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FLOOD SAFETY OF THE MIXED SPECTRUM SUPERHEATER

By
A. B. Reynolds

May 25, 1961

Atomic Power Equipment Department
General Electric Company
San Jose, California

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Atomic Power Equipment Department
General Electric Company

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ABSTRACT

Calculations are presented which show that the reactivity effect of flooding and unflooding the fast superheating section of the Mixed Spectrum Superheater can be made small by the addition of epithermal poisons to the superheater. The reactivity effects of flooding superheater sections ranging in size from 1.25 ft to 3.5 ft cubes and containing U-235 oxide or Pu-239 oxide fuel and various amounts of the epithermal poison europium were calculated. Reactivity changes during several postulated flooding processes are given. Methods for determination of fissile and fertile material and poison cross sections in the resonance region are discussed.

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SUMMARY

Calculations indicate that the flooding effect on reactivity in the fast oxide superheater section of the Mixed Spectrum Superheater can be rendered small by the addition of epithermal poisons. The reactivity effect of flooding the superheater was calculated as a function of the concentration of the epithermal poison europium in the superheater and as a function of the superheater core size. Both large power and small experimental U-235 fueled cores and a large power Pu-239 core with longer range economic potential were analyzed. For the Pu-239 fueled 3.5 ft. cubical superheater, 0.23 volume percent europium oxide must be added to the superheater to produce zero reactivity change between the unflooded and completely flooded superheater. For the 3.5 ft., 2.0 ft., 1.5 ft., and 1.25 ft. U-235 fueled superheaters, the corresponding volume percentages of europium oxide which must be added are 0.06%, .8%, 2.7%, and 4.4% respectively.

The effects of partially flooding the superheater by the uniform addition of water to the coolant volume and the effect of loss of steam pressure were calculated for a 3.5 ft Pu core and a 1.5 ft U-235 core. In both cases it was possible to choose a europium concentration which would limit the maximum reactivity addition from complete loss of pressure to complete flooding to less than one dollar.

Reactivity effects during the flooding process in which water floods the superheater exit steam pipe and rises up into the fast core were estimated for a 3.5 ft Pu core and a 1.5 ft U-235 core. The reactivity throughout this flooding process is negative whenever the reactivity of the completely flooded superheater is negative.

A high degree of flexibility through the use of various combinations of epithermal and thermal poisons is available to obtain a given reactivity response during a flooding or unflooding accident. Flexibility through use of various geometrical arrangements of the poison has not been studied. Further study is needed to specify the required reactivity response for the various possible mechanisms by which flooding or unflooding can occur.

Calculations were made using nineteen group diffusion theory. Cross sections above 9100 ev were obtained from Argonne National Laboratory fast reactor cross section compilations. In the resonance region cross sections for U-238, europium, and the fissile materials were obtained from the calculated Doppler broadened energy dependent resonance integrals. Statistical variation of both the unknown fission widths and the unknown neutron widths were taken into account for the fissile materials.

CONCLUSIONS

Analytical methods were developed for treating the resonance capture in the flooded superheater of the MSS in sufficient detail for an accurate study of the flooding problem.

Calculations indicate that:

1. The reactivity effect of completely flooding the superheater of a Mixed Spectrum Superheater can be made small or negative for a wide range of core sizes by the addition of a small amount of epithermal poison to the core.
2. For the flooding process equivalent to the uniform addition of water to the core and for the loss of steam pressure in the core, intermediate positive reactivity changes can be held below one dollar by proper choice of epithermal poison concentration.
3. The reactivity is negative throughout the flooding process in which water floods the superheater exit steam pipe and rises up into the fast core, provided the reactivity of the completely flooded core is negative.
4. A high degree of flexibility through the use of various combinations of epithermal and thermal poisons is available to obtain a required reactivity response during the flooding and unflooding of the superheater. Because of the several possible mechanisms by which flooding or unflooding can occur, further study is needed to specify exactly what this response should be.

I Introduction

The Mixed Spectrum Superheater (MSS) is a nuclear reactor contained within a pressure vessel in which both boiling of water and superheating of the resulting steam takes place. A schematic drawing of the reactor is shown in Figure 1. The reactor consists principally of an unmoderated, oxide-fueled, superheating core surrounded by a boiling water core which is very similar to that in a standard boiling water reactor. The neutron energy spectrum varies from a fast spectrum in the superheater core to an intermediate spectrum in a buffer region between the superheating and boiling sections to a thermal spectrum in the boiling water core--hence the name Mixed Spectrum Superheater. Water is boiled in the outer boiling water core and the steam-water mixture passes up through radial-type steam separators. The separated steam passes through steam dryers and then travels down a central pipe to the superheater core. After passing through the superheater, the superheated steam leaves the pressure vessel through the pipe which penetrates the bottom of the vessel.

In the safety analysis of the MSS an important hazard which must be considered is the accidental flooding or unflooding of the fast core. It is desirable to design the superheater so that the flooding or unflooding of the fast core always results in a near zero reactivity change. Preliminary calculations indicated that flooding the superheater would introduce a large positive increase in reactivity unless epithermal poisons were added to the core, and that negative reactivities could be achieved by the addition of relatively small amounts of epithermal poison (1). The study reported here represents a more detailed investigation of the nature of the flooding problem, particularly the effect of changes in resonance absorption and the flexibility available for controlling these changes by the use of epithermal poisons. The efficacy of epithermal poisons both in a full power MSS and in a small demonstration MSS was examined.

MIXED SPECTRUM SUPERHEATER

FIGURE 1

II Description of the Flooding Problem

Flooding the fast core is accompanied by two conflicting effects: first, the neutron leakage is reduced and, second, the neutron energy spectrum is sharply degraded. Decreasing the neutron leakage tends to increase the reactivity of the fast core. Degrading the spectrum increases the absorption rate in the resonance energy region which increases the parasitic capture rates of the fertile materials more than the fission rates of the fissile materials, thereby tending to reduce the reactivity. For the size fast cores considered in this investigation it is necessary to increase the parasitic resonance absorption by adding small amounts of resonance poisons to the superheater in order for the effect of the increased parasitic absorption to override the effect of the reduced leakage.*

It is of interest to study the flood safety of both a large full power reactor and a small demonstration reactor. The size and shape of the superheater in the 300 MW(e) MSS, which is described in Reference 1, is a 3.5 ft cube. The effects of flooding cores as small as a 1.25 ft cube were studied to determine how small a superheater core could be made flood safe. Both Pu-239 oxide and U-235 oxide were considered as fuels for the 3.5 ft core but only U-235 oxide was considered for the smaller cores. The volume fractions of fuel, stainless steel clad, and steam were 0.445, 0.318, and 0.237 respectively.

It is also necessary to consider the manner in which flooding of the fast core takes place for it is entirely conceivable that the reactivity change might reach a positive maximum at some point during the flooding process. Several possible procedures amenable to analysis can be postulated. First, flooding could occur uniformly throughout the fast core, equivalent analytically to allowing the water density to vary from zero to that for the completely flooded core. Second, water could pour into the steam pipe beneath the superheater core first flooding the bottom reflector and then the core itself. These two flooding mechanisms were studied for two size cores during this investigation. Third, an outside annular part of the fast core could flood before the central part or vice versa. This third procedure has not been studied.

Converse to the flooding problem is the loss of pressure in the fast core in which the steam density would be uniformly reduced. The reactivity effect of pressure loss was calculated together with the uniform flooding calculations.

The major analytical problem in this investigation was the accurate determination of fertile material, fissile material, and poison cross sections. Since flood safety depends on the competition between capture rates and leakage rates in the superheater, detailed attention to the evaluation of competing cross sections in the resonance energy range was necessary. Furthermore, the fertile, fissile, and poison material cross sections vary with their concentrations due to strong self-shielding effects, and it was necessary to account for this variation for all cases calculated.

* It should be noted that this poison does not act as a burnable poison since the resonance flux is appreciable only when flooding occurs. In normal operation the absorption by these poisons is quite low.

Eurpoium in the form of the oxide (Eu_2O_3) was selected as the resonance poison for this investigation because of its high resonance integral and, in particular, its concentration of strong resonances in the energy region between the 0.3 ev Pu-239 resonance and the 6.7 ev U-238 resonance. A comparison of the resonance integrals and costs of other possible resonance poisons is presented in Appendix B.

III Calculated Results

A. Complete Flooding of the Superheater

The results for completely flooded cores are presented in Figure 2 in which the criticality factor of each flooded core is plotted as a function of volume fraction of europium in the superheater. Curves are shown for four different size cubical cores varying from 1.25 ft to 3.5 ft on a side. For the 3.5 ft core results for both U-235 and Pu-239 fuel were obtained; for the smaller cores only U-235 fuel was considered. For each case the criticality factor of the unflooded core was unity. The results are plotted in Figure 2 and are summarized together with the critical enrichment for each core in Table I.

In all cases flooding the superheater to which no resonance poison was added introduces a positive reactivity change. As the core size is reduced, the increased leakage makes necessary higher U-235 enrichments so that relatively less parasitic absorption by U-238 occurs on flooding. Hence, considerably larger amounts of resonance poisons are needed for the smaller cores.

B. Partial Uniform Flooding of the Superheater

Results for uniformly partially flooded cores and for loss of superheater pressure are presented in Figure 3 for 3.5 ft Pu-239 fueled cores containing 0.2% and 0.3% europium in the superheater and a 1.5 ft U-235 fueled core containing 3.6% europium. In this figure changes in fast core reactivity from normal operating conditions are plotted as a function of the fraction of the coolant volume which is filled with water. The coolant volume fraction of unity corresponds to the superheater completely flooded with saturated water at 1000 psi; the value of 0.032 corresponds to normal operating conditions; and zero corresponds to complete loss of pressure. The reactivity unit dollar corresponds here to $\Delta k/k \approx .0034$ for the Pu-239 core and $\Delta k/k \approx .0075$ for the U-235 core. The dollar varies slightly with the fraction of coolant volume flooded as described in Appendix C, where more exact calculated values are given.

The radically different shapes of the curves in Figure 3 illustrate the complex nature of the competition between the leakage and the absorption as a function of energy spectrum and reactor size. The integrated power spectra and the leakage probabilities for three stages in the flooding process for both the 3.5 ft and 1.5 ft cores are given in Table II.

For the 3.5 ft core the reactivity change as flooding begins is negative because the effect of the increased parasitic resonance absorption due to the degraded spectrum outweighs the effect of the reduced neutron leakage. However, for the cases shown here this trend is reversed when flooding is about half complete. At this point further flooding results in degrading sufficient neutrons to thermal energy, where the thermal utilization is high, that the combined positive reactivity effects of high thermal utilization and reduced leakage outweigh the effect of increased resonance absorption. For the 0.2% europium case this positive effect was sufficient to render the core supercritical, whereas the resonance absorption was sufficient for the 0.3% europium case to sharply curtail this belated positive effect.

Figure 2

REACTIVITY EFFECT OF FLOODING A SUPERHEATER AS A
FUNCTION OF EUROPIUM CONCENTRATION AND CORE SIZE.
THE DIMENSION S REFERS TO THE LENGTH OF THE SIDE
OF A CUBICAL CORE. THE FUEL USED IS INDICATED IN PARENTHESES.

TABLE I

Results for Complete Flooding of the Superheater

Length of Side of Core (Ft)	Principal Fissile Isotope	Volume % Eu in Core	Critical Enrich. of Unflooded Core	Criticality Factor of Flooded Core
3.5	Pu-239	0	12.9	1.093
		0.25	13.3	0.995
		0.5	13.6	0.935
3.5	U-235	0	18.0	1.029
		0.25	18.5	0.949
		0.5	18.9	0.903
2.0	U-235	0	34.4	1.096
		0.5	35.0	1.027
		1.0	35.6	0.992
		2.5	37.0	0.926
1.5	U-235	0	52.3	1.134
		0.5	52.6	1.090
		1.0	52.9	1.063
		2.5	53.8	1.006
		5.0	54.7	0.940
1.25	U-235	0	68.8	1.150
		1.0	69.0	1.098
		2.5	69.2	1.050
		5.0	69.4	0.988

Figure 3
REACTIVITY DURING UNIFORM FLOODING

TABLE II

Integrated Power Spectrum and Leakage
in the Superheater During Uniform Flooding

Group	Lower Energy of Group	Power Spectrum ^a for 3.5 ft Pu Core at Various Stages in Flooding Proc.				Power Spectrum ^a for 1.5 ft U-235 Core at Various Stages in Flooding Process			
		Complete Loss of Pressure	Normal Operating Condition	Half Flooded	Comple. Flooded	Complete Loss of Pressure	Normal Operating Condition	Half Flooded	Comple. Flooded
1	2.25 Mev	.10	.10	.09	.08	.10	.10	.09	.08
2	1.35	.21	.21	.19	.16	.20	.20	.18	.16
3	.825	.27	.27	.24	.20	.31	.30	.25	.23
4	.5	.37	.36	.29	.23	.43	.42	.34	.29
5	.3	.45	.44	.33	.26	.53	.52	.40	.34
6	.18	.55	.53	.37	.28	.66	.64	.47	.39
7	.11	.64	.61	.40	.30	.77	.74	.53	.43
8	.067	.71	.67	.43	.31	.86	.83	.58	.47
9	.025	.82	.77	.47	.34	.95	.92	.66	.54
10	9100 ev	.90	.85	.51	.37	.99	.97	.75	.61
11	4000	.93	.88	.54	.39	.99	.98	.79	.65
12	1000	.97	.94	.61	.44	1.00	1.00	.87	.75
13	300	.99	.97	.70	.52	1.00	1.00	.93	.84
14	100	1.00	.99	.80	.63	1.00	1.00	.97	.92
15	30	1.00	1.00	.86	.71	1.00	1.00	.99	.97
16	5	1.00	1.00	.93	.82	1.00	1.00	1.00	.98
17	1.0	1.00	1.00	.95	.85	1.00	1.00	1.00	.99
18	0.4	1.00	1.00	.97	.90	1.00	1.00	1.00	.99
19	Thermal	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Leakage (All Groups)		.282	.266	.15	.11	.466	.456	.34	.28

^a Numbers show fraction of power generated above lower energy of group.

These results suggest that considerable flexibility is available for flattening the reactivity response during the flooding process by the addition of thermal poisons (e.g. gadolinium) and either less europium or a combination of epithermal poisons having resonances at different energies. An additional method to vary the effectiveness of the poison which has yet received no consideration is variation of the geometrical arrangement of the poison. Furthermore, it is still unclear whether or not a minimum in the reactivity curve is desirable; increased safety in an unflooding accident might result by having the initial reactivity change negative.

The part of the curves below 0.032 fractional flooding shows that loss of pressure from the 3.5 ft core produces a reactivity increase. Here the effect of hardening the spectrum overrides the effect of the increased leakage.

Quite the opposite behavior is observed for the 1.5 ft core. In this core the leakage is so great and the enrichment so high that even in the completely flooded core practically no neutrons survive to thermal energy. Hence, as flooding becomes complete, more neutrons are degraded into the highly poisoned resonance energy region and the reactivity decreases to its lowest value for the completely flooded case.

C. Partial Flooding by Water Flooding the Exit Steam Pipe and Rising Upward into the Superheater.

The effect of water flooding the exit steam pipe and rising upward into the bottom of the superheater could not be calculated directly with the symmetrical one-dimensional computer code in use. The effect of flooding the lower reflector and the bottom few inches of the core can be estimated, however, as half the effect of flooding both top and bottom parts of the core symmetrically. The reactivity effects of flooding the upper and lower reflectors and then progressively flooding the fast core symmetrically from the top and bottom toward the center are shown in Table III for both a 3.5 ft Pu-239 core and a 1.5 ft U-235 core. The volume fraction of europium was 0.3% in the 3.5 ft core and 3% in the 1.5 ft core. The reflector consisted of 35% by volume depleted uranium oxide, 25% stainless steel, 40% steam or water, and no epithermal poisons. The one-dimensional results for the completely flooded cores are not exactly consistent with the zero-dimensional results due to the more detailed treatment of the reflector in the one-dimensional case; qualitatively this inconsistency has little significance.

It is observed from Table III that, provided the reactivity of the completely flooded core is negative, the reactivity is also negative throughout the flooding process in which water floods the exit steam pipe and rises up into the core.

TABLE III

Reactivity Effect of Progressively Flooding the
Top and Bottom Portions of the Superheater

Size of Superheater	Portion of Superheater Flooded	k	ρ (dollars)
3.5 ft	None	1.000	0
	Top and bottom reflector	0.994	-1.8
	Top and bottom 2 inches of core	0.996	-1.1
	Top and bottom 4 inches of core	0.997	-0.9
	Top and bottom 8 inches of core	0.995	-1.5
	Entire superheater	0.981	-5.7
1.5 ft	None	1.000	0
	Top and bottom reflector	0.989	-1.4
	Top and bottom 4 inches of core	1.001	+0.2
	Entire superheater	1.002	+0.3

IV Analytical Methods

The criticality calculations were made using nineteen group diffusion theory. An "equivalent bare reactor" model was used for all calculations except the flooding of the top and bottom of the superheater (Section III C), for which one-dimensional calculations were necessary. Inelastic scattering into the top eleven energy groups was allowed. The Selengut-Goertzel approximation was used for hydrogen scattering for all groups. Leakage was calculated by assuming a single buckling for all energy groups. The extrapolated distances necessary for the buckling calculation were obtained from extrapolation to zero of the flux distributions obtained from 19 group one-dimensional calculations, in which both the 3.5 ft superheater and the boiling water core were present. In this way different extrapolation distances were obtained for the flooded and non-flooded cases. The use of a single buckling for all groups introduces little error since the extrapolated distances for all groups other than the lower three energy groups are very nearly constant. And the leakage in the three lower groups is small. The extrapolated distances calculated for the 3.5 ft superheater were used for all size cores since the extrapolated distance depends principally on the composition of the medium surrounding the superheater. Errors in extrapolation distances are more important for the smaller size cores resulting in less accurate quantitative results for the small superheaters.

Cross Sections

The accuracy of the energy dependent cross sections is particularly important for this flooding study since the delicate balance between increased capture rates and reduced leakage rates determines whether the reactor is flood safe.

Cross sections for the first ten of the nineteen groups (i.e. above 9100 ev) were taken directly from the 1958 Argonne National Laboratory fast reactor cross section set (2) except for europium. These first ten group cross sections were used for both the flooded and the non-flooded reactors. For the unflooded cases the percent fissions occurring in the first ten groups are ~ 84% for the 3.5 ft cores and 96% for the 1.5 ft cores (see Table II). For the flooded cases the corresponding percent fissions for the 3.5 ft and 1.5 ft cores are 37% and 56%. Cross sections for europium in groups 11-15 (30 ev - 9100 ev) were estimated from the curve in BNL-325, 2nd edition (3), assuming an 8 barn scattering cross section. Above 9100 ev, the europium absorption cross section was assumed equal to the holmium cross section which is plotted in BNL-325, 2nd edition, supplement No. 1 (4). Reasons for assuming these two cross sections to be equal are, first like europium and holmium samples give equal reactivity contributions in a fast reactor (5), and second, the europium-151 and holmium absorption cross sections between 0.2 Mev and 2 Mev are nearly equal (4).

In the resonance energy range, which includes groups 11 through 18 (9100 ev - 0.4 ev), cross sections were calculated from detailed analyses of the resonance integrals of the fertile and fissile materials and of the poisons. The absorption cross section for each group was defined as the ratio of the effective resonance integral in the group to the group lethargy width, i. e.

$$\sigma_1 = \frac{\int_{E_1}^{E_2} \sigma(E) \frac{\Sigma_s}{\Sigma(E)} \frac{dE}{E}}{\Delta u_1}$$

This expression is derived in Appendix A (see Eq. 6) and is based on the assumptions of widely spaced narrow resonances and a homogeneous medium. The methods used to calculate the effective resonance integral for each group are also described in detail in Appendix A. These methods involve the calculation of Doppler broadened resonance integrals for all resolved resonances in a homogeneous absorbing medium as presented by Dresner (6). The method used for the fissile materials is more complex than the method used for the fertile materials and poisons because of the statistical variation of the fission width superimposed on the statistical variation of the neutron width.

Since the self-shielding varies for each enrichment of fissile material, fertile material, and poison, resonance integrals and cross sections were calculated for a wide range of concentrations of each type of material. Hence, for each calculation included in this report, properly self-shielded cross sections were used for the U-235, Pu-239, U-238 and europium. The extent to which these cross sections varied as a function of concentration can be seen from Appendix D in which the multigroup constants for each material are listed.

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APPENDIX A

Calculation of Effective Cross Sections

Cross sections for the highest ten energy groups, above 9100 ev, (except for europium) were obtained directly from the Argonne National Laboratory cross section set (2). Cross sections for the heavy elements and poisons in the energy groups within the resonance energy region, defined here as the energy from .4 ev to 9100 ev, were calculated from an approximate form of the flux weighted average cross section. Because of the importance of the cross sections in this region, the assumptions involved in the approximation are examined in this appendix.

1. Derviation of the Approximate Form of the Flux Weighted Average Cross Section.

A homogeneous model was assumed for the resonance cross section calculations. This model is justified on the basis of the small chord length in the moderator relative to the neutron mean free path in the moderator. The mean chord length in the coolant passages between fuel rods is 0.12 cm. The mean free path of resonance neutrons in hot water is 1.0 cm.

The flux weighted average cross section in group 1 bounded by energies E_1 and E_2 is

$$\sigma_1 = \frac{\int_{E_1}^{E_2} \sigma(E) \phi(E) dE}{\int_{E_1}^{E_2} \phi(E) dE} \quad (1)$$

To obtain an expression for $\phi(E)$, we write the balance relation for energies below the fission source

$$\phi(E) \Sigma(E) = \sum_m \int_E^{E/(1-\alpha_m)} \phi(E') \Sigma_{sm}(E') \frac{dE'}{\alpha_m E'} \quad (E < E_s) \quad (2)$$

where the summation is over all materials m and the energy is below the fission source energies E_s .

For the assumption of widely spaced narrow resonances, the solution to this equation is

$$\phi(E) = \frac{S_p(E)}{\xi \Sigma(E)} \quad (3)$$

$$\text{where } p(E) = \exp - \int_E^{E_s} \frac{\Sigma_a(E')}{\xi \Sigma(E')} \frac{dE'}{E'}$$

$$\Sigma(E) = \Sigma_s(E) + \Sigma_a(E)$$

and $\Sigma_s(E)$ is the potential scattering cross section of all materials for which the maximum energy loss per collision near a given resonance energy is greater than the practical width of the resonance (i.e. narrow resonance scattering). (All scattering in the reactors considered here satisfies this condition except scattering by the heavy elements around the low energy U-238 resonances.)

For moderation only by hydrogen the solution (3) is exact for all energies in the resonance region even without the restriction of widely spaced resonances. For moderation by materials heavier than hydrogen the restriction of widely spaced resonances is important since the solution is invalid in the transient region immediately below an absorption resonance. The level spacing for the U-238 resonances is sufficiently large relative to the width of the transient region following scattering by oxygen and steel for the collision density to recover to its asymptotic distribution. However, the resonances of the fissile materials and the europium are closely spaced so that the effect of recovery following oxygen and steel scattering is uncertain. Even in the flooded reactor, only one third of the scattering is by hydrogen. Weinberg and Wigner (7) show that the integral results of this theory for heavy moderators are accurate (since the effects of transient oscillations in the collision density are averaged out over many randomly spaced resonances) except for the low energy region of resonance reactors where the collision density has dropped to a small fraction of its original value. Unfortunately, this is an important region for the present investigation; hence, the uncertainty in the treatment of the fissile material and poison absorption persists.

Substituting solution (3) into (1) gives

$$\sigma_1 = \frac{\int_{E_1}^{E_2} \sigma(E) \frac{Sp(E)}{\xi \Sigma(E)} \frac{dE}{E}}{\int_{E_1}^{E_2} \frac{Sp(E)}{\xi \Sigma(E)} \frac{dE}{E}} \quad (4)$$

It is assumed that $p(E)$ can be replaced by a constant value since its value varies only slightly over any given energy group. Hence we obtain the fundamental result for the cross section in a homogeneous medium;

$$\sigma_1 = \frac{\int_{E_1}^{E_2} \sigma(E) \frac{1}{\Sigma(E)} \frac{dE}{E}}{\int_{E_1}^{E_2} \frac{1}{\Sigma(E)} \frac{dE}{E}} \quad (5)$$

We now make a final assumption in order to put the above expression into a form which is readily calculable. We assume that, in the denominator, $\Sigma(E) = \Sigma_s$ where Σ_s is constant. This assumes very narrow resonances, i. e. the value of Σ_s is significant only over a small energy region. Equation (5) can then be written as

$$\sigma_1 = \frac{\int_{E_1}^{E_2} \sigma(E) \frac{\Sigma_s}{\Sigma(E)} \frac{dE}{E}}{\Delta u_1} = \frac{I_1}{\Delta u_1} \quad (6)$$

where the numerator is recognized as the effective resonance integral of the group. The error involved in replacing $\Sigma(E)$ by Σ_s in the denominator of (5) is small which can be shown by the following analysis. Let us assume that all the absorption in group 1 occurs in square wave resonances and that all these resonances cover a fraction $\Delta u/\Delta u_1$ of the lethargy width of the group. We will further assume that each square wave resonance has the same magnitude, σ . Conserving the effective resonance integral in the group, the magnitude of each resonance is $\sigma = \sigma_1 \Delta u_1/\Delta u$. For this model the ratio of the denominators in (5) and (6) is

$$\frac{\int_{u_2}^{u_1} \frac{1}{\Sigma(u)} du}{\int_{u_2}^{u_1} \frac{1}{\Sigma_s} du} = 1 - \frac{\Delta u}{\Delta u_1} \left(1 - \frac{\Delta u/\Delta u_1}{\frac{\Delta u}{\Delta u_1} + \frac{\sigma_1}{\sigma_p}} \right)$$

where σ_p is the potential scattering cross section per absorber atom. For $\Delta u \ll \Delta u_1^p$, i.e. for very narrow resonances, the error is negligible. For the 3.5 ft MSS superheater, large value of σ_1/σ_p were 0.1. For large values of $\Delta u/\Delta u_1$, say 0.2, and $\sigma_1/\sigma_p = 0.1$, the above ratio is 0.93.

2. Calculation of Resonance Integrals for Fertile Materials and Poisons.

The Doppler broadened resonance integrals for all resolved resonances were calculated according to Dresner's method for a homogeneous absorbing medium(6)*.

* The IBM 704 computer code RES, developed by Nordheim, et al (8), was used for both resolved and unresolved resonance integral calculations.

The contribution to the total resonance integral of each resonance was calculated from the relation (see Dresner's article for the derivation)

$$I_j = \int_{-\infty}^{\infty} \sigma_a \frac{\Sigma_s}{\Sigma} \frac{dE}{E}$$

$$= \frac{\sigma_p \Gamma_r}{E_j} J(\xi, \beta)$$

where σ_a = absorption cross section (barn).

σ_p = potential scattering cross section per absorber atom (barn).

Σ_s, Σ = potential scattering and total cross sections of the medium (cm^{-1}).

E_j = energy at the center of the resonance (ev).

$\Gamma_r, \Gamma_n, \Gamma$ = radiation, neutron, and total level width (ev).

$$J(\xi, \beta) = \int_0^{\infty} \frac{\psi}{\sqrt{1+\beta}} dx$$

$$\psi = \frac{\xi}{\sqrt{4\pi}} \int_{-\infty}^{\infty} \frac{\exp - \left[\frac{\xi^2}{4} (x-y)^2 \right]}{1+y^2} dy$$

$$x = \frac{2(E-E_j)}{\Gamma}$$

$$\xi = \sqrt{\frac{A\Gamma^2}{E_j kT}}$$

$$\beta = \sigma_p / \sigma_0$$

$$\sigma_0 = 4\pi\lambda^2 \frac{g\Gamma_n}{\Gamma}$$

g = statistical weight factor

A = atomic weight of absorber

Values of $J(\xi, \beta)$ are available as functions of ξ and β in both references 6 and 8.

The resonance integral of the i^{th} energy group is then the sum of the resonance integrals of the resonances within the group.

No attempt was made to account for self-shielding of resonances of one material by resonances of another material. Fortunately, none of the very large resonances in U-238 overlap resonances in U-235, Pu-239 or Eu. In the lower energy groups (groups 17 and 18) in which most of the resonance integral of europium lies no plutonium resonances are present (other than the tail of the 0.3 ev resonance) and little overlapping of resonances between U-235 and Eu occurs. In group 16 (5 ev to 30 ev) the overlapping between U-235 and europium is quite significant while overlapping between Pu-239 and europium again is not very important. The group 16 europium cross sections may be seriously overestimated in the presence of U-235 since the U-235 is fairly highly self-shielded; the resonance integral for 60% enriched U-235 is only $2/3$ that for 20% enriched. Above group 16 the self-shielding by both U-235 and Pu-239 decreases rapidly so that overlapping of resonances ceases to introduce significant errors.

The contribution of the unresolved resonances in U-238 was calculated by assuming a Porter-Thomas (or chi-squared) distribution of reduced neutron widths with one degree of freedom (9). The level spacing used in the calculation was 18.5 ev, which is the average level spacing in the resolved region.

Resonance parameters for U-238 were obtained from Rosen (10). Resonance parameters for europium were obtained from BNL-325, 2nd Edition, Supplement No. 1 (4).

3. Calculation of Resonance Integrals for Fissile Materials*

Total Absorption Integral

For those resonances in the fissile materials for which both Γ_f and Γ_n are known, the total absorption resonance integrals were calculated in the same manner as for the heavy elements and poisons except that " Γ " in the input to the computer code was replaced by the sum of Γ_f and Γ_n . In the case of Pu-239, the resonance parameters were obtained from BNL-325, 2nd Edition (3) in which fission widths are listed for all resonances for which the neutron widths are known. In the case of U-235, the resonance parameters were obtained from several sources (11, 12, 13, 14, 15) although fission widths were available for only a few of the resonances for which neutron widths were listed. In the unresolved resonance region, the Porter-Thomas distribution (i.e., a chi-squared distribution with one degree of freedom) (9) was assumed for the reduced neutron widths, and a chi-squared distribution with three degrees of freedom, as recommended by Bethe (16), was assumed for the fission widths.

For an accurate calculation of the contribution of the unresolved resonances to the resonance integral of the fissile isotopes it is necessary to carry out double integrations over the neutron width and fission width distributions. For this study a simpler approximate calculation was made in which

* A major portion of this calculational method was contributed by Dr. P. Greebler of APED.

the continuous distributions were replaced by a finite number of values of Γ_f and Γ_n . At the energy corresponding to the mid lethargy point of each energy group in the unresolved region, a resonance integral was calculated for each possible combination of Γ_n and Γ_f as if each combination was a resolved resonance. The resulting resonance integrals were then averaged. It was assumed that the average level spacing, D , was the same as that for the resolved resonances. The resonance integral for the i th group is given by the relation

$$I_i = \frac{\Delta E_i}{D} \bar{I}_j$$

where $\bar{I}_j$ = average resonance integral of a resonance occurring at the mid-lethargy point of group j .
 ΔE_i = energy width of group i .

For this study it was assumed that sufficient accuracy could be obtained by replacing the continuous distribution of reduced neutron widths with four values of Γ_n^0 . Fewer values of the fission width are needed for the same statistical accuracy since more degrees of freedom (c.f. $\nu=3$ for fission widths with $\nu=1$) result in less statistical variation in widths. Hence, the continuous fission width distribution was replaced by three values of Γ_f .

The detailed method used to obtain these values of Γ_n and Γ_f follows. The distributions $P(x, \nu/2)$ (where $x = \Gamma/\bar{\Gamma}$ for either the reduced neutron width or the fission width) were plotted versus the ratio x . The areas under these curves as a function of x , i.e. $\int_0^x P(x, 1/2)dx$ and $\int_0^x P(x, 3/2)dx$, were then determined graphically and plotted versus x . In order to obtain four values of $\Gamma_n^0/\bar{\Gamma}_n^0$ with approximately equal probabilities of occurrence, $\Gamma_n^0/\bar{\Gamma}_n^0$ values were chosen at values of $\int_0^x P(x, 1/2)dx$ of $1/8$, $3/8$, $5/8$, and $7/8$. Similarly values of $\Gamma_f/\bar{\Gamma}_f$ were chosen for values of $\int_0^x P(x, 3/2)dx$ of $1/6$, $1/2$, and $5/6$. The average of the values of $\Gamma_n^0/\bar{\Gamma}_n^0$ and $\Gamma_f/\bar{\Gamma}_f$ obtained by this method differed slightly from unity; hence they were normalized to an average value of unity. The unnormalized values (as determined graphically and listed under the column x) and the normalized values for $\Gamma_n^0/\bar{\Gamma}_n^0$ and $\Gamma_f/\bar{\Gamma}_f$ are shown in Table IV.

The average value for the reduced neutron width was the average of the Γ_n^0 in the resolved energy region. The calculation of the three values of $(\Gamma_f)_k$ at each energy from $(\Gamma_f/\bar{\Gamma}_f)_k$ is discussed below.

Evaluation of α and Fission Resonance Integrals

The fission resonance integral is related to the absorption resonance integral by the ratio of the capture to fission cross section, α ; hence,

$$I_f = \frac{I_a}{1 + \alpha}$$

TABLE IV

APPROXIMATE REDUCED NEUTRON WIDTH AND FISSION WIDTH DISTRIBUTIONS

k	$\int_0^x P(x, \frac{1}{2}) dx$	x	<u>normalized</u>		x	<u>normalized</u>	
			$(\Gamma_n^0 / \bar{\Gamma}_n^0) k$	$\int_0^x P(x, 3/2) dx$		$(\Gamma_f / \bar{\Gamma}_f) k$	
1	1/8	0.0275	0.0320	1/6	0.29	0.310	
2	3/8	0.240	0.279	1/2	0.80	0.854	
3	5/8	0.815	0.947	5/6	1.72	1.836	
4	7/8	2.36	2.742				

It was assumed that $\bar{\Gamma}_f$ was related to Γ_γ by α , such that $\Gamma_\gamma/\bar{\Gamma}_f = \alpha$.

Values of α for U-235 above 40 ev were obtained from a compilation of the experimental results of Kanne, et. al., (17), Kalashnikov, et. al., (18), and Diven (19). Values of α for Pu-239 for the unresolved resonance region were obtained from Molino, et. al., (20). For U-235 and Pu-239 in these energy ranges values of $(\Gamma_f)_k$ were calculated from the values of $(\Gamma_f/\bar{\Gamma}_f)_k$ in Table IV from the relation

$$(\Gamma_f)_k = (\Gamma_f/\bar{\Gamma}_f)_k \Gamma_\gamma/\alpha(E) \quad k = 1, 2, 3$$

Between .4 ev and 40 ev, values for both Γ_f and α for most of the U-235 resonances were unknown. In this energy range (groups 16-18), a value for α for U-235 was obtained for each group from the resonance integrals of those resonances for which Γ_f was known in the following manner. Let I_{aj} = absorption resonance integral for the j^{th} resonance for which Γ_{fj} and Γ_{nj} were both known. Then, $I_{fj} = I_{aj} \Gamma_{fj} / (\Gamma_{fj} + \Gamma_\alpha) =$ fission resonance integral for that resonance. The α for group i was then calculated as

$$\alpha_i = \frac{\sum_j (I_{aj} - I_{fj})}{\sum_j I_{fj}}$$

Finally, values for $(\Gamma_f)_k$ for those resonances in groups 16-18 for which Γ_f was unresolved were calculated from the relation

$$(\Gamma_f)_k = (\Gamma_f/\bar{\Gamma}_f)_k \Gamma_\gamma/\alpha_i \quad k = 1, 2, 3$$

4. Calculation of Thermal Poison Cross Sections

The absolute magnitude of the thermal cross sections in the fast reactor is of little importance since the core is essentially black to thermal neutrons and since most of the thermal neutron absorption occurs in the fissile material because of the high enrichments. For the study of the effect of flooding, however, the relative cross section of the thermal poison and the fissile material is important. Since most (~ 95%) of the scattering into the thermal group occurs from hydrogen, the virgin neutron source distribution in the thermal group is uniform in energy. The absorption cross section of the fuel and poison throughout the thermal group is sufficiently greater than the scattering cross section that it is a good approximation to assume that all first collisions of thermal neutrons result in captures by either fuel or poison. With these assumptions it is possible to calculate the average relative poison and fuel cross sections in the thermal group by the relation

$$\frac{\bar{\Sigma}_p}{\bar{\Sigma}_p + \bar{\Sigma}_{\text{fuel}}} = \int_0^{.414 \text{ ev}} \frac{\Sigma_p(E)}{\Sigma_p(E) + \Sigma_{\text{fuel}}(E)} \frac{dE}{.414}$$

*This fact which leads to the method used for calculating the relative poison and fissile material thermal absorption cross sections was pointed out to the author by Dr. B. Wolfe of APED.

where Σ_p = macroscopic poison absorption cross section
 Σ_{fuel}^p = macroscopic fuel absorption cross section

The ratio of the europium and the U-235 absorption cross sections did not vary appreciably with the europium to U-235 concentration ratios encountered in this study. Furthermore the thermal group was of minor importance even for flooded reactors. Hence, a constant value of $\sigma_a(Eu)/\sigma_a(fuel)$ of 4.4 was used for all calculations, including the plutonium-fueled cases. If gadolinium is used as the poison, the thermal group would have greater significance, and the ratio of gadolinium to fissile isotope cross sections does vary appreciably with the ratio of their concentrations.

APPENDIX B

Comparison of Resonance Poisons

For comparison the calculated resonance integrals of several rare earth poisons under similar reactor environments are presented in Table V. Each resonance integral was calculated for 2.5 volume percent of the material in the core, flooded conditions, and fuel and poison at 2500°F average temperature. Only those resolved resonances for which resonance parameters were given in BNL-325 (3) were used in the calculation. In addition to the resonance integrals, 2200 m/sec absorption cross sections and information about the cost of some of the poisons are given.

TABLE V

Resonance Integrals and Costs of Resonance Poisons

Poison ^a	Price ^b	Pricing Conditions	2200 m/sec. Absorption Cross Sec- tions (barns)	Resolved Resonance Integral (barns) (2.5% by volume in core)			
				0.4-1 ev	1-5 ev	5-30 ev	30-100 ev
Eu	\$590/lb	20-100 lb lots	4,300	1,123	584	261	
Gd	\$30-75/lb	lb to ton lots	46,000	100	106	61	17
Sm	\$20-40/lb	lb to ton lots	5,600	181	27	70	48
Dy	\$85-125/lb	lb to ton lots	950	-	237	192	35
Er			173	57	3	185	36
Ho			65	-	121	61	80
Lu			112	-	64	190	37
Hf	\$14-25/lb		105	-	313	160	3

a Oxides except for hafnium, which is elemental.

b Source of cost data: K. M. Mayer, Nucleonics, 167 (November, 1960)

APPENDIX C

Evaluation of the Dollar

The effective delayed neutron fraction, which is the fractional reactivity change corresponding to one dollar, was calculated from the relation,

$$\beta_{eff} = \gamma \frac{\sum_m \beta_m \nu_m F_m}{\sum_m \nu_m F_m}$$

where γ = relative effectiveness of delayed neutrons to prompt neutrons.

β_m = delayed neutron fraction of the m^{th} fissionable material

F_m = fraction of fissions occurring in material m

The ratio γ was calculated for both the flooded and unflooded 3.5 ft and 1.5 ft cores from the relation

$$\gamma = k_{delayed} / k_{prompt}$$

where $k_{delayed}$ and k_{prompt} are criticality factors calculated for an assumed delayed neutron fission spectrum and prompt neutron fission spectrum respectively. The energy spectrum for U-235 delayed neutrons (21) was used. For this spectrum 96% of the neutrons are born into energy group 5 and 4% are born into group 6.

The calculated values of β_{eff} are given in Table VI.

TABLE VI

Calculated Values of β_{eff}					
Core Size	Fuel	Coolant Condition	β	γ	β_{eff}
3.5 ft	Pu-239	nonflooded	0.0038	0.91	0.0035
3.5 ft	Pu-239	flooded	0.0035	0.93	0.0033
1.5 ft	U-235	nonflooded	0.0071	1.09	0.0077
1.5 ft	U-235	flooded	0.0070	1.04	0.0073

Values used for β_m were:

$$\beta(\text{Pu-239}) = 0.0022$$

$$\beta(\text{U-235}) = 0.0066$$

$$\beta(\text{U-238}) = 0.0158$$

APPENDIX D

Multigroup Constants

i	Energy group
X	Fission spectrum (fraction of fission neutrons born into each group)
E_L	Lower boundary of energy group
u	Lethargy at E_L ($u = 0$ at 10 Mev)
Δu	Group lethargy width
σ_{tr}	Transport cross section
σ_f	Fission cross section
σ_c	Capture cross section
σ_{er}	Cross section for neutrons removed to next lowest group by elastic scattering (except for hydrogen for which Selengut-Goertzel scattering was assumed.)
σ_{in}	Cross section for neutrons removed from the energy group by inelastic scattering (except for hydrogen-see footnote f, Table VIII)
$\sigma_{in,1 \rightarrow i+k}$	Transfer cross section for inelastic scattering from group i to group $i+k$
ν	Neutrons produced per fission
α	σ_c / σ_f

TABLE VII

Energy Group Structure

i	X	E_L	u	Δu
1	0.338	2.25 Mev	1.5	1.5
2	0.236	1.35	2.0	0.5
3	0.178	0.825	2.5	0.5
4	0.116	0.5	3.0	0.5
5	0.066	0.3	3.5	0.5
6	0.033	0.18	4.0	0.5
7	0.017	0.11	4.5	0.5
8	0.008	0.067	5.0	0.5
9	0.006	0.025	6.0	1.0
10	0.002	0.0091	7.0	1.0
11	--	4100 ev	7.8	0.8
12	--	1010	9.2	1.4
13	--	304	10.4	1.2
14	--	101	11.5	1.1
15	--	30.5	12.7	1.2
16	--	5.04	14.5	1.8
17	--	1.0	16.13	1.63
18	--	0.414	17.0	0.87
19	--	thermal	--	--

TABLE VIII

Cross Sections

Material	Conditions ^a	i	σ_{er}	σ_{in}	σ_{tr}	σ_c	σ_f	ν	α
Pu-239	$\sigma_p=370$ e=13% nonflooded	1	.014	1.349	4.6	0.04	2.00	3.18	0.02
		2	.024	1.000	4.6	0.08	1.95	3.09	0.04
		3	.044	0.700	5.3	0.09	1.86	3.02	0.05
		4	.056	0.400	5.9	0.12	1.75	2.97	0.07
		5	.081	0.230	7.1	0.17	1.70	2.95	0.10
		6	.106		8.5	0.27	1.68	2.93	0.16
		7	.124		9.5	0.40	1.73	2.92	0.23
		8	.142		10.8	0.54	1.80	2.91	0.30
		9	.080		12.3	0.74	2.00	2.91	0.37
		10	.110		16.4	1.06	2.25	2.91	0.47
		11	.106		12.9	1.34	2.58	2.91	0.50
		12	.061		14.7	2.17	3.87	2.91	0.56
		13	.071		22.9	4.70	7.55	2.91	0.62
		14	.077		36.0	10.7	16.4	2.91	0.65
		15	.071		43.3	19.0	15.3	2.91	1.24
		16	.047		53.8	16.8	28.0	2.91	0.60
		17	.052		31.0	6.0	15.0	2.91	0.40
		18	.100		193	53	130	2.91	0.41
		19			1475	480	986	2.91	0.49
Pu-239	$\sigma_p=550$ e=13% flooded	1-10 ^b							
		11			13.0	1.29	2.60	2.91	0.50
		12			14.9	2.21	3.95	2.91	0.56
		13			23.9	5.10	8.19	2.91	0.62
		14			40.2	12.3	18.9	2.91	0.65
		15			55.8	23.2	18.7	2.91	1.24
		16			63.6	20.5	34.1	2.91	0.60
		17-19 ^b							

TABLE VIII cont'd.

Material	Conditions ^a	i	σ_{er}	σ_{in}	σ_{tr}	σ_c	σ_f	ν	α
U-235	$\sigma_p=366$ $e=20\%$ flooded	1	0.009	2.300	4.5	0.10	1.30	2.77	0.08
		2	0.019	1.849	4.5	0.10	1.28	2.65	0.08
		3	0.039	1.150	4.8	0.10	1.25	2.58	0.08
		4	0.043	1.199	5.1	0.14	1.20	2.53	0.12
		5	0.069	0.700	6.3	0.19	1.28	2.51	0.15
		6	0.098	0.400	7.9	0.26	1.42	2.49	0.18
		7	0.130		9.6	0.32	1.60	2.48	0.20
		8	0.143		10.9	0.48	1.90	2.47	0.25
		9	0.075		12.2	0.69	2.30	2.47	0.30
		10	0.075		13.5	1.19	3.40	2.47	0.35
		11	0.106		13.9	1.19	2.67	2.43	0.445
		12	0.061		17.3	2.26	5.09	2.43	0.445
		13	0.071		23.9	4.49	9.46	2.43	0.475
		14	0.077		34.2	7.8	16.4	2.43	0.475
		15	0.071		56.3	20.4	25.9	2.43	0.79
		16	0.047		63.6	35.4	18.2	2.43	1.95
		17	0.052		34.8	9.8	15.0	2.43	0.65
		18	0.100		147	20	117	2.43	0.173
		19			270	38	222	2.43	0.173
U-235	$\sigma_p=244$ $e=30\%$ flooded	1-10 ^c							
		11	0.106		13.9	1.19	2.67	2.43	0.445
		12	0.061		17.3	2.26	5.07	2.43	0.445
		13	0.071		23.8	4.45	9.36	2.43	0.475
		14	0.077		33.6	7.6	16.0	2.43	0.475
		15	0.071		53.2	18.8	24.4	2.43	0.79
		16	0.047		57.5	31.4	16.1	2.43	1.95
		17	0.052		33.5	9.3	14.2	2.43	0.65
		18-19 ^c							

TABLE VIII cont'd.

Material	Conditions ^a	i	σ_{er}	σ_{in}	σ_{tr}	σ_c	σ_f	ν	α
U-235	$\sigma_p=183$ e=40%	1-10 ^c							
		11	0.106		13.9	1.19	2.67	2.43	0.445
	flooded	12	0.061		17.3	2.25	5.06	2.43	0.445
		13	0.071		23.7	4.41	9.28	2.43	0.475
		14	0.077		33.0	7.4	15.6	2.43	0.475
		15	0.071		50.8	17.6	23.2	2.43	0.79
		16	0.047		53.1	20.5	14.6	2.43	1.95
		17	0.052		32.3	8.8	13.5	2.43	0.65
		18-19 ^c							
U-235	$\sigma_p=122$ e=60%	1-10 ^c							
		11	0.106		13.8	1.18	2.65	2.43	0.445
	flooded	12	0.061		17.2	2.22	4.99	2.43	0.445
		13	0.071		23.1	4.22	8.88	2.43	0.475
		14	0.077		31.4	5.84	15.6	2.43	0.475
		15	0.071		47.6	15.9	21.7	2.43	0.79
		16	0.047		46.9	24.4	12.5	2.43	1.95
		17	0.052		30.8	8.2	12.6	2.43	0.65
		18-19 ^c							
U-238	$\sigma_p=40$ e=100%	1	0.014	2.87	4.7	0.015	0.59	2.65	
		2	0.023	2.44	4.5	0.06	0.45	2.55	
	nonflooded	3	0.062	1.20	5.0	0.13	0.003	2.47	
		4	0.083	0.44	5.5	0.14			
		5	0.102	0.47	6.7	0.13			
		6	0.127	0.55	8.2	0.15			
		7	0.150	0.55	9.6	0.20			
		8	0.171	0.40	10.9	0.30			
		9	0.099	0.05	12.2	0.40			
		10	0.108		13.5	0.61			

TABLE VIII cont'd.

Material	Conditions ^a	i	σ_{er}	σ_{in}	σ_{tr}	σ_c	σ_f	ν	α
U-238		11	0.106		10.4	0.44			
		12	0.061		10.9	0.85			
		13	0.071		11.0	1.03			
		14	0.077		11.8	1.80			
		15	0.071		12.5	2.48			
		16	0.047		14.1	4.14			
		17	0.052		10.3	.34			
		18	0.100		10.6	0.55			
		19			11.5	1.52			
		1-10 ^d							
		11	0.106		10.4	0.37			
		12	0.061		10.9	0.93			
		13	0.071		11.7	1.74			
		14	0.077		13.8	3.83			
		15	0.071		15.4	5.37			
		16	0.047		18.6	8.61			
		17-19 ^d							
		1-10 ^d							
		11	0.106		10.4	0.38			
		12	0.061		11.0	0.98			
		13	0.071		11.9	1.87			
		14	0.077		14.3	4.30			
		15	0.071		16.1	6.11			
		16	0.047		19.8	9.81			
		17-19 ^d							
U-238	$\sigma_p=163$ $e=45\%$ flooded	1-10 ^d							
		11	0.106		10.4	0.40			
		12	0.061		11.1	1.06			
		13	0.071		12.1	2.10			
		14	0.077		15.2	5.22			
		15	0.071		17.7	7.66			
		16	0.047		22.4	12.41			
		17-19 ^d							

TABLE VIII. cont'd.

Material	Conditions ^a	i	σ_{er}	σ_{in}	σ_{tr}	σ_c	σ_f	ν	α
Eu	v.f.=0.5% flooded	1			8.0	.03			
		2			8.1	.07			
		3			8.1	.11			
		4			8.2	.2			
		5			8.3	.3			
		6			8.5	.5			
		7			8.7	.7			
		8			9.1	1.1			
		9			10	2.0			
		10			12	4.0			
		11			19	11			
		12			32	24			
		13			58	50			
		14			82	74			
		15			155	147			
		16			250	242			
		17			531	523			
		18			2836	2828			
		19			1158	1150			
Eu	v.f.=1.0% flooded	1-14 ^e							
		15			152	144			
		16			238	230			
		17			463	455			
		18			2067	2059			
		19 ^e							

TABLE VIII cont'd.

Material	Conditions ^a	i	σ_{er}	σ_{in}	σ_{tr}	σ_c	σ_f	v	α
Eu	v.f.=2.5% flooded	1-14 ^e							
		15			143	135			
		16			199	191			
		17			366	358			
		18			1299	1291			
		19 ^e							
Eu	v.f.=5.0% flooded	1-14 ^e							
		15			128	120			
		16			172	164			
		17			295	287			
		18			941	933			
		19 ^e							
R ^f		1		2.5 ^f	0.83				
		2		2.5	0.83				
		3		4.0	1.33				
		4		5.6	1.87				
		5		7.0	2.34				
		6		8.9	2.97				
		7		10.4	3.47				
		8		13.8	4.60				
		9		15.9	5.30				
		10		18.4	6.13	0.0004			
		11		20	6.67	0.0007			
		12		20	6.67	0.001			
		13		20	6.67	0.002			
		14		20	6.67	0.004			
		15		20	6.67	0.007			
		16		20	6.67	0.015			
		17		20	6.71	0.035			
		18		20	6.74	0.067			
		19			21.00	0.132			

TABLE VIII Cont'd

Material	Group	σ_{er}	σ_{in}	σ_{tr}	σ_c
O	1	0.250		1.05	0.32
	2	0.348		1.60	
	3	0.871		3.60	
	4	0.726		3.00	
	5	1.379		5.70	
	6	0.811		3.35	
	7	0.789		3.26	
	8	0.789		3.26	
	9	0.394		3.26	
	10	0.394		3.26	
	11	0.570		3.64	
	12	0.326		3.64	
	13	0.380		3.64	
	14	0.415		3.64	
	15	0.380		3.64	
	16	0.253		3.64	
	17	0.280		3.64	
	18	0.524		3.64	
	19			3.64	
Cr	1	0.043	1.180	2.0	0.006
	2	0.086	0.826	2.0	0.006
	3	0.107		1.9	0.006
	4	0.157		2.2	0.006
	5	0.214		3.0	0.007
	6	0.249		3.5	0.008
	7	0.306		4.3	0.010
	8	0.378		5.3	0.010
	9	0.242		6.8	0.030
	10	0.142		4.0	0.014
	11	0.806		17.0	0.005
	12	0.122		5.9	0.009
	13	0.142		4.3	0.012

TABLE VIII Cont'd

Material	Group	σ_{er}	σ_{in}	σ_{tr}	σ_c
	14	0.155		4.3	0.025
	15	0.142		4.3	0.037
	16	0.095		4.4	0.135
	17	0.105		4.7	0.326
	18	0.196		4.9	0.624
	19			6.0	1.72
Fe	1	0.043	1.180	2.0	0.005
	2	0.086	0.650	2.0	0.005
	3	0.107	0.400	1.9	0.005
	4	0.157		2.2	0.005
	5	0.214		3.0	0.006
	6	0.249		3.5	0.007
	7	0.306		4.3	0.008
	8	0.378		5.3	0.008
	9	0.242		6.8	0.025
	10	0.142		4.0	0.012
	11	0.303		6.9	0.004
	12	0.201		8.0	0.007
	13	0.313		10.7	0.010
	14	0.364		11.4	0.020
	15	0.333		11.4	0.030
	16	0.222		11.5	0.110
	17	0.245		11.7	0.2648
	18	0.460		11.9	0.5097
	19			12.8	1.40
Ni	1	0.043	1.180	2.0	0.0065
	2	0.086	0.826	2.0	0.0065
	3	0.107		1.9	0.0065
	4	0.157		2.2	0.0065
	5	0.214		3.0	0.008
	6	0.249		3.5	0.009
	7	0.306		4.3	0.010
	8	0.378		5.3	0.010

TABLE VIII Cont'd

Material	Group	σ_{er}	σ_{in}	σ_{tr}	σ_c
	9	0.242		6.8	0.032
	10	0.142		4.0	0.016
	11	0.722		17.0	0.008
	12	0.413		17.0	0.013
	13	0.482		17.0	0.019
	14	0.525		17.0	0.038
	15	0.482		17.0	0.057
	16	0.321		17.1	0.209
	17	0.354		17.6	0.504
	18	0.664		17.9	0.967
	19			19.6	2.66

a. σ_p = potential scattering cross section per absorber atom, e = enrichment, and v.f. = volume fraction in fast core.

b. Same as Pu²³⁹, σ_p = 370

c. Same as U²³⁵, σ_p = 356

d. Same as U²³⁸, σ_p = 40

e. Same as Eu, v.f. = 0.5%

f. For hydrogen, the scattering cross section is listed under " σ_{in} "

TABLE IX

INELASTIC SCATTERING MATRICES

$\sigma_{in, i-d + k}$								
Material	<u>1</u>	<u>k=1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>
Pu ²³⁹	1	0.513	0.382	0.221	0.120	0.0648	0.0486	
	2	0.457	0.264	0.143	0.078	0.0385	0.0195	
	3	0.331	0.179	0.098	0.049	0.0245	0.0182	
	4	0.336	0.183	0.0917	0.0446	0.0340	0.0084	
	5	0.20	0.100	0.0512	0.0371	0.00884	0.00248	
	6	0.115	0.0591	0.0428	0.0101	0.00276		
U ²³⁵	1	0.881	0.660	0.382	0.207	0.115	0.0552	
	2	0.845	0.488	0.265	0.144	0.0712	0.0361	
	3	0.544	0.294	0.161	0.0805	0.0402	0.0299	
	4	0.576	0.314	0.157	0.0798	0.0582	0.0144	
	5	0.351	0.175	0.0896	0.0649	0.0155	0.00434	
	6	0.200	0.103	0.0744	0.0176	0.0048		
U ²³⁸	1	1.06	0.46	0.66	0.39	0.12	0.18	
	2	0.62	0.89	0.52	0.16	0.10	0.15	
	3	0.52	0.35	0.22	0.11			
	4	0.44						
	5	0.47						
	6	0.50	0.05					
	7	0.55						
	8	0.38	0.02					
	9	0.03	0.02					
Cr	1	0.747	0.433					
	2	0.492	0.163	0.099	0.055	0.011	0.006	
Fe	1	0.450	0.206	0.196	0.122	0.0802	0.0448	0.0814
	2	0.39	0.26					
	3		0.17	0.104	0.0592	0.0332	0.0332	
Ni	1	0.747	0.433					
	2	0.492	0.163	0.099	0.055	0.011	0.006	

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