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OF BRIGHTNESS MEASURING DEVICES

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OF BRIGHTNESS MEASURING DEVICES

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ABSTRACT

Attempts to measure high electron beam brightness at low values of beam energy are plagued by the effects of space charge forces. These forces can substantially lower the phase space acceptance of various brightness measuring devices. This report considers several models for the effects of space charge upon the acceptance of both the field free, double aperture system and the magnetic "emittance selector" and compares them for some recent experiments on ATA and the High Brightness Test Stand. Reasonably conservative correction factors for the acceptances of these devices are derived.

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INTRODUCTION

In attempts to measure the brightness of the beam just out of the injector on ETA, ATA and the HBTS, two different types of collimators have been employed. At the low energies characteristic of the various injector outputs the effects of space charge are significant and must be taken into account. The following analysis attempts to provide useful correction factors for collimator acceptance that will yield lower limits to the beam brightness.

FIELD FREE COLLIMATOR (DOUBLE APERTURE) ACCEPTANCE CALCULATION

The four-volume acceptance of this collimator was derived by Sessler in ELF Note 88 as

$$V_4 = \pi^2 R_p^4 / L^2 \quad (1)$$

where L is the collimator length and R_p is the pipe or aperture radius. The assumption used in the derivation of equation (1) is that the electron orbits in the collimator are just straight lines. The presence of appreciable space charge force in the collimator alters the nature of the electron trajectories. Therefore, we consider a slight generalization of one of Sessler's methods.

The definition of the four-volume acceptance for two apertures is

$$V_4 = \int dx_0 \int dy_0 \int dx_0' \int dy_0'$$

subject to the two constraints

$$x_0^2 + y_0^2 \leq R_p^2$$

$$x(L)^2 + y(L)^2 \leq R_p^2$$

where the subscript 0 denotes quantities at $z = 0$, which is taken to be at the first aperture.

In general, we may write $x(L)$ and $y(L)$ as

$$x(L) = \alpha x_0 + \delta x_0'$$

$$y(L) = \alpha y_0 + \delta y_0'$$

where α and δ depend on the differential equation of motion of the electron between the apertures but not on the initial conditions x_0, y_0, x_0' and y_0' . For the special case of straight line orbits $\alpha = 1$ and $\delta = L$.

To perform the integration we make the changes of variable

$$t \sin \phi = y_0$$

$$t \cos \phi = x_0$$

$$S \sin \phi = \alpha y_0 + \delta y_0'$$

$$S \cos \phi = \alpha x_0 + \delta x_0'.$$

The constraints then become

$$t^2 \leq R_p^2,$$

$$S^2 \leq R_p^2.$$

The Jacobian $|\partial(x_0, y_0, x_0', y_0')/\partial(t, \phi, S, \varphi)|$ is easily shown to be tS/δ^2 .

Thus the four-volume acceptance is

$$\begin{aligned} V_4 &= \int_0^{R_p} t dt \int_0^{2\pi} d\phi \int_0^{R_p} S dS \int_0^{2\pi} d\varphi / \delta^2 \\ &= \pi^2 R_p^4 / L^2 \end{aligned} \quad (3)$$

where the correction factor $\epsilon = (L/\delta)^2$.

SPACE CHARGE MODEL

If we assume a uniform density beam profile in the collimator we may write the equation of motion for the electrons as

$$x'' = 2I_1 / ((\beta\gamma)^3 I_0 a^2(z)) x \quad (4)$$

where I_1 is the current which passes through the first aperture, $I_0 = mc^3/e \approx 17$ kA and $a(z)$ is the beam edge radius between the apertures. A prime denotes differentiation with respect to z , the axial distance.

In the experiments performed with this collimator to date all indications are that the current I_1 is preserved throughout the collimator until it contacts the second aperture at which point the current I_2 emerges downstream. The model then depends on the choice of the function $a(z)$. We shall not attempt to find $a(z)$ self consistently but rather we will investigate certain choices for $a(z)$ which lead to exact solutions of equation (4) and hence exact solutions for the acceptance of the double aperture system. The full solutions for the various models are given in the appendix.

Case 1: $a(z) = a_0$, a constant

δ is readily shown to be $\sinh(kL)/k$ where

$$k^2 = 2I_1 / ((\beta\gamma)^3 I_0 a_0^2) \quad (5)$$

We can use this solution to place coarse upper and lower bounds on the effects of space charge. By choosing $a_0 = R_p$ we clearly overestimate the effects of

space charge unless $I_1 = I_2$, in which case $a(z)$ probably has a minimum $< R_p$ somewhere between the apertures. Fortunately, $I_1 \gg I_2$ in nearly all experiments performed to date.

We may obtain a clear lower bound to the effects of space charge if we set a_0 equal to the maximum beam radius $a(L)$. Under the assumption previously stated, that the beam profile is fiat, we have

$$a(L) = R_p (I_1/I_2)^{1/2} . \quad (6)$$

Thus the limits for ϵ are:

$$\text{Lower limit:} \quad \epsilon = (kL/\sinh(kL))^2 \quad k^2 = 2I_1 / ((B\gamma)^3 I_0 R_p^2) .$$

$$\text{Upper limit:} \quad \epsilon = (kL/\sinh(kL))^2 \quad k^2 = 2I_2 / ((B\gamma)^3 I_0 R_p^2) .$$

CONDITIONS FOR NEGLIGIBLE SPACE CHARGE EFFECTS

It is clear from these solutions that space charge effects will be negligible in all cases where $kL \ll 1$. If we define a current I_s as

$$I_s \equiv \frac{(B\gamma)^3 I_0 R_p^2}{2L^2}$$

then the condition for negligible space charge becomes $I_1 \ll I_s$.

Now in order that the second aperture completely collimate the beam in velocity space we must have $E/R_p > 2R_p/L$ where we have assumed an upright phase ellipse at the first aperture. Here E is the unnormalized edge emittance and R_p is the beam edge radius. For an ellipsoidal phase space volume the normalized brightness J_n is equal to $2I/E_n^2$ so that we may write the above condition as $(2I/J_n)^{1/2} / (B\gamma R_p) > 2R_p/L$. If we now assume that the incident beam

profile is flat we have $(R_0/R_p)^2 = I_1/I_2$ and we may express the collimation condition as $I_s > I_L$ where I_L is defined as

$$I_L \equiv 2(\beta\gamma)^2 J_n^2 R_p^4 / L^2 .$$

In order to satisfy both the conditions for collimation and negligible space charge we must have $I_s \gg I_L$ or equivalently, $J_n \ll \beta\gamma I_0 / (4R_p^2)$. This is very similar to a condition obtained in reference 1; however, satisfaction of this condition is necessary but not sufficient to insure negligible space charge effects.

Case 2: $a(z) = R_p \exp(\lambda z/2)$.

The use of equation (6) gives $\lambda = (1/L) \ln(I_1/I_2)$ and

$$c = \left[\frac{\lambda L/2}{I_0(\frac{2k}{\lambda}) K_0(\frac{2k}{\lambda} e^{-\lambda L/2}) - K_0(\frac{2k}{\lambda}) I_0(\frac{2k}{\lambda} e^{-\lambda L/2})} \right]^2$$

where k is given by equation (5) with $a_0 = R_p$. I_0 and K_0 are the modified Bessel functions of order zero.

Case 3: $a(z) = R_p (1+(z/\lambda)^2)$.

We use equation (6) to determine λ as $L/((I_1/I_2)^{1/2} - 1)^{1/2}$. And k is given by equation (5) with $a_0 = R_p$. We obtain

$$c = \left[\frac{\sqrt{k^2 \lambda^2 - 1} \cos \phi_0}{\sinh \left\{ \sqrt{k^2 \lambda^2 - 1} \left(\frac{\pi}{2} - \phi_0 \right) \right\}} \right]^2$$

where $\phi_0 = \tan^{-1}(\lambda/L)$.

Case 4: $a(z) = R_p(1+z/\lambda)$.

Equation (6) yields $\lambda = L/((I_1/I_2)^{1/2}-1)$ and k is given by equation (5) with $a_0 = R_p$. The correction factor is

$$c = \left[\frac{1}{2\lambda} \frac{\sqrt{1 + 4k^2\lambda^2}}{(1 + L/\lambda)^{1/2} \sinh \left\{ \frac{1}{2} \sqrt{1 + 4k^2\lambda^2} \ln(1 + L/\lambda) \right\}} \right]^2$$

COMPARISON OF THE VARIOUS MODELS

The correction factors obtained from the various models are shown in Table 1 for the following examples taken from recent ATA and HBTS experiments:

- A. HBTS: $I_1=40A$, $I_2=6A$, $L=27.9cm$, $R_p=.152cm$, $\beta\gamma=2.78$
- B. HBTS: $I_1=80A$, $I_2=15A$, $L=38.1cm$, $R_p=.237cm$, $\beta\gamma=2.99$
- C. ATA: $I_1=1000A$, $I_2=88A$, $L=38.0cm$, $R_p=.476cm$, $\beta\gamma=4.81$
- D. ATA: $I_1=200A$, $I_2=20A$, $L=38.0cm$, $R_p=.199cm$, $\beta\gamma=5.8$.

Table 1

Case Example	1(Upper)	1(Lower)	2	3	4
A	.701	.131	.390	.308	.448
B	.584	.0883	.288	.219	.333
C	.824	.153	.487	.399	.580
D	.914	.434	.727	.665	.782

REASONABLE LOWER LIMIT TO THE CORRECTION FACTOR

Given the assumption of a flat beam profile between the apertures a good lower limit to ϵ is given by case 4, that of a linear beam profile. This can be seen from the following argument. No matter what r' the beam has upon entrance to the first aperture, $a(0) = R_p$ (providing the beam radius is $> R_p$). And, for a fixed ratio of I_1/I_2 , $a(L)$ is determined. In the absence of focusing between the apertures a'' must be ≥ 0 as can be seen from the envelope equation. Therefore the actual beam edge is always within the linear profile of the model. This means that the space charge effects are underestimated by the linear profile model. Figure 1 depicts the situation for the various models.

SPACE CHARGE EFFECTS IN THE MAGNETIC COLLIMATOR

The four-volume acceptance of a long pipe in a uniform axial magnetic field for relativistic electrons was derived in ELF Note 84 as

$$V_4 = \pi^2 k_c^2 R_p^4 / 6 \quad (7)$$

where k_c is the cyclotron wavenumber $eB/(\beta\gamma mc^2)$. This result is valid in the absence of space charge where the electron trajectories are simple cyclotron orbits. We now consider a simple model of space charge in the collimator. We take the beam profile to be flat. Under this assumption the equations of motion become

$$x'' = k^2(z)x - k_c y' \quad \text{and}$$

$$y'' = k^2(z)y + k_c x'$$

where $k^2 = 2I(z)/((\beta\gamma)^3 I_0 R_p^2)$. If we define ξ as $x + iy$ then the two equations above may be combined to yield

$$\xi'' - ik_c \xi' - k^2(z)\xi = 0.$$

If we now define $\xi \equiv \Omega \exp(ik_c z/2)$ we obtain

$$\Omega'' + (k_c^2/4 - k^2(z))\Omega = 0. \quad (8)$$

We next consider a long collimator so that $I(z) = I$, the output current of the collimator. In this manner we insure that the space charge effects are being underestimated since, at least for small z , $I(z) > I$. Then if we define

$$\Omega \equiv \eta + i\lambda$$

we can write equation (8) as

$$\eta'' + k_e^2 \eta = 0 \quad (9)$$

$$\lambda'' + k_e^2 \lambda = 0 \quad (10)$$

where $k_e^2 \equiv k_c^2/4 - k^2$.

In ELF Note 102 the acceptance of a system satisfying equations of the form (9) and (10) was derived to be

$$V_4 = 2\pi^2 k_e^2 R_p^4/3. \quad (11)$$

Thus, in the space $(\eta_0, \eta_0', \lambda_0, \lambda_0')$ the four-volume acceptance is given by equation (11). Next we relate the four-volume in the space $(\eta_0, \eta_0', \lambda_0, \lambda_0')$ to that in the space (x_0, x_0', y_0, y_0') . We find

$$x = \eta \cos(k_c z/2) - \lambda \sin(k_c z/2)$$

$$y = \eta \sin(k_c z/2) + \lambda \cos(k_c z/2)$$

$$x' = -(k_c \eta/2) \sin(k_c z/2) - (k_c \lambda/2) \cos(k_c z/2) + \eta' \cos(k_c z/2) - \lambda' \sin(k_c z/2)$$

$$y' = (k_c \eta/2) \cos(k_c z/2) - (k_c \lambda/2) \sin(k_c z/2) + \eta' \sin(k_c z/2) + \lambda' \cos(k_c z/2).$$

With these relations the Jacobian $|\partial(x_0, y_0, x_0', y_0')/\partial(\eta_0, \lambda_0, \eta_0', \lambda_0')|$ may be shown to be equal to unity. Thus the acceptance with space charge is

$$V_4 = 2\pi^2(k_c^2/4-k^2)R_p^4/3 \quad (12)$$

We point out that if space charge is negligible equation (12) yields

$$V_4 = \pi^2 k_c^2 R_p^4/6$$

the result obtained for a magnetic collimator by a different method in ELF Note 84 (Equation (7)).

For parameters of interest V_4 as determined from equation (12) is substantially different from V_4 as given by equation (7). As an example we consider a recent ATA experiment with $R_p = 1$ cm, $B = 1$ kgauss, $\beta\gamma = 6.8$ and an output current of 2.29 kA. Equation (7) gives V_4 as 1.22×10^{-2} (rad-cm)² while Equation (12) gives $V_4 = 6.61 \times 10^{-3}$ (rad-cm)². The brightnesses inferred from these four volumes are respectively, 4×10^4 A/cm²-steradian and 7.4×10^4 A/cm²-steradian. The space charge has made a substantial difference and we may be reasonably confident that the brightness is at least 7.4×10^4 A/cm²-steradian.

REFERENCE

1. G. J. Caporaso and D. L. Birx, "Brightness Measurements on the Livermore High Brightness Test Stand," LLNL Report UCRL-91914, May 9, 1985. Submitted for publication in the proceedings of the 1985 Particle Accelerator Conference, Vancouver, BC, Canada, May 13-16, 1985.

FIGURE CAPTION

- Fig. 1 Diagram showing the beam incident on the double aperture system. The beam profiles corresponding to cases 1 through 4 are indicated. Both the upper and lower limits for case 1 are also shown.

APPENDIX

We write equation (4) as

$$x'' - (k/f(z))^2 x = 0 \quad (A1)$$

with $k^2 \equiv 2I_1/((\beta\gamma)^3 I_0 R_p^2)$.

Case 1: $f(z) = f_0$, a constant

We may write down the solution immediately as

$$x(z) = x_0 \cosh(kz/f_0) + x_0' f_0/k \sinh(kz/f_0).$$

For $a_0 = R_p$, $f_0 = 1$ while for $a_0 = R_p(I_1/I_2)^{1/2}$ we have $f_0 = (I_1/I_2)^{1/2}$.

Case 2: $f(z) = \exp(\lambda z/2)$

We have

$$x'' - k^2 e^{-\lambda z} x = 0 \quad (A2)$$

If we put $t = e^{-\lambda z}$ equation (A2) becomes

$$x'' + x'/t - k^2 x/(\lambda^2 t) = 0$$

where a prime now denotes differentiation with respect to t . This is one of the many variants of Bessel's equation. The solution is

$$\begin{aligned} x(z) = & (2kx_0/\lambda) [K_1(2k/\lambda) I_0(2k/\lambda e^{-\lambda z/2}) + I_1(2k/\lambda) K_0(2k/\lambda e^{-\lambda z/2})] \\ & + (2x_0'/\lambda) [I_0(2k/\lambda) K_0(2k/\lambda e^{-\lambda z/2}) - K_0(2k/\lambda) I_0(2k/\lambda e^{-\lambda z/2})] \end{aligned}$$

Case 3: $f(z) = 1+(z/\lambda)^2$

Equation (A1) becomes

$$x'' - k^2 x/[1+(z/\lambda)^2]^2 = 0 \quad (A3)$$

If we define $z \equiv \lambda \cot \phi$ equation (A3) becomes

$$x'' + 2 \cot \phi x' - k^2 \lambda^2 x = 0 \quad (A4)$$

where a prime now denotes differentiation with respect to ϕ . If we put $x = \Omega/\sin\phi$ we get

$$\Omega'' - (k^2\lambda^2 - 1)\Omega = 0$$

which is easily solved in terms of hyperbolic functions. The complete solution is

$$x(z) = x_0 \cosh [(k^2\lambda^2 - 1)^{1/2}(\pi/2 - \phi(z))]/\sin\phi(z) \\ + [\lambda x_0' / (k^2\lambda^2 - 1)^{1/2}] \sinh [(k^2\lambda^2 - 1)^{1/2}(\pi/2 - \phi(z))]/\sin\phi(z)$$

Case 4: $f(z) = 1+z/\lambda$

Equation (A1) becomes

$$x'' - k^2 x / [1+z/\lambda]^2 = 0 \quad (A5)$$

If we put $\zeta \equiv 1+z/\lambda$ we obtain

$$x'' - k^2 \lambda^2 x / \zeta^2 = 0 \quad (A6)$$

where the prime now denotes differentiation with respect to ζ . Equation (A6) is the Equidimensional differential equation with solutions of the form $x = \zeta^m$. It is easily solved to yield

$$x(z) = x_0 / (m_2 - m_1) [m_2 (1+z/\lambda)^{m_1} - m_1 (1+z/\lambda)^{m_2}] \\ + x_0' \lambda / (m_2 - m_1) [(1+z/\lambda)^{m_2} - (1+z/\lambda)^{m_1}]$$

where $m_1 = [1 + (1+4k^2\lambda^2)^{1/2}]/2$ and $m_2 = [1 - (1+4k^2\lambda^2)^{1/2}]/2$.

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Figure 1