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Abstract

In specific cases the critical beta ($\beta_c$) for stability against internal magnetohydrodynamic (MHD) ballooning modes can be improved significantly by taking into account the stabilizing influence of the ion diamagnetic drifts. This kinetic modification to the ideal MHD analysis together with radial corrections to the local theory are included in a calculation of $\beta_c$ as a function of toroidal mode number ($n$) for a particular tokamak equilibrium sequence of interest.
A realistic determination of the critical beta (ratio of plasma to magnetic pressure) for stability in tokamaks is very important both for fusion reactor design considerations as well as for understanding performance in present generation experiments. Previous ideal magnetohydrodynamic (MHD) studies [1-5] have indicated that with the possible exception of an $n = 1$ free boundary mode, the onset conditions for infinite-toroidal-mode-number ($n \to \infty$) internal ballooning modes lead to the most pessimistic estimates of this limiting value of beta ($\beta_c$). If radial modifications to the local theory are taken into account, the resultant "finite-n corrections" are found to be stabilizing [6]. As shown in Ref. 6, the $n$-dependence of $\beta_c$ for large values of $n$ (e.g., $n > 10$) can be quite strong for certain sequences of realistic equilibria. However, for such large values of $n$, kinetic modifications to the MHD analysis can also be very important [7-12].

In considering kinetic effects on MHD modes, most studies have focused on the stabilizing finite gyroradius contribution from the ion diamagnetic drifts. This acts directly against the interchange driving mechanism and becomes significant when $k_{\parallel}^2 \rho_i^2$ is comparable to $L_p/L_k$. Here $\rho_i$ is the ion gyroradius, and $L_p$ and $L_k$ are the respective scale lengths for the pressure gradient and field-line-curvature. Since the worst ideal MHD cases occur for the smallest values of $L_p$, significant modifications can occur even for small values of $k_{\parallel} \rho_i$. Radially local ballooning mode calculations have indicated that inclusion of this kinetic effect leads to significantly more optimistic values of $\beta_c$ [8,11]. On the other hand, it should be remembered that such an improvement is meaningful only if the ideal MHD $\beta_c$ at infinite-$n$ is much lower than that at moderately large values of $n$ [8]. As illustrated, for example, in Ref. 12, equilibrium sequences which do not exhibit this sensitive large-$n$-dependence show very little enhancement of $\beta_c$ due to kinetic effects. In the
present paper a procedure for carrying out a finite-n ballooning mode
calculation including kinetic contributions from the ion diamagnetic drifts is
described. When the analysis is applied to the relevant case of an
equilibrium sequence in which \( \beta_c \) has a strong large-n dependence, it is found
that the critical beta can indeed be significantly improved.

Following standard procedures [7], the lowest order ballooning mode
equation including diamagnetic drift effects can be expressed as

\[
\frac{B}{|\xi_1|^2} v_A^2 \nabla \cdot \left( \frac{|\xi_1|^2}{|B|} \nabla \cdot \phi \right) + \left( \omega^2 + \frac{2\omega_k \omega_p}{b} - \frac{\omega_p^2}{4} \right) \phi = 0
\]  

(1)

where \( \hat{n} \equiv \hat{B}/B \), \( v_A \) is the Alfvén speed \( (B^2/4\pi n m_i M_i)^{1/2} \), \( \hat{\omega} \equiv \omega - (\omega_{p1}/2) \),
\( b = |\xi_1|^2 \rho_i^{2/3} \), \( \omega_{pi} \equiv (cT/eB)\xi_1 \times \hat{v}_p \), \( \omega_k = (cT/eB)\xi_1 \times \hat{\xi}, \kappa \equiv \nabla \times \hat{\xi}, \) \( \xi \equiv \nabla \times \hat{\xi} \), and \( p \)
is the scalar pressure. The ideal MHD form of this equation is recovered by
taking \( \omega_{p1} \to 0 \). As discussed in some detail in Ref. 7, even in this
simplified limit, it is first necessary to assume that the mode frequency is
either small or large compared to the effective ion acoustic frequency \( \alpha \)
after a single second-order differential eigenmode equation can be obtained. To
arrive at the kinetic form given in Eq. (1), it is in addition necessary to
ignore wave-particle resonances (due, for example, to magnetic drifts) and
trapped-particle effects. The magnetic drift resonances in particular can
lead to residual growth rates on the drift-wave time scale [9,10]. These
kinetic instabilities of the shear Alfvén branch are similar in character to
other low frequency drift-type modes which are predicted to be unstable even
for \( \beta \to 0 \).

In the present paper attention is focused on the kinetic effects due to
the ion diamagnetic drift term, since this factor alone directly influences
the interchange driving mechanism of ideal MHD ballooning modes. It is
convenient to rewrite Eq. (1) in the form

\[ \frac{1}{4\pi} \frac{|k^*|^2}{B^2} \theta^* \psi + \frac{2}{B^2} p^*(\phi) (k^* x \hat{n}) + \rho^2 \frac{|k^*|^2}{B^2} - n^2 \left[ \frac{1}{2} \frac{\omega_p}{\omega_{ni}} (1+\tau) \right]^2 \frac{|k^*|^2}{B^2} \hat{\phi} = 0 \]  

(2)

with \( \tau \equiv T_e/T_i \) (taken constant with respect to \( \psi \)) and \( \omega_{pi} \) is the ion plasma frequency. As in the earlier paper [6], the perturbations have the eikonal form

\[ \phi = \phi(\theta, \psi, a, \epsilon) \exp[-i\omega t - iS(\alpha, \psi)/\epsilon] \]  

(3)

where \( \alpha \equiv \zeta - q\theta \), \( \zeta \) is the toroidal angle, \( \hat{B} = \nabla \times \psi \), \( \epsilon = 1/n \) is the finite-n expansion parameter. The eikonal \( S \) has the form

\[ S(\alpha, \psi) = Q_k + \int dq(q) k_q(\psi) , \]  

(4)

where \( k_q/\epsilon \) is a constant of order unity, and \( k_q/\epsilon \) is the radial wave-number in \( \alpha-q \) coordinates. Hence,

\[ \hat{k}_q \equiv k_q/Q_k \]  

(5)

with \( \hat{\theta}_k \equiv k_q/\alpha_k \).

In the ideal MHD calculation, Eq. (2) (with the last term dropped and \( \hat{\omega} = \omega \)) is numerically solved as an eigenvalue equation for \( \hat{\omega}^2 \) under the condition that \( \hat{\phi} \) be square-integrable on the domain \(-\alpha < \theta < \alpha\). The resultant dispersion relation is just \( \hat{\omega}^2 = \lambda(q, \hat{\theta}_k) \) with \( q(\psi) \) labelling the surfaces. Contours of constant \( \lambda \) (as a function of \( \hat{\theta}_k \) and \( q \)) can readily be
computed, and the familiar WKB quantization condition leads to the result

\[ n_c(\lambda) = n/dq \theta_k \]  

with \( n_c \) being the value of \( n \) at which the most unstable mode reaches stability. The denominator in Eq. (6) is just the area enclosed by a \( \lambda \)-constant contour in the \( q - \theta_k \) plane. Here, for the ideal MHD case, attention is focused on the \( \lambda = 0 \) case since this corresponds to marginal stability. However, for the present kinetically-modified analysis, the marginal point occurs for \( \lambda = -\omega_{pi}^2 / 4 \) (i.e., \( \hat{\omega} = 0 \) and \( \omega = \omega_{pi} / 2 \)). Now it becomes necessary to calculate and plot \( n_c \) as a function of \( \lambda \) and then to look for the intersections between this curve and the curve

\[ \lambda = -\frac{\omega_{pi}^2}{4} = -n^2 \left( \frac{c}{\omega_{pi}} \right)^2 (1+\tau)^2 \]  

Here it is assumed that the \( \phi \)-dependence of \( \omega_{pi} \) is negligibly small. An example of this procedure is illustrated in Fig. 1 where the ideal MHD (\( \lambda = 0 \)) value of \( n_c = 16 \) for \( \beta = 11\% \) is now changed by the kinetic correction to \( n_c = 23 \) and \( n_c = 40 \).

In Ref. 6 it was found that for the "Sequence II" series of equilibria, the \( n \)-dependence of \( \beta_c \) was very strong for large values of \( n \). Application of the kinetic analysis described gives the results summarized on Fig. 2. Here it is seen that for such classes of equilibria, the kinetic modifications can produce better than a factor of two improvement in \( \beta_c \) over the ideal MHD estimate (dashed curve). On the other hand, for equilibria where the large-\( n \)-dependence of \( \beta_c \) is weak (e.g., "Sequence I" of Ref. 6 and the cases described in Ref. 12), the \( \omega_{pi} \)-corrections tend to be insignificant.
In summary, it has been demonstrated in earlier work that kinetic effects due to ion diamagnetic drifts and radially nonlocal (finite-\(n\)) corrections can be important when calculating the critical \(\beta\) for stability against ideal MHD internal ballooning modes. In the present paper the procedure for simultaneously taking both types of effects into account is described, and the analysis is applied to a sequence of equilibria first studied in Refs. 5 and 6. As shown in Fig. 2, the \(\omega_{*1}\)-corrections can give better than a factor of two improvement in \(\beta_c\). Although it can be reasonably argued that this type of equilibrium sequence [5,6] may still be somewhat idealized, it has been noted that realistic equilibria for the ISX-B experiment exhibit similar characteristics [14]. It has also been shown that these \(\beta \sim 10\%\) equilibria can be sustained on a resistive timescale [15]. Finally, it should be noted that even if \(\omega_{*1}\)-effects can indeed significantly improve \(\beta_c\) for relevant scenarios, a realistic assessment of tokamak \(\beta\)-limits will still require an appropriate low-\(n\) analysis of free-boundary kink modes for such equilibria, together with a systematic study at high-\(n\) of energetic beam and alpha particle effects [16], as well as other kinetic modifications.

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References


Figure Captions

Fig. 1. The solid curve is a plot of $n_c$ [given in Eq. (6)] as a function of $\lambda$, and the dashed curve represents the kinetic relationship given in Eq. (7). The intersections of these curves give the appropriate values of critical $n$. For the example case shown here, $\beta = 11\%$, $n_c (\text{MHD}) = 16$, and $n_c (\text{KINETIC}) = 23, 40$.

Fig. 2. The dashed curve is a plot of $\beta_c$ versus $n$ as given in Ref. 6 for equilibrium "Sequence II." The solid curve represents the kinetically-modified results taking $n_o = 2.5 \times 10^{14} \text{ cm}^{-3}$. 
Fig. 1