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C00-2218-183

RADIAL ELECTRIC FIELDS FROM LARMOR RADIUS
EFFECTS IN THE FIELD-REVERSED THETA PINCH

BY

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B.S., National Tsing-Hua University, 1977

THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Nuclear Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1980

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YUAN HU, M.S.
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The generation of radial electric field from the different Larmor radii between the diffusing ions and electrons in Field-Reversed Theta Pinch (FRTP) has been evaluated by developing a new computer code, FLR. The code treats the background electrons as point particles, while for large-orbit ions the effects of finite Larmor radius are incorporated into an accurate density formulation. This is necessary since in a small device whose size is comparable to an ion gyroradius (e.g. FRTP) most ions will contribute to the density over a fairly large region of the plasma. The actual ion density at any point in the plasma represents a time-averaged contribution from all particles whose gyro-orbits pass through that point.

The FLR code has predicted the electric field strengths between 10^3 and 10^4 volts/cm, which is in good agreement with the experimental values of FRX-A measurements which correspond roughly to the order of 10^3 volts/cm.

Acknowledgements

The author would like to acknowledge her thesis adviser Professor Chan K. Choi for his guidance and encouragement, and to Professor George H. Miley for his valuable suggestions and discussions during the course of this thesis. In addition, she likes to express thanks for the constructive discussions given by Mr. Mark Campbell.

The author is greatly indebted to her beloved parents, and two brothers for their lasting support and encouragement.

Finally, a special note of thanks should also be given to Mrs. Chris Stalker for her assistance in the typing of this thesis manuscript.

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CHAPTER 1

ASPECTS OF FIELD-REVERSED THETA PINCH (FRTP)

1.1 Motivation and Significance of FRTP Study

From a fusion power standpoint the motivations for achieving field reversal are economic in nature: Field reversed systems, being intrinsically high beta ($\beta = \frac{\text{plasma particle pressure}}{\text{magnetic field pressure}}$) states, make very efficient use of the magnetic field. For the same reason they are compact; useful fusion power could be derived from field-reversed entities of only 5 or so orbit radii in dimensions. Finally, plasma confinement times in field-reversed units could be substantially enhanced as compared to confinement times in the comparable non-reversed states.

Advanced fuels, which are comprised of fusionable fuels other than D-T, require very high plasma temperatures and large regions of plasma essentially free of magnetic field void power losses due to synchrotron radiation, and, therefore, require the highest possible β . This can only be achieved when the plasma currents are strong enough to exclude or even reverse the external magnetic fields. Based on encouraging experimental results at LASL, the concept of Field-Reversed Theta Pinch (FRTP) appears to be an important alternative approach as a "compact torus" type reactor, either using D-T or advanced fuel.¹ Alternately, it might be viewed as a possible start-up approach for a Field-Reversed Mirror (FRM) reactor. Early experiments with theta pinches, in which powerful plasma currents are induced by a rapidly rising current in the external coil, were unsuccessful because those plasmas were sufficiently "dirty" that the plasma configuration was quickly destroyed by resistive instabilities. However, the interest in competing field reversal approaches has been

renewed by the experiments by Eberhagen² and by Es'kov, et al.³, using modern techniques, produced the reversed-field theta pinch configuration and showed that the reversed-field configuration (RFC) can achieve stable times of 25 to 50 microseconds which are more than an order of magnitude longer than is expected for the MHD and tearing mode growth rates. Similar experiments were undertaken at LASL (Linford, et al., 1978⁴) leading to the same conclusions about the stability: After the plasma has reached equilibrium, the RFC remains stable for up to 30 microseconds, followed by the rapid growth of the rotational $n = 2$ instability which terminates the confinement (Here n is the toroidal mode number). These theta-pinch plasmas are only a few Larmor orbits across, and thus still preserve the "compactness." In the following sections, the nature of field reversal (e.g., generation and stability of the reversal and the plasma rotation) will be discussed followed by the plasma transport in FR θ P and the electric field generation in Chapters 2 and 3, respectively. Summaries and the findings of the work will be illustrated in the conclusion.

1.2 Generation of Field Reversal

Figure 1 illustrates the arrangement of a field-reversed configuration in a theta pinch coil. In FR θ P experiments at Los Alamos⁴, the Kurchatov Institute³, and from earlier work by Eberhagen and Grossmann², a reversed field geometry is obtained by superimposing an axial bias field on the initial cold plasma, and then starting the main theta pinch discharge with reversed direction. The anti-parallel field situation was achieved for a cylindrical plasma sheet shown in Figure 2a. Subsequently in the first microsecond a tearing mode at the ends of the theta-pinch coil allows magnetic field lines to reconnect forming an elongated plasma torus.

Figure 1. Arrangement of Theta-Pinch Coil, Discharge Tube, and Reversed-Field Plasma Configuration.

Figure 2. Illustration of FRØP Start-Up.

Within a few microseconds the remaining annular plasma configuration contracted axially towards the midplane of the coil under the action of the enclosing field lines (Figure 2b). This produces a configuration shown in Figure 2c. Then the left-hand translation coil produces a much stronger field that drives the field-reversed plasma ring to the right. Finally it is ejected from the coil system into the guide field region (Figure 3). Axial translation is desirable because it would be difficult to make the closely-coupled theta pinch coil compatible with a minimum $-B$ geometry magnetic mirror coil set (i.e., guide field coils). The resulting closed field lines and confined plasma (Figure 4) has the same topology as in a low aspect ratio toroidal Z-pinch with an elongated cross section and with no stabilizing toroidal field. A separatrix in Figure 4 defines the plasma boundary inside of which field lines are closed and outside of which the magnetic field lines are open extending out to the discharge tube walls forming a natural divertor for the reversed-field configuration.

1.3 Stability

In general the stability of all field-reversed plasma configurations remains an open question. MHD stability is a major concern and is derived from the topological similarity between the FR θ P and the toroidal Z-pinch. In the absence of an external toroidal field, the toroidal Z-pinch is known to be unstable to violent "kink" modes that bend the plasma column, driving it to the walls of the vessel in microseconds. On the other hand the FR θ P has been shown to be remarkably stable in experiments, though this is not yet well understood theoretically.

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Figure 3. Axial Translation of FR0P.

Figure 4. Reversed-Field Configuration.

The unexpected MHD stability observed in the field-reversal experiments (FRX)^{2,3,4} has led to theoretical studies to resolve the issues on the field reversal. Some of these have demonstrated, again, the ideal MHD instability of these equilibria to the very localized modes in the infinite toroidal mode number limit ($n \rightarrow \infty$) by the energy principle approach. The kink mode may have a substantially reduced growth rate due to the highly elongated (elongation $\frac{b}{a} \gg 1$, b = axial length, a = the minor radius) cross section (Figure 4), where the growth rate is inversely proportional to the plasma length. Thus the geometry-modified growth rate may be too slow to see during the plasma life time.

Finite Larmor radius effects may also be important. In the theta pinch experiments and in the compact field-reversed mirror reactor design, $a/\rho_i \leq 5$ to 10, where ρ_i is the ion gyroradius whereby the MHD theory applicable to the Z-pinches may not apply. The introduction of the finite Larmor radius effects into plasma stability theory has caused speculation that finite gyroradius stabilizing effects are a general property of magnetically confined plasma. Whether these features actually account for the greater stability of theta pinches remains a matter of conjecture at this time. It is also possible that the observed plasma rotation may have a stabilizing effect.² All of these possibilities need careful theoretical consideration. Experimental investigation may become more practical in the future if the stable plasma lifetime can be extended long enough to observe these modes. Linford and co-workers at Los Alamos have exerted considerable effort⁴ in an attempt to duplicate the longer lifetime of 50 microseconds reported by Es'kov, et al. but have, thus far, been unsuccessful. The reason for the Russian lifetime³ is unclear but is thought to be due to their strong multipole barrier field.

Since the FRX profiles eventually rotated and then broke up with an $n = 2$ flute-like instability, D. Anderson, et al.⁵ have also used the LLL code, MALICE, to generate rotating equilibria and evolve these with time, later determine their stability properties. In doing so, they have observed a strikingly similar $n = 2$ instability in rotating plasmas, which is consistent with Linford's experiment.

Current efforts are aimed at addressing the finite Larmor radius stabilization question, calculation of growth rates of the finite n spectrum, and the control of the plasma rotation.

1.4 Plasma Rotation

The above experimental results indicate that the study of the stability properties of the FRX-produced plasma is dominated by the rotational $n = 2$ mode. Although rotation has long been thought to be the driving mechanism for the instability and an accurate prediction of the rotation speed needs to be reliably obtained, a satisfactory rotational instability theory does not exist.

FRØP-A (the smaller LASL experiment)⁴ has shown that the field-reversed plasma states can last for about 25 microseconds, at which point it is determined by the rapid growth of an $n = 2$ rotational instability. Meanwhile, measurements of the plasma rotation indicate that the plasma is formed with negligible angular momentum. The rotation velocity can be observed by the Doppler shift of the line radiation from carbon ions (C-V) which exist in small amounts in the plasma. However, the rotation source and angular acceleration rate are not well understood. The mechanism which causes the plasma to spin up to the $n = 2$ rotational instability threshold (critical rotation velocity) has not been positively identified.

However, three types of explanations have been proposed. First, the plasma may initially rotate and because of profile changes related to transport later reach the critical rotation at which point the instability begins. The magnitude of initial rotation for this explanation to hold requires the ions to carry a large fraction of the current initially. Since this is inconsistent with a number of theta-pinch experiments, this explanation seems unlikely.

The second possible contribution to plasma rotation might be the shorting out of the radial electric field at the walls.⁶ Line-tying of the open field lines shorts out the radial electric field causing the plasma on open field lines to rotate as in a standard theta pinch. This rotating plasma then causes the confined plasma to rotate by ion viscosity. This is unlikely to be the full explanation because the energy available in the open field region is much less than the final rotational energy of the plasma.

Finally, particle and flux loss from the closed field region might result in the loss of angular momentum from the system and corresponding plasma rotation. In Seyler's model⁷ the preferential loss of particles with negative angular momentum is considered to be the most plausible explanation for the spin-up of the plasma. These particles are found to be located either outside the separatrix, precisely the locations of anticipated large losses, or encircling the axis. Wang and Miley⁸ have shown that if the particles have negative angular momentum, they are untrapped (i.e., can leave the system). Particles are lost, resulting in a net gain of angular momentum by the confined plasma. Seyler⁷ found that the rotational velocity reaching the instability threshold requires the loss of one half to two thirds of the plasma. Also due to Eberhagen and

Grossmann, the observed rotation relies on plasma leaving the confined region carrying away momentum as a result of plasma radial diffusion across the magnetic field in the presence of a radial electric field.² (The electric field is established due to the different Larmor radii between the diffusing ions and electrons, which will be worked out in detail in Chapter 3.) The reaction force causes the confined plasma to spin in the opposite direction. In addition, resistive decay of the closed field flux corresponds to a net gain of mechanical angular momentum by the plasma.⁷

CHAPTER 2

PLASMA TRANSPORT IN FRØP

2.1 Introduction

Transport has been the least satisfactorily studied area both experimentally, and theoretically.

The rotational $n = 2$ mode, often observed to occur in experiments near the termination of plasma confinement, is presently thought to be the most plausible consequence of non-classical plasma transport. The rotation arises only after a significant part (i.e., one half or two thirds) of the plasma has escaped, implying that the basic limitation on plasma confinement is particle loss and is not due to the rotation itself. This leads to the motivation for the investigation of various confinement options.

2.2 Particle Confinement

It is assumed that the plasma length L is much larger than either the major radius R or the minor radius a , so that particle and heat losses are taken to be predominantly radial. We also assume that plasma variables are azimuthally symmetric ($\partial/\partial\theta = 0$) and independent of axial coordinate Z ($\partial/\partial Z = 0$). Only the variation of the plasma quantities with the radial coordinate r is treated. Though Linford, et al. never did any diagnostics to find out the density of neutral particles, it is viewed that the density of neutrals is less than the impurity level, which is 0.5%. This suggests that the plasma be fully ionized.

In the following discussion, then, a rigid rotor model⁴ is chosen to describe the radial behavior of the density and the magnetic field:

$$n(r) = n_m \operatorname{sech}^2 K[(r^2/R^2)-1] \quad , \quad (2.1)$$

$$B(r) = B_\infty \tanh K[(r^2/R^2)-1] \quad , \quad (2.2)$$

$$T_e(r) = T_e \quad , \quad (2.3)$$

$$T_i(r) = T_i \quad , \quad (2.4)$$

where

$$\begin{aligned} K &\equiv \tanh^{-1} (B_i/B_\infty), \\ B_i &\equiv -B(r=0) \text{ for } B(r=0) < 0, \\ B_\infty &\equiv B(r \gg R), \\ n_m &= n(r=R), \text{ and } R \text{ is the major radius.} \end{aligned} \quad (2.5)$$

Density and field profiles are plotted in Figure 5. The electron and ion temperatures are assumed to be uniform inside the separatrix.

By setting $B(r_s) = -B(0)$, where r_s is the separatrix radius, one finds that $r_s = \sqrt{2} R$. The minor radius is defined by Linford⁴ through the relation

$$n_m 2\pi R(2a) \equiv \int_0^\infty n 2\pi r dr \quad , \quad (2.6)$$

which yields $R/a = 4K/(1 + \tanh K)$.

Assuming axial uniformity of n and B within the separatrix, we define

$$N \equiv 2\pi \int_0^{r_s} dr r n(r) = \bar{n}(\pi r_s^2) = \pi r_s^2 n_m (\tanh K)/K, \quad (2.7)$$

$$W_e \equiv 2\pi(3/2)T_e \int_0^{r_s} dr r n(r) = \pi r_s^2 1.5 T_e n_m (\tanh K)/K, \quad (2.8)$$

$$W_i \equiv 2\pi(3/2)T_i \int_0^{r_s} dr r n(r) = \pi r_s^2 1.5 T_i n_m (\tanh K)/K, \quad (2.9)$$

2.2.1 Classical Diffusion in a Fully Ionized Plasma

The transverse-directed motion of the electrons and ions is given by

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$$n = n_m \operatorname{sech}^2 K \left(\frac{r^2}{R^2} - 1 \right)$$

$$B = B_o \tanh K \left(\frac{r^2}{R^2} - 1 \right)$$

$$K = \tanh^{-1} \left(\frac{B_i}{B_o} \right) \approx \frac{R}{2a}$$

$$a \equiv \frac{1}{2Rn_m} \int_0^\infty n r dr$$

Figure 5. Rigid Rotor Density and Magnetic Field Profiles.

$$\begin{aligned}
e\vec{E} + \frac{e(\vec{V}_e \times \vec{B})}{c} + T_e \frac{\nabla n_e}{n_e} + m_e v_{ei} (\vec{V}_e - \vec{V}_i) &= 0, \\
Ze\vec{E} + \frac{Ze(\vec{V}_i \times \vec{B})}{c} - T_i \frac{\nabla n_i}{n_i} - m_i v_{ie} (\vec{V}_i - \vec{V}_e) &= 0.
\end{aligned}
\tag{2.10}$$

For FRX-A parameters, the Debye length λ_D (evaluated using the average plasma density), which is 2×10^{-4} cm, is much smaller than the typical gradient scale lengths. Therefore, the neutrality of the plasma is conserved in the diffusion of charged particles. In finding the common solution of these equations we must keep in mind the neutrality condition $n_e \approx Zn_i \equiv n$ and the relation between the collision frequencies $n_e m_e v_{ei} = n_i m_i v_{ie}$ (a consequence of momentum conservation in the collisions). Solving the system in (2.10) it is an easy matter to find the directed particle velocities. The electron and ion velocities are the same in the direction of the density gradient:

$$\vec{V}_{e\perp} = \vec{V}_{i\perp} = -D_{\perp} \frac{\nabla_{\perp} n_e}{n_e}, \quad D_{\perp} = \frac{(T_e + \frac{1}{Z} T_i) v_{ei}}{m_e \omega_{ce}^2}, \tag{2.11}$$

where

$$v_{ei} = \frac{4\sqrt{2\pi} Z^2 e^4 n_i \ln \Lambda}{3\sqrt{m_e} T_e^{3/2}} \tag{2.12}$$

is the electron-ion collision frequency. ($\ln \Lambda$ is the Coulomb logarithm.⁹)

That is to say, the diffusion in a fully ionized plasma across a magnetic field is automatically ambipolar (as long as like-particle diffusion is neglected) regardless of the magnitude of the electric field in the plasma.

The electron and ion velocities in the direction perpendicular to the density gradient and the field are given by

$$\vec{v}_{e_T} = -\frac{c}{B} (\hat{b} \times \vec{E}) - \frac{cT_e}{eB} \frac{(\hat{b} \times \nabla n_e)}{n_e}, \quad (2.13)$$

$$\vec{v}_{i_T} = -\frac{c}{B} (\hat{b} \times \vec{E}) + \frac{cT_i}{ZeB} \frac{(\hat{b} \times \nabla n_i)}{n_i}, \quad (2.14)$$

here $\hat{b}$ is a unit vector in the direction of the magnetic field. These velocities determine the electron and ion drift that exists regardless of electron-ion collisions.

The diamagnetic current density, j_θ , in a fully ionized gas is given by

$$j_\theta = n_e e (\vec{v}_{i_T} - \vec{v}_{e_T}) = \frac{c}{ZB} (T_i + ZT_e) (\hat{b} \times \nabla n_e). \quad (2.15)$$

Particle confinement is governed by $\tau_p = N/[2\pi r_s (nV_r)_{r_s}]$. Assuming diffusive loss at the separatrix evaluated using Eq. (2.11), one finds

$$(nV_r)_{r_s} = - (D_\perp \frac{\partial n}{\partial r})_{r_s} = - \frac{c^2 n_1^p n(r_s) (T_e + T_i)}{B^2(r_s)} (\frac{\partial n}{\partial r})_{r_s}, \quad (2.16)$$

where n_1^p is the resistivity governing particle loss. Here in the above $Z = 1$ for a deuterium plasma is used, and the expression for classical resistivity is

$$n_1^p = \frac{4\sqrt{2\pi} e^2 \sqrt{m_e} \ln A}{3T_e^{3/2}}. \quad (2.17)$$

For FRX-A parameters (see Table 1 for a summary of plasma parameters), $\ln A \sim 12.25$ where the Spitzer expression for A was used. After calculation, we obtain $\tau_p \sim 1.2$ milliseconds. In conclusion, the classical cross field transport times are much too long, particle losses are too small and does not agree with experiment.

Table 1
FRX-A Parameters

$\vec{B}$ -field (kG)	3.0
Coil length (cm)	100
Peak density (cm^{-3})	2×10^{15}
Major radius, R(cm)	2 ~ 6
Minor radius, a(cm)	2
Length, 2b(cm)	35
Electron temp. (eV)	100
Ion temp. (eV)	200
Mirror ratio	1.2
τ_{life}	25 μ s

2.2.2 Bohm Diffusion

Bohm diffusion process is generally defined as

$$\tau_{\text{Bohm}} = \frac{a^2}{D_B} \quad (2.18)$$

where a is the minor radius, and D_B is the Bohm diffusion coefficient. For FRX-A parameters, $\tau_{\text{Bohm}} \sim 9.6$ microseconds.

The observed confinement time during the equilibrium phase of about 40 to 80 microseconds is longer than predicted for Bohm diffusion, but is substantially shorter than the 1.2 milliseconds of classical diffusion time. One candidate for enhanced transport is the like-particle (ion-ion) diffusion¹⁰ which can dominate electron-ion diffusion when the ratio of the ion gyroradius in the external field to the typical gradient scale lengths (i.e., $B/|\vec{\nabla}B|$, $n/|\vec{\nabla}n|$, etc.) is of the order of fourth root of the electron-ion mass ratio, e.g. $(m_e/m_i)^{1/4} \approx 0.1285$. Also possible is the anomalous transport due to the lower hybrid drift instability which is driven by relatively steep gradients on the plasma edge (radial separatrix point).¹¹ Consequently, we will choose the anomalous resistivity resulting from the lower hybrid drift instability to be tested.

2.2.3 The Lower Hybrid Drift Resistivity

The lower hybrid drift resistivity is given by

$$\eta_L^p = \frac{2\sqrt{2} \pi^{3/2}}{5\Omega_{ci}} \left(\frac{m_e}{m_i}\right)^{1/2} \frac{(T_i/T_e)^{5/4}}{1 + (T_i/T_e)} \frac{\Omega_{ci}^2}{\omega_{pi}^2} (\rho_i \ell_s)^2 \left[1 - \frac{1}{\sqrt{10}} \left(\frac{T_i}{T_e}\right)^{1/4}\right] \quad (2.19)$$

where Ω_{ci} is the ion cyclotron frequency and ω_{pi} is the ion plasma frequency. The quantity ℓ_s is the inverse gradient scale length, and ρ_i is the ion gyroradius. When ℓ_s is evaluated at the separatrix using a rigid rotor profile, $\tau_L \sim 342$ microseconds, implying that the value ℓ_s is sufficiently

small that particle loss times are larger than those observed experimentally. Hamasaki and Linford¹² have found that in fact the density drops very rapidly outside the separatrix, so ℓ_s should be larger than the value obtained from the rigid rotor profile. These uncertainties have led us to take the quantity $\ell_s \rho_i$ as a parameter which can be varied. Typically we have found that $\rho_i \ell_s = 2$ yield reasonably good agreement with the experimental data, consequently, τ_1 will be reduced to ~ 73 microseconds. Therefore, by our present experimental and theoretical understanding we see that the plasma contained in the closed field region gets diffused out into the open field region mainly due to microinstability-induced anomalous transport. However, particles on open field lines have been essentially unconfined in field-reversed experiments carried out to date. The diffused-out plasma leaks out along the open field lines, which enables the plasma edge to sustain a finite sharp gradient which, in turn, feeds the microinstabilities near the open field region. Then the microinstabilities will further enhance the radial plasma transport. In many cases on open field lines the magnetic field is slightly mirrored ($M \sim 1.2$) but not enough to significantly change the estimated free-streaming time of ions, i.e.,

$$\tau_{FS} = \frac{M L_{1/2}}{V_{ti}} \quad (2.20)$$

Here M is the magnetic mirror ratio, $L_{1/2}$ is the coil half-length, and V_{ti} is the ion thermal speed. For FRX-A parameters, $\tau_{FS} \sim 6.1$ microseconds. Thus, one possibility for improving particle confinement might dramatically increase τ_{FS} (e.g., by end plugs¹³ or a field reversed multiple mirror⁷), providing significant pressure on the separatrix and again reducing cross-field losses.

2.3 Thermal Conduction

The treatment of thermal conduction is uncertain because the temperature is constant inside the separatrix and is unknown outside the separatrix. In our modeling, it is evaluated by using the boundary layer model, where a uniform temperature profile is assumed except for a thin layer that is taken to be δ thick.

For ion thermal conduction we use

$$\tau_{th}^i = 1.5 \bar{n} T_i V / (q_r^i)_{r_s} S_A \quad (2.21)$$

where

$$(q_r^i)_{r_s} = - K_\perp^i (\partial T_i / \partial r)|_{r_s} = - K_\perp^i (T_i / \delta) \quad (2.22)$$

Here τ_{th}^i is the ion thermal conduction time, V/S_A is the plasma volume to surface ratio, δ is the temperature gradient scale length, and $K_\perp^i$ is the classical value for ion thermal conductivity given by⁹

$$\begin{aligned} K_\perp^i &= \frac{8(\pi m_i k)^{1/2} n_i^2 Z^2 e^2 c^2 \ln \Lambda}{3 B^2 T_i^{1/2}} \\ &= 1.48 \times 10^{-17} \frac{A_i^{1/2} Z^2 n_i^2 \ln \Lambda}{T_i^{1/2} B^2} \frac{\text{erg}}{\text{sec deg(K) cm}} \end{aligned} \quad (2.23)$$

where A_i is the atomic weight of positive ions. In Eqs. (2.22) and (2.23) temperatures are expressed in degrees Kelvin.

The value of δ is not known, however, an ion gyroradius temperature gradient scale length is chosen for the present studies. This model is chosen because the large ion orbits should lead to good thermal conductivity in the closed field region (and, consequently, a uniform plasma temperature).¹⁴ When $\delta = \rho_i$ is used, τ_{th}^i is found to be ~ 82.6 microseconds.

For electron thermal conduction we use

$$\tau_{th}^e = 1.5 \bar{n} T_e V / (q_r^e) r_s S_A, \quad (2.24)$$

where

$$(q_r^e) r_s = - K_1^e (\partial T_e / \partial r) |_{r_s} = - K_1^e (T_e / \delta) \quad (2.25)$$

Here K_1^e is the classical value for the cross field electron thermal conductivity given by¹⁵

$$K_1^e = \frac{18.64 (2\pi m_e k)^{1/2} n_i n_e Z^2 e^2 c^2 \ln \Lambda}{3 B^2 T_e^{1/2}} \quad (2.26)$$

In Eqs. (2.25) and (2.26) temperatures are expressed in degrees Kelvin.

When $\delta = \rho_i$ is chosen, electron thermal conduction time are so long (i.e., $\tau_{th}^e \sim 2$ milliseconds) that electron thermal conduction is unimportant.

Finally, we consider the electron-ion equilibration time, τ_{eq} . The Spitzer expression for energy equilibration time between Maxwellian ions and Maxwellian electrons is given by⁹

$$\tau_{eq} = \frac{3m_i m_e}{8(2\pi)^{1/2} n_e Z^2 e^4 \ln \Lambda} \left(\frac{T_i}{m_i} + \frac{T_e}{m_e} \right)^{3/2} \quad (2.27)$$

$$= \frac{3m_i m_e}{8(2\pi)^{1/2} n_e Z^2 e^4 \ln \Lambda} \left(\frac{T_e}{m_e} \right)^{3/2} \text{ sec} \quad (2.28)$$

$(1/\tau_{eq})$ is the average of the inverse of the electron-ion equilibration time, defined by

$$\begin{aligned} (1/\tau_{eq}) &= \frac{1}{N} 2\pi \int_0^R dr r n(r) / \tau_{eq}(r) \\ &= \left(\frac{8\sqrt{2\pi} m_e^{1/2} e^4 Z^2 \ln \Lambda}{3m_i} \right) \frac{n_m}{T_e^{3/2}} \left(1 - \frac{\tanh^2 K}{3} \right) \end{aligned} \quad (2.29)$$

For FRX-A parameters, $(1/\tau_{eq}) \approx 2.98 \times 10^4 \text{ sec}^{-1}$. From the conservation of energy we obtain

$$\begin{aligned} \frac{dT_i}{dt} &= (T_e - T_i) \left(\frac{1}{\tau_{eq}} \right) - \frac{T_i}{\tau_{th}} \\ &= -T_i \left(\frac{1}{\tau_{eq}} + \frac{1}{\tau_{th}} \right) + T_e \left(\frac{1}{\tau_{eq}} \right) \end{aligned} \quad (2.30)$$

In LASL experiments over the stable lifetime of the plasma, the electron temperature was essentially constant in both time and space at about 100 eV for radii interior to the separatrix.⁷ Therefore, assuming T_e is fixed, the ion temperature will decrease exponentially with the time constant $1/((\frac{1}{\tau_{eq}}) + \frac{1}{\tau_{th}})$, which is about 23.9 microseconds. During the 14 microseconds of equilibrium and instability phases, the ion temperature will decrease by about 40% which is in rough agreement with the experiment (~20%).¹⁶

CHAPTER 3 ELECTRIC FIELD GENERATION

Because of a radial electric field rotation arises away from a field null by the formula

$$\vec{v}_p = c \frac{\vec{E} \times \vec{B}}{B^2}, \quad (3.1)$$

where $\vec{v}_p$ is the drift velocity of any charged species, and $\vec{E}$ and $\vec{B}$ are the vector electric and magnetic field strengths, respectively. Such rotations of the plasma can greatly affect stability and transport.¹⁷

To calculate the magnitude of electric fields in plasmas we need to understand the various mechanisms of their generation. Electric fields can be set up by the charge separation, by nonequal transport rates, or by the fast fusion particles (FP's) losses. Petrie¹⁸ has shown that highly-energetic FP's in tokamak plasmas can have initial trajectories lying outside the plasma. Consequently, these particles can be permanently lost to the system. If neither the background electrons nor ions can cross magnetic surfaces fast enough to neutralize the charge left by the escaping fusion products, a radial electric field results. However, FP's leakage study is appropriate for reactor type plasmas, not for near-term experiments. Another way to generate the fields necessary to cause toroidal rotation is due to the different Larmor radii between the diffusing ions and electrons, which will be discussed in detail in Section 3.1.

Of equal importance are those damping mechanisms which tend to inhibit net charge accumulation inside the plasma. These plausible mechanisms include charge-exchange, magnetic field perturbations, perpendicular viscosity, and ion-neutral scattering --- etc.,¹⁸ Undamped plasma rotation may lead to $n = 2$ instabilities.

3.1 Finite Larmor Radius Effects and the Time-Averaged, Spatially Dependent Ion Density Equations

The finite orbit size of an energetic ion in the magnetic field gives rise to an immediate spreading of the ion density. One expects that this effect produces a spread of the ion density having a radial extent of approximately the order of the Larmor diameter. Thus, from these considerations, the effects of finite Larmor radius must be properly treated in order to accurately calculate a density picture.

The Theoretical Model:

Let the plasma be radially subdivided into concentric spatial shells. The spatial dependences are coupled due to the fact that the density at any point in the plasma represents a time-averaged contribution from all particles whose gyro-orbits pass through that point as indicated in Figure 6. The reason for doing this is that, in a small device whose size is comparable to an ion gyroradius (e.g., FRØP and FRM), most ions will contribute to the density over a fairly large region of the plasma, making a finite Larmor radius treatment mandatory for an accurate radial density profile. This is conveniently done by transforming the plasma guiding center densities (used in point particle models) into the plasma particle densities with an appropriate radial weighting function,²⁰ which will be worked out in the following section.

The ion distribution may be represented by a two-dimensional model. The magnetic field is assumed to be constant with respect to time. The plasma geometry is shown in Figure 6, where orbits are projected on the (r, ϕ) -plane. In a simple model we will assume that the ions displaced a distance, d , above the magnetic axis move in circular orbits of radius ρ . We now wish to compute the ion density at r due to all ions at r_0 which contribute to the density at r . Let R be the distance from the magnetic

axis to the center of an ion orbit. Now if we rotate the coordinate system so that the x axis coincides with a line drawn from the magnetic axis to the center of the circular orbit, then the equation for the circular orbit may be written in terms of the x, y coordinate system as

$$(x-R)^2 + y^2 = \rho^2 \quad (3.2)$$

The radial distance r is related to the x, y coordinate system by the equation

$$x^2 + y^2 = r^2 \quad (3.3)$$

Solving Equations (3.2) and (3.3) for x and y, one obtains the points of intersection of the orbit with the circle of radius r. The points of intersection are

$$\begin{aligned} x &= (R^2 + r^2 - \rho^2)/2R, \\ y &= \pm (1/2R)(2R^2r^2 + 2R^2\rho^2 + 2r^2\rho^2 - r^4 - R^4 - \rho^4)^{1/2}. \end{aligned} \quad (3.4)$$

Since the amount contributed to the density in an incremental element of volume between r and (r + dr) by an ion at r_0 is proportional to the amount of time the ion spends between r and (r + dr), the density at r is proportional to the path length dl where dl is given by the relation

$$dl = [(dx)^2 + (dy)^2]^{1/2} \quad (3.5)$$

Differentiating the expressions for x and y and substituting into Equation (3.5) gives

$$dl = \frac{2\rho r dr}{[4R^2r^2 - (R^2 + r^2 - \rho^2)^2]^{1/2}} \quad (3.6)$$

Therefore, the fraction of time spent between r and (r + dr) by an ion moving in a circle about a center at R is

$$\frac{2d\ell}{2\pi\rho} = \frac{2\pi r dr}{\pi^2 [4R^2 r^2 - (R^2 + r^2 - \rho^2)^2]^{1/2}} \quad (3.7)$$

Since the integral of this fraction over all r covered by the ion orbit must be unity, we have

$$\int_{r=R-\rho}^{r=R+\rho} \frac{2\pi r dr}{\pi^2 [4R^2 r^2 - (R^2 + r^2 - \rho^2)^2]^{1/2}} = 1 \quad (3.8)$$

This expression is easily verified by carrying out the integration.

The density of plasma guiding centers $D(R)$ may be defined as the number of ions/cm³ moving in circular orbits with centers at R . For a plasma isotropic in the azimuthal angle ϕ , it follows from Equation (3.7) that the relation between guiding center densities and the time-averaged plasma particle densities in cylindrical geometry is given by,

$$\begin{aligned} n(r) &= \int_{r-\rho}^{r+\rho} \int_0^{2\pi} \frac{D(R) d\phi R dR}{\pi^2 [4R^2 r^2 - (R^2 + r^2 - \rho^2)^2]^{1/2}} \\ &= \frac{2}{\pi} \int_{r-\rho}^{r+\rho} \frac{D(R) R dR}{[4R^2 r^2 - (R^2 + r^2 - \rho^2)^2]^{1/2}}, \end{aligned} \quad (3.9)$$

where $n(r)$ are the time-averaged plasma densities.

Rewriting Equation (3.9) in the form yields

$$n(r) = \frac{2}{\pi} \int \frac{R D(R) [U(R - |r-\rho|) - U(R - r - \rho)] dR}{[r^2 - (R-\rho)^2][(R+\rho)^2 - r^2]}, \quad (3.10)$$

Where $U(x)$ is the unit step defined by $U(x) = 1$ for $x \geq 0$, and $U(x) = 0$ for $x < 0$.

For calculation Equation (3.10) is used to define coefficients $f(r, r')$ which give the contribution to plasma density in a radial shell at radius r from unity guiding center density in a shell at radius r' . Hence,

$$n(r) = \sum_{r'} f(r, r') D(r'), \quad (3.11)$$

where r' sums over all shells to within one Larmor radius of r . The equation for f is

$$f(r, r') = \frac{1}{\alpha(r)} \int_{r-\Delta R/2}^{r+\Delta R/2} \left(\frac{2}{\pi} \int_{r'-\Delta R/2}^{r'+\Delta R/2} \frac{R[U(R-|x-p|)-U(R-x-p)]dR}{\{[x^2-(R-p)^2][(R+p)^2-x^2]\}^{1/2}} \right) 2\pi x dx, \quad (3.12)$$

where $\alpha(r)$ is the cross-sectional area of the radial shell of thickness ΔR centered at radius r .

3.2 The Computer Code: FLR

Section 3.1 covered the theoretical considerations of the finite Larmor radius effects on plasma particle densities. This section is concerned with the application of these concepts into a working code (FLR) developed by the author. The mechanics of FLR are relatively straightforward. The plasma is radially subdivided into equally spaced, concentric shells. As we have already noted in Section 3.1, the actual time-averaged ion density in each shell is determined by the sum of contributions to that particular shell from all ions whose gyro-orbits intercept it.

The radial electric field is generated due to the difference between the electron and the ion densities across the plasma radius (i.e., the electric field is found from Poisson's equation when we are given the charge density). Initially, this difference can be found by the method outlined in Section 3.1 for large-orbit ions to give a time-averaged, radial ion density profile which can be compared to the electron density to find the electric field. For calculating the difference between the electron and the ion densities in FLR code, it is convenient to treat

the background electrons as point particles. In this approach, therefore, the width of each shell must be much smaller than the average ion gyro-radii to discern the finite Larmor radius effects, but still narrow enough to be larger than the electron gyroradii.

Let the radial position of orbit center $\vec{R}$ be in the m^{th} shell. For convenience in FLR, we divide the orbit into the upper and the lower half-orbits respective to the line which both passes through the orbit center and is perpendicular to $\vec{R}$. In order to calculate the plasma density contribution in the k^{th} shell from the guiding center density in the m^{th} shell, one must first evaluate the fractional contributions (percent contributions) for each half orbit to each shell. As the ion moves along its orbit, it spends a certain amount of time in each shell it crosses. The time which the particle spends in, say, the i^{th} shell is proportional to the orbital path length at that particular shell. Then, the fractional contribution for each half orbit to the i^{th} shell is

$$f_m^i = \frac{2(\theta_i - \theta_{i-1})}{\pi} \quad , \quad i_{\min} \leq i \leq i_{\max} - 1 \quad , \quad (3.13)$$

where the 2 accounts for double-crossings of the shells. Here θ_i , illustrated in Figure 7, is given by

$$\theta_i = \begin{cases} \arcsin\left(\frac{r_i^2 - R^2 - \rho^2}{2Rp}\right) \quad , \quad \text{for upper half-orbit} \\ \frac{\pi}{2} - \arcsin\left(\frac{R^2 + \rho^2 - r_i^2}{2Rp}\right) \quad , \quad \text{for lower half-orbit,} \end{cases} \quad (3.14)$$

where $r_i = i(\Delta R)$ and $i_{\min} \leq i \leq i_{\max} - 1$. Note that $\theta_{i_{\min}-1} = 0$ and $f_m^{i_{\max}} = 1 - \sum_{i=i_{\min}}^{i_{\max}-1} f_m^i$. This process is thus repeated for each of the shells

Figure 7. Illustration of the Upper and Lower Half-Orbits Used in the FLR Code.

crossed by the orbit. After doing these, we obtain the contribution of a particle whose half-orbit radius is ρ and whose gyro-center is in shell m to all adjacent shells k where $\text{IFIX}(m - \frac{\rho}{\Delta R}) \leq k \leq \text{IFIX}(m + \frac{\rho}{\Delta R})$ and ΔR is the shell thickness. In the above, IFIX represents a function which converts its arguments from real values to integer values.

For convenience again, we consider that the density of plasma guiding centers in each of the shells is constant over the width ΔR (e.g., in shell m

$$N_m = \frac{\int_{(m-1)\Delta R}^{m\Delta R} D(R) 2\pi R dR}{\int_{(m-1)\Delta R}^{m\Delta R} 2\pi R dR}, \quad (3.15)$$

where N_m are the average guiding center densities in shell m).

Thus the plasma density contribution in the k^{th} shell from the guiding center density in the m^{th} shell, n_m^k , is then evaluated via

$$n_m^k = \left\{ \begin{array}{l} \text{fractional} \\ \text{contributions} \\ \text{to adjacent} \\ \text{kth shells} \end{array} \right\} \times N_m \times \left\{ \begin{array}{l} \text{the cross-sectional} \\ \text{area of the } m^{\text{th}} \\ \text{shell} \end{array} \right\} \quad (3.16)$$

$$\times \left\{ \begin{array}{l} \text{fraction of time} \\ \text{spent in upper} \\ \text{or lower half-} \\ \text{orbits (i.e., 50\%)} \end{array} \right\} \bigg/ \left\{ \begin{array}{l} \text{the cross-sectional} \\ \text{area of the } k^{\text{th}} \\ \text{shell} \end{array} \right\}.$$

Ultimately, sum contributions to each k^{th} shell to find total contribution to that particular shell from all adjacent shells.

So far the ion density distribution function has been completely determined. The generation of the radial electric field, then the plasma potential, is consequent upon the difference between our calculated time-averaged ion density and the assumed electron density across the radius. In this approach, the ion density formulation in fact requires using the

self-consistent ion orbit size which includes the electric field effects.

To see this, consider the equation

$$\rho_i = \frac{\sqrt{2m_i c^2}}{eB} (W_{\perp i} - e\phi)^{1/2}, \quad (3.17)$$

where $W_{\perp i}$ is the transverse energy of ions.

The orbits are dependent on particle energies and the equations discussed above in this chapter are applicable to any energy range, however, a major approximation made in the FLR code is that all ions are considered to be at the mean plasma energy.

The flow chart of the FLR code, a schematic for the electric field calculation, is shown in Figure 8. As shown, an initial orbit size is assumed with the constraint that it shall not be greater than the ion gyroradius in the vacuum field and the electric field for that particular gyroradius is evaluated. If $e\phi \leq W_{\perp i}$, the sequence is terminated by printing $\vec{E}$ and $n(r)$; if not, a new orbit size is then calculated as illustrated at the steps B and C (see Figure 8 for each assignment statement). This process is continued until not only the plasma potential is smaller than the mean transverse energy of ions but the width of each shell is smaller than the average ion gyroradii. At block A in Fig. 8, the transformation from the density of plasma centers to the time-averaged plasma particle densities is determined by the method outlined in this section.

Iterating on these equations, conditions, and the time-averaged density yields the self-consistent solution for the electric field and the ion gyroradii. Because the electric field is a rapidly varying function of radius, a very fine radial-grid mesh must be used. The number of shells in our code is approximately 7500.

Figure 8. FLR flow chart. At B, $a \equiv (\text{mean transverse energy of ions})/qE$.

CHAPTER 4

CONCLUSIONS

The actual electric field existing throughout the FR0P plasma is very difficult to determine, however, through plausible arguments via Equation (3.17) for the ion energies in FRX-A (~ 200 eV), plasma potential, ϕ , of the order of 200 volts is determined. The electric field, having magnitude approximately ϕ/δ with δ being the ion gyroradius, can be estimated. Therefore, electric field of the order of 10^3 volts/cm can be expected. Because the ion gyroradius is smaller than the typical gradient scale lengths (i.e., $B/|\vec{\nabla}B|$, $n/|\vec{\nabla}n|$, etc.), one expects a smooth electric field profile.

Equation (2.14) for ions can be expressed in the following forms

$$\frac{cE_r}{rB_z} + \frac{V_{\theta i}}{r} = \frac{c T_i}{ZerB_z} \frac{1}{n_i} \frac{\partial n_i}{\partial r} , \quad (4.1)$$

or

$$- \Omega_E + \Omega = \Omega^* , \quad (4.2)$$

where each term is an angular frequency. Here Ω_E is the $\vec{E} \times \vec{B}$ drift frequency, Ω is the rotational frequency, and Ω^* is the pressure-gradient drift frequency. For the rigid rotor profiles given by Equations (2.1) to (2.5), the pressure-gradient drift frequency can be written as

$$\Omega^* = - \frac{4K}{B_\infty R^2 Z} \frac{cT_i}{e} . \quad (4.3)$$

Within the accuracy of available measurements⁴, the probable range of Ω^* is $7 \times 10^5 < \Omega^* < 4 \times 10^6$ per second, and the experimentally observed critical carbon-V rotational velocity normalized to deuterium diamagnetic

drift velocity is ~ 0.4 . (At this normalized value of rotation the onset of the rotational instability occurs at times of 23 microseconds.⁴) That is,

$$\Omega_{\text{critical}} / \Omega^* \sim 0.4 \quad . \quad (4.4)$$

Thus, the electric field strengths between 10^2 and 10^3 volts per centimeter can be estimated from Equations (4.1) to (4.4) for the experimentally observed values of rotational frequencies of the order of 10^6 per second at times of 23 microseconds.

The FLR code has finally evaluated the distribution of the radial electric field as illustrated in Figure 9. In the FLR code we treat electron density profile as that of a rigid rotor. As for the accurate ion-density distribution function, it appears to be a rigid rotor profile. It takes only a small charge imbalance (typically one part in 10^6) to give rise to potentials of the order of kT/e ,²¹ which is 200 volts in FRX-A. Therefore, the difference between the electron and ion densities across the plasma radius is so small when compared to the number of the total densities that it cannot be demonstrated in the figure. Nevertheless, the sign of the spatial derivative of the radial electric field indicates whether the ion density is larger or smaller than the electron density. Thus, we find from Figure 9 that in the interior of the plasma the ion density is smaller than the electron density; however, near the separatrix the ion density is larger than the electron density which is consistent with the FLR effects. One expects that the FLR effects will produce a higher ion density having a radial dimension of approximately the order of the Larmor diameter, thus, there must be region toward the inside of the plasma with the corresponding lowering density in consequence of the particle conservation.

Figure 9. The electrostatic field estimated due to the finite Larmor radius effects is plotted vs. plasma radius r for the FRØP. Experimental values of FRX-A correspond roughly to the order of 10^3 volts/cm. R is the major radius, and r_s is the separatrix radius.

In fact for the current FROP system, we need to take into account for both the effect of transition between different classes of orbits where a particle may undergo through collisions as it diffuses and also the local magnetic field effect. These considerations are, however, quite complicated and require lengthy derivations in order to be included in the present work. It is the scope of the current study to sketch the nature of some of the aforementioned phenomena by considering the simple case as illustrated in Chapter 3. The current investigation is thus only to show that electric fields can be generated by the finite Larmor radius effects: Whether it is the only mechanism remains to be an open question.

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