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WASTE FORM DISSOLUTION IN BEDDED SALT

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ABSTRACT

Waste Form Dissolution in Bedded Salt

We devise a model for waste dissolution in bedded salt - a hydrologically tight medium. For a typical Spent UnReprocessed Fuel (SURF) emplacement, the dissolution rate will be diffusion limited and will rise to a steady state value after $t_{eq} \cong 250 (1 + (1 - \epsilon_0) K_D / \epsilon_0)$ (yrs) where ϵ_0 is the overpack porosity and K_D is the overpack sorption coefficient. The steady state dissolution rate itself is dominated by the solubility of UO_2 . Steady state rates between 5×10^{-5} and .5 (gm/yr) are achievable by SURF emplacements in bedded salt without overpack. And rates between 5×10^{-7} and 5×10^{-3} (gm/yr) with an overpack having porosity of 10^{-2} .

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WASTE FORM DISSOLUTION

in

BEDDED SALT

Introduction

The prediction of waste form dissolution rates depend on the chemistry of waste/water interaction, but also on molecular diffusion and hydrologic transport. In what follows, we shall derive a simple model for waste dissolution which depends upon the waste solubility, the molecular diffusion and certain hydrological parameters.

Although we have emphasized bedded salt as a medium, the model itself will be useable in other emplacement media as well. The model is applicable after the groundwater has entered the repository and the niche in which the canister is emplaced. Figure 1 shows the geometry of a waste canister with radius a_0 and length L_0 . The canister sits in a metal sleeve over which is fitted a concrete plug. The plug has been fitted to allow retrieval.

We model the escape of waste from the canister as transport through a set of consecutive one-dimensional transport paths. The first path of length L_1 corresponds to the region between the top of the canister and the bottom of the plug. The second corresponds to the concrete plug. The plug does not fit the sleeve well enough to insure complete closure, since the sleeve is expected to corrode rapidly anyway. A pathway of width w is assumed to be opened by this corrosion.

We assume that the waste is at saturation in the groundwater at the top of the canister. Further, at the flow of the repository, we assume that the flow is rapid enough that the concentration there is reduced to values much less than saturation.

WASTE FORM DISSOLUTION IN BEDDED SALT

Figure 1. Geometry of Waste Form Emplacement

We solve the advection diffusion equation for the circumstance just described and obtain an expression for the integrated flux of waste which would emerge from the canister emplacement hole. Two important regimes are identified: a diffusion dominated dissolution and an advection dominated one. The diffusion regime would be appropriate for bedded salt where flow from the host medium into the canister niche would be expected to be small.

In the diffusion regime, the solubility of the waste form the porosity of the overpack and the dimension of the emplacement niche are important determinants of the dissolution rate. The large uncertainty in waste form solubility produces a correspondingly large uncertainty in dissolution rate as well.

A transient analysis reveals that sorption of the overpack and overpack porosity can be used to control the approach to steady-state dissolution rates.

An estimate of the dissolution behavior of a SURF (Spent UnReprocessed Fuel) package is given as a numerical example of the modeling work to be described.

The Model

To construct the model, we assume that advective flow can occur in the canister emplacement hole through a crack system that may be connected to an aquifer. We assume that Q , the total flux of fluid within the emplacement hole is conserved and is independent of position.

Let z be the coordinate in the direction of the canister axis. We let $z = 0$ correspond to the top of the canister and $z = L$ correspond to the floor of the repository. The concentration of waste in groundwater $C(z)$ then satisfies the following set of coupled equations:

$$(1) \quad \theta \frac{\partial}{\partial t} C(z,t) + \frac{\partial}{\partial z} J(z,t) = 0$$

$$(2) \quad J(z,t) = [q(z) - D(z) \frac{\partial}{\partial z}] C(z,t)$$

where $J(z,t)$ is the local flux of waste in units of (m/y), $q(z)$ is the local D'Arcy flux, $D(z)$ is the diffusion coefficient and θ is the effective porosity.

Together with equation (1) and (2), which govern the dynamics of waste transport we assume the following boundary conditions. Near the top of the canister ($z=0$) we assume saturated conditions. That is, $C(0,t) = C_s$. At the floor of the repository, ($z=L$) we assume that the dissolved waste is carried away rapidly enough that $C(L,t) = 0$.

The quantities in equation (1) and (2) are related to more fundamental observables.

The effective porosity is defined by:

$$(3) \quad \theta \triangleq \epsilon_0 + g_0 (1-\epsilon_0) K_D$$

The local D'Arcy flux $q(z)$ is related to the total D'Arcy flux Q which is assumed over the entire the emplacement hole.

$$(4) \quad q(z) \triangleq \begin{cases} q_1 = Q/\pi a^2 & 0 \leq z \leq x_1 \\ q_2 = Q/[\pi w(2a-w)] & x_1 \leq z \leq L \end{cases}$$

Finally, the diffusion constant $D(z)$ is given by:

$$(5) \quad D(z) \triangleq \begin{cases} D_1 = \alpha q_1 + \alpha_m \epsilon_0 & 0 \leq z \leq \kappa_1 \\ D_2 = \alpha q_2 + \alpha_m \epsilon_0 & \kappa_1 \leq z \leq L \end{cases}$$

The parameters α and α_m together with others important to the dissolution problem are summarized below:

w	$\triangleq$	pathway width around concrete plug (m)
a	$\triangleq$	emplacement hole radius (m)
L	$\triangleq$	distance between top of canister and repository floor (m)
κ_1	$\triangleq$	distance between top of canister and bottom of plug (m)
κ_2	$\triangleq$	distance between bottom of plug and floor repository (m)
ϵ_0	$\triangleq$	overpack porosity (m^3/m^3)
g_0	$\triangleq$	overpack specific gravity (gm/m^3)
K_D	$\triangleq$	overpack sorption coefficient (gm/gm)
Q	$\triangleq$	Total D'Arcy flux through emplacement hole (m^3/yr)
α	$\triangleq$	longitudinal dispersivity in overpack (m)
α_m	$\triangleq$	molecular diffusion constant (m^2/yr)

Solution of the Model Equations - Steady State Analysis

In Appendix A, we have derived an expression for the steady-state dissolution rate with the boundary conditions stated in the preceding section. The expression, given by

$$(6) \quad R_D = \frac{\nu_o Q C_s}{\left[1 - \exp\left(-\frac{Q_1 x_1}{D_1} - \frac{Q_2 x_2}{D_2}\right) \right]}$$

is equation (A-17) of Appendix A. Equation (6) indicates a linear dependence on the waste form solubility C_s which depends very critically on the Eh and pH of the groundwater. If we look at the variability with flow rate we find that the dissolution rate behaves much like Fig. 2.

Fig. 2. Variation of dissolution rate R_D with flow rate Q .

We note that: $q_1 = Q/a_1$ and $q_2 = Q/a_2$ where a_1 and a_2 are the areas available for flow above the canister and past the plug respectively.

When $Q \ll \alpha_m \epsilon_0 \text{ Min } \left(\frac{a_1}{l_1}, \frac{a_2}{l_2} \right)$

we are in the diffusion limit and:

$$(7a) \quad R_D \sim \frac{\rho_0 C_s}{\frac{l_1}{a_1 \alpha_m \epsilon_0} + \frac{l_2}{a_2 \alpha_m \epsilon_0}}$$

When $Q \gg \frac{\epsilon_0 \alpha_m}{\alpha} \text{ Max } (a_1 a_2)$

we are in the advection limit and:

$$(7b) \quad R_D \sim \frac{\rho_0 C_s Q}{1 - \exp \left[- \left(\frac{l_1}{\alpha} + \frac{l_2}{\alpha} \right) \right]} \sim \rho_0 C_s Q$$

when $l_1 + l_2 \gg \alpha$.

The details of the hydrology and crack system around each canister dictates which of the two limiting equations (7a) or (7b) are applicable. We note, however, that since α is of the order of the grain size in the overpack the advection limit stated by equation (7b) will nearly always satisfy the condition $l_1 + l_2 \gg \alpha$ in which case the approximation $\rho_0 C_s Q$ may be used for the dissolution rate.

Since bedded salt is expected to be hydrologically tight ($Q=0$), the diffusion limit in our model will most likely be the appropriate approximation to apply.

We may estimate the dissolution rate in the diffusion limit using proposed design parameters for a SURF (Spent UnReprocessed Fuel) assembly. We use:

$$\begin{aligned}k_1 &= 1m \\q_1 &= .2m^2 \\k_2 &= 3m \\q_2 &= .06m^2\end{aligned}$$

A typical value for the molecular diffusivity is:

$$\alpha_m \sim .03 \text{ m}^2/\text{y}$$

Then:

$$(8) \quad R_D \sim 5 \times 10^2 \epsilon_0 C_s \text{ (gm/yr)}$$

When there is no overpack $\epsilon_0 = 1$, and for typical variability in C_s for UO_2 between 10^{-7} and 10^{-3} gm/gm we find:

$$5 \times 10^{-5} \text{ (gm/yr)} \leq R_D \leq .5 \text{ (gm/yr)}$$

With overpack in which $\epsilon_0 = .01$, $5 \times 10^{-7} \text{ (gm/yr)} < R_D < 5 \times 10^{-3} \text{ (gm/yr)}$.

Typical assemblies contain 2×10^5 gm UO_2 . Thus, the dissolution times may range between

$$4 \times 10^5 \leq t_D \leq 4 \times 10^9 \quad \text{with } \underline{\text{no}} \text{ overpack } \epsilon_0 = 1$$

$$4 \times 10^7 \leq t_D \leq 4 \times 10^{11} \quad \text{with overpack } \epsilon_0 = .01.$$

depending on UO_2 solubility.

Transient Analysis

In Appendix B, we have made estimates of the time required to reach a steady-state condition. We can illustrate the variables of importance by looking at case III of Appendix B (no concrete plug). In this case, an exact solution has been found. The solution is characterized by the time-like variable:

$$(9) \quad \tau = \frac{Dt}{L^2 \epsilon_0}$$

and the parameter

$$(10) \quad \gamma = \frac{QL}{2aD}$$

In bedded salt cases Q is very small and the transient response of the dissolution rate is most likely characterized by the curve labeled with $\gamma = 0$ in Fig. B-1 of the appendix. Steady-state has been reached for $\tau \approx 0.3$. The time required to reach steady-state is given, therefore, by the expression.

$$(11) \quad t_{eq} = 0.3 \frac{L^2 \epsilon_0}{a_m} = 0.3 \frac{L^2}{a_m} \left(1 + \frac{g_0}{\epsilon_0} (1 - \epsilon_0) K_D \right)$$

For our SURF assembly $L = 5m$, we find:

$$(12) \quad t_{eq} = 250 \left(1 + \frac{(1 - \epsilon_0)}{\epsilon_0} K_D \right)$$

By choosing an overpack material in which the ratio K_D/ϵ_0 is large, we may delay onset of steady-state dissolution rates. Typically, attainable values of K_D at 10^3 and ϵ_0 at 10^{-2} will delay the onset of steady-state dissolution by 2.5×10^7 . However, since ϵ_0 also enters into the steady-state rates we can reduce it by a factor of 10^2 as well.

CONCLUSIONS

We have derived a model for waste dissolution which is appropriate for waste emplacement in bedded salt--a hydrologically "tight" medium. For a typical SURF emplacement, the dissolution rate is diffusion limited and rises to a steady-state value in a time given by $t_{eq} \approx 250 \left(1 + \frac{(1-\epsilon_n)}{\epsilon_0} K_D \right)$ (yrs).

The steady-state dissolution rate is dominated by the solubility of UO_2 . Rates between 5×10^{-5} and .5 (gm/yr) are achievable by SURF emplacements in bedded salt without overpack. And rates between 5×10^{-7} and 5×10^{-3} (gm/yr) with an overpack with porosity of 10^{-2} .

APPENDIX A

SOLUTION OF THE MODEL EQUATIONS - STEADY STATE ANALYSIS

We transform both equations (1) and (2) and the boundary conditions under a Laplace transform in t and write the resulting differential equation in z as:

$$(A-1) \quad \frac{\partial}{\partial z} \bar{X}_k(z, s) = M_k \bar{X}_k(z, s) \quad \left\{ \begin{array}{ll} k = 1 & 0 \leq z \leq l_1 \\ k = 2 & l_1 \leq z \leq L \end{array} \right.$$

where $\bar{X}_k$ is the vector:

$$(A-2) \quad \bar{X}_k(z, s) \triangleq \begin{pmatrix} \bar{c}(z, s) \\ \bar{j}(z, s) \end{pmatrix} \triangleq \begin{pmatrix} \bar{c}(z, s) \\ \frac{1}{q_1} \bar{j}(z, s) \end{pmatrix}$$

and M_k the constant 2×2 matrix:

$$(A-3) \quad M_k = \begin{pmatrix} q_k/D_k - q_1/D_k \\ -qs/q_1 & 0 \end{pmatrix}$$

The formal solution to equation (A-1) is:

$$(A-4) \quad \left\{ \begin{array}{l} \bar{X}_1(z, s) = \exp[M_1 z] \bar{X}_1(0, s) \\ \bar{X}_2(z, s) = \exp[M_2(z - l_1)] \bar{X}_2(l_1, s) \end{array} \right.$$

Then, since the boundary conditions relate $\bar{X}_2(\ell_1, s)$ to $\bar{X}_1(\ell_1, s)$ via $\bar{X}_2(\ell_1, s) = B\bar{X}_1(\ell_1, s)$ where B is the diagonal matrix:

$$(A-5) \quad B = \begin{pmatrix} 1 & 0 \\ 0 & \frac{a^2}{w(2a-w)} \end{pmatrix}$$

we find:

$$(A-6) \quad \bar{X}_2(L, s) = \exp[M_2 \ell_2] B \exp[M_1 \ell_1] \bar{X}_1(0, s)$$

Now we have required $\bar{c}(L, s) = 0$ and $\bar{c}(0, s) = (1/s)c_s$ and we would like to obtain $\bar{J}(L, s)$. Equation (11) has the form:

$$\begin{pmatrix} 0 \\ \bar{J}(L, s) \end{pmatrix} = P(s) \begin{pmatrix} \frac{1}{s} c_s \\ \bar{J}(0, s) \end{pmatrix}$$

In terms of the components p_{ij} of the matrix P we find that:

$$(A-7) \quad \bar{J}(L, s) = -q_1 \frac{\det[P(s)]}{p_{12}(s)} \frac{c_s}{s}$$

$$\text{Now } \det[P(s)] = \exp \left[\text{Tr}[M_1 \ell_1] \right] \frac{a^2}{w(2a-w)} \exp \left[\text{Tr}[M_2 \ell_2] \right]$$

Then:

$$(A-8) \quad \bar{J}(L, s) = -\frac{a^2 q_1}{w(2a-w)} \frac{\exp \left[\frac{q_1 \ell_1}{D_1} + \frac{q_2 \ell_2}{D_2} \right]}{p_{12}(s)} \frac{c_s}{s}$$

The transient time dependence of $J(L, t)$ comes from the zeros of $p_{12}(s)$ while the steady state value of $J(L, t)$ is given by $\bar{J}(L, 0)$.

We have $\bar{J}(L, 0) = J_s(L)$ given by the expression:

$$(A-9) \quad J_s(L) = - \frac{a^2 q_1 c_s}{w(2a - w)} \frac{\exp \left[\frac{q_1 l_1}{D_1} + \frac{q_2 l_2}{D_2} \right]}{p_{12}(0)}$$

What remains is to compute $p_{12}(s)$. This is done by diagonalizing the matrices M_1 and M_2 . Let U_1 and U_2 be the diagonalizing transformations:

$$(A-10) \quad \Lambda_k^\pm = U_k^{-1} M_k U_k$$

Then:

$$(A-11) \quad P(s) = U_2 \exp[\Lambda_2(s) l_2] U_2^{-1} B U_1 \exp[\Lambda_1(s) l_1] U_1^{-1}$$

The eigenvalues of M_k are $\Lambda_k^\pm$; these are:

$$(A-12) \quad \Lambda_k^\pm = \frac{1}{2} \left[\lambda_k \pm u_k(s) \right] = \frac{1}{2} \left[\frac{q_k}{D_k} \pm \sqrt{\left(\frac{q_k}{D_k} \right)^2 + \frac{4\theta s}{D_k}} \right]$$

with

$$\lambda_k \triangleq \frac{q_k}{D_k}, \quad u_k \triangleq \sqrt{\frac{q_k^2}{D_k^2} + \frac{4\theta s}{D_k}}$$

The matrices U_k are given by:

$$(A-13) \quad U_k = \begin{bmatrix} 1 & 1 \\ \frac{-u_k - \lambda_k}{2\beta_k \lambda_k} & \frac{u_k + \lambda_k}{2\beta_k \lambda_k} \end{bmatrix}; \quad U_k^{-1} = \begin{bmatrix} \frac{u_k + \lambda_k}{2u_k} & \frac{-\beta_k \lambda_k}{u_k} \\ \frac{u_k - \lambda_k}{2u_k} & \frac{\beta_k \lambda_k}{u_k} \end{bmatrix}$$

where $\beta_k = \frac{q_1}{q_k} = \begin{cases} 1 & k = 1 \\ \frac{w(2a - w)}{a^2} & k = 2 \end{cases}$

A simple calculation gives:

(A-14)

$$p_{12}(s) = -2 \exp \left[\frac{\lambda_1 l_1 + \lambda_2 l_2}{2} \right] \left\{ \frac{\lambda_1}{u_1} \sinh \left(\frac{u_1 l_1}{2} \right) \cosh \left(\frac{u_2 l_2}{2} \right) + \frac{\lambda_2}{u_2} \sinh \left(\frac{u_2 l_2}{2} \right) \cosh \left(\frac{u_1 l_1}{2} \right) \right\}$$

where we have used the definition of β_k .

If we set $s = 0$, $u_k \rightarrow \lambda_k$ and equation (19) becomes:

$$(A-15) \quad p_{12}(0) = -2 \exp \left[\frac{\lambda_1 l_1 + \lambda_2 l_2}{2} \right] \sinh \left[\frac{\lambda_1 l_1 + \lambda_2 l_2}{2} \right]$$

Then from equation (14) and the definition of λ_k , we obtain the steady state material flux:

$$(A-16) \quad J_s(L) = \frac{a^2 q_1 c_s}{w(2a - w) \left[1 - \exp \left(-\frac{q_1 l_1}{D_1} - \frac{q_2 l_2}{D_2} \right) \right]}$$

and the dissolution rate is:

$$(A-17) \quad R_D = \frac{\rho_0 q c_s}{\left[1 - \exp \left(-\frac{q_1 l_1}{D_1} - \frac{q_2 l_2}{D_2} \right) \right]}$$

where ρ_0 is the density of water $\sim 10^6$ gm/m³.

APPENDIX B

TRANSIENT ANALYSIS

What remains is to do some analysis of the transients. That is, the time required for a steady state dissolution rate. There are two cases to consider: (1) $Q \gg D/L$ and (2) $Q \ll D/L$. In both cases we shall assume $\beta_2 \ll 1$.

Case 1:

The roots of $p_{12}(s)$ determine the transient behavior. Let us consider case (1) first: $q_1 \gg D_1/l_1$, $q_2 \gg D_2/l_2$. This may be an impossible case since $D_k = \alpha q_k + \frac{\alpha}{m} \epsilon_0$. But if $\alpha \ll l_1$, $\alpha \ll l_2$ we are in business. (Such would be the case with finely divided backfill.)

$$\text{Then: } \frac{q_k}{D_k} + \frac{1}{\alpha}, \text{ and } u_k + \frac{1}{\alpha} \sqrt{1 + \frac{4\theta s}{q_k}}, \lambda_k + \frac{1}{\alpha}$$

It is easy to show that the condition for $p_{12}(s) = 0$ is equivalent to:

$$(B-1) \quad \frac{1}{\sqrt{1 + \frac{4\theta s}{q_1}}} \tanh \left[\frac{l_1}{2\alpha} \sqrt{1 + \frac{4\theta s}{q_1}} \right] = - \frac{1}{\sqrt{1 + \frac{4\theta s}{q_2}}} \tanh \left[\frac{l_2}{2\alpha} \sqrt{1 + \frac{4\theta s}{q_2}} \right]$$

Now since $q_2 \gg q_1$ the largest negative value of s for which equation (B-1) is satisfied is controlled by the left hand side of the equation, namely:

$$(B-2) \quad \frac{l_1}{2\alpha} \sqrt{1 + \frac{4\theta s}{q_1}} \cong \frac{i\pi}{2}$$

Then:

$$(B-3) \quad s = -\frac{q_1}{4\theta\alpha} \left[1 + \frac{\pi^2 \alpha^2}{\lambda_1^2} \right]$$

is the approximate value of this largest root and the appropriate time constant:

$$(B-4) \quad \tau \sim \left| \frac{1}{s} \right| = \frac{4\theta\alpha}{q_1} \frac{1}{1 + \left(\frac{\pi\alpha}{\lambda_1} \right)^2}$$

Case II:

For the other case, $Q \ll D/L$, we look at the limit in which $q \rightarrow 0$.

$$\text{Then } D_k \rightarrow \alpha_m \epsilon_0, \quad U_k \rightarrow \sqrt{\frac{4\theta s}{\alpha_m \epsilon_0}}, \quad \lambda_k \rightarrow 0$$

To study this case we take the limit:

$$(B-5) \quad \lim_{q_1 \rightarrow 0} \frac{p_{12}(s)}{q_1} = - \left\{ \frac{1}{\sqrt{\frac{\theta s}{\alpha_m \epsilon_0}}} \sinh \left(\lambda_1 \sqrt{\frac{\theta s}{\alpha_m \epsilon_0}} \right) \cosh \left(\lambda_2 \sqrt{\frac{\theta s}{\alpha_m \epsilon_0}} \right) \right. \\ \left. + \frac{a^2}{w(2\alpha - w)} \frac{1}{\sqrt{\frac{\theta s}{\alpha_m \epsilon_0}}} \sinh \left(\lambda_2 \sqrt{\frac{\theta s}{\alpha_m \epsilon_0}} \right) \cosh \left(\lambda_1 \sqrt{\frac{\theta s}{\alpha_m \epsilon_0}} \right) \right\}$$

Then:

(B-6)

$$\lim_{q_1 \rightarrow 0} J(L, s) = \frac{c_s}{s} \frac{\sqrt{\frac{\theta s}{\alpha_m \epsilon_0}}}{\left[\frac{w(2a-w)}{a^2} \sinh \left(l_1 \sqrt{\frac{\theta s}{\alpha_m \epsilon_0}} \right) \cosh \left(l_2 \sqrt{\frac{\theta s}{\alpha_m \epsilon_0}} \right) + \cosh \left(l_1 \sqrt{\frac{\theta s}{\alpha_m \epsilon_0}} \right) \sinh \left(l_2 \sqrt{\frac{\theta s}{\alpha_m \epsilon_0}} \right) \right]}$$

when $w(2a-w) \ll a^2$ we find approximate poles at:

$$(B-7) \quad s = \begin{cases} -\frac{\alpha_m \epsilon_0}{\theta l_1^2} (k + 1/2)^2 \pi^2 & k = 0, 1, 2, \dots \\ -\frac{\alpha_m \epsilon_0}{\theta l_2^2} k^2 \pi^2 & k = 1, 2, \dots \end{cases}$$

From equation (30), the largest transient time constant is:

$$(B-8) \quad \tau_{\max} = \text{Max} \left[\frac{\theta l_1^2}{\epsilon_0 \alpha_m \pi^2} \frac{4}{2}, \frac{\theta l_2^2}{\epsilon_0 \alpha_m \pi^2} \frac{1}{2} \right]$$

which depends on the sizes of l_1 and l_2 .

Case III: No Concrete Plug

A final case of interest is one in which the concrete plug is not present (non-retrievable canister) and the niche is completely backfilled. Such

a case is obtained by setting $l_2 = 0$ and $w = a$. Then, $l_1 = L$, $\lambda_1 = q/D$. Equation (19) becomes:

$$u_1 = \sqrt{\left(\frac{q}{D}\right)^2 + \frac{4\theta s}{D}} \quad , \quad q = \frac{Q}{\pi a^2}$$

$$B-9 \quad p_{12}(s) = -2 \exp\left[\frac{qL}{2D}\right] \frac{q}{D} \frac{1}{\sqrt{\left(\frac{q}{D}\right)^2 + \frac{4\theta s}{D}}} \sinh\left[\frac{L}{2} \sqrt{\left(\frac{q}{D}\right)^2 + \frac{4\theta s}{D}}\right]$$

Then from equation (A-8) we find:

$$B-10 \quad J(L,s) = \frac{a^2 D c_s \exp\left[\frac{qL}{2D}\right] \sqrt{\left(\frac{q}{D}\right)^2 + \frac{4\theta s}{D}}}{w(2a-w) \sinh\left[\frac{L}{2} \sqrt{\left(\frac{q}{D}\right)^2 + \frac{4\theta s}{D}}\right]} \frac{1}{s}$$

$J(L,s)$ is the D'Arcy flux of solute past the infinitely thin concrete plug.

The total flux is:

$$B-11 \quad J_{TOT}(L,s) = \pi w(2a-w) p_0 J(L,s)$$

If we introduce the dimensionless parameter $\gamma = \frac{qL}{2D}$, the dimensionless time variable $\tau = \frac{Dt}{L^2}$, and the dimensionless s variable $\sigma = \frac{L^2 \theta}{D} s$, we find:

$$(B-12) \quad J_{TOT}(L, \tau) = \frac{\pi a^2 \rho_0 D c_s}{2L} e^{\gamma} \int_{-i\infty + c}^{i\infty + c} \frac{d\sigma}{\sigma} \frac{\sqrt{\gamma^2 + \sigma}}{\sinh \sqrt{\gamma^2 + \sigma}} e^{\sigma \tau}$$

Then, in the time domain, the total dissolution rate is:

$$(B-13) \quad R_D(\tau) = \frac{\pi a^2 \rho_0 D c_s}{2L} e^{\gamma} \left\{ \frac{\gamma}{\sinh \gamma} - F(\gamma, \tau) \right\}$$

where:

$$(B-14) \quad F(\gamma, \tau) = 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n\pi)^2}{\gamma^2 + (n\pi)^2} e^{-(\gamma^2 + (n\pi)^2)\tau}$$

$F(\gamma, \tau)$ represents the transient portion of the dissolution rate. Figure B-1 represents plots of $e^{\gamma} \left\{ \frac{\gamma}{\sinh \gamma} - F(\gamma, \tau) \right\}$, as a function of τ for several values of γ .

Figure 8-1. Plots of dimensionless waste dissolution rate:

$$\frac{LR_D(\tau)}{\pi a^2 D c_s} \text{ with } \tau = \frac{Dt}{L^2} \quad , \quad \gamma = \frac{qL}{2D}$$