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Forced Oscillations of the Los Alamos Scientific Laboratory's Dry Hot Rock Geothermal Reservoir

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FORCED OSCILLATIONS OF THE LOS ALAMOS SCIENTIFIC LABORATORY'S

DRY HOT ROCK GEOTHERMAL RESERVOIR

by

Y. C. Hsu

ABSTRACT

Equations of motion are derived for the Los Alamos Scientific Laboratory's geothermal reservoir consisting of an elastic half-space (rock), with an open vertical pipe from the surface connected at depth with a vertical penny-shaped fracture. This reservoir, which is filled with water (laminar flow through a pipe), is subjected to seismic waves and/or blast-like impulses applied downhole. Oscillations (responses) of the water column, of the downhole internal net pressure required to keep the fracture open, and of the fracture itself, are found for the reservoir.

If the fracture is continuously oscillated subject to pressure waves applied downhole and failure of the fracture surface occurs, it is usually through fatigue after a long period of time.

Of concern next is transient oscillation of the fracture when downhole blast-like impulses are applied. Of particular concern is the maximum amplitude, $\epsilon_{\max}$, of the fracture (crack) oscillation in early cycles. Generally, failure due to $\epsilon_{\max}$ is attributed singly to the strength of the fractured rock being exceeded.

Since the period, τ , of the oscillations is sensitive to the radius, R , of the fracture, experimental determination of τ , in conjunction with Eq. (20), will permit R to be evaluated accurately.

I. INTRODUCTION

Field data,¹ collected from geothermal hydraulic systems in Iceland, revealed certain types of oscillations of the water column, the downhole constant internal pressure required to open a fracture, and the fracture. One case occurred during injection of water through an adjacent borehole, with a period of 2700 s. The second case occurred in a natural spring, with a period of 600 s. Bodvarsson¹ considered a system consisting of an elastic half-space (rock) with an open borehole from the surface connected at depth with a horizontal circular fracture. This fracture, of radius R , was assumed to have a very small constant width. The system was filled with water to a static level slightly below the surface. Based on the above system, R can be predicted by using the formula derived in conjunction with field data of the period, τ .

Of interest here is forced oscillation of the Los Alamos Scientific Laboratory's (LASL) dry hot rock geothermal reservoir (system). The reservoir, which is filled with water, is subjected to seismic waves and/or blast-like impulses applied downhole. The period, τ , will be derived as a function of R . The forced oscillations will be found for the water column, the constant internal net pressure over the plane of the fracture required to open it, and the fracture itself.

II. LASL'S DRY HOT ROCK GEOTHERMAL RESERVOIR

As shown in Fig. 1, the reservoir consists of an elastic, isotropic, homogeneous half-space, with an open vertical pipe of radius r_c from the surface connected at depth, h , with a vertical penny-shaped fracture. The elastic material properties of the rock are denoted by the shearing modulus, μ , and

Fig. 1. The LASL's dry hot rock geothermal reservoir filled with water.

Poisson's ratio, ν . The static water level is slightly below the surface. This reservoir, which is filled with water (laminar flow through a pipe), is subjected to seismic waves and/or blast-like impulses applied downhole.

Since the fracture of radius R is opened by the constant internal net pressure over the circle $0 \leq r \leq R$, the fracture (crack) is ellipsoidal in cross-sectional shape,²

$$r^2/R^2 + w^2/6^2 = 1,$$

where $w = u_z|_{z=0}$ is the normal component of the surface deflection of the fracture and 6 is the width of an ellipsoidal depression of the fracture. The volume of water in the fracture (or the fracture volume) is defined by V .

III. EQUATIONS OF MOTION

Figure 2 shows the dynamic free-body diagram for the rigid column of water in an open vertical pipe. By using Newton's second law of motion, the equation of motion of the rigid column of water is derived as

Fig. 2. Dynamic free-body diagram of the rigid column of water in an open vertical pipe.

$$\underbrace{F_g - \pi r_t^2 P_t}_{\text{The resultant of an unbalanced force system acting on the rigid column of water.}} = \underbrace{\frac{d}{dt} \left[\left(\rho \pi r_t^2 (h-x) \right) \frac{dx}{dt} \right]}_{\text{The time rate of decrease of linear momentum of the water column.}} + \underbrace{\left(\rho \pi r_t^2 \frac{dx}{dt} \right) \frac{dx}{dt}}_{\text{The outward flux of linear momentum at the bottom of the vertical pipe or water column.}}, \quad (1)$$

The resultant of an unbalanced force system acting on the rigid column of water.

The time rate of decrease of linear momentum of the water column.

The outward flux of linear momentum at the bottom of the vertical pipe or water column.

where x = the vertical downward displacement of the rigid column of water from the static water level, considered positive,

ρ = the density of water,

F_g , the gravitational force = $\rho \pi r_t^2 (h-x)g$, (2)

P_t , the total downhole pressure at the bottom of the vertical pipe

$$= \rho g(h-x) + \Delta P_v + (P-S_k) + P_{P.W.}, \quad (3)$$

$\rho g(h-x)$ = the downhole pressure contributed only by the hydraulic head, $(h-x)$,

ΔP_v , the pressure drop³ for Hagen-Poiseuille laminar flow through a pipe, for relatively small radii and flow velocities

$$= [8\mu_v(h-x)/r_t^2] \dot{x}, \quad (4)$$

μ_v = viscosity of water,
 S_k = the least principal stress perpendicular to the fracture plane,
 P = the constant internal fluid pressure acting over $0 \leq r \leq R$,
 $(P-S_k)$, the constant internal net pressure required to open a fracture
 $= [3\mu\pi r_t^2 / 8(1-\nu)R^3]x$, (5)
 $P_{P.W.}$ = pressure waves and/or blast-like impulses, applied downhole.

Equation (5) will be derived as follows: the volume of the inflated fracture, V , is²

$$V = \frac{8(1-\nu)}{3\mu} (P-S_k)R^3, \quad (\mu = E/[2(1+\nu)])$$

Here $-(P-S_k)$ is defined as the restoring elastic pressure provided by the fractured rock. The elastance of the fracture, $dV/[d(P-S_k)]$, is

$$dV/[d(P-S_k)] = [8(1-\nu)/3\mu]R^3$$

This constant slope is also equal to

$$[8(1-\nu)/3\mu]R^3 = (V-0)/[(P-S_k)-0] = (\pi r_t^2 x)/(P-S_k)$$

which results in Eq. (5). Here Eq. (5) is derived by neglecting inertia of the rock.

Substituting Eqs. (2) to (5) into (1) gives, for small x ,

$$m\ddot{x} + c\dot{x} + k_e x = F(t), \quad (6)$$

where m , the mass of water column $= \pi r_t^2 h \rho$,

c , viscous damping constant in the vertical pipe $= 8\mu_v h \pi$, (7)

k_e , the equivalent spring constant for a reservoir $= 3\mu(\pi r_t^2)^2/[8(1-\nu)R^3]$, (8)

$F(t)$, the downhole exciting force due to pressure waves and/or blast-like impulses
 $= -(\pi r_t^2)P_{P.W.}$ (9)

Some parameters are defined as follows:

$$c^2/4m^2 = 16\nu_v^2/r_t^4, \quad (10)$$

$$k_e/m = 3\mu(\pi r_t^2)^2/[8(1-\nu)\rho h R^3], \quad (11)$$

$$\omega_n, \text{ the undamped natural circular frequency} = \sqrt{k_e/m}, \quad (12)$$

$$c_{cr}, \text{ the critical damping constant} = 2m\omega_n$$

. This form is derived based on

$$c_{cr}^2/4m^2 = k_e/m,$$

$$\zeta, \text{ the damping ratio } (c/c_{cr}) = 4(\nu_v)/\omega_n r_t^2, \quad (13)$$

TABLE I

ρ AND ν_v OF WATER IN TERMS OF WATER TEMPERATURE, T

$T(^{\circ}\text{C})$	$\rho \text{ (kg/m}^3\text{)}$	$\nu_v \times 10^6 \text{ (m}^2\text{/sec)}$
20	997.194	101.264 $\times 10^{-2}$
60	982.765	48.217 $\times 10^{-2}$
100	957.059	29.636 $\times 10^{-2}$

ω_d , the damped natural circular frequency

$$= \omega_n \sqrt{1-\zeta^2}, \quad (14)$$

ν_v , the kinematic viscosity of water

$$= \mu_v/\rho. \quad (15)$$

Table I contains some numerical values³ of ρ and ν_v for water. For the granite, $\nu = 0.25$, $E = 8 \times 10^{11}$ dynes/cm², and $\mu = 3.2 \times 10^{10}$ N/m². For a particular reservoir, $r_t = 2.389$ in. = 0.06 m, $h = 9260$ ft = 2822.45 m and $R = 50$ ft = 15.24 m. Table II contains some parameters in terms of T and this reservoir.

As shown in Table II, $c^2/4m^2 \ll k_e/m$ (or $\zeta \ll 1$) and the case of underdamping would occur. Hence, $x_h = e^{-(c/2m)t} (A \cos \omega_n t + B \sin \omega_n t)$ is simplified as $x_h \approx (A \cos \omega_n t + B \sin \omega_n t)$. Here, x_h is denoted as the homogeneous solution of Eq. (6) by neglecting the damping term $c\dot{x}$. Hence, Eq. (6) becomes, with small damping constant,

$$m\ddot{x} + k_e x = F(t) \quad (16)$$

Substituting Eq. (5) into (16) gives, with

$$\dot{S}_k = \dot{S}_k = 0,$$

$$\ddot{x} + \frac{3\mu(\pi r_t^2)}{8(1-\nu)\rho h R^3} x = \left[\frac{3\mu(\pi r_t^2)}{8(1-\nu)\rho h R^3} \right] (S_k - P_{P.W.}) \quad (17)$$

The relationship² between $(P-S_k)$ and θ was obtained as

$$P - S_k = \frac{\pi E}{4(1-\nu^2)} \left(\frac{\theta}{R} \right) \quad (18)$$

TABLE II

SOME PARAMETERS IN TERMS OF WATER TEMPERATURE, T , AND A GIVEN RESERVOIR

$T(^{\circ}\text{C})$	$\frac{c^2}{4m^2} \left[\left(\frac{\text{rad}}{\text{sec}} \right)^2 \right]$	$\frac{k_e}{m} \left[\left(\frac{\text{rad}}{\text{sec}} \right)^2 \right]$	$\omega_n \left[\frac{\text{rad}}{\text{sec}} \right]$	ζ
20	0.1210×10^{-5}	0.0186	0.1364	0.807×10^{-2}
60	0.0274×10^{-5}	0.0189	0.1375	0.323×10^{-2}
100	0.0103×10^{-5}	0.0193	0.1389	0.232×10^{-2}

from which it follows that $P - S_k$ is constant for constant values of the ratio (G/R) . Substituting Eq. (18) into (17) gives, with $F = [\pi E / \{4(1-\nu^2)R\}]G$,

$$\ddot{G} + \frac{3\mu(\pi r_t^2)}{8(1-\nu)\rho h R^3} G = -\frac{3r_t^2}{4\rho h R^2} P_{P.W.} \quad (19)$$

Equations (16), (17), and (19) are the equations of motion for the water column in the vertical pipe, the constant internal fluid pressure for opening the fracture, and the width of the fracture.

IV. MEASUREMENT OF τ AS A MEANS OF PREDICTING R

The form of $\omega_n = \sqrt{k_e/m}$ in Eq. (12) was derived based on Eq. (16). This form is also valid for Eqs. (17) and (19). The period, τ , of the oscillations of $x(t)$, $P(t)$, and $G(t)$, for one cycle is

$$\tau = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{8(1-\nu)h\phi R^3}{3\pi \mu r_t^2}} \text{ seconds.} \quad (20)$$

The undamped natural frequency is then

$$f = \frac{1}{\tau} \text{ Hz (cps)} \quad (21)$$

At $T = 20^\circ\text{C}$, $\rho \approx 10^3 \text{ kg/m}^3$. For the granite, $\nu = 0.25$ and $\mu = 3.2 \times 10^{10} \text{ N/m}^2$. As such, Eq. (20) becomes

$$\tau = 0.5 \times 10^{-3} (R/r_t) \sqrt{hR} \text{ seconds.} \quad (22)$$

For $r_t = 0.06 \text{ m}$ and $h = 2822.45 \text{ m}$, Eq. (22) gives $\tau = 26 \text{ s}$, 386 s , and 2356 s for $R = 15.24 \text{ m}$ (50 ft), 91.44 m (300 ft), and 304.8 m (1000 ft). For a given pipe and a certain depth, τ is proportional to $R^{3/2}$. Since τ is sensitive to R , experimental data on τ can determine R with accuracy. For a large fracture (for instance, $R = 1000 \text{ ft}$), there is a certain type of low-frequency water-column (or P or G) oscillation which can occur in the LASL geothermal reservoir.

V. FORCED OSCILLATIONS (RESPONSES) CAUSED BY $P_{P.W.}$

Rewrite Eq. (16) as

$$\ddot{x} + \frac{k_e}{m} x = -\frac{r_t^2 P_{P.W.}}{m} \quad (23)$$

The solutions of Eq. (23) will be found for case A, the downhole-applied seismic waves and/or earth tidal waves, and case B, the downhole-applied blast-like impulses. Then $P(t)$ and $G(t)$ can be determined from Eqs. (17) and (19), or obtained from Eqs. (5) and (18) in conjunction with the solution $x(t)$ obtained from Eq. (23).

A. Downhole-Applied Seismic Waves and/or Earth Tidal Waves

1. Simple Harmonic Pressure Wave. The experimental wave form is represented by

$$P_{P.W.} = P_1 \sin \omega t, \quad (24)$$

where the amplitude P_1 and the forcing circular frequency ω are determined from experimental data. Subjected to $P_{P.W.}$ in Eq. (24), the solution of Eq. (23) is

$$x = A \cos \omega_n t + B \sin \omega_n t - X \sin \omega t, \quad (25)$$

where $X = \pi r_t^2 P_1 / [m(\omega_n^2 - \omega^2)]$.

The first two terms are called transient, even though damping is absent in this case. Arbitrary constants A and B depend on initial conditions. After a period of time with any measurable energy dissipation in damping, the influence of these two terms is small. However, the last term is not affected by initial conditions and remains as long as the forcing function is applied. It is called the steady-state oscillation. Omit the transient oscillation and direct attention to the variation of the amplitude of the steady-state oscillation,

$$X / (\pi r_t^2 P_1 / k_e) = 1 / (1 - \omega^2 / \omega_n^2) \quad (26)$$

At low forcing frequencies, when $\omega \ll \omega_n$, the amplitude, X , is approximately equal to $\pi r_t^2 P_1 / k_e$. It is the static deflection which the elastic cracked rock would have if $\pi r_t^2 P_1$ were a static force. For $\omega < \omega_n$, the amplitude ratio defined in Eq. (26) is positive and the oscillation is in phase with the applied pressure wave.

At high forcing frequencies, when $\omega \gg \omega_n$, X is very small. It becomes less and less as the frequency ratio, ω / ω_n , is increased. For $\omega > \omega_n$, the

amplitude ratio is negative and the oscillation is out of phase with the applied pressure wave.

When $\omega = \omega_n$, very large oscillation amplitudes can result from very small pressure waves. This phenomenon is known as resonance.

2. Periodic But Not Simple Harmonic Pressure

Wave. The experimental wave form is represented by the Fourier series

$$P_{P.W.}(\omega t) = \frac{1}{2}A_0 + \sum_{j=1}^{\infty} A_j \cos j\omega t + \sum_{j=1}^{\infty} B_j \sin j\omega t. \quad (27)$$

Defining $P_j = \sqrt{A_j^2 + B_j^2}$ and $\tan^{-1} \alpha_j = A_j/B_j$, Eq. (27) becomes

$$P_{P.W.}(\omega t) = \frac{1}{2}P_0 + \sum_{j=1}^{\infty} P_j \sin(\omega t + \alpha_j). \quad (28)$$

The arbitrary coefficients, A_j and B_j , can be found by either direct or numerical integration, with the period $\omega t = 0$ to 2π ,,,

$$A_j = \frac{1}{\pi} \int_0^{2\pi} P_{P.W.}(\omega t) \cos j\omega t d(\omega t), \quad (29)$$

$$B_j = \frac{1}{\pi} \int_0^{2\pi} P_{P.W.}(\omega t) \sin j\omega t d(\omega t). \quad (30)$$

The magnitude $A_0/2$ is the average value of $P_{P.W.}$ over the period 2π ,

$$2\pi \left(\frac{A_0}{2}\right) = \int_0^{2\pi} P_{P.W.} d(\omega t). \quad (31)$$

The steady-state oscillation, x_s , of the water column subjected to the downhole-applied pressure waves in Eq. (28) is

$$x_s = - \left[\frac{\pi r_t^2 P_0}{2k_e} + \sum_{j=1}^{\infty} \frac{\pi r_t^2 P_j}{k_e (1 - (j^2 \omega^2 / \omega_n^2))} \sin(j\omega t + \alpha_j) \right]. \quad (32)$$

Substituting $x_s = -X \sin \omega t$ in Eq. (25) and x_s in Eq. (32) into Eq. (5), respectively, $(P-S_k)$ is then found. Substituting the above mentioned $(P-S_k)$

into Eq. (18) gives the steady-state fracture oscillation $G(\omega t)$. The fracture is continuously oscillated subject to the downhole-applied pressure waves. If failure of the fracture surface occurs, it is usually through fatigue after a long period of time.

B. Downhole Applied Blast-Like Impulses

1. The Sudden Pressure Wave Which Decays Exponentially With Time. The experimental wave form is represented by one exponential decaying step function,

$$P_{P.W.} = P_1 e^{-at}, \quad (33)$$

which can be adapted to a variety of blast-like impulses by changing the time constant a , which has the same units as ω .

The solution of Eq. (23) subject to $P_{P.W.}$ in Eq. (33) is

$$x = A \cos \omega_n t + B \sin \omega_n t - \frac{\pi r_t^2 P_1}{m a^2 + k_e} e^{-at}. \quad (34)$$

For rest conditions initially (i.e., $x_0 = 0$ and $\dot{x}_0 = 0$), Eq. (34) becomes

$$x = -X \left[e^{-at} + (a/\omega_n) \sin \omega_n t - \cos \omega_n t \right] \quad (35)$$

where the steady-state amplitude, X , is

$$X = \pi r_t^2 P_1 / \left[k_e (1 + a^2 / \omega_n^2) \right]. \quad (36)$$

It is reached for $at > 5$. The term of e^{-at} is then, the transient oscillation, which is not necessarily periodic. Figure 3 shows the oscillation of the

Fig. 3. The oscillation of a water column subject to an exponentially decaying step function.

water column subject to an exponentially decaying step function.

2. The Sudden Pressure Wave With Two Exponential Decaying Step Functions. The experimental wave form is represented by

$$P_{P.W.} = P_1(e^{-at} - e^{-bt}) \quad (b > a) \quad (37)$$

The solution of Eq. (23) subject to $P_{P.W.}$ in Eq. (37) is

$$x = A \cos \omega_n t + B \sin \omega_n t$$

$$- \frac{\pi r_t^2 P_1}{k_e} \left[e^{-at} / (1 + a^2 / \omega_n^2) - e^{-bt} / (1 + b^2 / \omega_n^2) \right] \quad (38)$$

For rest conditions of the water column initially ($x_0 = 0$, $\dot{x}_0 = 0$), Eq. (38) becomes

$$x = - \frac{\pi r_t^2 P_1}{k_e (1 + \frac{a^2}{\omega_n^2})} \left[\frac{a}{\omega_n} \sin \omega_n t - \cos \omega_n t + e^{-at} \right] + \frac{\pi r_t^2 P_1}{k_e (1 + \frac{b^2}{\omega_n^2})} \left[\frac{b}{\omega_n} \sin \omega_n t - \cos \omega_n t + e^{-bt} \right] \quad (39)$$

Here terms e^{-at} and e^{-bt} are transient oscillations of the water column. Figure 4 shows the oscillation of the water column subject to two exponentially decaying step functions.

Equations (5) and (18), in conjunction with Eq. (39), give the oscillations of $P(t)$ and $\theta(t)$. Of concern is the transient solution where a shock pressure wave is applied. Of particular concern is the

Fig. 4. The oscillation of a water column subject to two exponentially decaying step functions.

maximum amplitude of the fracture oscillation, $\theta(t)$, in early cycles. Generally, failure from transient oscillation of $\theta(t)$ is attributed to the strength of the cracked rock being exceeded.

VI. CONCLUSIONS

Equations of motion of the water column, x , the internal fluid pressure, P , required to keep the fracture open, and the width of fracture, θ , are derived in Eqs. (16), (17), and (19) for the LASL's dry hot rock geothermal reservoir, subject to the downhole-applied pressure waves and/or blast-like impulses. Oscillations, $x(t)$, $P(t)$, and $\theta(t)$, are found in Section V for the reservoir.

When reservoir fracture is continuously oscillated by the downhole-applied pressure waves, failure of the fracture surface--if it occurs--will usually be through fatigue after a long period of time.

Of concern next is transient oscillation of the fracture when downhole blast-like impulses are applied. Of particular concern is the maximum amplitude θ_{max} of the fracture oscillation $\theta(t)$ in early cycles. Generally, failure due to θ_{max} is attributed to the strength of the fractured rock being exceeded.

For $r_t = 0.06$ m, $h = 2822.45$ m, and granite, one obtains $\tau = 26$, 386, and 2356 s for $R = 15.24$, 91.44, and 304.8 m, respectively. Thus, the radius of the fracture can be calculated by using Eq. (20) in conjunction with experimental measurement of τ .

The present work can be extended to study the "waterhammer" problem by considering the elastic column of water, the elastic pipe, and the restoring force provided by the elastic, fractured, inflated rock. Theoretical results will serve to interpret experimental data of the "waterhammer" phenomenon.

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