**FINITE ELEMENT ANALYSIS
OF TIME-DEPENDENT INELASTIC DEFORMATION
IN THE PRESENCE OF TRANSIENT THERMAL STRESSES**

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ABSTRACT

A finite element formulation for the solution of time-dependent inelastic deformation problems for metallic structures, in the presence of transient thermal stresses, is presented in this paper. A rate formulation of the equations is used and any of a number of recently proposed combined creep-plasticity constitutive models with state variables can be adopted to describe material behavior. The computer program developed can solve planar (plane strain and stress) and axisymmetric problems. Using one of the above mentioned constitutive models, numerical results are presented for several illustrative problems, and comparisons of results, using either the quasi-steady or the unsteady diffusion equation for the determination of the temperature field, are carried out.

INTRODUCTION

Mathematical modelling of time-dependent inelastic deformation in metallic structures has traditionally been carried out by superposing elastic, time-independent plastic and time-dependent creep strains to obtain the total strain in a structure. Recently, several investigators (eg. [1-7], to quote a few) have proposed combined creep-plasticity models in which plastic and creep strains are combined into a single unified strain which is regarded as time-dependent. These theories also generally include certain internal or state variables to define the current deformation state of an element in the structure. The mathematical structure of many of the state variable models of inelastic deformation can be summarized by the following equations:

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^e + \dot{\epsilon}_{ij}^n + \dot{\epsilon}_{ij}^T \quad (1)$$

$$\dot{\epsilon}_{ij}^n = f_{ij}(\sigma_{ij}, q_{ij}^{(k)}, T) \quad (2)$$

$$\dot{q}_{ij}^{(k)} = g_{ij}(\sigma_{ij}, q_{ij}^{(k)}, T) \quad (3)$$

$$\dot{\epsilon}_{kk}^n = 0 \quad (4)$$

Here $\dot{\epsilon}_{ij}^e$, $\dot{\epsilon}_{ij}^n$ and $\dot{\epsilon}_{ij}^T$ are the elastic, non-elastic and thermal strain rates, respectively, σ_{ij} is the stress tensor, T the temperature and $q_{ij}^{(k)}$ are state variables. The number of state variables vary in the different models and they can be scalars or tensors. These state variables are assumed to completely characterize the present deformation state of the material, and the history dependence of the rate of non-elastic strain upto the current time is assumed to be completely taken into account by their current values. It is important to note that the rates of the non-elastic strain and state

variables at any time depend only on the current values of the stress, state variables and temperature.

The authors of this paper, together with other co-authors, have been interested for some time in the numerical solution of boundary value problems with material behavior governed by a constitutive model of the kind described above. A rate formulation of the governing differential equations has been used. This formulation takes advantage of the deterministic mathematical structure of the constitutive model. Spatial integration of the relevant equations have been carried out by the finite element or boundary element method [8-11] and time-integration by a march forward time-integration scheme with automatic time step control [12]. In these papers, two-dimensional isothermal inelastic deformation problems have been solved and the constitutive model due to Hart [1,2] has been used.

The subject of the present paper is the solution of inelastic boundary value problems in the presence of transient thermal stresses by the finite element method. In this analysis, the temperature field is obtained by solving the unsteady diffusion equation subjected to a prescribed temperature or heat flux boundary condition. The temperature rate field at any time determines the thermal strain rate field

$$\dot{\epsilon}_{ij}^T = \alpha \dot{T} \delta_{ij} \quad (5)$$

which is used together with the equilibrium and constitutive equations to obtain the displacement and stress rate field at this time. The time histories of the quantities of interest are then obtained by using a march forward time-integration scheme with automatic time step control.

FINITE ELEMENT FORMULATIONS

Diffusion equation

The diffusion equation for the temperature field has the form

$$k \nabla^2 T = c \frac{\partial T}{\partial t} - Q \quad (6)$$

subject to the boundary conditions

$$T = \bar{T} \quad \text{on part of the boundary } \partial S_T$$

$$k \frac{\partial T}{\partial n} = -q \quad \text{on part of the boundary } \partial S_q$$

In the above, T is the temperature, k the thermal conductivity, c is the specific heat/unit volume, Q is the heat source/unit volume and $\bar{T}$ and q are prescribed boundary temperature and boundary heat input per unit area respectively. The symbol ∇^2 is the Laplacian operator.

This equation is solved by using the method discussed by Zienkiewicz and Parekh [13]. A discretized version of the equation, obtained by a Galerkin type procedure, is

$$[H]\{T\} + [P]\{\dot{T}\} + \{F\} = 0 \quad (7)$$

where

$$[H] = \sum_E \int [\nabla N]^T k [\nabla N] dv$$

$$[P] = \sum_E \int [N]^T c [N] dv$$

and

$$\{F\} = -\sum_E \int [N]^T Q dv + \sum_{S_E} \int [N]^T q dS$$

with ∇ the gradient operator.

The shape functions, $[N]$, are defined in the usual way

$$T = [N]\{\hat{T}\} \quad (8)$$

where $\{\hat{T}\}$ are the nodal values of the temperature. The summation is carried out over all the elements, E is the element region and S_E refers only to those elements with an external surface with prescribed q .

This system of ordinary differential equations (7) for $\{\dot{T}\}$ are now solved by using a finite difference scheme in time which leads to the following equation

$$[H]\{T\}_{t+\Delta t/2} + [P](\{T\}_{t+\Delta t} - \{T\}_t)/\Delta t + \{F\} = 0 \quad (9)$$

where $[H]$, $[P]$ and $\{F\}$ are assigned their mid-interval values. With

$$\{T\}_{t+\Delta t/2} = (\{T\}_t + \{T\}_{t+\Delta t})/2,$$

a convenient scheme for integration is obtained as

$$\{T\}_{t+\Delta t/2} = ([H] + 2[P]/\Delta t)^{-1}(2[P]\{T\}_t/\Delta t - \{F\}) \quad (10)$$

and

$$\{T\}_{t+\Delta t} = 2\{T\}_{t+\Delta t/2} - \{T\}_t \quad (11)$$

In the above, t is time and Δt is a time interval.

Stress analysis equations

A rate form of the principle of virtual work can be written as [14]

$$\int_E \{\delta \epsilon\}^T \{\dot{\sigma}\} dV = \int_S \{\delta u\}^T \{\dot{i}\} dA \quad (12)$$

where $\{\dot{\sigma}\}$ and $\{\dot{i}\}$ are the rates of stress and prescribed surface traction and $\{\delta \epsilon\}$ and $\{\delta u\}$ are virtual strain and displacement vectors.

This leads to the equilibrium equations (see [11])

$$[K]\{\dot{\delta}\} + \{\dot{F}^n\} + \{\dot{F}^T\} + \{\dot{F}^B\} = 0 \quad (13)$$

where

$$[K] = \sum_E \int [B]^T [D] [B] dV$$

$$\{\dot{F}^n\} = -\sum_E \int [B]^T [D] \{\dot{\epsilon}^n\} dV$$

$$\{\dot{F}^T\} = -\sum_E \int [B]^T [D] \{\dot{\epsilon}^T\} dV$$

and

$$\{\dot{F}^B\} = -\sum_{S_E} \int [N]^T \{\dot{i}\} dS$$

In the above equation, $[K]$ is the stiffness matrix, $\{\dot{F}^n\}$ and $\{\dot{F}^T\}$ are the pseudonodal forces due to the nonelastic and thermal strain rates, and $\{\dot{F}^B\}$ is the equivalent nodal force due to the prescribed traction rate $\dot{i}$ on the boundary of the element. The volume integrals are evaluated over the

region of the element and the surface integral is evaluated over that part of the surface on which the traction is prescribed. The vector $\{\dot{\delta}\}$ denotes nodal displacement rates which are related to the displacement rate field through the shape function matrix $[N]$

$$\{\dot{u}\} = [N]\{\dot{\delta}\} \quad (14)$$

The local strain-displacement relation in terms of nodal displacement has the form

$$\{\dot{\epsilon}\} = [B]\{\dot{\delta}\} \quad (15)$$

and Hooke's law relating the elastic strain rates to the stress rates becomes

$$\{\dot{\sigma}\} = [D]\{\dot{\epsilon}^e\} = [D](\{\dot{\epsilon}\} - \{\dot{\epsilon}^n\} - \{\dot{\epsilon}^T\}) \quad (16)$$

Solution strategy

The initial distribution of state variables and temperature are prescribed and the initial nonelastic strains are set to zero. The initial distribution of displacements and stresses are obtained by solving the corresponding thermoelastic problem. The rates of nonelastic strains and state variables at the initial time are obtained from the constitutive equations (2-4) and the temperature rates are obtained from equations (10-11). These rates are used in equation (13) which is solved to give the initial rates of nodal displacements. Next, the rates of displacement, total strain and stress are obtained at every node from equations (14-16). These rates are now used to find the values of the variables at a new time Δt , and so on, and in this way the time histories of all the relevant variables are calculated.

Prescribed time steps Δt_T are used for the diffusion equation while the time steps for the displacement equation, Δt_u , are chosen automatically at each time by using an Euler type scheme with automatic time step control [12]. This procedure is illustrated below in terms of a single differential equation

$$\frac{dy}{dt} = F(y,t) \quad (17)$$

The value of $y(t+\Delta t_u)$ in terms of $y(t)$ is taken as

$$y(t+\Delta t_u) = y(t) + F(y,t)\Delta t_u \quad (18)$$

and the error at this step, used for automatic time step control is defined as

$$e = \Delta t_u |\nabla F| / |y(t)| \quad (19)$$

where $\nabla F = F(y,t) - F(y,t - \Delta t_{u\text{-previous}})$, is the first backward difference of F . Two error parameters $e_{\max}$ and $e_{\min}$ are initially prescribed. The algorithm proceeds as follows:

$e_{\max} < e$: replace Δt_u by $\Delta t_u/2$; recompute e

$e \leq e_{\max}$: accept Δt_u ; calculate $y(t+\Delta t_u)$

The time step for the next step, $\Delta t_{u\text{-next}}$, is decided according as

$e_{\min} < e \leq e_{\max}$: $\Delta t_{u\text{-next}} = \Delta t_u$

$e \leq e_{\min}$: $\Delta t_{u\text{-next}} = 2\Delta t_u$

The problems at hand, of course, involve systems of such equations for the rates of variables at all the nodes. In this case the error e_i for the i th variable is defined as an L^1 norm of the type

$$e_i = \frac{\Delta t_u \sum |vF_k|}{\sum |y_k(t)|} \quad (20)$$

where the summation extends over the values of the i th variable over all the nodes; and then

$$e = \max |e_i|$$

The algorithm then proceeds as described above.

Matching of the time steps when $\Delta t_T \neq \Delta t_u$ is carried out in the following way:

$\Delta t_T > \Delta t_u$: Interpolate temperature

$\Delta t_T < \Delta t_u$: Continue integration of the diffusion equation till $t + \Delta t_u$ is exceeded and then interpolate temperature between this time (say $\bar{t}$) and $\bar{t} - \Delta t_T$.

NUMERICAL RESULTS AND DISCUSSION

Harts constitutive model

The specific equations for Hart's constitutive model used in the numerical examples, are presented next. The model in tension is shown in Fig. 1. Material response in dilatation is assumed to be elastic. According to this model, the nonelastic strain is decomposed into two (time-dependent) components

$$\dot{\epsilon}_{ij}^n = \dot{\epsilon}_{ij}^a + \dot{\epsilon}_{ij}^p \quad (21)$$

where ϵ_{ij}^a is the anelastic strain, a stored strain which reflects the magnitude and direction of prior deformation history and ϵ_{ij}^p is the completely irrecoverable and path dependent permanent strain. The two state variables in the model are the anelastic strain and a scalar σ^* , called hardness, which is similar to an isotropic strain hardening parameter. The constitutive equations prescribe flow rules for the anelastic strain tensor and the permanent and nonelastic strain rates. Two auxillary tensors s_{ij}^a and s_{ij}^f are used, and their sum equals s_{ij} , the deviatoric part of the stress σ_{ij} . Certain scalar invariants of the relevant tensors are defined in the usual way (eg. $\sigma^a = \sqrt{(3/2)s_{ij}^a s_{ij}^a}$) and these are related to each other through the uniaxial equations (see Fig. 1)

$$\sigma^a = M\epsilon^a, \quad \dot{\epsilon}^n = \dot{\epsilon}_0 (\sigma^f / \sigma_0)^M \quad (22, 23)$$

$$\dot{\epsilon}^p = \dot{\epsilon}^* (\ln(\sigma^* / \sigma^a))^{-1/\lambda} \quad (24)$$

$$\dot{\epsilon}^* = \dot{\epsilon}_{ST}^* (\sigma^* / \sigma_s^*)^m \exp(Q/RT_B) \exp(-Q/RT) \quad (25)$$

$$\dot{\sigma}^* = \dot{\epsilon}_0^P \Gamma(\sigma^*, \sigma^a) \quad , \quad \Gamma(\sigma^*, \sigma^a) = (\beta/\sigma^*)^\delta (\sigma^a/\sigma^*)^{\beta/\sigma^*} \quad (26, 27)$$

Equation (22) represents a linear anelastic element, (23) a nonlinear dash-pot, (24) and (25) a "plastic" element and, finally, (26) and (27) describe strain hardening. The flow parameters are M , M , m , λ , $\dot{\epsilon}_0$ (at a reference stress level σ_0) and $\dot{\epsilon}_{ST}^*$ (at a reference hardness level σ_s^* and reference temperature T_B); β and δ are strain hardening parameters, R is the gas constant and Q the activation energy for atomic self diffusion. The material parameters, many of which are temperature dependent, are obtained from load relaxation and constant strain rate tension experiments. They are given in Appendix A of the EPRI report by Kumar and others [15].

Material parameters

The material parameters for 304 stainless steel at 100°C, used in some of the numerical calculations, are given below. Those at other temperatures are available in ref. [15]. In the examples that follow, these parameters are assumed to be temperature invariant for the sake of simplicity. However, temperature dependent material properties can be easily incorporated into the computer program.

$$E = 0.267 \times 10^8 \text{ psi} \quad \nu = 0.3$$

$$c/k = 144 \quad \text{s.in.}^{-2} \quad \alpha = .125 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$$

$$\lambda = 0.15 \quad M = 7.8 \quad m = 5$$

$$M = 0.1068 \times 10^9 \text{ psi}$$

$$\dot{\epsilon}_{ST}^* = 0.721 \times 10^{-41} \text{ s}^{-1} \quad \text{at } \sigma_s^* = 10 \text{ ksi} \quad \text{and } T_B = 373^\circ\text{K}$$

$$\dot{\epsilon}_o = 0.4967 \times 10^5 \text{ s}^{-1} \quad \text{at} \quad \sigma_o = 10 \text{ ksi}$$

$$\beta = 0.1965 \times 10^6 \text{ psi} \quad \delta = 1.33$$

$$Q/R = 32,500 \text{ }^\circ\text{K}$$

The initial values of the state variables are

$$\sigma^*(\underline{x}, 0) = \sigma_o^*$$

$$\epsilon_{ij}^a(\underline{x}, 0) = 0$$

Numerical results

The first problem considered is an uniform thick cylinder of radius ratio 1.5, inplane strain. The initial temperature throughout the cylinder is 100°C. The inside curved surface of the cylinder is heated at a constant temperature rate while the outside surface is kept at a fixed 100°C. The flat ends of the cylinder are assumed to be insulated. The FEM mesh used is shown in figure 2. Axisymmetric finite elements with triangular cross-sections are used and temperatures and displacements are interpolated quadratically in each triangle, ie the shape functions N in equations (8) and (14) are quadratic. Numerical results for strains and stresses are shown in figures 3-6. The value of time steps Δt_T in these calculations is 0.5 seconds and the error parameters for control of Δt_u are 10^{-3} and 10^{-4} respectively.

The variations of inside circumferential strain with inside temperature for various values of inside temperature rate are shown in figure 3. The quasi-steady solutions are obtained by using the equation $\nabla^2 T = 0$ for the temperature field. These compare very well with direct calculations from

ref [16]. At a given temperature, lower strains result from Hart's model compared to the thermoelastic solution. This is primarily due to redistribution of stress accompanying the generation of nonelastic strains. The strain rates are very sensitive to stress so that a moderate decrease in stress leads to a considerable reduction of strain rates and consequently accumulated strains. The quasi-steady and unsteady solutions are indistinguishable from each other for a low heating rate (7.2°C/hr) but are considerably different for a rapid heating rate (5.6°C/s). The significance of a slow or a fast rate, of course, is related to the time constant of the material in the diffusion equation. In the rapid heating example, use of the quasi-steady diffusion equation leads to overprediction of strain relative to that obtained by using the unsteady equation. Thus, in this case, use of the simplified model for the temperature field would lead to a conservative design.

The inside circumferential stresses are shown as functions of temperature for various heating rates in figure 4. Redistribution of circumferential stress in the cylinder for the quasi-steady and unsteady cases, for the rapid heating rate situation, are shown in figures 5 and 6. These figures are plotted on the same scale. The stress profiles are considerably different after a short time (2.6 seconds) but nearly coincide after a long time (38 seconds), showing that thermal transients have little effect on the long time stress distribution. The profiles are qualitatively similar to those obtained from classical plasticity and show two yield fronts, the first starting at the inside and the second at the outside of the cylinder.

Numerical results for a Gas Cooled Fast Breeder Reactor (GCFR) cladding tube are shown in figure 8 with the corresponding FEM mesh in figure 7. The tube is in plane strain, subjected to a steady internal pressure of 10 ksi

and a uniform heating rate of 5.6°C/s . The axisymmetric, quasi-steady option of the program is used in this case. The initial hardness for this cold worked tube is taken to be 95 ksi and the tube dimensions are representative of actual design [17].

The inside radial displacement at $z = 0$ is shown in figure 8, as a function of temperature. The initial elastic component of the displacement is about .407 mil and this is followed by a rapidly accumulated anelastic displacement of about .09 mil. The displacement then increases with temperature. Part of the displacement is due to thermal expansion and part of it is nonelastic. The rate of permanent deformation, $\dot{\epsilon}^P$, continues to increase with increasing temperature.

Typical CPU times on an IBM 370/168 computer are 80 seconds (quasi-steady) and 120 seconds (unsteady) for the uniform cylinder problems and 240 seconds for the nonuniform (GCFR) tube.

CONCLUSIONS

A finite element formulation has been presented in this paper for the analysis of inelastic deformation of metallic structures in the presence of transient thermal stresses. The computer program developed is capable of using any of a number of recently proposed combined creep-plasticity constitutive models of inelastic deformation. Numerical results are presented for some illustrative problems and comparisons of results using either the quasi-steady or the unsteady diffusion equation for the temperature field been carried out. It is found that in the examples studied, the effect of thermal transients on the long time stresses is minimal and use of the simpler quasi-steady equation leads to prediction of equal or larger nonelastic strains compared to those obtained by using the unsteady equation. The numerical results are obtained efficiently and accurately.

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Figure 1. Hart's model in tension.

Figure 2. Finite element mesh for uniform thick cylinder in plane strain.

Figure 3. Inside circumferential strain as a function of inside temperature for an uniform thick cylinder in plane strain. $b/a = 1.5$, $T_a(0) = 100^\circ\text{C}$, $T_b = 100^\circ\text{C}$, $\sigma_0^* = 17$ ksi.

Figure 4. Circumferential stress in element 1 (fig. 2) as a function of inside temperature in an uniform thick cylinder in plane strain. $b/a = 1.5$, $T_a(0) = 100^\circ\text{C}$, $T_b = 100^\circ\text{C}$, $\sigma_o^* = 17$ ksi.

Figure 5. Redistribution of circumferential stress in an uniform thick cylinder in plane strain. FEM-quasi-steady. $b/a = 1.5$, $T_a(0) = 100^\circ\text{C}$, $T_b = 100^\circ\text{C}$, $\sigma_o^* = 17$ ksi.

Figure 6. Redistribution of circumferential stress in an uniform thick cylinder in plane strain. FEM-unsteady. $b/a = 1.5$, $T_a(0) = 100^\circ\text{C}$, $T_b = 100^\circ\text{C}$, $\sigma_o^* = 17$ ksi.

Figure 7. Finite element mesh for GCFR tube in plane strain.

Figure 8. Inside radial displacement at $z = 0$ in a GCFR tube in plane strain as a function of temperature. Internal pressure = 10 ksi, $T(r, z, 0) = 25^{\circ}\text{C}$, $\dot{T}(r, z, 0) = 5.6^{\circ}\text{C/s}$, FEM-quasi-steady. $\sigma_0^* = 95$ ksi (cold worked specimen).