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TWO-PHASE PRESSURE DROP IN A NATURAL-
CIRCULATION BOILING CHANNEL

by

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NOMENCLATURE

Alphabetic

D_e	equivalent diameter (4 times hydraulic radius), ft.
f	measured "Darcy" friction factor, dimensionless; f_0 for smooth tube friction factors.
G	mass velocity, $\text{lb}/(\text{hr})(\text{ft}^2)$.
g_c	conversion factor, $(\text{lb}_m)(\text{ft})/(\text{lb}_f)(\text{sec}^2)$.
H_F	friction head loss, ft.
L	length, ft.
P	pressure, psf.
Q'''	power density, kw per liter of coolant.
R	local ratio of two-phase friction pressure drop gradient to liquid-phase friction pressure drop gradient when liquid is flowing at total mixture flow rate, dimensionless.
r_2	see eq. (6).
U	velocity, fps.
V	void measurement data, mv; V_f for "hot-full" data; V_e for "hot-empty" data.
x	ratio of steam flow rate to total mixture flow rate, dimensionless; also distance, ft.

Greek

α	steam volume fraction, dimensionless.
μ	dynamic viscosity, $\text{lb}/(\text{hr})(\text{ft})$
ξ	dummy variable.
ρ	density, lb/ft^3
x_{LO}^2	same as R (used by Martinelli-Nelson and Sher).

Subscripts

2,3	refers to pressure taps as designated in Figure 6.
A	acceleration
B	inception of boiling (saturation point).
C	refers to pressure tap designated in Figure 6.

e	exit.
F	friction.
f	saturated liquid.
g	saturated vapor.
H	hydrostatic.
LB	local boiling.
LBF	local boiling friction
LO	refers to liquid phase only flowing at total mixture flow rate.
m	mixture.
NB	nonboiling
S	static.
TP	two-phase.
TPF	two-phase friction.

TWO-PHASE PRESSURE DROP IN A NATURAL-CIRCULATION BOILING CHANNEL

by

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ABSTRACT

Experimental two-phase pressure drop data were obtained from a $\frac{1}{4} \times 2 \times 60$ -in. vertical, uniformly heated, test section. The local volume fraction of steam was measured simultaneously with the pressure drop, thus allowing separation of the terms for hydrostatic and acceleration pressure drops from the overall static pressure drop. The results are expressed in terms of an average two-phase friction factor multiplier,

$$\bar{R} = \Delta P_{\text{TPF}} / \Delta P_{\text{LO}} \quad ,$$

and are compared with the Martinelli-Nelson correlation, the Lottes-Flinn correlation, and a correlation combining that of Martinelli-Nelson and a flowrate parameter. The ranges of variables include: pressure from 150 to 600 psig; power density from 17.2 to 94.5 kw/liter of coolant; sub-cooling from 4.8 to 25.7°F; exit qualities from 0.009 to 0.065; exit steam volume fraction from 0.19 to 0.77; and velocities from 2.56 to 3.43 fps.

I. INTRODUCTION

Development of the boiling water reactor concept has required considerable study of the mechanism of two-phase flow. The major effort of the Heat Engineering Section has been devoted to two of the major problems associated with two-phase flow: (1) prediction of the pressure drop associated with a flowing two-phase mixture, and (2) prediction of the steam volume fraction. Accurate prediction of the steam volume fraction, hereafter called the void fraction, is desirable because the hydrodynamics of two-phase flow and the nuclear characteristics of a boiling reactor are strongly dependent upon the volume of steam present. The prediction of pressure drop is essential for estimating flow rates, and consequently the reactor power or void fraction, in natural-circulation systems.

The mechanism of two-phase flow and the resulting pressure drop has all but defied analysis because of the complex interaction between the phases. There are no truly satisfactory correlations available at this time that will predict accurately either the void fraction or the pressure drop over a wide range of variables. There are, however, considerable experimental data that have been gathered over the last decade that are applicable to a number of specific systems and flow conditions. Excellent surveys of the two-phase pressure drop are given in References (1) and (2).

The majority of the work on pressure drop has been done on adiabatic, two-component (gas-oil, air-water) flow in horizontal pipes. Very few data have been reported for heated vertical channels in which a change of phase is occurring, because of the inability to measure the local void fraction. Without knowledge of the void fraction, the hydrostatic, frictional, and acceleration components of pressure drop cannot be separated from the overall static pressure drop.

This study of two-phase flow phenomena utilized a natural-circulation system designed for a pressure of 600 psig, and employed a vertical, rectangular, heated test section. The system was unique in that it allowed simultaneous measurement of the overall static pressure drop and local void fraction along the axis of the boiling channel. An IBM-650 digital computer was utilized for reduction of the raw data.

This report is concerned primarily with the pressure-drop data. The void-fraction data obtained during these tests were reported in Reference (3).

II. EXPERIMENTAL APPARATUS

A. General Description

The boiling pressure-drop tests were conducted in a single-channel, natural-circulation system with a test section that was hydrodynamically similar to the EBWR flow channel. In this type of system, the steam generated is bled to a condenser and replaced with an equal weight of cold makeup feedwater, the remaining water being recirculated through the flow loop.

The essential elements of such a natural-circulation loop consist of a vertical heated section, an unheated riser section, a steam-water separator zone, a vertical downcomer section, and horizontal crossovers to connect the two vertical sections. The system is shown schematically in Figure 1 and pictorially in Figure 2. The system is described in detail in Reference (3).

FIG. 1
SCHEMATIC OF EXPERIMENTAL SYSTEM

FIG. 2
EXPERIMENTAL SYSTEM

Subcooled water entered the bottom of the test section and was heated sufficiently to induce boiling and produce a net generation of steam. The two-phase mixture passed through an unheated riser above the test section and was discharged to the separator where the vapor and liquid phases were separated by gravitational force; the steam flowed to the condenser and the water through the liquid crossover to the downcomer. Cold makeup water was added to the recirculating water at the junction of the upper crossover and downcomer. The subcooled mixture flowed down the downcomer, through the flow measurement venturi, and across to the test section. The driving head was provided by the density difference between the subcooled liquid in the downcomer and the two-phase mixture in the heated test section and riser.

A constant system pressure was maintained by throttling the steam to the condenser at a pressure ratio that was greater than the critical pressure ratio; thus, changes in condenser performance had no effect on the system.

B. Test Section

The rectangular test section, with a 0.258 x 2.06-in. cross section for flow and a heated length of $60\frac{1}{2}$ in., was weld-constructed of 0.109-in., Type 304 stainless steel. Two electrically insulated, $\frac{5}{8}$ -in., carbon steel backup plates supported the 2-in. span under pressure. Five static pressure taps, consisting of $\frac{1}{16}$ -in. ID stainless steel tubing, were installed on one of the narrow sides of the test section at locations of 15, 35, 45, 55 and 60 in. from the inlet. Electrical resistance heating was employed by passing a low-voltage, ac current through the test section. The entire assembly was covered with a 2-in. thickness of magnesia insulation and wrapped with an aluminum foil radiation shield. Figure 3 shows the test-section assembly.

FIG. 3
TEST SECTION ASSEMBLY

The riser section was 29 in. long and of the same cross section and material as the test section. Static pressure taps were installed $\frac{1}{4}$ in. and at $14\frac{23}{32}$ in. from the riser exit.

C. Instrumentation

Iron-constantan thermocouples with a calibrated accuracy of $\pm 0.5^\circ\text{F}$ were located just below the entrance to the test section, in the saturated steam zone in the separator, and in the makeup water storage tank. Operating experience has since indicated the advisability of using thermopiles of higher accuracy to measure the subcooling of the inlet water, as this has considerable influence on the calculated boiling length and quality.

The rate of circulation of liquid was measured with a 0.7 x 2-in. vertical venturi meter, located in the downcomer section, having a calibrated accuracy of $\pm 1\%$. The head corrections necessitated by the vertical installation were applied to the flow measurements. A 0.250-in. orifice and a 0.1105-in. orifice calibrated to an accuracy of $\pm 1\%$ were used to measure the steam flow and flow of makeup feedwater, respectively.

The pressure in the system was measured at the exit of the test section by means of a calibrated, Heise, precision, bourdon tube gauge having a reading accuracy of ± 1 psi. The measurements of differential static pressure drop between taps were obtained with well- and U-type manometers containing manometer fluids of specific gravity 1.235 and 1.728, and permitting a reading accuracy of ± 0.02 or ± 0.06 in. H_2O , respectively. The necessary corrections for the "cold leg" manometer were made to evaluate the actual static pressure drop.

Power was supplied to the test section by a 120-kva transformer and was controlled by three 37.5-kva saturable reactors connected in parallel. The secondary voltages ranged from 0 to 16 v while the current varied from 0 to 3250 amp. Measurements of voltage drop, current, and voltage were made with $\frac{1}{4}\%$ precision instruments.

D. Measurements of Void Fraction

The void fraction of the two-phase mixture was measured by the attenuation of gamma rays from a 0.085-Mev thulium (Tm^{170}) source. The unattenuated portion of gamma rays were detected by a photomultiplier tube with a NaI (thallium-activated) scintillation crystal. The tube signal was amplified by a linear current amplifier and recorded on an Esterline-Angus recorder. The gamma beam was collimated at the photomultiplier tube by a 1-in.-thick lead shield with an accurately machined window $\frac{1}{2}$ in. high by $\frac{3}{8}$ in. wide (Figure 4). Cooling coils were placed around the photomultiplier tube to maintain a constant temperature, since both the NaI crystal and photomultiplier tube are quite sensitive to temperature changes.

FIG 4
PHOTOMULTIPLIER TUBE COLLIMATING SHIELD

The gamma source and photomultiplier tube were aligned on a carriage so that the collimated beam would pass between the two backup plates, through the long dimension of the channel. The carriage was moveable along the axis of the channel. Void measurements were made at 11 positions along the axis, beginning $3\frac{1}{2}$ in. below the exit of the heated section, with 5-in. intervals between positions. Figure 5 illustrates the carriage in position and Figure 6 shows schematically the relative positions of the stations for measurement of voids and of the pressure taps.

After each test run, a "hot-full" (void fraction of zero) calibration point was taken at each position and at the end of each day a "hot-empty" (void fraction = 1.0) calibration point was obtained by draining the water from the system (normal operation permitted three test runs per day). Knowing the "hot-full" and "hot-empty" calibration points and assuming exponential attenuation, the void fraction at each position was calculated from the equation

$$\alpha = \frac{\ln (V/V_f)}{\ln (V_e/V_f)} \quad (1)$$

The derivation of eq. (1) is given in Reference (4). Typical curves of void fractions vs length are shown in Figure 7.

A complete discussion of the inaccuracies associated with the gamma-attenuation method is given in Reference (4). In general, the accuracy of uniformly distributed void fractions measured in the $\frac{1}{4} \times 2$ -in. channel can be shown to be within a maximum of ± 0.03 .

NOTE: ALL DIMENSIONS IN INCHES

FIG. 5-a
PLAN VIEW OF DENSITY CARRIAGE ASSEMBLY

FIG. 5-b
DENSITY CARRIAGE ASSEMBLY

FIG. 6
SCHEMATIC OF PRESSURE TAPS AND
VOID MEASUREMENT STATIONS

FIG. 7
TYPICAL CURVES OF VOIDS VS LENGTH (in) AT 150 psig

Petrick⁽⁵⁾ investigated the relative accuracy of the "one-shot" and traverse methods with plastic mockups of various two-phase flow patterns. It was found that the average error for both methods decreased with decreasing channel width. Comparison of the two methods with actual two-phase fluids showed very good agreement for a $\frac{1}{4}$ x 2-in. section.

A comparison of the "one-shot" voids with the integrated average voids (obtained by traversing the channel in a plane normal to its axis) is shown in Figure 8 for a system pressure of 150 psig. It may be seen that all but two data points fall within the theoretical maximum possible error due to measurement techniques.

III. DISCUSSION OF RESULTS

A. Equations for Pressure Drop

The pressure drop associated with a fluid flowing through a channel is expressed in general terms by the integrated form of Euler's equation

$$-\int_1^2 dP_S = P_{S1} - P_{S2} = \int_1^2 \rho dx + \int_1^2 \rho \frac{UdU}{g_c} + \int_1^2 \rho dH_F \quad (2)$$

The first term on the right-hand side represents the hydrostatic head, the second term the acceleration pressure drop, and the third term the pressure drop associated with friction.

For a two-phase mixture, with a relative velocity between the phases, the local density is expressed in terms of the local void fraction.

$$\rho_m = (1 - \alpha) \rho_f + \alpha \rho_g \quad ; \quad (3)$$

hence, the integral of the hydrostatic head term becomes

$$\Delta P_H = \int_1^2 \rho dx = [(1 - \bar{\alpha}) \rho_f + \bar{\alpha} \rho_g] L = \bar{\rho}_m L \quad (4)$$

Martinelli and Nelson⁽⁶⁾ derived an expression for the acceleration pressure drop when there is a relative velocity between the two phases. The acceleration pressure drop is thus expressed as

$$\Delta P_A = \int_1^2 \rho \frac{UdU}{g_c} = r_2 \frac{G^2}{g_c} \quad , \quad (5)$$

where

$$r_2 = \left[\frac{(1 - x)^2}{1 - \alpha} + \frac{x^2}{\alpha} \frac{\rho_f}{\rho_g} - 1 \right] \frac{1}{\rho_f} \quad (6)$$

The two-phase frictional pressure drop is commonly expressed as

$$\Delta P_{TPF} = \int_1^2 \rho dH_F = \frac{fG^2}{2g_c \rho D_e} \int_1^2 R dx = \bar{R} \frac{fG^2 L}{2g_c \rho D_e} \quad (7)$$

where

$$R = \frac{(dP/dL)_{TPF}}{(dP/dL)_{LO}}$$

B. Data-reduction Methods

The two-phase frictional pressure drop was determined from the test data by

$$\Delta P_{\text{TPF}} = \Delta P_S - \Delta P_A - \Delta P_H - \Delta P_{\text{LBF}} \quad . \quad (8)$$

The measurements of static pressure drop were taken between pressure taps 2 and C (Figure 6), a distance of 40 in., and the data reduced to give the frictional pressure drop component over the calculated net boiling length. The beginning of net boiling is defined as that point at which the enthalpy of the mixture equals the enthalpy of the saturated liquid. The results of this experimental work are expressed in terms of an average two-phase friction factor multiplier, calculated from

$$\bar{R} = \Delta P_{\text{TPF}} / \Delta P_{\text{LO}} \quad , \quad (9)$$

where ΔP_{LO} is the frictional pressure drop associated with the liquid phase only flowing at the total mixture flow rate.

The acceleration pressure drop was calculated from

$$\Delta P_A = \frac{G^2}{g_c} \left\{ \left[\frac{(1-x)^2}{1-\alpha} + \frac{x^2}{\alpha} \frac{\rho_f}{\rho_g} \right]_C - \left[\frac{(1-x)^2}{1-\alpha} + \frac{x^2}{\alpha} \frac{\rho_f}{\rho_g} \right]_2 \right\} \frac{1}{\rho_f} \quad . \quad (10)$$

by means of calculated qualities and measured void fractions.

The hydrostatic head term is composed of the hydrostatic head in the local boiling region, $(\Delta P_H)_{2-B}$, as well as that in the net boiling region, $(\Delta P_H)_{B-C}$; the hydrostatic head in the net boiling region was calculated from eq. (4) using the calculated boiling length, L_{B-C} , and a mean void fraction that was determined by integrating the local voids over the net boiling length.

In the local boiling region it was assumed that any steam bubbles present, as indicated by the void fraction measurements, were clinging to the wall, thereby leaving a subcooled liquid core (the bubbles leaving the wall being immediately quenched). Thus, the hydrostatic head term for the local boiling region was computed from

$$(\Delta P_H)_{2-B} = \bar{\rho}_{\text{NB}} L_{2-B} \quad , \quad (11)$$

where $\bar{\rho}_{\text{NB}}$ is the average density of the subcooled liquid core in the local boiling region.

Transverse void profiles in the local boiling region at a system pressure of 150 psig have since cast some doubt as to the validity of the above assumption at the lower pressures.

An error analysis indicates that if the local boiling mixture density calculated from eq. (3) were used instead of the liquid density, the friction factor multiplier $\bar{R}$ would be increased no more than an average 3.9% for the low power densities, and an average 2.7% for the high power densities, at 150 psig. The small error is the result of the relatively short local boiling length included between taps 2 and C. At 600 psig, however, the local boiling length is much longer, and the error analysis indicates a possible increase in $\bar{R}$ of 18.3% at low powers and 6.6% at high powers. It seems improbable that the maximum possible error would ever be realized, since the void traverse revealed that, although some vapor had worked its way to the center of the channel, the majority of the vapor was still located near the wall. Thus, the actual error introduced by the assumption should be considerably less than the possible error.

The local boiling friction pressure drop was obtained by multiplying the single-phase pressure drop by a local boiling friction factor multiplier:

$$\Delta P_{LBF} = \bar{R}_{LB} \frac{f L_{2-B} G^2}{2 g_c D_e \bar{\rho}_{NB}} \quad (12)$$

The local boiling friction factor multiplier was obtained from a plot of $(\bar{R} - 1)$ vs $\bar{\alpha}$ (see Figure 9) that provided an almost straight line and was easily extrapolated to the small $\bar{\alpha}$'s that were calculated for the local boiling region. The data points for the curve were obtained by calculating the $\bar{R}$'s and $\bar{\alpha}$'s between pressure taps 2-3 and 2-C, the values between taps 2-3 being calculated to give points in the lower $\bar{\alpha}$ range. The mean void fraction was chosen over the exit void fraction as the independent variable to reduce scatter in the low void region where the void uncertainty is the greatest.

The single-phase frictional pressure drop was calculated with the aid of smooth-tube friction factors. Periodic measurements were made of the actual single-phase friction factor in the test section and were found to agree within $\pm 3\%$ with the values from smooth tubes during the major portion of the test program. A rapid increase in the friction factors with time was observed during the latter stage of the program, as indicated by Figure 10. When the error became greater than 5 per cent, a correcting factor f/f_0 , taken from Figure 10, was applied to the smooth-tube friction factor for calculation of these points. These points are noted on the data sheets. The method used in reducing the data is presented in detail in Appendix II.

FIG. 9
PRESSURE DROP DATA VS. MEAN VOID FRACTION
FOR OBTAINING LOCAL BOILING FRICTION
FACTOR MULTIPLIER

FIG. 10
RATIO OF MEASURED FRICTION FACTOR TO SMOOTH TUBE FRICTION

An error analysis was made to evaluate the deviation in the two-phase friction factor multiplier that would result from errors in the measurement of the void fractions. The analysis was made with the aid of the computer data-reduction program by altering the input data of several representative runs and comparing the resultant answers with the original computations.

To establish the limits of error, conditions of high and low power density for both 150 and 600 psig were analyzed. The data were changed to give voids that were 5% and 20% less and 20% greater than the measured values. The results are shown below.

Pressure, psig	Power, kw/liter	$\% \bar{R} \text{ Error: } \left(\frac{\bar{R} - \bar{R}_{\text{data}}}{\bar{R}_{\text{data}}} \right) 100$		
		$\text{Void Fraction Error: } \left(\frac{\alpha - \alpha_{\text{data}}}{\alpha_{\text{data}}} \right) 100$		
		-20%	-5%	+20%
150	18	-10	-2.5	+6
	76	-10	+0.5	-20
600	17	-17	-4.5	+16
	94	-11	-2.2	+5

It is believed the measured void fractions are reliable to within 5%; consequently, the uncertainty induced in the two-phase friction multiplier by the void measurement would appear to be no greater than 5%.

C. Comparison with Martinelli-Nelson Correlation

The experimental results were compared with the correlation of Martinelli and Nelson,⁽⁶⁾ which is presently that most widely accepted for steam-water flow with vaporization. Figure 11 shows a fair agreement between the correlation and the data; however, the data appear to be diverging from the correlation at the higher qualities.

D. Comparison with Lottes-Flinn Correlation

Previous work at Argonne indicated a strong relationship between the two-phase friction multiplier and the void fraction. Lottes and Flinn⁽⁷⁾ proposed a simple annular-flow model (similar to the Martinelli *et al.*⁽⁸⁾ "turbulent-turbulent" flow model) in which the mean two-phase friction multiplier is expressed as a function of the exit steam void fraction rather than exit quality. The model is presented in Appendix I and shows that, for small qualities or low pressures, eq. A-8 reduces to the approximation

$$\bar{R} = \frac{1}{3} \left[1 + \frac{1}{1 - \alpha_e} + \left(\frac{1}{1 - \alpha_e} \right)^2 \right] \quad (13)$$

FIG. 11-a
COMPARISON OF DATA WITH
MARTINELLI-NELSON CORRELATION

FIG. 11-b
COMPARISON OF DATA WITH
MARTINELLI-NELSON CORRELATION

A comparison of the data with eq. (13) is presented in Figure 12. The experimental data were approximately 30% higher than predicted by the correlation. It may be seen that plotting $\bar{R}$ against the void fraction appears to eliminate the pressure parameter, at least over the range of variables tested.

The significant aspect of Figure 12 is the way the data follow the shape of the correlation curve. While there is a lack of quantitative agreement, the previously indicated dependence of the friction multiplier upon the void fraction is clearly seen. This strong dependence is more clearly seen in Figure 13 in which $\bar{R}$ is plotted against $\bar{\alpha}$. The data used for this comparison are the same data shown in Figure 9 and differ from the data shown in Figure 12 in that no correction has been made for the local boiling friction.

FIG. 12
COMPARISON OF DATA WITH LOTTES-FLINN CORRELATION

FIG. 13
ILLUSTRATION OF $\bar{R}$ DEPENDENCE UPON THE VOID FRACTION

Although the apparent dependence of the friction multiplier upon the void fraction may be seen in Figure 12, there appears to be a trend in the data that will make them pass through the line representing the Lottes-Flinn correlation at a void fraction of about 0.70. It is apparent that for the higher void fractions the correlation must give friction factor multipliers that are too large because the analytical model postulates extremely high liquid velocities in this range. This is clearly illustrated by the following example.

The ratio of the pressure gradient for steam to that of water, when both are flowing at the same rate, may be expressed approximately as

$$(\Delta P/\Delta L)_g/(\Delta P/\Delta L)_f \approx \left(\frac{\mu_g}{\mu_f}\right)^{0.25} \left(\frac{\rho_f}{\rho_g}\right) \quad (14)$$

This ratio is equivalent to $\bar{R}$ of a flowing two-phase mixture with a quality of 100% along the entire length of the channel. The values of $\bar{R}$ from the Lottes-Flinn correlation for an exit quality of 100% (equivalent to an average quality of 50% along the entire heated length) are shown below for comparison with the values obtained from eq. (14).

<u>Pressure,</u> <u>psig</u>	<u>$\bar{R}$ by</u> <u>eq. (14)</u>	<u>$\bar{R}$ from</u> <u>Lottes-Flinn</u>
150	87.3	7850
250	55.5	2920
400	35.9	1130
600	23.8	475

It may be seen that the Lottes-Flinn correlation gives values that are orders of magnitude higher even though the average quality is only one half of the quality implied by the use of eq. (14).

E. Flowrate Parameter Correlation

There is increasing evidence that a mass-velocity parameter is required to properly correlate two-phase pressure drop data. Sher⁽⁹⁾ reports a mass-velocity effect on local friction multiplier data for boiling in a vertical channel at 2000 psia. Petrick⁽⁵⁾ reported a similar effect for a nonboiling, air-water system at atmospheric pressure. Petrick's mass-velocity effects were obtained by two methods: (1) variable flow rate with constant flow area, and (2) variable flow area with constant flow rate. It was not possible to establish a mass-velocity effect for the natural-circulation tests reported here as the mass velocity was essentially constant at $G = 0.6 \times 10^6 \text{ lb}/(\text{hr})(\text{ft}^2)$.

Sher suggested the following method, hereafter referred to as Method I, for applying the mass-velocity parameter established at 2000 psia to lower pressures:

- (1) Use the Martinelli-Nelson correlation to obtain the ratio of ϕ_{LO}^2 at the pressure of interest to ϕ_{LO}^2 at 2000 psia, as a function of x (see Figures 14 and 15).
- (2) Multiply the ordinate of Figure 16 by this ratio for corresponding qualities to obtain a new family of curves.
- (3) Integrate the curve of interest to obtain the average value of the two-phase friction multiplier $\bar{R}$ as a function of exit quality.

FIG. 14a
MARTINELLI-NELSON ϕ_{LO}^2 VS. QUALITY FOR SMALL QUALITIES

FIG. 14b
MARTINELLI-NELSON ϕ_{L0}^2 VS. QUALITY, FOR SMALL QUALITIES

FIG. 15
RATIO OF MARTINELLI-NELSON ϕ_{L0}^2 AT PRESSURE OF INTEREST TO ϕ_{L0}^2
AT 2000 psia AS A FUNCTION OF QUALITY AND PRESSURE

FIG. 16
SHER'S FAMILY OF CURVES REPRESENTING BULK BOILING PRESSURE DROP DATA
VERTICAL UP-FLOW 2000 psia

A comparison of the experimental data with Method I is shown in Figure 17. It appears that Sher's mass-velocity parameter is adequate for these low qualities and pressures and provides a better prediction of the data than the Martinelli-Nelson correlation alone.

Another method of correlating pressure drop data has been suggested.⁽¹⁰⁾ A comparison of the data with the results from the use of this method (Method II) is shown in Figure 18.

The correlation in ANL-6063 is based upon two assumptions: (1) The Martinelli-Nelson curves of ϕ_{LO}^2 vs x represent the true two-phase friction factor multiplier for a mass velocity of $1 \times 10^6 \text{ lb}/(\text{hr})(\text{ft}^2)$ for all pressures; and (2) the void fraction, rather than the quality, should be used as the independent variable. The latter assumption is based upon the strong void dependency exhibited by the data in this test and in other adiabatic, two-phase, two-component tests reported in the literature. The first assumption is based upon the observations of Sher,⁽⁹⁾ who found that his correlation of ϕ_{LO}^2 for $G = 1 \times 10^6 \text{ lb}/(\text{hr})(\text{ft}^2)$ corresponded closely with the Martinelli-Nelson correlation for 2000 psia and qualities greater than 4%, and Petrick⁽⁵⁾ who found the Martinelli-Nelson correlation adequate for air-water data at atmospheric pressure with $G > 1 \times 10^6 \text{ lb}/(\text{hr})(\text{ft}^2)$.

It is seen from Figure 18 that Method II predicts values of $\bar{R}$ that are higher by an average 20% than the data indicate. This difference between the data and the correlation is due mainly to assumption (1), above, for it is obvious from Figure 11 that the Martinelli-Nelson correlation corresponds to friction multipliers for mass velocities of approximately $0.6 \times 10^6 \text{ lb}/(\text{hr})(\text{ft}^2)$ in these tests.

FIG. 17-a
COMPARISON OF DATA WITH
METHOD OF SHER (METHOD I)

FIG. 17-b
COMPARISON OF DATA WITH
METHOD OF SHER (METHOD I)

Method I appears to be a better correlation of the data than does Method II. It must be noted, however, that the range of variables obtained in these tests are not sufficient to provide a good test of any correlation, particularly one containing a mass-velocity parameter.

FIG. 18
COMPARISON OF DATA WITH CORRELATION IN ANL-6063

IV. CONCLUSIONS

1. The Lottes-Flinn correlation, utilizing void fractions rather than quality, is not adequate for predicting two-phase pressure drop in boiling channels over a wide range of conditions.

2. For void fractions less than 0.65, the average two-phase friction factor multiplier displays a dependence on void fractions quite similar to that proposed by the Lottes-Flinn Model. It is recommended that this dependence be explored more fully.

3. The Martinelli-Nelson correlation is improved over the range of this data by incorporating the mass velocity parameter reported by Sher. It is felt, however, that a mass-velocity parameter cannot be recommended until more data are available.

V DATA

A Summary of 150 psig Results

Run No	Power Density kw/liter	Velocity (Satd) fps	Subcooling ΔT OF	Exit Quality X_C	Exit Void α_C	Mean Void $\bar{\alpha}_B$ C	Boiling Length L_B C in	Static Pressure Drop (ΔP_S) _{2 C} psf	Hydrostatic Head (ΔP_H) _{2 C} psf	Acceleration Pressure Drop (ΔP_A) _{2 C} psf	Local Boiling Friction (ΔP_{fLB}) _{2 B} psf	Two Phase Friction (ΔP_{TPF}) _{B C} psf	Single-Phase Friction (ΔP_{fo}) _{B C} psf	Average Net Boiling Friction Multiplier $\bar{R}_B$ C
25	18.0	3.05	4.8	0.0096	0.332	0.160	33.6	193.5	158.9	7.3	2.6	24.7	11.7	2.12
26	18.0	3.12	4.5	0.0095	0.313	0.160	34.3	194.1	158.4	7.3	2.3	26.0	12.4	2.09
378	18.0	3.02	5.1	0.0093	0.414	0.206	32.5	199.8	152.9	10.7	4.8	31.4	18.2 (1.64)*	1.72
377	22.8	3.08	6.1	0.0118	0.484	0.268	33.2	202.2	142.8	14.3	4.9	40.2	19.4 (1.64)	2.07
31	22.7	3.31	5.9	0.0147	0.500	0.272	36.5	198.0	138.1	18.0	1.8	40.1	14.7	2.75
258	22.7	3.22	7.2	0.0137	0.510	0.303	33.1	198.1	137.6	18.4	3.4	38.8	13.8 (1.09)	2.80
26	37.3	3.27	8.1	0.0199	0.573	0.334	36.3	199.2	128.2	24.4	1.8	44.8	14.3	3.13
24	37.3	3.27	7.5	0.0207	0.656	0.406	37.7	200.3	113.7	33.1	1.4	52.2	14.9	3.50
30	47.0	3.14	9.9	0.0271	0.650	0.447	37.5	203.2	106.8	28.8	1.7	65.9	13.7	4.79
25	47.0	3.16	9.7	0.0272	0.709	0.470	37.8	202.2	102.4	38.8	1.4	59.6	14.0	4.25
331	47.0	2.74	11.0	0.0314	0.655	0.438	38.1	208.4	107.4	23.3	1.2	76.6	14.1 (1.28)	5.44
27	56.6	3.01	11.8	0.0349	0.709	0.479	38.4	203.2	99.48	35.5	0.9	67.3	13.0	5.17
33	56.6	2.98	12.2	0.0349	0.736	0.512	38.1	204.2	94.50	38.8	1.3	69.6	12.7	5.47
155	56.6	3.08	12.6	0.0328	0.674	0.462	37.0	200.2	105.4	32.3	1.9	60.6	13.1	4.63
243	56.6	2.76	13.4	0.0373	0.714	0.522	37.7	207.0	93.7	30.3	1.6	81.5	11.8 (1.07)	6.93
28	66.3	2.85	14.4	0.0433	0.784	0.541	38.6	203.9	88.2	47.5	0.7	67.5	11.9	5.68
36	66.3	2.97	14.7	0.0406	0.753	0.532	37.7	203.9	92.0	42.7	1.6	67.6	12.5	5.41
29	76.0	2.72	17.4	0.0518	0.736	0.541	38.5	207.2	88.4	33.8	0.9	84.1	11.0	7.67
38	76.0	2.71	16.6	0.0530	0.772	0.588	39.4	204.0	78.0	39.3	0.4	86.4	11.1	7.76
253	76.0	2.67	19.0	0.0515	0.715	0.516	37.5	198.7	95.1	30.5	1.6	71.5	11.1 (1.08)	6.45

Ratio of fffo applied to data

E Summary of 250 psig Results

Run No	Power Density kw/liter	Velocity (Satd) fps	Subcooling ΔT OF	Exit Quality X_C	Exit Void α_C	Mean Void $\bar{\alpha}_B$ C	Boiling Length L_B C in	Static Pressure Drop (ΔP_S) _{2 C} psf	Hydrostatic Head (ΔP_H) _{2 C} psf	Acceleration Pressure Drop (ΔP_A) _{2 C} psf	Local Boiling Friction (ΔP_{fLB}) _{2 B} psf	Two Phase Friction (ΔP_{TPF}) _{B C} psf	Single Phase Friction (ΔP_{fo}) _{B C} psf	Average Net Boiling Friction Multiplier $\bar{R}_B$ C
193	17.8	3.03	5.8	0.0090	0.306	0.153	29.5	186.5	158.3	6.8	3.7	17.8	10.0	1.76
226	17.8	2.96	5.9	0.0091	0.334	0.178	29.3	187.8	155.2	7.3	3.7	21.6	9.7 (1.05)	2.23
229	17.7	3.00	5.8	0.0091	0.336	0.191	29.6	187.4	153.2	7.3	4.0	22.8	10.1 (1.05)	2.26
235	17.8	3.02	5.9	0.0087	0.344	0.185	29.1	187.7	154.4	7.8	4.3	21.2	10.1 (1.06)	2.11
191	27.4	3.30	7.1	0.0141	0.460	0.278	32.9	189.7	137.8	14.6	3.5	33.8	12.9	2.61
238	27.4	3.23	7.5	0.0141	0.449	0.269	32.0	192.8	140.1	13.3	4.1	35.2	12.4 (1.06)	2.83
186	37.1	3.27	9.2	0.0199	0.485	0.249	33.9	192.0	140.9	16.8	2.5	31.8	13.1	2.43
225	37.1	3.11	8.7	0.0214	0.515	0.320	32.9	194.5	128.1	17.2	2.3	46.9	13.4 (1.05)	3.51
223	37.1	3.21	9.0	0.0208	0.508	0.337	34.6	203.3	126.7	16.3	3.0	57.3	13.2 (1.05)	4.33
234	37.1	3.23	9.8	0.0196	0.507	0.339	32.8	194.6	129.0	16.6	4.1	45.0	12.8 (1.06)	3.51
189	46.7	3.29	11.3	0.0253	0.580	0.385	34.3	195.3	119.9	23.1	3.4	48.9	13.4	3.66
185	56.4	3.26	13.3	0.0315	0.600	0.374	35.0	195.7	120.3	25.3	2.8	47.3	13.4	3.54
223	56.4	3.04	13.2	0.0351	0.617	0.425	36.4	199.4	109.9	23.0	2.1	64.3	12.6 (1.05)	5.09
224	56.4	3.04	13.4	0.0347	0.621	0.421	36.1	199.5	111.1	23.2	2.4	62.8	12.5 (1.05)	5.02
227	56.4	3.03	13.9	0.0343	0.597	0.417	35.5	199.4	112.8	20.9	2.8	63.0	12.3 (1.05)	5.14
188	66.1	3.08	15.6	0.0401	0.663	0.468	36.0	199.4	103.8	28.8	2.6	64.3	12.5	5.15
192	66.1	3.13	15.1	0.0399	0.632	0.447	36.3	198.9	106.4	26.1	2.3	64.1	12.9	4.95
237	66.1	2.96	16.6	0.0414	0.630	0.447	35.6	202.0	107.9	23.3	2.7	68.0	11.9 (1.06)	5.74
184	75.7	3.18	17.5	0.0444	0.693	0.462	35.9	197.5	105.0	35.6	2.7	54.2	13.1	4.13
236	75.8	2.84	20.6	0.0484	0.671	0.484	34.9	203.5	103.5	25.8	3.2	71.0	10.8 (1.06)	6.55
187	85.4	2.88	21.4	0.0559	0.715	0.513	36.2	203.3	96.1	32.7	2.3	72.2	11.1	6.49
240	85.4	2.77	21.3	0.0593	0.667	0.495	37.0	204.7	97.3	24.9	1.9	80.6	11.0 (1.07)	7.35
190	95.1	2.77	24.3	0.0654	0.723	0.541	36.6	203.9	90.7	31.7	2.2	79.3	10.5	7.57

Ratio of fffo applied to data

C Summary of 400 psig Results

Run No	Power Density kW/liter	Velocity (Satd) fps	Subcooling ΔT °F	Exit Quality X_C	Exit Void α_C	Mean Void $\bar{\alpha}_{B-C}$	Boiling Length L_{B-C} in	Static Pressure Drop $(\Delta P_{SD})_{B-C}$ psf	Hydrostatic Head $(\Delta P_{HD})_{B-C}$ psf	Acceleration Pressure Drop $(\Delta P_{AD})_{B-C}$ psf	Local Boiling Friction $(\Delta P_{fLB})_{B-C}$ psf	Two-Phase Friction $(\Delta P_{TPF})_{B-C}$ psf	Single-Phase Friction $(\Delta P_{fo})_{B-C}$ psf	Average Net Boiling Friction Multiplier $\bar{R}_{B-C}$
206	17.5	2.84	7.3	0.0086	0.289	0.165	24.5	178.3	154.8	5.1	5.1	13.2	7.0	1.91
213	17.5	2.82	7.1	0.0090	0.312	0.185	25.4	178.8	152.1	5.1	5.6	16.0	7.2 (1.05)	2.21
214	27.1	3.15	8.1	0.0150	0.364	0.216	30.6	181.3	144.0	8.5	3.9	25.0	10.5	2.38
213	27.1	3.12	8.5	0.0147	0.362	0.213	29.8	181.1	145.1	8.4	4.3	23.4	10.1	2.32
216	27.1	3.12	8.3	0.0149	0.369	0.227	30.1	181.8	143.0	8.6	4.1	26.1	10.3	2.54
196	36.8	3.31	10.3	0.0197	0.447	0.250	31.1	184.7	139.1	13.3	4.4	27.9	11.6	2.40
207	36.8	3.28	10.6	0.0196	0.440	0.262	30.6	184.3	138.2	12.6	4.7	28.9	11.3	2.56
218	36.8	3.22	10.0	0.0211	0.431	0.294	32.3	185.2	133.2	11.7	3.8	36.5	11.6 (1.05)	3.14
210	46.5	3.34	12.2	0.0256	0.468	0.299	32.3	186.0	131.2	14.7	4.0	36.0	12.3	2.92
215	46.5	3.26	13.2	0.0253	0.489	0.335	31.1	187.0	127.9	14.9	5.0	39.2	11.4	3.44
198	56.1	3.37	14.2	0.0317	0.543	0.361	33.2	188.3	121.3	19.6	4.0	43.4	12.7	3.41
212	56.1	3.31	15.3	0.0307	0.503	0.354	31.7	188.2	124.5	16.7	4.8	42.3	12.0	3.52
222	56.1	3.31	14.9	0.0327	0.543	0.375	35.4	190.0	119.8	17.8	4.0	48.4	11.9 (1.05)	4.06
194	65.8	3.29	16.8	0.0381	0.554	0.344	33.3	192.4	123.4	19.9	3.7	45.4	12.3	3.69
211	65.8	3.25	16.9	0.0366	0.544	0.379	33.4	190.6	118.3	18.6	3.7	50.0	12.2	4.10
221	65.8	3.18	16.8	0.0402	0.578	0.402	34.0	192.4	114.2	20.1	3.5	54.6	12.0 (1.05)	4.55
197	75.5	3.22	19.3	0.0447	0.620	0.441	33.6	192.7	109.2	25.0	3.9	51.6	12.1	4.52
212	75.5	3.15	19.8	0.0461	0.587	0.414	33.7	193.1	112.9	20.9	3.6	55.6	11.6	4.50
214	75.5	3.15	20.3	0.0453	0.576	0.408	33.2	193.0	114.8	19.6	4.2	54.5	11.5	4.73
220	75.5	3.09	19.8	0.0475	0.611	0.431	34.1	194.1	109.9	21.8	3.5	58.9	11.4 (1.05)	5.16
205	85.1	3.12	23.0	0.0515	0.622	0.431	33.3	194.6	111.4	23.8	3.9	55.5	11.2	1.95
205	85.1	3.14	22.2	0.0525	0.606	0.440	33.9	194.0	108.9	22.5	3.6	58.9	11.5	5.11
195	94.8	3.10	24.2	0.0604	0.647	0.421	34.6	196.7	110.4	25.9	3.3	57.1	11.5	4.97
203	94.8	3.04	24.5	0.0619	0.619	0.443	34.8	195.8	106.8	22.8	3.0	63.1	11.2	5.63
217	94.8	2.96	25.2	0.0634	0.620	0.452	34.7	197.5	105.5	21.8	3.0	67.2	10.8	6.22

Ratio of f/fo applied to data

D Summary of 600 psig Results

Run No	Power Density kW/liter	Velocity (Satd) fps	Subcooling ΔT °F	Exit Quality X_C	Exit Void α_C	Mean Void $\bar{\alpha}_{B-C}$	Boiling Length L_{B-C} in	Static Pressure Drop $(\Delta P_{SD})_{B-C}$ psf	Hydrostatic Head $(\Delta P_{HD})_{B-C}$ psf	Acceleration Pressure Drop $(\Delta P_{AD})_{B-C}$ psf	Local Boiling Friction $(\Delta P_{fLB})_{B-C}$ psf	Two-Phase Friction $(\Delta P_{TPF})_{B-C}$ psf	Single-Phase Friction $(\Delta P_{fo})_{B-C}$ psf	Average Net Boiling Friction Multiplier $\bar{R}_{B-C}$
160	17.2	2.56	7.0	0.0119	0.188	0.090	28.1	169.4	155.3	2.1	2.8	9.2	6.1	1.50
166	26.9	2.97	10.2	0.0148	0.244	0.139	26.0	170.2	150.9	4.2	4.7	10.4	7.5	1.38
169	26.9	3.03	9.8	0.0148	0.262	0.151	26.5	172.0	149.4	4.5	5.4	12.7	7.9	1.68
174	36.5	3.10	12.3	0.0209	0.314	0.191	28.1	171.4	143.9	6.1	7.8	13.6	8.8	1.90
177	36.5	3.14	11.3	0.0219	0.322	0.181	29.8	172.3	143.7	6.4	4.1	18.1	9.6	1.89
176	46.2	3.29	13.6	0.0265	0.366	0.215	30.0	174.7	139.6	8.9	4.5	21.8	10.5	2.09
232	46.2	3.25	13.4	0.0275	0.379	0.246	30.6	176.8	135.1	9.0	4.5	28.2	10.8 (1.06)	2.62
158	55.9	3.41	14.8	0.0329	0.397	0.239	31.8	177.6	135.0	10.7	4.1	27.8	11.5	2.42
161	55.9	3.35	15.0	0.0333	0.401	0.238	31.7	177.9	135.4	11.0	3.7	27.7	11.2	2.48
164	65.5	3.43	17.5	0.0379	0.426	0.270	31.4	179.5	131.4	12.7	4.4	31.1	11.7	2.66
167	65.5	3.43	17.6	0.0377	0.401	0.268	31.3	181.4	131.8	10.9	4.8	33.9	11.6	2.91
231	65.5	3.29	17.7	0.0404	0.458	0.292	32.1	181.3	127.7	13.0	4.1	36.5	11.6 (1.06)	3.15
172	75.2	3.43	20.6	0.0429	0.461	0.300	30.9	183.0	128.2	14.6	4.7	35.5	11.5	3.08
175	75.2	3.35	20.6	0.0446	0.460	0.289	31.4	181.0	129.0	13.8	4.2	33.9	11.3	3.00
165	84.9	3.41	21.7	0.0512	0.479	0.304	32.6	183.5	125.5	15.3	4.0	38.7	12.0	3.27
168	84.9	3.40	22.0	0.0510	0.474	0.304	32.3	184.3	125.9	15.0	4.2	39.2	11.9	3.31
159	94.5	3.42	23.7	0.0576	0.479	0.308	33.0	185.3	124.6	16.3	3.7	40.6	12.0	3.38
162	94.5	3.37	24.4	0.0578	0.521	0.325	32.7	183.5	122.7	18.5	3.6	38.7	11.6	3.33
230	94.5	3.23	25.7	0.0602	0.505	0.317	32.5	187.4	124.0	16.0	3.6	43.8	11.3 (1.06)	3.87

Ratio of f/fo applied to data

APPENDIX I

LOTES-FLINN TWO-PHASE FLOW MODEL

The flow model set forth for upward flow of a boiling mixture in a heated channel is an annular distribution of the liquid phase around a core of vapor, the liquid phase being in contact with the walls. It was proposed that the increased friction associated with two-phase flow could be attributed to the increasing frictional drag of the liquid phase on the walls; the result of the increasing liquid velocity along the channel as the volume percentage vaporized increases faster than the weight percentage vaporized:

$$\Delta P_{\text{TPF}} = \frac{f \rho_f}{2 g_c D_e} \int_0^L U_f^2 d\xi \quad . \quad (\text{A-1})$$

From a mass balance, the inlet velocity at the saturation point is related to the local liquid velocity by

$$\frac{U_f}{U_B} = \frac{1 - x}{1 - \alpha} \quad . \quad (\text{A-2})$$

The ratio of the local vapor velocity to the local liquid velocity that is, the (slip ratio) may similarly be expressed from a mass balance as

$$\frac{U_g}{U_f} = \left(\frac{x}{1 - x} \right) \left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{\rho_f}{\rho_g} \right) \quad . \quad (\text{A-3})$$

By combining eqs. (A-2) and (A-3) to eliminate the steam volume fraction, the local liquid velocity ratio is found to be

$$\frac{U_f}{U_B} = 1 + x \left(\frac{\rho_f U_f}{\rho_g U_g} - 1 \right) \quad . \quad (\text{A-4})$$

If for any given pressure a constant slip ratio is assumed, it is seen that the velocity ratio U_f/U_B is a linear function of x ; consequently, for uniform heat generation it will be linearly dependent upon the channel length.

An average two-phase friction factor multiplier is then defined as

$$\bar{R} = \Delta P_{\text{TPF}} / \Delta P_{\text{LO}} \quad . \quad (\text{A-5})$$

If eqs. (A-1) and (A-5) are combined, it is seen that

$$\bar{R} = \frac{1}{L} \int_0^L \left(\frac{U_f}{U_B} \right)^2 d\xi = \frac{1}{LU_B^2} \int_0^L U_f^2 d\xi \quad . \quad (A-6)$$

Recalling that U_f is a linear function of length, eq. (A-6) may be integrated to give

$$\bar{R} = \frac{1}{3} \left[1 + \frac{(1 - x_e)}{(1 - \alpha_e)} + \left(\frac{1 - x_e}{1 - \alpha_e} \right)^2 \right] \quad . \quad (A-7)$$

Elimination of x between eqs. (A-3) and (A-7) results in

$$\bar{R} = \frac{1}{3} \left[1 + \frac{1}{1 - \alpha_e(1 - \phi)} + \left(\frac{1}{1 - \alpha_e(1 - \phi)} \right)^2 \right] \quad , \quad (A-8)$$

where

$$\phi = \rho_g U_g / \rho_f U_f \quad .$$

It may be seen from eq. (A-7) that for small qualities, or from eq. (A-8) for low pressures [$(\rho_g/\rho_f) \gg 1$], these equations reduce to the approximation

$$\bar{R} = \frac{1}{3} \left[1 + \frac{1}{1 - \alpha_e} + \left(\frac{1}{1 - \alpha_e} \right)^2 \right] \quad . \quad (A-9)$$

In the case of adiabatic two-phase flow with no transfer of mass between the phases, it is evident from eq. (A-2) that the local pressure gradient ratio may be expressed as

$$R = \left(\frac{1-x}{1-\alpha} \right)^2 \quad . \quad (A-10)$$

APPENDIX II

SAMPLE CALCULATION

The pressure-drop and steam-volume-fraction data-reduction equations were programmed for computation on the IBM 650 Digital Computer. The program was divided into three parts: Part I for calculating fluid properties, temperatures, flow rates, quality and boiling length; Part 2 to give the void fraction and slip ratio; and Part 3 to calculate the average two-phase friction factor multiplier.

A sample calculation showing the equations used in this program is presented below. Raw data to be used in this calculation are listed in Table A-1 and are typical of that obtained throughout the tests. Table A-2 is a compilation of various constants and dimensions used throughout the calculation.

Table A-1

SYSTEM DATANomenclature

Run No.		158
Power, kilowatts	kw	30
Barometric Pressure	P_a	750 mm Hg
System Pressure	P_s	598.75 psig
Exhaust Steam Pressure	P_e	284.0 psig
Inlet Temperature	T_i	13.282 mv
Makeup Temperature	T_m	1.205 mv
Total Flow Manometer	R_t	7.21 in.
Correction for Total Flow Manometer	C_t	0.72 in.
Specific Gravity	S_t	1.728
Makeup Flow Manometer	R_m	5.5 in.
Correction for Makeup Flow Manometer	C_m	-0.1 in.
Specific Gravity	S_m	2.95
Steam Flow Manometer	R_s	3.7 in.
Correction for Steam Flow Manometer	C_s	-0.52 in.
Specific Gravity	S_s	13.57

PRESSURE-DROP DATA

Manometer _{2-c}	R_2	24.8 in.
Manometer Correction _{2-c}	C_2	0.35 in.
Specific Gravity	S_2	1.235
Manometer _{3-c}	R_3	14.7 in.
Manometer Correction _{3-c}	C_3	0.4 in.
Specific Gravity	S_3	1.235

Table A-1 (Cont'd.)

<u>VOID DATA</u>				
<u>Station*</u>	<u>L_j, in.*</u>	<u>V_f, v</u>	<u>V_e, v</u>	<u>V, v</u>
1'	53.70	0.555	0.910	0.555
2'	48.68	0.560	0.920	0.565
3'	43.66	0.565	0.930	0.580
4'	38.64	0.560	0.925	0.585
5'	33.62	0.560	0.920	0.600
6'	28.60	0.565	0.925	0.615
7'	23.58	0.570	0.930	0.630
8'	18.56	0.565	0.920	0.640
9'	13.54	0.560	0.915	0.645
10'	8.52	0.560	0.905	0.660
11'	3.50	0.570	0.935	0.700

* Refer to Figure 6.

Table A-2

$h_t = 7.15$ in.	Vertical leg correction for venturi meter
$C = 0.963$	Venturi coefficient
$A_2 = 0.00268$ ft ²	Venturi throat area
$V = 0.518$ liters	Test section volume
$L_T = 59.25$ in.	"Effective" heated length of test section
$L_2 = 44.75$ in.	Distance of Tap 2 from exit
$L_3 = 24.75$ in.	Distance of Tap 3 from exit
$L_C = 4.75$ in.	Distance of Tap c from exit
$D_e = 0.0377$ ft	Equivalent diameter of channel

Part 1

Table A-3 lists the system heat loss and various fluid properties that were expressed empirically as third-order polynomials.

Table A-3

<u>Nomenclature</u>		
System Pressure	P_s	612.2 psia
Saturation Temp	T_s	488.4°F
Inlet Temp	T_i	473.6°F
Makeup Water Temp	T_m	75.0°F
Subcooling	ΔT	14.8°F
Specific Heat	C_p	1.16 BTU/(lb)(°F)
Latent Heat	h_{fg}	728.7 BTU/lb
Density, sat H ₂ O	ρ_f	49.6 lb/ft ³
Density, Downcomer	ρ_D	50.3 lb/ft ³
Density, sat steam	ρ_g	1.33 lb/ft ³
Density, Makeup H ₂ O	ρ_m	62.2 lb/ft ³
Heat Loss	C_Q	3.634×10^3 BTU/hr

Venturi Pressure Differential:

$$\begin{aligned}
 P_t &= (R_t - C_t) (S_t - 1) \rho_m - h_t (\rho_m - \rho_D) \\
 &= \frac{(7.21 - 0.72) (1.728 - 1) 62.2 - 7.15 (62.2 - 50.3)}{12} \\
 &= 1.452 \text{ psf}
 \end{aligned}$$

Total Flow Rate:

$$\begin{aligned}
 W_t &= CA_2 (3600) \sqrt{2 g_c \rho_D \Delta P_t} \\
 &= (0.963) (0.00268) (3600) \sqrt{(64.4) (50.3) (1.452)} \\
 &= 2208 \text{ lb/hr}
 \end{aligned}$$

Net Heat Transfer:

$$\begin{aligned}
 Q_{\text{net}} &= (3413) \text{ kw} - C_Q \\
 &= (3413) 30 - 3634 \\
 &= 98,750 \text{ BTU/hr}
 \end{aligned}$$

Power Density:

$$\begin{aligned}
 Q''' &= \frac{Q_{\text{net}}}{V} = \frac{98,750}{(3413) (0.518)} \\
 &= 55.86 \text{ kw/Liter}
 \end{aligned}$$

Boiling Length:

$$\begin{aligned}
 L_B &= L_T \frac{Q_B}{Q_{\text{net}}} = L_T \frac{(1 - W_t C_p \Delta T)}{Q_{\text{net}}} \\
 &= 59.25 \left[1 - \frac{(2208) (1.16) (14.8)}{98,750} \right] \\
 &= 36.5 \text{ in.}
 \end{aligned}$$

Exit Quality:

$$\begin{aligned}
 X_e &= \frac{Q_B}{W_t h_{fg}} = \frac{60,843}{(2208)(728.7)} \\
 &= 0.0378
 \end{aligned}$$

Part 2

Steam Volume Fraction:

$$\alpha = \frac{\ln \frac{V}{V_f}}{\ln \frac{V_e}{V_f}} = \frac{\ln \frac{.615}{.565}}{\ln \frac{.925}{.565}} \quad (\text{Station } 6')$$

$$= 0.172$$

A running total of the area under the α -L curve was obtained by integrating the curve by the trapezoidal method. It is listed under the column

$\int_{L_1'}^{L_j'} \alpha dL$ in the summary of void data below.

Summary of Void Station Data

<u>Station</u>	<u>α</u>	<u>$\int_{L_1'}^{L_j'} \alpha dL \text{ (in.)}$</u>
1'	0	0
2'	0.018	0.045
3'	0.053	0.222
4'	0.087	0.572
5'	0.139	1.140
6'	0.172	1.920
7'	0.204	2.865
8'	0.256	4.020
9'	0.288	5.384
10'	0.342	6.966
11'	0.415	8.867

Mean Void Fractions:

Values of $\int \alpha dL$ at points between the void stations were determined as follows:

$$\begin{aligned} \int_{L_1'}^{L_2'} \alpha dL &= \int_{L_1'}^{L_2'} \alpha dL + \left(\int_{L_1'}^{L_3'} \alpha dL - \int_{L_1'}^{L_2'} \alpha dL \right) \frac{(L_2' - L_2)}{(L_2' - L_3')} \\ &= 0.045 + (0.222 - 0.045) \frac{(48.68 - 44.75)}{(48.68 - 43.66)} \\ &= 0.184 \end{aligned}$$

$$\begin{aligned}
 \bar{\alpha}_{2-3} &= \frac{\int_{L_1'}^{L_3} \alpha dL - \int_{L_1'}^{L_2} \alpha dL}{L_2 - L_3} \\
 &= \frac{2.645 - 0.184}{44.75 - 24.75} \\
 &= 0.123
 \end{aligned}$$

Similarly,

$$\bar{\alpha}_{B-C} = 0.239$$

$$\bar{\alpha}_{2-B} = 0.076$$

Void Fraction at Tap C:

$$\begin{aligned}
 \alpha_C &= \alpha_{10}' + (\alpha_{11}' - \alpha_{10}') \frac{(L_{10}' - L_C)}{5.02} \\
 &= 0.342 + (0.415 - 0.342) \frac{(8.52 - 4.75)}{5.02} \\
 &= 0.397
 \end{aligned}$$

Quality at Tap C:

$$\begin{aligned}
 X_C &= X_e \left(1 - \frac{L_C}{L_B} \right) \\
 &= 0.0378 \left(1 - \frac{4.75}{36.5} \right) \\
 &= 0.033
 \end{aligned}$$

Part 3

Static Pressure Drop:

$$\begin{aligned}
 \Delta P_s &= \left[(C_2 - R_2) (S_2 - 1) + (L_2 - L_C) \right] \frac{\rho m}{12} \\
 &= \left[(0.35 - 24.8) (1.235 - 1) + 40 \right] \frac{62.2}{12} \\
 &= 177.55 \text{ psf}
 \end{aligned}$$

Hydrostatic Head:

$$\begin{aligned}\Delta P_{HLB} &= \bar{\rho}_{LB} \frac{(L_2 - L_B)}{12} \\ &= 49.7 \frac{(44.75 - 36.5)}{12} \\ &= 34.17 \text{ psf}\end{aligned}$$

$$\begin{aligned}\Delta P_{HBC} &= \left[\rho_f - \bar{\alpha}_{B-C} (\rho_f - \rho_g) \right] \frac{(L_B - L_C)}{12} \\ &= \left[49.6 - 0.239 (49.6 - 1.33) \right] \frac{(36.5 - 4.75)}{12} \\ &= 100.8 \text{ psf}\end{aligned}$$

$$\begin{aligned}\Delta P_H &= \Delta P_{HLB} + \Delta P_{HB-C} = 34.17 + 100.8 \\ &= 134.97 \text{ psf}\end{aligned}$$

Acceleration Pressure Drop:

$$\begin{aligned}r_C &= \left[\frac{(1 - X_C)^2}{1 - \alpha_C} + \frac{X_C^2}{\alpha_C} \left(\frac{\rho_f}{\rho_g} \right) - 1 \right] \frac{1}{\rho_f} \\ &= \left[\frac{(1 - 0.033)^2}{(1 - 0.397)} + \frac{(0.033)^2}{(0.397)} \frac{(49.6)}{(1.33)} - 1 \right] \frac{1}{49.6} \\ &= 0.013145\end{aligned}$$

$$\begin{aligned}r_2 &= \left[\frac{1}{1 - \alpha_2} - 1 \right] \frac{1}{\rho_f} = \left[\frac{1}{(1 - 0.045)} - 1 \right] \frac{1}{49.6} \\ &= 0.00095\end{aligned}$$

$$\begin{aligned}\Delta P_A &= (r_C - r_2) \frac{G^2}{g_C} \\ &= (0.013145 - 0.00095) \frac{(6.08 \times 10^5)^2}{(32.2) (3600)^2} \\ &= 10.7 \text{ psf}\end{aligned}$$

Local Boiling Friction Pressure Drop:

$$\begin{aligned}\Delta P_{\text{LBF}} &= \bar{R}_{\text{LB}} f_0 \left(\frac{f}{f_0} \right) \frac{(L_2 - L_B)}{D_e} \frac{G^2}{2 g_C \bar{\rho}_{\text{LB}}} \\ &= (1.37) (0.0184) (1) \frac{(44.75 - 36.5)}{(0.0377) (12)} \frac{(6.08 \times 10^5)^2}{(3600)^2 (64.4) (49.7)} \\ &= 4.10 \text{ psf}\end{aligned}$$

NOTE: $\bar{R}_{\text{LB}}$ was obtained from Figure 9 for $\bar{\alpha}_{2-B} = 0.076$.

Single-Phase Friction Pressure Drop:

$$\begin{aligned}\Delta P_{\text{LO}} &= f_0 \left(\frac{f}{f_0} \right) \frac{(L_B - L_C)}{D_e} \frac{G^2}{2 g_C \rho_f} \\ &= (0.0184) (1) \frac{(36.5 - 4.75)}{(0.0377) (12)} \frac{(6.08 \times 10^5)^2}{(3600)^2 (64.4) (49.6)} \\ &= 11.50 \text{ psf}\end{aligned}$$

Two-Phase Friction Pressure Drop:

$$\begin{aligned}\Delta P_{\text{TPF}} &= \Delta P_S - \Delta P_H - \Delta P_A - \Delta P_{\text{LBF}} \\ &= 177.55 - 134.97 - 10.7 - 4.10 \\ &= 27.78 \text{ psf}\end{aligned}$$

Average Two-Phase Friction Factor Multiplier:

$$\bar{R} = \frac{\Delta P_{\text{TPF}}}{\Delta P_{\text{LO}}} = \frac{27.78}{11.50} = 2.42$$

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