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NEGATIVE REACTIVITY COEFFICIENTS

by

C. A. Heath

September 19, 1968

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POSSIBLE INSTABILITIES IN A THERMIONIC REACTOR WITH ALL NEGATIVE REACTIVITY COEFFICIENTS

by

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ABSTRACT

Previous analyses of the open-loop stability of thermionic reactors have used a model which considers only the fuel-emitter combination to predict system behavior. These models have indicated that, if all coefficients of reactivity in the system are negative, the system will be stable. An analysis of stability in a thermionic reactor has been performed which includes the delayed collector and structure coefficients. This analysis shows that, for certain ranges of coefficients, it is possible for a thermionic reactor to show open-loop oscillatory instability with all negative coefficients.

This phenomenon exists for small ($< 6 \times 10^{-7}/^{\circ}\text{K}$) negative values of the temperature coefficient of the emitter material. An increase in the magnitude of the negative coefficient of the collector and structure material does not improve the stability of the system and for some cases will lead to oscillatory instability. Proposed methods of increasing the negative coefficient of the emitter are discussed.

INTRODUCTION

The small size fast-spectrum reactor cores that are being considered for thermionic reactor application tend to have prompt reactivity coefficients that are small. The fuel and emitter combination of the thermionic diode is separated from the rest of the reactor by a considerable thermal lag, and the expansion coefficients of the fuel are small because of the subdivision within the core.

Because these prompt reactivity coefficients are small, they have little effect upon the design of control systems, but they do retain importance in possible accident and control-failure situations. In particular, it will be shown in this paper that a combination of large coefficients associated with the reactor coolant and structure and small coefficients associated with the fuel element can lead to a condition of oscillatory instability in the open-loop system.

LINEARIZED EQUATIONS TO REPRESENT THE REACTOR SYSTEM

The reactor kinetics may be described by

$$\frac{dN}{dt} = \frac{\rho - \beta}{\Lambda} N(t) + \sum_{i=1}^6 \lambda_i G_i(t) \quad (1)$$

$$\frac{dG_i}{dt} = \frac{\beta_i}{\Lambda} N(t) - \lambda_i G_i \quad (2)$$

where N is the neutron density, G_i is the concentration of the i^{th} group of delayed neutron precursors, ρ is the reactivity, Λ is the prompt neutron generation time, β is the delayed neutron fraction, and λ_i is the decay constant for the i^{th} precursor group.

The neutron kinetics equations may be simplified by representing the delayed neutron precursors as a single group and by employing the prompt-jump approximation,⁽¹⁾

$$\frac{N(t)}{\Lambda} = \frac{\lambda G(t)}{\beta - \rho} . \quad (3)$$

If this approximation is substituted into the precursor Eq. 2, the reactor kinetics may be represented by a single equation.

$$\frac{dG}{dt} = \frac{\rho N(t)}{\Lambda} . \quad (4)$$

The excess reactivity, ρ , is made up of two components representing external reactivity and internal temperature feedbacks

$$\rho = \rho_e + \alpha_k \delta T_k . \quad (5)$$

If Eq. 4 is linearized and the reactivity is represented by Eq. 5, we get

$$\delta \dot{G} = \frac{N_0}{\Lambda} \delta \rho_e + \frac{N_0}{\Lambda} \alpha_k \delta T_k . \quad (6)$$

Heat transfer between the components of the reactor system may be described in considerable detail by dividing the system into a number of discrete lumps each having a "lumped" temperature. The simplest representation is obtained by grouping the fuel and emitter structure together (region 1) and considering the rest of the system (region 2) as having a single temperature. This model is represented by

$$V_1 C_1 \frac{dT_1}{dt} + h_1 A_1 (T_1 - T_2) = V_1 \gamma N . \quad (7)$$

Where V is the volume, C is the heat capacity per unit volume, A is the area for heat transfer, h_1 is a heat transfer coefficient between the emitter and the rest of the system, and γ is a constant to convert neutron density to power density.

If r_1 is the radius of the emitter, Eq. 7 may be simplified to

$$\frac{dT_1}{dt} = \frac{\gamma N}{C_1} - \frac{2h_1}{r_1 C_1} (T_1 - T_2) . \quad (8)$$

The thermal balance of the rest of the system may be written as

$$V_2 C_2 \frac{dT_2}{dt} = N_{DS} h_1 A_1 (T_1 - T_2) - \epsilon \sigma A_R T_2^4, \quad (9)$$

where the last term in the equation represents radiation heat transfer from A_R , that portion of the power plant radiator area that dissipates the rejected heat from N_{DS} diodes connected to a single coolant duct.

These heat transfer equations represent an approximate model of the thermionic reactor system, but they are valuable in that they permit an analytical representation of a stability criterion. A more detailed model may be derived for a thermionic system with discrete temperatures for diode components, the reactor structure, reactor coolant, and radiator structure. Calculated results from the more detailed equations are compared to this simplified model in the latter part of this paper.

SYSTEM STABILITY REQUIREMENTS

The set of Eqs. 6, 8, and 9 may be linearized by assuming small perturbations around an operating point and neglecting second order terms. The resulting set of simultaneous linear differential equations have the form

$$\dot{\underline{v}}(t) = \underline{A} \underline{v} + \underline{B} u. \quad (10)$$

The stability of such a set of equations may be evaluated by examining the matrix $\underline{A}$. A well-known criterion for system stability⁽²⁾ requires that the equation

$$|\underline{A} - \underline{I} \omega| = 0 \quad (11)$$

have no solutions for ω which have positive real parts.

Using this stability criterion on the three equation set, it is possible to derive three inequalities which must be satisfied for the system to be stable. Equation 11 has the form

$$a_0 \omega^3 + a_1 \omega^2 + a_2 \omega + a_3 = 0, \quad (12)$$

where

$$a_0 = 1 \quad (13)$$

$$a_1 = \frac{\epsilon \sigma A_R T_{20}^3}{V_2 C_2} + \frac{2N_{DS} h_1 r_1 \pi L_1}{V_2 C_2} - \frac{\gamma N_0 \alpha_1}{C_1 \beta} + \frac{2h_1 N_{DS}}{r_1 C_1} \quad (14)$$

$$a_2 = \frac{-\epsilon \sigma A_R T_{20}^3 \gamma N_0 \alpha_1}{V_2 C_2 C_1 \beta} + \frac{\epsilon \sigma A_R T_{20}^3 2N_{DS} h_1}{V_2 C_2 r_1 C_1} - \frac{\gamma N_0 \alpha_1 2N_{DS} h_1 r_1 \pi L_1}{C_1 \beta V_2 C_2} - \frac{2N_{DS} h_1 r_1 L_1 \pi \gamma N_0 \alpha_2}{V_2 C_2 C_1 \beta} - \frac{N_0 \alpha_1 \gamma \lambda}{C_1 \beta} \quad (15)$$

$$a_3 = - \frac{N_0 \alpha_2 \gamma \lambda 2N_{DS} h_1 r_1 \pi L_1}{C_1 \beta V_2 C_2} - \frac{N_0 \alpha_1 \gamma \lambda}{C_1 \beta} \left(\frac{\epsilon \sigma A_R T_{20}^3}{V_2 C_2} + \frac{2N_{DS} h_1 r_1 \pi L_1}{V_2 C_2} \right) . \quad (16)$$

The required inequalities for stability are that the quantities a_1 , a_3 , and $(a_1 a_2 - a_0 a_3)$ all be greater than zero. By reference to Eqs. 14 and 16 it is immediately seen that, if α_1 is less than or equal to zero and if α_2 is less than zero, the coefficients a_1 and a_3 will always be positive.

The particular case of interest here is for the condition where α_1 and α_2 are both negative and where the absolute value of α_2 is much greater than that of α_1 . It is interesting to examine the requirement that $(a_1 a_2 - a_0 a_3)$ be positive for the case where α_1 is assumed to be zero. This approximation simplifies the algebra somewhat and permits us to write the requirement

$$\left(\frac{\epsilon \sigma A_R T_{20}^3}{V_2 C_2} + \frac{2N_{DS} h_1 r_1 \pi L_1}{V_2 C_2} + \frac{2h_1}{r_1 C_1} \right) \left(\frac{\epsilon \sigma A_R T_{20}^3 2h_1 N_{DS}}{V_2 C_2 r_1 C_1} - \frac{2h_1 N_{DS} r_1 L_1 \pi \gamma N_0 \alpha_2}{V_2 C_2 C_1 \beta} \right) + \frac{N_0 \alpha_2 \gamma \lambda 2N_{DS} h_1 r_1 \pi L_1}{C_1 \beta V_2 C_2} > 0 . \quad (17)$$

This inequality may be rewritten to express a requirement upon the value of α_2 , the temperature coefficient of reactivity for the "delayed" thermal lump

$$\begin{aligned}
 & -\alpha_2 \left[N_0 \gamma_{2N_{DS}} h_1 r_1 \pi L_1 \left\{ \lambda - \frac{\epsilon \sigma A_R T_{20}^3}{V_2 C_2} - \frac{2N_{DS} h_1 r_1 \pi L_1}{V_2 C_2} - \frac{2N_{DS} h_1}{r_1 C_1} \right\} \right] \\
 & < \frac{\epsilon \sigma A_R T_{20}^3 2N_{DS} h_1 \beta}{r_1} \left(\frac{\epsilon \sigma A_R T_{20}^3}{V_2 C_2} + \frac{2N_{DS} h_1 r_1 \pi L_1}{V_2 C_2} + \frac{2N_{DS} h_1}{r_1 C_1} \right) . \quad (18)
 \end{aligned}$$

The absolute value below which a delayed negative coefficient must fall is dictated by the mass of the thermal rejection system represented by $C_2 V_2$ in Eq. 18. If the large mass of the radiator associated with a string of diodes is neglected in the modelling of the system, then $C_2 V_2$ is considerably smaller.

SYSTEM STABILITY BY DIGITAL CALCULATIONS

The three-equation model of the previous section may be analyzed to obtain values of characteristic roots with a set of physical properties typical of a thermionic reactor system. The value of the delayed coefficient which results in instability when the prompt coefficient is equal to zero is on the order of -10^{-4} .

A more detailed description of a thermionic reactor power plant can be derived by considering individual components in a diode, the reactor structure and coolant, a coolant loop with appropriate delay times, and the spacecraft radiator. This approach provides a set of 16 equations describing the system. Typical values of temperature coefficients of reactivity used in the analysis are shown in Table 1.

With the reference values listed in Table 1, characteristic roots were calculated with changing values of the feedback coefficient of the reactor structure. The resulting root-locus plot is illustrated in Fig. 1. Although other roots now appear due to the additional equations, the crossover into the positive real part of the plane at a frequency of about 0.02 radians per second still appears. It is also seen that an increase in the

Fig. 1. Roots of open-loop plant using 16 equations with changing values of temperature coefficient of the structure, α_5

negative value of the delayed coefficient tends to destabilize the system as predicted by Eq. 18.

TABLE 1
TEMPERATURE COEFFICIENTS ($\frac{\delta k}{K}/^{\circ}\text{K}$) OF REACTIVITY
INVESTIGATED IN THE LINEAR STABILITY ANALYSIS

	<u>Reference Value</u>	<u>Range Investigated</u>
Fuel	0	
Emitter	-6×10^{-7}	$-6 \times 10^{-7} \rightarrow -6 \times 10^{-6}$
Collector	-4×10^{-6}	$-4 \times 10^{-6} \rightarrow -6 \times 10^{-6}$
Insulator	0	
Structure	-6×10^{-6}	$-1 \times 10^{-5} \rightarrow +6 \times 10^{-6}$
Coolant	0	

Figure 2 illustrates, however, that if the negative coefficient connected with emitter structure may be made more negative, the effect is to stabilize the system. Specifically, the relative values of reactivity feedback coefficients for the fuel and emitter versus the thermally separated structural coefficients determine the system stability. This conclusion may be reached analytically if one uses a simple two temperature system with a simple neutron equation

$$\frac{dn}{dt} = \frac{\rho n}{\Lambda} \quad (19)$$

Performing the linearization and using the Routh-Hurwitz criterion, the stability requirement takes the form

$$\alpha_1 - \alpha_2 \left(\frac{c_1 r_1^2}{c_2 (r_2^2 - r_1^2)} \right) < \left(\frac{c_1 \Lambda}{N_0 \gamma_1} \frac{2h_2 r_2}{c_2 (r_2^2 - r_1^2)} \right) \left(\frac{2h_1}{r_1 c_1} + \frac{2h_2 r_2 + 2h_1 r_1}{c_2 (r_2^2 - r_1^2)} \right) \quad (20)$$

Fig. 2. Roots of open-loop plant simulation using 16 equations with changing values of temperature coefficient of the emitter, α_2

This simplified formulation shows that negative values of fuel and emitter coefficients, represented by α_1 , will promote stability. On the other hand, a large negative coefficient connected with the reactor structure or coolant, represented by α_2 , can lead to instability.

APPLICATION OF THE STABILITY CRITERION TO THERMIONIC REACTOR SYSTEMS

In a small fast-spectrum reactor containing thermionic diodes, the reactivity coefficients associated with the fuel or the emitter will tend to be small. The fuel is divided into short pieces inside the core so that little effect is felt from fuel expansion. In addition, the calculated Doppler coefficients for both fuel and tungsten in the reactor design examined were much smaller than previously expected. The calculated spectrum in the core design examined was much harder than those previously studied at Gulf General Atomic. Two factors seem to have produced this effect. First, the thermionic cells used had a larger diameter and the fuel volume fraction is higher than that used in higher power level designs. Second, the details of the structure of the reflector region have been designed in considerably more detail than before and the effect on the reflector upon the neutron spectrum has been lessened considerably.

It should be pointed out that the frequency of the calculated instability is very low and causes no serious problem in controller design. In addition, the margin of instability as represented by the magnitude of the positive real part of the roots is small and stability is obtained with small adjustment of the feedback coefficients. However, it is generally required that open-loop reactor systems be completely stable, and this will probably be true for the thermionic reactor. Furthermore, the uncertainties connected with calculated Doppler coefficients are still significant so that the existence of a positive real root within the error margin of the coefficients should be avoided.

Since the calculated instability has been shown to be related to the mass of the heat rejection system, a careful modelling of the coolant loops and radiator is necessary. However, if the root locus of Fig. 1 persists with more detailed analysis, then, as indicated in Fig. 2, steps should be

taken to improve the negative coefficient of the fuel-emitter combination. This can be accomplished by the addition of more tungsten to the emitter wall and/or a softening of the spectrum in the core. Methods of core support, which require less niobium between the core and the beryllium oxide reflector, may result in a softer spectrum. Furthermore, reactor core designs for low electric power levels which contain hydride material will also have a softer spectrum. The Doppler coefficients in these systems will probably be much improved and possible open-loop instability will be prevented.