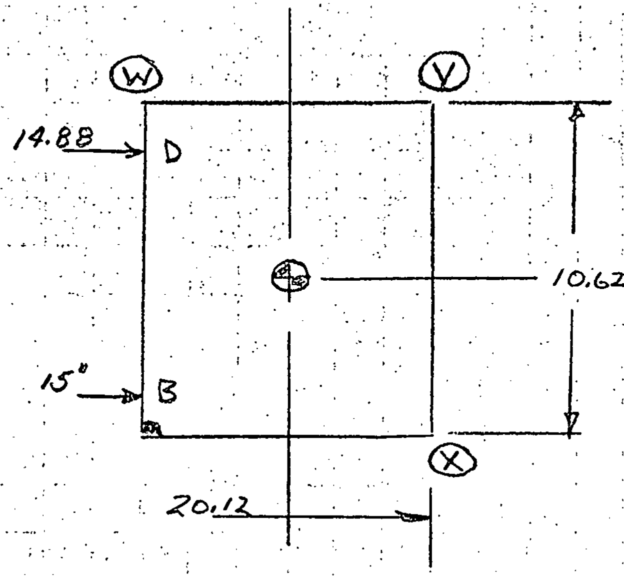
$w = -8025$ see pg.17

$$\sigma_2 = 14700 - 9450 \pm 16$$

$$\sigma_2 = 14700 + \begin{bmatrix} -9434 \text{ PSI INSIDE} \\ -9466 \text{ PSI OUTSIDE} \end{bmatrix}$$

6. PT. W —

DISCONTINUITY DEFLECTION



$\Delta R_W = \Delta R_{c_{DISC}} - 5.312\ \theta_{c_{DISC}}$

where $\Delta R_{c_{DISC}}$ is the AVERAGE DEFLECTION of INNER FACE of the FLANGE & θ_c IS ROTATION DUE TO DISCONTINUITY.

$\Delta R_{c_{DISC}} = 5870$ [See pg 16]

$\theta_{c_{DISC}} = 45$ " " "

$10^6 \Delta R_W = 5870 - 5.312(45)$

$10^6 \Delta R_W = 5630$

CIRCUMF. STRESS AT W —

$\sigma_2 = \dfrac{\Delta R_W}{R_W} \times E = \dfrac{5630}{14.88} \times 28.5 = 10800$ P.S.I.

7. AT PT. V —

a) AVERAGE DEFLECTION of OUTER SURFACE of FLANGE

$\Delta R_o = \dfrac{Pb}{E}\left(\dfrac{2a^2}{b^2-a^2}\right)$ REF: FORMULAS FOR STRESS & STRAIN ROARK Pg 232 CASE 17

$= \dfrac{(Q_B + Q_D - H_G)}{10.625}\ \dfrac{20.12}{28.5 \times 10^6}\left[\dfrac{2(15)^2}{(20.12)^2-(15)^2}\right]$

$= \dfrac{(43645 - 507 - 18237)(1.77)}{10.625} \times 10^{-6}$

$10^6 \Delta R_o = 4200$

b) DEFLECTION AT PT. V

$$10^6 \Delta R_V = 4200 - 5.312\,\Theta_o$$

$$= 4200 - 5.312\overset{239}{(45)}$$

$$10^6 \Delta R_V = 3961$$

$$\Delta R_{V_{TOTAL}} = 4200 \div \overset{3920}{5.312(737)}$$
$$= 280$$

c) STRESS at V –

$$\sigma_2 = \frac{\Delta R_V}{R_V} \times E = \frac{3961}{20.12} \times 28.5$$

$$\sigma_2 = \underline{\underline{5480 \text{ psi}}}$$

8) AT PT X – LOWER OUTER CORNER of UPPER RING

a) DEFLECTION AT X

$$10^6 \Delta R_X = \Delta R_0 + 5.312\,\Theta_C$$

$$= 4200 + .239$$

$$10^6 \Delta R_X = 4439$$

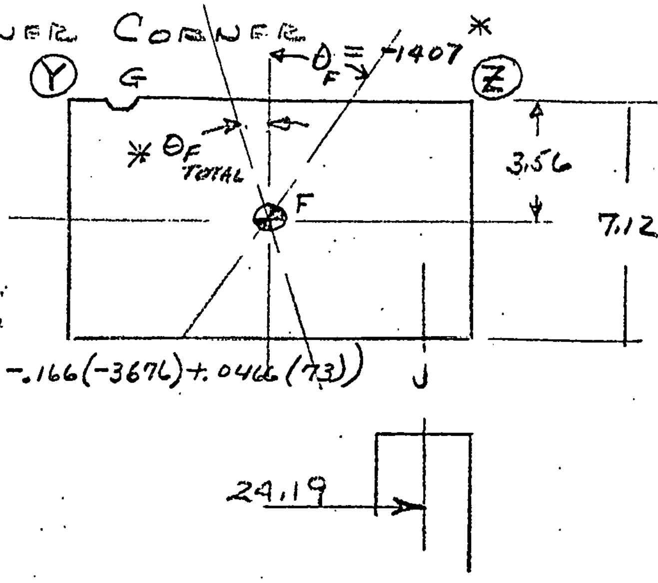
$$\Delta R_{X_{TOTAL}} = 4200 - \overset{3920}{5.312(737)}$$
$$= 280$$

b) STRESS AT X

$$\sigma_2 = \frac{\Delta R_X}{R_X} \times E = \frac{4439}{20.12} \times 28.5$$

$$\sigma_2 = \underline{\underline{6280 \text{ psi}}}$$

9) AT PT Y – VESSEL FLANGE INNER CORNER



$\Delta R_Y \approx \Delta R_G$

From Boundary Conditions

$$\Delta R_G = \Delta R_{F_G} - 3.78\ \theta_F$$

$$\Delta R_Y = \Delta R_{F_G} - 3.78(\overset{\theta_F = -1407}{\theta_F = -.111(18237) - .166(-3676) + .0466(73)})$$

$$\Delta R_Y = 3383 - \overset{5320}{3.78(-1407)} = 8703$$

STRESS AT Y

$$\text{Circumf: } \sigma_z = \frac{\Delta R_Y}{R_Y} \times E = \frac{8703}{14} \times 28.5$$

$$\sigma_z = 17{,}700 \text{ psi}$$

10) AT PT Z – VESSEL FLANGE OUTER CORNER

Deflection of outer surface of Flange $\approx \Delta R_{F_J}$

$$\Delta R_{F_J} = .1013\ \underset{(18237)}{H_G} - .174\ \underset{(-3676)}{Q_J} = 2490$$

$$10^6 \Delta R_Z = 2490 - 3.56\ \theta_F$$

$$= 2490 - \overset{+5000}{3.56(-1407)}$$

$$10^6 \Delta R_Z = 7490$$

STRESS AT Ⓩ

$$\sigma_z = \frac{7490}{24.19} \times 28.5$$

$$\sigma_z = \underline{\underline{8810 \text{ psi}}}$$

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* $\theta_{F_{TOTAL}}$ = TOTAL ROTATION of FLANGE DUE TO DISCONTINUITY FORCES + TEMP. DIST.
θ_F = ROTATION DUE TO DISCONTINUITY FORCES ONLY

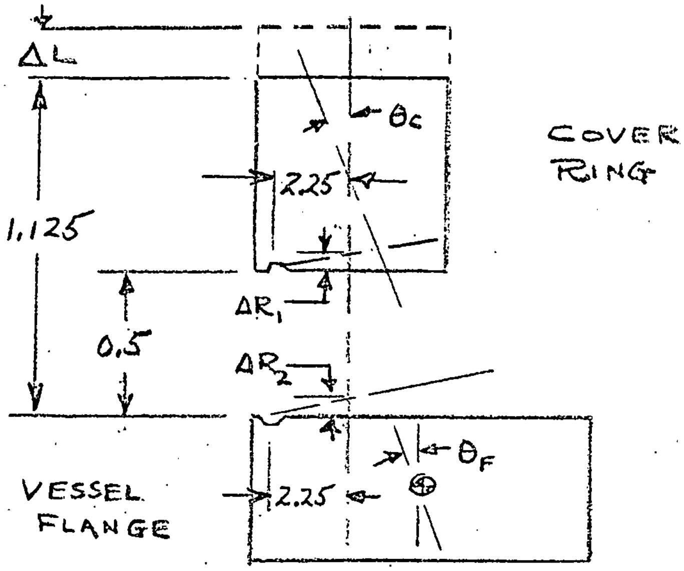
ALCO PRODUCTS INC.

BY ______ DATE ______ SUBJECT SM-1A VESSEL AND COVER ANALYSIS SHEET NO. 24 OF 28

CHKD. BY ______ DATE ______ JOB NO. ______

DEFLECTION of TOP & BOTTOM FLANGES ALONG CENTERLINE of STUD

$$10^6 \Delta R_1 = 2.25\,\theta_C{}^{*}$$
$$= 2.25\,(737)$$
$$10^6 \Delta R_1 = \underline{\underline{1660}}$$



$$10^6 \Delta R_2 = 2.25\,\theta_F{}^{*}$$
$$= 2.25\,(800)$$
$$10^6 \Delta R_2 = \underline{\underline{1800}}$$

* WHERE θ_C & θ_F ARE THE TOTAL ROTATION DUE TO DISCONTINUITY FORCES & THERMAL EXPANSION

DECREASE IN DISTANCE BETWEEN FLANGE SURFACES ALONG ℄ OF STUD = 1800 − 1660 = 140×10^{-6} IN.

THERMAL EXPANSION OF COVER RING & STUD —

$$\Delta L_{\substack{COVER\\RING}} = \alpha_{232}\, \underset{162}{\Delta T}\, L_{RING}$$

$$10^6 \Delta L = 6.5\,(232-70)(10.625)$$

$$10^6 \Delta L = \underline{\underline{11180}}$$

$$\Delta L_{STUD} = \alpha_{227}\, \Delta T\, L_{STUD}$$

$$= 5.5\,(227-70)(11.125)$$

$$10^6 \Delta L_{STUD} = \underline{9600}$$

THE DIFFERENCE IN THERMAL EXPANSION of THE COVER RING & STUD IS THE AMOUNT THE STUD IS STRETCHED.

$$\begin{array}{rl} 11180 \times 10^{-6} & \text{COVER RING} \\ -\ \ 9600 \times 10^{-6} & \text{STUD} \\ \hline 1580 \times 10^{-6}\ \text{IN.} & \text{STRETCH IN STUD} \end{array}$$

BUT DUE TO THE DIFFERENCE IN ROTATION of LOWER FLANGE & COVER RING, THERE IS A RELAXATION of 140×10^{-6} IN. . ALSO from FLANGE & COVER ANALYSIS CALCS, CASE "B" pg 31, THE INCREASE IN STUD LENGTH WAS 1428×10^{-6} IN.

∴ NET INCREASE IN STUD LENGTH for CASE C $= 1580 - 140$

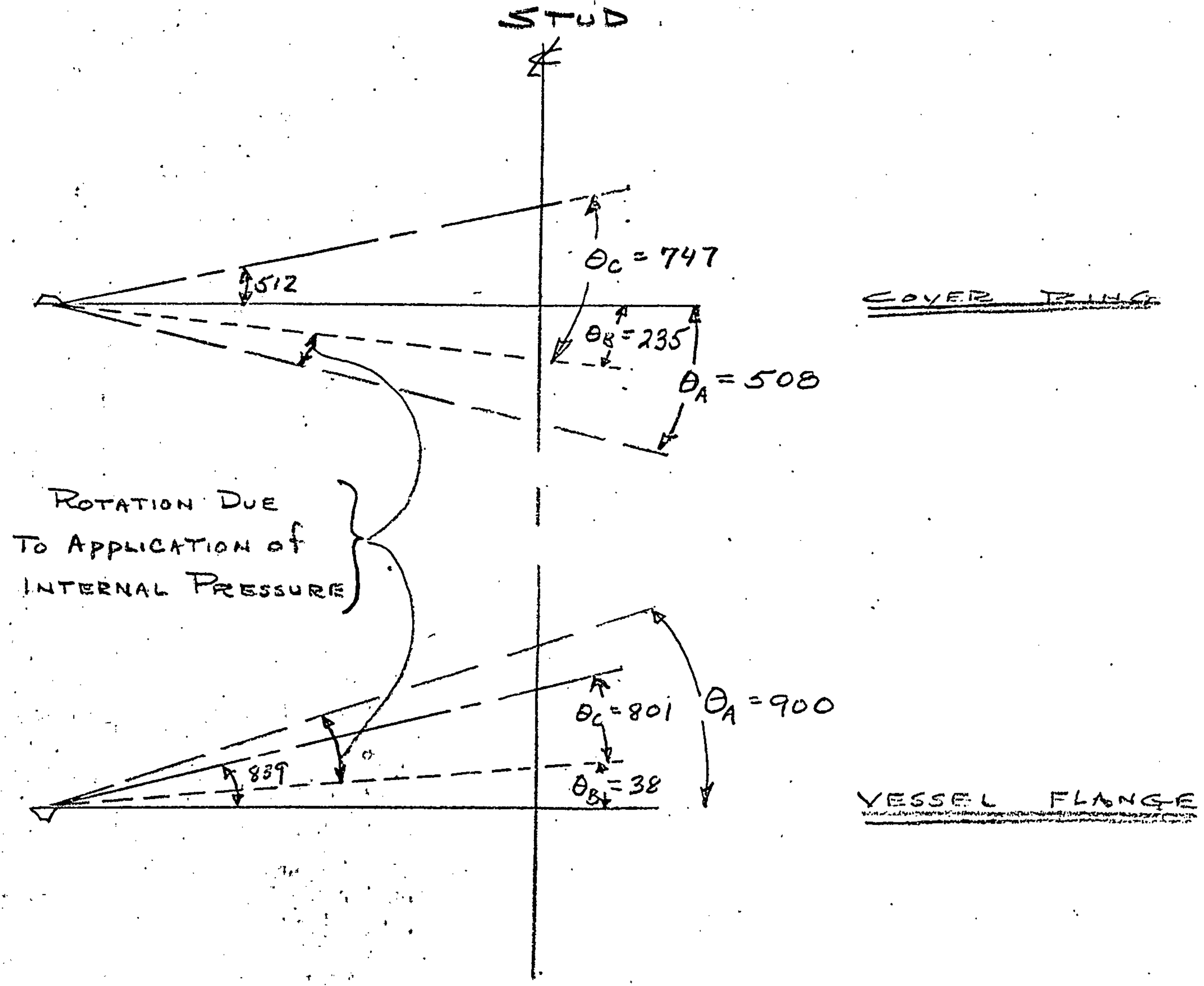
$$= \underline{1440 \times 10^{-6}\ \text{IN.}}$$

STRESS DUE to INCREASE IN LENGTH of STUD FOR CASE C $= \frac{\Delta L}{L} \times E$

$$= \frac{1440 \times 10^{-6}}{11.125} \times 29.5 \times 10^{6}$$

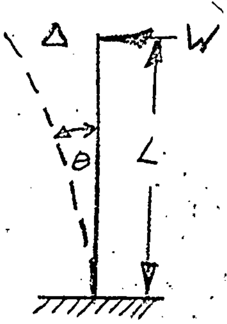
$$= \underline{3810\ \text{PSI}}\ (\text{TENSION})$$

BENDING STRESS IN STUD —



RESULTANT ROTATION of STUD ℄ = 839 − 512 = $\underline{\underline{327}}$

Subscripts A, B, C correspond to CASES A, B, C



HORIZONTAL DEFLECTION of STUD –

$$\Delta = L\theta$$
$$= (11.125)(327 \times 10^{-6})$$
$$\Delta = \underline{\underline{3620 \times 10^{-6} \text{ IN.}}}$$

FOR CANTILEVER BEAM –

$$\Delta = \frac{WL^3}{3EI} \quad \text{\&} \quad M_{MAX} = -WL$$

where W = APPLIED LOAD

$$W = \frac{\Delta\, 3EI}{L^3}$$

$$M_{MAX} = \frac{\Delta\, 3EI}{L^3} \times L$$

where $I = \frac{\pi r^4}{4}$
& MINOR STUD DIA = 2.59"
$r = 1.295$"

$$M_{MAX} = \frac{(3620 \times 10^{-6})(3)(29.5 \times 10^{6})(\pi)(1.295)^4}{4\,(11.125)^2}$$

$$M_{MAX} = \underline{\underline{5740 \text{ in.lb.}}}$$

$$\text{BENDING STRESS} = \frac{Mc}{I} = \frac{5740(1.295)(4)}{\pi(1.295)^4}$$

$$\text{BENDING STRESS} = \underline{\underline{3370 \text{ PSI}}}$$

BY______ DATE______ SUBJECT______ SHEET NO. 28 OF 28

CHKD. BY______ DATE______ JOB NO.______

STUD STRESS SUMMARY

	ΔL STUD (in)	INCREASE IN STUD STRESS (psi)
CASE "A"	..	20000
CASE "B"	1428×10^{-6}	3780
CASE "C" STEADY STATE THERMAL	1580×10^{-6}	3810
	-140×10^{-6}	3370 BENDING
TOTAL	2868×10^{-6} in	30960 psi

TRANSIENT STATE

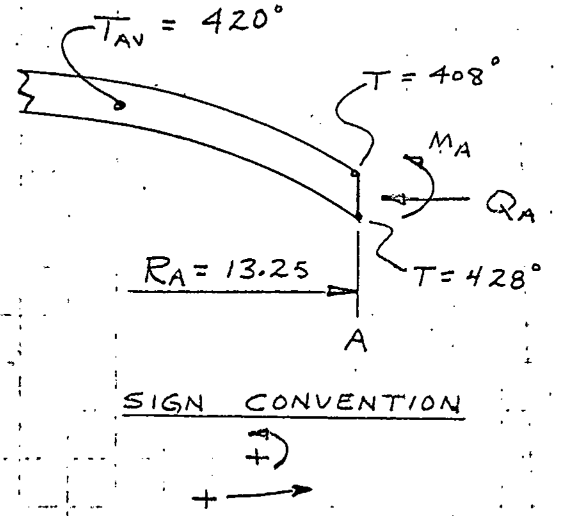
THERMAL STRESS ANALYSIS - COVER & FLANGE

Reference(8)(9): Matthews, F.T., "Transient State Temperature Distribution Through SM-1A Vessel Cover and Flange," Letter Report to D.W. McLaughlin, September 13, 1961.

Average temperatures of the cover and flange sections were extrapolated from a 6 hour heat-up at a 50°F per hour ramp to a 6 hours and 40 minutes heat-up, (see sheet 2)

COVER -

Deflection at A due to temperature distribution:



$$\Delta R_A = \alpha_{420} \Delta T R_A$$

$$= (6.9 \times 10^{-6})(420-70)(13.25)$$

$$10^6 \Delta R_A = (6.9)(350)(13.25) = +32,000$$

Referring to cover and flange stress analysis:

$$10^6 \Delta R_A = -.505\, Q_A + .187\, M_A$$

∴ For temp. distribution -

$$10^6 \Delta R_A = -.505\, Q_A + .187\, M_A + 32,000 \quad\text{———————} \quad (1)$$

Rotation at A due to temperature distribution:

$$M_{TH} = \frac{\Delta T}{2} \alpha \frac{E}{1-\nu} \frac{t^2}{6}$$

Theory of Plates & Shells
-Timoshenko

BY________ DATE ________

CHKD. BY ________ DATE ________

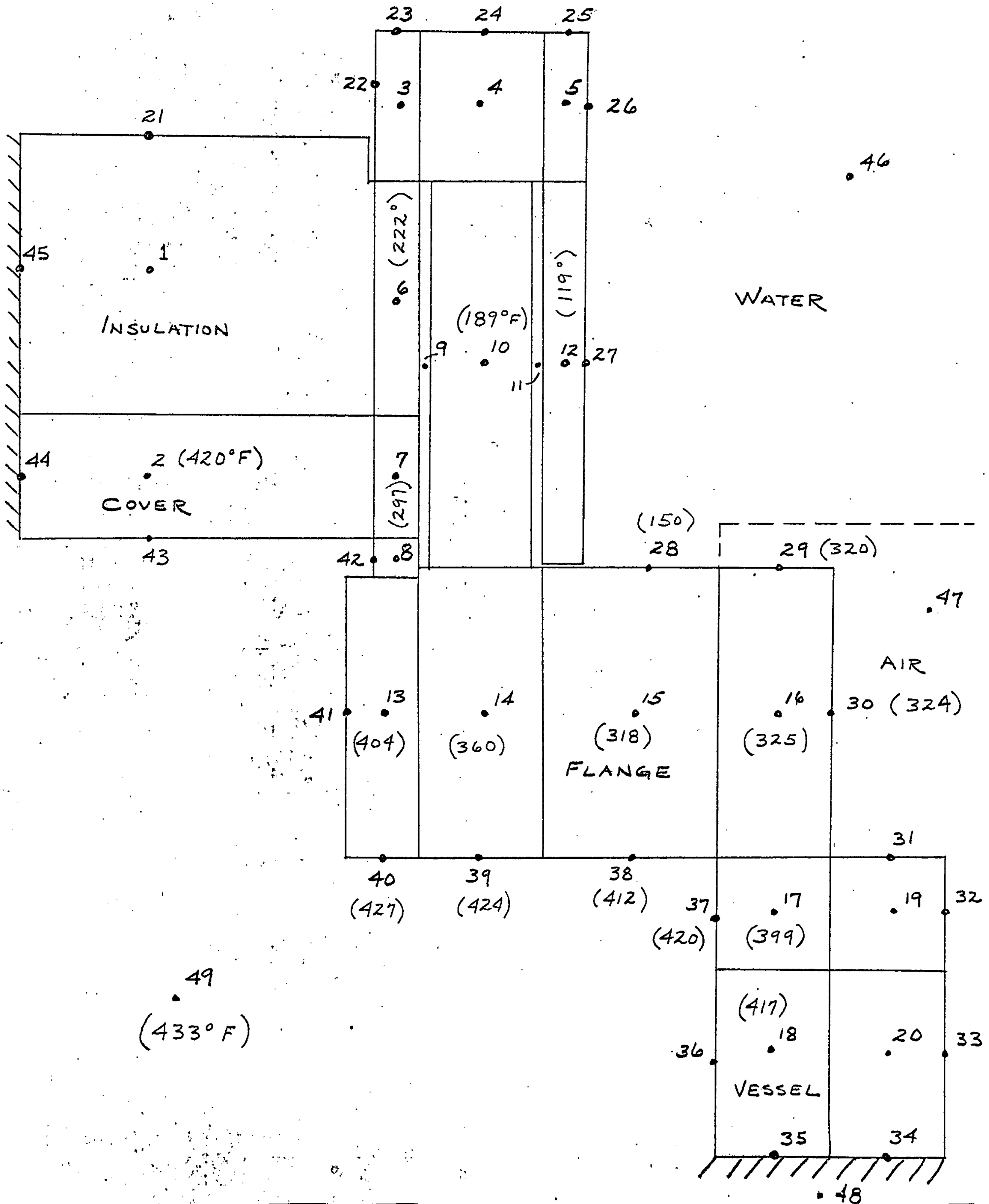
SUBJECT SM-1A Reactor

Vessel Flange Model

For Tiger II Code Run II

SHEET NO. 2 OF 27

JOB NO. ________



This equation is the equivalent bending moment at edge A which would produce the same effect as the temperature difference at the edge.

ΔT = Temperature difference across thickness at edge.

E and α are taken at the average temp. at the edge.

$$M_{TH} = \left(\frac{20}{2}\right)\left(6.9\times10^{-6}\right)\left(\frac{27.5\times10^{6}}{1-.3}\right)\left(\frac{(2.59)^2}{6}\right)$$

$$= 69\,(39.3)(1.12) = +\,3{,}030$$

Referring to cover and flange stress analysis :

$$10^{6}\theta_A = -.187\,Q_A + .1158\,M_A$$

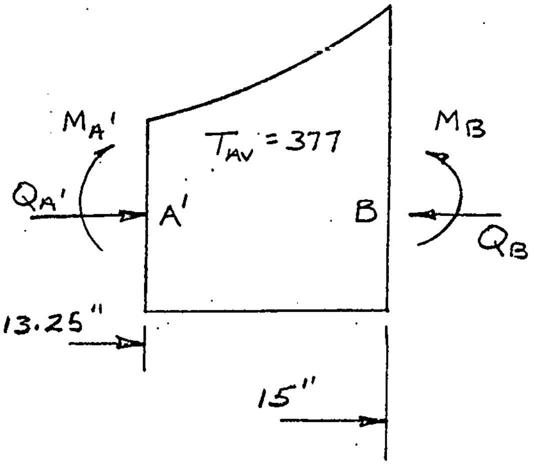
But due to temp. distribution there is an additional equivalent moment of + 3,030

$$\therefore\ 10^{6}\theta_A = -.187\,Q_A + .1158\,(M_A + 3{,}030)$$

$$10^{6}\theta_A = -.187\,Q_A + .1158\,M_A + 351 \quad \text{———————— (2)}$$

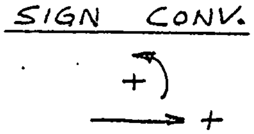
INTERMEDIATE RING -

Deflection at A' :



$$\Delta R_{A'} = \alpha\,\Delta T\,R_{A'}$$

The average temp. of this section was not calculated in the transient state analysis. Therefore, an approx. average temp. was gotten for this section by taking the temp. difference between the intermediate ring and the cover in the steady state condition and using it to obtain an approx. T_{AV} for the transient state.



T_{AV} of cover, steady state = 430°
T_{AV} of intermediate ring, steady state = 387°
43°

T_{AV} of cover, transient state = 420°F

T_{AV} of intermediate ring, transient state = 420° − 43° = 377°F

$$\Delta R_{A'} = \alpha_{377} (377-70)(13.25) = +27,700 \times 10^{-6}$$

Referring to cover and flange analysis:

$10^6 \Delta R_{A'} = 1.14\, Q_{A'} - 1.235\, Q_B$ becomes

$$10^6 \Delta R_{A'} = 1.14\, Q_{A'} - 1.235\, Q_B + 27,700 \quad \text{— — —} \quad (3)$$

Deflection at B:

$$\Delta R_B = \alpha_{377} (377-70)(15) = 6.8 \times 10^{-6} (307)(15)$$

$$10^6 \Delta R_B = +31,300$$

Referring to cover and flange analysis, ΔR_B becomes:

$$10^6 \Delta R_B = 1.09\, Q_A - 1.197\, Q_B + 31,300 \quad \text{— — — —} \quad (4)$$

Rotation at B:

From previous cover and flange analysis:

$$10^6 \theta_B = \theta_A + \Delta\theta_B = -.187\, Q_A + .1158\, M_A + .01208\, M_{A'}$$

Then for temperature distribution:

$$10^6 \theta_B = -.187\, Q_A + .1158\, (M_A + 3,030) + .01208\, M_{A'}$$

$$10^6 \theta_B = -.187\, Q_A + .1279\, M_A + 351 \quad \text{— — — — — —} \quad (5)$$

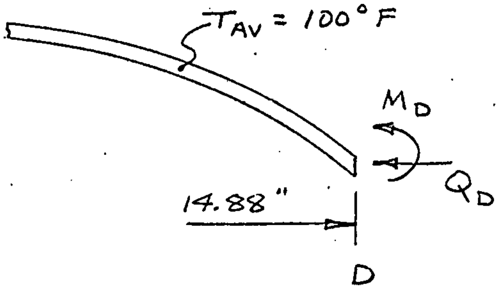
INSULATION COVER (S.S. 304) –

Deflection at D:

$\Delta R_D = \alpha_{100} \Delta T \, R_D'$

$= 8.51 \times 10^{-6} \, (100-70)(14.88)$

$10^6 \Delta R_D = +3{,}800$



SIGN CONV.

$\rightarrow +$

$+\circlearrowright$

From previous analysis:

$10^6 \Delta R_D = -40 \, Q_D$ which becomes

$10^6 \Delta R_D = -40 \, Q_D + 3800$

Also: $10^6 Q_D = -.0665 \, Q_A + .0299 \, Q_B + .0269 \, M_A$

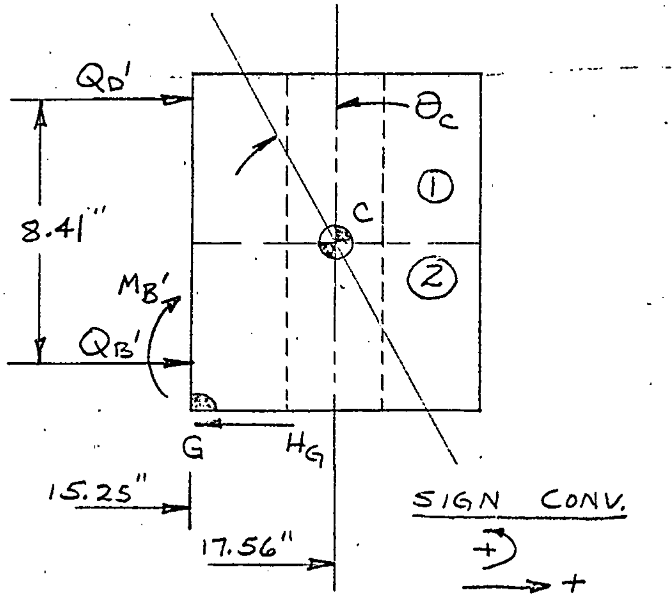
which becomes: $10^6 Q_D = -.0665 Q_A + .0299 Q_B + .0269 M_A - .025(31{,}300) + .025(8.41)(351)$

$10^6 Q_D = -.0665 \, Q_A + .0299 \, Q_B + .0269 \, M_A - 708$ ——— (6)

COVER RING –

$T_{AV①} = 166°F$
$T_{AV②} = 205°F$

T_{AV} for complete piece $= 192°F$



Deflection at D':

$\Delta R_{D'} = \alpha_{166} \, (166-70)(14.88)$

$10^6 \Delta R_{D'} = +9{,}000$

SIGN CONV.

$+\circlearrowright$

$\rightarrow +$

Deflection at B'

$\Delta R_{B'} = \alpha_{205}(205-70)(15) = 6.41 \times 10^{-6}(135)15$

$10^6 \Delta R_{B'} = +13,000$

Deflection at C

$\Delta R_c = \alpha_{192}(192-70)(17.56) = 6.4 \times 10^{-6}(122)(17.56)$

$10^6 \Delta R_c = +13,700$

Rotation at C:

$\theta_c = \frac{\Delta R_{B'} - \Delta R_{D'}}{8.41} = \frac{(13,000-9000)10^{-6}}{8.41}$

$10^6 \theta_c' = 476$

From previous analysis:

$10^6 \theta_c = .0263\,M_c$ which becomes

$10^6 \theta_c = .0263\,M_c + 476$

Taking sum of moments about C:

$M_c = 3.08\,Q_B - .855\,M_B - 4.8\,H_G - 4.11\,Q_D$

Substitue in θ_c above:

$10^6 \theta_c = .081\,Q_B - .0225\,M_B - .126\,H_G - .108\,Q_D + 476$

But $M_B = .883\,M_{A'}$ (for thermal condition $M_A' = M_A$)

$10^6 \theta_c = .081\,Q_B - .0199\,M_A - .126\,H_G - .108\,Q_D + 476$ —— —— (7)

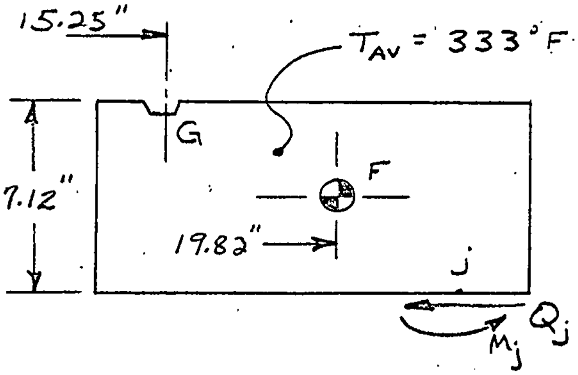
Deflection of cover ring at C

$10^6 \Delta R_c = .236(Q_B + Q_D - H_G)$ which becomes

$10^6 \Delta R_C = .236(Q_B + Q_D - H_G) + 13{,}700$ ——— ——— ——— — (8)

VESSEL FLANGE & CYLINDER -

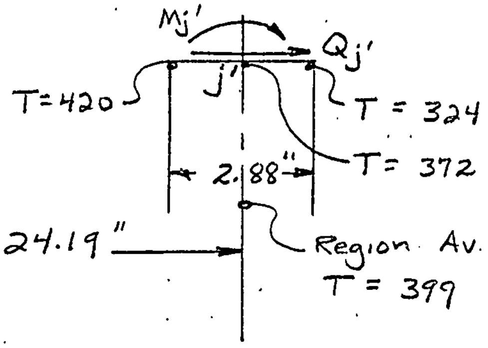
Deflections



ΔR_{F_G} - deflection of flange at gasket

$\Delta R_{F_G} = \alpha_{333}(333-70)(15.25)$

$10^6 \Delta R_{F_G} = +27{,}000$

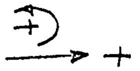


ΔR_{F_j} - deflection of flange at j

$\Delta R_{F_j} = \alpha_{333}(333-70)(24.19)$

$10^6 \Delta R_{F_j} = +42{,}700$

SIGN CONV.



ΔR_j - deflection of cylinder at j

$\Delta R_j = \alpha_{399}(399-70)(24.19)$

$10^6 \Delta R_j = +54{,}600$

From original equations

$10^6 \Delta R_{F_G} = .1483\, H_G - .186\, Q_j$ becomes

$10^6 \Delta R_{F_G} = .1483\, H_G - .186\, Q_j + 27{,}000$ ——— ——— ——— (9)

$10^6 \Delta R_{F_j'} = .1013 H_G - .174 Q_j$ becomes

$10^6 \Delta R_{F_j'} = .1013 H_G - .174 Q_j' + 42,700$ ———————— (10)

$10^6 \Delta R_j = .337 M_j + 2.19 Q_j$ becomes

$10^6 \Delta R_j' = .337 M_j + 2.19 Q_j' + 54,600$ ———————— (11)

Rotation of Flange

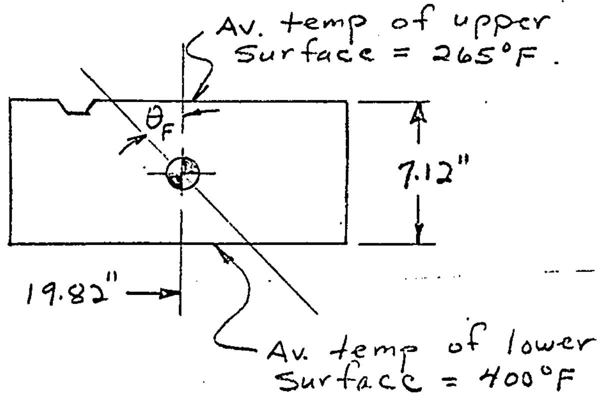
Rotation of flange due to temp. distribution

$\Delta R_{UPPER} = \alpha_{265} (265-70)(19.82)$

$10^6 \Delta R_{upper} = +25,300$

$\Delta R_{Lower} = \alpha_{400} (400-70)(19.82)$

$10^6 \Delta R_{Lower} = +45,000$



$$10^6 \theta_F = \frac{\Delta R_L - \Delta R_u}{7.12}$$

$$= \frac{45,000 - 25,300}{7.12}$$

$10^6 \theta_F = +2,765$

From previous analysis:

$10^6 \theta_F = .0382 M_F$ which becomes

$10^6 \theta_F = .0382 M_F + 2,765$

Also: $M_F = -2.9 H_G - 4.34 Q_j + 1.22 M_j$

Subst. in θ_F above

$10^6 \theta_F = -.111 H_G - .166 Q_j + .0466 M_j + 2,765$ ———————— (12)

Rotation of Cylinder

Equivalent bending moment at edge of cylinder which would produce the same effect as the temp. difference at the edge.

$$M = \frac{\Delta T}{2} \alpha \frac{E}{1-\nu} \frac{t^2}{6}$$

$$M = \left(\frac{420-324}{2}\right)\left(6.8 \times 10^{-6}\right)\left(\frac{27.8 \times 10^{6}}{.7}\right)\frac{(2.88)^2}{6}$$

$$M = -17{,}900$$

original sign convention $\overset{+}{\circlearrowright}$ but due to temp. diff. the shell rotates $\curvearrowright$
$\therefore$ M is negative.

From previous analysis:

$$10^6 \theta_j = -.104 M_j - .337 Q_j$$

But due to temp. distribution there is an additional equivalent moment of −17,900 at j which gives:

$$10^6 \theta_j = +.104\,(-M_j - 17{,}900) - .337 Q_j$$

$$10^6 \theta_j = -.104 M_j - .337 Q_j - 1{,}860 \quad \text{———————— (13)}$$

SUMMARY OF EQUATIONS

COVER

$$10^6 \Delta R_A = -.505\, Q_A + .187\, M_A + 32{,}000 \tag{1}$$

$$10^6 \theta_A = -.187\, Q_A + .1158\, M_A + 351 \tag{2}$$

INTERMEDIATE RING

$$10^6 \Delta R_{A'} = 1.14\, Q_{A'} - 1.235\, Q_B + 27{,}700 \tag{3}$$

$$10^6 \Delta R_B = 1.09\, Q_A - 1.197\, Q_B + 31{,}300 \tag{4}$$

$$10^6 \theta_B = -.187\, Q_A + .1279\, M_A + 351 \tag{5}$$

INSULATION COVER

$$10^6 Q_D = -.0665\, Q_A + .0299\, Q_B + .0269\, M_A - 708 \tag{6}$$

$$10^6 \Delta R_D = -40\, Q_D + 3{,}800 \tag{7}$$

COVER RING

$$10^6 \theta_C = .081\, Q_B - .0199\, M_A - .126\, H_G - .108\, Q_D + 476 \tag{8}$$

$$10^6 \Delta R_C = .236\,(Q_B + Q_D - H_G) + 13{,}700 \tag{9}$$

FLANGE

$$10^6 \Delta R_{F_G} = .1483\, H_G - .186\, Q_j + 27{,}000 \tag{10}$$

$$10^6 \Delta R_{F_j} = .1013\, H_G - .174\, Q_j + 42{,}700 \tag{11}$$

$$10^6 \theta_F = -.111\, H_G - .166\, Q_j + .0466\, M_j + 2{,}765 \tag{12}$$

CYLINDER

$$10^6 \Delta R_j = .337\, M_j + 2.19\, Q_j + 54{,}600 \tag{13}$$

$$10^6 \theta_j = -.104\, M_j - .337\, Q_j - 1{,}860 \tag{14}$$

BOUNDARY CONDITIONS

$$\Delta R_A = \Delta R_A' \qquad (15)$$

$$\Delta R_B = \Delta R_B' = \Delta R_C + 3.6\,\theta_B \qquad (16)$$

$$\theta_B = \theta_C \qquad (17)$$

$$\Delta R_G = \Delta R_B + 1.94\,\theta_B = \Delta R_{F_G} - 3.78\,\theta_F \qquad (18)$$

$$\Delta R_j' = \Delta R_j = \Delta R_{F_j} + 3.56\,\theta_j \qquad (19)$$

$$\theta_F = \theta_j \qquad (20)$$

$$Q_D = f(Q_B, Q_A, M_A) \qquad (21)$$

Subst. equations 1 & 3 in 15 gives

$$-1.645\,Q_A + .187\,M_A + 1.235\,Q_B = -4{,}300 \qquad (22)$$

Subst. equations 4, 5 & 9 in 16 gives

$$1.763\,Q_A - 1.433\,Q_B - .236\,Q_D + .236\,H_G - .46\,M_A = -16{,}340 \qquad (23)$$

Subst. equations 5 & 8 in 17 gives

$$-.187\,Q_A + .1478\,M_A - .081\,Q_B + .126\,H_G + .108\,Q_D = +125 \qquad (24)$$

Subst. equations 4, 5, 10 & 12 in 18 gives

$$.727\,Q_A - 1.197\,Q_B + .248\,M_A - .5683\,H_G - .441\,Q_j + .176\,M_j = -15{,}430 \qquad (25)$$

Subst. equations 13, 11 & 14 in 19 gives

$$.707\,M_j + 3.564\,Q_j - .1013\,H_G = -18{,}520 \qquad (26)$$

Subst. equations 12 & 14 in 20 gives

$$-.111\,H_G + .171\,Q_j + .1506\,M_j = -4{,}625 \qquad (27)$$

Subst. equation 6 in 21 gives

$$10^6 Q_D + .0655\,Q_A - .0299\,Q_B - .0269\,M_A = -708 \qquad (28)$$

Solving the above 7 equations (22) to (28) yields:

$Q_A = 46{,}551$

$M_A = 73{,}943$

$Q_B = 47{,}328$

$Q_D = -400$

$H_G = 14{,}111$

$Q_j = -989$

$M_j = -19{,}187$

Also from sheet 6

$M_B = .833\ (M_A) = 61{,}600$

Check of boundary conditions –

① $\Delta R_A = \Delta R_A'$

$$-.505\,Q_A + .187\,M_A + 32{,}000 = 1.14\,Q_A' - 1.235\,Q_B + 27{,}700$$

$$-.505(46{,}551) + .187(73{,}943) + 32{,}000 = 1.14(46{,}551) - 1.235(47{,}328) + 27{,}700$$

$$22{,}320 \approx 22{,}400$$

ΔR_A due to discontinuity forces only $= -9{,}680 \times 10^{-6}$

② $\Delta R_B = \Delta R_c + 3.6\,\theta_B$

$$1.09\,Q_A - 1.197\,Q_B + 31{,}300 = .236(Q_B + Q_D - H_G) + 13{,}700 + 3.6(-.187\,Q_A + .1279\,M_A + 351)$$

$$1.09(46{,}551) - 1.197(47{,}328) + 31{,}300 = .236\left[47{,}328 - 400 - 14{,}111\right] + 13{,}700 + 3.6\left[-.187(46{,}551) + .1279(73{,}943) + 351\right]$$

$$25{,}400 \approx 25{,}366$$

ΔR_B due to discont. forces only $= -5{,}900 \times 10^{-6}$

③ $\theta_B = \theta_c$

$$-.187\,Q_A + .1279\,M_A + 351 = .081\,Q_B - .0199\,M_A - .126\,H_G - .108\,Q_D + 476$$

$$-.187(46{,}551) + .1279(73{,}943) + 351 = .081(47{,}328) - .0199(73{,}943) - .126(14{,}111) - .108(-400) + 476$$

$$1{,}101 \approx 1{,}099$$

θ_c due to discont. forces only $= 623 \times 10^{-6}$

④ $\Delta R_B + 1.94\,\theta_B = \Delta R_{F_G} - 3.78\,\theta_F$

$25,400 + 1.94(1,101) = .1483\,H_G - .186\,Q_j + 27,000 - 3.78(-.111\,H_G - .166\,Q_j + .0466\,M_j + 2,765)$

$25,400 + 2,135 = .1483(14,111) - .186(-989) + 27,000 - 3.78[-.111(14,111) - .166(-989) + .0466(-19,187) + 2,765]$

$27,535 = 29,274 - 3.78(469)$

$27,535 \approx 27,499$

⑤ $\Delta R_j = \Delta R_{F_j} + 3.56\,\theta_j$

$.337\,M_j + 2.19\,Q_j + 54,600 = .1013\,H_G - .174\,Q_j + 42,700 + 3.56(-.104\,M_j - .337\,Q_j - 1,860)$

$337(-19,187) + 2.19(-989) + 54,600 = .1013(14,111) - .174(-989) + 42,700 + 3.56[(-.104)(-19,187) - .337(-989) - 1,860]$

$45975 \approx 45,958$

⑥ $\theta_F = \theta_j$

$-.111\,H_G - .166\,Q_j + .0466\,M_j + 2,765 = -.104\,M_j - .337\,Q_j - 1860$

$-.111(14,111) - .166(-989) + .0466(-19,187) + 2,765 = -.104(-19,187) - .337(-989) - 1860$

$469 \approx 465$

θ_F due to discont. forces only $= -2,296 \times 10^{-6}$

(7.) $Q_D + .0655\,Q_A - .0299\,Q_B - .0269\,M_A = -708$

$-400 + .0655(46{,}551) - .0299(47{,}328) - .0269(73{,}943) = -708$

$-400 + 3100 - 1{,}415 - 1{,}990 = -708$

$-705 \approx -708$

TOTAL DEFLECTIONS AT CORNERS of FLANGES —

Pt W

$$10^6 \Delta R_{W_{TOT}} = \Delta R_B - 8{,}91\, \theta_{B_{TOTAL}}$$

$$= 25400 - 8.91(1100)$$

$$10^6 \Delta R_{W_{TOT}} = 15600$$

Pt V

$$10^6 \Delta R_{V_{TOT}} = \Delta R_{V_{Th}} + \Delta R_{V_{DISC}}$$

$$10^6 \Delta R_V = \Delta R_{V_{Th}} + \Delta R_{O_{DISC}} - 5.13\, \theta_{C_{DISC}}$$

where $\Delta R_{V_{Th}} = \alpha_{166^\circ}\, \Delta T R_o$

$$\Delta R_{V_{Th}} = 6.3(166-70)(20{,}12) = 12200$$

and $\Delta R_{o_{DISC}} = 5440$ from pg 21

$$10^6 \Delta R_V = 12200 + 5440 - \overset{3310}{5.13(623)}$$

$$10^6 \Delta R_{V_{TOT}} = 14340$$

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Pt X

$$10^6 \Delta R_{x_{TOT}} = \Delta R_{x_{TH}} + \Delta R_{0_{DISC}} + 5.31\, \theta_{0_{DISC}}$$

where $\Delta R_{x_{TH}} = \alpha_{205} \Delta T R_0$

$$10^6 \Delta R_{x_{TH}} = 6.41\,(205-70)(20,12)$$

$$10^6 \Delta R_{x_{TH}} = 17400$$

$$10^6 \Delta R_x = 17400 + 5450 + 3310$$

$$10^6 \Delta R_{x_{TOT}} = 26160$$

Pt Z

$$10^6 \Delta R_{z_{TOT}} = \Delta R_{z_{TH}} + \Delta R_{F_{J_{DISC}}} - 3.56\, \theta_{F_{DISC}}$$

where $\Delta R_{z_{Th}} = \alpha_{333^\circ} \Delta T R_z$

$$10^6 \Delta R_{z_{Th}} = 6.7\,(333-70)(25.63)$$

$$10^6 \Delta R_{z_{Th}} = 45200$$

and $\Delta R_{F_{J_{DISC}}} = .1013\, H_G - .174\, Q_J$

$$= .1013(14111) - .174(-989) = 1602$$

$$10^6 \Delta R_{z_{TOT}} = 45200 + 1602 - 3.56\,(-2296)$$

$$10^6 \Delta R_{z_{TOT}} = 54972$$

Pt Y

$$10^6 \Delta R_{Y_{TOT}} \approx \Delta R_{G_{TOT}} = 27535$$

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BY RAC DATE 9/7/61 SUBJECT

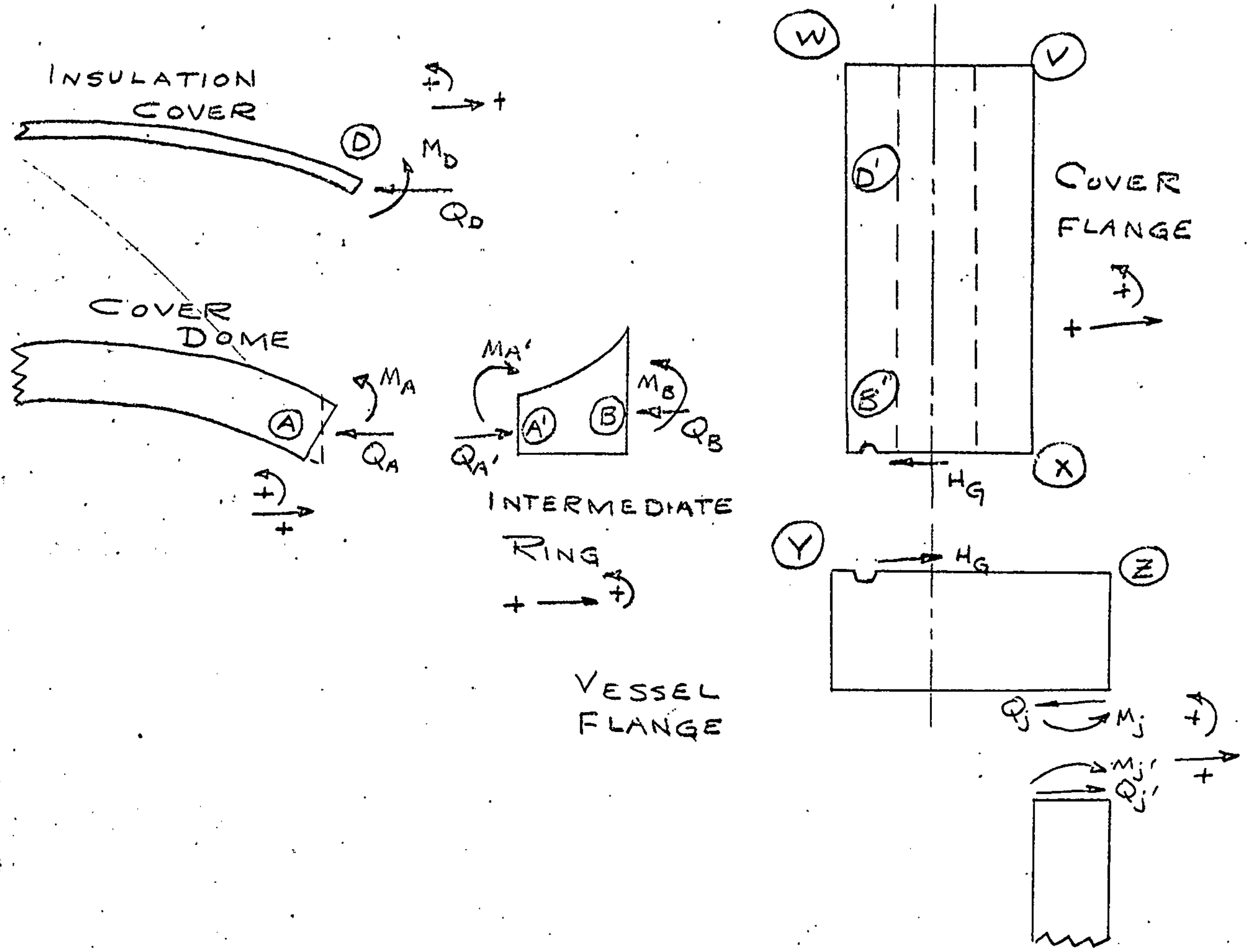
SHEET NO. 18 OF 27

CHKD. BY DATE

JOB NO.

SUMMARY - CASE D

TRANSIENT THERMAL STRESS FOR 50°/HR HEATUP



LOCATION		TOTAL DEFLECTION ΔR (10^{-6} IN)	TOTAL ROTATION θ (10^{-6} RAD)	STRESSES (PSI)			
ABOVE SK.	STRESS CHARTS			σ_2 CIRCUMF.	σ_1 AXIAL	σ_3 RADIAL	SHEAR
A	12	22,320	—	-32,100 OUTER -9,500 INNER	-51200 +24000	0	-
B		25,400	1,100	-20,620 INNER -1,780 OUTER	+22160 -40640	0	-
D	13	19,800	1,100	30,700	2130	0	-
G		27,535	—	—	—	—	18,800
J	11	45,975	469	-14 350 INNER -6,050 OUTER	-13 850 +13,850	0	—
V	14 OUTSIDE	14,340	1,100	3,040	—	—	—
W	14 INSIDE	15,600	1,100	8,500	—	—	—
X	16	26160	1,100	12,400	—	—	—
Y	15 INSIDE	27535	469	22,300	—	—	—
Z	15 OUTSIDE	54972	469	11,500	—	—	—

STRESSES

THE DEFLECTION DUE TO DISCONTINUITY FORCES IS USED TO FIND CIRCUMFERENTIAL STRESSES.

① AT POINT A $M_A = 73943$

AXIAL STRESS:

$$\sigma_1 = \sigma_{THRUST} + \sigma_{BENDING}$$

$$= -\frac{Q_A}{t_A} \pm \frac{6M_A}{h^2} = \overset{-13600}{-\frac{46551}{3.43}} \pm \overset{\pm 37600}{6\frac{(73943)}{(3.43)^2}}$$

$$\sigma_1 = \begin{cases} 24000 \text{ psi} & \text{INSIDE} \\ -51200 \text{ psi} & \text{OUTSIDE} \end{cases}$$

CIRCUMF. STRESS:

$$\sigma_2 = \frac{\Delta R_A}{R_A} \times E \pm \nu \frac{6M_A}{h^2}$$

$$= \frac{-9680}{13.25}(28.5) \pm .3(37600)$$

$$= -20850 \pm 11270$$

$$\sigma_2 = -9580 \text{ psi INSIDE}$$
$$-32150 \text{ psi OUTSIDE}$$

RADIAL STRESS: $\sigma_3 = 0$

② AT POINT B $M_B = .833\, M_A = 61600$

AXIAL STRESS:

$$\sigma_1 = \sigma_{Thrust} + \sigma_{BENDING}$$

$$= -\frac{Q_B}{t_B} \pm \frac{6M_B}{h^2} = \overset{-9240}{-\frac{47328}{5.12}} \pm \overset{\pm 31400}{6\frac{(61600)}{(3.43)^2}}$$

$$\sigma_1 = -40640 \text{ psi Outside}$$
$$22160 \text{ psi INSIDE}$$

CIRCUMF:
$$\sigma_2 = \frac{\Delta R_B}{R_B} E \pm \frac{\nu 6 M_B}{(h)^2}$$

$$= -\frac{\overset{-11200}{5900}}{10}(28.5) \pm .3\overset{9430}{(31400)}$$

$$\sigma_2 = \begin{matrix} -20630 \text{ PSI} & \text{OUTSIDE} \\ -1770 \text{ PSI} & \text{INSIDE} \end{matrix}$$

RADIAL STRESS: $\sigma_3 = 0$

✓ ③ AT POINT D $(M_D = 0)$

AXIAL STRESS: $\sigma_1 = \sigma_{Thrust} + \sigma_{BENDING}$

$$\sigma_1 = \frac{Q_D}{t_D} = \frac{400}{.1875} = 2130 \text{ PSI}$$

CIRCUMFERENTIAL: $\Delta R_D = -40 Q_D$

$$\sigma_2 = \frac{\Delta R_D}{R_D} \times E = -\frac{40(-400)}{14.88}(28.5)$$

$$\sigma_2 = 30700 \text{ PSI}$$

RADIAL STRESS: $\sigma_3 = 0$

✓ ④ SHEAR STRESS IN GASKET $= \frac{H_G}{.75} = \frac{14111}{.75} = 18800$ P.S.I.

✓ ⑤ AT POINT J —

AXIAL STRESS:

$$\sigma_1 = \frac{6 M_J}{h^2} = \frac{6(-19187)}{(2.88)^2} = \begin{matrix} +13850 \text{ psi} & \text{outer} \\ -13850 \text{ psi} & \text{inner} \end{matrix}$$

CIRCUMFERENTIAL:

$$\sigma_2 = \frac{E\omega}{a} \pm \frac{\nu 6 M_J}{h^2} = \overset{-10200}{-\frac{8625}{24.19}(28.5)} \pm .3\overset{4150}{(13850)}$$

$$\sigma_2 = \left[\begin{matrix} -14350 \text{ PSI} & \text{INSIDE} \\ -6050 \text{ PSI} & \text{OUTSIDE} \end{matrix}\right]$$

RADIAL:

$$\sigma_3 = 0$$

✓ ⑥ At point W (Inner Upper Corner of Cover Flange) –

$$\Delta R_W = \Delta R_c - 5.312\,\theta_c \qquad \text{where } \theta_c = 623$$

$$\Delta R_W = 7,740 - 3,310 = 4,430 \times 10^{-6}$$

$$\sigma_2 = \frac{\Delta R_W}{R_W} E = \frac{4,430}{14.88}(28.5) = 8,500 \text{ psi}$$

✓ ⑦ At point V. (Outer Upper Corner of Cover Flange) –

(a) Deflection of outer surface V–X of flange (ΔR_o)

$$\Delta R_o = \frac{Pb}{E}\left(\frac{2a^2}{b^2-a^2}\right)$$

Formulas for Stress and Strain – Roark
p. 232, case 17

$$10^6 \Delta R_o = \left(\frac{47,328-400-14,111}{10.625}\right)\left(\frac{20.12}{28.5}\right)\left[\frac{2(15)^2}{(20.12)^2-(15)^2}\right]$$

$$10^6 \Delta R_o = 3,080\,(1.77) = 5,450 \checkmark$$

(b) Deflection at point V –

$$10^6 \Delta R_V = \Delta R_o - 5.312\,\theta_c$$

$$10^6 \Delta R_V = 5,450 - 3,310 = 2,140 \checkmark$$

(c) Stress at point V –

$$\sigma_2 = \frac{\Delta R_V}{R_V} E = \frac{2,140}{20.12}(28.5)$$

$$\sigma_2 = 3,040 \text{ psi} \checkmark$$

✓ ⑧ At point X (Lower Outer Corner of Upper Ring) -

(a.) Deflection at X -

$$10^6 \Delta R_X = \Delta R_0 + 5.312\, \theta_c \qquad \theta_c = 623$$

$$10^6 \Delta R_X = 5,450 + 3310 = 8,760$$

(b.) Stress at X -

$$\sigma_2 = \frac{\Delta R_X}{R_X} E = \frac{8,760}{20.12}(28.5) = 12,400 \text{ psi}$$ ✓

✓ ⑨ At point Y (Vessel Flange Inner Corner) -

$\Delta R_Y \approx \Delta R_G$ From boundary conditions:

$$\Delta R_G = \Delta R_{F_G} - 3.78\, \theta_F$$

$$\therefore \Delta R_Y = 2274 - 3.78(-2296)$$

$$10^6 \Delta R_Y = 10,944$$

Stress at Y -

$$\sigma_2 = \frac{\Delta R_Y}{R_Y} E = \frac{10944}{14}(28.5) = 22,300 \text{ psi}$$

✓ ⑩ At point Z (Vessel Flange Outer Corner) -

$$10^6 \Delta R_Z = \Delta R_{F_J} - 3.56\, \theta_F$$

$$10^6 \Delta R_Z = 1603 - 3.56(-2296) = 9,763$$

Stress at Z -

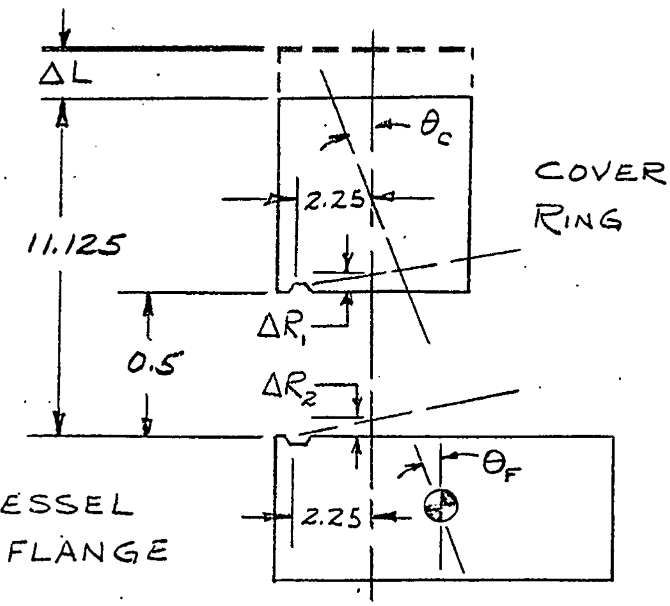
$$\sigma_2 = \frac{9,763}{24.19}(28.5) = 11,500 \text{ psi}$$

ALCO PRODUCTS INC.

BY________ DATE________ SUBJECT________ SHEET NO. 23 OF 27

CHKD. BY________ DATE________ JOB NO.________

DEFLECTION OF TOP & BOTTOM FLANGES ALONG CENTERLINE OF STUD



$10^6 \Delta R_1 = 2.25\,\theta_c$

$= 2.25(1100)$

$10^6 \Delta R_1 = 2{,}475$

$10^6 \Delta R_2 = 2.25\,\theta_F$

$= 2.25(469)$

$10^6 \Delta R_2 = 1{,}055$

Where θ_c and θ_F are the total rotation due to discontinuity forces and thermal expansion.

Increase in distance between flange surfaces along ℄ of stud $= 2475 - 1055 = 1{,}420 \times 10^{-6}$ in.

Thermal expansion of cover ring and stud:

① $\Delta L_{\text{COVER RING}} = \alpha_{192}\,\Delta T\, L_{\text{RING}}$

$10^6 \Delta L = 6.4(192-70)(10.625)$

$10^6 \Delta L = 8{,}300$

② $\Delta L_{\text{STUD}} = \alpha_{197}\,(197-70)(11.125)$

$10^6 \Delta L = 5.5(127)(11.125)$

$10^6 \Delta L = 7{,}770$

The difference in thermal expansion of the cover ring and stud is the amount the stud is stretched.

$$\begin{array}{rl} 8300 \times 10^{-6} & \text{cover ring} \\ - \; 7,770 \times 10^{-6} & \text{stud} \\ \hline 530 \times 10^{-6} \text{ in} & \text{stretch in stud.} \end{array}$$

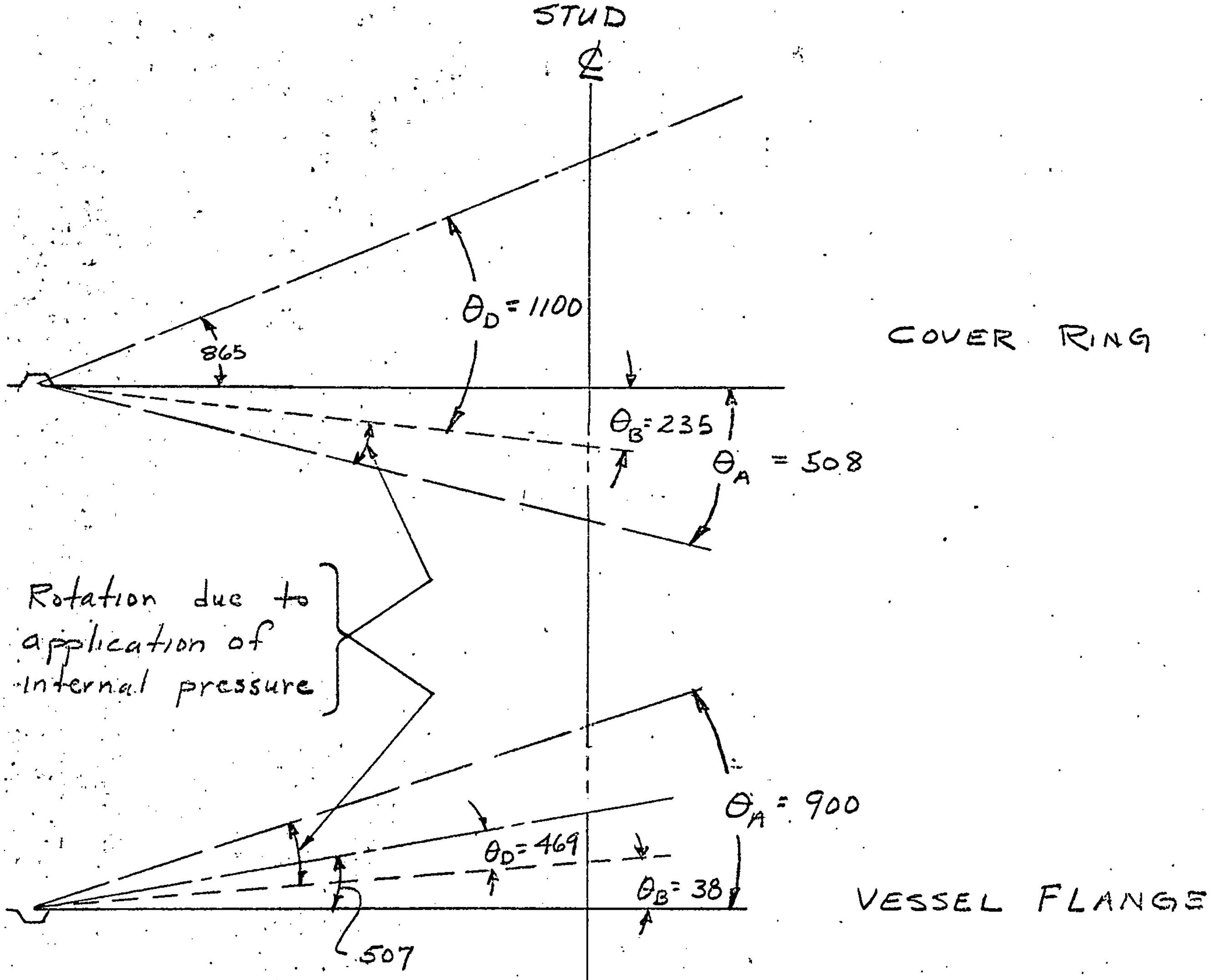
But due to the difference in rotation of lower flange and cover ring, there is an increase of 1420 x 10^{-6} in. Therefore, the total increase in stud length for the transient state (Case D) is (1,420 + 530) = 1,950 x 10^{-6} in.

Stress due to increase in length of stud $= \frac{\Delta L}{L} \times E$

$$= \frac{1950}{11.125}(29.5)$$

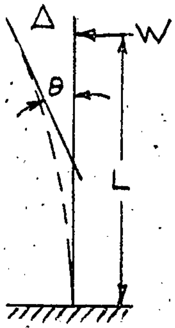
$$= 5,170 \text{ psi (Tension)}$$

BENDING STRESS IN STUD -



Resultant Rotation of Stud ℄ = 865 - 507 = 358

Subscripts A, B, D correspond to cases A, B & D.



Horizontal Deflection of stud -

$$\Delta = L\theta$$

$$= 11.125\,(358 \times 10^{-6})$$

$$\Delta = 3{,}980 \times 10^{-6} \text{ in}$$

For Cantilever Beam -

$$\Delta = \frac{WL^3}{3EI} \qquad M_{max} = -WL$$

where W = applied load

$$W = \frac{\Delta\, 3EI}{L^3}$$

$$M_{max} = \frac{\Delta\, 3EI}{L^3} \times L \qquad \text{where } I = \frac{\pi r^4}{4}$$

& Minor stud dia. = 2.59", r = 1.295"

$$M_{max} = \frac{3980 \times 10^{-6}\,(3)(29.5 \times 10^{6})\,\pi\,(1.295)^4}{4\,(11.125)^2}$$

$$M_{max} = 6{,}287 \text{ in-lbs}$$

$$\text{Bending Stress} = \frac{Mc}{I} = \frac{6287\,(1.295)(4)}{\pi\,(1.295)^4}$$

$$\text{Bending Stress} = \frac{32600}{8.84} = 3{,}690 \text{ psi}$$

STUD STRESS SUMMARY

	ΔL STUD (in)	INCREASE IN STUD STRESS (psi)
CASE "A"	—	20,000 (TENSION)
CASE "B"	1428×10^{-6}	3,780 (TENSION)
CASE "D"	1950×10^{-6}	5,170 (TENSION)
		3,690 (BENDING)
Total	$3{,}378 \times 10^{-6}$	32,640

ALCO PRODUCTS INC.

BY BJR DATE 9/11/61
CHKD. BY ______ DATE ______

SUBJECT SM-1-A REACTOR VESSEL STRESS ANALYSIS AE-90 TASK 6.9

SHEET NO. 1 OF 1
JOB NO. CONTR NO. AT(30-1)-2639

THERMAL STRESS IN VESSEL WALL DUE TO A TEMPERATURE TRANSIENT 50°/HR

FROM NAVY CODE SECT A.3.5 –

FOR INNER SURFACE – $\sigma_2 = \sigma_1 = -\frac{E\alpha(\Delta T_f)}{1-\nu} n_i$

FOR OUTER SURFACE – $\sigma_2 = \sigma_1 = \frac{E\alpha \Delta T_f}{1-\nu} n_o$

where $\Delta T_f = 50°F/HR$

$\left.\begin{matrix} n_i \\ n_o \end{matrix}\right\}$ stress factors for inside & outside surface from Figs A.3-5 & A.3-6 of NAVY CODE in which $\frac{12K}{ht} \approx 0$

$$\lambda = \frac{\varepsilon\theta}{t^2} = \frac{(.018)(3600)}{(2.88)^2} = 7.8$$

$\therefore\ n_i = .045$

$n_o = .02$

Then for INSIDE $\sigma_2 = \sigma_1 = -\frac{28.5\times10^6(6.9\times10^{-6})(50)(.045)}{(1-.3)}$

$\sigma_2 = \sigma_1 = -632$ psi

Then for OUTSIDE $\sigma_2 = \sigma_1 = \frac{28.5\times10^6(6.9\times10^{-6})(50)(.02)}{1-.3}$

$\sigma_2 = \sigma_1 = 281$ psi

ALCO PRODUCTS INC.

BY BJK DATE

CHKD. BY DATE 12/9/60

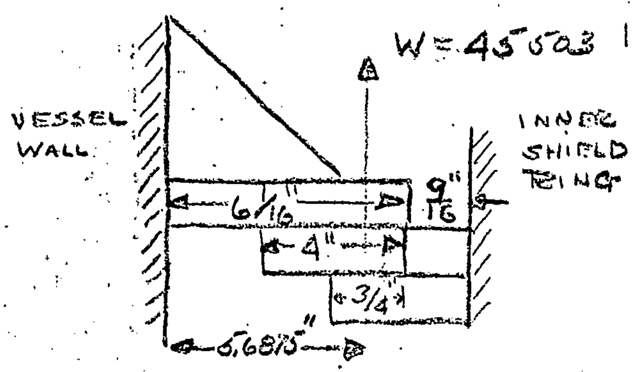
SUBJECT SM-1A STRESS ANALYSIS AE-76- TASK 6.9

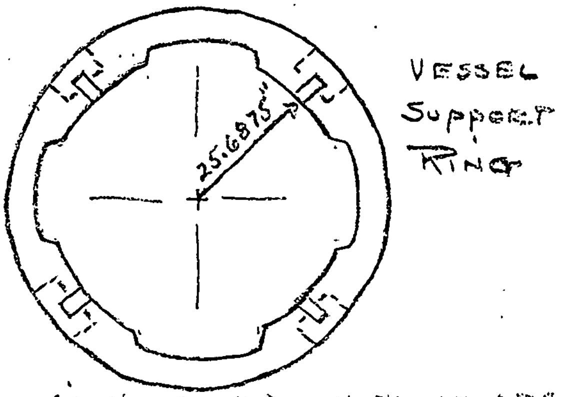
SHEET NO. 1 OF 5

JOB NO. CONTR. NO AT(30-1)-2639

STRESS DUE TO BENDING MOMENT APPLIED BY VESSEL SUPPORT RING TO VESSEL WALL DUE TO WEIGHT OF VESSEL, WATER, ETC.

1. DETERMINATION OF MOMENT APPLIED —





WEIGHT OF COMPLETE VESSEL WITH CORE STRUCTURE, THERMAL SHIELDS, COVER, INSULATION, INSULATION JACKET, WATER IN THE VESSEL, WATER IN UPPER SHIELD TANK SUPPORTED BY VESSEL IS 45503 lbs.

WEIGHT SUPPORTED BY ONE SEGMENT OF RING IN CONTACT WITH VESSEL WALL = $\frac{45503}{4}$ lbs.

MOMENT = $\frac{45503}{4} \times 5.6875''$ = 64,600 in.lb. APPLIED at each segment

2. DETERMINATION OF EQUIVALENT CIRCULAR AREA —

AREA of RING SEGMENT IN CONTACT WITH VESSEL = 28.05 IN^2 {AREA FROM C. Aaker

AREA of GUSSET (.875)(3) = 2.84 IN^2

30.89 IN^2

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CONSIDER THIS AREA AS A CIRCULAR AREA WITH RADIUS r_o OF AN ATTACHMENT TO THE VESSEL, IN ORDER TO OBTAIN BENDING STRESSES TRANSMITTED by The RING DUE TO VESSEL WEIGHT.

$$A = \pi r_o^2$$

$$r_o = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{30.89}{\pi}}$$

$$r_o = 3.14 \text{ in.}$$

3. PER NAVY CODE –

$$\beta = \frac{0.875 r_o}{a} \qquad \gamma = \frac{a}{t}$$

where a = radius of middle surface of shell = 24.875"
r_o = outer radius of attachment = 3.14"
t = thickness of shell = 2.875

$$\beta = \frac{0.875(3.14)}{24.875} = .1105 \qquad \gamma = \frac{24.875}{2.875} = 8.65$$

From A.5-6 $\frac{M_x}{M/a\beta} = .096$

A.5-7 $\frac{M_\phi}{M/a\beta} = .057$

A.5-8 $\frac{N_x}{M/a^2\beta} = .17$

A.5-9 $\frac{N_\phi}{M/a^2\beta} = .55$

ALCO PRODUCTS INC.

BY ________ DATE ________ SUBJECT ________ SHEET NO. 3 OF 5

CHKD. BY ________ DATE ________ JOB NO. ________

$a = 24.1875"$

$\beta = .1105"$

$t^2 = (2.875)^2 = 8.26$

$a\beta t^2 = 22.1$

$$\frac{M_x}{M/a\beta} \times \frac{6M}{a\beta t^2} = 6\frac{M_x}{t^2}$$

$$.096 \times \frac{6M}{22.1} = 6\frac{M_x}{t^2}$$

$$.0261\,M = 6\frac{M_x}{t^2}$$

$$\frac{M_\phi}{M/a\beta} \times \frac{6M}{a\beta t^2} = \frac{6M_\phi}{t^2}$$

$$.057 \times \frac{6M}{22.1} = 6\frac{M_\phi}{t^2}$$

$$.01546\,M = 6\frac{M_\phi}{t^2}$$

$t a^2\beta = (2.875)(24.1875)^2(.1105)$

$t a^2\beta = 186$

$$\frac{N_x}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N_x}{t}$$

$$.17 \times \frac{M}{186} = \frac{N_x}{t}$$

$$.000914\,M = \frac{N_x}{t}$$

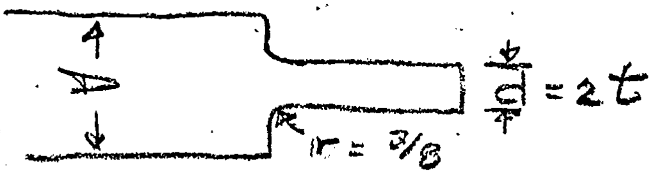
$$\frac{N_\phi}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N_\phi}{t}$$

$$.55 \times \frac{M}{186} = \frac{N_\phi}{t}$$

$$.00296\,M = \frac{N_\phi}{t}$$

STRESS CONCENTRATION FACTORS —

Assume the vessel support ring to be a stepped bar in bending with a shoulder radius of 3/8" (which is the size of the fillet weld where the ring is welded to the vessel)



where d = 1.5" = the thickness of the support ring

Ref. Fig. A.7-1 Pg. 92 Navy Code

$$\frac{r}{t} = \frac{.375}{.750} = \frac{1}{2}$$

from curve $K_m = 1.5$

$K_N = 1.8$

$$\sigma_\phi = K_N \frac{N_\phi}{t} \pm K_m \frac{6M_\phi}{t^2}$$

$$\sigma_\phi = 1.8(.00296 M) \pm 1.5(.01546 M)$$

$$\sigma_\phi = .00533 M \pm .0232 M$$

$$\sigma_\phi = .02853 M = .02853(64600) = 1840 PS$$

$$\sigma_\phi = -.0179 M = -.0179(64600) = -1156 ps$$

$$\sigma_x = K_N \frac{N_x}{t} \pm K_m \frac{6M_x}{t^2}$$

$$\sigma_x = 1.8(.000914M) \pm 1.5(.0261 M)$$

$$\sigma_x = .001642 M \pm .0391 M$$

$$\sigma_x = .0407 M = .0407(64600) = 2650 PSI$$

$$\sigma_x = -.0375 M = -.0375(64600) = -2420 psi$$

The GREATER of the two SETS of stresses IS SELECTED AS THE MAXIMUM STRESS due TO THE LONGITUDINAL MOMENT APPLIED TO THE VESSEL

MAX. STRESS = 2650 P.S.I.

ALCO PRODUCTS INC.

BY C.A. DATE 11/11/60 SUBJECT Support Ring Stresses AE 90 TASK 6.9

CHKD. BY DWM DATE 12/12/60

SHEET NO. 1 OF 7

JOB NO. CONTR. NO AT(30-1)-2639

Reactor Vessel Dwg F-41201-1-2

Based on Analysis by D.W. McLaughlin for The SM-1 Reactor Vessel, Dated 10/10/60.

The vessel support ring has a slower response to primary fluid temperature changes than the reactor vessel itself and acts to restrain the free thermal expansion of the vessel. In calculating the stresses in the vessel wall due to this restraint, the following procedure will be used:-

1. The ratio of the stiffness of the cylinder and ring will be determined.

2. The portion of the thermal expansion mismatch to be absorbed by the ring, will be calculated using the stiffness ratio.

3. The load requiring to deflect the ring by the amount obtained from (2) will be determined.

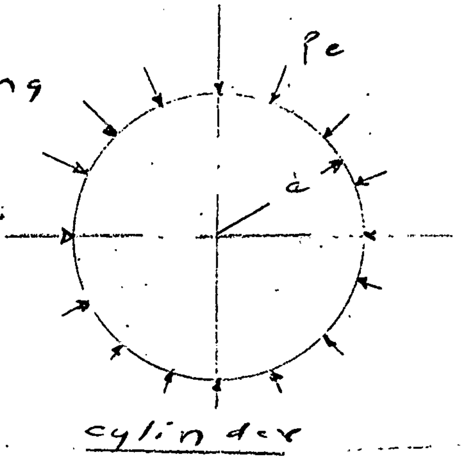
4. Vessel wall stresses will be calculated by the methods of the Navy Code, using the load from (3).

ALCO PRODUCTS INC.

BY C.A. DATE 11/13/60 SUBJECT Support Ring SHEET NO. 2 OF 7
CHKD. BY ______ DATE ______ JOB NO. ______
SM-1A

1. Stiffness of Ring and Cyl.

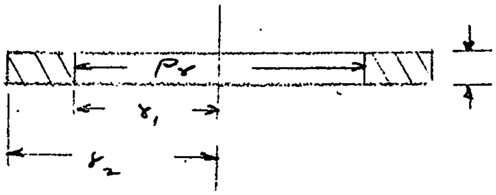
If we take the case of a long cylinder restrained by a ring, the deflections of the cylinder and ring are:



cylinder

$$y_{cyl.} = \frac{P_c \beta a^2}{2Eh} \qquad \text{where: } \beta^4 = \frac{3(1-\mu^2)}{a^2h^2}$$

$$y_{ring} = \frac{P_r r_1^2}{\ell E(r_2 - r_1)}$$



Ring

Equilibrium requires that:

$P_c = P_r$ where P = load per unit length of circumference. Now we determine the ratio of deflections (or stiffness) for the load P.

$$\frac{y_c}{y_r} = \frac{\ell E(r_2 - r_1)}{r_1^2} \times \frac{\beta a^2}{2Eh}$$

$$\frac{y_c}{y_r} = \frac{\ell \beta}{2h}\left(\frac{a}{r_1}\right)^2 (r_2 - r_1)$$

This ratio of deflections will be used in further calculations although in this particular case the ring is in contact with the cylinder over only about 45% of its circumference

$$\frac{4 \times 40^\circ}{360^\circ} = 44.4\%$$

2. Mismatch To be absorbed by Ring

Mismatch due To Thermal expansion:

$$y_c + y_r = \alpha \Delta T r_1$$

ΔT = difference between average Temperatures of ring and cylinder.

From P. 2

$$y_c = \frac{l\beta}{2h}\left(\frac{a}{r_1}\right)^2 (r_2 - r_1)\, y_r$$

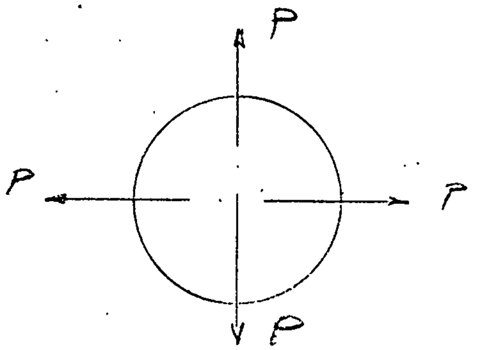
Combining Two equations -

$$y_r\left[1 + \frac{l\beta}{2h}\left(\frac{a}{r_1}\right)^2 (r_2 - r_1)\right] = \alpha \Delta T r_1$$

$$y_r = \frac{\alpha \Delta T r_1}{1 + \frac{l\beta}{2h}\left(\frac{a}{r_1}\right)^2 (r_2 - r_1)}$$ = Mismatch to be absorbed by ring.

3. Deflection of ring from local radial loads

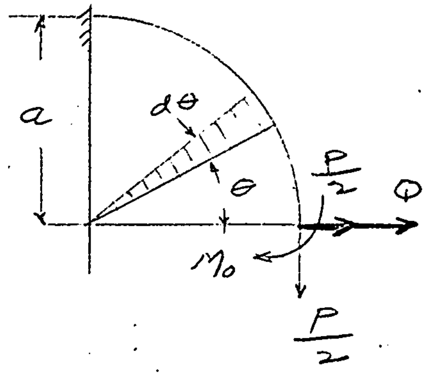
Using Castigliano's Theorem - M_0 may be determined.



$$\overset{\curvearrowright}{M_\theta} = M_0 + \frac{Pa}{2}\left[1 - \cos\theta - \sin\theta\right]$$

$$\frac{\partial M_\theta}{\partial M_0} = 1$$

$$ds = a\, d\theta$$



Rotation at M_0 is given by -

$$\phi_0 = \frac{1}{EI}\int_0^{\frac{\pi}{2}} M_\theta \frac{\partial M_\theta}{\partial M_0}\, ds$$

BY G.A. DATE 11/12/60 SUBJECT Support Ring SHEET NO. 4 OF 7

CHKD. BY DATE JOB NO.

SM-1A

From symmetry $\phi_0 = 0$

$$\int_0^{\frac{\pi}{2}} M_\theta \frac{\partial M_\theta}{\partial M_0} ds = 0$$

$$0 = \int_0^{\frac{\pi}{2}} \left\{ M_0 + \frac{Pa}{2}\left[1 - \cos\theta - \sin\theta\right] a \, d\theta \right.$$

$$= \left[M_0 a \theta + \frac{Pa^2}{2}\left(\theta - \sin\theta + \cos\theta\right)\right]_0^{\frac{\pi}{2}} = M_0 a \frac{\pi}{2} + \frac{Pa^2}{2}\left(\frac{\pi}{2} - 2\right)$$

$$\underline{\underline{M_0 = -Pa\left(\frac{1}{2} - \frac{2}{\pi}\right)}}$$

To find horizontal deflection at M_0, apply dummy load φ.

$$M_\theta = M_0 + \frac{Pa}{2}\left(1 - \cos\theta - \sin\theta\right) - \varphi a \sin\theta$$

$$\frac{\partial M_\theta}{\partial \varphi} = -a \sin\theta$$

$$ds = a \, d\theta$$

$$y_0 = \frac{1}{EI}\int_0^{\frac{\pi}{2}} M_\theta \frac{\partial M_\theta}{\partial \varphi} ds$$

$$= \frac{a^2}{EI}\int_0^{\frac{\pi}{2}} \left[M_0 \sin\theta + \frac{Pa}{2}\left(\sin\theta - \sin\theta\cos\theta - \sin^2\theta\right) + \varphi \sin\theta \right] d\theta$$

After integrating and setting $\varphi = 0$

$$y_0 = -\frac{a^2}{EI}\left[-M_0\cos\theta + \frac{Pa}{2}\left(-\cos\theta - \frac{\sin^2\theta}{2} - \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\right]_0^{\frac{\pi}{2}}$$

$$y_0 = \frac{a^2}{EI}\left[-M_0 - \frac{Pa}{4}\left(1 - \frac{\pi}{2}\right)\right]$$

But $M_0 = -Pa\left(\frac{1}{2} - \frac{2}{\pi}\right)$

$$y_0 = \frac{Pa^3}{EI}\left[\frac{1}{4} + \frac{\pi}{8} - \frac{2}{\pi}\right]$$

$$y_0 = .006 \frac{Pa^3}{EI} = y_v \text{ from P.3}$$

BY C.A. DATE 11/13/60 SUBJECT Support Ring SHEET NO. 5 OF 7

CHKD. BY ______ DATE ______ JOB NO. ______

SM-1A

4. Stress in Vessel wall due To Ring

Stresses are calculated using The Navy Code, Ref 1, for radial load on cylinder.

Load P is applied over a $1\frac{1}{2}" \times 18.7"$ area ignoring the area of the three reenforcing gussets at each lug.

Area = D × W

$A: 1\frac{1}{2}" \times 18.7" = \pi r_o^2$

$r_o = \sqrt{\frac{28.05}{\pi}} = 2.988$

$a = \frac{51\frac{1}{4} + 46}{4} = 24.3125"$

$t = 2\frac{5}{8}"$ thick.

$D = 1\frac{1}{2}"$

$R = \frac{25.6875 + 27.75}{2} = 26.72$

$W = 26.72" \times 2\pi \times \frac{40°}{360°} = 18.7"$

r_0 = radius of circle of same area as applied load area.

$\beta = \frac{0.875\, r_o}{a} = \frac{0.875(2.988)}{24.3125} = .1075$

$\gamma = \frac{a}{t} = \frac{24.3125}{2.625} = 9.262$

From Fig. A.5.4 & A.5.5

$\frac{M_\phi}{P} = .20$

$\sigma_{\phi_b} = \frac{6P}{t^2} \times .20$

$= \frac{6 \times .20 \times P}{2\frac{5}{8}^2}$

$= .1741\ P$

$\frac{N_\phi}{P/a} = 2.3$

$\sigma_{\phi_m} = \frac{P}{aT} \times 2.3$

$= \frac{2.3 \times P}{24.31 \times 2\frac{5}{8}}$

$= .0360\ P$

Combining Stresses

$\sigma_\phi = .1741P + .0360P = \underline{.2101\ P}$

ALCO PRODUCTS INC.

BY C.A. DATE 11/11/60 SUBJECT Support Ring SHEET NO. 6 OF 7

CHKD. BY ____ DATE ____ JOB NO. ____

SM-1A

Calculation of P from pp. 3 & 4

$a = 24.3125''$

$h = 2\frac{5}{8}''$

$r_1 = 25\frac{11}{16}''$

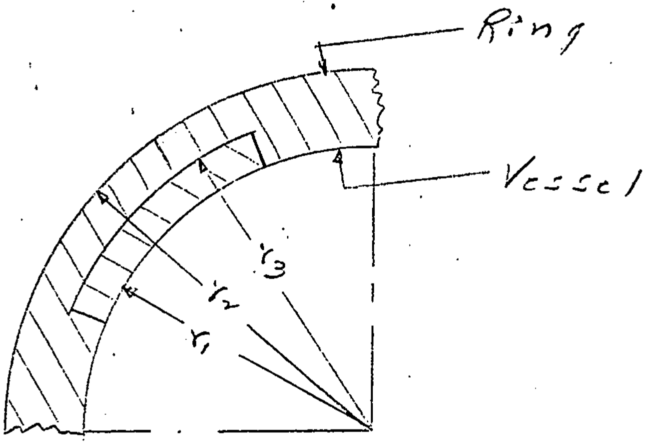
$r_2 = 31\frac{3}{4}''$

$r_3 = 27\frac{3}{4}''$

$\mu = .30$

$l = 1\frac{1}{2}''$

$E = 28 \times 10^6$



$\alpha = 6 \times 10^{-6}$ in./in./°F. (carbon steel)

$$\beta = \left[\frac{3(1-\mu^2)}{a^2h^2}\right]^{\frac{1}{4}} = \frac{1.284}{\sqrt{\underset{7.987}{63.82}}} = .1508 \qquad (P. 2)$$

$$I_{Avrg.} = \frac{1}{2}\left[\frac{l(r_2-r_1)^3}{12} + \frac{l(r_2-r_3)^3}{12}\right] = \overset{\frac{1\frac{1}{2} l}{12}}{\frac{1.5}{2\times 12}}\left[\overset{222.8\ +\ 64}{6\tfrac{1}{16}^3 + 4^3}\right]$$

$$= .0625\,(\overset{286.8}{222.8 + 64})$$

$$= \underline{\underline{17.925 \text{ in.}^4}}\ ✓$$

From P. 3

$$y_r = \frac{\alpha r_1 \Delta T}{1 + \frac{l\beta}{2h}\left(\frac{a}{r_1}\right)^2 (r_2 - r_1)} = \frac{6\times 10^{-6} \times 25.6875\ \Delta T}{1 + \left\{\frac{1.5 \times .1508}{2 \times 2\frac{5}{8}}\left(\frac{24.3125}{25.6875}\right)^2\left(31\frac{3}{4} - 25\frac{11}{16}\right)\right\}}$$

$$\frac{y_c}{y_r} = .0459 \times .8957 \times 6.0625$$

$$= .2492$$

(P. 2)

$$y_r = \frac{1.541 \times 10^{-4}\ \Delta T}{1 + (.2492)}$$

$$y_r = \underline{\underline{.00012337\ \Delta T}}\ ✓$$

Ratio $\frac{y_c}{y_r} = \underline{\underline{.2492}} \simeq \frac{1}{4}$ ∴ Ring takes most of deflection.

ALCO PRODUCTS INC.

BY C.A. DATE 11/11/60 SUBJECT Support Ring SM-1A. SHEET NO. 7 OF 7

CHKD. BY ______ DATE ________ JOB NO. ________

From P. 4 $y_o = y_r = \frac{.006\ Pa^3}{EI}$

$$.0001234\ \Delta T = \frac{.006\ Pa^3}{EI}$$

$$P = \frac{.0001234\ \Delta T\ EI}{.006\ a^3} = \frac{\overset{59{,}723}{.0001234 \times 2.7\times10^6 \times 17.925}\ \Delta T}{\underset{86{,}226}{.006 \times 24.3125^3}}$$

$$P = \underline{692.63\ \Delta T} \checkmark$$

From P.5 - combined stress in vessel wall

$$\sigma_\phi = .2101\ P = .2101 \times 692.63\ \Delta T$$

$$\sigma_\phi = \underline{145.52\ \Delta T} \checkmark$$

The same ΔT between the vessel wall and the support ring will be used as was used for SM-1 for a 50° F/Hr Temp Transient. This will be a close approximation since primary system temp & configuration are almost the same.

$$\sigma_\phi = 145.52\ (13.7)^{*} = 1990\ psi$$

LISTED
UNDER PT. 7
ON STRESS
CHART AS
THERMAL
TRANSIENT
STRESS

* AP NOTE 290 - RADIATION DAMAGE STUDY FOR SM-1 VESSEL

BY G.A./MR DATE 11/10/60 SUBJECT SM-1A STRESS ANALYSIS AE-90 TASK 6.9 SHEET NO. 1 OF 4
CHKD. BY DWM DATE 12/12/60 JOB NO. CONTR. NO. AT(30-1)-2639

STRESSES IN REACTOR SHELL DUE TO PIPING LOADS ON NOZZLES

REF: NAVY CODE - STRESSES FROM RADIAL LOADS & EXTERNAL MOMENTS ACTING ON SPHERICAL & CYLINDRICAL SHELLS. (SECT A.5)

1. $\beta = \frac{0.875\, r_o}{a}$ REF. A.5.3 NAVY CODE $\qquad \gamma = \frac{a}{t}$ REF A.5.4 NAVY CODE

r_o = outside radius outlet nozzle
a = radius middle surface of shell
t = thick. of shell = $2\,5/8''$

$r_o = \frac{19}{2} = 9.5''$ $\qquad a = \frac{51\frac{1}{4} + 46}{4} = 24.3125$

$\beta = \frac{0.875 \times 9.5}{24.3125}$ $\qquad \delta = \frac{24\frac{5}{16}}{2\frac{5}{8}} = \underline{\underline{9.26''}}$

$\beta = \underline{\underline{.3419}}$

FROM NAVY CODE, the stress is obtained from

$$\sigma_i = K_N \frac{N_i}{t} \pm K_M\, 6 \frac{M_i}{t^2} \qquad (1)$$

where:

σ_i = stress acting in the i direction
N_i = membrane force per unit length acting in i direction
M_i = bending moment " " " " " " "
t = thickness of shell
K_N = stress concentration factor on the membrane stress (obtained from FIG A.5.6 NAVY Code)
K_M = stress concentration factor on the bending stress (obtained from FIG A.5.6 NAVY Code)

BY ______ DATE ______

CHKD. BY ______ DATE ______

SUBJECT SM-1A STRESS ANALYSIS AF-912 TASK 6.9

SHEET NO. 2 OF 4

JOB NO. ______

The Longitudinal external piping moment is 45200 FT-lbs *. THE CIRCUMFERENTIAL MOMENT IS ZERO.

FROM NAVY CODE CURVES, for LONGITUDINAL EXTERNAL MOMENT, the MOMENTS are

$$\frac{M_x}{M/aB} = .0452 \quad (\text{FIG } A.5.6) \quad (\text{longitudinal}) \; (a)$$

$$\frac{M_\phi}{M/a^2B} = .034 \quad (\text{FIG } A.5.7) \quad (\text{circumferential}) (b)$$

The membrane force is

$$\frac{N_x}{M/a^2B} = .38 \quad (\text{FIG } A.5.8) \quad (\text{longitudinal}) \quad (c)$$

$$\frac{N_\phi}{M/a^2B} = 1.00 \quad (\text{FIG } A.5.9) \quad (\text{circumferential}) (d)$$

Per NAVY Code Multiply (a) + (b) above by $\frac{6M}{aBt^2}$

$$\frac{M_x}{M/aB} \times \frac{6M}{aBt^2} = \frac{6M_x}{t^2} = .0452 \times \frac{6M}{(24.3125)(.3419)2\frac{5}{8}} = \underline{\underline{.00474M}}$$

$$\frac{M_\phi}{M/a^2B} \times \frac{6M}{aBt^2} = \frac{6M_\phi}{t^2} = .034 \times .1048M = \underline{\underline{.00356M}}$$

* From piping calcs. by A. Pidwerbetsky 6/1/59

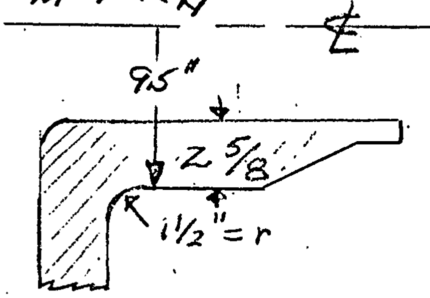
Also per: NAVY Code, Multiply (c) & (d) by $\frac{M}{ta^2\beta}$

$$\frac{N_x}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N_x}{t} = .38 \times \frac{M}{2\frac{5}{8} \times 24.3125^2 \times .3419} = \underline{\underline{.000715M}}$$

$$\frac{N_\phi}{M/a^2\beta} \times \frac{M}{ta^2\beta} = \frac{N_\phi}{t} = 1.00 \times .00188M = \underline{\underline{.00188M}}$$

STRESS CONCENTRATION FACTOR — K_M & K_N

(REF SECT A.7 NAVY CODE)



r = fillet radius

$T = .5D = r_0 = 9.5''$

$$\frac{r}{T} = \frac{1.5}{9.5} = .1578$$

FROM FIG A.7.3.2 - scale B

$K_{TENSION} = \underline{\underline{2.70}}$; $K_{BENDING} = \underline{\underline{2.27}}$

SUBSTITUTING ABOVE VALUES IN EQUATION (1),

The circumferential Stress due to Longitudinal MOMENT

$$\sigma_\phi = K_N \frac{N_\phi}{t} \pm K_M \frac{6M_\phi}{t^2}$$

$$= 2.70 \times .00188M \pm 2.27 \times .00356M$$

$$= .005076M \pm .008081M$$

where M = 45200 ft. lbs.

$$\sigma_\phi = \begin{cases} .01315(45200)(12) = \underline{7110\ PSI} \\ -.00305(45200)(12) = \underline{\underline{-1360\ PSI}} \end{cases}$$

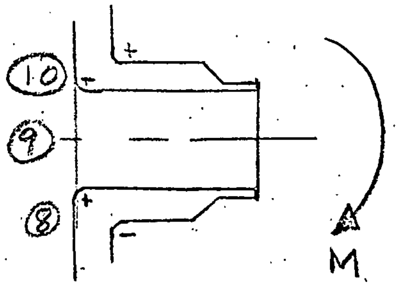
Again substituting in (1), the longitudinal stress due to a longitudinal moment –

$$\sigma_x = K_N \frac{N_x}{t} \pm K_m 6 \frac{M_x}{t^2}$$

$$\sigma_x = 2.70(.000715M) \pm 1.0(.00474M)$$

$$\sigma_x = .00193M \pm .00474M$$ where $M = 45200''\#$

$$\sigma_x = \begin{cases} +.01269M = +573.6\frac{\#}{ft^2} = \underline{\underline{6870 psi}} \\ -.00883M = -399.1\frac{\#}{ft^2} = -\underline{\underline{4780 psi}} \end{cases}$$



AT PT ⑨ $\sigma = 0$ Since Moment = 0

Using same subscripts as on stress summary charts

$$\sigma_x = \sigma_1$$
$$\sigma_\phi = \sigma_2$$

Then at ⑩ $\sigma_1 = \begin{cases} 6870 \text{ psi outside surface} \\ -4780 \text{ psi inside surface} \end{cases}$

$\sigma_2 = \begin{cases} 7110 \text{ psi outside surface} \\ -1360 \text{ psi inside surface} \end{cases}$

at ⑧ $\sigma_1 = \begin{cases} -6870 \text{ psi outside} \\ 4780 \text{ psi inside} \end{cases}$

$\sigma_2 = \begin{cases} -7110 \text{ psi outside} \\ 1360 \text{ psi inside} \end{cases}$

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BY B/R DATE

CHKD. BY DATE 12/15/60

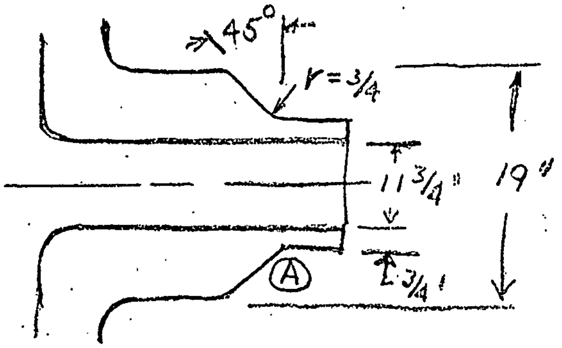
SUBJECT SM-1A STRESS ANALYSIS

AE 90 TASK 6.9

SHEET NO. 1 138 OF 2

JOB NO. CONTR. NO. AT(30-1)-2639

STRESS IN VESSEL NOZZLES DUE TO PIPE REACTIONS AT Ⓐ



1. STRESS CONCENTRATION FACTORS - REF. NAVY CODE FIG A.7-1 & A.7-2

From Fig A.7-1

$\frac{r}{t} = \frac{.75}{6.625} = .113$

K_o (tension) = 3.1

K_o (bending) = 2.67

From Fig. A.7-2

$\frac{r}{R} = \frac{.75}{3.875} = .193 \quad \& \quad \frac{\beta}{90} = \frac{45}{90} = \frac{1}{2}$

From curve $\frac{K'-1}{K_o-1} = .74$ (a)

Substiting values for K_o in (a)

for Bending $K' = .74(2.67-1)+1 = 2.24$

for Tension $K' = .74(3.1-1)+1 = 2.55$

ALCO PRODUCTS INC.

BY B/R DATE ______ SUBJECT SM-1A STRESS ANALYSIS SHEET NO. 2 OF 2

CHKD. BY ______ DATE ______ JOB NO. ______

For the axial stress –

$$\sigma_a = \frac{P}{a} \pm \frac{Mc}{I}$$

where axial Force = 0

Bending Moment } = 45200 ft.#

$$\therefore \sigma_a = \pm \frac{45200\,(6.25) \times 12}{\frac{\pi}{4}\left(6.625^4 - 5.875^4\right)} \times K'$$

$$= 2.24 \times \frac{45200\,(6.25)(4)}{\pi\,(750)} \times 12$$

$$\underline{\underline{\sigma_a = \pm 12{,}900\,psi}}$$

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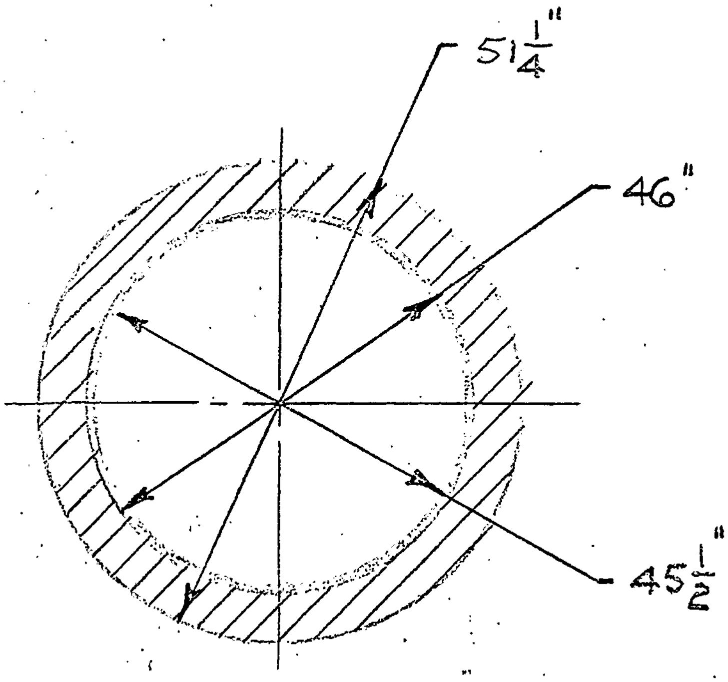
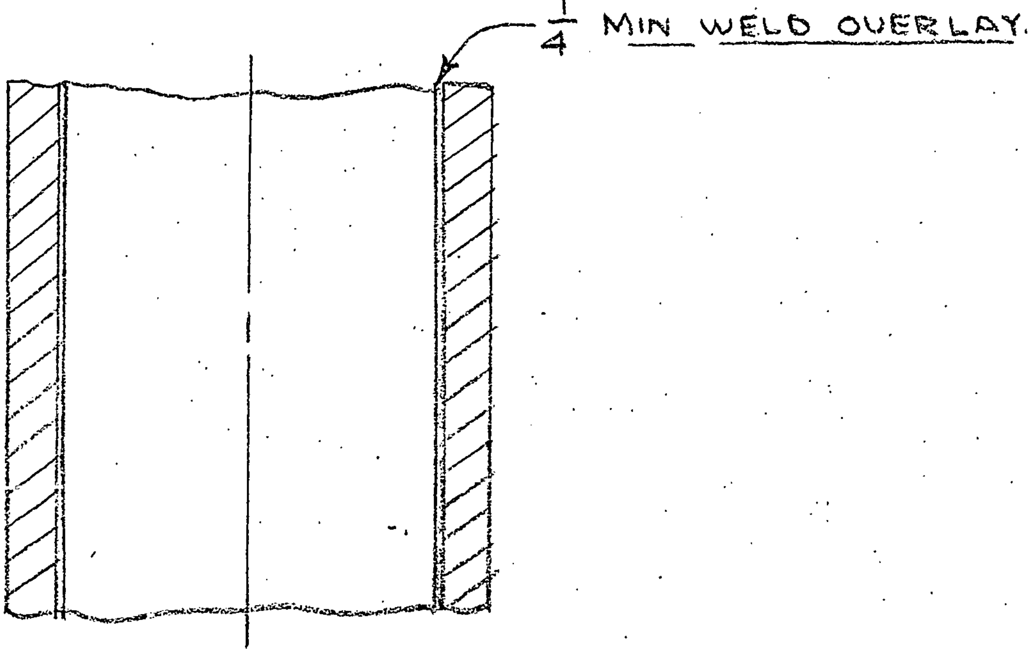
BY DMV DATE [illegible] SUBJECT CLADDING STRESS SHEET NO. 1 OF 18
CHKD. BY BJR DATE [illegible] SM-1A REACTOR VESSEL JOB NO. CONTR NO
(DWG F-41201-1-2) AT(30-1)-2639
AE 90 TASK 6.9



1. STRESS AFTER WELDING
2. STRESS AFTER HEAT-TREAT (STRESS-RELIEVE)
3. STRESS AFTER HYDRO. TEST.
4. NORMAL OPERATING STRESS.



ALCO PRODUCTS INC.

BY DMV DATE 1/18/61 SUBJECT CLADDING STRESS SHEET NO. 2 OF 18
CHKD. BY BJR DATE 9/15/61 SM-1A REACTOR VESSEL JOB NO. CONTR. NO.
AE-90 TASK 6.9 AT(30-1)-2659

SM-1A DETAIL INFORMATION

PREHEAT TEMPERATURE (WELDING) —— 250°F

STRESS RELIEVE TEMPERATURE —— 1150°F

LENGTH STRESS RELIEVE TEMP. HELD —— 7 HRS (APPR)

HYDRO TEST PRESSURE —— 2400 psi

OPERATING TEMPERATURE —— 434.3°F (AVE)

VESSEL MATERIAL —— SA-350 GR# LF-1

CLAD MATERIAL —— 304 S.S.

I.

a. STRESS IN CLAD DUE TO WELDING

REF. " STRESS IN CLAD STEELS IN HIGH TEMP. CORROSIVE ENVIRONMENTS" - W.H. FUNK, LUKENS STEEL CO.

VERIFICATION OF CLADDING STRESS FORMU$\overset{L}{\wedge}$AE - DMcL 11-10-60

$$S_c = + \frac{E_c}{L(1-\mu)} \frac{\delta_c - \delta_b}{1 + \frac{E_c t_c}{E_b t_b}}$$

$$\delta_c = \alpha_c . \Delta T_c L$$

$$\delta_b = \alpha_b \Delta T_b L$$

① $$\therefore \; S_c = + \frac{(\alpha_c \Delta T_c - \alpha_b \Delta T_b) E_c}{(1-\mu)\left[1 + \frac{E_c t_c}{E_b t_b}\right]}$$

IF IT IS ASSUMED $\Delta T_c = \Delta T_b$ (THIS IS NOT THE TRUE CASE BUT IT WILL GIVE A CONSER$\overset{VA}{\wedge}$TIVE STRESS)

② THEN $$S_c = + \frac{(\alpha_c - \alpha_b)\Delta T_c E_c}{(1-\mu)\left[1 + \frac{E_c t_c}{E_b t_b}\right]}$$

IF IT IS ASSUMED $E_c = E_b$ (THIS AGAIN IS NOT A TRUE CASE & WILL GIVE A SLIGHT INCREASE IN STRESS BY THE RATIO OF E_c/E_b IN THE DEMONINATOR

③ THEN $$S_c = + \frac{(\alpha_c - \alpha_b)\Delta T_c E_c}{(1-\mu)\left[1 + \frac{t_c}{t_b}\right]}$$

FROM THE TEMP. PATTERN THRU THE CLAD & BASE METAL WHICH WAS APPROXIMATED (SEE FIGURE)

$\alpha_c = 10.3 \times 10^{-6}$ IN/IN-°F @ 1370°F *

$\alpha_b = 6.6 \times 10^{-6}$ IN/IN-°F @ 337°F

$E_c = 21 \times 10^6$ @ 1370°F *

$t_c = .25''$

$t_b = 2.625$

USING EQ. ③

$$S_c = + \frac{\overbrace{(10.3 \times 10^{-6} - 6.6 \times 10^{-6})\, 21 \times 10^{6}}^{77.6}\ \Delta T_c}{\underbrace{.7 \left[1 + \frac{.25}{2.625}\right]}_{.767}}$$

$$S_c = + 101\ \Delta T_c$$

THE YIELD STRENGTH OF THE CLAD MATERIAL AT ROOM (80°F) TEMP. IS 30,000 psi (R.HC. - CURVE 11-29-60)

SUB. S_{YIELD} FOR S_c.

$$\Delta T_{c\,MAX} = \frac{30,000}{101}$$

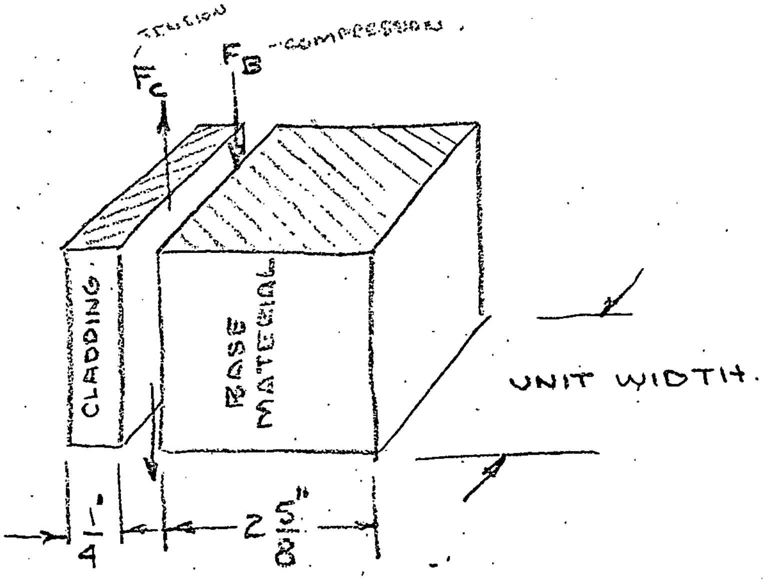
$$= \underline{\underline{297°F}}$$

* THE TEMP. AT WHICH THE VALUES OF α & E ARE DETERMINED, WILL HAVE LITTLE EFFECT ON THE ΔT TO CAUSE YEILD STRESSES IN THE CLAD.

IT IS KNOWN THAT DURING WELDING THE CLADDING MATERIAL WILL SEE A ΔT MUCH IN EXCESS OF 300°F THERE FORE IT CAN BE ASSUMED THAT THE CLADDING STRESS AFTER WELDING (COLD - ROOM TEMP.) WILL BE THE YIELD STRENGTH OF THE CLADDING MATERIAL. (@ 80°F)

+ 30,000 psi (TENSION)

b. STRESS IN BASE MATERIAL DUE TO WELDING



$F_c = -F_b$

ASSUME UNIFORMLY DISTRIBUTED LOAD, ACROSS CLADDING & BASE MATERIAL.

$F_c = S_c A_c = -S_b A_b = -F_b$

FOR A UNIT WIDTH $A_c = t_c$ & $A_b = t_b$.

$S_c t_c = -S_b t_b$.

$$S_b = - S_c \frac{t_c}{t_b}$$

$$= -30{,}000 \frac{.25}{2.625}$$

$$= -2{,}860 \text{ psi} \quad (\text{COMPRESSION.})$$

II. STRESS INDUCED DURING STRESS RELIEVE

a. CLADDING STRESS

S.R. TEMP – 1150°F

YIELD STRENGTH @ 1150 = 12,500 psi

① $\Delta T = T_{INITIAL} - T_{FINAL}$

∴ HEATING TO 1150°F WILL CAUSE A COMP. STRESS IN CLADDING MATERIAL

② FROM (I) ∴ THE ΔT GOING TO S.R. TEMP. WILL CAUSE A THEORETICAL STRESS IN CLAD. OF APPROX. $(30{,}000 \times {}^{1150}/_{300}) = 115{,}000$ psi

③ THE CLADDING WILL BE STRESSED FROM +30,000 TO –12,500 A RANGE OF 42,500 psi

④ ∴ THE CLADDING WILL GO FROM THE COLD TENSILE YIELD STRESS TO THE HOT (1150) COMPRESSIVE YIELD STRESS

⑤ ON COOLING FROM 1150 TO ROOM TEMP (80) THE REVERSE OF THE ABOVE WILL TAKE PLACE & THE CLADDING WILL RETURN TO THE COLD TENSILE YIELD STRESS OF (30,000 psi).

RESULTANT CLADDING STRESS AFTER S.R.

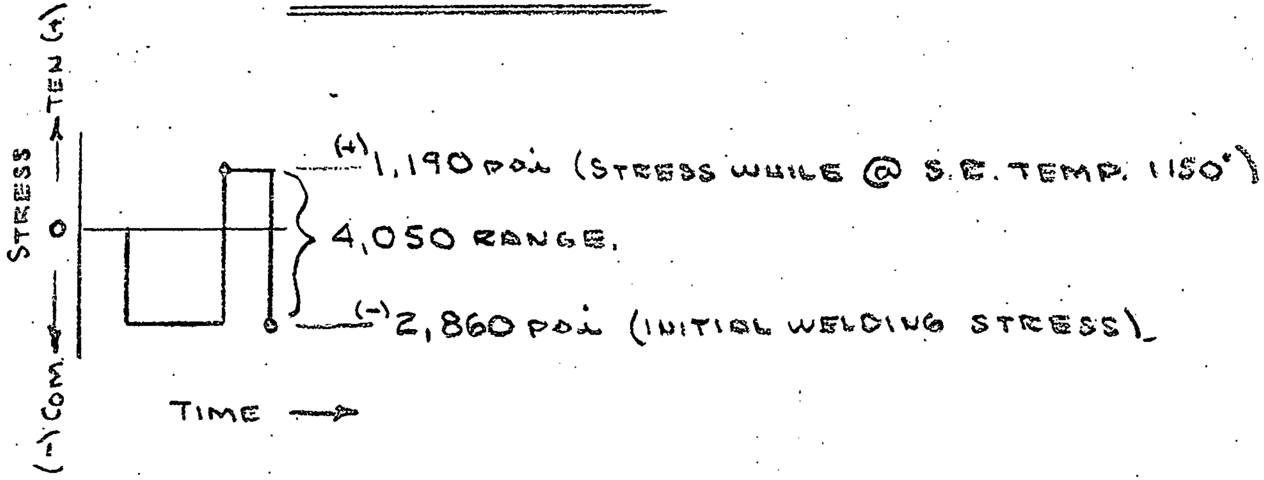
+ 30,000 psi

b. BASE MATERIAL STRESS

$$S_b = -S_c \frac{t_c}{t_b}$$

$$S_b = -(-42,500)\frac{.25}{2,625}$$

$$= +4,050 \text{ psi}$$



SINCE THE STRESS IN THE BASE MATERIAL AT S.R. TEMP (1150) IS SO LOW (1,190 psi) NO RELAXATION WILL TAKE PLACE & BASE MATERIAL WILL RETURN

BY DATE SUBJECT SHEET NO. 8 OF 18

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CHKD. BY DATE JOB NO.

TO VALUE PRIOR TO S.R. - WHICH IS -2,860 psi COMPRESSION.

-2,860 psi (COMPRESSION)

BY______ DATE______ SUBJECT______ SHEET NO. 9 OF 18 148

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III. EFFECT OF HYDROSTATIC TEST PRESSURE

TEST PRESSURE - 2400 psi -

HYDRO STATIC TEST IS DONE WITH COLD WATER (60-100°)

RATIO OF VESSEL WALL THICKNESS TO INSIDE RADIUS.

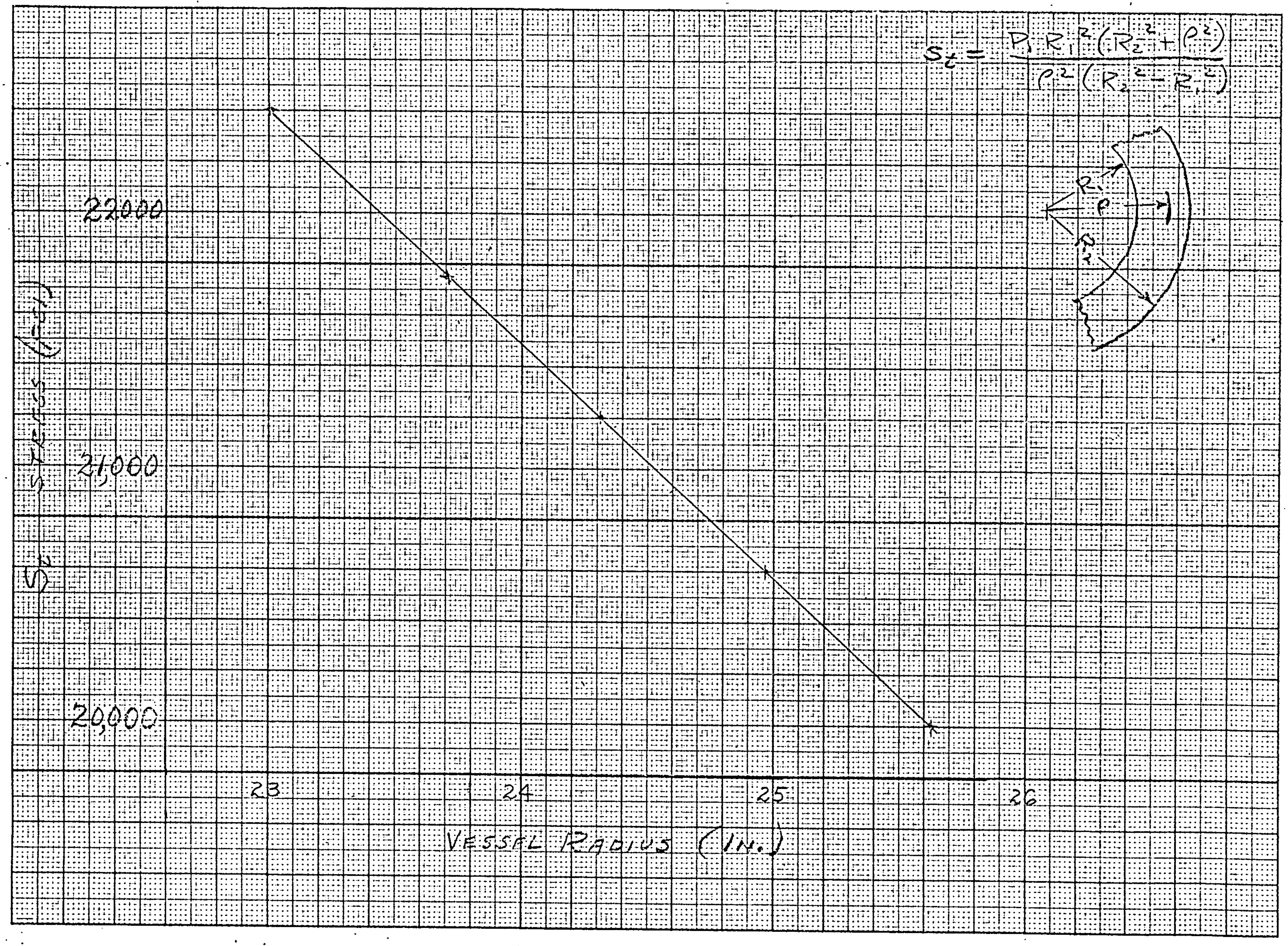
$$\frac{2.625}{23} = \frac{x}{10}$$

$$= 1.14/10$$

THE VESSEL APROACHES BEING A THIN CYLINDER. THE FOLLOWING CURVE IS PLOT OF WALL STRESS VS. DISTACE FROM THE VESSEL ℄.

IT CAN BE SEEN THAT THE AVERAGE STRESS IS 21,200 AND THE MAXIMUM STRESS IS 5½% ABOVE THE AVERAGE.

THE PRESSURE STRESS WILL CAUSE TENSILE LOADS IN THE VESSEL WALL & SINCE THE CLADDING IS ALREADY AT ITS TENSILE YIELD STRESS IT IS ASSUMED THAT THE CLADDING DOES NOT CARRY ANY OF THE HYDRO STATIC TEST PRESSURE LOAD.



$$S_t = \frac{P_1 R_1^2 (R_2^2 + \rho^2)}{\rho^2 (R_2^2 - R_1^2)}$$

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THE EFFECT OF PROOF TEST VESSEL STRESS ON CLADDING STRESS.

1. UNIT STRAIN-VESSEL WALL

$$\delta = \frac{S}{E}$$

$$= \frac{21.2 \times 10^3}{29 \times 10^6}$$

$$= \underline{\underline{.73 \times 10^{-3}}}$$

2. SINCE THE CLADDING & VESSEL WALL ARE ONE PIECE THE CLADDING UNIT STRAIN WILL ALSO BE.

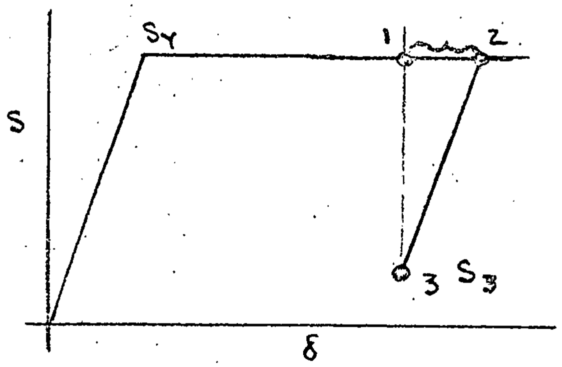
$$\underline{\underline{.73 \times 10^{-3}}}$$

3. UNIT STRAIN OF CLADDING, AT YIELD POINT (COLD)

$$\delta = \frac{E}{S}$$

$$= \frac{30 \times 10^3}{29 \times 10^6}$$

$$= \underline{\underline{1.035 \times 10^{-3}}}$$



1. CLADDING PRIOR TO PROOF TEST.
2. DURING APPLICATION OF PRESS.
3. AFTER PROOF TEST.

$$+\frac{S_Y}{\delta_Y} = \frac{S_3}{\delta_Y - \delta_{1-2}}$$

$$+E = \frac{S_3}{\delta_Y - \delta_{1-2}}$$

$$+29\times10^{6} = \frac{S_3}{(1.035-.73)\times10^{-3}}$$

$$\begin{array}{r} -30{,}000 \\ +\ 8850 \\ \hline (-)\,21{,}150 \end{array}$$

$$S_3 = 29\times10^{6}\times .305\times10^{-3}$$

$= \underline{+8{,}850\ \text{psi}}$ (TENSION) RESULTANT STRESS IN CLADDING AFTER PROOF TEST.

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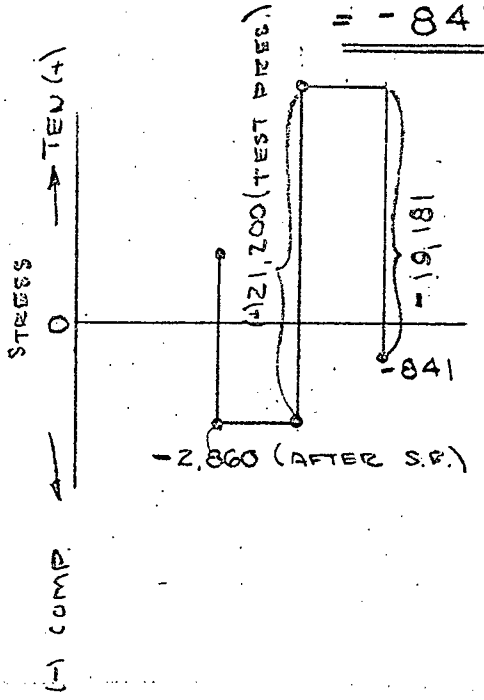
THE EFFECT OF REDUCTION OF CLADDING STRESS ON VESSEL WALL STRESS.

$$S_b = -S_c \frac{t_c}{t_b}$$

$$= -8{,}850 \frac{.25}{2.625}$$

$$\underline{\underline{= -841}} \quad \text{(COMPRESSION)}$$



$$\begin{array}{r} 2{,}860 \\ -\ 841 \\ \hline 2019 \end{array}$$

$$\begin{array}{r} 21{,}200 \\ -\ 2019 \\ \hline (-)19{,}181 \end{array}$$

IV

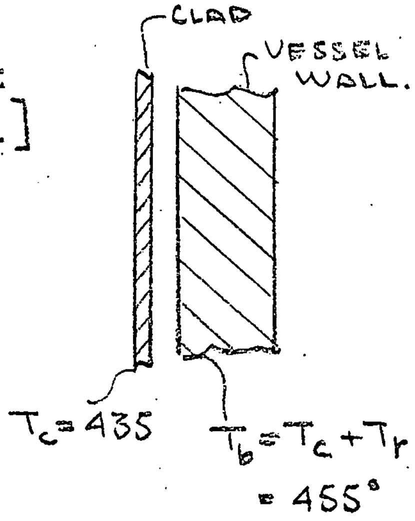
NORMAL OPERATING STRESS

a. CLADDING STRESS CAUSED BY GOING TO OPERATING TEMPERATURES.

AVERAGE COOLANT TEMP. 434.3 (435°F)

TEMP INCREASE IN VESSEL WALL-γ HEATING -
ASSUME $T_r = 20°F$

1. HEATING TO OPERATING TEMP. WILL CAUSE COMP. STRESS IN CLAD. $\left[\Delta T = T_{INITIAL} - T_{FINAL}\right] = -\Delta T$



$$S_c = -\frac{(\alpha_c \Delta T_c - \alpha_b \Delta T_b) E_c}{(1-\mu)\left[1 + \frac{E_c t_c}{E_b t_b}\right]}$$

SUB. $\Delta T_b = \Delta T_c + T_r$

$$S_c = -\frac{\left[\alpha_c \Delta T_c - \alpha_b(\Delta T_c + T_r)\right] E_c}{(1-\mu)\left[1 + \frac{E_c t_c}{E_b t_b}\right]}$$

$\Delta T_c = 355$
$T_r = 20°$
$E_c = 26.5 \times 10^6$ @ 435°
$E_b = 28.5 \times 10^6$ @ 455
$\alpha_c = 9.4 \times 10^{-6}$ @ 435
$\alpha_b = 6.85 \times 10^{-6}$ @ 455
$t_c = .25''$
$t_b = 2.625$

{SOLVE EQ. IN TERMS OF T_r SO THAT CORRECT VALUE MAY SUBSTITUTED.

$S_{YIELD} = 19,500$ @ 435

$$S_c = -\frac{\left[9.4\times10^{-6}\times355 - 6.85\times10^{-6}\left(355+T_r\right)\right]26.5\times10^{-6}}{.7\left[1+\frac{26.5\times.25}{28.5\times2.625}\right]}$$

$$= -\frac{\left[3340 - 2430 - 6.85\,T_r\right]26.5}{.7\left[1.085\right]}$$

$$= -\frac{23200 - 181\,T_r}{.76}$$

$$= -238\left(128 - T_r\right)$$

ASSUMING $T_r = 20°F$

$$\therefore\ S_c = -238(108)$$

$$= \underline{-25{,}700\ \text{psi}}\ \text{(COMPRESSION)}$$

THE CLADDING HAS A STRESS OF + 8,850 psi (TENSION) PRIOR TO GOING TO OPERATING TEMP.

$$\begin{array}{rl} -25{,}700 & \text{COMP} \\ +\ 8{,}850 & \text{TENSION} \\ \hline -16{,}850 & \text{COMP.} \end{array}$$

$\underline{-16{,}850\ \text{psi}}$ STRESS IN CLADDING AFTER GOING TO OPERATING TEMP.

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b. VESSEL WALL STRESS CAUSED BY GOING TO OPERATING TEMP.

$$S_b = -S_c \frac{t_c}{t_b}$$

$$= -(-25,700) \frac{.25}{2.625}$$

$$= + 2,440 \text{ psi} \quad (\text{TENSION})$$

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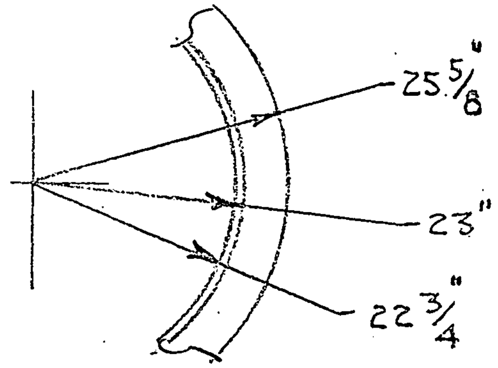
VESSEL WALL & CLADDING STRESS CAUSED BY GOING TO OPERATING PRESSURE (1200 psi)



$$S_{t_{INSIDE}} = \frac{P R_1^2 (R_2^2 + \rho^2)}{\rho^2 (R_2^2 - R_1^2)}$$

$\rho = R_1$

$$= \frac{P (R_2^2 + R_1^2)}{R_2^2 - R_1^2}$$

$$= \frac{1200 \times 1188}{143}$$

$$= \underline{\underline{9{,}860 \text{ psi}}}$$

$R_1 = 22.75$; $R_1^2 = 516$

$R_2 = 25.625$ $R_2^2 = 659$

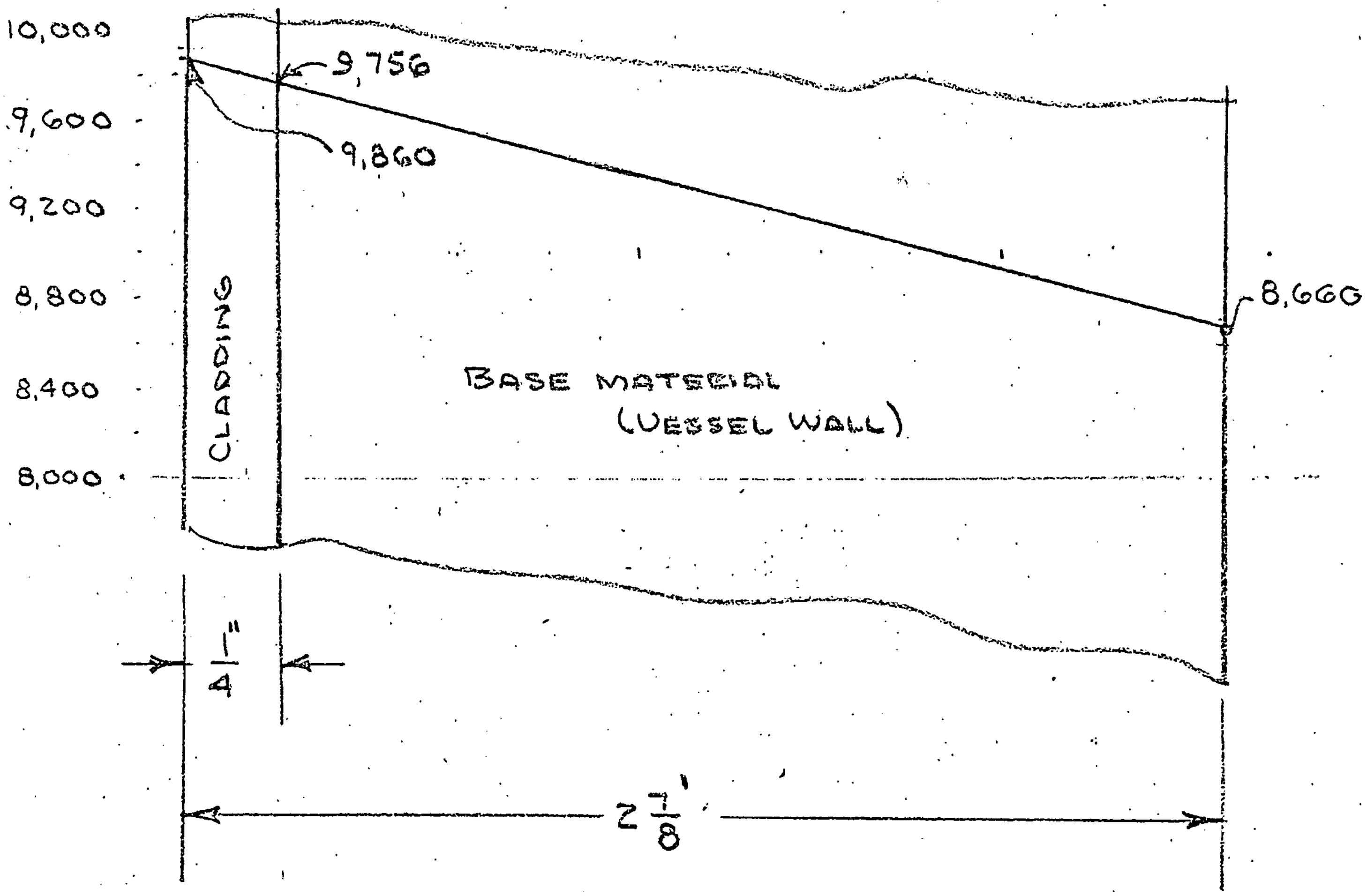
$P = 1200$

$$S_{t_{OUTSIDE}} = \frac{P R_1^2 (R_2^2 + R_2^2)}{R_2^2 (R_2^2 - R_1^2)}$$

$$= \frac{2 P R_1^2}{(R_2^2 - R_1^2)}$$

$$= \frac{2 \times 1200 \times 516}{143}$$

$$= \underline{\underline{8{,}660 \text{ psi}}}$$



9,860
8,660
1200

9860
104
9,756

$\frac{1200}{2.875} \times \frac{1}{4} = 104.2$

AVG. CLADDING STRESS = $\frac{9,860 + 9,756}{2}$ = +9808 psi

AVG. VESSEL WALL STRESS = $\frac{9,756 + 8660}{2}$ = +9208 psi

SUMATION OF STRESSES

CLADDING STRESS

$\Sigma S_c =$ +30,000 (WELDING)
~~-42,500~~ (STRESS RELIEVE)
~~+42,500~~
-21,150 (PROOF TEST)
-25,700 (OPERATING TEMP)
+ 9,808 (OPERATING PRESS.)

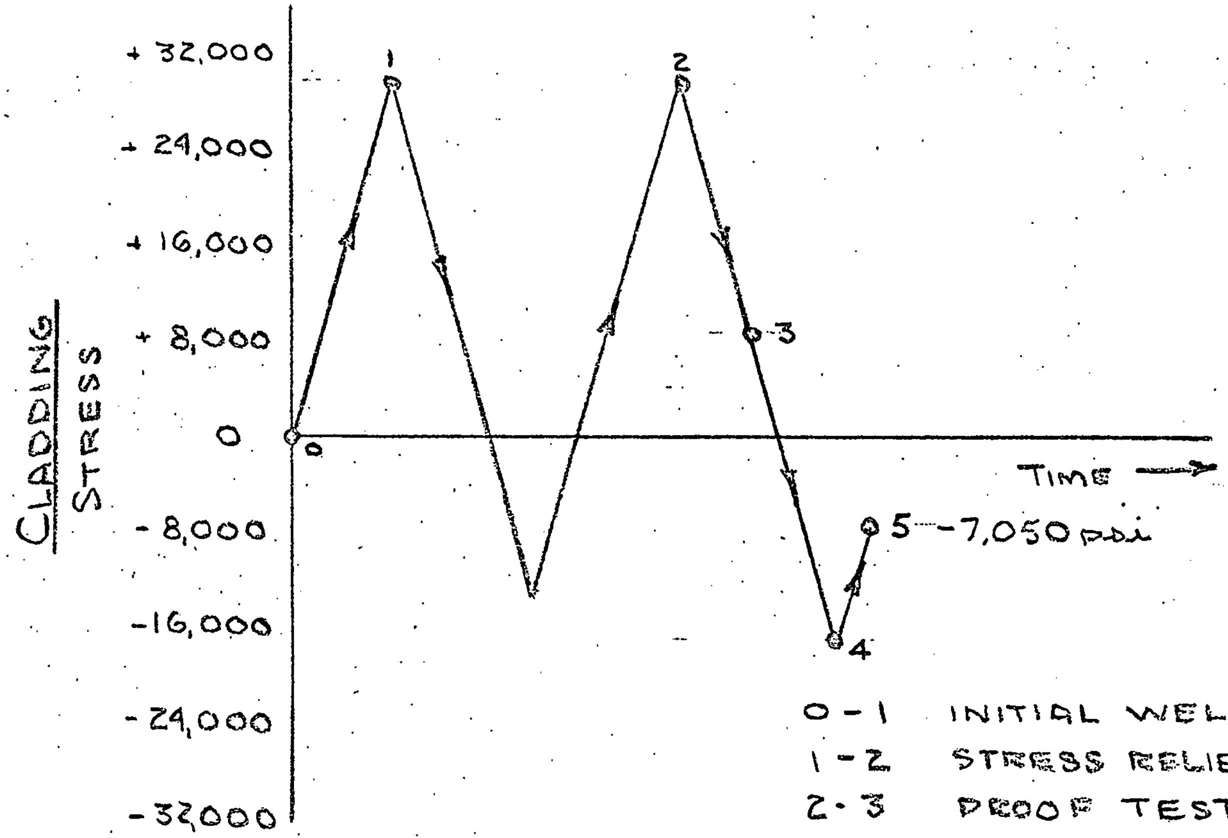
-7,050 psi (COMPRESSION)

BASE MATERIAL STRESS

$\Sigma S_b =$ -2860 (WELDING)
~~+4050~~ (STRESS RELIEVE)
~~-4050~~
+21,200 (PROOF TEST)
-19,181
+2,440 (OPERATING TEMP)
+9208 (OPERATING PRESS.)

+10,807 psi (TENSION)

CHANGE IN STRESS DUE TO CLADDING = **+1600 PSI.**



0 - 1 INITIAL WELDING
1 - 2 STRESS RELIEVE CYCLE
2 - 3 PROOF TEST
3 - 4 OPERATING TEMP
4 - 5 OPERATING PRESS

POINT 5 - STRESS AT OPERATING CONDITIONS.