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9700 South Cass Avenue
Argonne, Illinois 60439

**STUDIES OF AXIAL-LEAKAGE SIMULATIONS FOR HOMOGENEOUS
AND HETEROGENEOUS EBR-II CORE CONFIGURATIONS**

by

K. N. Grimm and D. Meneghetti

EBR-II Division

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ABSTRACT

When calculations of flux are done in less than three dimensions, leakage-absorption cross sections are normally used to model leakages (flows) in the dimensions for which the flux is not calculated. Since the neutron flux is axially dependent, the leakages, and hence the leakage-absorption cross sections, are also axially dependent. Therefore, to obtain axial flux profiles (or reaction rates) for individual sub-assemblies, an XY-geometry calculation delineating each sub-assembly has to be done at several axial heights with space- and energy-dependent leakage-absorption cross sections that are appropriate for each height. This report discusses homogeneous and heterogeneous XY-geometry calculations at various axial locations and using several differing assumptions for the calculation of the leakage-absorption cross section. The positive (outward) leakage-absorption cross sections are modeled as actual leakage absorptions, but the negative (inward) leakage-absorption cross sections are modeled as either negative leakage absorptions ($\pm B^2$ method) or positive downscatter cross sections [the $\Sigma_g(1+g)$ method].

I. INTRODUCTION

Experimental Breeder Reactor No. II (EBR-II) has a heterogeneous core region (containing driver subassemblies and fueled and structural experimental subassemblies), upper and lower steel axial reflectors, a radial steel reflector, and a surrounding fertile radial blanket. Because of computer storage requirements and iteration-time constraints, flux calculations are not done in three dimensions for this heterogeneous system. Instead, two-dimensional calculations are done, with the third (axial) dimension modeled by

leakage-absorption cross sections. To obtain heterogeneous axial flux (or reaction-rate) profiles, multiple heterogeneous two-dimensional calculations have to be done with the appropriate space- and energy-dependent leakage-absorption cross sections.

Reference 1 describes how current run-to-run transport calculations are performed and the problems associated with doing flux calculations in less than three dimensions. It was seen that problems can arise in multigroup analysis when leakages in the noncalculated dimension are modeled by leakage-absorption cross sections. The upper-energy-group leakages are usually positive (outward), whereas the lower-energy-group leakages are usually negative (inward). Positive leakages imply positive leakage absorptions in a DB^2 type of leakage model, whereas negative leakages imply negative leakage absorptions. These negative leakage absorptions can, in turn, lead to errors in the iterative calculation of flux and eigenvalue.

Reference 1 introduced two alternative methods of modeling inward axial leakages: use of either additional fission sources (χ_g method) or additional downscatter sources [$\Sigma_g(1+g)$ method]. That reference also compared flux spectra from RZ calculations to those from 1D cylindrical calculations, using in both instances the two leakage-approximation methods.

Whereas Ref. 1 dealt with transport-theory analysis, this report emphasizes use of diffusion-theory analysis in determining the effects of heterogeneity. This report compares fluxes and eigenvalues from RZ calculations to those from the 1D cylindrical calculations. It also compares fluxes and eigenvalues from XY calculations which use leakage-absorption cross sections from RZ calculations to those from an XYZ reference calculation. In the studies reported, axial leakage absorptions based on the XYZ reference configuration were used in an XY simulation of another core loading configuration, and the results for that simulation were compared with those from an XYZ calculation of the latter configuration. This was an attempt to see if "generic" leakage absorption cross sections can be used for similar calculations.

The primary reason for changing from transport-theory analysis to diffusion-theory analysis for these studies is that comparisons can more easily be made between heterogeneous two-dimensional (XY) results, using one of the above modeling methods, and heterogeneous three-dimensional (XYZ)

results with the latter analysis. Reference 1 showed that the two approximation methods [the χ_g method and the $\Sigma_s(1+g)$ method] gave approximately the same errors. This was expected, because both methods were designed to model the same phenomenon. In this report, results using only the $\Sigma_s(1+g)$ method will be presented. (The χ_g method was not used, because the diffusion-theory code used throughout this report would have had to been reprogrammed to accept the regional fission spectra inherent in that method.) In addition, since the diffusion-theory code used allows direct use of both positive and negative leakage absorptions, results obtained from using these leakage absorptions will also be presented.

In the diffusion-theory code used throughout this analysis, leakage absorptions are input by the use of buckling (B^2) values. The input buckling values are multiplied by the in-situ diffusion coefficient (D) to obtain the leakage-absorption cross section (DB^2). Results obtained from leakage-absorption cross sections calculated in this way will be referred to as results from the $\pm B^2$ method. The main focus of this report will, however, be on results from the $\Sigma_s(1+g)$ method, because, as will be shown later, in most cases the results using the $\Sigma_s(1+g)$ method are more accurate than those from the $\pm B^2$ method.

The core configuration used as a reference in this report is based on a representative current loading similar to EBR-II run 105A. It has about 6-1/2 rows of a core region, 3-1/2 rows of a radial reflector region, and 6 rows of a radial blanket region. Figure 1 depicts the configuration. Except where otherwise indicated, the half-height geometrical configuration used in most of the RZ calculations is the upper half of the full-height RZ description shown in Fig. 2. This also was the basis of the XYZ axial demarcations. The XY calculations were quarter-core calculations (sector A plus half of sector B) with reflective boundary conditions at the quarter boundaries. The same quarter core was used in the XYZ calculations. Figure 3 shows this geometry.

The 29-energy-group neutron cross-section set for the calculations was generated by the MC² code,² using the P_0 transport approximation and based on ENDF/B Version I data. Table I shows the 29-energy-group structure. Calculations (XYZ, RZ, XY, and 1D cylindrical) were done with the CITATION diffusion-theory analysis code,³ assuming a total reactor power of 57.25 MWt.

When flux spectra calculated in 2D RZ and 1D cylindrical geometry are compared, they will be compared for cylindricized rows 1, 5, 9, and 12. Rows 1 and 5 have been chosen as representative core rows, row 9 as a representative radial reflector row, and row 12 as a representative radial blanket row.

Fig. 1. Core Loading Configuration for EBR-II Run 105A

Fig. 2. Geometrical Configuration for RZ Reference Calculation

Fig. 3. Geometry and Numbering Convention
Used in XY Quarter-core Analysis

Current run-to-run transport-theory calculations of EBR-II neutronics in XY geometry use single energy-independent buckling values in the core region (0.0023 cm^{-2}), the radial reflector region (0.00085 cm^{-2}), and the radial blanket region (0.0023 cm^{-2}). These buckling values give satisfactory eigenvalues and generally satisfactory reaction rates except for very-low-energy reactions. Therefore, they are used as a starting point for the diffusion-theory calculations. When they are used in a 1D cylindrical calculation, they give an error of 1.32% in the system eigenvalue (relative to the 2D eigenvalue).

TABLE I. Twenty-nine-group-set Energy Structure Used in Neutronic Calculations for EBR-II Runs

Group No.	Upper Energy, eV	Lethargy Width
1	0.19999+8 ^a	0.5
2	0.60653+7	0.5
3	0.36788+7	0.5
4	0.22313+7	0.5
5	0.13534+7	0.5
6	0.82085+6	0.5
7	0.49787+6	0.5
8	0.30197+6	0.5
9	0.18316+6	0.5
10	0.11109+6	0.5
11	0.67379+5	0.5
12	0.40868+5	0.5
13	0.24788+5	0.5
14	0.15034+5	0.5
15	0.91188+4	0.5
16	0.55308+4	0.5
17	0.33546+4	0.5
18	0.20347+4	0.5
19	0.12341+4	0.5
20	0.74852+3	0.5
21	0.45400+3	0.5
22	0.27536+3	0.5
23	0.16702+3	0.5
24	0.10130+3	0.5
25	0.61442+2	0.5
26	0.37267+2	1.0
27	0.13710+2	1.0
28	0.50435+1	1.0
29	0.18554+1	1.0

^aRead 0.10000+8 as 0.10000×10^8 .

^bLower energy of group 29 = 0.68256+0.

Figures 4-7 show the errors in midplane flux for representative core rows 1 and 5, reflector row 9, and blanket row 12. These figures show that the major error in flux spectra, when using positive, constant bucklings, results from the out-leakage of low-energy neutrons when really there should be an in-leakage of those neutrons. In this case, the RZ calculations indicate that core in-leakage starts in energy group 13. This is apparent in Table II, which also shows that the energy group for which in-leakage starts varies from region to region. The results also show that the high-energy flux in the radial reflector region (row 9) is too large, which indicates that the radial-reflector buckling value is too small and can cause an excessive eigenvalue.

II. THEORETICAL CONSIDERATIONS

The groupwise axial leakage (L) from centered mesh point i,j,k to centered mesh point $i,j,k-1$ is calculated as follows. Using Fick's law, the boundary groupwise axial leakage is calculated from the groupwise flux at centered mesh point i,j,k and the groupwise flux at the boundary. The boundary groupwise axial leakage is also calculated from the groupwise flux at centered mesh point $i,j,k-1$ and the groupwise flux at the boundary. Equating these leakages (i.e., requiring that there is no neutron pile-up on the boundary) and eliminating the boundary groupwise fluxes gives the groupwise leakage (L_g):

$$L_g(z_{k-1}) = \frac{2A_{i,j}}{\frac{\Delta z_k}{D_{i,j,k}^g} + \frac{\Delta z_{k-1}}{D_{i,j,k-1}^g}} (\phi_{i,j,k}^g - \phi_{i,j,k-1}^g), \quad (1)$$

where $A_{i,j}$ is the normal cell area, and $\phi_{i,j,m}^g$, $D_{i,j,m}^g$, and Δz_m are, respectively, the cell centered groupwise flux, the cell diffusion coefficient, and the cell height for energy group g and the m th cell in the

Fig. 4. Midplane volume-integrated Flux in EBR-II Core Row 1 for 2D RZ Reference Calculation and 1D Calculation with Constant Buckling

Fig. 5. Midplane Volume-integrated Flux in EBR-II Core Row 5 for 2D RZ Reference Calculation and 1D Calculation with Constant Buckling

Fig. 6. Midplane Volume-integrated Flux in EBR-II Reflector Row 9 for 2D RZ Reference Calculation and 1D Calculation with Constant Buckling

Fig. 7. Midplane Volume-integrated Flux in EBR-II Blanket Row 12 for 2D RZ Reference Calculation and 1D Calculation with Constant Buckling

TABLE II. Regional Bucklings for 2.65-cm (one-mesh-interval) Axial Slice at Core Midplane

Energy Group	Buckling, cm^{-2}			
	Core Row 1	Core Reflector Row 5	Row 9	Blanket Row 12
1	2.86-3 ^{a,b}	2.76-3	3.00-3	1.48-3
2	2.82-3	2.72-3	2.99-3	1.43-3
3	2.82-3	2.73-3	2.92-3	1.55-3
4	2.78-3	2.69-3	2.75-3	1.74-3
5	2.71-3	2.62-3	2.40-3	1.83-3
6	2.66-3	2.57-3	2.13-3	1.70-3
7	2.38-3	2.29-3	1.81-3	1.54-3
8	2.13-3	2.03-3	1.52-3	1.38-3
9	1.90-3	1.77-3	1.34-3	1.28-3
10	1.44-3	1.30-3	1.11-3	1.16-3
11	8.23-4	7.23-4	9.25-4	1.04-3
12	1.47-4	6.80-5	7.49-4	9.66-4
13	-1.52-3	-1.21-3	4.60-4	7.93-4
14	-3.00-3	-2.44-3	3.49-4	7.23-4
15	-4.31-3	-3.62-3	2.96-4	6.86-4
16	-5.99-3	-4.92-3	2.37-4	6.39-4
17	-6.47-3	-5.21-3	1.83-4	5.77-4
18	-1.47-2	-9.82-3	-1.39-6	4.28-4
19	-3.26-2	-1.80-2	-1.70-4	3.23-4
20	-6.41-2	-2.77-2	-3.53-4	1.75-4
21	-9.42-2	-3.36-2	-5.34-4	2.55-5
22	-1.22-1	-3.71-2	-7.16-4	-1.31-4
23	-1.66-1	-3.91-2	-9.03-4	-2.91-4
24	-2.08-1	-4.32-2	-1.10-3	-4.74-4
25	-2.43-1	-4.01-2	-1.30-3	-7.23-4
26	-2.79-1	-4.81-2	-1.68-3	-1.06-3
27	-2.68-1	-6.30-2	-2.08-3	-1.61-3
28	-1.43-1	-9.57-2	-2.45-3	-1.89-3
29	-1.83-1	-8.04-2	-3.04-3	-2.37-3

^aRead 2.86-3 as 2.86×10^{-3} .

^bConstant-buckling cases assume: $2.30-3 \text{ cm}^{-2}$ in core region, $8.50-4 \text{ cm}^{-2}$ in reflector region, and $2.30-3 \text{ cm}^{-2}$ in blanket region.

axial direction. The leakage-absorption cross section (DB^2) for any given region (R) is then calculated by summing the top- and bottom-cell leakages in a region and then using

$$D_R^g B_{R,g}^2 \phi_R^g V_R = L_g(R), \quad (2)$$

where D_R^g , $B_{R,g}^2$, ϕ_R^g , and V_R are, respectively, the region (R) and energy-group (g) diffusion coefficient, buckling, average flux, and volume. The region groupwise leakage, $L_g(R)$, is the sum of the region top-cell and bottom-cell axial leakages, since internal cell-to-cell leakages cancel.

The theoretical basis for the $\Sigma_s(1 \rightarrow g)$ method was discussed in Ref. 1. Repeated here for completeness is the equation used to calculate the values of $\Sigma_s(1 \rightarrow g)$ when the leakage-absorption cross section (DB^2) is negative. (Note that this relationship is just a flow-conservation equation.) The equation is

$$\Sigma_s^R(1 \rightarrow g) = \frac{\sum_{i \in R} |L_g(i)|}{\sum_{i \in R} \phi_i^R V_i} = \frac{\sum_{i \in R} D_i^g |B_{g,i}^2| \phi_i^g V_i}{\sum_{i \in R} \phi_i^R V_i}, \quad (3)$$

where the sum is over all mesh points (i) in the region R. The region-dependent pseudo-scattering cross-section sets so defined are represented by corresponding fictional isotopes in the cross-section library used in these neutronics calculations.

The group diffusion equations are

$$\begin{aligned} & -D_{r,g} \nabla^2 \phi_{r,g} + \left(\Sigma_{a,r,g} + \sum_n \Sigma_{s,r,g \rightarrow n} + D_{r,g} B_{r,g}^2 \right) \phi_{r,g} \\ & = \sum_n \left[\Sigma_{s,r,n \rightarrow g} + \frac{\chi_g}{k} (\nu \Sigma_f)_{r,n} \right] \phi_{r,n}, \end{aligned} \quad (4)$$

where

- g, n = energy-group indices,
- $D_{r,g}$ = diffusion coefficient at position r for energy group g,
- ∇^2 = Laplacian operator,

- $\phi_{r,g}$ = neutron flux at position r for energy group g ,
 $\Sigma_{a,r,g}$ = macroscopic absorption cross section at position r
 for energy group g ,
 $\Sigma_{s,r,g \rightarrow n}$ = macroscopic outscatter cross section at position r for
 scattering from energy groups g to n ,
 $B_{r,g}^2$ = buckling at position r for energy group g ,
 χ_g = fission spectrum,
 k = effective multiplication factor,

and

- $(\nu\Sigma_f)_{r,n}$ = macroscopic production cross section for fission
 at position r for energy group n .

When the $\Sigma_s(l \rightarrow g)$ method is used for negative leakage absorptions, normal mixing methods add $\Sigma_s(l \rightarrow g)$ to the inscatter macroscopic cross section, $\Sigma_{s,r,l \rightarrow g}$, for energy group g . In addition to modifying the inscatter macroscopic cross section, mixing automatically increments the outscatter cross section, $\Sigma_{s,r,l \rightarrow n}$, for energy group l . Therefore, to conserve the total cross section in energy group l , we will define a group- l absorption cross section for the pseudo-scattering isotopes as

$$\Sigma'_{a,r,l} = - \sum_n \Sigma_s(l \rightarrow n). \quad (5)$$

The group- l macroscopic total cross section is thereby conserved, i.e.,
 (absorption + pseudo-absorption) + (scattering + pseudo-scattering) =
 (absorption + scattering), or

$$\left[\Sigma_{a,r,l} - \sum_n \Sigma_s(l \rightarrow n) \right] + \left\{ \sum_n [\Sigma_{s,r,l \rightarrow n} + \Sigma_s(l \rightarrow n)] \right\} = \Sigma_{a,r,l} + \sum_n \Sigma_{s,r,l \rightarrow n}. \quad (6)$$

Therefore, for each region there is a pseudo-scattering isotope which has a group- l absorption cross section $-\sum_n \Sigma_s(l \rightarrow n)$ and downscatter cross sections from group l $[\Sigma_s(l \rightarrow n)]$. All of the other cross-section values

for the pseudo-scattering isotope are zero. All regional isotopes are added onto the regular isotope library and mixed (with a 1.0 number density).

The CITATION computer code does not explicitly use in-group scattering. This implies that the $\Sigma_g(1 \rightarrow n)$ method cannot be used to model a net inflow of neutrons in energy group 1 by a $\Sigma_g(1 \rightarrow 1)$ pseudo-scattering cross section. Therefore, if the $\Sigma_g(1 \rightarrow g)$ method is to be used in these situations, regions that have a net inflow in energy group 1 have to be modeled by using a minus leakage-absorption cross section in group 1 and the appropriate $\Sigma_g(1 \rightarrow g)$ pseudo-scattering cross section in the other energy groups having a net inleakage. In these situations, the group-1 absorption for the pseudo-scattering isotopes should not include the $\Sigma(1 \rightarrow 1)$ cross section.

When the $\Sigma_g(1 \rightarrow g)$ method is used, the flux times volume (area times 1 cm) of the n-dimensional calculation has to be compared with a flux times volume (area times axial height) used in the (n+1)-dimensional calculation. The axial height used for the flux-times-volume calculations has to be the same as the height for which the leakage absorptions were calculated. In essence, the flux calculated using the $\Sigma_g(1 \rightarrow g)$ method is the average flux in the axial interval associated with the leakage-absorption calculation.

III. RESULTS

The quantitative comparisons between the reference n-dimensional flux spectra and (n-1)-dimensional spectra calculated using leakage absorptions are presented here in four different sections. The first section (III.A) details the differences between a reference RZ calculation and a 1D calculation when either a full-core-height slice or a partial-core-height slice is assumed. Although the primary emphasis of this report is on heterogeneous effects, in Sec. III.A some row-wise homogeneous comparisons between 2D RZ and 1D cylindrical calculations are presented. These were carried out, using diffusion theory, as a basis for the subsequent XY- and XYZ-geometry diffusion-theory analyses and for comparisons with transport-analysis results of Ref. 1.

The second section (III.B) details the effects of one-subassembly heterogeneity. It shows the differences between a modified RZ reference calculation in which a sodium subassembly (sodium in a hex can) is in row 1 and a 1D full-core-height-slice calculation (and partial-core-height-slice calculation) in which a sodium subassembly is also in row 1 but for which the leakage absorptions were obtained from the reference RZ calculation (described in Sec. III.A). The section also presents the same types of results as above, but for a steel structural subassembly instead of a sodium subassembly in row 1. Although these heterogeneity effects involve only a subassembly in row 1, they should reflect the order of magnitude of error associated with any single-subassembly in-core heterogeneity.

The third section (III.C) details the differences between a homogeneous XYZ reference calculation and a homogeneous XY calculation when either a full-core-height slice (Sec. III.C.1) or a partial-core-height slice (Sec. III.C.3) is assumed. Section III.C.2 compares the XY flux spectra calculated using leakage absorptions derived from an RZ reference calculation with the spectra calculated using the leakage absorptions based upon the reference XYZ calculation. These row-wise homogeneous XY and XYZ flux calculations are in a sense an initial first step before full XYZ heterogeneous calculations are considered (in Sec. III.D). They show the errors inherent in an XYZ simulation using XY methods when the leakage-absorption cross sections are calculated from either XYZ fluxes or RZ cylindrical fluxes.

The fourth section (III.D) compares flux spectra for heterogeneous loadings. Section III.D.1 compares XY-geometry flux spectra calculated using leakage absorptions derived from the RZ reference calculation with those from a reference XYZ-geometry heterogeneous calculation. Section III.D.2 details differences in XY flux spectra when the heterogeneous loading configuration differs from the reference heterogeneous loading configuration, and the leakage absorptions used are based upon the reference RZ calculation. This section illustrates the applicability of reference leakage-absorption cross sections to nonreference-loading calculations.

A. Reference Core Calculations in RZ Geometry

1. One Slice Consisting of Full Core Height

In this case, the full core height is modeled as though it were one axial interval. Table III shows representative leakage absorptions obtained from the reference RZ axial calculation when the calculational slice is the full core height (34.49 cm). It shows that inleakage dominates after energy group 0 in the core region and that the energy level at which the transition from outleakage dominance to inleakage dominance decreases in going outward from the core to the radial blanket. Table IV shows, respectively, the group (1-g) macroscopic transfer cross section due to actual scattering, and Table V shows the group (1-g) macroscopic transfer cross section $[\Sigma_g(1 \rightarrow g)]$ due to inleakage simulation. Comparison of Tables IV and V shows that the pseudo-scattering dominates the (1-g) downscattering when there is inleakage.

The 1D cylindrical flux calculations using these leakage absorptions were done two ways. The first way used the positive and negative leakage absorptions as leakage absorptions. As discussed in Sec. I, this will be called the $\pm B^2$ method. The second way used the positive leakage absorptions as leakage absorptions and the negative leakage absorptions as $\Sigma_g(1 \rightarrow g)$ pseudo cross sections. Figures 8-11 show the flux errors, for both approaches, relative to the RZ calculation, in the reference rows. The results show that although both methods for calculating flux are identical theoretically, the iteration scheme's use of a positive in-scatter source calculated from an already converged current iterated flux gives less error than use of a negative DB^2 -type loss. This superiority of the $\Sigma_g(1 \rightarrow g)$ method over the $\pm B^2$ method is also reflected by the respective eigenvalues. The eigenvalue error using the $\pm B^2$ method is -0.070% whereas the eigenvalue error using positive leakage-absorptions in conjunction with the pseudo-scattering cross section [the $\Sigma_g(1 \rightarrow g)$ method] is +0.005%.

TABLE III. Regional Leakage-absorption Cross Sections for a 31.49-cm (full-core-height) Axial Slice

Energy Group	Cross Section, cm ⁻¹			
	Core Row 1	Core Reflector Row 5	Row 9	Blanket Row 12
1	1.90-2 ^a	2.04-2	7.32-3	4.78-3
2	1.69-2	1.74-2	6.47-3	4.26-3
3	1.41-2	1.45-2	5.55-3	3.94-3
4	1.12-2	1.16-2	4.80-3	3.66-3
5	9.19-3	9.26-3	4.39-3	2.13-3
6	6.80-3	6.78-3	3.36-3	2.28-3
7	4.82-3	4.58-3	2.42-3	1.69-3
8	3.72-3	3.55-3	2.12-3	1.33-3
9	2.48-3	2.28-3	1.50-3	1.03-3
10	1.15-3	8.25-4	1.27-3	8.36-4
11	-2.59-4	-4.92-4	8.31-4	6.92-4
12	-1.54-3	-2.00-3	7.41-4	5.75-4
13	-5.04-3	-5.01-3	6.73-4	5.27-4
14	-6.58-3	-6.58-3	2.16-4	4.38-4
15	-9.37-3	-9.83-3	1.17-4	3.72-4
16	-1.45-2	-1.51-2	9.30-5	3.61-4
17	-1.49-2	-1.62-2	6.28-5	5.48-4
18	-2.81-2	-2.80-2	-9.01-6	6.71-4
19	-5.75-2	-5.66-2	-1.10-4	3.18-4
20	-8.51-2	-8.66-2	-2.12-4	1.60-4
21	-9.54-2	-9.77-2	-3.05-4	6.28-5
22	-1.51-1	-1.58-1	-3.98-4	-5.91-5
23	-1.67-1	-1.78-1	-4.93-4	-2.28-4
24	-1.82-1	-1.89-1	-6.02-4	-7.08-4
25	-2.54-1	-2.68-1	-7.07-4	-1.07-3
26	-2.34-1	-2.46-1	-9.02-4	-8.05-4
27	-2.21-1	-2.27-1	-1.11-3	-1.95-3
28	-1.07-1	-1.14-1	-1.34-3	-8.27-4
29	-1.55-1	-1.60-1	-1.57-3	-1.53-3

^aRead 1.90-2 as 1.90 x 10-2.

TABLE IV. Regional Macroscopic Downscatter Cross Sections ($\Sigma_{s,1-g}$)

Energy Group	Cross Section, cm^{-1}			
	Core Row 1	Core Row 5	Reflector Row 9	Blanket Row 12
1	a	a	a	a
2	7.48-3 ^b	8.46-3	1.34-2	4.96-3
3	7.55-3	8.05-3	1.47-2	7.00-3
4	1.06-2	1.11-2	2.14-2	1.38-2
5	1.07-2	1.11-2	2.01-2	1.78-2
6	8.40-3	8.56-3	1.40-2	1.65-2
7	5.49-3	5.55-3	7.84-3	1.33-2
8	3.29-3	3.33-3	3.77-3	1.05-2
9	1.84-3	1.87-3	1.64-3	7.44-3
10	9.38-4	9.58-4	6.69-4	4.37-3
11	4.33-4	4.43-4	2.62-4	2.19-3
12	1.85-4	1.90-4	1.00-4	9.78-4
13	7.38-5	7.59-5	3.74-5	4.06-4
14	2.71-5	2.78-5	7.95-6	1.59-4
15	9.05-6	9.33-6	1.54-6	5.75-5
16	2.94-6	3.06-6	0.0	2.09-5
17	9.66-7	1.02-6	0.0	7.34-6
18	2.24-7	2.35-7	0.0	1.70-6
19	0.0	2.10-10	0.0	0.0
20	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	0.0
22	0.0	0.0	0.0	0.0
23	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0
25	0.0	0.0	0.0	0.0
26	0.0	0.0	0.0	0.0
27	0.0	0.0	0.0	0.0
28	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0

^aCITATION does not use the ingroup scattering cross section.

^bRead 7.48-3 as 7.48×10^{-3} .

TABLE V. Regional Pseudo-scattering Cross Sections [$\Sigma_s(1+g)$] for 34.49-cm (full-core-height) Axial Slice

Energy Group	Cross Section, cm^{-1}			
	Core Row 1	Core Reflector Row 5 Row 9		Blanket Row 12
1	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0
11	2.57-3 ^a	4.77-3	0.0	0.0
12	9.05-3	1.13-2	0.0	0.0
13	1.99-2	2.15-2	0.0	0.0
14	1.27-2	1.36-2	0.0	0.0
15	7.22-3	7.78-3	0.0	0.0
16	5.03-3	5.48-3	0.0	0.0
17	1.53-3	1.66-3	0.0	0.0
18	1.01-2	1.12-2	3.07-4	0.0
19	1.05-2	1.15-2	2.56-3	0.0
20	8.04-3	8.68-3	3.68-3	0.0
21	6.04-3	6.48-3	4.23-3	0.0
22	4.80-3	5.22-3	4.46-3	2.92-4
23	3.79-3	4.16-3	4.56-3	8.77-4
24	3.13-3	3.48-3	4.47-3	2.42-3
25	2.56-3	2.84-3	4.21-3	2.26-3
26	3.69-3	4.17-3	7.14-3	7.09-4
27	2.38-3	2.69-3	5.22-3	1.44-3
28	1.24-3	1.36-3	3.08-3	2.52-4
29	8.29-4	9.00-4	1.93-3	4.00-4

^aRead 2.57-3 as 2.57×10^{-3} .

Fig. 8. Percent Error in Flux (relative to RZ reference calculation) When Using a 1D Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ and $\pm B^2$ Methods: Full Core Height, Core Row 1

Fig. 9. Percent Error in Flux (relative to RZ reference calculation) When Using a 1D Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ and $\pm B^2$ Methods: Full Core Height, Core Row 5

Fig. 10. Percent Error in Flux (relative to reference RZ calculation) When Using a 1D Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ and $\pm B^2$ Methods: Full Core Height, Reflector Row 9

Fig. 11. Percent Error in Flux (relative to reference RZ calculation) When Using a 1D Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ and $\pm B^2$ Methods: Full Core Height, Blanket Row 12

2. Multiple Slices Across Full Core Height

In this case, the core is divided axially into 13 slices. Each axial mesh interval (2.653 cm), or node, of the core in the RZ reference calculation was made into a slice. Table VI shows representative leakage-absorption values in row 1 for the mid-core and top-of-core axial slices. Also shown, for comparison, are the values for the whole-core slice. The table shows that leakage absorption is a strong function of axial slice, so care must be taken that the set of leakage-absorption cross sections being used corresponds to the correct calculational slice.

Figures 12-15 show the flux errors from the $\Sigma_g(1 \rightarrow g)$ method for the reference rows. As with the calculation for the full core height, the errors associated with each slice are small. The calculated eigenvalue errors for the middle slice and the top slice are +0.010% and -0.003%, respectively. (Recall that the whole-core eigenvalue error is +0.005%.) Thus the eigenvalues calculated for partial core slices are similar to that for the whole-core slice (and also the RZ calculation).

3. Noncore Axial Slice

One-dimensional calculations of a single-node axial slice were also done for noncore regions. Of the two slice calculations considered, one was in the lower reflector region, and one was in the upper reflector region. The leakage-absorption cross sections for these calculations were obtained from a full-height RZ reference calculation. The leakage-absorption cross sections in the upper energy groups were negative in both of these calculations because of the strong axial leakage from the core region (all the core axial-slice calculations done previously had positive leakage-absorption cross sections in the upper energy groups).

These calculations also were done with both the $\Sigma_g(1 \rightarrow g)$ method and the $\pm B^2$ method. However, in the $\Sigma_g(1 \rightarrow g)$ method for these calculations, the negative leakage-absorption cross section for the first group had to be modeled by a minus buckling (the $\pm B^2$ method) value, because ingroup scattering, including, therefore, $\Sigma(1 \rightarrow 1)$ scattering, is not directly used in the CITATION diffusion-theory code. This negative leakage-absorption cross section leads to a group-1 negative effective transport cross section (absorption + downscatter + leakage absorption) in some regions of these

TABLE VI. Regional Leakage-absorption Cross Sections for Various Axial-slice Configurations in Core Row 1

Energy Group	Cross Section, cm^{-1}		
	One-mesh Interval, Midplane	One-mesh Interval, Top of Core	Whole Core
1	1.14-2 ^a	4.79-2	1.90-2
2	9.82-3	4.37-2	1.69-2
3	9.01-3	3.26-2	1.41-2
4	8.26-3	2.12-2	1.12-2
5	7.85-3	1.35-2	9.19-3
6	6.37-3	8.14-3	6.80-3
7	5.19-3	3.84-3	4.82-3
8	4.26-3	2.50-3	3.72-3
9	3.09-3	1.03-3	2.48-3
10	2.12-3	-9.79-4	1.15-3
11	1.10-3	-2.91-3	-2.59-4
12	1.67-4	-4.71-3	-1.54-3
13	-2.02-3	-9.65-3	-5.04-3
14	-2.95-3	-1.13-2	-6.58-3
15	-3.11-3	-1.67-2	-9.37-3
16	-3.88-3	-2.55-2	-1.45-2
17	-3.04-3	-2.80-2	-1.49-2
18	-1.32-2	-3.45-2	-2.81-2
19	-2.83-2	-6.52-2	-5.75-2
20	-5.10-2	-9.07-2	-8.51-2
21	-7.19-2	-9.92-2	-9.54-2
22	-8.00-2	-1.61-1	-1.51-1
23	-1.04-1	-1.72-1	-1.67-1
24	-1.35-1	-1.87-1	-1.82-1
25	-1.44-1	-2.61-1	-2.54-1
26	-1.92-1	-2.39-1	-2.34-1
27	-1.97-1	-2.26-1	-2.21-1
28	-1.51-1	-1.07-1	-1.07-1
29	-1.22-1	-1.60-1	-1.55-1

^aRead 7.48-3 as 7.48×10^{-3} .

Fig. 12. Percent Error in Flux (relative to RZ reference calculation) When Using a 1D Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Top and Middle Nodal Slices of 13-node Problem, Core Row 1

Fig. 13. Percent Error in Flux (relative to RZ reference calculation) When Using a 1D Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Top and Middle Nodal Slices of 13-node Problem, Core Row 5

Fig. 14. Percent Error in Flux (relative to RZ reference calculation) When Using a 1D Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Top and Middle Nodal Slices of 13-node Problem, Reflector Row 9

Fig. 15. Percent Error in Flux (relative to RZ reference calculation) When Using a 1D Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Top and Middle Nodal Slices of 13-node Problem, Blanket Row 12

problems for both the $\Sigma_s(1+g)$ method and the $\pm B^2$ method. Other than in group 1, no other negative leakage-absorption cross sections are involved in the $\Sigma_s(1+g)$ method, so there are no other energy groups with negative effective transport cross sections when using that method.

When the $\pm B^2$ method was used, the first few energy groups in some regions had negative effective transport cross sections. Even though these calculations involved some negative effective cross sections, they ran to completion. The flux errors were the largest in the higher energy groups because of the negative effective cross sections. Even though the $\pm B^2$ method resulted in more negative values of effective transport cross section, the fluxes calculated using the $\Sigma_s(1+g)$ method had more relative error. This is because the flux error resulting from the group-1 negative effective transport cross section influences more of the energy-dependent flux calculation, through the pseudo-scattering sources, than the negative effective transport cross sections do in the $\pm B^2$ method. This flux error is also reflected in the system eigenvalue errors.

The eigenvalue error for a lower-reflector-slice calculation relative to the full-height RZ calculation is -7.36% when the $\Sigma_s(1+g)$ method is used and -3.21% when the $\pm B^2$ method is used. (Recall that the $\pm B^2$ buckling method for a whole-core slice gave an eigenvalue error of -0.070% whereas the $\Sigma_s(1+g)$ method gave an error of +0.005%.) With these relatively large errors in eigenvalue and flux that result from the negative transport cross sections, it is not certain that the superiority of one method over another can even be discussed.

Two other approaches, neither of which was explored, might be used to assure that there are no negative transport cross sections. The first would be to change the computer code used in these calculations so that in-group scattering is used throughout the calculation. This would eliminate any need for negative buckling inputs, and the negative absorption cross section in the group-1 pseudo-scattering isotopes would be balanced by the positive pseudo-scattering cross sections in the out-scatter scattering matrix. The second method that could be used to eliminate negative transport cross sections is the χ_g method that was discussed in Ref. 1. This method modifies the fission spectra rather than modifying any cross-section values. Hence no

negative cross sections would ever be involved. In fact, the calculations for the lower-reflector and upper-reflector slices discussed in Ref. 1 were done with the χ_g method and not with the $\Sigma_s(1+g)$ method.

B. Effects of Heterogeneity in One Subassembly

This section is divided into four parts. The first two parts deal with whole-core- and partial-core-slice calculations when the heterogeneity is a sodium channel in the core row 1 position. The second two parts deal with whole-core- and partial-core-slice calculations when the heterogeneity is a reflector (structural) type of subassembly in core row 1. The structural-type heterogeneity will probably be the most prevalent in EBR-II analysis, but the basic heterogeneity effects will be more pronounced with the sodium channel.

1. Whole-core Slice with Sodium Channel in Row 1

The calculations for this case were 1D cylindrical calculations for the reference configuration except that the driver in row 1 was replaced by a sodium channel throughout the 34.49-cm-long axial region of the core. The sodium in the channel was smeared with the steel of the row-1 hex can. Since sodium has a large diffusion coefficient and a smaller absorption cross section than that of either a driver, reflector, or blanket subassembly, replacing a subassembly by a sodium channel allowed study of the effect of simulating a high-leakage heterogeneity.

To determine the error resulting from this change, a series of calculations was done. The first calculation done was the reference RZ calculation modified with a sodium channel, instead of a driver, in row 1. The fluxes from this sodium-channel-modified reference calculation were used as the comparison fluxes.

The first 1D calculation done used the whole-core positive leakage-absorption cross sections and pseudo-scattering $[\Sigma_s(1+g)]$ cross sections for the same modified reference loading. The calculation will be called the $(DB^2)_{Na}$ conservation case, because the leakage-absorption cross section for the whole core is being conserved in it. For this case, the maximum calculated groupwise flux error in row 1 is 0.13%, and the associated calculated eigenvalue error is 0.0013%.

The second 1D calculation done used the whole-core positive leakage-absorption cross sections and pseudo-scattering cross sections for the original (unmodified) reference loading. This calculation will be called the $(DB^2)_0$ conservation case. Figure 16 shows the row-1 groupwise flux spectra for the modified RZ reference calculation and the 1D $(DB^2)_0$ conservation case. Flux spectra, rather than flux errors, are shown, because the errors can be quite large. Also shown in this figure, for reference purposes, is the row-1 groupwise flux for the unmodified reference case with a driver subassembly in row 1. [In this figure, the flux for the $(DB^2)_{Na}$ conservation case cannot be distinguished from that for the modified RZ reference case.] The eigenvalue error for the $(DB^2)_0$ conservation case, relative to that for the modified reference case, is +0.097%.

Fig. 16. Full-core-height Volume-integrated Flux in EBR-II Core Row 1 for RZ Reference Calculation, Sodium-channel-modified RZ Reference Calculation, and for 1D Calculation with Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and Assuming Either $(DB^2)_0$ Conservation or $D_{Na}B_0^2$ Conservation

A third calculation was done in which the positive buckling B_0^2 (obtained by dividing the positive leakage-absorption cross section from the unmodified reference case by the diffusion coefficient, D_0 , for row 1 of that case) multiplied by the diffusion coefficient, D_{Na} , of the sodium channel was used in the energy groups with a positive leakage absorption. In groups that had a negative leakage absorption, the reference pseudo-scattering cross sections were multiplied by the ratio D_{Na}/D_0 . This calculation conserves the product of reference buckling and D_{Na} in all energy groups. The flux spectra for this case, called the $D_{Na}B_0^2$ case, is also shown in Fig. 16. The eigenvalue error for the case is -0.168%.

Although not obvious in the figure, the row-1 high-energy flux in the $(DB^2)_0$ conservation case is somewhat higher than that for the modified reference case. The comparisons of high-energy spectra in row 1 plotted in Fig. 16 show that the high-energy leakage-absorption cross section that should be used for a subassembly-sized heterogeneity in EBR-II is not a strong function of the subassembly composition. Instead, it is a function of the composition of the subassembly and its neighbors. Although this is not as true as for low-energy inleakage, it is still more valid than using the diffusion coefficient for the heterogeneous subassembly.

Table VII shows the activity errors, which are related by cross sections to flux-spectra errors, for the three cases under consideration. The actual errors shown in parentheses include cancellation of positive and negative errors whereas the absolute errors sum the absolute values of the errors in groupwise activity.

The ^{238}U fission activity is the only high-energy activity shown. All the other activities shown are low-energy capture activities. Table VII, while showing the same qualitative results as Fig. 16, also shows that the errors in activity are not nearly as large as the absolute errors in flux. Figure 17 shows the effects of a single-subassembly change on the error in radial flux spectra for each conservation case. The figure shows that the effects of a major subassembly change are felt only a few rows away, at most.

TABLE VII. Absolute Error in Row-1 Activities for Several Leakage-absorption Assumptions: Sodium Channel, Full Core Height

Case	Absolute Error in Activity in Row 1, %			
	^{238}U Fission	^{10}B Capture	^{181}Ta Capture	^{135}Xe Capture
Cons. $(\text{DB}^2)_{\text{Na}}$ ^a	0.01 (-0.01) ^b	0.05 (0.04)	0.05 (0.01)	0.06 (-0.06)
Cons. $(\text{DB}^2)_0$	0.47 (0.47)	2.53 (-1.75)	5.88 (-5.35)	16.54 (-16.54)
Cons. $\text{D}_{\text{Na}}\text{B}_0^2$	1.30 (-1.30)	2.60 (1.54)	6.33 (5.50)	24.86 (24.86)

^aIn this case, the leakage-absorption cross sections were calculated from the RZ reference with a sodium channel in row 1.

^bActual error (with cancellations).

Fig. 17. Radial Maximum Absolute Groupwise Error in Flux (relative to sodium-channel-modified RZ reference calculation) for 1D Calculations with Variable Leakage Absorptions Modeled by the $\Sigma_s(1-g)$ Method and Assuming Either $(\text{DB}^2)_{\text{Na}}$ Conservation or $(\text{DB}^2)_0$ Conservation

2. Multiple Slices Across Core Height with a Sodium Channel in Row 1

In the preceding section, the full core height of 34.49 cm was considered as the calculational slice for the sodium channel. In this section, calculations are shown for two partial-core-height slices with the sodium channel in row 1. The calculations are for a one-mesh-interval (2.653-cm)-high slice (or node) at the core midplane and a one-mesh-interval (2.653-cm)-high slice at the top of the core. The whole-core-slice calculations discussed in the preceding section will represent something in between these two limiting situations.

Figures 18 and 19 plot the row-1 flux spectra for the mid-node calculation and the top-node calculation. Each figure shows the spectra calculated for (a) the modified RZ reference configuration; (b) the modified 1D reference configuration, using the $\Sigma_s(1 \rightarrow g)$ method with the $(DB^2)_O$ conservation assumption; and (c) the original RZ reference configuration with a driver subassembly in row 1. The spectra based on the $(DB^2)_{Na}$ conservation assumption were also calculated, but because the relative error is so small in these spectra, they cannot be distinguished from the modified reference spectra in the figures. As with the whole-core-slice calculations, the only appreciable error is in the lower energy groups where the effective subassembly diffusion coefficient is most dependent on the actual composition of the subassembly. In a later section (III.D.1), a scheme is tried to modify somewhat the leakage-absorption cross sections, and hence the pseudo-scattering cross sections, for the $(DB^2)_O$ conservation case to reflect the actual composition of the row-1 subassembly and of the surrounding subassemblies.

Table VIII shows the row-1 absolute activity errors for the cases under consideration. As with the results for whole-core-slice activities, the results show that errors in integral quantities are much less than groupwise errors. When the $(DB^2)_{Na}$ conservation assumption is used, the relative eigenvalue errors are 0.0018% for the mid-core node and -0.0073% for the top-of-core node. When the $(DB^2)_O$ conservation assumption is used, the relative

Fig. 18. Middle-core-node (middle node of 13 axial nodes) Volume-integrated Flux in EBR-II Core Row 1 for RZ Reference Calculation, Sodium-channel-modified RZ Reference Calculation, and for a 1D Calculation with Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and Assuming $(DB^2)_0$ Conservation

Fig. 19. Top-core-node (top node of 13 axial nodes) Volume-integrated Flux in EBR-II Core Row 1 for RZ Reference Calculation, Sodium-channel-modified RZ Reference Calculation, and for a 1D Calculation with Variatie Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and Assuming $(DB^2)_0$ Conservation

TABLE VIII. Absolute Error in Row-1 Activities for Several Leakage-absorption Assumptions: Sodium Channel, Mid- and Top-of-core Slices

Case	Node	Absolute Error in Activity in Row 1, %			
		^{238}U Fission	^{10}B Capture	^{181}Tl Capture	^{135}Xe Capture
Cons. $(\text{DB}^2)_{\text{Na}}$ ^a	Mid	0.002 (0.002) ^b	0.054 (-0.053)	0.16 (-0.16)	1.29 (-1.29)
Cons. $(\text{DB}^2)_{\text{O}}$	Mid	0.86 (0.86)	1.29 (-0.45)	3.19 (-2.42)	84.32 (84.32)
Cons. $(\text{DB}^2)_{\text{Na}}$	Top	0.209 (-0.209)	0.222 (0.199)	0.288 (0.275)	0.281 (0.281)
Cons. $(\text{DB}^2)_{\text{O}}$	Top	7.17 (-7.17)	9.75 (7.93)	17.32 (16.27)	28.90 (28.90)

^aIn this case, the leakage absorption cross sections were calculated from the RZ reference with a sodium channel in row 1.

^bActual error (with cancellations).

eigenvalue errors are 0.060% for the mid-core node and 0.20% for the top-of-core node. These eigenvalue errors, the previously shown activity errors, and the previously shown flux spectra show that the high-energy flux has more relative error in the top-of-core-node calculations and that the low-energy flux has more relative error in the mid-core-node calculations.

3. Whole-core-slice Calculation with Structural Subassembly in Row 1

In this case, 1D whole-core-slice cylindrical calculations were done with the reference configuration in which the driver in row 1 was replaced with a structural subassembly in the 34.49-cm-long core region. The upper and lower reflector regions are similar to the structural-type channel, because they are a mixture of steel and sodium. The in-core composition of the structural subassembly assumed was identical to that of a radial-reflector subassembly. As with the sodium-channel case described previously, the structural-channel case followed along the same basic calculational path. First an RZ calculation was done for the reference configuration with a

structural subassembly substituted in row 1. Then a series of 1D calculations was done with the structural subassembly in row 1, first with leakage-absorption cross sections calculated from the structural-subassembly-modified reference RZ calculation and then with the leakage-absorption cross sections calculated from the basic reference RZ calculation in which a driver subassembly was in row 1. These will be called respectively the $(DB^2)_{sr}$ conservation case and, as before, the $(DB^2)_o$ conservation case. The case corresponding to the $D_{Na} B_O^2$ case (where only the reference buckling was conserved) was not calculated, because the sodium-channel results showed that $(DB^2)_o$ conservation assumption resulted in more accurate flux spectra and eigenvalues.

Figure 20 compares the flux spectra in row 1 for the structural-channel-modified RZ calculation and the flux spectra calculated by 1D analysis using the $(DB^2)_o$ conservation assumption. The $(DB^2)_{sr}$ conservation calculation was also done, but as with the results for the sodium-channel modification shown previously, the results cannot be distinguished on the figure from those for the RZ modified reference configuration. (The row-1 flux spectra from the RZ basic reference calculation is also shown for reference in Fig. 20.) Except for the spectra in the very-low-energy groups, the difference between the RZ-calculated modified reference spectra and the flux spectra calculated using the $(DB^2)_o$ conservation assumption is visually indistinguishable.

Table IX lists the relative errors in activities. From this table and Fig. 20, it is seen that the $(DB^2)_o$ conservation assumption gives very good results for the average core spectra and average core reaction rates. The relative eigenvalue error for the $(DB^2)_{sr}$ conservation case is 0.0003%, and the eigenvalue error for the $(DB^2)_o$ conservation assumption is 0.052%.

Figure 21 shows the effects of a structural subassembly in row 1 on the error in radial flux spectra for each conservation case. Even though there is a considerable amount of scatter in the curves, it is seen that the maximum relative absolute error in flux spectra is always less than 1% when the $(DB^2)_{sr}$ conservation assumption is used and is also less than 1% about four rows or more from the structural subassembly when the $(DB^2)_o$ conservation assumption is used.

Fig. 20. Full-core-height Volume-integrated Flux in EBR-II Core Row 1 for RZ Reference Calculation, Structural-channel-modified RZ Reference Calculation, and for 1D Calculation with Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and Assuming $(DB^2)_o$ Conservation

TABLE IX. Absolute Error in Row-1 Activities for Two Leakage-absorption Assumptions: Structural Channel, Full Core Height

	Absolute Error in Activity in Row 1, %			
	^{238}U <u>Fission</u>	^{10}B <u>Capture</u>	^{181}Ta <u>Capture</u>	^{135}Xe <u>Capture</u>
Cons. $(DB^2)_{sr}$ ^a	0.059 (-0.059) ^b	0.10 (-0.068)	0.25 (-0.23)	0.82 (-0.82)
Cons. $(DB^2)_o$	1.62 (-1.62)	0.80 (0.16)	1.26 (0.67)	0.84 (0.84)

^aIn this case, the leakage-absorption cross sections were calculated from the RZ reference with a structural channel in row 1.

^bActual error (with cancellations).

Fig. 21. Radial Maximum Absolute Groupwise Flux Error (relative to structural-channel-modified RZ reference calculation) for 1D Calculations with Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and Assuming Either $(DB^2)_{sr}$ Conservation or $(DB^2)_o$ Conservation

4. Multiple Slices Across Core Height with a Structural Subassembly in Row 1

As with the calculations for multiple slices with a sodium channel in row 1, analogous calculations were done for the structural-subassembly channel. Considered in the calculations were a 2.653-cm-high single mesh interval at the core midplane and a similar interval at the top of the core. The whole-core-slice calculation of Sec. III.B.3 will represent an axial average somewhere between these two limiting situations.

Figures 22 and 23 plot the flux spectra in row 1 for the mid-node and top node of the core. Each figure shows the spectra calculated for the RZ modified reference configuration, calculated using the $(DB^2)_o$ conservation assumption, and, for comparison, calculated for the original RZ reference configuration with a driver subassembly in row 1. As with all the other cases, the $(DB^2)_{sr}$ conservation case gave a spectra different by less than a few percent from the spectra for the modified reference configuration. The

Fig. 22. Middle-core-node (middle node of 13 axial nodes) Volume-integrated Flux in EBR-II Core Row 1 for RZ Reference Calculation, Structural-channel-modified RZ Reference Calculation, and for 1D Calculation with Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and Assuming $(DB^2)_0$ Conservation

Fig. 23. Top-core-node (top node of 13 axial nodes) Volume-integrated Flux in EBR-II Core Row 1 for RZ Reference Calculation, Structural-channel-modified RZ Reference Calculation, and for 1D Calculation with Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and Assuming $(DB^2)_0$ Conservation

results show the same basic pattern seen for the $(DB^2)_o$ conservation cases with the sodium channel, i.e., the error in the high-energy flux spectra at both the mid-node and the top node is relatively small, and the error in the low-energy flux spectra is small at the mid-node and high at the top node.

It is seen from these figures and others, especially Fig. 18, that although the $(DB^2)_o$ conservation assumption leads to a more accurate calculated low-energy spectra than when using simply the low-energy driver spectra, the spectra are not much different than the low-energy driver spectra. This is not surprising, because the $(DB^2)_o$ conservation assumption uses the low-energy leakage-absorption cross sections from the RZ calculation for the unmodified reference core.

Table X shows the absolute row-1 activities. The corresponding eigenvalue errors are 0.0039% for the mid-node and -0.0064% for the top node when the $(DB^2)_{sr}$ conservation method is assumed and 0.011% for the mid-node and 0.14% for the top node when the $(DB^2)_o$ conservation assumption is used. The magnitudes of the eigenvalue errors show that the top core node has a larger error than the mid-core node, and hence has a larger error in the high-energy flux spectra. However, Figs. 18 and 22 show that the mid-core node has a larger error in the low-energy flux spectra.

TABLE X. Absolute Error in Row-1 Activities for Two Leakage-absorption Assumptions: Structural Channel, Mid- and Top-of-core Slices

Case	Node	Absolute Error in Activity in Row 1, %			
		^{238}U Fission	^{10}B Capture	^{181}Ta Capture	^{135}Xe Capture
Cons. $(DB^2)_{sr}$ ^a	Mid	0.011 (0.011) ^b	0.047 (-0.021)	0.10 (-0.078)	4.08 (-4.08)
Cons. $(DB^2)_o$	Mid	0.84 (-0.84)	0.72 (-0.68)	1.39 (-1.36)	55.8 (-55.8)
Cons. $(DB^2)_{sr}$	Top	0.46 (-0.46)	0.18 (0.061)	0.18 (0.075)	0.19 (-0.14)
Cons. $(DB^2)_o$	Top	5.15 (-5.15)	6.65 (16.12)	12.17 (11.90)	17.97 (17.97)

^aIn this case, the leakage-absorption cross sections were calculated from the RZ reference with a structural channel in row 1.

^bActual error (with cancellation).

C. Calculations in XY Geometry for Homogeneous Core

The calculations described in Sec. III.A and III.B used RZ leakage-absorption cross sections with 1D cylindrical geometry to model RZ axial leakage. This section considers XY flux calculations of whole-core slices, in which the axial leakage-absorption cross sections are from either an XYZ calculation of a homogeneous core with row-smeared compositions (Sec. III.C.1) or an RZ flux calculation having those row-smeared compositions (Sec. III.C.2). Also considered, in Sec. III.C.3, are XY calculations of partial-core-height slices.

The three-dimensional XYZ fluxes used for the reference values were generated from a calculation which modeled a half-height, quarter-core reactor with azimuthally smeared subassemblies. Computer-storage considerations enabled only a half-height XYZ calculation to be done. The top half of the reactor, without the control rods, was used as the half-height model. (See Fig. 2 for the overall configuration.) Figure 3 shows the quarter-core XY configuration (with the appropriate boundary conditions used in all the XY and XYZ calculations). Because of the vacuum boundary conditions on the top and left boundaries, the ignoring of some peripheral blanket subassemblies, and the increased mesh spacing of the peripheral subassemblies, the errors that will be shown for the blanket rows may not be indicative of the error that would be obtained if all subassemblies were accounted for with the same mesh spacing. Row 7, the row with a mixture of fueled and nonfueled subassemblies, was smeared in the sense that all fueled-subassembly locations contained the average composition of the drivers in that row and all nonfueled subassemblies contained the average composition of the nonfueled subassemblies in that row. The XYZ leakage-absorption cross sections were obtained from this calculation.

With 29 energy groups, there can be 3509 subassembly-averaged groupwise flux errors in 121 subassemblies. Since it would be impractical to show a representative sample of these errors, two alternatives were considered: (1) show the groupwise maximum absolute flux errors in a subassembly, and (2) show the absolute error in reaction rates in a subassembly. The first method reduces to a tractable number the large quantity of errors, but it may give an unrealistic and pessimistic slant to the data. For example, if the error is 15% in one energy group and is 1% in all other energy groups, the error would

be shown as 15%. To circumvent this type of problem, the second method (absolute error in reaction rates in a subassembly) will be used. The absolute error in reaction rates is considered as the sum, over energy, of the absolute groupwise errors. The actual reaction-rate error will not be shown, because positive and negative errors can cancel each other to give an overly optimistic indication of the energy-dependent error. The ^{238}U fission reaction rate will be used as an indication of high-energy error, and the ^{181}Ta capture reaction rate will be used as an indication of low-energy error.

1. Full-core-height Homogeneous XY Calculations Using Axial Leakage Absorptions from an XYZ Calculation

In this case, subassembly-dependent leakage-absorption cross sections were obtained from a hexagonal row-smeared 1/8-core XYZ calculation and used in each subassembly of a hexagonal row-smeared 1/4-core XY calculation. The XY calculational geometry was identical to that for the core region of the XYZ calculation.

Figures 24 and 25 respectively show the absolute errors for the ^{238}U fission and ^{181}Ta capture activities when the full core height is modeled as an axial slice and the leakage-absorptions are modeled by the $\pm B^2$ method and are subassembly-dependent. Figures 26 and 27 plot the absolute errors when the $\Sigma_s(1+g)$ method is used to model negative leakage absorption. When the $\pm B^2$ method is used for each subassembly, the eigenvalue error is -0.12%; when the $\Sigma_s(1+g)$ is used, the eigenvalue error is +0.01%.

The results with use of row-smeared bucklings are plotted in Figs. 28-31. (The values for the row-smeared bucklings were obtained from the individual-subassembly XYZ calculations, not from RZ cylindrical calculations.) Figures 28 and 29 respectively show the absolute errors in ^{238}U fission and ^{181}Ta capture when the $\pm B^2$ method is used to model the row-smeared leakage absorptions; Figs. 30 and 31 show the corresponding absolute errors when the $\Sigma_s(1+g)$ method is used with the row-smeared leakage absorptions. The resulting eigenvalue errors are -0.19% when the $\pm B^2$ method

Fig. 24. Absolute Error in ^{238}U Fission Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\pm B^2$ Method: Full Core Height, Individual Subassembly Leakage Absorptions from XYZ Homogeneous Reference

Fig. 25. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\pm B^2$ Method: Full Core Height, Individual Subassembly Leakage Absorptions from XYZ Homogeneous Reference

Fig. 26. Absolute Error in ^{238}U Fission Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Full Core Height, Individual Subassembly Leakage Absorptions from XYZ Homogeneous Reference

Fig. 27. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Full Core Height, Individual Subassembly Leakage Absorptions from XYZ Homogeneous Reference

Fig. 28. Absolute Error in ^{238}U Fission Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\pm B^2$ Method: Full Core Height, Row-smeared Leakage Absorptions from XYZ Homogeneous Reference

Fig. 29. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\pm B^2$ Method: Full Core Height, Row-smeared Leakage Absorptions from XYZ Homogeneous Reference

Fig. 30. Absolute Error in ^{238}U Fission Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1-g)$ Method: Full Core Height, Row-smeared Leakage Absorptions from XYZ Homogeneous Reference

Fig. 31. Absolute Error in ^{181}Ta Fission Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1-g)$ Method: Full Core Height, Row-smeared Leakage Absorptions from XYZ Homogeneous Reference

is used and +0.02% when the $\Sigma_g(1+g)$ method is used. Both the activity errors and the eigenvalue errors are less with the latter method when either individual-subassembly or row-smeared leakage absorptions are used. This same result was shown in Sec. III.A.1 for 1D cylindrical calculations using leakage-absorption cross sections from a corresponding RZ reference calculation.

A study of the activity and eigenvalue errors from the $\Sigma_g(1+g)$ method reveals that the errors are less with the use of individual-subassembly leakage absorptions than with row-smeared leakage absorptions. This is to be expected, because the effects of axial leakage are better defined by 121 energy-dependent values of leakage absorption for the individual subassemblies than by 14 row-smeared energy-dependent values of leakage absorption. Nevertheless, the errors for each method are still relatively small (less than a few percent for absolute activity and 0.2% for eigenvalue).

In practice, of course, one would not do XY calculations with leakage absorptions calculated from the identical XYZ configuration. The results from such a calculation are presented here only to validate the methods used and gauge inherent errors.

2. Full-core-height Homogeneous XY Calculations Using Axial Leakage Absorptions from an RZ Calculation

Since the activity and eigenvalue errors are relatively small when row-smeared leakage-absorption cross sections from an XYZ calculation are used in a homogeneous XY calculation, it would be of interest to see how much more error, if any, would be introduced in homogeneous XY calculations if the leakage-absorption cross sections were from an RZ calculation. (These RZ leakage-absorption cross sections are certainly much easier to calculate than row-smeared XYZ values.) In the series of XY calculations, discussed here, the leakage absorptions used in a homogeneous XY calculation were obtained from a row-smeared, half-core-height RZ calculation. The row-smeared atom number densities in the core slice of the RZ calculation were identical to the hexagonal row-smeared densities in the XY calculation.

Figures 32 and 33 respectively plot the absolute errors in ^{238}U fission and ^{181}Ta capture activities when the full core height is modeled as an axial slice and the row-smear leakage absorptions modeled by the $\pm B^2$ method are obtained from the RZ reference calculation. Figures 34 and 35 show the corresponding absolute errors from the $\Sigma_s(1+g)$ method. These figures show

Fig. 32. Absolute Error in ^{238}U Fission Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\pm B^2$ Method: Full Core Height, Row-smear Leakage Absorptions from RZ Homogeneous Reference

Fig. 33. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\pm B^2$ Method: Full Core Height, Row-smear Leakage Absorptions from RZ Homogeneous Reference

again that the error is less when the $\Sigma_s(1+g)$ method is used instead of negative leakage absorptions to model inward leakage. As with the XYZ buckling cases discussed in the preceding section, the eigenvalue error is smaller with the $\Sigma_s(1+g)$ method—+0.067% vs 0.110% with the $\pm B^2$ method. Both the activity errors and the eigenvalue errors from the $\Sigma_s(1+g)$ -method calculations in this section and in the preceding section give approximately the same relatively small errors. Therefore, the use of leakage absorptions calculated from an RZ calculation, for use in an XY calculation, do not introduce significant additional error due to geometrical effects.

Fig. 34. Absolute Error in ^{238}U Fission Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Full Core Height, Row-smearred Leakage Absorptions from RZ Homogeneous Reference

Fig. 35. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Full Core Height, Row-smearred Leakage Absorptions from RZ Homogeneous Reference

3. Partial-core-height Homogeneous XY Calculations Using Axial Leakage Absorptions from an RZ Calculation

An XY full-core-height model was used in the two preceding sections, III.C.1 and III.C.2. Therefore, only the axially averaged fluxes would be obtained. Partial-core-height calculations can also be done in XY geometry as long as the height for the leakage-absorption calculation is core-slice height. (Recall that no matter what the calculational slice is, the product of the calculated 2D flux and the area is to be compared with the axially integrated flux over the calculational slice.) Because leakage-absorption values are strongly dependent on axial position, the calculational slice cannot be smaller than an axial mesh interval. In addition, the calculational-slice boundaries must be aligned with mesh-interval boundaries.

In this section, two different axial slices will be considered: (1) a mid-core one-mesh-interval slice, and (2) a top-of-core one-mesh-interval slice. These cases, and the full-height cases shown previously, should indicate the range of errors expected in multi-slice calculations within the axial range of the core. Only the results for the $\Sigma_s(1+g)$ method will be shown, because that method gives lower absolute errors than the $\pm B^2$ method for core slices. Cylindrical buckling values will be used, because they have been shown (see Sec. III.C.1 and III.C.2) to give results similar to those for the individual subassembly and the row-wise radial smear of the individual subassembly.

Figures 36 and 37 compare the mid-slice, top-slice, and whole-core-slice absolute errors for ^{238}U fission and ^{181}Ta capture rates, respectively. The errors for the top core slice are somewhat higher than those for the other slices, but, in general, all errors are less than about 4%. The eigenvalue errors for the mid-core slice, top-core slice, and whole-core slice are, respectively, 0.064%, -0.032%, and 0.067%. These results show that XY calculations for partial-core-height slices can be done with RZ cylindrical bucklings to give activity errors of less than a few percent and eigenvalue errors in the range of $\pm 0.1\%$.

Recall, from Sec. I, that constant bucklings (0.0023 cm^{-2} for the core, 0.0085 cm^{-2} for the radial reflector, and 0.0023 cm^{-2} for the radial blanket) are assumed in run-to-run transport calculations. Figures 38 and 39 plot the absolute errors in ^{238}U fission and ^{181}Ta capture activities when one-mesh-interval leakages for a mid-core slice are modeled by the above constant bucklings and when space- and energy-dependent leakage absorptions are used with the $\Sigma_s(1+g)$ method to model inward leakages. The results are the same as seen in Sec. I, i.e., the high-energy quantities have less error than the low-energy quantities. The eigenvalue error is 1.31% when using these constant bucklings and 0.064% when using variable bucklings with the $\Sigma_s(1+g)$ method. These eigenvalue errors are also consistent with those from the 1D calculations (see Sec. I).

Fig. 36. Absolute Error in ^{238}U Fission Activity (relative to XYZ homogeneous reference calculation) for Various Axial Core Slices When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Row-smeared Leakage Absorptions from RZ Homogeneous Reference

Fig. 37. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ homogeneous reference calculation) for Various Axial Core Slices When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Row-smeared Leakage Absorptions from RZ Homogeneous Reference

Fig. 38. Absolute Error in ^{233}U Fission Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Constant Bucklings: Mid-core Slice

Fig. 39. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ homogeneous reference calculation) When Using an XY Model with Constant Bucklings: Mid-core Slice

D. Calculations in XY Geometry for Heterogeneous Core

In all previous sections, the 1D cylindrical and 2D XY calculations were based on row-smeared compositions. The $\Sigma_g(l \rightarrow g)$ method worked very well on those homogeneous problems. The question that naturally arises is: How well can these methods do with heterogeneous problems?

Before calculations of heterogeneous problems are described, a few words will be said about what will be called a heterogeneous core loading. In such a loading, each subassembly will not have its own distinct atom number densities. Instead, for simplicity, all driver, partial-driver, control, radial-reflector, and radial-blanket subassemblies will have their own respective row-smeared atom densities for the indicated quadrant. All fueled and nonfueled experimental subassemblies will, however, have their own distinct compositions.

The first two parts of this section (Sec. III.D.1 and III.D.2) consider XY simulations of heterogeneous whole-core and partial-core slices, using the row-dependent, cylindrical leakage-absorption cross sections for the reference case. This is the same approach that was first discussed in Sec. III.B. As in that section, the use of these row-dependent leakage absorptions, modeled by the $\Sigma_g(l \rightarrow g)$ method, for all the subassemblies in a given row is called the $(DB^2)_0$ conservation assumption. Section III.D.1 also gives the results of auxiliary calculations to modify these row-wise leakage absorptions for each subassembly in an attempt to account for differing compositions of both that subassembly and of its nearest neighbor subassemblies.

The next two parts of this section (Sec. III.D.3 and III.D.4) discuss analogous whole-core-slice and partial-core-slice heterogeneous calculations for a quadrant of a different EBR-II heterogeneous core loading (run 121A) than discussed anywhere else in this report, but using the same leakage absorptions as the reference configuration. The object of these calculations is to see how much additional error can be expected if leakage-absorption cross sections based on one loading configuration are used in calculations for a similar, but different, core loading configuration. These calculations use the $(DB^2)_0$ conservation assumption in conjunction with the $\Sigma_g(l \rightarrow g)$ method. Section III.D.3 also discusses whole-core-slice calculations in which only the reference-case bucklings (B_0^2) are conserved. The leakage absorptions (DB^2)

are then calculated by multiplying the reference-case bucklings by the row-smeared diffusion coefficient of the different core-configuration loading. This procedure will be called the $D_{121}B_0^2$ conservation, because the diffusion coefficient is from the modified reference loading configuration, which was based on EBR-II run 121A, and the bucklings are based on the reference core-loading configuration (run 105A).

Something akin to this type of conservation was first discussed, for a single-subassembly heterogeneity, in Sec. III.B.1. The calculations described here were done to see if the same conclusions reached for a single-subassembly heterogeneity using a $D_{121}B_0^2$ type of conservation assumption apply to situations of multiple heterogeneity.

1. Simulation of Reference Full-core-height Heterogeneous XYZ Configuration Using XY Geometry and RZ Leakage Absorptions

The heterogeneity question was tested with two different XY whole-core-slice calculations. The first calculation assigned to each subassembly row-wise group leakage-absorption cross sections obtained from the reference RZ calculation (for run 105A). Use of this procedure is called the $(DB^2)_0$ conservation assumption, as before. The second calculation assigned to each subassembly a leakage-absorption cross section that was dependent upon the subassembly composition, location of the subassembly in a row, and the types of surrounding subassemblies. The position dependence of leakage absorption was determined by the spatial fraction that the hexagonal subassembly occupies in the neighboring cylindricized rows. The subassembly leakage-absorption cross section was set equal to the sum of the products of row fraction times the row-smeared leakage-absorption cross section obtained from the RZ reference calculation. The leakage-absorption cross sections were then modified to reflect the subassembly composition and the compositions of the six surrounding subassemblies. This was done by first determining the driver and structure fractions for the six surrounding subassemblies.

The subassembly-dependent leakage-absorption cross section along with driver and structure fractions for the surrounding subassemblies were then used in a linear interpolation scheme to find an adjusted subassembly leakage-absorption cross section. The interpolation scheme was based on reference leakages of either a driver or a structural subassembly surrounded by (a) all drivers, (b) half drivers and half reflector subassemblies, or (c) all reflector subassemblies. This scheme, in effect, tried to modify the reference cylindrical leakage-absorption values to account for the actual hexagonal-subassembly geometry and the effects of leakage from neighbors. This method will be called the positional-interpolation assumption.

Figures 40 and 41 plot the absolute error in ^{238}U fission and ^{181}Ta capture activities from these two calculations. These results show that the errors based on the $(\text{DB}^2)_0$ conservation assumption and the positional-interpolation assumption are similar. The first scheme, the $(\text{DB}^2)_0$ conservation assumption, conserves the homogenized leakage-absorption cross section $(\text{DB}^2)_0$ in all energy groups. Both that assumption and the positional-interpolation assumption treat positive and negative leakage effects the same way.

Fig. 40. Absolute Error in ^{238}U Fission Activity (relative to XYZ heterogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Full Core Height, Row-smeared Leakage Absorptions from RZ Homogeneous Reference

Fig. 41. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ heterogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Full Core Height, Row-smeared Leakage Absorptions from RZ Homogeneous Reference

The $(DB^2)_0$ conservation assumption and the positional-interpolation assumption give eigenvalue errors of +0.336% and +0.428%, respectively. These eigenvalue errors and Figs. 40 and 41 show there is no significant difference between results from the two assumptions. In fact, the only major difference between them seems to be the amount of work involved in calculating the leakage-absorption cross sections. With the $(DB^2)_0$ conservation assumption, only row-wise leakage-absorption cross sections are calculated (14 in this case). With the positional-interpolation assumption, these 14 row-wise leakage-absorption cross sections, represented by pseudo-isotopes, are transformed into 121 pseudo-isotopes. For this reason, the $(DB^2)_0$ assumption is preferable to the positional-interpolation assumption.

2. Simulation of Partial-core-height Heterogeneous XYZ Configuration Using XY Geometry and RZ Leakage Absorptions

In the preceding section, whole-core heterogeneous calculations were shown, using either the $(DB^2)_0$ conservation assumption or the positional-interpolation assumption, for EBR-II run 105A. It was shown there that the $(DB^2)_0$ conservation assumption was preferable to the positional-interpolation assumption. For this reason, the partial-core-slice heterogeneous calculations presented in this section use the $\Sigma_s(1+g)$ method with only the $(DB^2)_0$ conservation assumption. Calculations were done for two one-mesh-interval axial slices, one at the core midplane, the other at the top of the core. Also presented, for comparison with the midplane-slice calculations, are midplane quantities from calculations where constant bucklings of 0.0023 cm^{-2} in the core and blanket regions and 0.00085 cm^{-2} in the reflector regions are assumed.

Figures 42 and 43 plot the absolute error in ^{238}U fission and ^{181}Ta capture activities, respectively, when leakages for a heterogeneous one-mesh-interval core-midplane axial slice in run 105A are modeled by the stated constant buckling and by space- and energy-dependent leakage absorptions obtained with the $\Sigma_s(1+g)$ method and the $(DB^2)_0$ conservation assumption. The

Fig. 42. Absolute Error in ^{238}U Fission Activity (relative to XYZ heterogeneous reference calculation) When Using an XY Model Either with Row-smeared Variable Leakage Absorption Modeled by the $\Sigma_s(1+g)$ Method and $(\text{DB}^2)_0$ Conservation from RZ Homogeneous Reference or with Constant Bucklings: Mid-core Slice

Fig. 43. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ heterogeneous calculation) When Using an XY Model Either with Row-smeared Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and $(\text{DB}^2)_0$ Conservation from RZ Homogeneous Reference or with Constant Bucklings: Mid-core Slice

corresponding eigenvalue errors are +0.021% with constant buckling and +0.285% with the $\Sigma_s(1+g)$ method. The smaller error in eigenvalue from constant buckling is not a surprise, because the constant buckling values were obtained from comparisons between eigenvalues from a 1D cylindrical calculation and an RZ calculation. Figures 42 and 43 show, however, that the activities, and hence fluxes, are more accurate when space- and energy-dependent leakage absorptions are used.

Figures 44 and 45 plot the absolute error in ^{238}U fission and ^{181}Ta capture activities, respectively, when leakages for heterogeneous one-mesh-interval core-midplane and top-of-core slices in run 105A are modeled with space- and energy-dependent leakage absorptions using the $\Sigma_s(1+g)$ method and the $(\text{DB}^2)_0$ conservation assumption. (Shown also, for comparison, are the whole-core-slice errors.)

Fig. 44. Absolute Error in ^{238}U Fission Activity (relative to XYZ heterogeneous reference calculation) When Using an XY Model with Row-smeared Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and $(\text{DB}^2)_0$ Conservation from RZ Homogeneous Reference: Mid-core, Top-of-core, and Whole-core Slices

Fig. 45. Absolute Error in ^{181}Ta Capture Activity (relative to XYZ heterogeneous reference calculation) When Using an XY Model with Row-smeared Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and $(\text{DB}^2)_0$ Conservation from RZ Homogeneous Reference: Mid-core, Top-of-core, and Whole-core Slices

The peaks in the top-of-core slice of the core region (subassemblies 1-37) are mainly in structural-experiment and partial-driver subassemblies. These are the types of subassemblies that should have the largest errors, because their compositions vary the most from the row-smeared average subassembly compositions that were used in calculating the leakage absorption using the $(\text{DB}^2)_0$ conservation assumption. These peaks are more prevalent in the results for the top-of-core slice, because the ratio of low-energy neutrons to high-energy neutrons is higher in that slice than in the mid-core slice, and low-energy neutrons "see" heterogeneities better than high-energy neutrons.

An additional effect of using row-smeared leakage absorptions derived from RZ calculations can also be seen in Figs. 44 and 45. The cyclically periodic variation of the error is due to the geometric effect of using a cylindricized row-wise quantity to model a system in which hexagonal interfaces separate the rows. A subassembly at a corner of the hexagonal row

is farther from the cylindrical average subassembly, and a subassembly in the center of the flat is closer to the core center than the cylindrical average subassembly. This "corner/flat phenomenon" is also seen in the results for constant buckling in the mid-core slice shown in Figs. 42 and 43.

The eigenvalue errors are, respectively, +0.368% for the top-of-core slice, +0.334% for the whole-core slice, and +0.285% for the mid-core slice. The whole-core eigenvalue error should be, and is, between the other two eigenvalue errors.

3. Simulation of a Nonreference Full-core-height Heterogeneous XYZ Configuration Using XY Geometry and RZ Leakage Absorptions

This section will discuss calculations in which the leakage absorptions, and hence the pseudo-scattering cross sections, obtained from the reference RZ calculation are used to simulate the axial-leakage effects in a nonreference-configuration XY calculation. These calculations should give an indication of the feasibility of using a "generic" set of leakage-absorption cross sections for similar, though not identical, EBR-II run configurations.

The reference calculations in the previous sections were based on EBR-II run 105A. In this section, whole-core-slice XY heterogeneous calculations will be done for a quadrant (sector A and part of sector B) of run 121A, using the run-105A RZ row-smear leakage-absorption cross sections and pseudo-scattering cross sections. Run 121A was chosen for these additional calculations because its loading configuration is similar to that for run 105A. Figure 46 shows the loading patterns for the first eight rows of the corresponding quadrants for runs 105A and 121A. Not shown in the figure, but included in the calculation, are rows 9 and 10, which are radial reflector rows, and rows 11-16, which are radial blanket rows. In the first six rows of each configuration within the given quadrant, the numbers of fueled and nonfueled subassemblies in each row are the same. The actual atom densities of smeared fuel and experimental structural subassemblies may be different in

C - CONTROL ROD	K - CORE DUMMY
D - MARK-II DRIVER FUEL	R - SS REFLECTOR
P - 1/2 DRIVER FUEL, 1/2 SS	S - SAFETY ROD
	X - EXPERIMENTAL SUBASSEMBLY

Fig. 46. Heterogeneous Core Loadings in Quadrant of Rows 1-8 in EBR-II Runs 105A and 121A Used for Calculation

a row, and the subassemblies may be in different locations in a row, but the total number of fueled and nonfueled subassemblies in a row is the same. Row 7 in run 105A has five fueled and five nonfueled subassemblies, and in run 121A has six fueled and four nonfueled subassemblies. In each of the configurations, rows 8-10 contain radial reflector subassemblies and rows 11-16 contain the radial blanket subassemblies, which are row-smearred in the calculations.

Two different calculations using differing axial-leakage assumptions were made for the run-121A whole-core-slice analysis. The first calculation used the run-105A row-smearred leakage-absorption cross sections and pseudo-scattering isotopes. This assumption is what has been called

previously the $(DB^2)_0$ conservation assumption. For the input buckling values, the second calculation used the product of the run-105A positive bucklings (which are the run-105A positive leakage-absorption cross sections divided by the row-smeared diffusion coefficient) and the ratio of the run-121A row-smeared diffusion coefficient and the subassembly-dependent diffusion coefficient of run 121A, i.e.,

$$B_{\text{input}}^2 = \left[(DB^2)_{\text{Run 105A}}^{\text{Smear}} / D_{\text{Run 105A}}^{\text{Smear}} \right] \left(D_{\text{Run 121A}}^{\text{Smear}} / D_{\text{Run 121A}}^{\text{Heterogeneous}} \right).$$

This was done so that when the diffusion code internally calculated the leakage-absorption cross sections by multiplying the input buckling by the in-situ heterogeneous diffusion coefficient, the resultant positive leakage absorption was equal to the run-121A row-smeared diffusion coefficient multiplied by the run-105A positive buckling. The run-105A pseudo-scattering cross sections (representing the negative leakage-absorption cross sections) were modified by multiplying them by the ratio of the run-121A row-smeared diffusion coefficient and the run-105A row-smeared diffusion coefficient, i.e.,

$$\Sigma_s(1+g)_{\text{input}} = \Sigma_s(1+g)_{\text{Run 105A}} \left(D_{\text{Run 121A}}^{\text{Smear}} / D_{\text{Run 105A}}^{\text{Smear}} \right).$$

Using these values gives a leakage-absorption cross section, whether positive or negative, equivalent to the product of the run-121A row-smeared diffusion coefficient and the run-105A row-smeared buckling. This assumption is called the $D_{121} B_0^2$ conservation assumption.

Figures 47 and 48 plot respectively the sums of the absolute group errors in ^{238}U fission activity and ^{181}Ta capture activity calculated with the above assumptions. The maximum error sums are in the range of a few percent. As always, the actual error in reaction rate will be smaller than, or equal to, those shown in the figures, because the figures show absolute error whereas actual errors have some positive/negative cancellation. The corresponding eigenvalue errors are -0.64% for the $(DB^2)_0$ conservation

Fig. 47. Absolute Error in ^{238}U Fission Activity (relative to run-121A XYZ heterogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Full Core Height, Row-smeared Leakage Absorptions from Run-105A RZ Homogeneous Reference

Fig. 48. Absolute Error in ^{181}Ta Capture Activity (relative run-121A XYZ heterogeneous reference calculation) When Using an XY Model with Variable Leakage Absorptions and the $\Sigma_s(1+g)$ Method: Full Core Height, Row-smeared Leakage Absorptions from Run-105A RZ Homogeneous Reference

assumption and -0.59% for the $D_{121} B_0^2$ conservation assumption. The results also show that there is not much difference between the errors from these assumptions. Since the errors are similar, the $(DB^2)_0$ conservation assumption is preferable because of the fewer calculations involved in obtaining the leakage-absorption cross sections.

4. Simulation of Nonreference Partial-core-height Heterogeneous XYZ Configuration Using XY Geometry and RZ Leakage Absorptions

In the preceding section whole-core heterogeneous calculations were shown for run 121A when the space- and energy-dependent leakage absorptions were obtained from a run-105A RZ calculation. These calculations used the $\Sigma_s(1+g)$ method with either $(DB^2)_0$ conservation or $D_{121} B_0^2$ conservation for determining the leakage absorption. It was stated that the $(DB^2)_0$ conservation assumption was the preferred assumption because, for approximately the same error, it involved less calculational effort. Therefore, in this section on calculations for mid-core and other core slices, only results using the $(DB^2)_0$ conservation assumption are shown for calculations involving the $\Sigma_s(1+g)$ method. As with the run-105A heterogeneous calculations presented earlier, only the results for one-mesh-interval (one-node) slices in the selected core regions will be presented. These single-node calculations for run 121A were normalized to run-105A power for a single node, since, in this study, run 105A is the only configuration for the reference RZ calculation.

Figures 49 and 50 respectively plot the absolute errors in ^{238}U fission and ^{181}Ta capture activities when the leakages for the mid-core slice are modeled by either constant bucklings (0.0023 cm^{-2} in the core and blanket, 0.00085 cm^{-2} in the reflector) or space- and energy-dependent leakage absorptions from the run 105A RZ calculation. For the space- and energy-dependent leakages, the $\Sigma_s(1+g)$ method was used with the $(DB^2)_0$ conservation assumption. The mid-core-slice activity errors look very similar to those for run 105A (see Figs. 42 and 43). Comparing these activities for the two runs shows that the constant-buckling method gave similar results (as should be expected). It is also seen that using the so-called "generic" leakage absorptions for similar loading configurations leads to similar errors. The corresponding eigenvalue errors are +0.741% with constant buckling and -0.436% with space- and energy-dependent leakage.

Fig. 49. Absolute Error in ^{238}U Fission Activity (relative to run-121A XYZ heterogeneous calculation) When Using an XY Model Either with Row-smeared Variable Leakage Absorptions Modeled by the $\Sigma_s(1-g)$ Method and $(\text{DB}^2)_0$ Conservation from Run-105A Homogeneous Reference or with Constant Bucklings: Mid-core Slice

Fig. 50. Absolute Error in ^{181}Ta Capture Activity (relative to run-121A XYZ heterogeneous calculation) When Using an XY Model Either with Row-smeared Variable Leakage Absorptions Modeled by the $\Sigma_s(1-g)$ Method and $(\text{DB}^2)_0$ Conservation from Run-105A Homogeneous Reference or with Constant Bucklings: Mid-core Slice

Figures 51 and 52 respectively plot the absolute error in ^{238}U fission and ^{181}Ta capture activities when the run-121A leakages for the one-mesh-interval mid-core and top-of-core slices are modeled with space- and energy-dependent leakage absorptions obtained from the reference run-105A calculation and the net inleakage neutrons are modeled by the $\Sigma_s(1-g)$ method with the $(\text{DB}^2)_0$ conservation assumption. (Also shown, for comparison, is the absolute error for a whole-core slice.) As with the results for run 105A, the experimental structural subassemblies gave the largest top-of-core error. However, at least for the high-energy ^{238}U fission activity, the structural-type subassemblies in the run-121A core region gave less error than the structural-type subassemblies in the run-105A core region (from which the leakage absorptions were calculated). The situation is reversed in the reflector and blanket regions, where, for the ^{238}U fission activity, the run-121A errors were less than the run-105A errors. Overall, the error

Fig. 51. Absolute Error in ^{238}U Fission Activity (relative to run-121A XYZ heterogeneous calculation) When Using an XY Model with Row-smeared Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and $(\text{DB}^2)_0$ Conservation from Run-105A RZ Homogeneous Reference: Mid-core, Top-of-core, and Whole-core Slices

Fig. 52. Absolute Error in ^{181}Ta Capture Activity (relative to run-121A XYZ heterogeneous calculation) When Using an XY Model with Row-smeared Variable Leakage Absorptions Modeled by the $\Sigma_s(1+g)$ Method and $(\text{DB}^2)_0$ Conservation from Run-105A RZ Homogeneous Reference: Mid-core, Top-of-core, and Whole-core Slices

introduced by using leakage absorptions calculated from a similar run-type configuration introduces minor additional error. The eigenvalue errors for run 121A are -1.118% for the top-of-core slice, -0.614% for the whole-core slice (average node), and -0.436% for the mid-core slice. This is the same error ordering as was seen for run 105A. The eigenvalue errors for run 121A are somewhat larger than those for run 105A, but they are still relatively small.

IV. SUMMARY AND CONCLUSIONS

When planar flux calculations are used to calculate inherently three-dimensional fluxes, leakage-absorption cross sections must be used to model the other dimension. This study examined the effects due to geometry, heterogeneity, and use of different methods to both calculate and model leakage-absorption cross sections that are both space-dependent and energy-dependent. The results from diffusion-theory analysis were presented for 1D simulations of 2D RZ-geometry calculations, 2D XY-geometry simulations of 3D XYZ-geometry calculations, multi-slice core calculations, homogeneous and heterogeneous calculations, varying assumptions for the calculation of the problem-input leakage-absorption values (i.e., smeared values, individual-subassembly values, etc.), and the use of leakage absorptions calculated from one run-type configuration and used for calculation of another similar run-type configuration.

Four conclusions can be drawn from these diffusion-theory results using the CITATION code:

1. In regions where the net axial leakage is outward in energy group 1, the results for the $\Sigma_g(1 \rightarrow g)$ method are better than those for the $\pm B^2$ method.
2. Smeared leakage absorptions (as opposed to individual-subassembly leakage absorptions) are preferable because, for approximately similar errors, they reduce the amount of preliminary calculations and the data-storage requirements. Both XYZ row-smeared and RZ row-smeared leakage-absorption cross sections were evaluated. The cross sections obtained by both methods gave approximately the same activity and eigenvalue errors. RZ row-smeared leakage absorptions are preferable, however, because space and energy mesh considerations in the XYZ calculations limit the number of regions that can be described axially and also because RZ buckling values are easier to calculate.
3. When heterogeneous calculations are done, the row-smeared positive leakage-absorption cross sections and pseudo-scattering cross sections used should be such that the reference-case leakage cross section, $(DB^2)_0$, should be conserved. Several other methods were considered. They all gave about the same relative errors. However, the $(DB^2)_0$ conservation scheme was the simplest method to implement.

4. For partial axial core slices and where the calculations used leakage absorptions from the analogous RZ calculation, the absolute value of the eigenvalue error was less than 0.5%. For calculations of such slices and where the XY configuration is not the same as the reference, the absolute value of the eigenvalue error was less than 1.0%. The corresponding absolute errors in reaction rates of ^{238}U fission and ^{181}Ta capture were less than 5%.

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