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INFLUENCE OF DIAMAGNETIC DRIFTS ON CRITICAL
BETA IN TOKAMAKS

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Abstract

In specific cases the critical beta (β_c) for stability against internal magnetohydrodynamic (MHD) ballooning modes can be improved significantly by taking into account the stabilizing influence of the ion diamagnetic drifts. This kinetic modification to the ideal MHD analysis together with radial corrections to the local theory are included in a calculation of β_c as a function of toroidal mode number (n) for a particular tokamak equilibrium sequence of interest.

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A realistic determination of the critical beta (ratio of plasma to magnetic pressure) for stability in tokamaks is very important both for fusion reactor design considerations as well as for understanding performance in present generation experiments. Previous ideal magnetohydrodynamic (MHD) studies [1-5] have indicated that with the possible exception of an $n = 1$ free boundary mode, the onset conditions for infinite-toroidal-mode-number ($n \rightarrow \infty$) internal ballooning modes lead to the most pessimistic estimates of this limiting value of beta (β_c). If radial modifications to the local theory are taken into account, the resultant "finite-n corrections" are found to be stabilizing [6]. As shown in Ref. 6, the n-dependence of β_c for large values of n (e.g., $n > 10$) can be quite strong for certain sequences of realistic equilibria. However, for such large values of n, kinetic modifications to the MHD analysis can also be very important [7-12].

In considering kinetic effects on MHD modes, most studies have focused on the stabilizing finite gyroradius contribution from the ion diamagnetic drifts. This acts directly against the interchange driving mechanism and becomes significant when $k_{\perp}^2 \rho_i^2$ is comparable to L_p/L_{κ} . Here ρ_i is the ion gyroradius, and L_p and L_{κ} are the respective scale lengths for the pressure gradient and field-line-curvature. Since the worst ideal MHD cases occur for the smallest values of L_p , significant modifications can occur even for small values of $k_{\perp} \rho_i$. Radially local ballooning mode calculations have indicated that inclusion of this kinetic effect leads to significantly more optimistic values of β_c [8,11]. On the other hand, it should be remembered that such an improvement is meaningful only if the ideal MHD β_c at infinite-n is much lower than that at moderately large values of n [8]. As illustrated, for example, in Ref. 12, equilibrium sequences which do not exhibit this sensitive large-n-dependence show very little enhancement of β_c due to kinetic effects. In the

present paper a procedure for carrying out a finite- n ballooning mode calculation including kinetic contributions from the ion diamagnetic drifts is described. When the analysis is applied to the relevant case of an equilibrium sequence in which β_c has a strong large- n dependence, it is found that the critical beta can indeed be significantly improved.

Following standard procedures [7], the lowest order ballooning mode equation including diamagnetic drift effects can be expressed as

$$\frac{B}{|\vec{k}_\perp|^2} v_A^2 \hat{n} \cdot \nabla \left(\frac{|\vec{k}_\perp|^2}{B} \hat{n} \cdot \nabla \hat{\phi} \right) + (\hat{\omega}^2 + \frac{2\omega_{k1}\omega_{*p}}{b} - \frac{\omega_{*pi}^2}{4}) \hat{\phi} = 0 \quad (1)$$

where $\hat{n} \equiv \vec{B}/B$, v_A is the Alfvén speed $(B^2/4\pi m_i M_i)^{1/2}$, $\hat{\omega} \equiv \omega - (\omega_{*pi}/2)$, $b = |\vec{k}_\perp|^2 \rho_i^2/2$, $\omega_{*p} \equiv (cT/eBp)\vec{k}_\perp \cdot \hat{n} \times \nabla p$, $\omega_k = (cT/eB)\vec{k}_\perp \cdot \hat{n} \times \vec{k}$, $\kappa \equiv \hat{n} \cdot \nabla n$, and p is the scalar pressure. The ideal MHD form of this equation is recovered by taking $\omega_{*pi} \rightarrow 0$. As discussed in some detail in Ref. 7, even in this simplified limit, it is first necessary to assume that the mode frequency is either small or large compared to the effective ion acoustic frequency before a single second-order differential eigenmode equation can be obtained. To arrive at the kinetic form given in Eq. (1), it is in addition necessary to ignore wave-particle resonances (due, for example, to magnetic drifts) and trapped-particle effects. The magnetic drift resonances in particular can lead to residual growth rates on the drift-wave time scale [9,10]. These kinetic instabilities of the shear Alfvén branch are similar in character to other low frequency drift-type modes which are predicted to be unstable even for $\beta \rightarrow 0$.

In the present paper attention is focused on the kinetic effects due to the ion diamagnetic drift term, since this factor alone directly influences the interchange driving mechanism of ideal MHD ballooning modes. It is

convenient to rewrite Eq. (1) in the form

$$\left\{ \frac{1}{4\pi} \vec{B} \cdot \nabla \frac{|\vec{k}_\perp|^2}{B^2} \vec{B} \cdot \nabla + \frac{2}{B^2} p'(\psi) (\vec{k}_\perp \times \vec{k} \cdot \hat{n}) + \rho \omega^2 \frac{|\vec{k}_\perp|^2}{B^2} - n^2 \left[\frac{c p'(\psi)}{2 \omega_{pi} (1+\tau)} \right]^2 \frac{|\vec{k}_\perp|^2}{B^2} \right\} \hat{\phi} = 0 \quad (2)$$

with $\tau \equiv T_e/T_i$ (taken constant with respect to ψ) and ω_{pi} is the ion plasma frequency. As in the earlier paper [6], the perturbations have the eikonal form

$$\phi = \hat{\phi}(\theta, \psi, \alpha, \varepsilon) \exp[-i\omega t - iS(\alpha, \psi)/\varepsilon] \quad (3)$$

where $\alpha \equiv \zeta - q\theta$, ζ is the toroidal angle, $\vec{B} = \nabla\alpha \times \nabla\psi$, $\varepsilon \approx 1/n$ is the finite- n expansion parameter. The eikonal S has the form

$$S(\alpha, \psi) = \alpha k_\alpha + \int dq(\psi) k_q(\psi), \quad (4)$$

where $k_\alpha \equiv \varepsilon n$ is a constant of order unity, and k_q/ε is the radial wave-number in α - q coordinates. Hence,

$$\vec{k}_\perp = k_\alpha [\nabla\zeta - q\nabla\theta - (\theta - \theta_k) \nabla q] \quad (5)$$

with $\theta_k \equiv k_q/k_\alpha$.

In the ideal MHD calculation, Eq. (2) (with the last term dropped and $\hat{\omega} = \omega$) is numerically solved as an eigenvalue equation for ω^2 under the condition that $\hat{\phi}$ be square-integrable on the domain $-\infty < \theta < \infty$. The resultant dispersion relation is just $\omega^2 = \lambda(q, \theta_k)$ with $q(\psi)$ labelling the surfaces. Contours of constant λ (as a function of θ_k and q) can readily be

computed, and the familiar WKB quantization condition¹³ leads to the result

$$n_c(\lambda) = \pi / \oint dq \theta_k \quad (6)$$

with n_c being the value of n at which the most unstable mode reaches stability. The denominator in Eq. (6) is just the area enclosed by a λ -constant contour in the $q - \theta_k$ plane. Here, for the ideal MHD case, attention is focused on the $\lambda = 0$ case since this corresponds to marginal stability. However, for the present kinetically-modified analysis, the marginal point occurs for $\lambda = -\omega_{*p1}^2/4$ (i.e., $\hat{\omega} = 0$ and $\omega = \omega_{*p1}/2$). Now it becomes necessary to calculate and plot n_c as a function of λ and then to look for the intersections between this curve and the curve

$$\lambda = -\frac{\omega_{*p1}^2}{4} = -n^2 \left(\frac{cp'}{2\omega_{p1}(1+\tau)} \right)^2 \quad (7)$$

Here it is assumed that the ψ -dependence of ω_{*p1} is negligibly small. An example of this procedure is illustrated in Fig. 1 where the ideal MHD ($\lambda = 0$) value of $n_c \approx 16$ for $\beta \approx 11\%$ is now changed by the kinetic correction to $n_c \approx 23$ and $n_c \approx 40$.

In Ref. 6 it was found that for the "Sequence II" series of equilibria, the n -dependence of β_c was very strong for large values of n . Application of the kinetic analysis described gives the results summarized on Fig. 2. Here it is seen that for such classes of equilibria, the kinetic modifications can produce better than a factor of two improvement in β_c over the ideal MHD estimate (dashed curve). On the other hand, for equilibria where the large- n -dependence of β_c is weak (e.g., "Sequence I" of Ref. 6 and the cases described in Ref. 12), the ω_{*p1} -corrections tend to be insignificant.

In summary, it has been demonstrated in earlier work that kinetic effects due to ion diamagnetic drifts and radially nonlocal (finite- n) corrections can be important when calculating the critical β for stability against ideal MHD internal ballooning modes. In the present paper the procedure for simultaneously taking both types of effects into account is described, and the analysis is applied to a sequence of equilibria first studied in Refs. 5 and 6. As shown in Fig. 2, the ω_{*1} -corrections can give better than a factor of two improvement in β_c . Although it can be reasonably argued that this type of equilibrium sequence [5,6] may still be somewhat idealized, it has been noted that realistic equilibria for the ISX-B experiment exhibit similar characteristics [14]. It has also been shown that these $\beta \sim 10\%$ equilibria can be sustained on a resistive timescale [15]. Finally, it should be noted that even if ω_{*1} -effects can indeed significantly improve β_c for relevant scenarios, a realistic assessment of tokamak β -limits will still require an appropriate low- n analysis of free-boundary kink modes for such equilibria, together with a systematic study at high- n of energetic beam and alpha particle effects [16], as well as other kinetic modifications.

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Figure Captions

Fig. 1. The solid curve is a plot of n_c [given in Eq. (6)] as a function of λ , and the dashed curve represents the kinetic relationship given in Eq. (7). The intersections of these curves give the appropriate values of critical n . For the example case shown here, $\beta = 11\%$, $n_c(\text{MHD}) \approx 16$, and $n_c(\text{KINETIC}) \approx 23, 40$.

Fig. 2. The dashed curve is a plot of β_c versus n as given in Ref. 6 for equilibrium "Sequence II." The solid curve represents the kinetically-modified results taking $n_0 = 2.5 \times 10^{14} \text{ cm}^{-3}$.

Fig. 1

Fig. 2