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UNIFICATION THROUGH A SUPERGROUP

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ABSTRACT

The representations of supergroups provide the most precise fit to the known set of fundamental physical matter fields. $SU(2/1)$ describes the unified weak-electromagnetic interactions and $SU(5 + k/1)$ contains $SU(3)$ color and 2^k exact "generations". Moreover, $SU(2/1)$ predicts that the 4th states of the lepton multiplets (i.e. ν_R^0) de couple, whereas they don't in quark multiplets and $SU(n/1)$ predicts that the color degree of freedom is an $SU(3)$. We provide all relevant constructions.

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1. SU(2/1) and the quantum numbers of the spinor fields

It has been suggested ¹⁾²⁾ that the supergroup SU(2/1), when gauged in a way appropriate to an internal supergauge, generates the unified electroweak (or asthenodynamic) interaction. The resulting Lagrangian roughly corresponds to a highly restricted Salam-Weinberg $SU(2)_L \times U(1)$ model.

We have applied a theorem stating that the assignment of different gradings ³⁾, within the spinor field vector space v , between v_L (left) and v_R (right) chiralities, ensures that all matrices of $su(2)_L \otimes u(1)$ be supertraceless.

$$\text{str } M = \text{tr } M_L - \text{tr } M_R = 0 \quad (1.1)$$

Proof: (a) The electric charges of any components are reflected

$$Q(v_L^m) = Q(v_R^m)$$

$$\therefore \text{str } M \{Q\} = 0$$

$$(b) \text{tr } M_L \{su(2)_L\} = 0, \quad M_R \{su(2)_L\} \equiv 0$$

$$(c) Q = I_3^L + \frac{1}{2}U \quad (1.2)$$

(I_3^L is the third component of the weak left-handed isospin, U is the weak hypercharge). Thus, since (b) implies $\text{str } M \{I_3^L\} = 0$, we have

$$\text{str } M \{U\} = 0$$

Postulating that supertracelessness reflects the presence of a supergroup, we have assumed that this is SU(2/1). The defining (irreducible) representation $\mathcal{D}(1,1)$ in the notation of Scheunert et al ⁴⁾ (which we denote as SNR in this paper) carries the eigenvalues

$$I_3^L = \text{diag}(1, -1/2, 0), \quad U = \text{diag}(1, 1/2) \quad (1.3)$$

$$\text{fitting the antileptons } \left((\overline{e}_L^-)^+, (\overline{\nu}_L^0)_R / (\overline{e}_R^-)^+ \right)$$

Note that in ref. ¹⁾ we had picked the lepton eigenvalues for our definitions. Our present choice (1.3) provides for positive eigenvalues. It has the additional advantage of fitting the SNR notation $\mathcal{D}(1, u, i_{3\max}^L)$ characterizing the eigenvalues of the state with highest U within the set with highest I_3^L . The SU(2/1) hypothesis scores highly in providing us with a (star-hermitean) irreducible representation. $\mathcal{D}(1/6, 1/2)$ fitting the quarks

$$(u_L^{2/3}, d_L^{-1/3} / d_R^{-1/3}, u_R^{2/3}) \text{ precisely } ^5),$$

$$\left. \begin{aligned} \mathcal{D}(1/6, 1/2; I_3^L) &= \text{diag} (1/2, -1/2 / 0, 0) \\ \mathcal{D}(1/6, 1/2; U) &= \text{diag} (1/3, 1/3 / -2/3, 4/3) \end{aligned} \right\} \quad (1.4)$$

Note that the finite representations of $SU(2/1)$ are those of $SL(2/1)$, and had been listed ⁴⁾ by SNR prior to the formulation of our theory. The definition of unitarity for $SU(2/1)$ is based on (id. is the identity matrix)

$$M^+ = (M^T)^x, \quad M^+ M = \text{id.}$$

where M^T is ordinary transposition, and x denotes complex conjugation with inversion of the order of Grassmann generating-elements within each matrix-element. As long as we do not have a complete quantum theory of internal supergauge, we do not know how relevant Unitarity and hermiticity are in our representations. Condition (1.1) ensures super unimodularity

$$\text{sdet } M := \det (M_R) \cdot \det (M^{-1})_L = 1 \quad (1.5)$$

Note that the 4-dimensional representation in (1.4) depends on a parameter ⁶⁾ fixing the $\mathcal{U}_{\text{mid}} = 1/6$ SNR eigenvalue. This parameter b translates the eigenvalues of $\mathcal{D}(U)$, without affecting the dimension. Taking, for reasons that we shall touch upon later, $b = 0$ for the leptons (and more generally, $\mathcal{U}_{\text{mid}} = b - 1/2$)

$$\left. \begin{aligned} \mathcal{D}(-1/2, 1/2; I_3^L) &= \text{diag} (1/2, -1/2 / 0, 0) \\ \mathcal{D}(-1/2, 1/2; U) &= \text{diag} (-1, -1 / -2, 0) \end{aligned} \right\} b = 0 \quad (1.5)$$

as befits $(\nu_L^0, e_L^- / e_R^-, \nu_R^0)$

we get for $b = 1$, the 4 antileptons $((e_L^-)^+, (\nu_L^0)_R / (\nu_R^0)_L, (e_R^-)_L)$ given by $\mathcal{D}(1/2, 1/2)$.

The quarks (1.4) correspond to $b = 2/3$, and the antiquarks

$$\left((\overline{d}_L^{-1/3})_R, (\overline{u}_L^{2/3})_R / (\overline{u}_R^{2/3})_L, (\overline{d}_R^{-1/3})_L \right)$$

to $b = 1/3$ or $\mathcal{D}(-1/6, 1/2)$.

Note that the representations in which the absolute values of the two SNR quantum numbers coincide, or alternatively in which b is an integer, are reducible. They are not completely reducible, but $b = 1$ has the 3-dimensional set (1.3) as an invariant subspace. Integer b implies integer U (and even U for ν_R), i.e. integer electric charges Q , as fixed by (1.2). $SU(2/1)$ may thus be said to predict that for integer charges, one state

$(v_R^0 \text{ or } \overline{(v_R^0)_L})$ disconnects.

2. Physical interpretation of the vector-space statistics

Our introduction of $SU(2/1)$ was motivated by the finding ⁷⁾ that the ghosts X^a , introduced in the quantization procedure of a Yang-Mills theory, correspond to proper geometric "classical" objects, natural to the Principal Fiber Bundle description ⁸⁾. We refer the reader to the rederivation of the Becchi-Rouet-Stora equations ⁹⁾,

(w_μ^a is the Yang-Mills field potential, ψ^m a matter field)

$$s w_\mu^a = (D_\mu X)^a \quad (2.1)$$

$$s X^a = -\frac{1}{2} [X, X]^a \quad (2.2)$$

$$s v^m = [X, v]^m =: \psi^m \quad (2.3)$$

$$s^2 w_\mu^a = 0, \quad s^2 X^a = 0, \quad s^2 v^m = 0 \quad (2.4)$$

in which the ghosts X^a are seen to represent the "vertical" pieces of the complete connection on the principal bundle $P(M, G)$. We use equations (2.3) to define an "effective" matter-ghost ψ^m , with statistics opposed to those of v^m . Note that in the original derivation (10),

$$s = -\frac{\partial}{\partial \lambda} \delta_B \quad (2.5a)$$

in its action on w_μ^a , X^a and ψ^m . Here δ_B represents an ordinary gauge transformation, in which the parameter $\alpha^a(x)$ has been factorized into two fermionic factors,

$$\alpha^a(x) = X^a(x) \lambda \quad (2.5b)$$

where λ is a constant Grassmann anticommuting element. In our classical treatment ^{1), 5)} we extended equations (2.1 - 2.4) to the case of a supergroup (with a detailed presentation in ref. ¹¹⁾). All multiplets take on new components with opposite grading (assuming that G in (2.1 - 2.4) is the maximal even subgroup of the supergroup $\mathcal{G}$). We now write

$$\begin{pmatrix} w_\mu^a \\ \chi_\mu^i \end{pmatrix} = \epsilon_\mu \begin{pmatrix} \chi^a \\ w^i \end{pmatrix} \quad (2.6)$$

$$s \begin{pmatrix} v^m \\ \psi^r \end{pmatrix} = \begin{pmatrix} \psi^m \\ v^r \end{pmatrix} \quad (2.7)$$

where $\epsilon_\mu = s^{-1} D_\mu$, a formal solution of (2.1).

χ_μ^i is a new vector-ghost (fermionic, like $D_\mu \chi^i$ in (2.1)

and w^i is the Goldstone-Higgs multiplet. In $su(2/1)$ it

has the correct phenomenological assignment!

We use latin letters for physical fields and greek for ghosts. The indices a, i and m, r represent even-odd components in the algebra and in the matter representations respectively. The action of s may be inverted, as can be seen in (2.5a). Equation (2.7) and its inverse allow us to interpret the representations of $sl(2/1)$. Each of $b = 0, 1$ and $b = 2/3, 1/3$ have to appear twice, with inverted statistics, as on both sides of (2.7). In the first, the field v^m corresponding to the highest weight is a physical chiral spinor and the components with opposite gradings are ghost-fields ψ^r related by s to the physical v^r in the second representation. Similarly, s acting on v^m generates the ghosts ψ^m which fill out the highest weight chirality in that second representation. This is known as our "ghost interpretation" of the supergroup and its representation. We shall not deal here with the gauge-fields (2.6), but note that our classical treatment ^{1), 5)} yielded $\frac{4}{3}g^2$ for the w^4 coupling, and 250 GeV for the Higgs field's mass. We also had $\sin^2 \theta_w = \frac{1}{4}$ as the unrenormalized Weinberg angle, with a conjecture that it is not renormalized except for the $SU(2/1)$ breakdown itself (i.e. ~ 100 GeV rather than 10^{15} GeV of GUTs).

Without a quantum treatment, these results should be considered as tentative. We encountered some difficulties in dealing with the gauge-multiplet. The Killing metric

$$g_{AB} = \text{str} (M_A M_B) \quad (2.8)$$

is not positive-definite in the g_{uu} even direction (that of the $U(1)$ in the even subgroup $SU(2)_L \times U(1)$). It is also anti-symmetric in the odd directions (whereas w^4 requires a symmetric metric). We deal with these issues elsewhere and prefer to concentrate on the kinematical aspects, which are as spectacular in $su(2/1)$ as they were in the original unitary symmetry hadron $su(3)$.

However, our ghost interpretation is not the only one which has been suggested. Other interpretations have been proposed, based on extending space-time by additional Bose ²⁾¹²⁾ or Fermi dimensions ^{13,14,15)}, etc. The results presented in the rest of this paper are independent of the ghost conjecture and can be adopted in either interpretation.

3. The fundamental representations of $sl(n/m)$

In a recent study ⁶⁾, we have constructed a class of "fundamental" representations of $sl(n/m)$. These resemble the representations we used ¹⁷⁾ to describe the supergravity gauge multiplet. For $m \neq 1$, they are given by (k as an integer)

$$\Lambda^0(V) \otimes S^k(X) \oplus \Lambda^1(V) \otimes S^{k-1}(X) \oplus \dots \oplus \Lambda^n(V) \otimes S^{k-n}(X) \quad (3.1)$$

where $\Lambda(V)$ is the Grassmann manifold constructed on the fermi subspace $V = \Lambda^1$ of the defining vector-space $W = V + X$. The S^{k-i} is the space of homogeneous polynomials of degree $k-i$ on X^* and $S^j = 0$ for $j < 0$. In the special case $m = \dim X = 1$, (3.1) can be replaced by

$$\Lambda^0(V) \otimes F^b \oplus \Lambda^1(V) \otimes F^{b-1} \oplus \dots \oplus \Lambda^n(V) \otimes F^{b-n} \quad (3.2)$$

where $b \in \mathbb{C}$ and F^{b-i} is the one dimensional space of possibly multivalued functions defined on X and homogeneous of degree $b-i$. The dimensionality of (3.2) is 2^n , the same as that of $\Lambda(V)$. For (3.1) we recover the defining representation by taking $k = 1$ since we are left with

$$\Lambda^0(V) \otimes X \oplus \Lambda^1(V) \otimes \mathbb{C}$$

In extended Supergravity the odd generators behave like $\underline{n}$ of $SO(n)$ and have helicity $j_z = \frac{1}{2}$. One starts with the up-graviton with $j_z = 2$ and acts with the odd annihilation operators once, getting n $j_z = 3/2$ states behaving like $\underline{n}$ etc.

Here, working with the simpler $SL(n) \supset SU(2/1)$, we have altogether 3 parameters: 2 in picking the eigenvalues of U in the defining representation, and the quantum number b . We are given the highest weight, i.e. u_{mid} . Assuming for $sl(n/1)$

$$I_3^L = \text{diag} \left(\frac{1}{2}, -\frac{1}{2}, 0, \dots, 0/0 \right) \quad (3.3)$$

$n - 2$ zeros

and

$$U = \text{diag } (\alpha, \alpha, (\beta) \dots, (\gamma) / T) \quad (3.4)$$

n-5 3 times
times

$$T = 2\alpha + (n-5)\beta + 3\gamma$$

Note that if the $u_{\max}$ is the highest value of U in the representation b and if the lowest value is $-u_{\max}$ (in a self conjugate representation) we have $\int u_{\max} = \frac{n-1}{2} T$ (3.5)

$$b = \frac{n-1}{2} \quad (3.6)$$

and $u_{\max} = 2$ from phenomenology

E.G. (3.6) reduces our freedom of choice to 2 parameters! To construct the states of the representation and denoting by E the set of diagonal operators I_3^L , U etc., we have for their eigenvalues

$$\Lambda^0 : e_{\max} \text{ (given)}$$

$$\Lambda^1 : (i^{\text{th}} \text{ state}) e_{\max} - T(\epsilon) + \epsilon_i$$

$$\Lambda^2 : (i, j^{\text{th}} \text{ state}) e_{\max} - 2T(\epsilon) + \epsilon_i + \epsilon_j$$

E_i are the eigenvalues of E in the defining representation etc.

We have sketched here the mechanism for the construction of the representation's vector space. We refer the reader to ref. 6) for proofs and for the actual construction of the representation matrices.

4. Unification: precise generations and the prediction of su(3) color.

It is remarkable that with such a restricted system, the results should again be so spectacular. Using the choice

$$\left. \begin{aligned} &sl(n/1), n-5 = k \geq 0; \quad sl(n/1) \supset su(2/1) \oplus su(3)_c \\ &b = \frac{n-1}{2} \\ &U = \text{diag} \left(\left(\frac{-k}{4+k}, \frac{-k}{4+k}, \left(\frac{4}{4+k} \right)_{k \text{ times}}, \left(\frac{4-2k}{3(4+k)} \right)_{3 \text{ times}} \mid \frac{4}{4+k} \right) \right) \\ &I_3^L = \text{diag} \left(\left(\frac{1}{2}, -\frac{1}{2}, (0)_{k+3 \text{ times}} \mid 0 \right) \right) \end{aligned} \right\} \quad (4.1)$$

we get a 2^n -dimensional representation containing precisely 2^k "generations" of the basic 16 fermion fields ψ^m and their 16 matter ghosts $\bar{\psi}^m$. The discrete s operator in (2.7) is thus embedded in the superalgebra for $n \geq 5$. This set of 32 fields make up the $b = 2$ representation of $sl(5/1)$, which is itself, reducible

(though not completely reducible, a common case in superalgebras). This feature of reducibility and an invariant $2n-1$ subspace occurs for all $sl(n/1)$ if b is an integer. In the case of $sl(5/1)$, in approaches which do not distinguish between fields and ghosts, the irreducible 16-dim. invariant subspace already contains a full set of fields. In either case, the fields sometimes appear through their charge-conjugates, as in the Georgi-Glashow $su(5)$. However, if we take $sl(6/1)$ as the basic physical set, its irreducible 64-dim. $b = \frac{5}{2}$ representation is charge-conjugation symmetric and contains all 16 fields, their ghosts and the charge-conjugate states of both. Depending on the treatment, this could be regarded as one, two or four generations. C-symmetry might be important for the removal of Adler-Bell-Jackiw anomalies.

We have introduced color $SU(3)$ through the step $sl(2/1) \rightarrow sl(5/1)$. Our quantum numbers (4.1) produce an $su(5)$ which does not coincide with the Georgi-Glashow $su(5)$ assignments, since our $U \not\subset su(5)$ of $sl(5/1) \supset su(5) \times u(1)$ as can be seen in (4.1). However, it is also possible in $sl(5/1)$ to identify U with an $su(5)$ quantum number as in the Georgi-Glashow model, as was suggested by A. Salam and J. G. Taylor¹⁶⁾⁶⁾.

Beyond this spectacularly unique fit with the generations issue, it has also been shown¹⁸⁾ that the supergroup structure requires color to be represented by $su(3)$. Indeed, these fundamental representations impose a unique sub-division in the eigenvalues of U in (4.1), generating the $su(3)_{\text{color}}$ group commuting with $sl(2/1)$, if we require that the color group $su(r)$ appear in representation $\underline{r}$ for all quarks.

One important result of such a unification is the fact that all the matter fields (and their ghosts, in that interpretation) can be considered as composite. The fundamental constituent is the fractionally-charged $n+1$ dimensional primitive field with n fermions and one ghost. The charges will depend on the final selection for k .

In the multigeneration systems beyond $sl(5/1)$, the correlation between chirality and the grading vanishes.

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