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ISOTOPES — INDUSTRIAL TECHNOLOGY

FEASIBILITY STUDY OF A NEW MASS FLOW SYSTEM

Final Report

By
J. W. Haffner

January 31, 1962

Armour Research Foundation
Illinois Institute of Technology
Chicago, Illinois

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FEASIBILITY STUDY OF A NEW MASS FLOW SYSTEM

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Contract No. AT(11-1)-578
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FINAL REPORT

by

ARMOUR RESEARCH FOUNDATION
of Illinois Institute of Technology
Technology Center
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Report written by
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January 31, 1962

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This is the final report on ARF Project 1167 entitled, "Feasibility Study of a New Mass Flow System." This report covers the period from June 1, 1960 to November 31, 1961, and was conducted for the U. S. Atomic Energy Commission, Division of Isotopes Development, under Contract No. AT(11-1)-578, Project Agreement No. 5.

The author wishes to express his appreciation for the valuable assistance of those who contributed to the success of this program. The work of W. K. Genthe of Badger Meter Manufacturing Company especially deserves recognition, as he was responsible for much of the actual fabrication and testing of the gauge. C. A. Stone was a continuing source of ideas which contributed materially. G. M. Burgwald participated in the early phases of the program as well as prior work on self referencing density gauges which led to this research. C. M. Groom of ARF and W. Strobel of Badger Meter were responsible for much of the electronic circuitry. For the help of these and other individuals who worked on the program, grateful acknowledgement is made.

Respectfully submitted,

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FEASIBILITY STUDY OF A NEW MASS FLOW SYSTEM

ABSTRACT

A program of research and development activity was undertaken to demonstrate the feasibility of a new mass flow metering technique. A function (ρv^2) of flow velocity (v) and density (ρ) is measured by means of passing a fluid through a curved section of pipe and measuring the resulting rotational torque. This signal is multiplied by the fluid density (ρ) which is measured by means of a self-referenced, gamma ray absorption densitometer. The square root of this product is extracted to yield a readout for mass flow rate. Mass flow rate is integrated to provide a readout for mass flow quantity. The initial design goal of 3/4 per cent accuracy was attained and verified by an extensive series of tests. The results of these and other tests demonstrated the feasibility of the proposed technique and established it as a new mass flow metering principle.

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FEASIBILITY STUDY OF A NEW MASS FLOW SYSTEM

I. INTRODUCTION

The development of a mass flow metering technique universally suitable for all types of fluids has been the object of widespread study for many years. Patent literature shows attempts at solution of the mass flow problem since the beginning of the United States Patent Office, none of which have proven generally successful although some have shown promise in limited applications.

There are only ten general mass flow measurement techniques given in the patent and periodical literature. Inspection of many hundreds of patents revealed that each could be catalogued in one of these ten categories. They are:

1. Angular Momentum or Coriolis Force
2. Gyroscopic - rotating and vibrating
3. Magnus Effect
4. Twin Turbine
5. Combination of Venturi and Turbine
6. Flow Column Acceleration
7. Driven Streamline Bodies
8. Constant Displacement Circulating Pumps
9. Boundary Layer Heat Transfer
10. Combination of Positive Displacement or Inferential Volumetric Flow Meter with a Densitometer.

The tenth category above has never enjoyed notable success primarily because of the inadequacies of previous densitometer design. Only the first, and second techniques have found limited application in industry. The chief difficulty has been the size, complexity and fragility of these instruments. Further, none of these fulfill all of the desirable performance requirements industry places on mass flow instrumentation.

1. Accuracy better than $\pm 0.1\%$ of reading.
2. Ability to measure:
 - a) Homogeneous flow
 - b) Slurries
 - c) Highly corrosive fluids
 - d) Multiphase fluids.
3. Completely "sanitary" construction - no sharp corner or inaccessible cavities, able to be easily cleaned.
4. Low Pressure Drop.
5. Simple Mechanical Construction.
6. Extreme Ruggedness in Service.
7. No requirement for compensation for fluctuations in temperature, voltage level or frequency.
8. Reliable, stable calibration.
9. Cost between 750 and 1,750 Dollars.
10. Low Pressure Drop - minimum stream interruption.
11. Flow range 25:1.

On the basis of this background information and our knowledge of widespread specific industrial and commercial need for improved mass flow and

density measuring equipment, Armour Research Foundation and Badger Meter Manufacturing Company entered into a program of research and development in an effort to demonstrate the feasibility of a new mass flow metering technique.

II. DESCRIPTION OF THE MASS FLOW MEASUREMENT TECHNIQUE

A. Flow Element Design Features

1. General Description

An elastically isolated S-shaped section of conduit generates a torque about its axis of suspension when flow is caused to pass through it. As shown in Figure 1, this torque provides a relationship between flow rate and density which can be used to measure mass flow rate, volumetric flow rate, or density if one of these variables is known.

The torque has the same sign regardless of the direction of flow, so the flow element is bi-directional. For a known density, torque is directly proportional to the square of mass flow rate. Therefore, a measurement of the square root of this torque and of density provides a measurement of mass flow rate.

The flow range is equal to the square root of torque range, so a 20:1 flow range requires a 400:1 range in the torque measurement. This fact, together with an accuracy requirement of the order of 0.05 per cent on torque measurement, means that a very sensitive measurement technique must be used. A null balanced servo was selected which has excellent threshold, linearity, and dynamic characteristics, as described in the following.

Torque due to the passage of flow through the S-tube is resisted by a coil spring wound up by a torque motor. The torque motor is driven by an amplifier which receives its input from a synchro whose rotor is connected to the S-tube axis as shown in Figure 2 and 3. This

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$$dV = \pi r^2 R d\theta$$

$$dM = \pi r^2 R \rho d\theta$$

$$dF = \frac{v^2}{R} dM$$

$$dF_h = \frac{v^2}{R} \rho \pi r^2 R \cos \theta d\theta$$

$$F_h = v^2 \rho \pi r^2 \int_0^{\pi/2} \cos \theta d\theta$$

$$F_h = v^2 \rho \pi r^2$$

$$v = Q / \pi r^2 = Q / A$$

$$W = \rho Q$$

$$F_h = \rho Q^2 / A = W^2 / \rho A$$

$$T = 2 F_h L = 2 \rho Q^2 L / A = 2 W^2 L / \rho A$$

$$W^2 = \frac{\rho A}{2L} T \text{ and } Q^2 = \frac{A}{2\rho L} T$$

where:

dV = element of volume

dM = element of mass

dF = element of centripetal force

r = pipe radius

R = pipe bend radius

v = average velocity of the flow stream

 ρ = densitydF_h = horizontal component of dFd θ = element of angle

Q = volumetric flow rate

A = pipe cross section area

W = mass flow rate

T = torque about O

L = effective lever arm

FIGURE 1. DERIVATION OF TORQUE PROPORTIONAL TO FLOW RATE

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$$T = 2 \frac{W^2}{PA} L$$

$$T = 2 \frac{PQ^2}{A} L$$

FIGURE 2. S-TUBE TORQUE SERVO SCHEMATIC

FIGURE 3. S-TUBE FLOW ELEMENT WITH SIDE HOUSING REMOVED

closed servo loop constrains the S-tube to its null position, and provides a linear relationship between torque and the angular position of the torque motor shaft. The angular position of the torque motor shaft is read by means of the two dials shown in Figure 4. These dials provide a coarse and fine readout. The coarse dial rotates one full turn for maximum flux rate while the fine dial rotates ten turns. Hence the torque readout is in terms of per cent maximum torque. A ten turn potentiometer is geared to the fine dial for transmission of a voltage proportional to torque.

The minimum flow rate at a given accuracy is limited by friction in the S-tube support bearings. The maximum flow rate is limited by interleave clearances of the coil spring and the maximum continuous torque rating of torque motor.

The entire spring-S-tube-synchro assembly is immersed in a sealed oil bath to cancel the effects of line pressure, and pressure losses along the S-tube. The enclosure shown in Figure 3 is closed by a mating oil tight side cover as shown in Figure 4. Limited rotational freedom about the S-tube's central axis is provided by means of very flexible brass bellows which connect the S-tube's extremities to the inlet and outlet conduits. Pressure in the oil bath surrounding the S-tube is maintained equal to the average of inlet and outlet pressure by means of small diameter tubing connecting the inlet and outlet with the pressure chamber as shown in Figure 9.

FIGURE 4. MASS FLOW METER IN TEST STAND

2. Torque Measurement Servo

a. Servo Components

The torque measurement servo sketched in Figure 2 is made up of the following components:

S-tube

The S-tube is fabricated from PVC, polyvinyl chloride, a rugged plastic. Plastic was chosen for its radiation transparency (low density). The center section is square in cross section to present a more nearly consistent wall thickness to the radiation beam.

Torsion Spring

The torsion spring is made from 15 turns of 1" by 0.062" 18-8 stainless steel, cold rolled 40-50 per cent reduction, stress relieved at 525° for 24 hours after coiling, with an active length of 212.5". The spring is so constructed that 2.34 revolutions will result in 4 foot pounds of torque for a spring constant equal to 1.71 foot pounds per revolution. This spring constant is linear within ± 0.05 per cent at 20° C.

Torque Motor

The torque motor is a two phase Louis Allis type PJIT pancake torque motor, 1812 frame. It will deliver 4 foot pounds of torque at 220 VAC, 3.5 amperes per phase. The shaft is extended from the rear housing to drive readout gearing and dials, and a vernistat.

Synchro Error Generator

The synchro error detector is an Oster type 4003-20 which has a sensitivity of 200 millivolts per degree at 12 VAC excitation.

Using a second remote synchro as excitation makes it possible to shift instrument null and zero residual torque preload on the spring.

Torque Measurement Servo Amplifier

The S tube amplifier is a Cutler-Hammer No. E818215A and uses a pair of silicon controlled rectifiers driven from a magnetic amplifier, and a DC power supply. Both phases of the torque motor are excited by the amplifier. This provides minimum heat dissipation in the motor but results in a squared transfer function between motor torque and amplifier input voltage.

Flow Element Housing

The flow element housing is comprised of a pair of aluminum castings bolted together. Bearing and synchro mounts are bolted to machined pads which are part of the castings.

b. Torsion Calibration

The accuracy of the S-tube measurement is directly influenced by the precision with which the spring constant of the torsion spring has been measured. Proper interpretation of the flow element output dials requires that the relationship between torque motor angular displacement and S-tube torque be accurately known.

1) Torsion Spring Calibration Method

With the flow element disassembled as shown in Figure 3, precisely known torques were applied to the spring lever arm which is rigidly connected to the S-tube. These torques were generated by

means of a light weight cord attached to the lever arm, passed over a precision bearing, and supporting a pan upon which sealer's weights were placed. These weights are known to a precision of 0.02 per cent and provide weights from 1/4 ounce to 30 pounds in 1/4 ounce increments. As each weight was added, current was increased to the torque motor until the lever arm returned to its null position. At each torque value, the torque motor shaft rotation was read to the nearest 0.5 degree within a total possible rotation of 1080 degrees for a precision of 0.05 per cent.

2) Measured Torsion Spring Constant

Table 1 is a typical calibration run at 80° F, taken with 4 ounce (0.0874 ft. lb.) and 1 pound (0.3594 ft. lb.) increments. Averaging the values of k for the ten highest torque loadings yields a spring rate of 0.004741 ft. lbs. per degree of rotation or 0.0512 ft. lbs. per per cent of rotation at 80° F. Figure 5 demonstrates the good linearity of the torsion spring over all its range.

c. Torque Measurement Servo Amplifier

The servo amplifier responds to inputs from the synchro whose rotor is driven by the S-tube and whose stator is excited from an externally mounted reference synchro pictured in Figure 6. The rotor of the reference synchro can be positioned in increments of seconds of arc by means of the gear reduction shown also in Figure 6. The servo amplifier is pictured in Figure 7 and is comprised of components arranged as

TABLE I

Sealers Weight at 4.3125 lever arm lbs.	Torque T ft. lbs.	Percent Rotation (100% max)	Spring Rotation Degrees	k Torque/deg.
0	0	0	0	---
1/4	.0874	1.80	19.44	.00449
1/2	.1798	3.60	38.80	.00463
3/4	.2699	5.37	58.00	.00465
1	.3594	7.08	76.46	.00470
1.52	.524	10.70	115.6	.00453
2.02	.726	14.16	152.9	.004748
3.02	1.085	21.20	229.0	.004738
4.02	1.443	28.15	304.0	.004747
5.02	1.805	35.07	378.8	.004765
6.02	2.163	42.40	457.9	.004724
7.02	2.521	49.35	532.9	.004731
8.02	2.881	56.23	607.3	.004744
9.02	3.240	63.19	682.5	.004747
10.02	3.600	70.35	759.8	.004738
11.02	3.960	77.60	838.1	.004725
Average k for 10 last readings				.004741

FIGURE 6. REFERENCE SYNCHRO

FIGURE 7. TORQUE MEASUREMENT SERVO AMPLIFIER

in Figure 8. The silicon controlled rectifier output stage excites both windings of the torque motor to generate a restoring torque on the S tube proportional to the square of the angular deviation of the S-tube from its null position. The synchro used to detect S-tube rotation has a sensitivity of 200 millivolts per degree. The amplifier which drives the torque motor delivers maximum power to the torque motor at an input voltage of 100 millivolts. Therefore, the greatest angular deflection described by the S tube at maximum static flow rate is 30 minutes of arc. Each flexible coupling elongation E is given by:

$$E = R\theta = \frac{3(0.5)}{57.3} = 0.0262''$$

where:

$$R = 3''$$

$$\theta = 30 \text{ minutes or } 1/2 \text{ degree}$$

At 1/2 maximum flow rate, coupling elongation is: $0.0262/4$ or $0.00655''$.

The square relationship between amplifier output voltage and torque constitutes a variable gain which can cause servo instability problems at high flow rates. At maximum flow rate, this gain equals 0.0425 foot pounds of torque per volt of amplifier output. At minimum flow rate, this gain drops to 0.021 ft. lb/volt, lower by a factor of two. Total loop gain must be adjusted for a closed loop damping factor ξ greater than 0.5 at high flow rates. The dynamic characteristics of the motor-spring-S-tube combination were evaluated. The viscous drag of the oil did not provide sufficient damping for acceptable transient response. A DC tachometer was geared

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GREEN	GN
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to the torque motor shaft to provide high level rate feedback. Adjustment of the gain of this feedback channel allows a selection of response from underdamped to heavily overdamped, and response times from milliseconds to 20-30 seconds.

Closing of the S-tube servo loop through the 100 watt drive amplifier for fully automatic operation demonstrated that the loop was stable for low amplifier gains only. Instability at high gains was brought about by a low frequency lag term in the amplifier transfer function. In order to take best advantage of the self-nulling feature of the S-tube loop, it is necessary to operate at as high a forward gain as possible. To determine what type of passive compensation network would stabilize the system at high gain, the dynamics of the loop were investigated using block diagrams and plots of attenuation and phase shift as a function of frequency. This analysis indicated that another lag at still lower frequency would stabilize the system but would also increase the response time. A simple lag network was constructed and placed at the input to the servo amplifier.

This network stabilized the loop and brought about adequate loop performance for constant flow rates. Full scale travel time from zero to maximum flow rate is about two seconds. Full scale travel time from maximum to zero is about 20 seconds. This difference is caused by the fact that the amplifier responds only to in-phase input signals. When returning to zero, the synchro output voltage is out of phase, and the return of the torque motor to zero is provided by the torsion spring operating against the viscous damping provided by the tachometer sub-loop.

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$$P_3 = \frac{P_1 + P_2}{2}$$

FIGURE 9. PRESSURE EQUALIZATION DIAGRAM

3. Pressure Equalization

a. Effects of Pressure Drop Along the S-Tube

Due to pressure loss around the two 90° bends in the S-tube, shown in Figure 9, the pressure at the inlet P_1 does not equal the pressure P_2 at the outlet. It was therefore considered that an error torque may exist equal to:

$$E_{T1} = R A (P_1 - P_2) \quad (1)$$

where:

A = Effective area over which the total pressure differential acts.

R = Effective bend radius.

However, P_1 and P_2 are conversant with the oil bath surrounding the S tube through the pressure equalization tubing.

Therefore, a pressure P_3 exists in the oil bath equal to:

$$P_3 = \frac{P_1 + P_2}{2} \quad (2)$$

which is the arithmetic average of P_1 and P_2 . For our purposes, the casting which forms the outer container for the oil bath is rigid and the oil is virtually incompressible (0.1% volume reduction at 100 psi).

Define ΔP :

$$\Delta P = P_1 - P_3 = P_3 - P_2 \quad (3)$$

The force due to P_1 is $P_1 A$. It is resisted by a force $P_3 A$. The force due to P_2 is $P_2 A$, resisted by the same counter force $P_3 A$.

$$\begin{aligned}\text{Torque about 0} &= P_1 AR - P_3 AR + P_2 AR - P_3 AR \\ &= AR (P_1 - P_3 + P_2 - P_3)\end{aligned}\quad (4)$$

But equation (3) states that:

$$P_1 - P_3 - P_3 + P_2 = 0.$$

Therefore, the torque about 0 is seen to be zero and the error torque considered in equation (1) does not exist.

b. Effects of Variations in Effective Bend Radii

Differences in the velocity profile across the pipe at the entrance to the two 90° bends of the S-tube may cause differences between their mean radii R . For all flow rates under consideration flow will be turbulent. Additional eddies and recirculations can be expected to form as the flow passes over the elastic couplings and around the bends. Accordingly it is difficult to predict what the effective flow profile across any pipe section will look like. For any one configuration it would be expected that the effective radii differences would be independent of pressure and would vary repeatably with flow rate, if at all. In this event the flow rate output signal of the meter would depart from a perfect square function in a repeatable fashion and could be compensated. Pressure differences caused by losses due to this local secondary flow produce a torque reaction which is negated exactly as in a) above. No appreciable error from this cause was observed during the test program.

c. Effects of Unequal Coupling Cross Section Area

Unequal pressures P_1 and P_2 will cause unequal areas at the couplings as one expands and the other contracts radially. A two-fold error is possible as a result.

1) An error torque can arise due to unequal pressure-area products near the inlet and outlet.

$$\frac{\Delta P}{2} A \frac{P_1}{P_3} \neq \frac{\Delta P}{2} A \frac{P_2}{P_3} \quad (5)$$

where $A_1 = A \frac{P_1}{P_3}$ and $A_2 = A \frac{P_2}{P_3}$.

2) An error torque can also arise due to forces transmitted to the S-tube due to axial stressing of the coupling in both compression and tension.

Neither of these error torques are compensated by the oil bath. These possible errors can be minimized however, by the proper selection of coupling characteristics. The coupling must have the following elastic properties:

- Radial spring constant still enough that a 1 psi pressure will cause only negligible radial strain; but flexible relative to the stiffness of the container casting.
- Axial spring constant very flexible so as to minimize 2. above.

The pressure drop along the S-tube at 10 GPM is about 0.02 psi, at 100 GPM it is 2.0 psi. Flexible couplings having these characteristics are the brass

bellows used in the experimental model. Again, no appreciable errors due to these effects were noticed during the testing program.

d. Alternate Pressure Equalization Devices

It is necessary for proper operation of the flow element that average pressures interior and exterior to the S-tube be equal. Several techniques for providing pressure equalization were investigated. A special pressure chamber was constructed for these tests so that the flow element itself did not have to be successively disassembled and reassembled. Two devices evaluated in this pressure tight chamber are sketched in Figures 10 and 11. The bellows assembly proved to be too stiff and a net pressure difference of 12 psi existed when P_1 was set at 85 psi. A very flexible diaphragm 4 inches in diameter proved satisfactory. No pressure differential was observed. The S-tube flow element under test makes use of neither bellows nor diaphragm, but rather uses pressure lines directly conversant with the flow-stream and oil filled chamber, as shown in Figure 9. This has resulted in small amounts of test fluid entering the chamber and small amounts of oil entering the flow stream. For the tests, neither situation was troublesome.

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FIGURE 10. BELLOWS PRESSURE EQUALIZER

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TOLERANCE ON DECIMAL DIMENSIONS $\pm .005$

TOLERANCE ON ANGULAR DIMENSIONS $\pm 0^{\circ} 30$ MIN.

TOLERANCE ON FRACTIONAL DIMENSIONS $\pm .010$

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FIGURE 11. DIAPHRAGM PRESSURE EQUALIZER

B. Densitometer Design Features

1. General Description

As sketched in Figures 12 and 13, the measurement of density is accomplished by means of comparing radiation from a standard source with that from another source after its rays are absorbed by traversal across the S-tube and through the rotary wedge. The radiation attenuation due to the fluid for a fixed geometry is:

$$e^{-\left(\frac{\mu}{\rho}\right) \rho x}$$

where

$\frac{\mu}{\rho}$ = mass attenuation coefficient

ρ = fluid density

x = effective radiation path through the fluid

Similarly the radiation attenuation due to the rotary aluminum wedge is

$$e^{-\left(\frac{\mu}{\rho}\right)' \rho' x'}$$

where the primes refer to the corresponding quantities for aluminum.

The servo loop controls the angular position of the wedge so that the total attenuation of the fluid and of the wedge remains constant. If the quantities appearing in the exponents can be considered as constants (independent of radiation energy) including buildup effects due to scattering, then the angular position of the wedge is linearly proportional to the fluid density. This was checked in various static and dynamic tests, and in practice was verified to a very good approximation for the range of densities

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SHAFT POSITION AND
VOLTAGE PROPORTIONAL
TO MIXTURE DENSITY

FIGURE 12. WEDGE DENSITOMETER SCHEMATIC DIAGRAM

examined (see IV-D) so long as the composition is constant. A composition effect was encountered which is discussed in Section IV. D. 1.

The same detector is used to measure radiation from both sources, so time must be shared between them. The dual radiation beams must be chopped so that one reaches the detector while the other is attenuated and vice versa. Various techniques were examined for chopping the radiation beams. One approach involved a rotating shield with alternate sections open. This turned out to be mechanically too cumbersome in practice. Another idea was to mount the radioactive sources on vibrating reeds. This was discarded because of the possibility of source damage. The final design employed hevimet shields mounted on the ends of electromagnetically-driven reeds with the sources held stationary behind the shields. In each case 60 cps was used, which meant each source was "on" $1/120$ of a second at a time. The scintillation crystal alternately sees radiation from one source transmitted through the flow stream and the calibrated wedge, and then from the other (reference) source. The sources were Pm^{147} beta-excited x-ray sources.

The scintillation crystal fluoresces under the effect of incoming gamma rays. Visible fluorescence in the crystal is "viewed" by the photomultiplier tube and converted into an electron current at the output of the tube. This current provides a net AC error voltage which is amplified and used in a servo motor to drive the calibrated wedge so as to null out the error voltage. The position of the motor shaft is proportional to density.

By virtue of the closed loop configuration which balances out factors affecting both sources the same way, the density measurement is independent of various in line voltage, frequency shifts, photomultiplier tube sensitivity, amplifier gain changes, radioactive decay of the gamma sources, or radioactivity entrained in the flow stream so long as the components in the loop perform correctly.

The small circular windows shown in Figure 13 provide the path of the gamma rays across the center of the S-tube. The centerline of the S-tube, the radiation path, and the axis of rotation of the S-tube are mutually perpendicular. It is necessary for accuracy during transients in density that flow rate and density be measured as near to the same point as possible. The actual mounting of the density servo components indicated in Figure 13 is shown in Figures 14 and 15.

2. Radiation Modulation Technique

One of the two reeds tuned to 60 cps resonance and carrying a hevimet shield is pictured at rest in Figure 16 and vibrating in Figure 17. Placed behind the hevimet paddle in each case is a sealed Promethium 147 beta excited x-ray source containing 9.7 curies of beta ctivity. Orientation of these sources within each reed assembly is shown in Figure 18. The sources are totally enclosed within a fixed lead shield except for a collimating aperture in line with the source and the vibrating shield. As bursts of x-ray photons strike the scintillation crystal alternately from one source and then the other, the putput of the photomultiplier is a train of impulses whose

FIGURE 14. DENSITY SERVO DRIVE AND READOUT

FIGURE 15. DENSITY SERVO RADIATION DETECTOR AND FILTER

FIGURE 16. RADIATION MODULATOR, AT REST

FIGURE 17. RADIATION MODULATOR, VIBRATING

REED PLATE REVISION
FOR Pr-147 SOURCES

#685- 2X SCALE

RHS- 5-31-61

FIGURE 18. SOURCE PLACEMENT IN REED ASSEMBLY

number per unit time varies in a periodic fashion as shown in Figure 19.

When this signal is filtered so as to integrate the impulses and retain only the 60 cycle per second alternative voltage component, the sinusoidal wave form of Figure 20 results. This signal is the servo actuating voltage fed to the amplifier and represents an off-null situation, i. e., a condition where considerably more radiation from one source is reaching the detector than from the other.

The reeds themselves are made from strongly ferromagnetic stainless steel and are so designed that stresses due to vibration are uniformly distributed over the reed's length. Displacements are such that the maximum stresses are well below the fatigue limits for the steel used.

To demonstrate the reliability of the reed radiation chopper, a preliminary model was constructed and has been vibrating continuously since the first month of this program. The vibrating reed itself does not make contact with any other member, so wear problems do not exist. These facts combine to provide a radiation sampling technique which should be quite simple and very reliable.

Vibration is sustained by applying an alternating magnetic field to the ferromagnetic material of the reed. If the reed is to be vibrated at line frequency 60 cycles per second, the reed must have an undamped natural frequency equal to 60 cycles per second in order to minimize input power necessary to sustain the vibrations. The equations which describe the

FIGURE 19. PM TUBE OUTPUT, UNFILTERED

FIGURE 20. PM TUBE OUTPUT, FILTERED

the motion and the damped and undamped natural frequencies of the elastic reed are the basis of reed design and are derived as follows.

Let a force F be applied to the mass. This force will be resisted by a force:

$$F = k \theta + J \frac{d^2 \theta}{dt^2} + B \frac{d \theta}{dt}$$

where

$J = Ml^2$, the inertia of the mass

k = spring constant of reed

B = viscous damping coefficient of reed.

In Laplace notation:

$$F(s) = k \theta(s) + J s^2 \theta(s) + B s \theta(s)$$

$$\theta(s) = \frac{F(s)}{J s^2 + B s + k} = \frac{A/J}{s^2 + \frac{B}{J} s + \frac{k}{J}} F(s)$$

The characteristic equation $s^2 + \frac{B}{J} s + \frac{k}{J}$ is of the form $s^2 + 2 \xi w_o s + w_o^2$

wherein ξ is the damping factor and w_o is the undamped natural frequency.

The damping factor is given by

$$2 \xi w_o = \frac{B}{J}$$

$$w_o = \sqrt{\frac{k}{J}}$$

$$\xi = \frac{B}{2J} \sqrt{\frac{J}{k}}$$

In terms of ξ and w_o , the damped natural frequency is

$$w_d = \sqrt{1 - \xi^2} w_o$$

$$w_d = \sqrt{1 - \frac{B^2}{4Jk}} w_o$$

The damped natural frequency is slightly lower than the undamped natural frequency. The energy absorbed by the damping restraint of the reed and surrounding air is supplied by the magnetic field. Therefore, the mechanical design frequency used for selection of M, k, and l, must be the damped natural frequency which will be several cycles per second higher than sixty.

The thickness and width of the reed for a given θ and l are selected on the basis of the stress - fatigue characteristics of the reed material.

3. Statistical Accuracy and Stability Considerations

For a modulating rate (reed vibration rate) of 60 cycles per second, each source is exposed 1/120 second at a time. If the voltage pulse height at the photomultiplier tube output is to be representative of absorption, a very large source would be required. If however, the pulses are integrated over a longer period of time, the statistical accuracy is improved and a relatively smaller source is adequate. There are several

techniques by which this integration can be accomplished. One technique employed by Armour Research Foundation and described in ARF 1-152-3 Quarterly Report No. 1 is a "commutating system." In this system the output of the photomultiplier tube is commutated at the low frequency f' , equal to the radiation chopper frequency. This commutation separates pulses due to the reference source and pulses due to the measuring source. Each pulse train, constituting a DC level is integrated in an RC network and sampled again at 60 cycles per second to provide the input to a servo amplifier. This procedure is illustrated in Figure 21.

A linear block diagram for this system can be drawn as follows:

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FIGURE 21. COMMUTATING DENSITY SERVO SCHEMATIC DIAGRAM

where:

- K_{RS} : Reference source intensity in photons per period $1/(2f)$.
- K_D : Crystal-photomultiplier tube sensitivity in volts per photon.
- RC : Integrator time constant in seconds.
- K_a : Amplifier gain in volts per volt.
- K_m : Motor torque constant in dyne cm per volt.
- B_m : Motor viscous damping coefficient in dyne cm second per radian.
- J_{eq} : Equivalent total inertia of the motor rotor and load in gm. cm².
- K_g : Gear ratio in rad per radian.
- K_{W-MS} : Transfer function of wedge and measuring source combination in photons per radian-second.

For purposes of considering the dynamics of the closed

loop, the block diagram can be redrawn as follows:

where

$$K_G = \frac{K_a K_m K_{W-MS} K_D}{B_m K_g}$$

$$T_1 = \frac{J_{eq}}{B_m}$$

$$T_2 = RC$$

$$\frac{C(s)}{E(s)} = \frac{K_G}{s(1 + T_1 s)(1 + T_2 s)} \approx \frac{K_G}{s(1 + T_2 s)} \quad \text{for } T_2 \gg T_1$$

$$\frac{C(s)}{R(s)} \approx \frac{K_G}{T_2 s^2 + s + K_G} = \frac{K_G/T_2}{s^2 + \frac{1}{T_2} s + \frac{K_G}{T_2}}$$

The denominator is of the form

$$s^2 + 2 \xi w_o s + w_o^2 \quad \text{wherein}$$

ξ : Damping factor

w_o : Undamped natural frequency.

Hence

$$w_o = \frac{K_G}{T_2} \quad \text{and} \quad \xi = \frac{1}{2} \frac{1}{T_2 K_G}$$

If RC is selected to provide adequate statistical accuracy and K_G is selected for critical damping, the servo response will have the form:

$$C(t) = C_o (1 + w_o t) e^{-w_o t}$$

and $w_o = \frac{K_G}{T_2}$

Thus it can be seen that for large T_2 , w_o is small and t must be very large for the exponential to approach zero. This provides a long time constant.

Commutating and sampling of the error signal requires rather cumbersome circuit, so in the interest of simplicity, another damping

technique has been investigated. If a tachometer is added to the motor shaft as shown in Figure 12, a long time constant can be provided without commutation and sampling.

A linear block diagram for the system of Figure 12 can be drawn as follows:

where K_T : Tachometer Sensitivity
in volts seconds per radian

This diagram can be simplified to the following for consideration of dynamic response.

$$K_1 = \frac{K_a K_m}{B_m}$$

$$T_1 = \frac{J_{eg}}{B_m}$$

$$K_2 = \frac{K_D K_{W-MS}}{K_g}$$

$$\frac{C(s)}{E(s)} = \frac{K_1}{s(1 + T_1 s + K_1 K_T)}$$

$$\approx \frac{K_1}{(1 + K_1 K_T)s} \text{ for } T_1 \text{ small}$$

$$\frac{C(s)}{E(s)} = \frac{K_1}{K_1 K_2 + (1 + K_1 K_T)s}$$

$$= \frac{1 + \frac{K_1}{K_1 K_T}}{s + \frac{K_1 K_2}{1 + K_1 K_T}}$$

The response of this loop will be of the form:

$$c(t) = C_0 e^{-\frac{K_1 K_2}{1 + K_1 K_T} t}$$

For large K_T , $\frac{K_1 K_2}{1 + K_1 K_T}$ will be small, and a large t is required for the exponential to approach zero. The similarity in analyzed response between the commutator system and the tachometer system is noticeable.

4. Wedge and Density Range Considerations

a. Source Intensity

The actual configuration of the radiation path is

given below:

Source

As discussed in Section I-B. 1., the detector (NaI(Tl) crystal and the photomultiplier tube) counting rate as a function of fluid density is

$$N = N_o e^{-\left(\frac{\mu}{\rho}\right) \rho x} \pm \sqrt{N}$$

where

N = counting rate with fluid present

N_o = counting rate with no fluid present

$\frac{\mu}{\rho}$ = mass attenuation coefficient (a property of the fluid composition)

ρ = fluid density

x = effective path length of the radiation in the fluid.

In order to compute the counting rate it was necessary to know the above quantities. This, in turn, required knowledge of the source strength (number of curies), the energy (Mev), and type of radiation emitted. The types of radiation directly available from radioactive sources include alpha particles, beta particles, and gamma rays. These, in turn, may be used to give rise to secondary nuclear radiation such as bremsstrahlung and neutrons. The densities and total mass of material to be traversed obviously require gamma rays or bremsstrahlung.

Considerations of change in counting rate per unit change in density for water and water solutions employed in this research require a low energy source of electromagnetic radiation. Preliminary calculations, later verified by experiment led us to choose the promethium 147 beta excited x-ray source developed for DID under contract No. AT(11-1)-657. The effective energy of the bremsstrahlung source was chosen low enough so that a small change in fluid density (ρ) would cause a detectable change in radiation attenuation, yet high enough so that a reasonable source intensity would provide satisfactory counting statistics ($\frac{\mu}{\rho}$ increases rather rapidly as the effective energy decreases for energies of practical interest here). The final choice was for an effective energy of ~ 80 kev.

The choice of source strength (9.7 curies) rested upon expected counting rate. Unfortunately the efficiency of conversion from beta particles to bremsstrahlung ($\sim 2 \times 10^{-4}$) was lower than expected with the result that the counting rate was $\sim 30,000$ counts/second with the fluid pipe empty and the rotary aluminum wedge at its thinnest position. With the rotary wedge at its thickest point the counting rate was $\sim 20,000$ counts/second. Thus, the range of fluid densities which could be accommodated by the particular wedge used corresponded to an effective counting rate of $\sim 10,000$ counts/second. This meant that for a one second time constant the maximum accuracy attainable was 1 per cent, neglecting all other errors. The present technology is certainly adequate for increasing the accuracy by an order of magnitude.

The actual choice of Pm^{147} was based upon the high specific activity, the rather low energy of the bremsstrahlung emitted, the reasonable price and moderate half-life. Each of the two sources was a pressed pellet approximately 1/8 of an inch in diameter and 42 mils thick sealed into one end of an aluminum container 0.28 inches in diameter and approximately 3/4 of an inch long. The other end of each container was threaded and screwed into the back of the reed assembly (see Figures 16 and 17).

b. Calibrated Wedge Dimensions

The fluids which were used in our tests are various solutions of potassium carbonate in water having densities varying from unity to 1.54. The wedge has been constructed for use with flow density variations between 0.9 and 1.6. The S-tube center is square in cross section with a side dimension of 3.378 cm.

$$3.04 < \rho x < 5.419 \text{ gm/cm}^2$$

$$\Delta \rho x = 2.379$$

For aluminum $\rho = 2.7$ and

$$1.125 < x < 2.01 \text{ cm. or}$$

$$\Delta x = 0.885 \text{ cm. or } 0.317 \text{ inches.}$$

The wedge was constructed as shown in Figure 22, varying in thickness from 0.125 to 0.485 inches.

FIGURE 22. ROTARY WEDGE

C. Square Root Servo and Integration Design Features

S-tube torque is proportional to the square of mass flow rate as derived in Figure 1. The position of the S-tube torque motor shaft is proportional to torque, so the rotor voltage of a potentiometer whose stator carries a fixed voltage and whose rotor is driven by the torque motor will also be proportional to torque. If this voltage is used to excite the stator of another potentiometer whose rotor is being driven by the wedge shaft of the density servo, this potentiometer's rotor voltage will be proportional to the product of torque and density as shown in Figure 23. To determine mass flow rate W , it is necessary to extract the square root of ρT . This is accomplished by means of the square root servo shown in Figure 23.

The voltage ρT that appears at the input of the amplifier is amplified and applied to the control windings of the servo motor. The servo motor drives the wiper arms of the two Vernistat potentiometers until the voltage appearing on the wiper arm of the second potentiometer is of an equal magnitude and an opposite phase to that of the input signal. The servo loop will be in a null condition at this time and no motion will be present unless the input signal changes.

The feedback voltage that appears at the null transformer is proportional to the square of the motor shaft position, therefore, the motor shaft position is proportional to the square root of the input signal.

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FIGURE 23. SQUARE ROOT SERVO AND INTEGRATOR SCHEMATIC DIAGRAM

FIGURE 24. SQUARE ROOT SERVO AND INTEGRATOR

This shaft reads directly in mass flow rate. If this shaft is used to position the carriage of a ball and disc integrator whose disc is rotated at a constant rate by a synchronous motor, the integrator output shaft will rotate at a rate proportional to mass flow rate. When used to drive a rotary counter, a readout for mass flow quantity results. The torque and density potentiometers are mounted within their respective servo loops. The square root servo and integrator are shown in Figure 24.

The root extraction was performed within an accuracy of ± 0.11 per cent over the flow range, a figure which reflects a combination of loading, linearity, and resolution errors. The performance of this servo is described in Figure 25 which shows the linear relationship between mass flow rate and the square root of the product of torque and density: $W = k \sqrt{\rho T}$. Note that, for convenience, the abscissa and parameters are in terms of T and ρ rather than $\sqrt{T}$ and $\sqrt{\rho}$. One pound per minute corresponds to 2.85 root servo units.

The amplifier used in this servo is a standard 12 watt vacuum tube power amplifier with a voltage gain of 200 volts per volt and a maximum saturation output voltage of 115 volts. The servo motor is a two phase 115 VAC, 60 cps Oster-type 5203, size 18. The vernistats are Perkin Elmer types 2 BE and 231 E having terminal linearities of 0.05% and 0.05% respectively. The ball and disc integrator and counter is a Librascope Model 25 Linear Integrator. Potentiometers on the flow element torque motor shaft and the density servo shaft, shown in Figure 23, are Helipot 10-turn types

LV-7603, 1000 Ohms and LV-7603 RS, 30,000 Ohms respectively. The 1,000 Ohm pot has a linearity tolerance of ± 0.05 per cent, and the 30,000 Ohm pot ± 0.025 per cent.

III. TEST AND EVALUATION PROGRAM AND TEST RESULTS

A. Summary of Mass Flow Meter Test Results

To demonstrate the feasibility of this new mass flow metering technique, the test program was so planned that the original equations for predicted performance could be verified and the accuracy of measurement of mass flow rate and mass flow quantity could be established. The equation derivations shown in Figure 1 predict that:

$$W^2 = \frac{A}{2L} \rho T \text{ slugs}^2/\text{sec}^2 \quad (1)$$

For the 1-1/2 inch I. D. S-tube having $A = 1.767 \text{ in.}^2$, $r = 0.75 \text{ in.}$ and L , the effective lever arm taken as 4 in:

$$W^2 = 0.01841 \rho T \quad (2)$$

Test results verified the form of this equation and gave:

$$W^2 = 0.01644 \rho T \quad (3)$$

indicating that the estimated value of L in equation (1) should be increased by 12 per cent to 4.48 inches.

The somewhat ideal accuracy and range goals for the mass flow measurement were set at 0.25 per cent of reading over a 20:1 range of flow rates. In order that the root mean square of the accuracies of each of the four system components meet the accuracy goal, the measurement of $T \propto W^2/\rho$ made by the S-tube flow element must be better than

0.4 per cent. The measurement of ρ must be better than 0.16 per cent, and the accuracies of the square root servo and the ball and disc integrator must both be better than 0.16 per cent. These figures constituted the design goals at the beginning of the program.

Test results have shown the mass flow rate and mass flow quantity measurements to be accurate to within ± 0.5 per cent of reading over a 3:1 range of mass flow rates and ± 1.5 per cent of reading over a 5:1 range. Mass flow rate is measured to within ± 0.75 per cent of full scale over a 10:1 range. The density measurement was accomplished with an accuracy of ± 0.2 per cent of reading for densities between 0.997 and 1.100 gm cm⁻³, and ± 0.33 per cent of reading for densities between 0.997 and 1.500 gm cm⁻³. These results demonstrate the feasibility of the technique and are considered very encouraging particularly when they are compared with the ranges and accuracies of other mass flow and density devices which use different measurement principles, as for example, those described in the introduction to this report.

Accuracy at the flow rate extremities was limited to S-tube bearing friction at low flow rates and by interleave clearances in the torsion spring at high flow rates. The S-tube technique for measuring W^2/ρ should be capable of good accuracy over a broader range of flow rates if S-tube torque can be measured adequately over a wider range. Note that a 20:1 range of mass flow rates requires an accurate torque measurement over a 400:1 range of torques.

A limitation in the density measurement was the speed of response to density transients. In order to achieve requisite accuracy using the available Promethium 147 sources, the measurement time constant was set at about 1 minute. Density servo response time can be reduced to several seconds while retaining acceptable accuracy by use of a more intense radioactive source for the measurement beam.

B. Mass Flow Calibration and Test Equipment

Figure 26 illustrates the equipment used to control flow conditions and to provide a high precision measurement of mass flow rate and quantity for use as a calibrating and testing standard.

1. Control of Test Fluid Properties

Temperature, pressure, and density of the flowing medium were carefully controlled by means of equipment in the flow test bed.

The air compressor, receiver, and pressure regulator provide a pressure head over the fluid in the pressure vessel adequate to sustain steady flows between 0 and 1000 pounds per minute for periods of time adequate to fill the weight tank. The heating system, cooling system, and associated thermostats maintain fluid temperature at preset settings between 0°C and 65°C.

The density of the test fluid was controlled by means of addition of a solute, potassium carbonate, to the fluid. Densities between 1.0 and 1.46 were achieved using these solutions.

The addition of K_2CO_3 to the water changes the absorption coefficient as a function of density. Figure 27 is a plot of this composition effect and shows an 18.5 per cent change in μ for a 54 per cent change in ρ . Because the composition effect is a nearly linear function of density, its effect was cancelled by means of proper calibration of the densitometer. The small departure from linearity was evaluated as to its effect on density measurement accuracy, and was found to have a negligible effect (see Figure 36).

FIGURE 26. MASS FLOW METER TEST BED

% K_2CO_3
by weight

2. Flow Rate Selection and Control System

Test fluids used to evaluate the mass flow meter are conducted through a closed recirculating flow loop. This loop contains a 500 gallon pressure vessel used as reservoir for fluid and pressurized air. Air, from the compressor, reservoir, and pressure regulator, maintain a nearly constant pressure over the surface of the fluid in the pressure vessel regardless of the fluid level. This enables fluid static pressure to be set and maintained at any selected value up to 150 psia. As the fluid flows through the test meter during a test run, fluid from the pressure vessel is forced out and a slight air pressure "droop" eventually occurs in the vessel due to the dead zone of the air pressure regulator. Without additional flow rate stabilization, this pressure droop would cause a decrease in flow rate. Since it is desirable to test the flow meter at constant flow rates over a period of time, additional flow rate stabilization is necessary. This stabilization is provided by a closed servo loop consisting of a positive displacement meter which provides a DC voltage proportional to flow rate, (Figure 28), and amplifier, and a motor driven modulating valve, (Figure 29). A flow rate is selected by applying a DC voltage to the amplifier opposite in polarity to that of the positive displacement meter output. This voltage is set in using the flow rate selector, shown in Figure 30. The servo amplifier will drive the modulating valve to a position which provides a flow rate such that the P. D. meter output voltage cancels the selected input voltage. If this

quiescent condition were disturbed by a pressure drop occurring in the flow supply, the flow rate control servo would automatically adjust the position of the modulating valve, so as to maintain a constant flow rate at the selected value. A coarse (2 %) measurement of flow rate is provided by a Rotameter as shown in Figure 31. No variation in flow rate can be detected using the Rotameter when the aforementioned pressure droop occurs.

3. Flow Timing and Diversion System

Test accuracy is achieved by precision timing of the flow from the test meter conducted into a tank whose contents can be weighed with precision, the tank at the right in Figure 32. During each test, a flow rate is established first with the test meter effluent conducted to the dump tank at the left in Figure 32. This is accomplished by means of a V-pan divider mounted between the two tanks and a solenoid actuated diverter valve* which rapidly diverts the flow across the edge of the V-pan as shown in Figure 33. Next, a test time is selected on the timer, and the timer is switched on. At the moment the timer starts, the diverter valve switches flow into the weigh tank and the ball and disc integrator in the test meter is switched on. At the end of the preselected time, the flow is rediverted to the dump tank, and the test meter integrator is switched off. It is essential that no

*This valve is a "diverter valve" designed in accordance with National Bureau of Standards procedures shown in "Liquid-Flowmeter Calibration Techniques." Transaction of the ASME, October 1958, Page 1369-1379.

FIGURE 28. FLOW RATE REFERENCE METER

FIGURE 29. FLOW RATE MODULATING VALVE

FIGURE 30. FLOW RATE SELECTOR AND TIMER

FIGURE 31. ROTAMETER FOR COARSE FLOW RATE MEASUREMENT

pressure transient be generated and reflected to the meter under test when flow is switched to and from the weigh tank. This requirement is met by the diverter design described.

4. Flow Weighing and Weight Indication System

Weight of the contents of the tank at the right in Figure 32 is determined by means of a load cell system. The entire weigh tank is suspended from a single load cell, shown in Figure 34, whose output is measured by means of a servo balanced bridge which provides a numerical readout for weight. The readout servo unit is shown in Figure 35. The load cell and servo readout has been calibrated in five ranges and provides the following accuracies:

<u>Range:</u>	0 - 400 lbs	<u>Accuracy:</u>	<u>+ 0.4 lbs</u>
	400 - 800 lbs		<u>+ 0.8 lbs</u>
	800 - 1,200 lbs		<u>+ 1.2 lbs</u>
	1,200 - 1,600 lbs		<u>+ 1.6 lbs</u>
	1,600 - 2,000 lbs		<u>+ 2.0 lbs</u>

The load cell servo also has a precision DC tachometer which provides a voltage proportional to flow rate. This voltage provides a qualitative comparison for the test meter rate output voltage. The servo also drives the rotor of a 0.05 per cent linearity potentiometer whose output voltage can be used to measure instantaneous error of the test meter integrated output.

FIGURE 32. WEIGH AND DUMP TANK

FIGURE 33. FLOW DIVERSION ASSEMBLY

FIGURE 34. LOAD CELL SUPPORT FOR WEIGH TANK

FIGURE 35. LOAD CELL SERVO READOUT

C. Calibration and Test Procedure

1. Densitometer Calibration Procedure

Thirteen 1-gallon samples of water solutions of K_2CO_3 graduated in density from 0.9970 to 1.4615 were prepared and used for static calibration of the densitometer. The densities of these solutions were determined in two ways: 1. --- by regulating K_2CO_3 and water concentration by weight during mixing, and using concentration tables for density; and 2. --- accurate weighing of 100 cm^3 of fluid using a precision balance. All portions of the density servo were mounted in place on the flow element housing. The reading of the density servo dials, when the S-tube was filled with water was noted (Figure 14). Then the K_2CO_3 solutions were added sequentially and successive readings of the readout dials noted for each solution density. From these data a calibration factor was derived which relates dial reading and density. This factor was used in the calculation of the overall mass flow constant for the test model.

2. Mass Flow Meter Calibration Procedure

With reference to Figures 2, 12, and 23, calibration of the complete mass flow system was accomplished by means of the following sequence of steps:

a) S-tube Servo Alignment

For zero flow rate of water through the S-tube, a small torque was applied to the torsion spring by introducing an offset voltage

from the reference synchro. This provides a better null condition for the S-tube servo and in no way affects the measurement of torque. Torque readout dials on the torque motor (Figure 4) were set to zero as was the voltage output from the torque potentiometer.

b) Root Servo and Integrator Alignment

For a zero voltage output from the torque potentiometer, the voltage from the density potentiometer is zero multiplied by the density of water, or zero. The root servo output dial was adjusted for zero reading. The carriage of the ball and disc integrator was adjusted for zero rotation of the counter. Rotations of the torque potentiometer were introduced artificially and the directions of rotation of all elements were observed and corrected for proper phasing.

c) Mass Flow Rate and Quantity Calibration

The density of the test fluid was ascertained by accurately weighing 100 cm^3 of the solution. A sequence of flow rates from low to high values was selected using the flow rate controller. Readings of the S-tube torque motor shaft position, the density servo shaft position, and the square root servo shaft position were noted for each flow rate. Flow was diverted from the dump tank to the weigh tank for an accurately known period of time Δt , during which period the ball and disc integrator was switched on. Using the load cell system for each value of flow rate and density, the weight of the fluid ΔW delivered to the weigh tank was noted

as was the number of digits ΔI counted by the output counter of the ball and disc integrator. Division of ΔW and ΔI yielded the meter factor for the mass flow meter in units of pounds per integrator unit. Division of ΔW by ΔT provided a calibration of the root servo output in terms of mass flow rate, given in units of root servo units per lb. min.⁻¹

3. Mass Flow Test Procedure

The testing procedure was very similar to that discussed above for calibration. Test runs were made over a range of flow rates and densities and data collected relative to S-tube torque T , density readout ρ , square root serve readout W , and integrator readout M , for each flow rate and density. These readings were compared with those taken using the timer and load cell system. In this fashion, the values of mass flow rate and mass flow quantity measured by the experimental meter were compared with the standard values measured by the load cell and timer. Differences between experimental and standard values were calculated and percentage errors were determined as functions of flow rate and density.

D. Results of Density and Mass Flow Testing

1. Densitometer Performance

Figures 36 and 37 describe the performance of the densitometer servo when tested with a sequence of solutions with densities between $0.9970 \text{ gm cm}^{-3}$ and $1.4615 \text{ gm cm}^{-3}$. The measurement made by the servo was in agreement with the calibration standard within ± 0.2 per cent over most of the range and within ± 0.33 per cent over the entire range. This is equivalent to about ± 0.003 units of specific gravity or metric density. The random nature of the error indicates that the precision of the density standard (a balance and graduate) was probably of the same order of magnitude as that of the servo measurement.

Readings were made for each solution after a period of several minutes had elapsed to allow the heavily damped servo to reach quiescence. At quiescence the readout dials moved through small angles ($\pm 2^\circ$) in a random fashion due to the statistical nature of the servo actuating signal. Readings, Drdg reported in Figure 36, were the average values at each quiescent point.

The effect of the variation of solution μ as a function of density influenced the accuracy slightly. Its effect can be predicted by considering that the servo acts to maintain the radiation transmitted from the reference source equal to that transmitted from the measurement source through the wedge and S-tube, or

Solution #	ρ_C	Drdg	ρ_M	$\%E\rho_M$	Drdg	ρ_{Mc}	$\%E\rho_{Mc}$
1	0.9970	87.5	0.9980	+0.10	87.5	0.9980	+0.10
2	1.0025	86.8	1.0026	+0.01	86.8	1.0026	+0.01
3	1.0120	85.6	1.0105	-0.15	85.6	1.0105	-0.15
4	1.0225	83.8	1.0223	-0.02	83.8	1.0223	-0.02
5	1.0435	80.8	1.0421	-0.13	80.0	1.4021	-0.13
6	1.0695	76.7	1.0691	-0.04	76.7	1.0691	-0.04
7	1.0948	73.3	1.0915	-0.30	73.3	1.0915	-0.30
8.	1.1225	68.5	1.1230	+0.04	68.5	1.1230	+0.04
9	1.1437	65.4	1.1434	+0.03	65.4	1.4340	+0.03
10	1.1700	61.3	1.1704	+0.03	61.3	1.1704	+0.03
11	1.2465	49.3	1.2494	+0.23	49.1	1.2507	+0.34
12	1.3055	40.4	1.3079	+0.18	40.1	1.3099	+0.34
13	1.4615	18.2	1.4540	-0.51	17.2	1.4606	-0.06

DENSITOMETER

PERFORMANCE

DATA

- Figure 36 -

KEY:

ρ_C = Calibration density measured by balance and graduate
 Drdg = Densitometer dial reading (wedge angle of rotation)
 ρ_M = $1.00 + (0.00658)(87.2 - \text{Drdg})$
 $\%E\rho_M$ = $100 (\rho_M - \rho_C) / \rho_C$ = Percent Accuracy
 Drdg_C = Densitometer dial reading corrected for composition effect
 ρ_{Mc} = $1.00 + (0.00658)(87.2 - \text{Drdg}_C)$
 $\%E\rho_{Mc}$ = $100 (\rho_{Mc} - \rho_C) / \rho_C$ = Percent Accuracy

$\text{Drdg} \propto \text{Wedge Position}$

$$I_{Mo} e^{-(\mu_w \rho_w x_w + \mu_s \rho_s x_s)} = I_{Ro} e^{-(\mu_a \rho_a x_a)}$$

$$\mu_w \rho_w x_w + \mu_s \rho_s x_s = \mu_a \rho_a x_a \quad \text{if } I_{Mo} = I_{Ro}$$

$$K_1 \mu_s \rho_s = K_2 - K_3 x_w$$

$$\rho_s = \frac{K_2 - K_3 x_w}{K_1 \mu_s}$$

where

w subscripts denote the rotary wedge

s subscripts denote the S-tube

a subscripts denote a fixed absorber in the reference beam

If μ_s in the expression for ρ_s were a linear function of density, no effect on the measurement of ρ_s would result. However, according to Figure 27, μ_s drops off to slightly lower values than given by the linear approximation. The servo mistakes the increased radiation due to the reduction in μ_s for increased radiation due to a reduction in ρ_s and as a result reads slightly too low. Figure 36 demonstrates that this composition effect is noticeable only for appreciable concentrations of K_2CO_3 . In general the readings would require compensation only if the weight composition of the fluid being measured were to change radically during the measurement. For mass flow meter tests (and most industrial applications) no correction is necessary.

2. Mass Flow Meter Performance

A sequence of 120 mass flow test runs was carried out during the calibration and test program. Results of 51 of these runs are tabulated in Figures 38 and 39. For each test run, the test fluid with a known density, passed through the flowmeter at a prescribed flow rate for a precisely known period of time. The weight of fluid indicated by the meter was compared with the weight indicated by the load cell. The mass flow rate indicated by the meter was compared with the value calculated by dividing the load cell weight by the time increment. These comparisons are tabulated as per cent accuracies in Figures 38 and 39. The test results show a mass flow rate accuracy of about ± 0.75 per cent of full scale for mass flow rates between 75 and 750 pounds per minute, and a mass flow quantity accuracy of ± 1.5 per cent of reading for mass flow rates between 150 and 750 pounds per minute. Accuracy at the low flow rates was limited by torque servo dead zone brought about by friction in the S-tube support bearings. When water was used as test fluid, accuracy was maintained to high flow rates limited only by interleave clearances in the torsion spring, i. e., the spring wound tight at flow rates over 850 pounds per minute. When the K_2CO_3 solute was added to increase fluid density, dissolved air tended to come out of solution at high flow rates because reservoir pressure was several times higher than meter static pressure at these flow rates. With the air out of solution, the test fluid became a

Test #	Test Time min.	Mass Flow Rate: Standard W_S lb/min	Mass Flow Rate: Meter W_M lb/min	$W_M - W_S$ lb/min	Accuracy: Mass Flow Rate % of Full Scale	Accuracy: Mass Flow Rate % of Reading	Mass Flow Quantity: Standard M_S lbs.	Mass Flow Quantity: Meter M_M lbs.	$M_M - M_S$ lbs.	Accuracy: Mass Flow Quantity % of Reading
1	5.5	77.54	78.16	+0.62	+0.08	+0.80	426.5	429.9	+3.4	+0.80
2	5	78.02	84.70	+6.68	+0.92	+8.56	390.1	423.5	+33.4	+8.56
3	5	78.68	83.56	+4.88	+0.67	+6.20	393.4	417.8	+24.4	+6.20
4	3	160.5	160.8	+0.3	+0.04	+0.19	481.5	482.4	+0.9	+0.19
5	3	159.5	162.4	+2.9	+0.40	+1.82	478.5	487.2	+8.7	+1.82
6	3	160.2	159.9	-0.3	-0.04	-0.19	480.5	479.6	-0.9	-0.19
7	2	240.4	239.2	-1.2	-0.16	-0.52	480.8	478.3	-2.5	-0.52
8	2	241.2	242.5	+1.3	+0.18	+0.52	482.5	485.0	+2.5	+0.52
9	2	233.0	231.9	-1.1	-0.15	-0.47	466.0	463.8	-2.2	-0.47
10	1.5	320.7	319.9	-0.8	-0.11	-0.27	481.1	479.8	-1.3	-0.27
11	1.5	321.0	320.1	-0.9	-0.12	-0.29	481.5	480.1	-1.4	-0.29
12	1.5	314.5	311.3	-3.2	-0.44	-1.00	471.7	467.0	-4.7	-1.00
13	1	386.8	388.0	+1.2	+0.16	+0.31	386.8	388.0	+1.2	+0.31
14	1	393.7	390.1	-3.6	-0.50	-0.91	393.7	390.1	-3.6	-0.91
15	1	394.5	388.2	-6.3	-0.87	-1.60	394.5	388.2	-6.3	-1.60
16	1	484.6	481.1	-3.5	-0.48	-0.72	484.6	481.1	-3.5	-0.72
17	1	474.1	473.7	-0.4	-0.06	-0.08	474.1	473.7	-0.4	-0.08
18	1	477.8	475.4	-2.4	-0.33	-0.50	477.8	475.4	-2.4	-0.50
19	1	569.5	567.2	-2.3	-0.31	-0.40	569.5	567.2	-2.3	-0.40
20	1	563.2	560.2	-3.0	-0.41	-0.53	563.2	560.2	-3.0	-0.53
21	1	563.7	562.6	-1.1	-0.15	-0.20	563.7	562.6	-1.1	-0.20
22	1	643.7	647.5	+3.8	+0.52	+0.59	643.7	647.5	+3.8	+0.59
23	1	652.1	648.3	-3.8	-0.52	-0.58	652.1	648.3	-3.8	-0.58
24	1	642.3	653.4	+11.1	+1.53	+1.73	642.3	653.4	+11.1	+1.73
25	1	732.5	737.7	+5.2	+0.71	+0.71	732.5	737.7	+5.2	+0.71
26	1	725.2	725.9	+0.7	+0.10	+0.10	725.2	725.9	+0.7	+0.10
27	1	723.5	741.1	+17.6	+2.43	+2.43	723.5	741.1	+17.6	+2.43

TEST RESULTS: MASS FLOW AT DENSITY = 0.9970

Test #	Density ρ gm/cm ³	Mass Flow Rates: Standard W_c lb/min.	Mass Flow Rate: Meter W_m lb/min.	$W_m - W_c$ lb/min	Accuracy Mass Flow Rate: % of Full Scale	Accuracy Mass Flow Rate: % of Reading	Mass Flow Quantity: Standard M_c / lb.	Mass Flow Quantity: Meter M_m / lb.	$M_m - M_c$ lb.	Accuracy Mass Flow Quantity: % of Reading
1	0.9970	160.5	160.8	+0.3	+0.04	+0.19	481.5	482.4	+0.9	+0.19
2	1.080	159.6	162.6	+3.0	+0.40	+1.86	478.9	487.8	+8.9	+1.86
3	1.120	174.9	174.9	0.0	0.00	0.00	524.8	524.8	0.0	0.00
4	1.165	164.6	165.5	+0.9	+0.12	+0.57	493.8	496.6	+2.8	+0.57
5	0.997	240.4	239.2	-1.2	-0.16	-0.52	480.8	478.3	-2.5	-0.52
6	1.030	240.6	241.2	+0.6	+0.08	+0.23	481.2	482.3	+1.1	+0.23
7	1.120	248.2	250.8	+2.6	+0.35	+1.09	496.3	501.7	+5.4	+1.09
8	1.165	246.6	245.8	-0.8	-0.11	-0.28	493.1	491.7	-1.4	-0.28
9	0.997	320.7	319.9	-0.08	-0.11	-0.27	481.1	479.8	-1.3	-0.27
10	1.080	329.1	328.1	-1.0	-0.13	-0.30	493.7	492.2	-1.5	-0.30
11	1.120	334.7	330.9	-3.8	-0.51	-1.16	502.1	496.3	-5.8	-1.16
12	1.165	334.4	336.5	+2.1	+0.28	+0.62	501.6	504.7	+3.1	+0.62
13	0.997	368.8	388.0	+1.2	+0.16	+0.31	386.8	388.0	+1.2	+0.31
14	1.080	412.3	411.0	-1.3	-0.17	-0.32	412.3	411.0	-1.3	-0.32
15	1.120	414.9	411.6	-3.3	-0.44	-0.79	414.9	411.6	-3.3	-0.79
16	1.165	407.5	402.4	-5.1	-0.68	-1.25	407.5	402.4	-5.1	-1.25
17	0.997	484.6	481.1	-3.5	-0.48	-0.72	484.6	481.1	-3.5	-0.72
18	1.085	490.8	500.6	+9.8	+1.31	+1.99	490.8	500.6	+9.8	+1.99
19	1.120	508.1	508.1	0.0	0.00	0.00	508.1	508.1	0.0	0.00
20	1.165	509.7	509.1	-0.6	-0.08	-0.12	509.7	509.1	-0.6	-0.12
21	0.997	569.5	567.2	-2.3	-0.31	-0.40	569.5	567.2	-2.3	-0.40
22	1.085	575.6	611.2	+35.6	+4.74	+6.18	575.6	611.2	+35.6	+6.18
23	1.120	588.8	599.5	+10.7	+1.43	+1.82	588.8	599.5	+10.7	+1.82
24	1.165	600.0	602.7	+2.7	+0.36	+0.45	600.0	602.7	+2.7	+0.45

TEST RESULTS: MASS FLOW OF TEST FLUIDS WITH VARIOUS DENSITIES

foam with considerably altered density. The fluid to foam transformation happened sporadically and introduced severe transients in all three of the servos spoiling the accuracy of measurement for tests at high flow rate - high density. This, of course, was a circumstance peculiar to the test fluid and not to the meter. It should be noted that the meter would probably have faithfully measured even such severe mass transients if the response time of the density servo were decreased through use of more intense Promethium 147 sources.

It is notable that no fundamental limitations to accuracy are evident in the technique itself. The only limitations encountered were those brought about in the accessory instrumentation. Thus it is reasonable to expect that improved accuracy can be had by providing higher performance torque, density, square root, and integration measurements.

The mass flow meter calibration curve is shown in Figure 40. Originally, mass flow rate squared W^2 was predicted to be a linear function of ρT as shown in the equation derivation of Figure 1. This was verified experimentally and is the key fact demonstrating the feasibility of the S-tube technique for mass flow metering.

V. CONCLUSIONS

The feasibility and ability of a new mass flow principle has been established by the embodiment of this principle in a laboratory instrument and subsequent measurement of the accuracy, stability and general performance. These measurements have more than fulfilled our early hopes and expectations and we believe that the goals of the program have been reached. As with most research, however, new questions have been raised methods for further improvement in performance have been visualized and further variations of principle have been evolved.

The design of the laboratory instrument in its final form is somewhat complicated and could be simplified. Our objective was to put together a system which was able to clearly demonstrate the principle and no great effort was expended in achieving a well engineered model. As indicated throughout the report, flow element changes, source improvement, etc., should all yield even better performance. Further, if this principle is to be applied to a specific mass flow problem, additional design and further development is clearly indicated since the optimum instrument will reflect the densities, flow rates, etc., of the specific material(s) to be metered. It is our hope that this report will serve to interest those groups with mass flow metering problems to the point where this additional work will be undertaken and the application of these techniques to technology and industry will become a reality.