REACTOR- NOISE ANALYSIS IN THE TIME DOMAIN

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FOREWORD

This book is part of the continuing Critical Review Series published by the Division of Technical Information of the AEC. This series presents discussions on the state of the art in specific and limited scientific fields of interest, identifying significant contributions, both published and unpublished. Whenever possible, critical evaluations and alternative avenues will be given. Hopefully, the authors will take advantage of the opportunity to present a multitude of facts in an orderly manner and to synthesize new concepts out of the contributions of many.

We would be extremely interested in your comments concerning this book or others in the series. We hope that in the spirit of the program you will be as constructively critical as you can.

A handwritten signature in black ink, reading "John M. Sullivan". The signature is written in a cursive style with a large, stylized "J" and "S".

John M. Sullivan, Chief
Scientific Publications Branch
Division of Technical Information

PREFACE

This book is dedicated to Professor Ugo Farinelli and Dr. Charles E. Cohn, two important men in its development.

Ugo Farinelli introduced me to nuclear noise and correlation methods while I was writing my doctoral thesis under his direction. Later he suggested, encouraged, and supported a full program of theoretical and experimental studies in Italy, and he proposed that I make a long-term visit to one of the main laboratories of the United States to refine the fundamentals of reactor-noise analysis. My gratitude to Charles E. Cohn transcends any formal expression. With his continuous assistance, critical cooperation, and extensive knowledge of the subject, he helped not only in the writing and editing of the whole analysis but also in supervising the collection and organization of the material. I thank both of these gentlemen for the entire production.

Certainly, advancements and developments in reactor-noise time analysis will occur after this work goes into publication; and these cannot, of course, be included in this survey. Data collection will continue without interruption. I shall be grateful to any reader of this critical review who will inform me of his opinion on any subject, being as caustic as he likes. Private communications, memorandums, technical reports, and articles on reactor-noise time analysis and related techniques regarding both past experiments not quoted in this book and future activity will be greatly appreciated. For the sake of gathering information on this particular field, an accumulation point may be of general help to interested scientists. Please send at least one copy of all your works to me at

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INTRODUCTION

The quality and quantity of the growing number of theoretical studies and practical applications based on the observation of neutron fluctuations in a nuclear reactor emphasize the importance of this subject in reactor kinetics and dynamics.

In many laboratories around the world, groups of scientists work on reactor-noise analysis on a full-time schedule. Their contribution is clearly visible in the continually improving degree of sophistication of the theoretical treatments, in the highly automatized versions of experimental techniques and data processing codes, and in the number of special-purpose instruments now commercially available.

One of the most impressive facts concerning reactor-noise methods is the multiple differentiation and branching of the one or two pioneering procedures into several special techniques for the methodology, the data collection, and the interpretation of the results.

Today a bibliography on reactor-noise analysis is no longer an acceptable guide to the state of the art, since the quantity of published literature would make a neophyte approach to the subject extremely confusing. In order to partially avoid this confusion, we have restricted our presentation to the techniques related and applied to spontaneous fluctuations in the time domain and have disregarded the Fourier-conjugated methods that imply the observation and analysis of the same phenomena in the frequency domain. This review describes the evolution of the art treated in Chap. 6 of J. A. Thie, *Reactor Noise*, ANS-AEC Monograph, Rowman and Littlefield, Inc., New York, 1963.

Although a bibliography on reactor noise is adequate for theoretical reference sources, it is often short on practical indications for performing experiments or choosing the best formula for interpreting them. For this reason, we dedicate this book to the experimenters.

This review endeavors to synthesize and describe the state of the art at present by dividing the most-used techniques into four categories, each of which features substantial differences concerning either the measuring apparatus or the elaboration of data and the extraction of information on the reactor parameters.

The subject is approached in *medias res*: no general definitions or mathematical introductions to nuclear reactor kinetics, noise theory, and stochastic processes are given; these can be found in more nearly complete books. It would be worthwhile to read the fundamental *Physics of Nuclear Kinetics* by G. R. Keepin, *Reactor Noise* by J. A. Thie, and the vast comprehensive treatment in *Random and Pseudorandom Noise Techniques in Nuclear Systems* by R. E. Uhrig before encountering the specific details of the present survey. In addition, a familiarity with reactor operation and basic nuclear instrumentation is necessary.

The final formulas of each procedure are not derived but are simply described and interpreted according to an experimental perspective, since the main emphasis is given to the methodology, the efficiency, and the more or less difficult realization of the techniques.

An extensive chronological reference list is reported for each category to provide a sufficiently complete documentation of the theoretical background of the methods and of the most widely known results obtained in kinetic parameter measurements.

LIST OF SYMBOLS

A	Counting rate (counts/sec)
b(t)	Binary (0 or 1) variable
$B = \frac{eDK_p^2}{2\alpha t^2}$	Probability of detecting a correlated neutron pair per unit time interval at time $t = 0$
$C_{ii}(\tau)$	Autocorrelation function for a time lag τ
$C_{ij}(\tau)$	Cross-correlation function for a time lag τ
$\text{Cov}(\Delta t)$	Covariance of the number of events that occurred in the time interval Δt
$D = \frac{\langle \psi \rangle^2 - \langle \psi \rangle}{\langle \psi \rangle^2}$	Diven fission parameter
F	Fission rate (fissions/sec)
$f_k(\Delta t)$	Frequency of samples containing k events in a time interval Δt
G(s)	Probability distribution generating function
I	Amplitude of a spontaneous burst of neutrons
K	Effective multiplication factor
K_p	Prompt multiplication factor
N	Total number of samples

$N_k(\Delta t)$	Number of samples with content equal to k events in a time interval Δt
$N_{\text{even}}, N_{\text{odd}}$	Number of samples containing an even (or odd) number of events
$n(\Delta t), n$	Number of events in a time interval Δt
$n(t + j \Delta t),$ $n_j(\Delta t)$	Number of events that occurred in the time interval $t + (j - 1) \Delta t$ to $t + j \Delta t$
n_0	Amplitude of a spontaneous burst above the mean power level
$p(t) dt$	Probability that, after a pulse at time $t = 0$, the next pulse arrives between t and $t + dt$
$p_0(\Delta t), p_0(t)$	Probability of no counts in a time interval Δt or the probability that, after a time $t = 0$ chosen at random, the first pulse arrives any time after t
$p_k(\Delta t)$	Probability that a sample contains k events in a time interval Δt
$p_{\text{even}}, p_{\text{odd}}$	Probability that a sample contains an even (or odd) number of events
$P(t) dt$	Probability that, after a pulse at time $t = 0$, a pulse arrives between t and $t + dt$
P_1, P_2, P_3, P_4	Probability of detecting a $++$, $-+$, $--$, and $+-$ pair, respectively
$\text{Pol}(\tau)$	Polarity correlation function for a time lag τ
$R(r) = \frac{C_{hk}(r)}{C_{hk}(0)}$	Normalized correlation function ($h =$ or $\neq k$)
RR	Repetition rate of bursts of intensity I (bursts/sec)
S	Source strength
T	Total measurement time
$U(\Delta t)$	Ratio of the expected number of correlated neutron triplets to the expected number of random neutron triplets in a time interval Δt
$V(\Delta t)$	Relative variance of the number of events in the time interval Δt
$W(\Delta t), W_i$	Correlated relative variance of the difference between successive samples of duration Δt and its asymptotic prompt values ($i = 1$) and delayed values ($i = 2, 3, \dots, 7$)
$X(\Delta t), X_i$	Third-order correlation parameter and its asymptotic prompt value ($i = 1$) and delayed values ($i = 2, 3, \dots, 7$)
$Y(\Delta t), Y_i$	Correlated relative variance of the number of counts in a time interval Δt and its asymptotic prompt value ($i = 1$) and delayed values ($i = 2, 3, \dots, 7$)
$Z(\Delta t), Z_i$	Ratio of the expected number of correlated neutron pairs to the expected number of random neutron pairs in a time interval Δt and its prompt value ($i = 1$) and delayed values ($i = 2, 3, \dots, 7$) for a time interval $\Delta t = 0$

$\$ = R/\beta$	Reactivity (dollar units)
Σ	Summation symbol
Π	Product symbol
$\langle \rangle$	Expectation operator
α	Prompt-neutron-decay eigenvalue, (Rossi alpha)
α_i	Reactor-transfer-function poles (for $i = 1$ it coincides with α)
β	Effective total fraction of delayed neutrons
γ, γ'	Pal-Mogilner-Zolotukhin correlation parameters ($\equiv 1$ for purely random distribution)
Δt	Sampling time interval
ϵ	Detector efficiency (counts/fission)
l	Prompt-neutron lifetime
Λ	Prompt-neutron generation time
ν	Number of neutrons emitted per fission
$\pi(t) dt$	Probability that, after a time $t = 0$ chosen at random, the first pulse arrives between t and $t + dt$
$\rho = \frac{K - 1}{K}$	Reactivity
σ	Nondimensional source strength
s	Generating-function dummy variable
$\text{sign } x(t)$	Polarity (+ 1 or -1) variable
τ	Time lag
τ_f, τ_s	Time constants
t	Time
$\langle t_{op} \rangle$	Average time interval between a random origin and the first-arriving pulse
$\langle t_{pp} \rangle$	Average time interval between consecutive pulses

GENERAL CONSIDERATIONS ON REACTOR-NOISE TIME-ANALYSIS METHODS

The first fundamental assumption of each method is that the reactor is in a steady-state operating condition, i.e., critical or subcritical in equilibrium with a uniform source. This assumption may be dispensable for some procedures, but it represents a useful schematization.

The neutron population is observed through a suitable detector whose output is either integrated or simply counted in the time domain; the neutron energy is usually disregarded. The instrumentation for these measurements can be the common neutron-monitoring apparatus of the reactor (pulse counters and

current chambers) with the exception of those cases in which high detection efficiencies are needed and some in-core instrumentation must be added.

A common feature among the procedures is that all the techniques are based on observing the prompt-neutron chains that decay because of the prompt subcritical conditions. Their decay constants determine the order of magnitude of the times involved in such analysis; they are approximately

100 msec	for D_2O and graphite reactors
10 msec	for H_2O and organic reactors
1 msec	for thermal plutonium reactors
10 μ sec	for reflected fast reactors
1 μ sec	for bare fast reactors

Since all the techniques have a statistical derivation, the whole analysis is based, not on single events, i.e., time separation between two neutrons, but on observing and storing a large amount of data.

Two approaches are possible. One is strictly a time analysis based on the localization of the arriving neutrons on a time axis; the other is an integral type that consists in counting the number of neutrons detected in fixed time intervals and collecting the data as a time sequence, $n_j(t + j \Delta t)$, or as suitable reductions of it. The symbol n_j denotes the number of pulses that occurred (or the integrated output) in time interval $t + (j - 1)\Delta t$ to $t + j \Delta t$, where t is an instant in reactor life, Δt is the time interval adopted, and j is an integer ($j = 1, 2, 3, \dots, N$) characterizing the sequence.

Four different types of procedure accommodating 15 methods can be distinguished according to how these outputs are manipulated:

1. Methods based on time-correlation analysis among the pulses, in which the various correlation functions can be calculated off-line or approximated by a suitable arrangement of the measuring apparatus (delayed-coincidence system, time analyzer, or multiscaler with repetitive cycles). These include

- the Rossi-alpha method
- the endogenous pulsed source method
- the autocorrelation and cross correlation method
- the time interval from a random origin method
- the pulse to pulse time interval method

Although they belong to the time-analysis category, these last two techniques will be included in group 3 for the sake of a more continuous and logical sequence and for a better understanding of the presented information.

2. Methods based on analysis of the moments of the distribution $\{n_j\}$, in which a calculation limited to the second or third order is generally sufficient to give the desired information. These include

- the Feynman variance method
- the expected number of correlated-neutron-pairs method

- the Bennett variance method
- the third-order analysis and the expected number of correlated-neutron-triplets method
- the covariance method

3. Methods based on differential analysis of the distribution $\{n_j\}$, in which the experimental data are compared with some theoretical probability profiles to find the fitting parameters. These include

- the zero-count probability method
- the Polya-model method
- some intermediate methods (the zero-count to one-count probability-ratio method, the Furuhashi-Izumi probability method, the even-count or odd-count probability method)

4. Methods based on correlation analysis among the signs of the fluctuations of the instant or integrated counting rate around its mean value, in which the interpretation is in terms of the autocorrelation function for a one-detector output and in terms of cross correlation or covariance of the data for a two-detector output. These include

- the polarity autocorrelation and cross-correlation method
- the covariance method via polarity detection

All the described techniques determine the values of time-behavior parameters, e.g., prompt-neutron decay constants, as well as of static parameters, e.g., reactivity and power, and replace or extend the now obsolete original reactor-physics procedures that were developed with the first multiplying assemblies.

For a nuclear system, the prompt period at criticality, namely, the ratio of the prompt-neutron lifetime to the effective total fraction of delayed neutrons, is one of the most important parameters for comparison between theory and experiment since it allows basic data concerning cross sections, spectral distributions and discretization into groups, importance functions, etc., to be checked directly with measurement results. This homogeneous comparison between calculated and observed α offers valid information in addition to that gained from the critical experiments. For subcritical systems the parameter α is at least as expressive as reactivity, the difference being that α is observable whereas reactivity is not. This factor suggests a different approach to all the reactivity-related coefficient measurements, i.e., fuel elements, control rods, temperature, moderator level, which can be extended to highly subcritical systems unlike all other reactivity-based procedures.

All the mathematical developments are generally based on a bare monoenergetic diffusion-theory model; the prompt-neutron kinetics approximation is often adopted.

Such a simplified treatment does not imply an unawareness of or disregard for reactor-noise theoreticians and their more complex and sophisticated models. Obviously the validity of the methods as practical tools remains intact and effective. Problems of a more subtle interpretation of the results come after the possibility of measuring reactor parameters is verified.

The current encounters with reactor engineering and related exigencies make these reactor-noise techniques a challenging, and still not sufficiently diffused, field. They provide more confidence in what the macroscopic complexity of a nuclear system allows to be known for immediate use. The increasing research makes this review of the state of the art only an intermediate step toward the more extended information that still has to be communicated.

TIME-CORRELATION ANALYSIS AMONG THE NEUTRON-DETECTION PULSES

This part deals with methods of time analysis of the pulse sequence from a neutron detector placed in a reactor system. The main purpose of time-correlation analysis is to localize the arriving pulses on a time scale and to measure part of or all of the time intervals among two or more pulses. The various methods differ in the way such an analysis is triggered and information is collected.

The start may be randomly chosen at any moment of the reactor life or may be initiated by a neutron detection. After a time origin has been set this way, one or more events may be recorded in their time sequence after the start. They may occur in the same detector that generated the start pulse or in another one.

Either the very next pulse or all the pulses coming from the moment of the start pulse until the end of a sweep cycle might be of interest. Since this type of analysis must be repeated many times, the triggers might occur for every second (or third) pulse, for another randomly chosen trigger (e.g., the end of a sweep cycle), or with the first neutron pulse coming after the cycle has been completed.

The different time distributions of pulses, recorded according to the proposed approaches, are then interpreted in relation to the corresponding theory. The measuring apparatus can either give directly the final data to be mathematically treated or give a set of raw data which has to be processed before being fed into the final data-reduction routine.

ROSSI-ALPHA METHOD

The Rossi-alpha method, named for Bruno Rossi, who suggested the experiment during the early days of Los Alamos, is based on the observation of prompt-neutron decay, namely, the time distribution of counts following a certain triggering pulse due to randomly chosen first neutron detection.

Counts uncorrelated with the initial one are likely to occur any time after the triggering event and therefore have a constant probability of detection. Counts correlated with the initial one, i.e., neutrons belonging to the same fission chain, exhibit a time-decreasing probability of occurrence because of the subcritical prompt multiplication factor. Therefore an investigation of the time behavior of this detection probability yields a determination of one or more reactor kinetic characteristics, i.e., the prompt reactor period, the ratio of the effective total fraction of delayed neutrons to the prompt-neutron lifetime, the reactivity of the system, the power level, and so forth. This type of experiment can be used as a valid alternative for the pulsed-neutron measurements, mainly at criticality and in slightly subcritical conditions, where the former procedure becomes experimentally unreliable.

The technique is especially profitable when applied to fast systems in low-power operating conditions.

It can be extended, though with less convenience and efficiency, to thermal reactors if proper modifications are made (see Experimental Procedure). It cannot be used in non-zero-power reactors and becomes impractical when the spontaneous neutron source exceeds 0.2 neutron per prompt-neutron lifetime, i.e., in plutonium systems.

Mathematical Formulation

When $P(t) dt$ is the total probability of detecting a neutron in the interval t to $t + dt$, given that there has been a first detection at $t = 0$, then

$$P(t) dt = A dt + B e^{-\alpha t} dt \quad (1)$$

where $A = F\epsilon$ = detector counting rate

F = fissions per unit time

ϵ = detection efficiency (counts/fission)

$B = \epsilon D K_p^2 / 2\alpha l^2$ = probability of detecting a correlated neutron per unit time (at time $t = 0$)

$D = (\langle \omega^2 \rangle - \langle \omega \rangle^2) / \langle \omega \rangle^2$ = Diven fission parameter¹

K_p = prompt multiplication factor

α = Rossi alpha or reciprocal of the prompt period

l = prompt-neutron lifetime

ν = neutrons emitted per fission

The decay constant α can be written as

$$\alpha = \frac{1 - K_P}{l} = \frac{1}{l} \frac{\beta - \rho}{1 - \rho} = \frac{\beta}{\Lambda} (1 - \$) \quad (2)$$

where β = effective total fraction of delayed neutrons

ρ = reactivity

$\$$ = reactivity (dollar units)

Λ = prompt-neutron generation time

Equation 1 was derived by Orndoff² according to a heuristic approach to the problem but has been later confirmed by Matthes,⁶ Borgwaldt and Sanitz,⁹ Borgwaldt and Stegemann,¹⁷ Babala,²⁶ and Iijima³⁸ through rigorous mathematical treatments.

Bryce^{13,19} reached a conclusion in contrast with Eq. 1 for the correlated term B. He obtained

$$B' = \frac{\epsilon D}{2\Lambda} \quad (3)$$

The main feature of the equation is the independence from α and therefore reactivity, whereas the original B is inversely proportional to the degree of subcriticality.

Since both conclusions are supported by experimental investigations, the discrepancy remains unexplained,^{17,37} although there is a general trend toward greater confidence in the Orndoff formulation.

An important consideration applicable to both formulas is that the uncorrelated term depends on the power level (or source level in the case of subcritical measurements), whereas the correlated term does not. This implies that the smaller the power (source) level, the larger the correlated to uncorrelated ratio, and hence reactor parameters are better obtained.

Experimental Procedure

The original instrumentation employed by Orndoff² was a system of 10 coincidence circuits, whose operation can be clearly understood from Fig. 1. The reactor power level and the detection efficiency were set at appropriately low values in order to prevent significant overlapping of the neutron chains observed.

Since Orndoff's experiment, the Rossi-alpha designation has often been applied to several investigations more or less related to the original one. They could still be interpreted in terms of Eq. 1 if suitable modifications of practical and interpretative nature are applied to each case. Modifications can be made by the adoption of two detectors, by the replacement of the obsolete cabled

Fig. 1. Block diagram of the measuring apparatus of the Rossi-alpha method (delayed-coincidence system).

coincidence system with a multichannel time analyzer, and by the choice of triggering pulse.

The situation can be summarized by the following points:

1. The adoption of a separate input-2 from a second detector avoids some possible troubles due to the time-resolution requirements on a counting apparatus employing only one detector. This modification clearly illustrates that the measuring device actually clocks the time intervals between one pulse from detector 1 (trigger) and the pulses from detector 2 that come after the trigger pulse and within the time interval established by the sum of the coincidence delays. If no pulse arrives within this limit, an anticoincidence occurs in the last block of the apparatus. It is also evident that, if such a system is employed, more than one point can be collected in the time sequence after the trigger.

2. If the same two-detector system is counted by a stopwatch time analyzer (operating in the time-of-flight mode, i.e., time sweep triggered by a pulse and stopped by the following one), a more flexible and a higher resolution time analysis can be achieved, but some modification of the original theory is caused by the different instrumentation. The stopwatch procedure is a hybrid technique between the Rossi-alpha method and the pulse to pulse time interval method. In fast reactor experiments and low-detection efficiencies, the three of them coalesce. The subject is discussed in Part III. Details can also be found in Ref. 46. The stopwatch time-analyzer device works by sweeping a certain number of channels, namely, time intervals, after a start signal represented by the triggering pulse from detector 1. The first-arriving pulse from detector 2 after the trigger acts as a stop signal, adds a count to the channel being swept at that instant, and resets the sweep to the zero-channel, making it ready for a new triggering event from detector 1. This way only one point of the time sequence after the trigger can be recorded, and the interrupted sweeping causes the later channels to be examined less often than the early ones. Brunson et al.,³ who first adopted such measuring apparatus, found the proper correction to be made in the experimental data to account for these circumstances. Thus the probability of detecting a neutron in the n th channel is

$$P_n = \frac{c_n}{c_0 + \sum_{i=1}^N c_i}$$

where c_i is the number of counts in the i th channel, N is the total number of channels, and c_0 is the number of sweeps during which no event has been recorded.

3. The use of a multiscaler apparatus allows all channels to be examined on every sweep, but each recorded point may cause the analyzer to undergo 2 through 20 μsec^* of inhibition because of the memory storage cycle. When the

*This value is essentially dependent on the modernity of the instrumentation adopted. Recently, multiscalers working alternately on two scalers instead of one as data register feature a channel-advance dead time well below 1 μsec .

Rossi-alpha technique is applied to fast reactors, this limitation is serious, and the stopwatch approach is greatly preferred.

4. When the Rossi-alpha technique is applied to thermal reactors, the use of a second detector does not yield any substantial improvement, and generally only one detector is employed. Because of the slower decay of the neutron chains, a multiscaler apparatus can also be adopted with only minor corrections caused by the channel-to-channel dead-time effects (see Fig. 2). The sweep is triggered by

Fig. 2. Block diagram of the measuring apparatus of the Rossi-alpha method (time-analyzer or multiscaler system).

an arbitrary pulse, and the following pulses are recorded in the channels according to their time of arrival, only one per channel per sweep for a time analyzer (more than one for a multiscaler), until the entire sequence of channels has been completed. The first-arriving pulse after the sequence has been completed makes the system begin a new cycle. The analysis for this type of procedure is different from the original Orndoff experiment in which the pulse sequence after *each pulse* was analyzed in its time distribution with respect to the triggering one. The stopwatch technique includes this modification since the second pulse, assumed to come from the same detector instead of from detector 2, is used only to stop the watch, and no measurement of time interval is performed between this pulse and the third pulse, i.e., the trigger of the next start-stop sequence. If half of the time correlations are lost in fast reactor measurements, an even greater amount of them are lost in thermal reactor measurements unless a proper low detection efficiency is chosen.

Effects of this kind have been theoretically treated by Babala,²⁶ Borgwaldt,²⁵ and Iijima³⁸ and carefully checked by Edelmann,³⁷ who comparatively analyzed a tape-recorded sequence through a time analyzer and a simulation of the Orndoff apparatus for different detection efficiencies to observe the substantial discrepancies that occurred in different analyses of the same phenomena. He noticed that, as the triggering-detector efficiency decreases, the time-analyzer experiment and the delayed-coincidence experiment tend to coincide.

5. An interesting addendum to the experimental procedure has been made by Mihalcz²⁷ for his Rossi-alpha measurements in fast assemblies. He employed

the two-detector stopwatch technique in such a way that pulses from detector 2 were delayed with respect to those from detector 1. In this way the events in detector 2 that occurred before the triggering pulse from detector 1 could be observed as well, and the symmetrical counterpart of the time cross-correlation function (see cross-correlation method, p. 19) of the two counting sequences could also be measured. The complete time distribution is expected to be

$$P(t') dt' = A dt' + B e^{\alpha t'} dt' \quad (4)$$

where $t' = t - t_d \leq 0$, t_d is the delay time, and the coefficients A and B are the same as in Eq. 1. A double measurement of α can therefore be performed with a single run.

6. Other sources of systematic errors in the process may include the following: (1) the experimental data do not follow the law expressed by Eq. 1 if the time-analyzer channel width is not very short with respect to $1/\alpha$ and (2) account must be taken of the delay between the triggering event and the actual start of the sweep, which is defined by the first-arriving pulse from an external (or internal) clock oscillator. This delay is obviously variable since there is no correlation between detected neutrons and clock pulses, and it is particularly serious for stopwatch measurements in fast systems.

These effects have been theoretically analyzed by Cohn,¹⁸ who also suggested the appropriate experimental conditions to circumvent these difficulties.

Interpretation of Data

The experimental data are processed through a least-squares fit to Eq. 1. Although the determination of α constitutes the main purpose of this measurement, some information about static parameters can be obtained by evaluating the coefficients A and B from the same fit. Obviously a meaningful interpretation of them in terms of reactor kinetic quantities is seriously affected by the previously discussed discrepancy concerning the correct formulation of the term B .

The versions of the Rossi-alpha technique described have been applied to measure the prompt-neutron time-decay eigenvalue in several types of reactors. The measurements concerned bare and reflected fast uranium and plutonium assemblies, e.g., Orndoff,² Brunson et al.,^{3,10} Karam,^{1,2,1,4,3} Mihalczko,^{1,2,27} Weale et al.,^{1,4} Bergstrom et al.,¹⁵ as well as thermal reactors with different reflectors and moderators, e.g., Starr and Price,⁵ Stribel,⁸ Bryce,¹³ Edelmann et al.,²² and Suwalski.²³

The interpretation of the experimental data includes a number of controversial topics that continue to captivate the attention of the experimenters in the several independent measurements related to the Rossi-alpha technique. The

validity of the involved assumptions and the degree of sophistication of the mathematical background needed to interpret the data are still often discussed.

The list of controversial topics can begin with the experiments on thermal reactors.

Generally, the processed data are well represented by Eq. 1, but in some cases the presence of a second exponential and therefore a time behavior of the type

$$P(t) dt = A dt + B e^{-\alpha t} dt + C e^{-\alpha' t} dt \quad (5)$$

has to be assumed to explain a time distribution of the counted pulses that differs slightly from Eq. 1.

Since α' is usually much greater than α , it is more direct to relate this faster time-decay to neutrons that have spent part of their lifetime in the reflector; however, no satisfactory results have been reached for reflector constants unless pulsed experiments were performed and interpreted through a formulation structurally similar to Eq. 5. Suwalski²³ describes in detail the problems in fitting the experimental data to Eq. 5.

The channel widths used in thermal reactor measurements are too large to sufficiently resolve the presence of the second exponential, whose existence is a minor disturbance in the first few channels. On the other hand, shorter gate widths would make the accumulation of data, already extremely time consuming in thermal reactor experiments, prohibitive in length and doubtful in effectiveness.

The second-exponential phenomenon is more evident in fast reactors as the observations by Brunson et al.¹⁰ and Karam^{11,21,43} testify. In these cases the second time-decay eigenvalue α' can be greater or even smaller than the first one. The first situation occurs when depleted uranium or metal reflectors surround the multiplying assembly, whereas the second type may occur if reflectors containing moderating elements, e.g., beryllium, are used. The latter case implies further problems, which are treated by Cohn,⁷ on the physical meaning of these eigenvalues.

Although the previously discussed problem of accuracy regarding thermal reactors arises even in the case of fast systems, the presence of at least one additional time-decay constant seems to be in better evidence. Karam²¹ appropriately focuses on the spurious physical meaning of this eigenvalue, which is probably an averaged approximation of a multimode-behavior spectrum of constants. Kistner,¹⁶ Otsuka and Iijima,²⁰ and Borgwaldt²⁸ have written theoretical papers on the matter.

If the eigenvalues α and α' are space independent in several experiments, the amplitudes A, B, and C are substantially affected by the separation of the two detectors, by their efficiency near or inside the reactor, and by the experimental techniques used. For example, stopwatch and time-analyzer-multiscaler procedures neglect part of the correlation among neutron pulses and therefore

underestimate B and C. Although the value of α and of α' is not biased by such a modified approach, B and C may be so damped that the observed time behavior is fairly constant, and α and α' are no longer deducible.

Space effects have been observed which contrast with some apparently logical but not empirically proven statements. These space effects can be summarized in two points:

1. It is not true that the closer the two detectors are placed, the higher the probability of detecting correlated events. In fact, the position of the terminating counter plays the fundamental role, and this probability is therefore proportional to the flux in which the terminating detector is placed. For a single-detector experiment also, this probability is proportional to the flux where the detector is located.

2. The ratio C/B increases as the distance between the two detectors increases; this effect is not necessarily connected with the fact that one of the two detectors is placed closer to the reflector.

A critical measurement of α gives the ratio β/Λ ; if the Rossi-alpha experiments are interpreted in terms of reactivity, additional determinations of α in various subcritical configurations are commonly suggested together with the use of Eq. 2. In fact, the extrapolation of α to zero gives the value of the dollar unity. Such a procedure allows the critical to prompt-critical interval to be determined although only subcritical measurements have been made. However, Eq. 2 has failed in fast reflected assemblies. The number of failures is significant as well as their common characteristics.

Brunson et al.¹⁰ measured α for various subcritical reactivities previously determined by the conventional period technique. They noticed that the values of α fell below those predicted by the linear relation indicated by Eq. 2. As a result the extrapolated value of the dollar was consistently larger than that used to calibrate the reactivity scale, usually derived from stable period measurements.

Weale et al.¹⁴ found in their experiments that, although the relation between α and reactivity can be well represented by straight lines, the observed values do not extrapolate to the expected value of β on the reactivity scale. An excess ranging from 20 to 50% was evident. Several explanations were proposed by the authors.

Bergstrom et al.¹⁵ tried to interpret the same observed discrepancy through a new definition and calculation of the prompt-neutron lifetime derived from the two-point kinetic model proposed by Cohn.⁷ They reached a better correspondence between theory and experiment, although they asserted that a completely successful comparison certainly needed a more sophisticated theoretical treatment of the problem.

As a recent conversation with Brunson³⁹ on the state of the art testifies, no satisfactory resolution of the enigma (briefly synthesized as "the dollar paradox") has yet been achieved. This situation creates serious doubt and uncertainty about the interpretation of results obtained from a technique that is

currently one of the most widely used methods in reactor physics laboratories for measuring kinetic parameters.

"ENDOGENOUS PULSED SOURCE" METHOD

The method discussed in this section is based on observation of the time distribution of counts following a certain triggering pulse that occurs when the spontaneous fluctuations of the reactor power reach a preselected value above the mean level.

Since these endogenous neutron fluctuations can be described in terms of an equivalent pulsed source with variable intensity and repetition rate, the technique has been named the "endogenous pulsed source" method (eps).

If the appearance of spontaneous bursts can be considered essentially random and due to sudden fission-rate variations, the proliferation and the decay of the bursts from the attained peak occur according to the reactor kinetic equations and therefore must be characterized by the fundamental decay eigenvalue α .

The eps technique can be considered intermediate between the purely statistical Rossi-alpha experiment and the classical pulsed-source technique, showing similarities to both of them in the experimental procedure and in the interpretation of data and sharing part of the advantages of both.

The basic improvement, due to the preselection device, appears essentially in the increased signal-to-background ratio since only decay chains of conspicuous amplitude are time analyzed and therefore constant or almost constant runs are avoided.

The eps technique allows statistical experiments to be performed in thermal reactors for reasonably acceptable measurement times whereas the conventional Rossi-alpha method is widely known to be prohibitively time consuming in such systems. The eps experiment is also practical compared with pulsed-neutron measurements since the expensive apparatus of the pulsed source is no longer necessary. Furthermore, setting up the experiment and managing the reactor are greatly simplified without the presence of the commonly bulky pulsed source. However, the latter advantages are features of most neutron-noise-analysis experiments.

The technique can possibly be extended with similar benefits and improvements to fast reactors by applying a suitable preselection device to the triggering detector used for Rossi-alpha measurements.

Mathematical Formulation

If we define the neutron density as $n(t)$, we can statistically describe the time behavior of this quantity after each spontaneous burst as

$$n(t) = n_0 e^{-\alpha t} + \langle n \rangle \quad (6)$$

The quantity $\langle n \rangle$ is the mean value of counts per channel width Δt and represents the constant background of all the bursts; it can be expressed in two ways according to reactor-operating conditions as

$$\langle n \rangle = F \epsilon \Delta t \quad (7)$$

for a critical reactor, and

$$\langle n \rangle = \frac{S \epsilon \Delta t}{\langle \phi \rangle (1 - K)} \quad (8)$$

for a subcritical reactor in equilibrium with a uniform source, where S is the effective source strength and K is the multiplication factor.

The amplitude of the burst above the mean level of counts is given by

$$n_0 = (M - 1) \langle n \rangle \quad (9)$$

where $M - 1$ is the signal-to-background ratio, whose magnitude is set by a suitable device (see Experimental Procedure).

A closer analysis of the phenomenon concerning these positive bursts and a suitable interpretation of Eq. 6 lead to the conclusion that the spontaneous neutron fluctuations are equivalent to a pulsed source with the following pulse-intensity and repetition-rate characteristics:

$$I \geq (M - 1) \langle n \rangle \quad (10)$$

$$RR = \frac{\sum_{k=0}^{\infty} p_k}{\Delta t} \quad (11)$$

The quantity p_k is the probability of counting k pulses in a time interval Δt if $\langle n \rangle$ is the mean value of counts; its explicit formulation will be described in Part III, Analysis of the Probability of Neutron Detection.

Values of I and RR calculated by Pacilio,^{33,44} together with a general formula to compute the time necessary to collect a given number of burst decays in such an experiment, are given in Refs. 33 and 44. The values show the noteworthy improvement in measurement duration and statistical accuracy with respect to the traditional Rossi-alpha technique in the case of thermal reactor experiments. A high detection efficiency has been assumed, which is likely to be obtained only through in-core instrumentation unless small systems are under observation.

Experimental Procedure

The eps method has been independently applied by Chwaszczewski et al.,^{32,34} Pacilio,^{33,36,44} Borgwaldt,³⁵ and Edelman,⁴⁵ who each engaged in different experimental approaches but looked for the same effects.

It has previously been emphasized that the improvement introduced by the eps technique is essentially due to a more meaningful selection of the decays to be analyzed. Thus the preselection and trigger device constitutes the most important part of the whole measuring apparatus, whose other components are identical with those employed in the traditional Rossi-alpha experiment with a single detector and a multiscaler that can record a practically unlimited number of pulses per channel per sweep. Figure 3 shows the relation of the burst monitor to the measuring apparatus.

Fig. 3. Block diagram of the measuring apparatus of the endogenous pulsed source method.

Four different types of burst monitors have been used:

1. **Rate Meter.** The output of the neutron-counting chain is fed into a fast counting rate meter that samples the neutron population. The instrument is adjusted to give an output signal as soon as a certain preset value of counting is reached. The threshold adopted must be changed according to the various reactor power levels and the efficiency obtainable for a given position of the detector (see Ref. 44).

2. **Compensated Rate Meter Pair.** Two counting rate meters are employed with highly different time constants. If no burst is present, the output from each of them compensates the other; the occurrence of a burst is clearly detected by the fast rate meter and ignored by the slow one so that the uncompensated

signal can trigger the time analyzer. The time constants must satisfy the inequality

$$\tau_f < 1/\alpha < \tau_s$$

which is a stringent condition for a profitable burst selection. The quantities τ_f and τ_s are the time constants of the fast (f) and slow (s) rate meter.^{32,34}

3. Digital Preselector. The output of the neutron-counting chain is digitally counted on a time basis equal to the channel width Δt of the time analyzer and reset to zero after each sampling. Since $\langle n \rangle$ can be measured by a separate scale, the preselector threshold can be set on the desired $(M - 1)\langle n \rangle$ value so that it will give an output pulse every time a count equal to or greater than $(M - 1)\langle n \rangle$ is recorded. The output pulse from the preselector triggers the multiscaler (see Refs. 33 and 44).

4. Coincidence System. A pulse from the detector opens a coincidence gate for a certain duration depending on the counting frequency selected. If a second pulse comes within this time interval, another coincidence gate is opened, to await a third pulse. The arrival of this third pulse within the fixed gate makes the coincidence circuit give an output that triggers the multiscaler. The number of pulses (three in this case) divided by the entire duration of the opened coincidence gates (two in this case) gives the minimum repetition rate selected. Additional coincidence stages can be used to obtain a more selective choice of bursts (see Refs. 35 and 44).

Other devices could be applied to the described burst-monitoring function. In general, two fundamental features must be present: (1) the minimum information required to decide whether or not a burst is rising and (2) a fast response and triggering of the multiscaler if a burst is rising.

Both characteristics are desired to obtain the maximum signal-to-background ratio, i.e., an analysis of each sufficiently great burst from the very peak of it. This analysis is difficult to obtain because the first multiscaler channel has a meaningless content since its arbitrary width can range from 0 to Δt . In fact, the data collection starts immediately after the trigger pulse, and in most instruments the pulse has no correlation with the clock pulse train that acts as a channel-commutation switch.

The use of a delay device can be suggested to obtain a data collection analogous to that of pulsed experiments, i.e., preburst background, rise, peak, and decay. If thermal (or fast) reactor experiments are planned, the delay must be of the order of milliseconds (or microseconds).

Experimental Data and Results

The experimental data are fit to Eq. 6. Generally, only the value of α is interpreted in terms of reactor parameters, mainly to determine the reactivity of the system analyzed.

Chwaszczewski et al.^{32,34} have performed measurements of this type at four negative reactivities in a thermal system. The results show satisfactory agreement with those obtained through pulsed-neutron experiments under the same conditions.

Pacilio^{33,36,44} has measured α at criticality and α at negative reactivities in the interval from 0 to \$13 for a thermal reactor. Furthermore, he has investigated the perturbation of α due to the preset signal-to-background ratio. The results have shown good agreement between the eps method and pulsed measurements over the whole range of reactivities, whereas the serious limitations of some of the static techniques, particularly inverse multiplication, are clearly emphasized by comparison.

The results of the eps measurements by Borgwaldt³⁵ and Edelmann⁴⁵ on a fast-thermal system are as yet unpublished.

AUTOCORRELATION AND CROSS-CORRELATION FUNCTIONS OF THE NEUTRON COUNTING RATE

As stated in the introduction, a time-behavior analysis of a nuclear system can be achieved by recording the counting rates $n_j(t + j \Delta t)$ ordered in their time sequence according to the increasing values of the integer j .

Assume in particular that the quantities $n(t)$ and $n(t + \tau)$ are under observation for one or more values of the lag time, τ . If the phenomena involved are random and stationary, the average product over the whole ensemble of pairs $n(t)$ and $n(t + \tau)$, namely, $\langle n(t) n(t + \tau) \rangle$, must clearly be identical for any value of τ .

Conversely, if a time evolution takes place in the system, this product contains information about the behavior of the system from time t to time $t + \tau$ and therefore assumes a different magnitude according to the value of τ involved.

An experimental procedure based on this approach is one of the many tools under the name of *methods of correlation analysis*; these methods have been applied to various systems to measure their dynamic characteristics.

When applied to nuclear reactors, the general formulation of the correlation analysis problem sets a fundamental theoretical basis for all the types of experiments described in this book, although some of them were originally proposed and interpreted on the basis of more heuristic treatments of the specific problems.

The conclusions deducible from this more general approach confirm the validity of the early theory and experiments and furthermore allow other techniques to be profitably applied to nuclear reactors for kinetic and dynamic parameter measurements.

The presentation is limited here to the most ad litteram and immediate application of these techniques, i.e., the computation of the previously introduced average product of two time-shifted counting rates, also called the autocorrelation or cross-correlation functions.

The methods described in the other sections of the book start from the same initial step but describe more developed and sophisticated procedures.

Mathematical Formulation

The correlation function $C(t_1, t_2)$ of the counting rates $n_i(t)$ and $n_j(t)$ is defined as the expected value of the product of the counting rate at times t_1 and t_2 , or

$$C(t_1, t_2) = \langle n_i(t_1) n_j(t_2) \rangle$$

Generally, the correlation function is dependent on the lag $\tau = t_2 - t_1$ between the two counting operations so that

$$C(\tau) = \langle n_i(t) n_j(t + \tau) \rangle$$

The autocorrelation function implies the use of a single detector ($i \equiv j$); the cross-correlation function is based on the use of at least two detectors ($i \neq j$).

The general form of $C_{ij}(\tau)$ in the case of a neutron detector placed inside or near a nuclear assembly has been calculated by many authors, e.g., Velez,⁴ Matthes,⁶ and Borgwaldt and Stegemann.^{1,7} It is

$$C_{ii}(\tau) = A(A + B e^{-\alpha \tau}) \quad (\tau > 0^+) \quad (12)$$

where A and B are the same constants as those used in Eq. 1.

$$C_{ij}(\tau) = A_i(A_j + B_j e^{-\alpha \tau}) \quad (13)$$

where the subscripts i and j refer to the corresponding detector efficiencies.

A comparison of Eqs. 1, 12, and 13 shows a substantial identity other than the proportionality factor A (or A_i) which normalizes the corresponding physical quantity differently. The similarity between the two final equations reflects essentially identical phenomenology and analogous data processing although different experimental apparatus and reduction of data are apparently employed. The similarity can be explained by a closer examination of the correlation algorithm.

Assume that the string of data in Table 1 has been recorded: the symbol 0 means no count, and the symbol 1 indicates the presence of one count in the channel. Forty channels of width Δt have been explored, and their binary content can be denoted by $b(t + j \Delta t)$ with $j = 0, 1, 2, \dots, N - 1$ ($\equiv 39$).

The autocorrelation function implies the computation of the following algorithm:

$$C_{ii}(k \Delta t) = \frac{1}{N - k} \sum_{j=1}^{N-k} b[t + j \Delta t] b[t + (j + k) \Delta t] \quad (14)$$

This operation is equivalent to comparing the original string of data with an identical string shifted back a number of intervals equal to $k = 1, 2, \dots$, i.e., the string A, B, C, ..., respectively, and to counting the number of channels with a simultaneous one-pulse content on both strings.

The number obtained is the quantity $C_{ii}^*(k \Delta t) = NC_{ii}(k \Delta t)$; its calculation is explicitly performed in Table 1, where an asterisk denotes the terms giving a nonvanishing contribution to $C_{ii}^*(k \Delta t)$ for each string.

The described algorithm is physically equivalent to counting the number of delayed coincidences between the two strings for a delay time $k\Delta t$ implied in the analyzed string and the adopted channel width Δt . It is also equivalent to determining the time interval between every pulse of the original string and all the following pulses and to counting the number of pairs separated by a time interval t .

This type of analysis is performed by the Orndoff² delayed-coincidence system; in fact, it measures the autocorrelation function of the neutron-counting rate simultaneously for 10 successive delay times $\Delta t, 2\Delta t, \dots, 10\Delta t$.

The discussion clarifies how the original physical meaning of the Rossi-alpha experiment (actually a real correlation-analysis procedure) is distorted when time analyzers are used as measuring devices and how much information on the time sequence of pulses is lost because of the adoption of this discontinuous logic.

The validity of the physical equivalence between the two techniques holds even for strings of data with more than one pulse per channel and can be easily extended to a cross-correlation experiment if two detectors of different efficiency are assumed in Eqs. 12 and 13.

Theoretical treatments of this technique have been outlined by Borgwaldt and Stegemann¹⁷ and Stegemann.³⁰ Working with a two-detector experiment, Sheff²⁴ developed interesting considerations about spatial effects which are particularly useful for cases in which the one-point approximation is no longer valid.

Experimental Procedure and Interpretation of Data

Although the autocorrelation and cross-correlation methods are the most direct and easily understood, the practical application of such experiments requires an off-line, a longer, and a more elaborate treatment of the collected data than does the classical Rossi-alpha experiment whose output is already in its definitive form.

Table I

INTERPRETATION OF THE AUTOCORRELATION ALGORITHM APPLIED TO A GIVEN STRING OF DATA

Lag	Neutrons detected
Original string of data	1 1 1 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 1 0 0 1 0 0 0 0 0 0
Shifted data:	
String A, $\tau = \Delta t$	1 1 1 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 1 0 0 1 0 0 0 0 0 0 * * * *
String B, $\tau = 2 \Delta t$	1 1 1 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 1 0 0 1 0 0 0 0 0 0 * * *
String C, $\tau = 3 \Delta t$	1 1 1 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 1 0 0 1 0 0 0 0 0 0 * * *
String D, $\tau = 4 \Delta t$	1 1 1 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 1 0 0 1 0 0 0 0 0 0 * *
String E, $\tau = 5 \Delta t$	1 1 1 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 1 0 0 1 0 0 0 0 0 0 *
String F, $\tau = 6 \Delta t$	1 1 1 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 0 1 0 0 0 0 0 0 1 1 1 0 0 1 0 0 0 0 0 0 *

*The asterisk denotes the simultaneous presence of a pulse in the original data and in the analyzed data. If the number of asterisks is counted for each string of data, then

$$C_{11}^*(\Delta t) = 6 \quad C_{11}^*(4 \Delta t) = 1$$

$$C_{11}^*(2 \Delta t) = 4 \quad C_{11}^*(5 \Delta t) = 1$$

$$C_{11}^*(3 \Delta t) = 3 \quad C_{11}^*(6 \Delta t) = 1$$

which is proportional to the autocorrelation-function dependence on the lag τ .

The successive steps of the experimental procedure are summarized as follows:

1. Read a large number of points of neutron detection vs. time dependence in the form of the string of data in Table 1.
2. Duplicate these data for the autocorrelation function computation (in the cross-correlation analysis, the second string is obtained from reading the output of the second detector).
3. Calculate the shifted product and sum as indicated in Eq. 14 for several values of τ .
4. Fit the obtained data according to Eqs. 12 and 13 to determine the numerical values of A, B, and α .

Applications of this technique have been made in critical and subcritical thermal assemblies by Nomura et al.⁴² using cross-correlation analysis and by Dragt³¹ using autocorrelation analysis.

The experiment by Nomura et al.⁴² is particularly interesting because it describes how the cross-correlation technique gives significant results even in the case of low detection efficiencies ϵ_1 and ϵ_2 and therefore allows measurements to be profitably realized without the necessity of special in-core instrumentation.

The advantages of the two-detector technique can be extended to other experiments in the time domain as well as in the frequency domain and can be one of the most important developments toward applying neutron-noise analysis to all kinds of nuclear assemblies without any limitation concerning the neutron spectrum and the reactor power level.

Further details of this general application of noise techniques are given in Part IV, where the polarity correlation techniques are presented and discussed.

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II ANALYSIS OF THE MOMENTS OF NEUTRON DISTRIBUTION

The general theory of inherent fluctuations of the neutron population in a reactor operating in a steady state is treated extensively in the material listed in the reference section. It provides a valuable mathematical background for understanding the phenomenology involved in observing the birth and death process of neutrons in nuclear systems from the viewpoint of the stochastic nature of these events.

Furthermore, these references suggest the proper conditions for setting up experimental procedures and extracting information about the macroscopic reactor kinetic parameters.

A noteworthy fraction of these works listed in the reference section emphasizes the theoretical aspects of the problem and consequently cannot be quoted with immediate reference to the essentially pragmatic presentation of this review.

An essential list for theoretical reference can be found in the reference section, especially items 1 to 18.

FEYNMAN VARIANCE METHOD

The Feynman variance method, named for Richard P. Feynman, who suggested the theoretical basis during the early days of Los Alamos, is based on

integrating the output* of a neutron detector placed inside or near a nuclear reactor for a time interval Δt .

The sampling of the neutron population under steady-state operating conditions must be repeated a large number of times to obtain a suitable ensemble $\{n_j(\Delta t)\}$ of data to use for calculating the moments of the neutron distribution. In the Feynman variance procedure, the computation of only the first- and second-order moments is necessary.

A particularly significant parameter of this distribution is the relative variance, defined as the ratio of the absolute variance (i.e., the square of the standard deviation) to the mean value. This quantity is unity for a purely Poissonian process; in nuclear reactor experiments it is greater than unity because of the correlation existing among neutrons belonging to the same decay chain.

The excess term of the relative variance of counts depends on the reactor kinetics and the characteristics of the detection equipment as well as on the time interval Δt employed for sampling the neutron population. An experimental observation of the dependence of the variance on Δt leads to the determination of the required reactor parameters.

The Feynman variance technique can be profitably applied to thermal systems but suffers from prohibitive limitations in all cases involving low detection efficiencies, $\epsilon \ll (1 - K_p)^2$, and therefore is practically useless for fast reactor experiments. It offers, however, a valid alternative to the conventional Rossi-alpha technique if applied to thermal reactors since it can be fast and precise under the proper conditions.

Mathematical Formulation

If n is the value obtained from counting a neutron detector placed inside or near a reactor, then

$$V(\Delta t) = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle} = 1 + \sum_1^7 Y_i \left(1 - \frac{1 - e^{a_i \Delta t}}{|\alpha_i| \Delta t} \right) \quad (15)$$

where V = relative variance

$\langle n \rangle, \langle n^2 \rangle$ = first- and second-order moment of the distribution of counts

Δt = sampling interval

*This terminology is used instead of "counting a neutron detector" for the sake of generality. The extension of all the pulse-counter methods treated in this section to the corresponding current-chamber methods is straightforward since it only implies a proportionality factor in the counting rates. This factor is essentially due to the electrical charge transferred per neutron detected.

The details of the current-chamber technique are extensively discussed under the Bennett variance method in Part II since Bennett originally employed this technique in his modified variance experiments.

α_i = neutron-population-decay eigenvalues ($i = 1$ for the prompt part; $i = 2, 3, \dots, 7$ for the delayed part)

Y_i = asymptotic value of the relative variance of the i th group, essentially dependent on detector efficiency and reactivity (i th group decay eigenvalue)

Some of these parameters have other properties and meanings, namely,

$$0 < Y_1 < Y_2 < Y_3 < \dots \quad (16)$$

$$Y_1 = \frac{\epsilon D K_p^2}{(1 - K_p)^2} \left(\sim \frac{\epsilon D}{\beta^2} \text{ at criticality} \right) \quad (17)$$

$$|\alpha_1| = \alpha \quad (18)$$

$$\alpha \gg |\alpha_2| > |\alpha_3| > \dots \quad (19)$$

Equation 15 has been derived by Feynman¹⁹ and de Hoffman²⁰ for only the prompt-neutron ($i = 1$) contribution to the variance and has been extended by Bennett²⁹ to include the effects of delayed neutrons.

Matthes,³⁰ Albrecht,³¹ Johnson,³⁴ Borgvaldt and Stegemann,⁴⁵ and Babala⁴⁷ have also derived Eq. 15.

A discussion of the physical meaning of Eq. 15 will help outline its various applications, based essentially on the range of values chosen for the time interval Δt and the accuracy obtainable from each different procedure.

Recalling the distinction between the prompt- and delayed-neutron contributions to the correlated relative variance, we notice the following characteristics.

1. Until the condition $|\alpha_2| \Delta t \ll 1$ holds, Eq. 15 can be regarded as equivalent to

$$V(\Delta t) = 1 + Y_1 \left(1 - \frac{1 - e^{-\alpha \Delta t}}{\alpha \Delta t} \right) = 1 + Y \quad (20)$$

because of Eqs. 17, 18, and 19. This is commonly called prompt-variance analysis since the delayed neutrons affect the variance only in terms of random contribution.

2. The closer to criticality the measurements are made, the stronger the effect of delayed neutrons manifests itself in Eq. 15.

3. As soon as $\Delta t \sim 1/|\alpha_2|$, the delayed-neutron terms have more effect on variance in comparison with the prompt terms.

Although it is evident from Eq. 20 that a good determination of Y can be achieved if a high-sensitivity counter is located in a high-efficiency position inside the reactor, it is less evident that the accuracy of the value of Y is strongly dependent on the number N of elements of the ensemble $\{n_j(\Delta t)\}$ used to

calculate the moments of the distribution. In fact, the quantity N does not appear explicitly in Eqs. 15 and 20 since they only express the expected variance. Harris²⁵ first faced this problem and made a number of significant tests to study the characteristics of the data ensemble and of their relative variance. Pacilio^{41,49} showed that, if the correlation between successive samplings is neglected, the relative standard deviation on Y is

$$f = \frac{\partial Y}{Y} = \left\{ \frac{1}{N} \left[\left(4 + \frac{3}{\langle n \rangle} \right) \frac{1}{Y} + \left(2 + \frac{3}{\langle n \rangle} \right) + \frac{Y}{\langle n \rangle} \right] \right\}^{\frac{1}{2}} \quad (21)$$

and it can be used to establish a compromise between the rapidity and the precision of the entire procedure.

A combined parametric study of Eqs. 15 and 21 allows the following statements to be made:

1. The ensemble $\{n_j(\Delta t)\}$ does not need to be ordered according to the time sequence of its detection process; thus the ensemble $\{n_j(\Delta t)\}$ can be suitably transformed into the frequency distribution $\{N_k\}$ where N_k is the number of samples with content equal to $k = 0, 1, 2, \dots$ pulses and

$$\sum_0^{\infty} N_k = N$$

As shown under Experimental Procedure, this consideration leads to faster and more effective data collection.

2. The dependence of Y on α is sensitive in the region $\alpha \Delta t < 1$; then it tends to vanish for increasing Δt . If α is measured, limitation in the time interval is suitable, which still allows a determination of Y , without any asymptotic prompt-variance measurement (i.e., for $\alpha \Delta t \gg 1$).

3. The accuracy f depends very weakly on $\langle n \rangle$ but strongly on N ; this suggests, together with point (2), that a large number of short samplings leads to a much better accuracy than a small number of long samplings.

4. Equation 15 indicates that at criticality

$$\lim_{\alpha \gamma \rightarrow 0} V(\Delta t) = \infty$$

for any value of Δt . This circumstance, which reflects the possibility that the power level of the critical assembly might increase without any upper limit, is not likely to occur in practice since the flux is always controlled and kept finite by a suitable feedback operation. The mathematical question of the divergence of $V(\Delta t)$ is discussed and clarified by Dragt.⁴⁶

Experimental Procedure

Unlike the Rossi-alpha experiment, where the early coincidence system is also the only one that correctly measures the time correlation among the

neutron pulses despite the flexibility and the better performances of modern instrumentation, the efficiency of the Feynman variance experiment has been greatly improved by the adoption of new equipment and by some changes in the measuring philosophy.

At present, at least four different types of data collection can be considered. Three of them have been used in several experiments without any chronological trend of preference, and the fourth one, still untried, combines some of the advantages of the other three.

As illustrated in Fig. 4, the differences are in the final stage of the measuring apparatus since the neutron detector and the preamplifier—amplifier—discriminator block are obviously indispensable in all cases. The various

Fig. 4. Block diagram of the measuring apparatus of the Feynman variance method. A different type of data collection is considered in each of lines 1, 2, 3, and 4.

possibilities are discussed in the order corresponding to the number of the connection shown in Fig. 4.

1. Gated-scaler method. The output pulses are counted on a fixed time basis Δt and stored in the sequence form $\{n_j(\Delta t)\}$, where $j = 1, 2, \dots, N$ denotes the progressive order of collection. In the measurements by Feynman et al.,²² McCulloch,²⁶ Kurusyna,³³ and Lindeman and Ruby⁵⁴ only one value of Δt has been adopted.

Other authors, Luckow and Churchill,^{23,24} Tachon,²⁷ Clouet d'Orval et al.,²⁸ Pal et al.,³⁶ Gotoh,³⁹ and Bergström et al.,⁵⁷ have repeated the measuring procedure for several values of Δt to observe the complete dependence of the variance of counts on the gate width.

In the early experiments the gated scaler was followed by a printer so that the content of each gate analyzed could be written down. This operation, which interrupts the data collection process since the two functions cannot be performed simultaneously, can now be done in about 10 msec (compared with the approximately 1 sec of the previous devices), which still represents a dead interval between two consecutive observations.

A multiscaler with the number of channels reaching 4000 is often used today for this operation. When the sweep has covered the entire number of channels, the data content is printed or punched out. The time needed for this purpose is clearly subtracted from the reactor-fluctuation observation.

If the fundamental role of the number of gates, N , and its required order of magnitude (at least $\geq 10^5$ for the values of Δt in the millisecond range) are recalled in regard to the efficiency and the accuracy of the variance-method experiment, it is evident that the writing time seriously affects the measurement duration.

The basic criticism of the gated-scaler method of data collection is that the time sequence of the gates, namely, the order indicated by the integer j , is *not needed at all* for the calculation of the moments of the neutron distribution. It is not worthwhile to waste measurement time to record the ordered sequence of data, since a more compact treatment of the data and a cumulative output allow further off-line calculations to be even simpler and less time consuming.

Such advantages are available with the following apparatus.

2. Distributive method. The output pulses are recorded in distribution form, i.e., according to the frequency distribution $\{N_k(\Delta t)\}$ of the gates, where N_k indicates the number of gates with content equal to $k = 0, 1, 2, \dots$ pulses.

The instrumentation can consist of a multichannel time analyzer as suggested by Farinelli⁴⁰ or a fast-acting electronic step switch as proposed by Stegemann.⁴⁵ The two devices are practically equivalent both in logic and in operation.

The output pulses are fed into the address register (channel-commutation switch), which proceeds one step for each arriving pulse. The end of the gate Δt , marked by an external timer, is fed into the data register to add a count to the channel swept at the moment at the end of Δt and to reset the address register.

The $(k + 1)th$ channel contains the number of gates that have exhibited the occurrence of k pulses, and the N_k vs. k spectral distribution is directly obtained. This type of output is easy to reduce since the moments of the observed distribution can be simply calculated as

$$\langle n^m \rangle = \frac{1}{N} \sum_0^{\infty} k^m N_k$$

The data collection is practically continuous since no time is wasted to print or punch between two successive gates. Hence the time required to observe an experimental profile composed by N samples is given by $T = N\Delta t$. The storing process can be especially fast for the prompt-variance analysis ($\Delta t \sim 1$ msec) since 10^5 data can be collected in 1 min 40 sec and 10^6 data in 16 min 40 sec.

Edelmann et al.,⁴³ Pacilio and Ferrari,⁴⁴ and Babala and Ogrin⁵⁶ have experimented with the described apparatus.

Methods 1 and 2 suffer a common type of inconvenience, which is usually minor and irrelevant but which affects the rigorous approach to a fluctuation analysis experiment. The analysis of the V vs. Δt dependence is intended as a parametric study of the variance as a function of the counting time interval adopted to observe the same fluctuation phenomena. With methods 1 and 2, fixed time intervals are used to sample different intervals of the reactor life, although the desired procedure would be to use variable time intervals to observe the same time sequence of neutron pulses.

All problems concerning stationary operating conditions become highly involved. A solution to this complication is suggested by the early experiments of Feynman et al.,²² which lead to the recorder and gated-scaler method.

3. Recorder and gated-scaler method. The output pulses are recorded for a sufficiently long time. The recording allows the same sequence of information about the reactor fluctuations to be analyzed for all the differently chosen time intervals Δt . This procedure avoids possible discrepancy among the determinations of $V(\Delta t)$ made on different data.

In the experiments by Feynman et al.,²² the pulses were recorded on a continuously moving film that photographed the screen of an oscilloscope into which the output signal was fed. Today improved systems, e.g., punched paper tape or magnetic tape, serve the same purpose.

The recorded tape is then sampled as in method 1. Measurements of this type have been made by Albrecht³¹ and Johnson.³⁴ A possible alternative that avoids the use of the tape recorder is based on the adoption of a very short time interval Δt so that the content of successive samples can be added to form samples related to longer intervals whose amplitude is a multiple of the chosen time basis Δt . Turkcan and Dragt⁵³ have used this method.

Identical considerations and criticisms as those for method 1 apply to these modified versions of the gated-scaler method.

Probably the most effective, rapid, and compact data collection can be obtained if the tape recorder is inserted as an intermediate step in the distributive method (2). The procedure denoted by line 4 in Fig. 4 is then adopted. It combines the advantages of all the previous techniques and can also be profitably used for analyzing the neutron-detection-probability distribution treated in Part III.

Generally, the operation of the apparatus is checked by counting pulses from a random source, e.g., a long-half-life radioactive substance. The calculation of the moments of the observed distribution allows them to be compared with the values predicted by the Poisson distribution; in particular, the relative variance of counts should be equal to unity for any value of Δt with a standard deviation given by

$$\sigma = \left[\frac{2}{N} \left(1 + \frac{1}{2\langle n \rangle} \right) \right]^{1/2}$$

The agreement between theory and experiment is usually good for all the measurements made in the $\Delta t > 1$ msec region but breaks down below that value and leads to a value of $V(\Delta t)$ which is smaller than unity and which continuously decreases with decreasing Δt .

This effect, which often gives physically unexplainable negative values of Y in the reactor experiments, can be explained in terms of dead time of the entire measuring apparatus. In fact, in the millisecond region, high counting rates are necessary to obtain significant statistics, namely, a mean value of the order of unity. Then the counting chain tends to damp all the actual moments of the neutron distribution as well as to modify the true distribution profile since it is likely to subtract one or more pulses from the content of some of the analyzed gates because of its inadequate resolution time.

This phenomenology has been studied and experiments with random- and nonrandom-emitting sources have been made by Pacilio,^{48,61} who also suggested a way to restore the true distribution profile and therefore to calculate the correlation actually involved between successive counts.

A reactor operating at a very low power, a good detection efficiency (10^{-3} to 10^{-4} for uranium systems), and a large number of samples are sine qua non conditions for the Feynman variance experiment. It is interesting that two experiments performed on plutonium systems by McCulloch and Clouet d'Orval et al.²⁸ were essentially possible because the high-level of the spontaneous fission could be partially compensated by a reduced detection efficiency and a still sufficiently large value of Y due to the much smaller β^2 of the plutonium fuel [see Eq. 20].

Interpretation of Data

The Feynman variance experiment can be interpreted in many ways; i.e., it may give information about several different parameters because of its multiple

dependence on fundamental reactor physics quantities [see Eq. 15].

The reference list shows that the prompt decay constant, prompt-neutron lifetime, the total fraction of delayed neutrons, the dispersion of neutrons in a nuclear fission, the reactivity of subcritical assemblies, and the reactor power level can be determined by such a procedure. Information about delayed-neutron parameters can be also reached through this procedure.

Not all the previously described quantities can be independently determined, and the evaluation of one of them from the experimental data is possible if some of the others are already known. The following experimental approaches and the consequent choice of parameters clarifies the measurement philosophy.

A logical approach is followed in this presentation rather than a chronological one (the latter is indicated by the reference list). Furthermore, the various types of measurements are divided according to the range of values of $\alpha \Delta t$ in which the experiments have been made, namely, the interval of values chosen for the time interval Δt of sampling.

Prompt-Variance Analysis. The designation prompt-variance analysis implies the necessary condition

$$|\alpha_2| \Delta t \ll 1 \quad (22)$$

for all the adopted time intervals. This assumption leads to Eq. 20.

The relation V vs. Δt is experimentally observed, and the value of α at criticality or in subcritical reactivity conditions is derived from a least-squares fit to Eq. 20. The same fit gives meaningful values of Y_1 if a sufficiently wide range of Δt values has been analyzed to allow Eq. 20 to be significantly different from a linear behavior valid for $\alpha \Delta t \ll 1$. Under this assumption

$$Y(\Delta t) = \frac{1}{2} Y_1 \alpha \Delta t + o[(\alpha \Delta t)^2] \quad (23)$$

which allows only the product αY_1 to be determined. Measurements of this type have been made by many authors. The symbol $o(x)$ indicates a quantity that behaves as x in first approximation.

The early experiments by Feynman et al.²² showed a rather qualitative agreement between Eq. 20 and the measurement data and led to an underestimate of the value of the critical alpha with respect to the ratio of β to Λ obtained through theoretical computations.

Luckow and Churchill^{23,24} obtained better agreement between the results of variance experiments and the values of β/l predicted or measured in other ways. A large discrepancy was found, however, in the power-level measurements.

The efficiency of the detector was derived from a determination of Y_1 at criticality obtained from the same experimental data set and from separate evaluations of D and β . Since ϵ and the mean counting rate were known, the absolute power level could be calculated and compared with the corresponding

values from other predictions and measurements. The results of Luckow and Churchill were below the expected values in all cases by a factor of 2 or 3.

Since the general validity of the method is beyond any cavil as other recent, extremely accurate measurements testify, the discrepancy apparently can be explained in terms of the perturbation introduced by the detector placement. The flux profile of the reactor is probably seriously altered by the presence of the instrument, and the consequent power-level determination is derived from an incorrect evaluation of the detector efficiency.

Since these first unsuccessful experiments, the variance measurements have been widely used for measuring kinetic parameters but are rarely used for power determinations although some papers [see Refs. 38 and 52] propose the revival of such a method.

The long list of prompt-variance experiments includes Tachon,^{2,7} Pal et al.,³⁶ Gotoh,³⁹ Pacilio,⁴² Edelmann et al.,⁴³ Pacilio and Ferrari,⁴⁴ Dragt and Turkcan,⁵³ and Babala and Ogrin.⁵⁶ The results obtained are usually consistent with determinations of the same parameters by conventional kinetic, pulsed-neutron, transfer-function, and other statistical methods.

The sensitivity and the accuracy of the procedure have been more and more improved by adopting high-efficiency in-core instrumentation, by collecting a large number (approximately one million) of samples, and by determining several experimental points in the $V(\Delta t)$ vs. Δt plot mainly in the strongly alpha-sensitive range of Δt values.

The Feynman variance experiment does not seem to be very sensitive to a higher mode or to reflector eigenvalue presence. Since these larger decay-constant modes must be time integrated over the interval Δt , the addendum they give to the variance is likely to be negligible, compared with the order of magnitude given by the fundamental mode.

Asymptotic Prompt-Variance Analysis. The designation asymptotic prompt-variance analysis implies the necessary double condition

$$\frac{1}{\alpha} \ll \Delta t \ll \frac{1}{|\alpha_2|} \quad (24)$$

for all adopted time intervals.

Generally just one Δt is actually used, and several runs are made to increase the precision of the measurement. The described condition cannot be achieved for heavy water and graphite systems where β/l is of the order of magnitude $|\alpha_2|$, and a true prompt analysis is possible only for very subcritical conditions.

If Eq. 24 holds, Eq. 15 becomes

$$V(\Delta t) = 1 + \frac{\epsilon DK_p^2}{(1 - K_p)^2} \quad (25)$$

$$V(\Delta t) \cong 1 + \frac{\epsilon D}{\beta^2} \quad (\text{at criticality}) \quad (26)$$

where the second term on the right hand side of the equation is underestimated by an amount equal to $\epsilon D/\beta^2 (\alpha \Delta)^{-1}$. Equations 25 and 26 can be used for different interpretations, as the references show.

The various approaches can be summarized as follows in terms of the measured quantity:

1. Dispersion D of the number of neutrons in a nuclear fission. This quantity is the Diven fission parameter (see Ref. 21) and is given by the relation

$$D = \frac{\langle \nu^2 \rangle - \langle \nu \rangle}{\langle \nu \rangle^2}$$

where ν is the number of neutrons emitted per fission and the bra-kets have the usual meaning of average.

The value of D has been measured by Feynman et al.²² through a critical determination of $V(\Delta t)$ for a large value of $\alpha \Delta t$ (~ 44). The value of ϵ previously measured and the value of β estimated by calculations were used to deduce D from Eq. 26.

The value obtained for D is in good agreement with that measured by Diven et al.²¹ by a direct analysis of fission events. Similar measurements have been made by Kurusyna.³³

2. Effective total fraction of delayed neutrons. McCulloch²⁶ used a similar procedure to determine the value of β for a plutonium system. He independently measured the detector efficiency, and he assumed the value of the parameter D to be equal to that obtained by Diven et al.²¹

Despite a large probable error, the results agreed well with previous measurements and calculations.

3. Reactivity of subcritical configurations. If Eq. 25 is used for variance experiments at subcritical conditions along with Eq. 26 at criticality, the reactivity of the analyzed assemblies can be immediately determined from the value of the ratio

$$\frac{Y_{\text{crit}}}{Y} = (1 - \beta)^2$$

Hence the determination of reactivity in dollar units is based on a calibration of the variance at criticality, just as a reactivity determination from values of α is based on a measurement of β/l and the assumption of a linear dependence of α on reactivity (see Eq. 2). The method requires no additional condition on the lifetime invariance with respect to reactivity as in the α vs. reactivity linearity assumption, however the detector efficiency is supposed to be constant throughout all the subcritical configurations.

The values of reactivity obtained from the asymptotic prompt variance experiment are in fairly good agreement with equivalent pulsed-neutron data over a range of subcritical reactivities through $\$3.5$. The measurements are by Lindeman and Ruby.⁵⁴

Prompt- and Delayed-Variance Analysis. A complete experimental check of Eq. 15 can be obtained if the range of the time intervals Δt is extended from milliseconds to 10 sec. Although a significant measurement of all the 14 reactor parameters involved is unreliable, i.e., the determination of the seven amplitudes Y_j and the seven eigenvalues α_j in the case of a six delayed-neutron groups scheme, such analysis of the variance can clarify the role of the delayed-neutron contribution to the variance and guarantee the correctness of the theoretical approach.

Derived from an experimental observation of 100 points of the V vs. Δt dependence, the results of Albrecht³¹ testify that no more than four values of the quoted parameters (two for prompt and two for delayed neutrons) can be measured with adequate precision.

Johnson³⁴ made an intermediate measurement by adopting an upper limit of 2 sec for the time intervals and subtracting the delayed-term contribution in order to treat the corrected data as it is treated in the prompt-variance analysis for determining β/l .

The experiments were performed in slightly subcritical conditions, and the value of β/l was obtained from a linear extrapolation of the α vs. $\$$ observed dependence.

EXPECTED NUMBER OF CORRELATED-NEUTRON-PAIRS METHOD

An interesting alternative to the variance experiment is based on determining the ratio of the expected number of correlated-neutron pairs to the expected number of random-neutron pairs in a given time interval. This parameter is strictly related to the instantaneous and mean counting rate and is quite close to the correlated relative variance. It can also profitably replace the correlated relative variance in some experimental circumstances where the variance approach can be unproductive, i.e., for subcritical measurements. The procedure is unsuitable for non-zero-power and fast systems.

The method is derived from a skillful analysis of some of the inconvenient aspects of the correlated relative variance, but it has its independent physical meaning and phenomenology. Actually the procedure is operationally new only in regard to the treatment of the data since the measuring apparatus and the data collection can be essentially identical with those adopted for the conventional variance experiment.

Some considerations about the theoretical formulation of the prompt-variance analysis described by Eq. 20 help in understanding the advantages of the procedure. As previously emphasized, the main role is played by the quantity Y , which, however, suffers from the following shortcomings:

1. It is not a real reactor parameter since its magnitude depends on detector sensitivity and location inside the reactor, i.e., flux profile.

2. Its value is small in the time region where the maximum number of correlated-neutron pairs is expected; i.e., its value is small in the range of time intervals $\Delta t \ll 1/\alpha$ where its dependence on the prompt-neutron-decay eigenvalue is strong and conversely becomes large where the dependence is no longer sensitive, i.e., for increasing Δt .

3. Its value is small if detector efficiencies are low with respect to the overall fission rate; i.e., instrumentation is far away from the flux peak or has low sensitivity.

4. It has such a dependence on reactivity that it decreases with respect to reactivity more rapidly than $(1 - \beta)^{-1}$, i.e.,

$$Y(\Delta t) \sim \frac{\epsilon D}{2\beta l} \frac{\Delta t}{(1 - \beta)} \quad (\text{for } \alpha \Delta t \ll 1)$$

since increasing values of α demand shorter intervals if α is to be accurately determined. Thus the Feynman variance method cannot be applied to highly subcritical systems, since small Y prohibits any analysis.

5. In some assemblies, such as natural-uranium and light-water lattices, even the asymptotic and maximum value Y_1 is of the order of 0.01 and practically useless for any significant interpretation.

Mathematical Formulation

If n is the value obtained from integrating the neutron-detector output over the time interval Δt , then

$$\frac{\langle n^2 \rangle - \langle n \rangle}{\langle n \rangle^2} = 1 + \sum_1^7 Z_i \left(\frac{|\alpha_i| \Delta t - 1 + e^{\alpha_i \Delta t}}{\alpha_i (\Delta t)^2} \right) \quad (27)$$

Equation 27 was derived by Furuhashi and Izumi.^{55,58}

Under the assumption $|\alpha_2| \Delta t \ll 1$, a prompt analysis is realized and Eq. 27 becomes

$$\frac{\langle n^2 \rangle - \langle n \rangle}{\langle n \rangle^2} = 1 + \frac{DK_p^2}{F(1 - K_p)^2} \left(\frac{\alpha \Delta t - 1 + e^{-\alpha \Delta t}}{\alpha (\Delta t)^2} \right) = 1 + Z \quad (28)$$

Some mathematical properties connected with the Feynman correlated term Y , or its asymptotic value Y_1 , hold for the quantity Z :

$$Z(\Delta t) \equiv \frac{Y(\Delta t)}{\langle n \rangle} \quad (29)$$

$$\lim_{\alpha \Delta t \rightarrow 0} Z(\Delta t) = \frac{DK_p^2}{2F\alpha^2} = \frac{Y_1 \alpha}{2F\epsilon} \quad (30)$$

The physical meaning and the basic characteristics of the parameter Z , partially in contrast with the quantity Y , can be summarized as follows:

1. The term Z can be written

$$\frac{\langle \binom{n}{2} \rangle - \frac{\langle n \rangle^2}{2}}{\frac{\langle n \rangle^2}{2}}$$

and is therefore equal to the ratio of the expected number of correlated-neutron pairs to the expected number of random-neutron pairs. The symbol $\binom{n}{2}$ indicates the Newton's binomial coefficient, while $\langle \binom{n}{2} \rangle$ obviously is the expected number of all the possible pairs.

2. The value Z vanishes for a Poissonian process, as does the value Y , because of the absence of any correlated pair in that process. This is equivalent to the fact that Z vanishes if no multiplication exists (see Eq. 28).

3. In direct contrast to Y , Z is large in the range of time intervals $\Delta t \ll 1/\alpha$ and has a strong dependence on the value of α , as can be seen from the following formula obtained from Eq. 28 for $\alpha \Delta t \ll 1$:

$$Z(\Delta t) \approx Z(0) \left(1 - \frac{\alpha \Delta t}{3} \right) \quad (31)$$

4. For very short time intervals, Z does not vanish, in contrast to Y , since no correlation is detectable for vanishing sampling intervals. Conversely, Z remains finite and tends to a maximum since the chances of correlated pairs increase with respect to the chances of random pairs. The limit value $Z(0)$ coincides with the B/A of the Rossi-alpha experiment (see Eq. 1) and therefore offers a significant theoretical basis and the possibility of a comparison between the two techniques.

5. For very long time intervals still in the prompt-neutron approximation, Z tends to vanish since the number of correlated pairs saturates the interval and practically all random pairs are detected.

6. The term Z is a characteristic physical parameter of the reactor since it does not depend on detector location and sensitivity but depends only on reactivity and power (source) level as well as on the sampling interval Δt .

7. The dependence of Z on reactivity is also given by Eq. 31 and appears to be a direct function of α , i.e., the slope of the behavior, since the initial value $Z(0)$ is approximately constant for decreasing reactivities. It can be written as

$$Z(C) \approx \frac{D\langle\nu\rangle}{2SI}$$

where S is the effective source strength due to spontaneous fission and any other additional source.

A typical comparison between Z and Y vs. Δt for a light-water system is illustrated in Fig. 5.

Fig. 5. Comparison between Z and Y vs. Δt for $\epsilon = 10^{-3}$ count/fission, $l = 35$ μ sec, $D = 0.8$, and $F\alpha = 2 \times 10^7$ fissions/sec².

If the detector efficiency is decreased, $Y(\Delta t)$ decreases proportionally, whereas $Z(\Delta t)$ remains unaffected. Decreasing ϵ , however, makes the mean value decrease and therefore affects the precision of Z . This is given by

$$\frac{\Delta Z}{Z} = \left\{ \frac{1}{N} \left[\left(4 + \frac{3}{\langle n \rangle} \right) \frac{1}{\langle n \rangle Z} + \left(2 + \frac{4}{\langle n \rangle} \right) + 2Z \right] \right\}^{\frac{1}{2}}$$

For $Z \approx 1$ and $\langle n \rangle \gtrsim 10$ the relative error is of the order of $N^{-\frac{1}{2}}$ (N is the number of samples) and is dependent slightly on Z and negligibly on $\langle n \rangle$. Conversely, $\Delta Z/Z$ significantly increases with $\langle n \rangle$ in the $\langle n \rangle \leq 1$ region.

Further questions related to this subject have been treated by Pacilio.⁶⁰

Experimental Procedure and Interpretation of Data

The criterion

$$Z \geq Y \quad (\text{if } F\epsilon \leq \alpha)$$

derived from Eq. 30 can be adopted to help decide which of the two procedures is more suitable since measurements are made in the $\alpha\Delta t \sim 1$ region.

The greatest term, either Z or Y , indicates the corresponding method to be applied.

Small detection efficiencies, i.e., $\epsilon \ll (1 - K_p)^2$, will prohibit using the Furuhashi-Izumi procedure although ϵ does not explicitly appear in Z . The increased value of F adopted to maintain α in the suitable range for the desired $\Delta Z/Z$ and N makes Z too small for significant analysis.

If an adequate efficiency is available, the Feynman variance appears to be more reliable at critical and slightly subcritical conditions, whereas the Furuhashi-Izumi method can replace it in more-subcritical configurations.

The experimental procedure is actually based on collecting counting data as in the conventional variance experiment and not on detecting neutron pairs and investigating their degree of correlation or independence as the name of the technique implies.

Since the expected total number of pairs and the expected number of random pairs are strictly related to the first- and second-order moments of the counting variable $n(\Delta t)$ distribution, the number of correlated pairs can easily be deduced by the difference.

The behavior $Z(\Delta t)$ vs. Δt can be actually observed in a range of time intervals smaller than that concerning $Y(\Delta t)$ vs. Δt . The latter must be extended to the $\alpha\Delta t > 1$ region because the linear behavior of Eq. 23 does not allow α and Y_1 to be separately determined. However, such does not occur for Z vs. Δt since even in the $\alpha\Delta t < 1$ range α and $Z(0)$ can be separately obtained as shown in Eq. 31.

The expected number of correlated-neutron-pairs method has been applied by Furuhashi and Izumi^{5,59} to highly subcritical bare and reflected light-water-moderated lattices. Since the sampling interval was as low as 65 nsec, a distributive method essentially analogous to line 2 of Fig. 4 has been adopted to collect data for $N \lesssim 10^9$ within a reasonable measurement time.

Pulses from the preamplifier-amplifier stage were fed into a count to pulse height converter and analyzed successively through a time analyzer in its pulse-height-analysis version. Pulses from an external timer were used to reset the count to pulse-height converter at every other Δt . In this way the distribution $\{N_k(\Delta t)\}$ was directly obtained and manipulated for the moment calculation.

Different detectors in various locations have checked the independence of Z of detection efficiency; experiments have proven it valid. A preliminary control

of the measuring apparatus, as well as of the influence of the dead time on the observable profile $\{N_k(\Delta t)\}$, has been made by counting pulses from a random source.

BENNETT VARIANCE METHOD

The Bennett variance method, named for Edgar F. Bennett, who suggested and performed the experiment in 1960, is not simply another alternative to the conventional variance technique; rather, it exhibits advantages and possibilities beyond those of the conventional variance technique.

The procedure is based on calculating the mean square value of the difference of integrated flux over two consecutive, equal time intervals. The quantity obtained contains as much information about the reactor parameters as the variance but does not give the mathematical property of divergence at criticality, which is characteristic of the quantity Y .

In this method a current chamber originally was used as a neutron detector instead of the commonly adopted pulse counter. Either instrument can be used in this procedure as well as in the other methods discussed in Part II. The measuring apparatus will be described, including the use of current chambers and integrating circuits. This type of data collection can be inserted in Fig. 4, line 1 or 3 and therefore applies to all the other methods.

The technique can be applied to thermal reactors. It cannot be used for fast reactors, however, because of the limitations imposed by the required high detection efficiency, as in the conventional variance experiment.

Mathematical Formulation

If the definition of $n_j(t + j\Delta t)$ is generalized to mean: integral of detector output over the interval $t + (j - 1)\Delta t$ to $t + j\Delta t$ in order to also include the current-chamber experiment, then

$$\frac{\langle (n_{j+1} - n_j)^2 \rangle}{2\langle n \rangle} = 1 + \sum_1^7 W_i \left(1 - \frac{\frac{3}{2} + \frac{1}{2} e^{2\alpha_i \Delta t} - 2e^{\alpha_i \Delta t}}{|\alpha_i| \Delta t} \right) \quad (32)$$

where the bra-kets $\langle \rangle$ indicate the operation of averaging over the ensemble of the N samples ($j = 1, 2, \dots, N$).

Equation 32 was derived by Bennett²⁹ in its complete form including the delayed-neutron contribution. Pal³⁵ reached an analogous conclusion via a different approach.

Equation 32 contains many mathematical and physical implications that can be summarized as follows:

1. The first term of the right-hand side parentheses is of Poissonian nature, whereas the second term contains the correlation between successive gates due to the presence of neutron decay chains. If the right-hand side is assumed to be equal to unity, the formulation of the left-hand side can be transformed: if n_{j+1} and n_j are uncorrelated, their subscripts can be dropped and the following equations are allowed:

$$\langle (n_{j+1} - n_j)^2 \rangle = 2\langle n^2 \rangle - 2\langle n \rangle^2 = 2\langle n \rangle$$

Thus the left-hand side of Eq. 32 is identically equal to unity for any value of Δt .

2. Until the condition $\lambda_2 \Delta t \ll 1$ is valid, Eq. 32 can be approximated by

$$\frac{\langle (n_{j+1} - n_j)^2 \rangle}{2\langle n \rangle} = 1 + \frac{\epsilon DK_p^2}{(1 - K_p)^2} \left(1 - \frac{\frac{3}{2} + \frac{1}{2} e^{-2\alpha \Delta t} - 2e^{-\alpha \Delta t}}{\alpha \Delta t} \right) = 1 + W \quad (33)$$

Some very close similarities between W and Y are noticeable: they both tend to vanish for very short channel widths (or integration intervals), and they both tend to the asymptotic value Y_1 for long channel widths defined by the double condition $\lambda_2 \Delta t \ll 1 \ll \alpha \Delta t$. The first result is easily interpreted; the second one needs clarification. Successive wide time intervals are practically uncorrelated since only a very small fraction of the neutron chains begins in one gate and dies in the successive gate. This consideration allows the left-hand side of Eq. 32 to be rewritten as

$$\langle (n_{j+1} - n_j)^2 \rangle = 2\langle n^2 \rangle - 2\langle n \rangle^2 = 2\langle n \rangle(1 + Y_1)$$

so that

$$\lim_{\substack{\alpha \Delta t \gg 1 \\ \lambda_2 \Delta t \ll 1}} W(\Delta t) = \lim_{\substack{\alpha \Delta t \gg 1 \\ \lambda_2 \Delta t \ll 1}} Y(\Delta t) = Y_1$$

3. When α is measured from experimental data interpreted according to Eq. 32, an upper limit for the channel width (or integration interval) is convenient since W , as well as Y , has a strong dependence on $\alpha \Delta t < 1$ zone which tends to vanish for longer time intervals.

4. The relative statistical error that affects $1 + W$ is given by

$$\frac{\delta(1 + W)}{(1 + W)} = \left[\frac{2}{N} \left(1 + \frac{1 + W}{2\langle n \rangle} \right) \right]^{1/2}$$

and is weakly dependent on $\langle n \rangle$ but strongly on N , one of the fundamental parameters of the measurement, as in the conventional variance experiment.

Measuring Methodology

Depending on the type of experiment involved, two completely different measuring apparatus must be considered, although the final output can be identical; one is based on a pulse counter and is likely to be applied to zero-power experiments: the other, with the use of a current chamber, can be applied to non-zero-power as well as to zero-power measurements.

The detailed description that follows relates to the block diagram of Fig. 6.

Fig. 6. Block diagram of the measuring apparatus for the Bennett variance method. Pulse-counter and current-chamber experiments are considered.

1. Pulse-counter method. The entire procedure can be the same as that for the Feynman variance experiment except that the output is necessarily in sequence form. The time sequence $\{n_j\}$ of the collected data is now fundamental information to be used to minimize the data collection time.

An alternative version using a tape recorder may or may not be taken into account with or without the inconveniences discussed previously.

The number N of samples must be large, and therefore the measurement time is likely to be long even for time intervals of about 1 msec unless suitable recording circuits are continuously applied and operated. Because it carries the information chronology, the distributive method cannot be applied unless a more sophisticated data collection is adopted, e.g., the distributive method of the difference between consecutive samples.

A gated scaler and recording circuit have been used by Turkcan and Dragt.⁵³

2. Current-chamber method. The basic instrument is a gamma-compensated current chamber of the type generally adopted as a flux monitor in nuclear reactor control. The chamber output is fed into a d-c amplifier and then into a double RC integrating circuit. The two parts of this circuit work alternately to make an integration of the detector output possible in actually successive intervals, i.e., without any lag, as theory imposes. At the end of each fixed time interval, the result of the integration is displayed via a suitable recorder.

An output in string form is obtained which is ready to be treated according to the left-hand side of Eq. 32.

Different time intervals can be obtained by changing the integrating-circuit time constants or by adding consecutive intervals to obtain an equivalent string of data in time intervals whose amplitude is a multiple of the chosen minimum Δt . A measuring apparatus of this type has been used by Bennett²⁹ and by Mogilner and Shvetsov.⁵²

The integration interval chosen is of the order of 10 sec. for both experiments so that completely different purposes were intended in contrast with those pursued through the short Δt pulse-counter technique (see Interpretation of Data).

The measuring apparatus can be checked by counting pulses from a random-emitting source, e.g., a long-half-life radioactive substance. The calculation of the left-hand side of Eq. 32 must lead to unity for all the adopted time intervals and for a standard deviation derived from the formula given in item 4, page 45, in which W equals zero.

Because of dead time for very short gates it can happen that the left-hand side of Eq. 32 is smaller than unity or, in the case of reactor experiments, $1 + W$ is smaller than unity, both of which are physically meaningless.

The role of the dead time of the measuring apparatus may be of fundamental importance in introducing this negative correlation and must be carefully considered.

Interpretation of Data

The Bennett variance experiment was originally intended as a procedure for measuring the absolute power level of the reactor. As previously stated, the term W actually contains the same type and amount of information as the correlated relative variance Y . In the prompt-neutron domain, W is smaller than Y , showing that the correlation between successive differences is weaker than that between the counts themselves.

Depending upon the equipment and the choice of the time interval in which the relation W vs. Δt is observed, two kinds of analysis can be considered.

Prompt-Neutron Analysis. Prompt-neutron analysis implies $k_2 \Delta t \ll 1$ and leads to Eq. 33.

The dependence of W on the chosen Δt is experimentally observed, and the values of α at criticality and in subcritical conditions are derived from a least-squares fit of the data to Eq. 33. The same fit gives a value of Y_1 if a sufficiently wide range of values for Δt has been adopted so that the dependence of W on Δt is far from quadratic. For $\alpha \Delta t \ll 1$, Eq. 33 becomes

$$W(\Delta t) \approx \frac{1}{3} Y_1 (\alpha \Delta t)^2 + o[(\alpha \Delta t)^3]$$

and only the product $\alpha^2 Y_1$ can be determined. The symbol $o(x)$ indicates a quantity that behaves as x in first approximation.

The parameters α and Y_1 can be separately determined even in the $\alpha \Delta t \ll 1$ region if the Bennett experiment is coupled with the Feynman experiment or with the third-order-moment (see the Third-Order Analysis) experiment in the same region. The Bennett experiment gives the value of $\alpha^2 Y_1$ and the other two experiments give the value of αY_1 , so that the separate values of α and Y_1 are obtainable.

Measurements of prompt-neutron variance have been made by Turkcan and Dragt,^{5,3} and the results agree closely with the conventional variance experiment. The number of samples was of the order of one million and allowed an adequate precision to be obtained.

Delayed-Neutron Analysis. If the integration time is chosen to be so long (~ 10 sec) that the terms with subscript $i = 6$ and $i = 7$ practically dominate the others, the analysis of the relation W vs. Δt is actually based only on the delayed neutrons, and even the first unity term in Eq. 33 can be neglected, at least for sufficiently high detector efficiency. Obviously a measurement of this type is not oriented to any kinetic parameter determination but constitutes a capable tool for measuring the reactor power in terms of absolute fission rate. Measurements of this type have been made by Bennett,^{2,9} Bennett and Long,^{32,37} and Mogilner and Shvetsov.^{5,2}

The accuracy is poor compared with the precision usually obtainable in other reactor parameter determinations, but it is comparable to that obtained via calorimetric, calibrated-source, or activation procedures.

The consistency among the results of the various techniques is satisfactory in the majority of cases.

THIRD-ORDER ANALYSIS AND THE EXPECTED NUMBER OF CORRELATED-NEUTRON-TRIPLETS METHOD

Further recent investigations on the neutron fluctuation phenomena have considered the possibility of reaching additional information via a third-order-moment analysis of the distribution of counts and via the determination of the

ratio of the expected number of correlated-neutron triplets to the expected number of random-neutron triplets. Both procedures can be regarded as extensions of the second-order-movement analysis previously discussed under the Feynman variance method and the expected number of correlated-neutron-pairs method.

For instance, the mathematical formulation shows that from a strictly theoretical point of view experiments of the third-order type yield β and Λ (or ℓ) separately without any need of static measurements. This would really be an asset in reactor parameter measurements.

Several types of analysis have been proposed according to the several combinations of third-order moments that can be adopted in manipulating the collected data.

The only experiment performed for third-order analysis has shown that for the moment the usefulness of this analysis is not greater than that offered by the second-order methods and that a separate determination of β and Λ is based on the detection of events (a triplet of neutrons originated from the same fission event) which is too rare and infrequent to make the analysis possible, at least in thermal reactors.

Thus at present the third-order experiment is just another type of analysis which can be done on the same set of data used for the second-order experiments and which in some cases, e.g., those in which a very high efficiency is available, can allow a more adequate precision to be reached in comparison with the various variance procedures.

Mathematical Formulation

If n is the number of counts from a neutron detector in a time interval Δt , then

$$\frac{\langle n(n-1)(n-2) \rangle - 3\langle n \rangle \langle n(n-1) \rangle + 2\langle n \rangle^3}{\langle n \rangle} = \sum_{i=1}^7 X_i \left(1 + e^{a_i \Delta t} - 2 \frac{1 - e^{a_i \Delta t}}{|a_i| \Delta t} \right) \quad (34)$$

Dragt⁵⁰ obtained Eq. 34 together with analogous third-order moment relations reported and partially discussed in this section.

Only Eq. 34 is analyzed since it is supported by experimental investigations. Some mathematical and physical implications of Eq. 34 can be noted:

1. If the left-hand side of Eq. 34 is related to a Poissonian distribution, the following consequences are valid:

$$\langle n(n-1) \rangle = \langle n \rangle^2$$

$$\langle n(n-1)(n-2) \rangle = \langle n \rangle^3$$

Thus the left-hand side vanishes, and Eq. 34 is identically satisfied since even the right-hand side is zero because of the assumption of no correlation.

2. Under the assumption $|\alpha_2| \Delta t \ll 1$, Eq. 34 can be approximated by

$$\frac{\langle n(n-1)(n-2) \rangle - 3\langle n \rangle \langle n(n-1) \rangle + 2\langle n \rangle^3}{3\langle n \rangle} = \frac{\epsilon^2 D^2 K_p^4}{(1-K_p)^4} \left(1 + e^{-\alpha \Delta t} - 2 \frac{1 - e^{-\alpha \Delta t}}{\alpha \Delta t} \right) = X \quad (35)$$

A comparison of X with the quantities Y and W introduced in the previous methods shows some connecting properties among these three terms as expressed by

$$\lim_{\alpha \Delta t \rightarrow 0} Y(\Delta t) = \lim_{\alpha \Delta t \rightarrow 0} W(\Delta t) = \lim_{\alpha \Delta t \rightarrow 0} X(\Delta t) = 0 \quad (36)$$

$$\lim_{\substack{\alpha \Delta t \gg 1 \\ |\alpha_2| \Delta t \ll 1}} Y^2 = \lim_{\substack{\alpha \Delta t \gg 1 \\ |\alpha_2| \Delta t \ll 1}} W^2 = \lim_{\substack{\alpha \Delta t \gg 1 \\ |\alpha_2| \Delta t \ll 1}} X = Y_1^2 \quad (37)$$

The limit is only approximate with a relative error for defect equal to $2(\alpha \Delta t)^{-1}$.

The meaning of Eq. 36 is evident; however, that of Eq. 37 can be derived from some properties of the probability distribution function of the detection process extensively discussed in Part III. In the prompt-neutron approximation,

$$\begin{aligned} \langle n \rangle \langle n(n-1) \rangle &= \langle n \rangle^3 + \langle n \rangle^2 Y_1 \\ \langle n(n-1)(n-2) \rangle &= \langle n \rangle^3 - 3\langle n \rangle^2 + 2\langle n \rangle \\ &= \langle n \rangle^3 + 3\langle n \rangle^2 Y_1 + 3\langle n \rangle Y_1^2 \end{aligned}$$

so that Eq. 37 is satisfied.

With sufficiently high detection efficiency and not very subcritical systems so that $Y_1 \gg 1$, the third-order analysis allows the reactor parameters to be derived from a quantity greater than that obtainable via second-order procedures and is likely to give better precision. The expected error affecting X is still uncalculated.

Other third-order formulas (in the prompt-neutron approximation) that can be used to check the experimental data are:

$$\frac{\langle (n - \langle n \rangle)^3 \rangle}{\langle n \rangle} \cong 1 + 3 Y_1 \left(1 - \frac{1 - e^{-\alpha \Delta t}}{\alpha \Delta t} \right) + 3 Y_1^2 \left(1 + e^{-\alpha \Delta t} + 2 \frac{1 - e^{-\alpha \Delta t}}{\alpha \Delta t} \right) \quad (38)$$

$$\frac{\langle (n_i - \langle n \rangle)(n_{i+j} - \langle n \rangle)(n_{i+k} - \langle n \rangle) \rangle}{\langle n \rangle} \cong Y_1^2 (\cosh \alpha \Delta t - 1) e^{-\alpha k \Delta t} \quad (\text{for } k > j > 0) \quad (39)$$

$$\frac{\langle (n_i - \langle n \rangle)^2 (n_{i+j} - \langle n \rangle) \rangle}{\langle n \rangle} \cong \frac{\langle (n_i - \langle n \rangle)(n_{i+j} - \langle n \rangle)^2 \rangle}{\langle n \rangle} \cong Y_1 \frac{\cosh \alpha \Delta t - 1}{\alpha \Delta t} \times e^{-\alpha j \Delta t} + Y_1^2 \frac{1 - e^{-\alpha \Delta t}}{\alpha \Delta t} (e^{\alpha \Delta t} - 1 - \alpha \Delta t) e^{-\alpha j \Delta t} \quad (40)$$

$$\frac{\langle n_i n_{i+j} (n_i - n_{i+j}) \rangle}{\langle n \rangle} \cong \frac{1}{3} \frac{\epsilon^2 \nu (\nu - 1)(\nu - 2)}{\psi (1 - K_p)^3} (\alpha \Delta t)^2 e^{-\alpha j \Delta t} (1 - e^{-\alpha j \Delta t}) \quad (\text{for } \alpha \Delta t \ll 1) \quad (41)$$

Equation 41 is valid under the condition that $\alpha \beta \gg 1$, which is evidently satisfied only with fast reactors.

All these formulas as well as Eq. 35 are approximate versions of more complicated theoretical dependences that include additional terms. However, these terms have negligible numerical importance and are likely to be unobservable in the related experiments.

Knowing the time-independent coefficient of Eq. 41 and knowing Y_1 allows β to be determined directly, and therefore an independent measurement of Λ from the ratio β/α at criticality can be obtained. The stringent necessity of a very high efficiency and the difficult realization of this task in fast reactor experiments are contrasting factors in such an experiment, which potentially can yield extremely valuable information.

Experimental Procedure and Interpretation of Data

The data collection is exactly the same as in the second-order-moment experiments, and therefore an identical measuring apparatus is implied. The various alternatives, e.g., the four lines in Fig. 4, will give the same results even

in this case. For instance, if data are treated according to Eqs. 35 and 40, all four lines in Fig. 4 are possible, but lines 2 and 4 are strongly recommended because of their compact and easily treated data collection.

On the contrary, if the formalism of Eqs. 39, 40, and 41 is adopted, only the procedures of lines 1 and 3 can be used since the output data are needed in string form, namely, their time sequence of detection must be known. Measurements of the third-order type have been made by Turkcan and Dragt.^{5,3} The method represented by line 1 was essentially applied. The total number of gates used were chosen consistently large.

The values of reactor parameters are derived by a fit of the observed X vs. Δt dependence; the fit is obviously based on Eq. 35 if the time intervals are chosen according to the prompt-neutron approximation. The values of α and Y_1 can be obtained separately only if the X vs. Δt dependence is significantly far away from parabolic behavior.

However, if $\alpha \Delta t \ll 1$, then

$$X(\Delta t) = \frac{1}{2} Y_1^2 (\alpha \Delta t)^2 + o[(\alpha \Delta t)^3]$$

and only the product αY_1 can be determined.

Measurements of the third-order type have been made by Turkcan and Dragt,^{5,3} whose final conclusions and comments can be summarized as follows:

1. The results obtained by the third-order-moment analysis confirm the theoretical statements as far as Eq. 35 is concerned.

2. The values of α and Y_1 obtained from this procedure agree well with homologous values obtained via Feynman variance experiments and Bennett variance experiments.

3. The precision in α and Y_1 was worse than that in the second-order analysis, even in the cases of $Y_1 \gg 1$. This result discourages the use of this technique, but perhaps the whole procedure needs to be optimized.

A possible alternative to the third-order-analysis method based on Eq. 34 can be obtained via a determination of the ratio U of the expected number of correlated-neutron triplets to the expected number of random-neutron triplets. If the left-hand side of Eq. 35 is divided by $\langle n \rangle^3$, the parameter U is derived as

$$U = \frac{\langle \binom{n}{3} \rangle - \langle n \rangle \left[\langle \binom{n}{2} \rangle - \frac{\langle n \rangle^2}{2} \right] - \frac{\langle n \rangle^3}{6}}{\frac{\langle n \rangle^3}{6}} \quad (42)$$

where $\langle \binom{n}{3} \rangle$ is the expected total number of triplets, $\langle n \rangle \left[\langle \binom{n}{2} \rangle - \frac{\langle n \rangle^2}{2} \right]$ is the expected number of triplets consisting of a correlated pair and a random neutron, and $\langle n \rangle^3/6$ is the expected number of random triplets.

When expressed in terms of reactor parameters and a sampling interval, the parameter U becomes

$$U = \frac{3D^2 K_p^4}{F^2 (1 - K_p)^4} \left[\frac{1 + e^{-\alpha \Delta t}}{(\Delta t)^2} - \frac{2(1 - e^{-\alpha \Delta t})}{\alpha (\Delta t)^3} \right] \quad (43)$$

With respect to X it has all the advantages that Z exhibits with respect to Y , i.e., independence from ϵ , large value in the $\Delta t < 1/\alpha$ region where the dependence on α is sensitive, etc. A comparison between X and U vs. Δt is given in Fig. 7.

Fig. 7. Comparison between X and U vs. Δt for $\epsilon = 10^{-3}$ count/fission, $l = 35 \mu\text{sec}$, $D = 0.8$, and $F\alpha = 2 \times 10^7$ fissions/sec².

Equations 42 and 43 have been derived by Furuhashi and Izumi.⁵⁸ No practical realization of the procedure has been made. The experimental approach can be assumed to be analogous to that used for the neutron-pair technique. Further details on the matter are discussed by Pacilio.⁶⁰

COVARIANCE METHOD

The covariance experiment is based on simultaneously counting two neutron detectors placed inside or near a nuclear reactor for a time interval Δt . This sampling of the neutron population must be repeated many times to obtain the ensemble $\{n_1(\Delta t), n_2(\Delta t)\}$ of data to use for the calculation of the covariance of

the two counting variables $n_1(\Delta t)$ and $n_2(\Delta t)$ from detector 1 and 2, respectively.

The covariance is defined as the difference between the average product of the variables and the product of their mean values. It vanishes if the two quantities are uncorrelated; otherwise, it is a good index of the degree of coherence or correlation between them.

Cohn⁵¹ recently suggested the covariance experiment, emphasizing advantages of this technique not obtainable with the use of a single detector and the computation of the variance. The adoption of two counters relieves the stringent necessity of a high detection efficiency and allows these techniques to be applied to fast systems also, unlike the major part of all the variance-type experiments. Actually, if the errors due to the finite number of samples are to be minimized, the detection efficiency cannot be too low.

No measurements of this type are known, or at least no published matter on the subject is available.

The present technique constitutes an intermediate step toward the more elaborate but also more generally applicable procedures extensively treated in Part IV.

Mathematical Formulation

If $n_1(\Delta t)$ and $n_2(\Delta t)$ are the neutron pulses from two simultaneously counted detectors in a given time interval Δt , then

$$\begin{aligned} \text{Cov}(\Delta t) &= \langle (n_1 - \langle n_1 \rangle)(n_2 - \langle n_2 \rangle) \rangle \\ &= \langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2 \rangle \\ &= \sum_j B_j \left(1 - \frac{1 - e^{\alpha_j \Delta t}}{\alpha_j \Delta t} \right) \end{aligned} \quad (44)$$

where the role and physical content of B_j is analogous to that of Y_j in Eq. 15.

Equation 44 can be easily obtained from one of the many papers listed in the references if the presence of two detectors with different efficiency and the consequent modifications are considered. If these changes are made, Eq. 44 can be derived from the heuristic approach of the early work by Feynman¹⁹ and de Hoffmann,²⁰ a thorough treatment has been given by Borgwaldt and Stegemann.⁴⁵ No author has explicitly considered Eq. 44 as a starting point for an applicable technique, however.

An analysis of the properties of Eq. 44 leads to the following conclusions:

1. If the left-hand side of Eq. 44 is related to a bivariate Poissonian distribution for the involved variables, then

$$\langle n_1 n_2 \rangle = \langle n_1 \rangle \langle n_2 \rangle$$

so that $\text{Cov}(\Delta t) \equiv 0$ for any value of Δt and Eq. 44 is identically satisfied since even the right-hand side vanishes because of the assumption of no correlation.

2. Under the prompt-neutron approximation $\alpha_2 |\Delta t| \ll 1$, Eq. 44 can be approximated by

$$\frac{\text{Cov}(\Delta t)}{\langle n_1 \rangle} = \frac{\epsilon_2 D K_p^2}{(1 - K_p)^2} \left(1 - \frac{1 - e^{-\alpha \Delta t}}{\alpha \Delta t} \right) \quad (45)$$

or

$$\frac{\text{Cov}(\Delta t)}{\langle n_2 \rangle} = \frac{\epsilon_1 D K_p^2}{(1 - K_p)^2} \left(1 - \frac{1 - e^{-\alpha \Delta t}}{\alpha \Delta t} \right) \quad (46)$$

where ϵ_1 and ϵ_2 denote the detector efficiencies.

When compared with the classical Feynman variance formulation (see Eq. 20) concerning the single-detector experiment, Eqs. 45 and 46 show the absence of the Poissonian relative variance that contributes an additional term equal to unity to the quoted formula of the autovariance, or simply variance, of counts.

As expected, the adoption of two detectors eliminates this uncorrelated term, gives only the required coherent amount of information, and so enables the experiment to be profitably performed in conditions of low detection efficiency.

Considering $\epsilon_1 = \epsilon_2 = \epsilon$ for simplicity, the new quantity $\text{Cov}(\Delta t)$ is not merely equal to $V(\Delta t) - 1$ of the corresponding one-detector experiment, as a simple mathematical passage would suggest. The unit value to subtract is affected by an error, given by Eq. 21, that can be large enough to hide the information concerning the correlated excess term, especially if $\epsilon < \beta^2$.

This situation does not occur in the covariance experiment, which avoids this bias term and gives directly the amount of the required effect due to the presence of the neutron chain.

Since the number of collected samples is finite, however, a term due to the contribution of uncorrelated events is still present in the error which affects $\text{Cov}(\Delta t)$.

3. The following limit properties, strictly related to those exhibited by $V(\Delta t)$, hold for $\text{Cov}(\Delta t)$:

$$\lim_{\alpha \Delta t \rightarrow 0} \text{Cov}(\Delta t) = 0 \quad (47)$$

$$\lim_{\substack{\alpha \Delta t \gg 1 \\ |\alpha_2| |\Delta t| \ll 1}} \text{Cov}(\Delta t) = \langle n_1 \rangle Y_1^{(2)} = \langle n_2 \rangle Y_1^{(1)} \quad (48)$$

where $Y_1^{(1)}$ and $Y_1^{(2)}$ denote the asymptotic correlated prompt relative variances of counts from detectors 1 and 2, respectively.

Measuring Methodology and Interpretation of Data

The mathematical formulation of the method immediately suggests that a practical covariance experiment is based on doubling the measuring apparatus of the classical Feynman experiment.

Two counting-chain lines of the type illustrated in the upper part of Fig. 6 are sufficient for the procedure. A necessary condition is that a common time marker triggers both gate scalars so that the successive time intervals Δt are opened simultaneously for the two counting operations.

The distributive method cannot be applied in the covariance method since the two distributions $\{n_1(\Delta t)\}$ and $\{n_2(\Delta t)\}$ are not required and only the joint distribution $\{n_1(\Delta t), n_2(\Delta t)\}$ is needed for the off-line data computation. The joint distribution could be directly recorded if a two-parameter analyzer were used.

A useful check of the measuring-apparatus conditions can be made by counting pulses from a random-emitting source. The calculation of the covariance of counts ought to lead to a zero value for any Δt .

Short gates, i.e., $\Delta t \lesssim 1$ msec, are expected to introduce a certain discrepancy, in comparison with the theoretical forecast, in terms of a bias (usually negative) in the value of $\text{Cov}(\Delta t)$. The dead time of the counting apparatus is one of the major causes of this effect. Other effects (usually in terms of positive bias) may be due to electronic noise, instability of some transducers, and other factors.

Even the interpretation of data follows the previously discussed variance procedures. Once again the experimental routine is based on repeating the same data collection for a certain number of values of Δt . The choice of the Δt range determines the purposes of the analysis. Recall that the following intervals and denominations can be adopted as extensions of the single-detector philosophy:

$\alpha \Delta t < 1$	prompt covariance analysis
$\alpha \Delta t \gg 1 \gg \alpha_2 \Delta t$	asymptotic prompt covariance analysis
$\alpha_2 \Delta t > 1$	delayed covariance analysis

The mathematical implications, the equations on which the least-squares codes are based, and the extraction of the reactor kinetic parameters are basically analogous to similar subjects described under Interpretation of Data in the various variance procedures.

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III ANALYSIS OF THE PROBABILITY OF NEUTRON DETECTION

Part II showed that the variance of counts from a neutron detector is one of the most expressive quantities related to reactor kinetic parameters. The variance, as its definition implies, was calculated from the standard deviation and the mean value of counts.

Another possible way of reaching the same quantity can be a determination of the probability $p_k(\Delta t)$ of counting k pulses in a time interval Δt via a measurement of the corresponding experimental frequencies $f_k(\Delta t)$.

For an uncorrelated (Poisson) process, $p_k(\Delta t)$ is known to be an explicit function of only the mean number $\langle n \rangle$ of counts in the interval Δt . For an experiment in a nuclear system, the probability $p_k(\Delta t)$ is likely to be a function of both $\langle n \rangle$ and Y since the presence of a nonvanishing correlated term Y in the relative variance of counts implies a greater amount of fluctuation in comparison with those of a purely random variable.

The methods based on the probability of a neutron detection actually impose a double task. The first task is theoretical and concerns the adoption of a suitable probability distribution function $\{p_k(\Delta t)\}$ whose characteristics could match the observable frequency distribution $\{f_k(\Delta t)\}$ of the neutron counts. At present two slightly different probability profiles are usually recognized as fitting the required characteristics and are employed for theory-experiment comparisons. The second part regards the data processing and the relations between the observed $f_k(\Delta t)$, for one or more values of k , and the quantity Y . Many methods have been suggested; the related literature indicates that Y can be determined if only the values of f_0 and $\langle n \rangle$ are known (zero-probability method)

or if the values of f_0 , f_1 , and $\langle n \rangle$ are known (zero-to-one-probability ratio method).

Other techniques are based on determining the value of Y from a complete comparison between $\{p_k(\Delta t)\}$ and $\{f_k(\Delta t)\}$ (Polya-model method) or from the probability of an even or an odd count.

Alternately, if f_0 , f_1 , and f_2 , are known, the Furuhashi-Izumi parameter Z can be determined so that a probability experiment can again be interpreted in terms of a variance-type procedure.

In this section two other methods are described (time interval from a random origin and pulse to pulse time interval) that actually belong under the discussion of time-correlation analysis. Their presentation, however, is more accessible and is better understood if it follows the probability methods.

ZERO-COUNT PROBABILITY METHOD

The zero-count probability technique is based on the determination of the relative variance of counts from a neutron detector by measuring the fraction of empty samples for different values of the sampling interval Δt to the total number of samples collected during the experiment.

Measurements need be taken at extremely low counting rates so that the probability of no counts can be significantly different from zero, and measurements are most likely to be applicable to thermal systems where adequate detection efficiencies are available.

The procedure is straightforward, unlike some other variance methods, and needs very little reduction of data since assembled data are given directly by the measuring apparatus.

Once the relative variance of counts has been obtained, the procedure has the same characteristics and implications as the methods described in Part II.

Mathematical Formulation

If $p_0(\Delta t)$ is the probability of no counts in a time interval Δt , then

$$p_0(\Delta t) = \exp \left\{ - \left[\frac{2A \Delta t}{1 + \gamma} + \frac{A}{B} \log \frac{(\gamma + 1)^2 - (\gamma - 1)^2 e^{-\alpha \gamma \Delta t}}{4\gamma} \right] \right\} \quad (49)$$

where A and B have been given in Eq. 1 and the symbol γ denotes

$$\gamma = (1 + 2Y_1)^{1/2}$$

and is seen to be related to Eq. 20.

Equation 49 has been formulated by Pal⁵ and subsequently by Mogilner and Zolotukhin⁷ and is valid in the prompt-neutron time domain, namely, $|\alpha_2| \Delta t \ll 1$.

Often the probability of no counts is indicated as

$$p_0(\Delta t) = (1 + Y)^{-\langle n \rangle / Y} \quad (50)$$

where the dependence of the right-hand side of the equation on Δt is contained in $\langle n \rangle$ and Y , as Eq. 20 implies. Equation 50, given by Mogilner and Zolotukhin^{2,3,4} can be essentially regarded as an approximation of Eq. 49.

The problems of the meaning and the limitations involved in adopting Eq. 50 instead of Eq. 49 have been extensively discussed by Mogilner and Zolotukhin,⁷ Pal,⁶ Pacilio,^{1,3,22} and Dragt and Turkcan,¹⁵ and no agreement has been reached yet. The theoretical implications are not completely clear at the present, and a range of values for some parameters, i.e., $\alpha \Delta t$, Y_1 , and reactivity, is still not rigorously conceived for which the approximation of Eq. 49 with Eq. 50 is allowed. Figure 8 shows that the disagreement is not so dramatic, and the two equations seem to be consistent in a range wider than expected.

Equations 49 and 50 have a number of characteristics that can be summarized as follows:

1. For a purely random process, the terms Y and $\gamma-1$ tend to vanish and the following limit holds:

$$\lim_{\substack{Y \rightarrow 0 \\ \gamma \rightarrow 1}} p_0(\Delta t) = e^{-\langle n \rangle}$$

for any value of Δt . The probability of no counts for a Poisson distribution is immediately recognizable. Actually the limit holds for very low detection efficiencies and for measurements in extremely subcritical configurations.

2. For fixed reactivity and detector efficiency, Eq. 50 depends only on the power level.

$$p_0(\Delta t) = e^{-\beta \langle n \rangle} \quad (51)$$

where

$$\beta = \log \frac{1 + Y}{Y}$$

which can be used as the point of departure for an experimental procedure or for a check of the validity of the approximation of Eq. 49 with Eq. 50.

Fig. 8. The zero-count probability $p_0(\Delta t)$ as a function of the time interval Δt for $\alpha = 200 \text{ sec}^{-1}$ and $A = 1000 \text{ counts/sec}$.

3. The statistical error on Y calculated from Eq. 50 is given by

$$\frac{\delta Y}{Y} = \left\{ \frac{1}{N} \frac{\frac{1+Y}{Y} \log(1+Y) + \frac{(1/p_0) - 1}{\log(1/p_0)} - 2}{\log \frac{1}{p_0} \left[1 - \frac{Y}{(1+Y) \log(1+Y)} \right]^{21}} \right\}^{1/2}$$

which can be approximated with Eq. 21 for small values of $\langle n \rangle$, i.e., $\langle n \rangle \lesssim 3$.

Experimental Procedure

Measuring apparatus of different types can be used in an experiment to determine the relative variance of counts from a neutron detector by the zero-count probability method. Possibly the most straightforward procedure is that shown in Fig. 9 based on an anticoincidence circuit. In this procedure the pulses from the

Fig. 9. Block diagram of measuring apparatus for the zero-count probability method (anticoincidence system).

discriminator are fed into the anticoincidence which gives an output only if during the interval marked by an external timer a neutron pulse does not arrive. The pulse finishing the interval resets the anticoincidence and prepares it for the same operation in the successive interval.

At the end of the procedure, the number of output pulses gives the value N_0 of samples with no pulse content. The total number N of samples is given by the ratio of the total measurement time T to the time interval Δt .

The gated scaler is used to determine the mean value $\langle n \rangle$ of counts over measurement time T . The operation is repeated for various values of Δt . The gated scaler has been used by Mogilner and Zolotukhin^{4,7} for measurements on a thermal system. The duration of the experiment is of the order of minutes and allows approximately 10^6 samples to be stored so that a fairly good accuracy can be obtained (see Eq. 52).

An effective alternative to this method is based on the use of the apparatus shown in Fig. 4. Although all four lines lead to a determination of the required data, i.e., essentially N_0 , N , and $\langle n \rangle$, lines 2 and 4 offer the final data already in assembled form and do not need any further data reduction of the sequence of counting intervals to pick up those samples with no counts. Line 4 also exhibits the advantage of allowing the same reactor history to be analyzed in terms of different Δt .

This alternative procedure has been profitably used by Mogilner and Zolotukhin⁷ to reduce reactor time for such a zero-count probability experiment. The operating conditions for both apparatus can be checked by the usual

test of counting a random source. In this case the zero-count probability should equal $\exp(-\langle n \rangle)$ for any value of Δt used.

For very short time intervals ($\Delta t \lesssim 1$ msec), the ratio of N_0 to N is likely to be greater than $\exp(-\langle n \rangle)$ because of dead-time effects that make some intervals with content equal to one or more pulses reduce to intervals with no counts because no sufficient time resolution is available. Effects of this type have been discussed and experimentally checked by Pacilio.¹¹ In the case of experiments on a reactor, the value of Y obtained from uncorrected data, i.e., from a value of N_0 spuriously increased, as well as the values of Y_1 and α , can be seriously increased.

Interpretation of Data

The standard operation of measuring the ratio of the number N_0 of empty samples to the total number N of samples must be repeated for different values of the time interval Δt . The unknown quantity Y is then found by solving the transcendental equation

$$\log(1 + Y) = HY \quad (52)$$

where the constant H is given by

$$H = \frac{1}{\langle n \rangle} \log \frac{N}{N_0}$$

Equation 52 can be solved only if $H < 1$, that is, if $N_0 > Ne^{-\langle n \rangle}$. The trivial solution $Y = 0$ is obviously disregarded.

From the observed dependence of Y on Δt , the values of α and Y_1 are derived through the usual least-squares fit to Eq. 20. Mogilner and Zolotukhin^{3,4} have applied the zero-count probability procedure to various subcritical assemblies to measure α vs. inverse multiplication and α vs. number of fuel elements removed, to extrapolate them to the critical value β/Λ , and also to determine the absolute power level.

Comparisons of the zero-count probability method with the frequency analysis of the reactor power spectral density (see Ref. 1, Bibliography, p. 98), showed satisfactory agreement within the limits of error.

Pacilio^{10,12} has applied the zero-count probability method to a critical thermal system to measure β/Λ and the detector efficiency. The procedure has been repeated for several time intervals Δt and at various fission-rate levels to check some of the fundamental equations of the method. The final results agreed well with data obtained by other statistical techniques. The precision of the method has been particularly improved by collecting a rather large number of samples and a suitably dense distribution of experimental points of the Y vs. Δt plot in the $\alpha \Delta t \lesssim 1$ region.

Other measurements of this type have been made by Bruna et al.⁸ to determine the critical moderator level and the critical value of α and by Lindeman and Ruby¹⁴ to determine reactivity in subcritical assemblies.

Dragt and Turkcan^{15,16} commented about the possible misinterpretations in the data treatment. For small values of $\alpha \Delta t$ (see Eq. 22) the Y vs. Δt relation reduces to a linear one, and the values of α and Y_1 cannot be separately obtained. Dragt and Turkcan discussed several examples of the least-squares-fit problem in relation to their experiments in subcritical assemblies.

POLYA-MODEL METHOD

The Polya-model method is based on determining the relative variance of counts from a neutron detector by directly comparing the frequency distribution of counts, i.e., the ensemble of the fractions f_k of samples with content equal to $k = 0, 1, 2, \dots$ pulses to the total number of samples, with the theoretical distribution function.

This method can be well approximated by the probability profile of the Polya urn model, which was introduced by G. Polya to describe some aspects of the theory of games and the aftereffects of contagious diseases. The method is obviously still efficiency sensitive and can be applied only to thermal systems where the relative variance of counts is significantly greater than unity even in the prompt-neutron time domain.

The method has some advantages compared with other related statistical techniques, namely, the Feynman variance method and the zero-count probability method, since it avoids some of the uncertainties that could affect them. The Feynman variance method calculates Y as the difference between the total relative variance and the random component of the relative variance equal to unity. Noise components not related to the neutron-population-fluctuation phenomena might be included, and they cannot be eliminated unless the actual distribution of neutrons is known. The zero-count probability method is too restrictive since only one point, f_0 , of the experimental distribution is used to derive the variance of counts and is therefore affected by all the possible consequences of a zero-dimensional measurement.

Mathematical Formulation

If $p_k(\Delta t)$ is the probability of counting k pulses in a time interval Δt , then

$$p_k = \frac{\langle n \rangle^k}{k!} (1 + Y)^{-\frac{\langle n \rangle}{Y}} - k \prod_{h=0}^{k-1} \left(1 + \frac{hY}{\langle n \rangle} \right) \quad (53)$$

or in simpler recursion formulas

$$p_k = \frac{\langle n \rangle + (k-1)Y}{k(1+Y)} p_{k-1} \quad (54)$$

$$p_0 = (1+Y)^{-\langle n \rangle/Y} = e^{-\langle n \rangle/Y \log(1+Y)}$$

where the dependence on Δt , present in $\langle n \rangle$ and Y , has been omitted for simplicity.

Equation 53 has been formulated by Mogilner and Zolotukhin,^{4,7} justified and discussed by Pal⁶ and Pacilio,^{1,3} and proved valid in the limitation of prompt-neutron kinetics.

Equation 53 is actually an approximation of a more general formula introduced by Pal⁵ and later by Mogilner and Zolotukhin.⁷ This formula does not directly express the probability of k counts in a time interval Δt but is given in the form of a probability-distribution generating function whose derivatives yield p_k .

The successive derivatives lead to such complicated analytical expressions that they are practically useless for any theory-experiment comparison, especially since significant probability profiles are obtained for $5 \lesssim \langle n \rangle \lesssim 10$, which means that the theoretical p_k and the observable frequencies f_k for $k \lesssim 20$ (up to 20th-order derivatives!) must be compared. No equations for p_k are available for the Pal-Mogilner-Zolotukhin distribution. For this reason, the Polya-model distribution, sometimes called a binomial negative distribution, has been adopted as an approximate version of the Pal-Mogilner-Zolotukhin distribution, primarily because it has some suggestive recursion properties that make it easy to handle and calculate.

The question of the validity and the limitation of such an approximate approach are extensively discussed in the papers by Mogilner and Zolotukhin,⁷ Pal,⁶ Pacilio,^{1,3} and Dragt and Turkcan.^{1,5} The problem, obviously an extension of the problems of the zero-count probability method, concerns the different limitations for some of the leading parameters of the experiment, namely $\alpha \Delta t$, Y_1 , and reactivity.

The mathematical and physical characteristics of Eq. 53 are as follows:

1. In a purely uncorrelated process, the quantity Y vanishes and Eqs. 54 have the limits

$$p_k = \frac{\langle n \rangle}{k} p_{k-1}$$

and

$$p_0 = e^{-\langle n \rangle}$$

so that

$$P_k = \frac{\langle n \rangle^k}{k!} e^{-\langle n \rangle}$$

which is the Poisson distribution. Actually the limit holds for very low detection efficiencies and for experiments in extremely subcritical configurations.

2. The Pal—Mogilner—Zolotukhin probability profile and the probability profile obtainable from the Polya model have the same first- and second-order moments—the same mean value and absolute variance and therefore the same relative variance. The higher-order moments may be different (for a certain value of $\alpha \Delta t$ the third-order moments also coincide), but they affect the extreme tails of a distribution, which can be suitably cut from the experimental data since the poor statistics of the third-order moments make them rather insignificant.

3. The statistical error in Y is given by

$$\frac{\delta Y}{Y} = \left\{ \frac{1}{N} \left[\frac{1}{Y} \left(4 + \frac{3}{\langle n \rangle} \right) + 2 + \frac{3}{\langle n \rangle} + \frac{Y}{\langle n \rangle} \right] \right\}^{\frac{1}{2}}$$

and is seen to be strongly dependent on N and weakly dependent on $\langle n \rangle$. A parametric study of the preceding formula has been done to find the order of magnitude of the quantities which optimize the experiment for a desired precision.

Experimental Procedure

The measuring apparatus shown in Fig. 4 is evidently useful even for the Polya-model method. The distributive method is well suited to give directly the frequency distribution of counts in terms of the quantities $N_k(\Delta t)$ that denote the number of pulses in the $(k + 1)$ th channel of the multiscaler, i.e., the number of time intervals with content equal to k pulses.

The two alternatives offered by lines 2 and 4 in Fig. 4 and all the consequent considerations are still valid and are applicable even to this procedure. Lines 1 and 3 are not suitable for this method since they need too much data reduction to transform the raw output from the time-sequence form to the distributive form.

The measuring apparatus can be checked by counting a random source and comparing the frequency distribution of counts with the Poisson probability profile. If such a check test is tried in the millisecond region (or below), the observed frequency distribution may depart considerably from the expected Poisson distribution; the tails run below the expected profile, and the central part of the curve runs above it. The damping of high repetition rates and the consequent damping of the true mean value, both due to the dead time of the measuring apparatus, cause the deformation and the back-shift of the experimental profile with respect to the expected one.

Suitable corrections have been suggested and applied to restore the original distribution, i.e., the true moments of the observed counting variable. Extensive studies on both random and correlated events have been made by Pacilio^{1,1} and by Furuhashi.^{2,3}

Interpretation of Data

After the frequency distribution $\{N_k\}$ of samples for different values of the time interval Δt has been collected, the problem of calculating Y from the experimental data arises. A procedure based on a least-squares fit can be adopted; the optimum values of $\langle n \rangle$ and Y are those which minimize the quantity

$$\chi^2 = \sum_0^{\infty} \frac{(N_k - N_{Pk})^2}{N_{Pk}}$$

i.e., the value of $\langle n \rangle$ and Y obtainable from equations

$$\frac{\partial \chi^2}{\partial \langle n \rangle} = \frac{\partial \chi^2}{\partial Y} = 0$$

This least-squares fit approach has been suggested and applied by Mogilner and Zolotukhin.⁴ The method, analytically rigorous and elegant, is actually quite cumbersome if the theoretical formulation of the p_k (Eq. 53) is recalled.

An alternate treatment minimizes the quantity

$$\sum_0^{\infty} w_k (\beta_k - b_k)^2$$

where

$$\begin{aligned} b_k &= \frac{p_k - p_{k-1}}{p_{k-1}} \\ &= \frac{\langle n \rangle - (K + Y)}{K(1 + Y)} \end{aligned}$$

and the corresponding observed quantity β_k is

$$\beta_k = \frac{N_k - N_{k-1}}{N_{k-1}}$$

If the weights w_k are assumed identically equal to unity and if the value of $\langle n \rangle$ is determined in another way, e.g., by a gated scaler operating during the whole measurement time, the relative variance of counts can be calculated as

$$1 + Y = \frac{(\langle n \rangle + 1)^2 \sum \frac{1}{k^2} - 2(\langle n \rangle + 1) \sum \frac{1}{k} + \#(k)}{(\langle n \rangle + 1) \left(\sum \frac{1}{k^2} + \sum \frac{\beta_k}{k} \right) - \sum \beta_k - \sum \frac{1}{k}}$$

where $\#(k)$ indicates the number of values of k used in the sums of the right-hand side of the preceding equation.

This last alternate data treatment, used by Pacilio,^{9,10} led to a value of the variance and a determination of α and Y_1 in satisfactory agreement with the results obtained for the same quantities through the zero-count probability, the Feynman variance, and the endogenous pulsed source methods.

The observed distribution coincided with the Polya-model probability profile with a precision better than 1% even for a wide range of the involved parameters.

Ruby¹⁸ recently fitted some reactor data to various distribution functions and found the best consistency when they were compared with the negative binomial distribution.

OTHER RELATED PROBABILITY METHODS

A certain number of commonly used procedures intermediate between the zero-count probability method and the Polya-model method are based on simple and synthetic properties of the negative binomial distribution and of the Pal-Mogilner-Zolotukhin distribution summarized as follows.

Zero- to One-Count Probability Ratio Method

If the recursion formula of Eq. 53 is limited to $k = 1$,

$$\frac{\langle n \rangle p_0}{p_1} = 1 + Y$$

which suggests a useful method to determine the relative variance of counts. This procedure has been applied by Bruna et al.,⁸ Lindeman and Ruby,¹⁴ and Turkcan and Dragt.¹⁶

Furuhashi-Izumi Probability Method

If the ratio of p_0 to p_1 yields the quantity $1 + Y$, a quantity typical of the Feynman variance experiment, a particular combination of p_0 , p_1 , and p_2 leads to the determination of the index $1 + Z$ introduced by Furuhashi and Izumi¹⁷ in the expected number of correlated-neutron-pairs method.

From Eq. 53, the equation

$$\frac{2p_0p_2}{p_1^2} = 1 + Z$$

can be obtained, which establishes a measuring procedure of the parameter Z for different time intervals Δt . From the Z vs. Δt graph, the quantities α and B/A can be obtained through the usual least-squares fit methods.

Both the zero- to one-count probability ratio method and the Furuhashi-Izumi probability method are limited. They need more complicated equipment than that used for the zero-count probability experiment (namely, lines 2 or 4 of Fig. 4 instead of the anticoincidence circuit of Fig. 6), and they simultaneously use only a few points of the observed frequency distribution $\{N_k\}$ to reach a significant value of Y or Z . The complete frequency profile $\{N_k\}$ can be determined by the Polya-model method in the same time and with the same apparatus, and certainly the Polya-model method gives more accurate values of Y and Z .

These disadvantages are not exhibited by another procedure, the even- or odd-count probability method, which uses simple equipment and still gives useful information about the relative variance of counts in a certain time interval Δt . This method is based on the probability of counting an even or an odd number of neutron pulses in a given time interval Δt .

Even- or Odd-Count Probability Method

From the basic relation between the probability p_k and its generating function $G(s)$,

$$p_k = \frac{1}{k!} \left(\frac{\partial^k G(s)}{\partial s^k} \right)_{s=0}$$

The following equation is valid for any type of discrete distribution:

$$P_{\text{even}} - P_{\text{odd}} = \sum_0^{\infty} \frac{(-1)^k}{k!} \left(\frac{\partial^k G(s)}{\partial s^k} \right)_{s=0} = G(-1) \quad (55)$$

where P_{even} (or P_{odd}) is the probability of an even (or odd) number of pulses, i.e., the sum of the probabilities of counting 0, 2, 4, ..., $2k$, ... (or 1, 3, 5, ..., $2k+1$, ...) pulses in the time interval Δt .

If Eq. 55 is applied to various generating functions, then, for a Poisson distribution,

$$p_{\text{even}} - p_{\text{odd}} = e^{-2\langle n \rangle} \quad (56)$$

for a Pal-Mogilner-Zolotukhin distribution,

$$p_{\text{even}} - p_{\text{odd}} = \exp \left[- \left\{ \frac{4A \Delta t}{1 + \gamma} + \frac{A}{B} \log \frac{(\gamma' + 1)^2 - (\gamma' - 1)^2 e^{-\alpha \gamma' t}}{4\gamma'} \right\} \right] \\ \text{[with } \gamma' = (1 + 4Y_1)^{1/2}] \quad (57)$$

and for a Polya distribution,

$$p_{\text{even}} - p_{\text{odd}} = (1 + 2Y)^{-\langle n \rangle / Y} \quad (58)$$

where Eq. 56 can also be obtained as a limit of Eqs. 57 and 58 for $Y \rightarrow 0$ and $\gamma' \rightarrow 1$, i.e., for $\epsilon \ll (1 - K_p)^2$.

The comparison between Eqs. 57 and 58 is plotted in Fig. 10.

The equations suggest a method based on the measurement of the quantities $N_{\text{even}} - N_{\text{odd}}$ (N_{even} is the number of samples with content equal to an even number of pulses) and $\langle n \rangle$. This method leads to a determination of Y vs. Δt , namely, of α and Y_1 , through the transcendental relation

$$\log(1 + 2Y) = QY$$

where

$$Q = \frac{1}{\langle n \rangle} \log \frac{N}{N_{\text{even}} - N_{\text{odd}}}$$

for the Polya-distribution. If the Pal-Mogilner-Zolotukhin distribution is assumed to be a distribution of samples, a least-squares fit to Eq. 57 vs. Δt yields the values of α and γ .

The advantage of this technique consists in straightforward collection of data, ignoring the numerical content of each analyzed counting interval Δt and in this way being realizable by a one-bit counter. The measuring apparatus can consist of a flip-flop circuit into which the discriminator output is fed. The flip-flop is reset by an external timer, and its output may be read just in terms of N_{even} (or N_{odd}) since the simple relations hold that

$$N_{\text{even}} - N_{\text{odd}} = 2N_{\text{even}} - N = N - 2N_{\text{odd}}$$

As usual the entire system can be checked by counting a random source and verifying the corresponding relation between p_{even} , p_{odd} , and $\langle n \rangle$.

The even- or odd-count probability procedure has been proposed and successfully applied by Turkcan and Dragt.^{15,16} The results obtained agree with

Fig. 10. The probability $p_{\text{even}} - p_{\text{odd}}$ as a function of the time interval Δt for $\alpha = 200 \text{ sec}^{-1}$ and $A = 1000 \text{ counts/sec}$.

results from other techniques although the precision is inferior with respect to other methods. The even- and odd-count probability method has introduced a new approach for the statistical techniques, based on the possibility of measuring the same parameters as previously discussed procedures with an approximately comparable precision, although the information collected is limited to a minimum (only one bit per sample).

For a Poisson distribution, the mean value can be determined, even if only the parity of the events (even or odd number of counts in a given time interval) is recorded and any other numerical information is disregarded.

In a Pal—Mogilner—Zolotukhin or in a Polya distribution, the relative variance of counts can be obtained by simply recording the parity of events, independently from their time sequence, and the long-time averaged mean value, which can be separately measured by a gated scaler operating during the whole measurement time.

This new approach has other interesting applications in the polarity techniques, based on the collection of only the sign of the fluctuations of a steady-state operating reactor power level with respect to the mean value of the fluctuations. Even in this case, some fundamental reactor physics quantities, such as the autocorrelation function, the crosscorrelation function, and the covariance of counts, can be obtained as well, although the numerical analysis of the neutron-population phenomena is completely bypassed and only one bit of information per sample (i.e., the polarity, the sign of the difference between the instant level and the average level of reactor flux) is recorded.

These methods are extensively treated in Part IV.

TIME INTERVAL FROM A RANDOM ORIGIN METHOD

The time interval from a random origin method is based on the determination of the time interval distribution between a randomly chosen instant in the reactor life and the first-arriving pulse from a neutron detector.

The basic difference in this method, compared with the Rossi-alpha or autocorrelation experiment, lies essentially in when the time analysis is made to start, i.e., in how the triggering pulse is chosen.

The Rossi-alpha experiment needs a neutron pulse to make the analyzer start the sweep, whereas this method can be triggered at random, i.e., the sequence of triggering pulses must obey a Poissonian law.

This experiment also differs from the zero-probability method in which the analysis concerns the probability of no counts in a given time interval and therefore the probability of a count any time later. This method implies the additional condition that one count occurs at the end of the given interval.

The method is efficiency sensitive and therefore cannot be applied to fast reactors where the relative variance of counts from a neutron detector is significantly different from unity (the variance of a random process) only for extremely high (usually unavailable) detection efficiency.

Mathematical Formulation

If $\pi(t) dt$ is the probability that after a time $t = 0$ chosen at random the first pulse arrives between t and $t + dt$, then

$$\pi(t) dt = 2A p_0(t) \frac{\gamma + 1 + (\gamma - 1) e^{-a\gamma t}}{(\gamma + 1)^2 - (\gamma - 1)^2 e^{-a\gamma t}} dt \quad (59)$$

where $p_0(t)$ is the probability of no counts in the time interval 0 to t , i.e., the probability that, after a time $t = 0$ chosen at random, the first pulse arrives at any time greater than t .

Equation 59, derived by Babala,¹⁹ holds in the prompt-neutron kinetics approximation because of the range of the time intervals involved. The following remarks about Eq. 59 clarify the physical meaning of the experiment:

1. The dependence of $\pi(t)$ on t is plotted in Fig. 11 for various values of Y_1 , i.e., for various degrees of correlation. The case involving $Y_1 = 0$ (pure Poisson process) is also plotted for comparison and appears to be a simple exponential.

2. In a purely random process, the probability $\pi(t) dt$ reduces to the product of two independent terms: the probability of no counts in the interval t (e^{-At}) and the probability of one count between t and $t + dt$ ($A dt$). Thus

$$\pi(t) dt = e^{-At} A dt$$

This result can be obtained from Eq. 59 by setting $\gamma = 1$, for very low detection efficiencies and for very subcritical configurations. In both cases the detector can no longer distinguish a significant number of neutrons belonging to the same decay chain and so regards them all as uncorrelated.

3. The term $\pi(t)$ is strictly related to $p_0(t)$ under fairly general assumptions. If the definition of $\pi(t)$ and $p_0(t)$ are recalled, then

$$\int_0^t \pi(\tau) d\tau$$

expresses the probability that after a time $t = 0$ chosen at random the first pulse arrives between 0 and t , whereas $p_0(t)$ is the probability that the same event occurs between t and infinity. Therefore

$$p_0(t) + \int_0^t \pi(\tau) d\tau = 1$$

which can be used to formulate $\pi(t)$ as a function of $p_0(t)$, i.e.,

$$\pi(t) = - \frac{\partial p_0(t)}{\partial t} \quad (60)$$

If Eq. 50 is used to replace $p_0(t)$, Eq. 59 is promptly obtained.

4. The average time interval $\langle t_{op} \rangle$ between a random origin and the first-arriving pulse is given by

$$\begin{aligned} \langle t_{op} \rangle &= \int_0^\infty t \pi(t) dt \\ &= \int_0^\infty p_0(t) dt \geq 1/A \end{aligned} \quad (61)$$

Fig. 11. The probability density $\pi(t)$ as a function of the time interval t for $\alpha = 200 \text{ sec}^{-1}$ and $A = 1000 \text{ counts/sec}$.

The last relation holds only in the terms of equality for a Poisson process. For correlated events the larger the degree of correlation Y_1 and the prompt decay constant α , the larger $\langle t_{op} \rangle$ is with respect to $1/A$. Some numerical examples are plotted in Fig. 12.

The physical meaning implied in the inequality

$$\langle t_{op} \rangle > 1/A \quad (62)$$

seems contradictory with the fact that the average time interval between

Fig. 12. Average time interval between a random origin and the first-arriving pulse as a function of the degree of correlation for $A = 1000$ counts/sec and for various values of the prompt-neutron decay eigenvalue α .

consecutive pulses is always equal to $1/A$ for any degree of correlation among the pulses (see next method when $\langle t_{pp} \rangle$ is introduced).

It could be concluded that, if the average time interval between two consecutive pulses is equal to $1/A$, the average time interval between any instant following the first pulse and the second pulse would be smaller than $1/A$.

Actually, Eq. 62 is a consequence of the way the neutron pulses are distributed on the time axis because of the fission events. Assume that the average counting rate is A and therefore that the average time interval between two successive pulses is $1/A$: visualize that the neutron pulses occur as clusters of counts alternated with long empty intervals, as the platykurtic properties of the Polya profile with respect to the Poisson profile imply; i.e., intervals with no counts and with a large number of counts are much more probable than in a purely random process.

Thus, if the triggering of the time analysis is chosen at random, with no correlation to the clusters of counts, it is more likely to be followed by one of the empty valleys than to precede an immediately arriving pulse.

The larger the correlation involved, the more accentuated is the flattening of the distribution of counts. Therefore, because of an increased number of intervals with no counts or with $k \gg A$ at pulses, the quantity $\langle t_{op} \rangle$ is easily seen as increasing.

Similarly the larger the value of α (i.e., the shorter the lifetime of the chain consisting of a cluster of fissions), the longer are the intervals with no pulses and therefore the larger is the value of $\langle t_{op} \rangle$ with respect to $1/A$.

5. The difference in the physical meaning and dimensions of $p_0(t)$ and $\pi(t)$ is pointed out by normalization and by their boundary conditions [e.g., $\pi(t)$ is a probability per unit time, whereas $p_0(t)$ is not]:

$$\begin{aligned} p_0(0) &= 1 & \lim_{t \rightarrow \infty} p_0(t) &= 0 & \int_0^{\infty} p_0(t) dt &= \langle t_{op} \rangle \\ \pi(0) &= A & \lim_{t \rightarrow \infty} \pi(t) &= 0 & \int_0^{\infty} \pi(t) dt &= 1 \end{aligned}$$

Further questions related to items 4 and 5 have been extensively treated by Pacilio.²²

Experimental Procedure and Interpretation of Data

The measuring apparatus must clock the time interval between a random origin and the first-arriving pulse. A time analyzer clearly fits this purpose since it must be triggered at random. The pulses emitted by a long-half-life radioactive substance may be one trigger. Actually, the time-analyzer sweep does not start with that external pulse but with the first-arriving pulse from the internal oscillator after the pulse from the radioactive substance (or any other random pulse generator).

Since there is no correlation between the triggering pulse and the sweep-start pulse, the delay introduced between the two pulses does not affect the correctness of the procedure.

When the first pulse arrives from the neutron detector, the analyzer adds a count in the channel swept at that moment and resets the address register to prepare it for a new cycle. This new cycle begins with the first-arriving pulse from the random-emitting source after the register has been reset.

Thus only one event per sweep is recorded, and the time distance of a pulse from the origin of the time axis (zero-channel) is actually the time interval from a random origin. The data collection is repeated for a large number of cycles and accumulated in the time analyzer.

The procedure is effective in thermal reactor experiments, where the Rossi-alpha method must usually be discarded, and it is faster and more direct than the zero-count probability method since only one run is needed to describe the entire $\pi(t)$ vs. t behavior.

A three-parameter least-squares fit to Eq. 59 yields the values of A , γ , and α from which information on the basic reactor kinetics quantities is derived. As is evident from Eq. 59, the dependence of $\pi(t)$ on these values is more complicated than in the Rossi-alpha experiment.

Austin et al.²¹ have applied the time interval from a random origin method under the modified designation "waiting time-alpha method." Their results

were consistent with the determination of the same prompt-neutron eigenvalue made by the conventional Rossi-alpha technique and some pulsed-neutron measurements.

PULSE TO PULSE TIME INTERVAL METHOD

The pulse to pulse time interval method is based on the determination of the time interval distribution between consecutive pulses, or, more precisely, on the probability that after a pulse at $t = 0$, the *successive* pulse comes between t and $t + dt$.

The basic difference in this method and the Rossi-alpha experiment lies in the distribution analyzed; the distribution of counts following a certain triggering pulse (a neutron detection) is analyzed in the Rossi-alpha experiment, and the distribution of the pulse immediately following the trigger is analyzed in this method.

Thus the probability expressed in the pulse to pulse time interval method differs from that for the Rossi-alpha method because it contains the additional condition that no pulse occurs in the time interval 0 to t . For very low detector efficiency and counting rates, the two experiments tend to coincide.

From a theoretical viewpoint the pulse to pulse time interval method can be regarded to be as effective as the Rossi-alpha procedure for fast reactor experiments but to be greatly preferred for thermal experiments since the Rossi-alpha method is rather time consuming and the autocorrelation method needs elaborate off-line computation.

Another advantage of the pulse to pulse time interval technique is that it is not extremely limited when only low detection efficiency and high power (source) level are involved, unlike most other statistical methods. Even in such extreme cases, the final formula still contains significant information about the required reactor parameters.

Mathematical Formulation

If $p(t) dt$ denotes the probability that after a pulse at $t = 0$, the next pulse comes within the interval t to $t + dt$, then

$$p(t) dt = C_1(t) dt + C_2(t)e^{-\alpha\gamma t} dt \quad (63)$$

where the quantities $C_1(t)$ and $C_2(t)$ are given by

$$C_1(t) = 4Ap_0(t) \left[\frac{\gamma + 1 + (\gamma - 1)e^{-\alpha\gamma t}}{(\gamma + 1)^2 - (\gamma - 1)^2 e^{-\alpha\gamma t}} \right]^2 \quad (64)$$

and

$$C_2(t) = \frac{16B p_0(t)\gamma^2}{[(\gamma + 1)^2 - (\gamma - 1)^2 e^{-\alpha\gamma t}]^2} \quad (65)$$

Equation 63, derived by Bábala,¹⁹ is clearly valid in the prompt-neutron approximation because of the range of the time intervals involved. Some characteristics of Eq. 63 can be summarized as follows:

1. Interestingly the pulse to pulse time interval method and the time interval from a random origin method coincide in a single procedure if the observed time sequence of pulses is of random nature. In this case the two events that trigger the time analysis have the same stochastic properties.

2. For a purely random process, the probability $p(t) dt$ reduces to

$$e^{-At} A dt$$

which is obtained from Eq. 63 by setting the second term equal to zero and $\gamma = 1$ in the first term, i.e., by canceling any degree of correlation among the pulses. The formula thus obtained clarifies the meaning of the probability $p(t) dt$ in terms of the composition of the probability of no counts in a time interval t (e^{-At}) and the probability of one count between t and $t + dt$ ($A dt$).

3. The probability $p(t) dt$ depends sensitively on both the power (source) level and the detector efficiency and yields useful information even in conditions prohibitive to other statistical techniques.

Assume, for instance, that the reactor is operating at high power (source) level conditions that make the Rossi-alpha or autocorrelation experiment useless for parameter measurements. In this case

$$\lim_{A \gg B} p(t) dt = C_1(t) dt \quad (66)$$

which is still related to the required reactor parameters.

Now assume that a very low detection efficiency is available and therefore all the efficiency-limited methods (see Parts II and III) are unapplicable. In this case

$$\lim_{Y_1 \ll 1} p(t) dt = e^{-At}(A + Be^{-\alpha t}) \quad (67)$$

which is a result comparable to the final formula of the Rossi-alpha or autocorrelation method with the exception of an exponential as a multiplying factor.

For fast reactor experiments the exponential is approximately equal to unity because of the low counting rate and the very small values of t ($\sim 10 \mu\text{sec}$); in

this case the two experiments practically coincide. This value of the exponential also explains why the stop-watch technique introduced by Brunson et al.¹ works like the Rossi-alpha method in fast experiments although it is closer to a pulse to pulse time interval experiment. However, the stop-watch technique cannot be completely assimilated to the pulse to pulse time interval, because of the discontinuous data collection (see Experimental Procedure of the Rossi-alpha Technique, and compare it with the logic needed for this method).

Actually, Eqs. 66 and 67 are just limiting cases, and Eq. 63 must be regarded as the fundamental formula of the procedure. Nevertheless, Eqs. 66 and 67 show that even in extreme conditions the pulse to pulse time interval method can give meaningful information about reactor parameters.

4. The dependence of $p(t)$ on t is plotted in Fig. 13 for various values of Y_1 , including $Y_1 = 0$, which gives the simple exponential behavior concerning a random process. Four related probability methods, i.e., Rossi-alpha, zero-count probability, random origin, and pulse to pulse time interval methods, are compared in Figs. 14A to 14D, where the basic event characteristic of the four different probabilities is schematically illustrated. The physical meaning of the time interval t is specified for each case.

5. The quantity $p(t)$ is closely related to $\pi(t)$ and therefore to $p_0(t)$ under slightly more restrictive conditions than those necessary to formulate $\pi(t)$ as a function of $p_0(t)$.

If a process is assumed for which the correlation among pulses is symmetrical in time, $\pi(t) dt$ is also the probability that after a pulse arrives between 0 and dt an empty interval t follows.

The same probability can be expressed as a product of the probability of one count between 0 and dt ($A dt$) by the probability of the next pulse occurring at any time greater than t . The latter can be calculated as

$$1 - \int_0^t p(\tau) d\tau$$

the integral being the probability that after a pulse at time $t = 0$ the next pulse arrives between 0 and t . Thus

$$\pi(t) dt = A dt [1 - \int_0^t p(\tau) d\tau]$$

which can be used to formulate $p(t)$ as a function of $\pi(t)$ and therefore of $p_0(t)$,

$$p(t) = -\frac{1}{A} \frac{\partial \pi}{\partial t} = \frac{1}{A} \frac{\partial^2 p_0(t)}{\partial t^2}$$

If Eqs. 50 and 59 are used to replace $p_0(t)$ and $\pi(t)$, respectively, Eq. 63 is obtained.

Fig. 13. The probability density $p(t)$ as a function of the time interval t for $\alpha = 200 \text{ sec}^{-1}$ and $A = 1000 \text{ counts/sec}$.

6. The average time interval between two successive pulses is given by

$$\begin{aligned}
 \langle t_{pp} \rangle &= \int_0^{\infty} t p(t) dt \\
 &= 1/A \int_0^{\infty} \pi(t) dt \\
 &= 1/A
 \end{aligned}$$

for any degree of correlation in the events.

Fig. 14. Comparison of phenomenology among four correlation methods.

7. Boundary conditions for $p(t)$ depend on reactor parameters opposite to those concerning $\pi(t)$. Normalization conditions do not. Therefore

$$p(0) = A + \frac{1}{2} \alpha Y_1 \quad \lim_{t \rightarrow \infty} p(t) = 0 \quad \int_0^{\infty} p(t) dt = 1$$

Further considerations on $p(t)$ are reported in detail by Pacilio.²²

Experimental Procedure and Interpretation of Data

The measuring apparatus must clock the time interval between consecutive pulses. The previously described stop-watch time analyzer can be used, but half of the data is lost, as explained in Part I. For a full utilization of the arriving information, an ad hoc adaptation of a time analyzer must be used which can work as follows:

The sequence of counts coming from the discriminator is fed into the time analyzer so that the first-arriving pulse triggers the sweep of the channels. Particular attention must be given to correcting the delay effect introduced by the completely random relation between the arrival of the triggering pulse and the first pulse from the analyzer internal timer, which actually begins the sweep.

When a second pulse occurs, the analyzer adds a count to the channel activated at that instant, resets the address register, and triggers another sweep. This three-part operation subtracts time in all three phases from the effective interval between successive pulses and yields delay effects that have to be taken into account to restore the true time correlation among the pulses.

Thus only one event per sweep is recorded, and the suitably corrected time distance of a pulse from the origin of the time axis (zero-channel) is actually the separation between that pulse and the preceding one. The process is repeated many times and accumulated in the time analyzer.

No correction formula is available at the moment for the various delay effects.

Babala and Ogrin²⁰ used the method and noticed that the procedure is rather effective in thermal reactor experiments. It replaces the time-consuming Rossi-alpha method and is faster and more flexible than the zero-count probability method since, unlike that procedure, a single run is needed to describe the entire $p(t)$ vs. t behavior.

Apart from the already discussed systematic errors introduced by the measuring apparatus, the pulse to pulse time interval method needs more work in the data treatment if a least-squares fit to Eq. 15 is tried. Four parameters are present in the expression of $p(t)$ as compared to the three parameters of $P(t)$ in Eq. 1, and the dependence on the most important of them, α and γ , is more complicated than that of a single exponential plus background.

The values of α determined through this procedure by Babala and Ogrin²⁰ are accurate and in good agreement with values measured via the Feynman variance and the zero-count probability methods.

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IV CORRELATION ANALYSIS AMONG THE SIGNS OF THE FLUCTUATIONS

POLARITY AUTOCORRELATION AND CROSS-CORRELATION METHOD

The polarity correlation method is based on the determination of the autocorrelation (or cross-correlation) function of the output of one (or two) neutron detector, where this output is not numerical but is simply +1 or -1 depending on whether the amount of detector fluctuations is above or below the mean value, at the instant of output.

The method is particularly applicable to non-zero-power systems where pulse counters cannot be adopted because of their limited resolution time. Furthermore, the cross-correlation technique ameliorates the inconvenience of low detection efficiencies in fast reactors; thus the procedure can be extended to all types of nuclear systems.

The method needs a polarity monitor (a detector of the sign of the difference between the fluctuations and their mean value) and the usual data reduction typical of the correlation algorithm, based essentially on the operations of delay, multiplication, and addition. These operations are obviously greatly simplified if the involved quantities are only binary variables with unit module.

Under specified conditions of the fluctuations, the polarity correlation function can be expressed in terms of the numerical correlation function and therefore as a function of reactor kinetic parameters.

Mathematical Formulation

If $x_1(t)$ and $x_2(t)$ are the compensated counting rates from two neutron detectors inside or near the reactor, the numerical cross-correlation function and the polarity cross-correlation function are defined as

$$C(\tau) = \langle x_1(t) x_2(t + \tau) \rangle \quad (68)$$

$$\text{Pol}(\tau) = \langle \text{sign } x_1(t) \text{ sign } x_2(t + \tau) \rangle \quad (69)$$

respectively. The mean values $\langle x_1(t) \rangle$ and $\langle x_2(t) \rangle$ are assumed to vanish, and the new symbols denote

$$\text{sign } x(t) = \begin{cases} +1 & [\text{if } x(t) \geq 0] \\ -1 & [\text{if } x(t) < 0] \end{cases}$$

The dependence of $C(\tau)$ on reactor parameters is given by Eq. 13.

If the amplitude distribution of $x_1(t)$ and $x_2(t)$ is assumed to be Gaussian, the two functions are related by

$$\sin \frac{\pi}{2} \text{Pol}(\tau) = \frac{C(\tau)}{C(0)} = R(\tau) \quad (70)$$

Veltman and Kwakemaak¹ derived Eq. 70, and Dragt³ has applied it to reactor noise analysis.

The more general case of an Edgeworth distribution for the amplitudes of $x_1(t)$ and $x_2(t)$ has been treated by Pacilio,⁶ who found that Eq. 70 is a particular aspect of the following relation, valid for any skew and nonmesokurtic profiles.

$$\begin{aligned} \text{Pol}(\tau) = & \frac{2}{\pi} \arcsin R + \frac{2(1-R^2)^{-1/2}}{\pi 2!} (-Rb_{20} + 2b_{11} - Rb_{02}) \\ & + \frac{2(1-R^2)^{-3/2}}{\pi 4!} [(3R - 2R^3)b_{40} - 4b_{31} + 6Rb_{22} - 4b_{13} \\ & + (3R - 2R^3)b_{04}] + \dots \end{aligned} \quad (71)$$

where the dependence of R on the lag τ has been omitted for simplicity in the right-hand side of the equation and the quantities b_{rs} are the coefficients of the bivariate Edgeworth series which vanish for a Gaussian-probability surface. These coefficients are essentially the differences between the actual $(r+s)$ th-order moments of the observed distribution and the corresponding ones expected for a standardized Gaussian-probability solid.

Heins et al.² obtained Eq. 71 in the particular case of $x_1(t)$ and $x_2(t)$ as non-zero-mean variables.

Since $R(\tau)$ is usually small compared to unity, only the first-order corrections must generally be taken into account; they are either constant or proportional to $R(\tau)$ and thus do not perturb the time dependence, because of the exponential structure of $R(\tau)$, thus allowing an unbiased value of α to be derived.

Experimental Procedure

Although the polarity technique has been introduced essentially to simplify the process of quantization necessary when continuous time signals must be digitized, the procedure is also likely to be applied to output signals already in digital form, e.g., number of pulses.

Generally, the procedure samples, at a specified rate, the current from an ion chamber, located inside or near the reactor. The output of the integrating circuit is compensated by the long-time-averaged mean value (obtained, for instance, through a long-time-constant counting-rate meter) and gives a +1 or a -1 reading depending on whether the outcoming signal is positive or negative.

If on-line compensation is not possible, quantization to a polarity signal can be realized off-line by a computer once the various samples have been suitably digitized. Dragt³ in his autocorrelation measurements on a thermal reactor adopted the off-line approach.

An improved procedure, presented by Cohn,^{4,5} is based on an instant sampling of the reactor fluctuations. The noise is separated from the d-c component, amplified, and then fed into a Schmitt trigger adjusted to switch at zero volts. The trigger output is sampled in every desired period.

In this way the neutron fluctuations are sampled by two detectors and then cross-correlated. A block diagram of half of the equipment used by Cohn in his experiments on a fast reactor is shown in Fig. 15. An on-line computer has been used to obtain faster and more compact data reduction.

Fig. 15. Block diagram of the measuring apparatus for the polarity correlation method (instant-sampling method). Only one detector line is represented.

The question of the more-or-less Gaussian nature of the involved variables has been experimentally checked by Dragt.³ The frequency distribution of data, i.e., the ensemble of the values of $x(t)$, has been observed and compared with a normal distribution with satisfactory agreement.

Therefore Eq. 70 can be regarded as valid for reactor noise.

Interpretation of Data

The polarity technique has been used in two experiments—an autocorrelation experiment on a subcritical thermal reactor by Dragt³ and a cross-correlation experiment on a critical fast reactor by Cohn.⁵ The conclusions reached from these experiments can be summarized as follows:

1. The comparison between the numerically calculated autocorrelation function and that derived from the polarity autocorrelation function is very successful since no significant discrepancy has been found between the two behaviors as functions of the time lag τ . The standard deviation of $\text{Pol}(\tau)$ is only 1.5 times that of $R(\tau)$, as a simple differentiation of Eq. 70 would imply.

2. In fast reactors, where the detection efficiencies are rather low, $\text{Pol}(\tau)$ and $R(\tau)$ are essentially proportional so that $P(\tau)$ can be directly processed through a least-squares fit to Eq. 13. Cohn⁵ has found that $\text{Pol}(\tau)$ was better fitted by two exponentials plus background. The smaller observed eigenvalue, β/Λ , was analogous to the result obtained by extrapolating a series of subcritical pulsed-neutron experiments to criticality.

3. The time necessary to calculate the autocorrelation or cross-correlation function constitutes the major part of the experiment since the data collection is extremely rapid, especially in fast reactor measurements. An on-line computer spends about one hundred times as long to process a sequence of data as it does to collect it. Thus the computer is accepting about 1% of the available polarity data from the reactor. This situation arises since the correlation function is obtained from the polarity correlation, which, although more rapidly computed than the numerical correlation because of the one-bit data, is still a fairly elaborate algorithm to be handled by the computer.

This inefficiency could be circumvented by a wired-program cross correlator constructed to accept the data continuously from the reactor while simultaneously calculating the correlation function. Cohn⁵ has suggested such a device and discussed some of the involved physical and economic problems.

COVARIANCE METHOD VIA POLARITY DETECTION

The covariance experiment has been discussed in Part II as a two-detector procedure. Its application is improved if the covariance of a joint distribution of two variables is used to optimize the data-collection process.

Assume that a two-detector output is integrated over a time interval Δt and that the joint distribution of these outputs is $\{x_1(\Delta t), x_2(\Delta t)\}$.

This method is based on the fact that no numerical information about the two variables is indispensable for the calculation of the covariance since only knowing the sign of the two variables with respect to their mean values and using

some properties of the probability-distribution function allow simpler and faster determination of the covariance. The difference between this polarity sampling and that introduced in the preceding section is illustrated in Fig. 16, where the terms "integrated" and "instant" sampling have been used for the two procedures, respectively.

Let $\{\text{sign}[x_1(\Delta t) - \langle x_1(\Delta t) \rangle], \text{sign}[x_2(\Delta t) - \langle x_2(\Delta t) \rangle]\}$ indicate the joint distribution of the polarity variables. Four pairs are clearly possible according to various combinations of signs, i.e., ++, +-, --, and +-. Denote with P_1, P_2, P_3 , and P_4 the probability of appearance of each pair, respectively.

If the two variables are positively (or negatively) correlated, P_1 and P_3 (P_2 and P_4) are greater than P_2 and P_4 (or P_1 and P_3) according to the amount of coherence involved, which is clearly the covariance of the two variables for the adopted integration interval Δt .

Since experimental quantities very close to P_i ($i = 1, 2, 3, 4$) can be measured in terms of the observable frequencies of the four types of pairs and since the dependence of the covariance on reactor kinetic parameters previously described can be considered, the experiment can be interpreted as soon as the dependence of P_i on the covariance $\text{Cov}(\Delta t)$ is known.

Being a two-detector experiment, the covariance method is not limited by low-efficiency detection; however, because of the finite number of samples, the detection efficiency cannot be too low. This method can be applied to fast-reactor-noise analysis, unlike all the other variance procedures.

Since the detection of only the sign of the variables is needed, a pulse counter is dispensable, and therefore the technique is expected to be applicable even to non-zero-power reactors.

Mathematical Formulation

If $x_1(\Delta t)$ and $x_2(\Delta t)$ are the integrated outputs of two simultaneously observed detectors over a time interval Δt , $\langle x_1(\Delta t) \rangle$ and $\langle x_2(\Delta t) \rangle$ are their mean values over the entire ensemble of samples, and two new variables are introduced:

$$\text{sign}(i) = \text{sign}[x_i(\Delta t) - \langle x_i(\Delta t) \rangle] = \begin{cases} +1 & [\text{if } x_i(\Delta t) \geq \langle x_i(\Delta t) \rangle] \\ -1 & [\text{if } x_i(\Delta t) < \langle x_i(\Delta t) \rangle] \end{cases} \quad (i = 1, 2)$$

The joint distribution $\{\text{sign}(1), \text{sign}(2)\}$ can be considered. The pair may exhibit four possible combinations, namely ++, +-, --, and +-, whose probabilities can be denoted P_1, P_2, P_3 , and P_4 , respectively.

If the joint-distribution function of $x_1(\Delta t)$ and $x_2(\Delta t)$ is assumed to be a bivariate normal distribution, then

(a) POLARITY SAMPLING (INSTANT)

(b) POLARITY SAMPLING (INTEGRATED)

Fig. 16. Difference between instant polarity sampling (polarity correlation function method) and integrated polarity sampling (covariance method via polarity detection).

$$P_1 = \frac{1}{4} + \frac{1}{2\pi} \arcsin [\text{Cov}(\Delta t)]$$

$$P_2 = \frac{1}{4} - \frac{1}{2\pi} \arcsin [\text{Cov}(\Delta t)]$$

$$P_3 = \frac{1}{4} + \frac{1}{2\pi} \arcsin [\text{Cov}(\Delta t)]$$

$$P_4 = \frac{1}{4} - \frac{1}{2\pi} \arcsin [\text{Cov}(\Delta t)]$$

where $\text{Cov}(\Delta t)$, the covariance of the variables, has been explicitly formulated in Eq. 44. If the 2 variables are uncorrelated, then $P_1 = P_2 = P_3 = P_4 = 1/4$. This is illustrated in Fig. 17, where the geometrical meaning of the P_i can be visualized as the volume contained in each of the four superior octants of the joint probability-distribution-function representation. Zero-mean variables have been adopted, i.e., $x' = x - \langle x \rangle$. If $\text{Cov}(\Delta t) \neq 0$, the probability solid is modified (see Fig. 18) as much as the amount of coherence involved. That is,

$$\text{Cov}(\Delta t) = \sin \frac{\pi}{2} [P_1 + P_3 - (P_2 + P_4)] \quad (72)$$

The new symmetry planes $\xi_1 = 0$ and $\xi_2 = 0$ are also visible in Fig. 18.

Equation 72 has been derived by Pacilio.^{7,8} He considered the more representative case of a bivariate Edgeworth distribution, which can describe quasi-Gaussian probability solids affected by skew and nonmesokurtic properties. The following equation is a general relation between the P_i ($i = 1, 2, 3, 4$) and $\text{Cov}(\Delta t)$:

$$\begin{aligned} (P_1 + P_3) - (P_2 + P_4) &= \frac{2}{\pi} \arcsin (\text{Cov}) + \frac{1}{\pi} (1 - \text{Cov}^2)^{1/2} \\ &\times [-b_{20} \text{Cov} + 2b_{11} - b_{02} \text{Cov}] \\ &+ \frac{2}{4! \pi} (1 - \text{Cov}^2)^{-3/2} \{ (3 \text{Cov} - 2 \text{Cov}^3) \\ &\times b_{40} + 4b_{31} - 6b_{22} \text{Cov} + 4b_{13} \\ &+ (3 \text{Cov} - 2 \text{Cov}^3) b_{04} \} + \dots \end{aligned} \quad (73)$$

The dependence of Cov on Δt has been omitted for simplicity.

Few comments on the choice of the joint-distribution function are needed. The Edgeworth distribution seems to be particularly appropriate in the case of $x_1(\Delta t)$ and $x_2(\Delta t)$ as integrated outputs from the two detectors. The actual

Fig. 17. Contours of equiprobable values and the vertical profile for an uncorrelated bivariate Gaussian distribution (profile of rotation for the symmetrically generated surface).

one-dimensional probability distribution of $x_i(\Delta t)$ ($i = 1, 2$) is known to be the Pal-Mogilner-Zolotukhin distribution (see Part III, Polya-model method) which tends to the Poisson distribution for small detection efficiencies, i.e., $\epsilon \ll (1 - K_p)^2$, and to a Gaussian profile for large values of $\langle x_i(\Delta t) \rangle$.

Fig 18. Contours of equiprobable values for a correlated Gaussian distribution. Different values of covariance are assumed.

Thus the nonvanishing skewness (asymmetry due to the finite value of $\langle x_i(\Delta t) \rangle$) and kurtosis (flattening of the central top and enhanced tails of the distribution profile) coefficients can take into account the described deformations and still retain the flexible and effective formalism based on the numerous integral properties of the normal curves and surfaces.

Pacilio⁶ has studied the problem in detail, together with some possible experimental situations.

Experimental Procedure and Interpretation of Data

The measuring apparatus needed for such a covariance experiment is similar to that used for other statistical methods. Since a covariance experiment via polarity detection has not yet been realized until now, the following experimental scheme and description are suggestions chosen from one of many possible ways.

The noise can be separated from the d-c component, amplified, and then integrated by one of the two alternatively working RC integrating circuits (as in the Bennett variance experiment). The polarity, the sign of the integrated output, is then recorded in sequence form.

An identical procedure parallels this one for the other detector so that the simultaneous one-bit information about the sign of the integrated noise with respect to the mean fluctuation level is recorded.

A system can be studied which would give directly the value of the four frequencies corresponding to the four possible pair outputs.

For the interpretation of data, once the value of $\text{Cov}(\Delta t)$ has been found for several values of Δt , the rest of the data reduction and treatment follows the usual track of the covariance experiments, i.e., the determination of α and Y_1 through a least-squares fit to Eq. 73.

A kind of normalization of the covariance data is needed, however, which can be realized although non-numerical information on $\langle x_1(\Delta t) \rangle$ and $\langle x_2(\Delta t) \rangle$ is known. If a steady power (source) level is maintained throughout the whole set of measurements (and this is a necessary condition), then $\langle x_1(\Delta t) \rangle$ and $\langle x_2(\Delta t) \rangle$ vary linearly with the integration interval Δt ; therefore a suitable normalization of the quantities $\text{Cov}(\Delta t)/\langle x_i(\Delta t) \rangle$ must be realized, for instance, in terms of a multiplication factor $\Delta t/\Delta t^*$, where the time basis Δt^* can be chosen in an arbitrary way.

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A SYNOPSIS

OF NEUTRON STATISTICS METHODS

A synopsis of reactor-noise time-analysis procedures is given in Table 2. The procedures are grouped by category. The number of pulse counters needed for each experiment, the possible use of current chambers, the type of reactor (T for thermal, F for fast) to which the procedure can be applied, and the reactor power and detector efficiency needed to perform each experiment are indicated

Table 2 SYNOPSIS OF REACTOR-NOISE TIME ANALYSIS PROCEDURES

Type of analysis	Number of pulse counters	Possible use of current chambers	Type of reactor	Fission rate (F) and detector efficiency (ϵ) specifications
Correlation analysis among neutron pulses				
Rossi-alpha method (delayed coincidences, stopwatch)	2	No	F	Very low F
Rossi-alpha method (time-analyzer)	1	No	T	Very low F
Endogenous pulsed source method	1	No	T, F	Large ϵ
Autocorrelation analysis	1	Yes	T, F	
Cross-correlation analysis	2	Yes	T, F	
Time interval from a random origin	1	No	T	Large ϵ
Pulse to pulse time interval	1	No	T, F	
Analysis of the moments of the counting distribution				
Feynman variance method	1*	Yes	T	Large ϵ
Correlated-neutron-pair method	1*	Yes	T	Large ϵ
Bennett variance method	1	Yes	T	Large ϵ
Third-order analysis and correlated-neutron-triplets method	1*	Yes	T	Large ϵ
Covariance analysis	2*	Yes	T, F	
Analysis of the probability of a neutron detection				
Zero-count probability method	1	No	T	Large ϵ
Polya-model method	1	No	T	Large ϵ
Intermediate methods (Even- or odd-count, etc.)	1	No	T	Large ϵ
Correlation analysis among the signs of the power level fluctuations				
Polarity autocorrelation analysis	1	Yes†	T	
Polarity cross-correlation analysis	2	Yes†	T, F	
Covariance analysis via polarity detection	2	Yes†	T, F	

*Pulse counter preferred although either pulse counter or current chamber can be used.

†Current chamber preferred although either current chamber or pulse counter can be used.

in the table. In the column giving the possible use of current chambers, the word "yes" implies the possibility of extending the method to non-zero-power systems (from plutonium assemblies to power reactors), which is one of the focal problems in the possible development of the present time-analysis techniques.

CONCLUSIONS

Reactor-noise-analysis techniques were originally based on sampling the neutron population in the time domain. In this way, they were implicitly restricted to the zero-power reactor class because of the physical repetition rate of the pulse counters used in such measurements. Furthermore, these methods were all limited by low detection efficiency (with the exception of the Rossi-alpha procedure) and these were unsuitable for fast reactor noise analysis.

The extension of these types of measurements, or, better, a kind of Fourier transform of them, to the frequency domain provided the opportunity of obtaining a meaningful noise analysis even in non-zero-power reactors through the use of current chambers, band-pass filters, and the application of autocorrelations and cross-correlation techniques for the elaboration of data. A frequency range wider than that stimulated by spontaneous fluctuations and a larger noise amplitude in certain frequency regions were sometimes excited by the external modulation of neutron wave forms inserted into the systems.

The frequency domain approach, however, featured some disadvantages such as the necessity of special equipment, the inapplicability to fast reactors, the time-consuming data reduction, and the fact that the reactor transfer function obtained by analog techniques is more difficult and less precisely interpreted in terms of roll-off frequencies than a time decay can be in terms of multiple exponential constants. The last disadvantage is particularly critical for reflected or multizone reactors since they need an especially accurate analysis.

These disadvantages and particular attention to reactor-noise physics by a constantly increasing number of authors and laboratories have again spotlighted the time-analysis theory and experiments and have contributed many new methods and suitable refinements of old ones.

The most recent trend in noise analysis considers a discrete noise field to be sampled in the time domain. This return to the study of individual pulses must be intended essentially as an extension of the original approach, motivated by new theoretical and technological opportunities that now allow a profitable application of noise techniques to any type of reactor.

Now that the fundamental limitations of this original approach seem to be circumvented, the application of time-analysis methods becomes broader and seriously competitive with that of the frequency analysis, whose developments in the last few years have been less impressive, at least in regard to methods for measuring reactor kinetic parameters.

The main elements of the current situation are as follows:

1. The improved characteristics of analog to digital converters that realize the quantization of a signal even when a pulse counter cannot be used directly, i.e., in all systems from plutonium assemblies to power reactors.
2. The time cross-correlation techniques between two detectors which bypass the limitation of low efficiency in fast reactors and give only the essential term of the noise, the one containing the required information about the decay eigenvalues.
3. The possibility of using on-line computers instead of time analyzers, which would allow a more compact and rapid treatment of data, as well as a higher sampling rate and an almost unlimited accumulation of data, together with a quasi-continuous check of the correctness of the experiment.
4. The development of new techniques in which only the sign of the fluctuations (with respect to their mean value) is needed to determine the time-correlation function and therefore the reactor parameters.

A comparison between the bibliography of Chap. 6 of *Reactor Noise*, by J. A. Thie, and the references listed in this review gives the best idea of the nature and the quantity of the developments in the past six years.

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