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FM-CW MILLIMETER WAVE RADAR TECHNIQUES**

BY

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**PLASMA PHYSICS
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Plasma Density Measurements Using FM-CW Millimeter Wave Radar Techniques

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Abstract

Modified FM-CW radar techniques using swept millimeter-wave oscillators are useful for determining when a particular density has been reached in a plasma. Narrowband measurements on the Princeton Large Torus (PLT) demonstrate the suitability of these techniques for controlling high-power auxiliary plasma heating systems. Broadband measurements using these same techniques are proposed, by which the density profile could be determined.

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Introduction

Present methods of measuring plasma density require millimeter-wave or far infrared phase shift interferometers.¹ High plasma densities such as found in tokamaks require the use of high frequencies to avoid cutoff and minimize refraction in these transmission measurements. Data from a large number of interferometers are needed to estimate the density profile. The possibility that plasma density profiles could be determined from reflection measurements has been mentioned by Artsimovich² and Cano and Cavallo.³

Plasma density profiles in the ionosphere have been determined for many years from reflection measurements.⁴ Such measurements now routinely yield "ionograms," which are plots versus frequency of the apparent altitude of reflection (proportional to round trip group delay). The density profile is easily obtained from these data by a simple inversion, provided the density variation is monotonic with height.

We propose a similar determination of plasma density profiles in tokamak and related devices using broadband reflectometer measurements of round trip delay versus frequency. These data are normally obtained in ionospheric sounding by pulse techniques. For measurements on tokamak plasmas, which involve much shorter delays and much higher frequencies, FM-CW radar techniques⁵ are more suitable.

In FM-CW radar, the transmitter is swept in frequency. The round trip delay is converted to an easily measured beat frequency. We propose to sweep the transmitter over very wide frequency ranges using broadband sweepable millimeter wave oscillators. The change of the beat frequency within a sweep then gives directly the desired delay versus frequency data.

Following an outline of the basic FM-CW radar techniques, we describe preliminary narrowband measurements on the Princeton Large Torus (PLT). These

measurements demonstrated that narrowband reflectometers can be used to determine the presence of safe densities for turning on auxiliary high power plasma heating systems. They also encourage a trial of broadband measurements aimed at density profile determinations. We review the inversion procedure by which the density profile may be obtained from such measurements, discuss some details of practical implementation, and then suggest how to minimize various errors.

I. Basic Configuration

A schematic of the FM-CW reflectometer used on PLT is shown in Fig. 1. The same basic configuration would also be used for broadband measurements. A backward wave oscillator (BWO) generates a time-dependent frequency $f_o(t)$ which is sent to both the plasma and the "local oscillator" (reference) port of the receiver. Path length differences cause the relative phase ϕ_p of the reference and the reflected signal at the mixer to change with time. The beat frequency $f_B(t)$ at the mixer output is

$$f_B(t) \equiv \frac{1}{2\pi} \frac{d\phi_p}{dt} = \left(\frac{1}{2\pi} \frac{\partial \phi_p}{\partial f_o} \right) \frac{df_o}{dt} \quad (1a)$$

or

$$f_B(t) = \tau_p(f_o) f_o'(t) . \quad (1b)$$

Here the round trip group delay into the plasma is

$$\tau_p(f_o) \equiv \frac{1}{2\pi} \frac{\partial \phi_p}{\partial f_o} , \quad (2)$$

and $f_o'(t) \equiv df_o/dt$.

Since the BWO frequency is nearly a linear function of delay line voltage, a triangular output from the waveform generator causes the derivative $f_0'(t)$ to be nearly constant in magnitude. The beat frequency is then proportional to the delay. The desired reflection from the plasma occurs at the critical density, where the plasma frequency equals the BWO frequency. A bandpass filter at the receiver output suppresses beat frequencies f_B caused by spurious reflections, provided such reflections occur with delays significantly different from the delay of the desired reflection.

II. Narrowband Measurements

As an initial test of these techniques, preliminary measurements were made on PLT. Since the available windows were narrowband, the BWO was swept over the narrow range 33.5 to 35.5 GHz. Both transmit and receive waveguides passed without tapers through separate windows to horns located at the bottom of the vacuum vessel and directed vertically. The horns were oriented to propagate the ordinary wave. The bandpass filter and frequency to voltage (F-V) converter (in this case a General Radio Model 1142-A discriminator) were located in the control room. With this arrangement, the bandpass filter also eliminated low frequency pickup on the cable from the amplifiers located near PLT. The passband on the filter was set at 50-80 kHz. An RC filter with a 0.3 ms time constant was necessary to suppress residual carrier present at the discriminator output.

Since the delay variations within a narrowband sweep are small, we examined instead the long term variations in delay from sweep to sweep during a plasma shot. While the density profile cannot be obtained in this manner, the presence of a density corresponding to 35 GHz can be determined. The sweep over approximately 2 GHz was accomplished every 2 ms, so that the beat

frequency should be about 1 kHz per nanosecond of round trip delay. The round trip waveguide length to PLT was about 15 meters, for which the delay near 35 GHz in WR28 waveguide is about 60 ns. The cross-sectional diameter of PLT is about 1 meter, corresponding to a round trip delay of about 7 ns in the absence of plasma. The reflection from the top wall of PLT produced a beat frequency of about 67 kHz. This frequency dropped to approximately 60 kHz during a plasma shot, as shown in Fig. 2a.

The envelope of the signal at the input to the F-V converter is shown in Fig. 2b. The amplitude of this envelope decreased approximately 50% during the plasma shot, corresponding to a loss of 6 dB. That is, the plasma reflected 25% of the transmitted energy into the receiving horn.

Rapid 7 kHz ripples in the envelope were also observed in the absence of plasma. These are too rapid to be visible on the time scale of Fig. 2b. At a sweep rate of 1 GHz/ms, the frequency of these ripples corresponds to a delay of 7 ns, which is the round trip delay inside the vacuum vessel. These ripples are therefore caused by the interference of energy making two round trips within PLT with the desired one round trip signal. The ripples were 6 dB in peak to peak amplitude, corresponding to interference that is about 10 dB below the desired signal. Placing angled reflectors near the transmitting and receiving horns should eliminate such multiple reflections both with and without plasma.

The line averaged density through the plasma center as measured by 2 mm microwave interferometry is shown in Fig. 3. Since the density in most of the PLT plasma during most of the shot was considerably greater than $1.5 \times 10^{13} \text{ cm}^{-3}$ (corresponding to a 35 GHz cutoff frequency for the ordinary mode), the transmitted energy was reflected near the plasma edge during most of the shot. As confirmed by Fig. 2a, the round trip delay should thus show none of

the large variations within the main part of the shot that are visible in Fig. 3. Consistent with the rapid rise and fall of the line averaged density, the 7 kHz change in frequency in Fig. 2a occurred in less than 20 milliseconds at the beginning and end of the plasma shot.

A practical narrowband system for control of auxiliary high power plasma heating systems would use a swept solid state oscillator. These are small and can be swept in microseconds if necessary. The transmitter center frequency would be the plasma frequency at the critical "safe" density.

III. Broadband Measurements

A. Theoretical Basis

For profile measurements, the delay versus frequency within each broadband sweep must be analyzed. The delay $\tau(f_0)$ is determined by the round trip phase ϕ_p according to (2). The component of ϕ_p due to the plasma is, within the geometrical optics approximation, an integral along the propagation direction x of the refractive index $n(f, x)$:

$$\phi_p(f) = \frac{4\pi f}{c} \int_0^{x_c(f)} n(f, x) dx \quad (3)$$

Note that in this expression the edge of the plasma is $x = 0$, the center is $x = a$, and the cutoff point $x_c(f)$ is given by $n(f, x_c) = 0$.

For propagation in the ordinary mode, with electric field parallel to a constant magnetic field,

$$n(f, x) = \left[1 - \left(\frac{f_p}{f} \right)^2 \right]^{1/2} \quad (4)$$

where the plasma frequency $f_p(x)$ in GHz is related to the density $N(x)$ in units of 10^{14}cm^{-3} by

$$f_p = 89.79 [N(x)]^{1/2} . \quad (5)$$

Using Eqs. (2), (3), and (4) we find for the ordinary mode

$$\tau_p(f) = \frac{2}{c} \int_0^{x_c(f)} \left[1 - \left(\frac{f_p}{f} \right)^2 \right]^{-1/2} dx . \quad (6)$$

To illustrate the type of delay variations $\tau_p(f)$ that could be expected, let us consider a parabolic density profile, which is often assumed for tokamaks. In this case,

$$N(x) = N_0 \left[1 - \frac{(a-x)^2}{a^2} \right] , \quad (7)$$

where $N_0 \equiv N(a)$ and $N(0)=0$, and the integration of (6) according to (7) is straightforward, yielding the result⁴

$$\tau_p(f) = \frac{a}{c} F \ln \left(\frac{1+F}{1-F} \right) . \quad (8)$$

Here

$$F = f/f_{po} \quad (9)$$

is the ratio of the frequency to the plasma frequency f_{po} at the center of the plasma $x = a$. Figure 4 is a plot of (8) normalized to $2a/c$, the round trip delay in free space for propagation over a distance a . For low frequencies,

$$\tau_p \approx \frac{2a}{c} F^2 , \quad F \ll 1 . \quad (10)$$

Figure 4 shows that (10) is a good approximation for $F < 0.5$. Near the plasma center, where the profile slope is small, the delay is large but remains less than $2(2a/c)$ for frequencies up to 97% of f_{po} .

Use of the phase integral in (3) neglects the breakdown of geometric optics (the WKB approximation) near the turning point $x = x_c$. Full wave calculations⁶ show, however, that this neglect is usually insignificant. For a linear density profile, for example, the approximation in (3) differs from the full wave result only by $\pi/2$, regardless of the slope of the profile. There is no error in the delay, which depends on the derivative of (3). For a parabolic density profile, the delay expression in (8) is accurate except when the frequency f is very close to f_{po} . The correct delay remains finite instead of rising to infinity as Fig. 4 would indicate.

More generally, the integral in (6) can be inverted so that the density profile can be calculated from delay versus frequency data $\tau_p(f)$. Equation (6) can be converted into a Abel integral equation whose solution is:⁴

$$x(f_p) = \frac{c}{\pi} \int_0^{f_p} \frac{\tau_p(f) df}{\sqrt{f_p^2 - f^2}} \quad (11)$$

Here $x(f_p)$ is the position at which the density satisfies (5), that is, the position at which reflection occurs for energy of frequency f_p . For some purposes, it is convenient to write $f = f_p \sin\beta$ to transform (11) into the form:

$$x(f_p) = \frac{c}{\pi} \int_0^{\pi/2} \tau_p(f_p \sin\beta) d\beta \quad (12)$$

Equations (11) and (12) hold for arbitrary profiles, provided they are monotonic.

B. Practical Implementation

For accurate profile determinations to cover the full density range of tokamak plasmas, one or more very broadband sweepable oscillators are required. The oscillators best fitting this description are millimeter wave backward wave oscillators (BWO's). Practical BWO sweep rates allow complete plasma profiles to be measured over periods as short as 2 milliseconds. Data

can be obtained from both the ascending and descending portions of a triangular waveform. The output of the F-V converter could be sampled every 0.1 millisecond. Alternatively, a frequency counter with a fast computer dump could be used. Both F-V converters and frequency counters operate over a wide dynamic range in signal amplitude.

Oversized waveguide would be particularly useful in these measurements. The long distances between instrumentation and plasma devices generally require the use of oversized waveguide to reduce propagation losses. Use of such waveguide also reduces the delay dispersion, which would add a spurious frequency-dependent delay component in (1) and reduce the effectiveness of the bandpass filter at the receiver output. An overall baseline calibration including the dispersion can be made by closing a shutter next to the vacuum window. Wedge shaped windows have very flat broadband frequency response and can be oriented to minimize direct antenna-to-antenna coupling. Open-ended oversized waveguide itself is a very good broadband antenna. Finally, to cover a larger frequency band, the outputs of 2 or more simultaneously swept BWO's can be combined in the oversized waveguide.

C. Sources of Error

The frequency (or delay) resolution of the signal shown in Fig. 2a is limited by undesired variations within each 2 ms sweep, as evidenced by the thickness of the oscilloscope trace. These variations are due to frequency variations in the BWO power reaching the reference port of the mixer. Such variations could be reduced by use of a leveling loop on the BWO output. These variations are also observed to be antisymmetric about the end points of each sweep. Hence by averaging the data corresponding to each millimeter wave frequency on the up and down portions of adjacent sweeps, these spurious variations can be eliminated.

Plasma motions (density fluctuations) will produce a doppler shift term f_d in (1). For a target in free space with velocity $v_p = dL/dt$, the doppler frequency shift would be

$$f_d = \left(-\frac{1}{2\pi} \frac{\partial \phi_p}{\partial L} \right) \frac{dL}{dt} = \frac{2f_o}{c} v_p \quad (13)$$

In a plasma the situation is more complicated, because the velocity of propagation is affected by the spatially varying refractive index within the plasma. Nevertheless, the doppler shift f_d is clearly independent of the sign of the sweep $f_o'(t)$, which appears in the first term of (1). If f_d is smaller than $\tau_p f_o'(t)$ in magnitude, then f_d on up and down sweeps alternately increases and decreases the measured magnitude $|f_B|$. Hence slowly varying doppler shifts caused by relatively slow plasma motion can also be removed by the same averaging used to remove the effects of BWO power variations.

Rapid plasma motions can also cause spurious beat frequencies. If the motions are fast enough, however, the bandpass filter can suppress the resulting sidebands. Only large plasma motions with periods on the same order as the sweep speed can cause large errors.

To obtain a complete density profile using the inversion in (11) or (12) requires, in principle, delay versus frequency data from DC to the plasma frequency at the plasma center f_{p0} . Such data may be incomplete due to cyclotron absorption or incomplete frequency coverage by available BWO's. The effects of missing data are examined next.

Since the tokamak toroidal field varies primarily in the horizontal direction, cyclotron absorption is more serious for horizontal than for

vertical propagation. In tokamaks with non-circular cross section or those in which the plasma is compressed or distorted horizontally, however, propagation along the horizontal mid-plane is required in order to avoid severe refraction of the wave. An estimate of the seriousness of cyclotron absorption for these measurements can be obtained from Fig. 5, which shows the plasma frequency f_p for parabolic profiles and the electron cyclotron frequency f_B for the standard operation of Princeton's PDX machine. For example, about 3 GHz of data between 42.5 and 45.5 GHz taken from the outside wall is lost due to cyclotron absorption when the central density is $0.5 \times 10^{14} \text{ cm}^{-3}$. Increasing the density further moves the region where absorption occurs closer to the plasma edge and consequently decreases the frequency range subject to absorption. The effect of cyclotron absorption can be handled either by interpolation of the data, or by taking data from antennas on both the inside and outside walls.

Because of a high frequency limit on available EWO's or because absorption near the center of the plasma may be severe, delay data may not be available for $f_p > F_0 f_{p0}$, where $F_0 \lesssim 1$. In that event, x cannot be determined from (11) or (12) for densities $N(x)$ greater than $F_0^2 N_0$.

The effect of missing data at low frequencies $f < f_{\min}$ is more complicated. Nevertheless, $\tau_p(f)$ in (6) is zero at $f = 0$, and is small at low frequencies. For reasonable profiles, (10) shows that $\tau_p(f)$ increases roughly as f^2 for low frequencies. Hence for small $f_{\min}/f_{p0}$ we can simply use (10) in (11) or (12) for frequencies less than $f_{\min}$. The accuracy of the profile determined in this way depends upon how severely the actual profile departs from parabolic near the plasma edge. Further discussion on obtaining $x(f_p)$ from $\tau_p(f)$ data can be found in reference 7.

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Fig. 1. Schematic of FW-CW Radar Circuit for Plasma Density Measurements.

(PPPL-804313)

Fig. 2. (a) Output of Frequency-to-Voltage Converter during PLT shot. (b) Envelope of Signal entering Frequency-to-Voltage Converter.

(PPPL-804314)

Fig. 3. PLT line average density along a central vertical chord for same shot as Fig. 2, as determined by a 140 GHz interferometer.

(PPPL-804366)

Fig. 4. Normalized round trip delay vs frequency for propagation in a plasma with radius a and parabolic density profile. The plasma frequency at the center of the plasma is f_{p0} .

(PPPL-804365)

Fig. 5. Cyclotron resonance f_B and plasma frequencies f_p for PDX. Major radius = 1.40 m, minor radius = 0.45 m. Toroidal field at center plasma = 2T. Parabolic density profiles.