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By

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OCTOBER 1985

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PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CEO-3073.

GENERALIZED BANANA-DRIFT TRANSPORT

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ABSTRACT

The theory of tokamak ripple transport in the banana-drift and ripple-plateau regimes is extended in a number of directions. The theory is valid for small values of the toroidal periodicity number n of the perturbation, as well as for the moderate values ($n \sim 10 - 20$) previously assumed. It is shown that low- n perturbations can produce much greater transport than the larger- n perturbations usually studied. In addition, the ripple perturbation is allowed arbitrary values of poloidal mode number m and frequency ω , making it applicable to the transport induced by MHD modes. Bounce averaging is avoided, so the theory includes the contributions to transport from all harmonics of the bounce frequency, providing a continuous description of the transition from the banana drift to the ripple-plateau regime. The implications of the theory for toroidal rotation in tokamaks are considered.

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I. INTRODUCTION

Previous theories of ripple transport in tokamaks [1-5] have assumed that n , the toroidal periodicity number of the ripple perturbation, is in the moderate range ($n \sim 10 - 20$) of the number of toroidal field coils, whose discrete character gives rise to the ripple perturbation. However, tokamak axisymmetry is also broken by low- n perturbations ($n \sim 1$). These may arise either from machine asymmetries, or from low- n MHD modes generated by the plasma itself. The theory developed in this paper generalizes the moderate- n theory of "banana-drift" transport [5] to apply to perturbations of this type. One of principal points of the work is that the transport arising from the low- n perturbations can be much larger, for reasonable parameters, than that from the moderate- n ripple considered to date. ("Moderate- n " here is used in contrast to the "low- n " perturbations already mentioned, and "high- n " perturbations ($n \sim 100$) arising from magnetic microturbulence, which also produces transport [6,7].)

One of the motivations for this work was to find a viable mechanism to explain the equilibration time for toroidal rotation which has been observed [8] in tokamaks under neutral beam heating. The toroidal torque which may be expected from the moderate- n theories was believed to be too small to provide sufficiently rapid saturation of the rotation rate. Here, it will be shown that the torque from moderate- n perturbations may, in fact, be sufficiently large, while that from low- n perturbations is easily large enough.

The theory presented here generalizes previous work in a number of directions, in response to the physical considerations just discussed. The theory is valid for small (as well as moderate) n , as already noted. The ripple perturbation is given a time dependence $\delta B \sim \exp(-i\omega t)$, with ω real but otherwise arbitrary, to allow for the time dependence of internally generated

perturbations. Another generalization made which is mathematically quite simple, but which can be important physically, is to allow the perturbation to have arbitrary helicity [in arbitrary in $\delta B \sim \exp(i n \phi - m \theta)$], again needed to be applicable to MHD modes. (Here, ϕ and θ are the toroidal and poloidal angles, respectively.) For $m \neq 0$, one finds that in place of the single parameter q_n arising in previous theory, now two parameters naturally arise, q_n and $q_N \equiv q_n - m$. (q is the safety factor.) This can be significant, because MHD modes tend to localize on flux surfaces where q_N is small, even for $q_n \gg 1$.

The theory is also generalized to apply to a rotating plasma. For a beam-heated tokamak such as TFTR, the plasma may have a toroidal rotation velocity u_ϕ which can be a large fraction of the sound speed, $u_\phi \lesssim c_s = 3 \times 10^5$ m/sec (TFTR parameters). Therefore, the fraction of a ripple period through which a toroidally trapped particle precesses in a bounce period is not small, as assumed in Ref. 5, and transport from a (near) resonance of the toroidal drift frequency Ω_ϕ of a particle and harmonics l of its bounce frequency Ω_b can be appreciable. The present theory includes these contributions. In the associated resonance condition,

$$\omega = l \Omega_\phi + m \Omega_b \quad ,$$

$\omega = 0$ and $l = 0$ were assumed in Ref. 5.

In Ref. 9, the contribution to transport from such bounce resonances is also calculated, restricted to moderate n and to $\omega = 0$ (but for a broader range of collisionality than the present work), by averaging the kinetic equation over a bounce period to obtain a mapping, and then solving a finite difference equation for the particle distribution f . In the present work, in

contrast, this intermediate step of discretizing the transport process is avoided, and the (differential) kinetic equation is instead solved directly, without bounce averaging, in a manner fully analogous to the differential approach used in transport calculations in simpler situations, e.g., in symmetric tokamaks [10], or in the bounce-averaged ripple calculation of Ref. 5. This is achieved by use of "eigenfunctions of the Liouville operator" [11], which make trivial the integrations along unperturbed trajectories which commonly arise in transport calculations. The approximate "eigenfunctions of the collision operator" used in Ref. 9 to represent f form that subset of the Liouville eigenfunctions which are independent of bounce phase θ_b . Because for $l \neq 0$, f is not independent of θ_b , the eigenfunction set in Ref. 9 is not sufficient to represent the bounce-resonant contributions to f . As will be seen, however, the effect of this variation of f with θ_b is small as long as $(l/2qN) \ll 1$. For the case of moderate- n and $m = 0$ studied in Ref. 9, this approximation holds up to larger values of l than are likely to be of interest. For low qN , however, the θ_b -dependence has appreciable effect at low values of l , and so must be kept.

Though it resembles the more restrictive calculation of Ref. 5, because it is not bounce averaged, the present calculation not only generalizes the transport coefficients in the banana-drift regime, but is valid into the ripple-plateau regime as well, providing a continuous description of the variation of the transport coefficients in the transition between the two regimes.

The remainder of the paper is organized as follows. In Sec. II we preview the results of the calculation to follow, describing the modifications of previous theory that result when the generalizations achieved by the present work are carried out. The formal derivation of these results is

presented in Sec. III. The theory is illustrated in Sec. IV by considering its implications for toroidal rotation in beam-heated tokamaks. In Sec. V we summarize the work of the preceding sections.

II. OVERVIEW

Before proceeding to the formal calculation, we briefly review the results of earlier ripple transport theory, and indicate the ways in which those results are modified by the present theory.

In Fig. 1 are depicted the basic features of tokamak neoclassical theory. Figure 1(a) shows the familiar three collisionality regimes for a symmetric tokamak [10], the banana regime [$D_{bn} = (v/v_*)D_{bp}$] at low collision frequency v , the Pfirsch-Schlüter regime [$D_{ps} \approx (\epsilon^{3/2} v/v_*)D_{bp}$] at high- v , and the intermediate banana-plateau regime, with diffusion coefficient D_{bp} :

$$D_{bp} \approx q^2 \rho^2 v_* \epsilon^{-3/2} . \quad (1)$$

[In these expressions for D , numerical coefficients of order unity are ignored.] Here, $\rho = v/\Omega$ is the particle gyroradius, $v_* \equiv v \epsilon^{3/2}/qR$, ϵ is the inverse aspect ratio, and $\Omega \equiv eB/Mc$ is the gyrofrequency of the species in question.

The symmetric "branch" shown in Fig. 1(a) is given for purposes of comparison to the ripple branch in Fig. 1(b), of which the high- and low- v banana-drift regimes (D_{-1} and D_1 , respectively) of interest to us here comprise the middle two of the four regimes shown. For this branch, transport comes from radial diffusion of the "banana centers" (i.e., the bounce-averaged position) of the toroidally trapped particles. Not shown in Fig. 1, nor considered in this paper, is a second ripple-induced branch, arising from

particles which entrap in the ripple wells [2]. The lowest- v "stochastic" regime (D_{st}) [4] and the highest- v "ripple-plateau" regime (D_{rp}) [3] have transport described by

$$D_{st} \approx D_{rp} \approx (qn \langle \delta^2 \rangle / \epsilon^2) D_{bp} \quad , \quad (2)$$

where $\langle \delta^2 \rangle$ means an appropriate flux surface average of the square of the ripple amplitude δ . The intermediate two (banana drift) regimes may be written

$$D_{-1} = (v_*/v)(qn)^{-2} D_{rp} = \frac{\epsilon^{1/2} \hat{r}^2}{(qn)^3 v_t} \quad , \quad (3)$$

and

$$D_1 \approx v v_* (R/u_E)^2 D_{rp} = \frac{v_t \epsilon^{1/2} \hat{r}^2 (qn)}{(n\Omega_\phi)^2} \quad . \quad (4)$$

Here, $u_E \equiv -cE_r/B \equiv R\Omega_E$ is the (predominantly poloidal) $E \times B$ drift due to the radial electric field E_r , and $v_t \equiv v/(2\epsilon)$. The toroidal precession frequency Ω_ϕ is assumed dominated by the $E \times B$ contribution, $\Omega_\phi = -\Omega_E$. $\hat{r} \equiv \delta q n v_{B0}$ is the amplitude of the radial drift velocity of bananas [5] due to a given ripple perturbation of amplitude δ [cf. Eq. (31)], with $v_{B0} \equiv \mu B/(eQr)$, and μ the particle magnetic moment.

We recall the physical origin of expressions (3) and (4): Banana-trapped particles, comprising a fraction $\epsilon^{1/2}$ of all particles, move radially due to the ripple at velocity $\dot{r} \sim \hat{r}$. However, $\dot{r}$ oscillates with the ripple phase, so that, for $qn \gg 1$, only the fraction $(qn)^{-1/2}$ of the bounce period when a particle is crossing the ripple wells at its banana tips is not cancelled in

the bounce-averaged radial velocity $\dot{\bar{r}}$. For the "collision-dominant" case D_{-1} , a banana thus takes a radial step $\Delta r = \dot{\bar{r}}/v_{\text{eff}} = \dot{\bar{r}}(qn)^{-1/2}/v_{\text{eff}}$, where $v_{\text{eff}} \equiv v_t(qn)^2$ is the effective collision frequency for decorrelation with the ripple phase. In the "precession-dominant" case D_1 , the radial step is instead controlled by the time to precess across a ripple period, $\Delta r = \dot{\bar{r}}/(n\Omega_\phi)$. Using these expressions in the heuristic formula $D \sim \epsilon^{1/2} v_{\text{eff}}(\Delta r)^2$ yields Eqs. (3) and (4).

Now we can describe the modifications of these expressions which result when the generalizations indicated in Sec. I are made. The single $l = 0$ term in Eqs. (3) or (4) is replaced by a sum over l :

$$D_{\text{bd}} = \sum_l D_{\text{bd}}^l, \quad (5)$$

where the l -th contribution D_{bd}^l is the smaller of the resonant (D_{-1}^l) and nonresonant (D_1^l) forms, which generalize Eqs. (3) and (4):

$$D_{-1}^l = \epsilon^{1/2} (\dot{\bar{r}} G_l)^2 / v_{\text{eff}}, \quad (6)$$

and

$$D_1^l = v_{\text{eff}} \epsilon^{1/2} (\dot{\bar{r}} G_l)^2 / (n\Omega_\phi + l\Omega_b - \omega)^2. \quad (7)$$

One notes that the factor $(n\Omega_\phi)$ in Eq. (4) is replaced by a full resonant denominator in Eq. (7). The effective collision frequency is also modified,

$$v_{\text{eff}} = v_t [(qn)^2 + (l/2)^2], \quad (8)$$

manifesting the modification for $(l/2qN) \gtrsim 1$ noted in Sec. I, as well as the importance of the distinction between qn and qN . The functions G_l , defined in Eq. (34), are for most pitch angles well-approximated by Bessel functions of the first kind, $G_l = J_l$. They are essentially the mode-particle "coupling coefficients" [7,12], expressing in a succinct fashion the effect of a given mode on a particle with a specified unperturbed orbit.

The J_l have the well-known asymptotic forms

$$J_l(z < l) \approx (z/2)^l / l! \quad , \quad (9)$$

$$J_l(z > l) \approx (2/\pi z)^{1/2} \cos(z - l\pi/2 - \pi/4) \quad , \quad (10)$$

where $z = qN\theta_T$ in the present context and θ_T is the poloidal azimuth of the turning point of a given particle. Thus, for the theory of Ref. 5, in which $l = 0 = m$ and $qn \gg 1$, one sees from Eq. (10) that G_l reduces to $J_0 \sim (qn)^{-1/2}$. Making this substitution in Eqs. (6) and (7), one recovers just the factors in Eqs. (3) and (4) which arise from the integral over the last ripple well, discussed in the heuristic derivation given for Eqs. (3) and (4). For the case $qN \sim 1$, it is no longer true that $\vec{r}$ contributes to $\vec{r}$ only near θ_T , and for the same reason, Eq. (10) is no longer valid when $z \lesssim l$. The G_l are the appropriately generalized coupling, for all z and l .

Figure 2 depicts the qualitative features of the result described by Eqs. (5)-(7), plotting D versus Ω_ϕ (or equivalently, versus E_r). Shown are the resonance peaks for $l = 0, \pm 1$, and ± 2 . The height of the l -th peak is D_{-1}^l . For $l \lesssim qN$, this scales with δ , qn , qN , and ν as

$$D_{-1}^l \sim (\delta^2/\nu) (qn)^2 / (qN)^3 \quad , \quad (l \lesssim qN) \quad . \quad (11)$$

For $l \gtrsim qN$, the peak heights $\sim G_x^2$ fall off rapidly, in accordance with Eq.

(9). The width $\delta\Omega_\phi$ of each peak, obtained by equating D_{-1}^0 with D_1^0 , is

$$\delta\Omega_\phi = v_{\text{eff}}/n \quad (12)$$

while the spacing $\Delta\Omega_\phi$ between peaks is

$$\Delta\Omega_\phi = \Omega_b/n \quad (13)$$

Therefore,

$$\delta\Omega_\phi/\Delta\Omega_\phi = v_{\text{eff}}/\Omega_b \quad (14)$$

In order for the situation depicted in Fig. 2 to hold, in which successive l -peaks act essentially separately as Ω_ϕ is varied, one must have $\delta\Omega_\phi/\Delta\Omega_\phi < 1$. Since from Eq. (11) the height of each peak scales as $1/v$, in the regime $\delta\Omega_\phi/\Delta\Omega_\phi < 1$, the total D on resonance scales as $1/v$. For $\delta\Omega_\phi/\Delta\Omega_\phi > 1$, however, the peaks in Fig. 2 overlap, with roughly $\delta\Omega_\phi/\Delta\Omega_\phi \sim v$ peaks contributing at any given Ω_ϕ , each of strength $D_{-1}^0 \sim v^{-1}$. Their sum, therefore, will be independent of v . From Eq. (14), therefore, one expects $D \sim 1/v$ (on resonance) for $v_{\text{eff}} < \Omega_b$, and $D \sim v^0$ for $v_{\text{eff}} > \Omega_b$. This is precisely the transition indicated in Fig. 1 from the collision-dominant banana-drift regime to the plateau regime. That is, though the calculation to follow is done in a manner formally quite parallel to that of Ref. 5 for the banana-drift regime, because it avoids bounce averaging, it is also valid into the ripple-plateau regime.

From Eq. (14) it is also apparent that, because $v_{\text{eff}} \sim (qN)^2$, the separate-peak case (in which the collision-dominant banana-drift regime holds) applies up to much larger v for $qN \sim 1$ than for $qN \gg 1$.

Assuming $N = n$, one sees from Eq. (11) that the peak height D_{-1}^2 scales as δ^2/n , so that for fixed δ the individual low- n peaks will be larger by an order of magnitude than those for moderate n . Moreover, δ tends to be larger for low- n perturbations than for moderate n . For TFTR, the average ripple from the toroidal field (TF) coils halfway out in minor radius is $\delta \approx 0.05\%$. In comparison, it has been calculated [8] that a misalignment of a TF coil by a bolt width (0.25 cm) in ISX would generate an $n = 1$ perturbation with $\delta \approx 0.3\%$, an order of magnitude larger. An order of magnitude difference in ripple is what one expects if the perturbations have radial components which are equal in magnitude, because the low- n perturbation can then take a correspondingly larger radial step before reversing direction. Perturbations due to low- n MHD modes can be larger still. The large radial extent of such modes can produce a ripple of amplitude [13] $\delta \approx 1\text{--}3\%$. Thus the potential transport from such perturbations is orders of magnitude above that from moderate- n ripple.

III. KINETIC THEORY

We wish to evaluate the radial flux density $\Gamma(g)$ of any function $g(K)$ of the particle kinetic energy $K \equiv E - e\phi$. The usual fluxes of interest are the particle flux $\Gamma(g=1)$ and the heat flux $\Gamma(g=K)$.

In terms of any set $z = [z^1, z^2, \dots, z^6]$ of independent variables parametrizing the six-dimensional phase space for a single particle, Γ may be written

$$\Gamma = (\Delta V)^{-1} \int_{\Delta V} d^6 z J_z g(K) \dot{r} f \quad (15)$$

Here, f is the particle distribution function (species label s suppressed), $\dot{r}$ is the radial drift velocity, and $J_z \equiv |\partial(x,p)/\partial z|$ is the phase space Jacobian for the variable set z . $\Delta V \approx (2\pi)^2 r R \Delta a$ is the volume of a toroidal shell, centered at the radius r of interest, and of thickness Δa small compared with the minor radius a at the plasma edge, and " ΔV " in $\int_{\Delta V}$ confines the phase space integral to that volume.

We may parametrize real space x as in Ref. 5 by flux coordinates (ψ, θ, ϕ) , with $2\pi\psi$ the toroidal flux contained within a flux surface, and toroidal and poloidal angles ϕ and θ chosen to make field lines "straight" in a flux surface, i.e., to make the safety factor $q \equiv d\psi/d\chi$ a function of ϕ only. Thus, $\phi_0 \equiv \phi - q\theta$ is constant along a field line, the " α - β coordinate" conjugate to ψ . Instead of ψ , it is sometimes convenient to label flux surfaces with the minor radial variable $r(\psi) \equiv (2\psi/B_0)^{1/2}$, with B_0 the field strength on axis.

In Eq. (15), f is as usual determined by the Boltzmann equation,

$$(\partial_t + L_H)f = Cf \quad (16)$$

Here, $\partial_t \equiv \partial/\partial t$, C is the collision operator, and the Liouville operator $L_H \equiv \dot{z}^i \partial_i$ propagates particles along a collisionless trajectory, with $\partial_i \equiv \partial/\partial z^i$. Taking as a coordinate set $z_1^i = (x, \theta, \phi_0, E, \mu, \phi_g)$ or $z_2^i = (x, \theta_b, \phi_0, E, \mu, \phi_g)$ (recall θ_b is the bounce phase), the time derivatives $\dot{E}$ and $\dot{\mu}$ of the energy E and magnetic moment μ are zero in L_H . Additionally, we neglect classical transport, by neglecting the variation of f with respect to gyrophase ϕ_g in L_H . Then we write $L_H = L_{HO} + \dot{r} \partial_r$, where

$$\begin{aligned}
 L_{HO} &= \Omega_{\phi} \partial_{\phi_0} + \dot{\theta} \partial_{\theta} \\
 &= \Omega_{\phi} \partial_{\phi_0} + \Omega_b \partial_{\theta_b}
 \end{aligned}
 \tag{17}$$

is the "unperturbed" Liouville operator, propagating particles along the projection into a flux surface of their collisionless orbits. The first form for L_{HO} in Eq. (17) is for the z_1 -system, and the second for the z_2 -system. For toroidally trapped particles, $\Omega_{\phi} = \dot{\phi}_0$ is the toroidal precession frequency, already introduced. As already mentioned, we shall assume it is dominated by the $E \times B$ contribution $\Omega_{\phi z} = -q\Omega_E$, which is approximately independent of pitch angle and bounce phase. This neglect of the VB-contribution is valid for $e\tilde{\phi}/eT \gg 1$.

As usual, we solve the kinetic Eq. (16) by introducing an ordering, assuming that $\Delta r \ll a$, where Δr is the radial step size discussed in Sec. II. We thus write

$$f = f_0 + f_1 + \text{higher order},$$

where the unperturbed distribution function $f_0 = f_0(E, r)$ satisfies

$$Cf_0 = 0 = L_{HO}f_0. \tag{18}$$

The linearized kinetic equation for f_1 is then

$$(\partial_t + L_{HO})f_1 + \dot{r} \partial_r f_0 = Cf_1. \tag{19}$$

Before proceeding to solve Eq. (19) in the resonant and nonresonant collisionality regimes, we assemble expressions for the quantities one sees from Eq. (15) and (16) or (19) will be needed to compute Γ . Bounce averaging Eq. (19), one would obtain the "banana kinetic" equation, the starting point for the transport calculation in Ref. 5. Here, we do not want to bounce average, but to develop the theory in a manner in which the corresponding bounce-averaged expressions are apparent.

We model the magnetic field strength by

$$B(r, \phi, \theta) = B_0 (1 - \epsilon \cos \theta - \delta \cos \eta) , \quad (20)$$

where the ripple phase is $\eta \equiv n\phi - m\theta - \omega t = n\phi_0 + qN\theta - \omega t$. We thus consider a single-component (i.e., a single n, m) perturbation $\delta B = -B_0 \delta \cos \eta$. We also take the ripple amplitude δ to be a function of r only. The generalizations to more than one component, and to a δ depending on θ as well as r , are straightforward.

We introduce the well-depth parameter y (called x^2 in Ref. 5),

$$y(r, E, \mu) \equiv (K/\mu B_0 - 1)/2\epsilon + 1/2 , \quad (21)$$

and denote by z_1 and z_2 the same coordinate sets as the z_1' and z_2' systems, but with y replacing μ . Using Eqs. (20) and (21), the parallel velocity $v_{\parallel}$ in the unperturbed field ($\delta \rightarrow 0$) may be written

$$v_{\parallel} = \sigma u = \sigma u_{a_1}(\theta, y) . \quad (22)$$

Here, $\sigma \equiv v_{\parallel}/|v_{\parallel}|$ is the sign of $v_{\parallel}$, and $u(r, \theta, E, \mu) \equiv [2(K - \mu B)/M]^{1/2}$ its

magnitude. u has amplitude $\hat{u} \equiv 2(\mu B_0 \epsilon / M)^{1/2}$, and its θ dependence and dominant pitch-angle dependence given through $a_1(\theta, y) \equiv [y - \sin^2(\theta/2)]^{1/2}$. From Eq. (22), one sees that $y = 0$ for banana particles most deeply trapped, ($u = 0$ at $\theta = 0$), $y = 1$ for those barely trapped ($u = 0$ at $\theta = \pi$), and $y > 1$ for passing particles. From this one also notes that $y = \sin^2(\theta_T/2)$.

As usual in calculations of ripple transport, we model the collision operator by the Lorentz form,

$$C_L f = \frac{v}{B} \frac{\partial_\mu M u \partial_\mu f}{\partial_E M u} = \frac{v}{a_1} \frac{\partial_y a_1 \partial_y f}{a_1'} \quad (23)$$

which scatters in pitch angle (measured by μ or y) only. In the second form in Eq. (23),

$$a_1' \equiv \partial_y a_1 = [y - \sin^2(\theta/2)]^{-1/2}/2 \quad .$$

The first form in Eq. (23) may be written in more standard fashion by using $\partial_E(Mu) = u^{-1}$. The forms written resemble most closely their bounce-averaged counterparts. By "bounce averaged" here, one means the time average, or ∂_b average, of the function F in question:

$$\langle F \rangle_t \equiv \oint \frac{d\theta}{2\pi} F = \left(\int_t \frac{d\ell}{u} F \right) / \tau_b \quad , \quad (24)$$

with $\tau_b = \left(\int_t \frac{d\ell}{u} \right) \equiv 2\pi/\Omega_b$ the toroidal bounce time, and " $\int_t$ " denoting the integral along a field line across the toroidal well. (Because " $\oint$ " in $\oint d\theta$ means an integral from zero to 2π , we avoid the ambiguity of the notation $\oint d\theta$,

which might mean $\int_0^{2\pi} d\theta$, or an integral over a full bounce period. Instead, for θ and l we use the notation $\int_{\sigma} \int_t$ for an integral over a bounce period. The factor $\int_{\sigma}$ in subsequent formulae numerically contributes just a factor of 2, but serves as a useful reminder of what is being represented.) Operating with $\langle \rangle_t$ on Eq. (23), one obtains

$$\begin{aligned} \bar{C}_L f &= \frac{v}{B} \frac{\partial \mu}{\partial E} \frac{\partial f}{\partial I} \\ &= \frac{v}{A_1} \frac{\partial A_1}{\partial y} \frac{\partial f}{\partial y} \end{aligned} \quad (25)$$

familiar from bounce-averaged theory. Here,

$$\begin{aligned} I(\tau, E, \mu) &\equiv \int_{\sigma} \int_t \frac{dl}{2\pi} \mu u \\ &= \hat{I} A_1(y) \end{aligned} \quad (26)$$

is the bounce action in the unperturbed field, with amplitude $\hat{I} \equiv ML_{ct} u$, and $L_t \equiv dl/d\theta = qR$ the toroidal connection length. The principal pitch-angle dependence enters I through the function $A_1(y)$, defined by

$$\begin{aligned} A_1(y) &\equiv \int_{\sigma} \int_t \frac{d\theta}{2\pi} [y - \sin^2(\frac{\theta}{2})]^{1/2} \equiv \langle \int_{\sigma} a_1 \rangle_{\theta} \\ &= \frac{4}{\pi} [E(y^{1/2}) - (1-y)K(y^{1/2})] \end{aligned} \quad (27)$$

Here, $\langle F \rangle_{\theta} \equiv (2\pi)^{-1} \int_t d\theta F$ is the θ -average of F , and K and E are the complete elliptic integrals. The final form given for A_1 is only valid for $y < 1$, the pitch-angle range of interest here. A_1 in Eq. (27) is π^{-1} times

the function of the same name in Ref. 5. Along with A_1 , we also introduce A_2 and A_3 , used in Ref. 5:

$$\begin{aligned} A_2(y) &= 2A_1' = \left\langle \sum_{\sigma} 2 a_1' \right\rangle_{\theta} \\ &= \frac{4}{\pi} K(y^{1/2}) \quad , \end{aligned} \quad (28)$$

$$A_3(y, qN) \equiv \left\langle \sum_{\sigma} 2a_1' \cos(qN\theta) \right\rangle_{\theta} = 2A_4' \quad , \quad (29)$$

where

$$A_4(y, qN) \equiv \left\langle \sum_{\sigma} a_1' \cos(qN\theta) \right\rangle_{\theta} \quad (30)$$

also arises naturally in the bounce-averaged theory (as will be seen).

A correspondance analogous to that between the unaveraged and averaged Eqs. (23) and (25) for C can also be given for $\dot{r}$. In Ref. 3 it is shown that $\dot{r}$ may be decomposed into two parts, one contributing to ripple transport (which vanishes in the limit of axisymmetry), and a second which produces the axisymmetric contribution, which we discard here. The remaining ripple contribution is given by

$$\begin{aligned} \dot{r} &= -(q/M\Omega r) \partial_{\phi_0} \langle \mu u \rangle / \partial_E \langle \mu u \rangle \\ &= \dot{r} \sin \eta \quad . \end{aligned} \quad (31)$$

Applying Eq. (24) to the first form for $\dot{r}$ in Eq. (31), one finds the bounce-averaged expression [5]:

$$\begin{aligned}\ddot{r} &= -(q/M\Omega r)\partial_{\phi_0} I/\partial_E I \\ &= \hat{r}(A_3/A_2)\sin(n\phi_0 - \omega t)\end{aligned}\quad (32)$$

The inversion of L_{HO} called for by kinetic theory is facilitated by using the second form for L_{HO} in Eq. (17), because the frequencies $\Omega_{\phi,b}$ there are (approximately) independent of the phases ϕ_0 and θ_b . Therefore, we write $\ddot{r}$ in Eq. (31) in the Fourier decomposition

$$\ddot{r} = \hat{r} \sum_{\ell=-\infty}^{\infty} G_{\ell} \sin \eta_{\ell}, \quad (33)$$

where $\eta_{\ell} \equiv n\phi_0 + \ell\theta_b - \omega t$, and

$$G_{\ell} [qN\theta_T(y), y] = \oint \frac{d\theta_b}{2\pi} \exp(-i\ell\theta_b) \exp[iqN\theta(\theta_b, y)] \quad (34)$$

defines the coupling coefficients introduced in Sec. II. For y appreciably less than 1, $\theta(\theta_b, y)$ goes to the harmonic limit

$$\theta(\theta_b, y) \approx \theta_T \sin \theta_b, \quad \theta_T/(y) = 2\arcsin y^{1/2}. \quad (35)$$

Using this in Eq. (34) along with the familiar Bessel function representation

$$J_{\ell}(z) \equiv \oint \frac{d\theta}{2\pi} \exp(-i\ell\theta) \exp(iz\sin\theta) \quad (36)$$

yields

$$G_l(qn\theta_T, Y < 1) \rightarrow J_l(qn\theta_T) . \quad (37)$$

From Eq. (34) and the oddness of $\theta(\theta_D, Y)$ in θ_D , one can show that the G_l 's share a number of important properties of the J_l 's for all Y , including the identities

$$G_{-l} = (-1)^l G_l , \text{ and } G_l^* = G_l .$$

Bounce averaging Eq. (33), only the $l = 0$ term survives:

$$\bar{r} = \hat{r} G_0 \sin(n\phi_0 - \omega t) . \quad (38)$$

Comparing this with (32), one is led to the identity

$$G_0 = A_3/A_2 , \quad (39)$$

a result which may be shown directly using Eqs. (28), (29), and (34).

Lastly, for Eq. (15) we need an expression for J_2 . Using $d^3 p \approx 4\pi M d\epsilon \mu B/u$, $d\mu/dy \approx -2\epsilon\mu$, and $\mu B = K$, one finds

$$J_{z1} = 4\pi M^{3/2} r R K^{1/2} \epsilon^{1/2} \int_0^1 a_1' \quad (40)$$

for the z_1 - system. Then, using

$$\hat{\theta} = u/L_t = \hat{u}/(2L_t a_1') , \quad (41)$$

$$\hat{\theta}_b = (\partial_E I)^{-1} = \hat{u}/(2L_t A_1') , \quad (42)$$

one has

$$\partial\theta/\partial\theta_b = \dot{\theta}/\dot{\theta}_b = A'_1/a'_1 \quad . \quad (43)$$

Using Eq. (43) in Eq. (40), we obtain J_z for the z_2 - system:

$$J_{z2} = 4\pi M^{3/2} r_{RK}^{1/2} \epsilon^{1/2} A'_1 \quad . \quad (44)$$

From Eqs. (40) and (28), one also sees that

$$\bar{J}_{z1} \equiv \langle J_{z1} \rangle = J_{z2} \quad . \quad (45)$$

Thus, $J_{z2} = \bar{J}_{z1}$ is independent of the angle variables (ϕ_o, θ_b) .

A. Nonresonant Case

We are now ready to solve Eq. (19) for f_1 . We begin with the nonresonant ("precession-dominant") case, in which the first term in Eq. (19) is large compared with the collision term. As usual in such calculations of low- ν transport, a subsidiary ordering in ν is introduced, writing

$$f_1 = f_1^0 + f_1^1 + \text{higher order} \quad .$$

Using this in (19), one finds

$$f_1^0 = -\delta r \partial_r f_o \quad , \quad (46)$$

and

$$f_1^1 = \int_0^t dt_0' C f_1^0, \quad (47)$$

where $\delta r \equiv \int_0^t dt_0' r$. Here and in Eq. (47), $\int dt_0'$ denotes an integral along an unperturbed orbit, i.e., one propagated by the operator $d/dt_0 \equiv (\partial_t + L_{H_0})$ in Eq. (19). The eigenfunctions of this operator have the form $F(I) \exp(i \ell \cdot \theta - \omega t) \equiv F(y, E, x) \exp(i \ell_1 \phi_0 + \ell_2 \theta_b - \omega t)$, with F , ℓ_1 , ℓ_2 , and ω for the moment arbitrary. Equation (33) expresses i as a sum of these eigenfunctions, and so using it, δr is easily solved for:

$$\delta r = \sum_{\ell=-\infty}^{\infty} r_{\ell} \cos \eta_{\ell}, \quad (48)$$

where $r_{\ell} \equiv \hat{r} G_{\ell} / (\omega - \ell \Omega_b - m \dot{\phi})$.

Again, as is usual in low- v transport, one can avoid evaluating f_1^1 in Eq. (47) by transforming Eq. (15) to a "symmetrized" form, in which the positive definite character of the transport coefficients is more manifest. In Appendix A, we show that (15) may be written

$$\Gamma_1 = (\Delta v)^{-1} \int_{\Delta v} d\mathbf{z} J_z g(\partial_r f_0) \left(\frac{v_t \mu u^2}{2\epsilon \mu B} \right) (\partial_y \delta r)^2, \quad (49)$$

where the subscript "1" on Γ refers to its linearity in v .

Equation (49) calls for $(\partial_y \delta r)$, which we compute using Eq. (48). The y -derivative is at constant θ , and therefore acts both on the amplitudes r_{ℓ} and on θ_b in η_{ℓ} :

$$\partial_y \delta r = \sum_{\ell=-\infty}^{\infty} [r_{\ell}' \cos \eta_{\ell} - (\ell \theta_b') r_{\ell} \sin \eta_{\ell}] \quad (50)$$

where $r'_l \equiv \partial_y r_l(y, E, r)$ and $\theta'_b \equiv \partial_y \theta_b(\theta, y)$. The term in $\ell \theta'_b$ in Eq. (50) is the origin of the enhancement of v_{eff} discussed in Secs. I and II. One has

$$\theta'_b = -(\partial \theta / \partial y)_{\theta_b} / (\partial \theta / \partial \theta_b)_y \approx \tan \theta_b / 2y. \quad (51)$$

For the final form here, we have used the harmonic approximation (35) for the bounce motion, valid for all but the most shallowly trapped banana particles. Even in the immediate vicinity of $y = 1$, $\theta'_b(\theta_b)$ maintains the same symmetries in θ_b as $\tan \theta_b / 2y$ (e.g., odd about $\theta_b = 0$ and π , singular at $\theta_b = \pm \pi/2$, zero at $\theta_b = 0$ and π), so this approximation retains most of the physics.

Putting Eq. (50) in (49) and using the z_2 -system, one finds

$$\begin{aligned} \Gamma_1 = & - \int dK dy (4\pi M^{3/2} K^{1/2} e^{1/2} a_1^2) g \partial_{r_0} f_0(2v_t) \\ & \times \oint \frac{d\theta}{2\pi} \oint \frac{d\phi}{2\pi} a_1^2 \sum_{l, l'} [...]_l [...]_{l'}, \end{aligned} \quad (52)$$

where $[...]_l$ denotes the expression in square brackets in (50), evaluated at the indicated l -value. Doing the ϕ_0 integral first yields for the second line in (52)

$$\begin{aligned} & \frac{1}{2} \oint \frac{d\theta}{2\pi} a_1^2 \sum_{l, l'} \{ [r'_l r'_{l'} + r_l r_{l'} \ell \ell' (\theta'_b)^2] \cos(\ell - \ell') \theta_b \\ & + [r'_l (r_{l'} \ell' \theta'_b) - (r_l \ell \theta'_b) r'_{l'}] \sin(\ell - \ell') \theta_b \} \end{aligned} \quad (53)$$

In the harmonic approximation, one has

$$a_1^2 \equiv y - \sin^2(\theta_T/2) \approx y \cos^2 \theta_b, \quad (54)$$

and using this, expression (53) reduces to a single sum over ℓ :

$$\begin{aligned} \frac{y}{4} \sum_{\ell} \{ [(r_{\ell}')^2 + r_{\ell+}' r_{\ell-}'] + [(r_{\ell}^{\ell}/2y)^2 + \{ r_{\ell+}^{\ell}/2y \} (r_{\ell-}^{\ell}/2y)] \\ [r_{\ell+}' (r_{\ell-}^{\ell}/2y) + r_{\ell-}' (r_{\ell+}^{\ell}/2y)] \} \quad , \end{aligned} \quad (55)$$

with $\ell_{\pm} \equiv \ell \pm 1$. Replacing expression (55) for the second line in Eq. (52) yields the desired expression for Γ_1 , corresponding to Eqs. (5) and (7) in Sec. II. Defining the diffusion coefficient $D(K)$ for energy shell K (at the radius r in question) by

$$\Gamma = -\sqrt{2} \, 4\pi \, M^{3/2} \int dK \, K^{1/2} \, gD(K) \, \partial_r f_0, \quad (56)$$

this correspondence can be made precise:

$$D_1^{\ell}(K) = \int_0^1 dy \, A_1 \, 2^{-3/2} \, \varepsilon^{1/2} \, v_{\ell} \{ \dots \} \quad , \quad (57)$$

where $\{ \dots \}$ denotes the expression in curly brackets in expression (55). Here, we have used $A_1 y \approx A_1$, which is valid in the harmonic approximation. It also correctly reproduces the result one finds, without invoking the harmonic approximation, in the derivation of the bounce-averaged result, i.e., the result one finds by neglecting all but the $\ell = 0$ term in Eq. (50). Then the only θ_b -dependence entering Eq. (53) is through a_1^2 , so the θ_b -integration may be exactly evaluated, using Eq. (27) and (43):

$$\oint \frac{d\theta}{2\pi} b a_1^2 = A_1 / 2A_1' .$$

This yields the bounce-averaged result

$$\overline{D}_1(K) = \int_0^1 dy A_1 2^{-3/2} \epsilon^{1/2} v_t(r_0')^2 , \quad (58)$$

which generalizes the result of Ref. 5 to $\omega \neq 0$ and arbitrary qN . If one assumes the successive l -peaks are well separated (cf. Fig. 2 and the discussion in Sec. II), then the dominant terms in Eq. (55) will be those having only that l -value of the nearest resonance, namely, the first term in the first and second square-bracketed factors in expression (55). That is,

$$D_1^l(K) = \int_0^1 dy A_1 2^{-3/2} \epsilon^{1/2} v_t \left[(r_l')^2 + (r_l l / 2y)^2 \right] , \quad (59)$$

of which Eq. (58) is just the special case $l = 0$. The y -derivative in r_l' acts principally on $G_l[qN\Omega_T(y)]$ in Eq. (48); so that

$$r_l' \sim qN r_l . \quad (60)$$

Thus, using Eq. (60), one readily sees from the term in square brackets in Eq. (59) the origin of the two terms in square brackets in Eq. (8). For $l \neq 0$ and in the anharmonic regime $y \sim 1$, the y -derivative acting on $\Omega_b = A_1'$ in Eq. (48) becomes large. However, the singular contribution this produces in the limit $y \rightarrow 1$, as well as the one arising from the term $(r_l l / 2y)^2$ in Eq. (59) in the limit $y \rightarrow 0$, are both eliminated from the physical result, because in those regions of singular D_1^l , the resonant expression D_{-1}^l becomes appropriate.

It is worth noting the resemblance of the result given by Eqs. (56) and (59) [or by the heuristic form in Eq. (7)] to expressions for the linear susceptibility χ . The latter has the structure [12]

$$\chi \sim \sum_{\underline{\ell}} \int d\underline{I} G_{\underline{\ell}}^2 \frac{\underline{\ell} \cdot \partial f_0 / \partial \underline{I}}{(\omega - \underline{\ell} \cdot \underline{\Omega})^2},$$

where, as already indicated, $\underline{I}$ represents the constants of the collisionless motion, and $\underline{\Omega}$ the characteristic collisionless frequencies. Thus, the factor $\underline{\ell} \cdot \partial f_0 / \partial \underline{I}$ corresponds to the factor $\partial_{\underline{r}} f_0$ in Eq. (56), and the $G_{\underline{\ell}}$ in the $r_{\underline{\ell}}$ in Eq. (59) are a special case of the coupling coefficients $G_{\underline{\ell}}$ here. Making the replacement $\omega \rightarrow \omega - i\nu_{\text{eff}}$ in χ , one has

$$\text{Im} \chi \sim \sum_{\underline{\ell}} \int d\underline{I} \frac{G_{\underline{\ell}}^2 (\underline{\ell} \cdot \partial f_0 / \partial \underline{I}) \nu_{\text{eff}}}{(\omega - \underline{\ell} \cdot \underline{\Omega})^2}$$

for $\nu_{\text{eff}} \ll |\omega - \underline{\ell} \cdot \underline{\Omega}|$, and

$$\text{Im} \chi \sim \sum_{\underline{\ell}} \int d\underline{I} \frac{G_{\underline{\ell}}^2 (\underline{\ell} \cdot \partial f_0 / \partial \underline{I})}{\nu_{\text{eff}}}$$

for $\nu_{\text{eff}} \gg |\omega - \underline{\ell} \cdot \underline{\Omega}|$. The former of these has just the form of the nonresonant flux Γ_1 just calculated, and the latter has the form of the resonant flux Γ_{-1} , whose formal derivation follows. This resemblance between $\text{Im} \chi$ and Γ might have been expected from the start, since Γ is a radial current induced in response to a specified perturbation of the plasma fields, and should thus be calculable from $\hat{r} \cdot \chi$. The real part of the response, corresponding to f_1^0 , vanishes under the flux surface average.

B. Resonant Case

Now we solve Eq. (19) in the resonant ("collision-dominant") case. We thus assume we are sufficiently near resonance, for a specified value of l , that the first term in Eq. (19) is negligible. In that limit, Eq. (48) implies that the component f_l^0 having the resonant l -value in the decomposition $f_1^0 = \sum_l f_l^0$, of Eq. (46) dominates the contributions $f_{l'}^0$, for $l' \neq l$. Thus, though from Eq. (23) the collision term Cf_1 in Eq. (19) couples all l' -values to the equation for f_l , we assume that the dominant contribution to the l -component $(Cf_1)_l \equiv (2\pi)^{-2} \int d\phi_0 \int d\theta_b \exp[-i(n\phi_0 + l\theta_b)] Cf_1$ of Cf_1 comes from the "diagonal" element $(Cf_l)_l$.

We begin by computing this element. Replacing f in Eq. (23) by $\hat{f}_l = \hat{f}_l(I) \exp i(l\theta_b - \omega t)$, one has

$$Cf_l = \exp[i(l\theta_b - \omega t)] v_t [(\partial_y + i l \theta_b') \hat{f}_l + (a_1/a_1') (\partial_y + i l \theta_b')^2 \hat{f}_l] \quad (61)$$

Just as occurred in Eq. (50), ∂_y in C acts both on the y -dependence in the amplitude $\hat{f}_l(I)$ of f_l , and on $\theta_b(\theta, y)$ in the phase-dependent factor. Taking the l -component of Eq. (61) and making use of the oddness of θ_b' in θ , yields

$$(Cf_l)_l = \exp[i(l\theta_b - \omega t)] v_t \left[\frac{\partial_y A_1 \partial_y}{A_1'} - l^2 \frac{a_1}{a_1'} \langle \theta_b' \rangle^2 \right] \hat{f}_l \quad (62)$$

Thus, for $l = 0$, the action of C is the same as the bounce-averaged form in Eq. (25). Evaluating the second term in square brackets in Eq. (62) in the harmonic approximation, using Eqs. (51) and (54), yields

$$\langle (a_1/a_1') (\theta_b')^2 \rangle_t = 1/4Y \quad (63)$$

Using Eqs. (62) and (63), Eq. (19) gives the basic equation for the resonant amplitude $\hat{f}_\ell$:

$$(\nu_t/A_1') \partial_{y_1} \partial_y \hat{f}_\ell - (\nu_t \ell^2/4y) \hat{f}_\ell = \hat{r}_\ell \partial_r f_0, \quad (64)$$

where $\hat{r}_\ell \equiv \hat{r} G_\ell/21$. As in Eq. (60), we compare the size of the two (collisional) terms on the left side of Eq. (64) by making the replacement $\partial_y \rightarrow qN$. Then one sees that for $y > y_\ell \equiv (\ell/2qN)^2$, the first term dominates, while the second term is dominant for $y < y_\ell$.

We obtain an approximate solution for $\hat{f}_\ell$ by neglecting the smaller collisional term in each of the two regimes. The solution is depicted in Fig. 3. In the regime $y < y_\ell$, where the pitch-angle dependence of ∂_y is the dominant collisional effect, the solution of Eq. (64) is trivial:

$$\hat{f}_\ell = -(\hat{r}_\ell/\nu_t) \partial_r f_0 (4y/\ell^2). \quad (65)$$

In this regime, the small-argument behavior of $G_\ell = J_\ell$ in Eq. (9) is appropriate, so that $\hat{f}_\ell$ scales with y as $y \theta_T^\ell \sim y^{(\ell/2+1)}$. As shown in Fig. 3, above $y = 0$, $\hat{f}_\ell$ rises in this fashion into the intermediate region $y \approx y_\ell$, where it turns over and begins large-argument oscillatory behavior appropriate to the regime $y > y_\ell$.

In the case $y > y_\ell$, the second term in Eq. (64) may be neglected, and the solution proceeds as in bounce-averaged theory, which is just the special case $\ell = y_\ell = 0$. We first generalize Eqs. (29) or (39) and (30) to arbitrary ℓ :

$$A_{3\ell}(y, qN) \equiv A_2 G_\ell = \left\langle \int_0^\pi 2a_1' \cos(qN\theta - \ell\theta_b) d\theta \right\rangle_0, \quad (66)$$

$$A_{4l}(y, qN) \equiv \frac{1}{2} \int_{y_l}^y dy' A_{3l}(y', qN) \quad . \quad (67)$$

Then the solution of Eq. (64) for $y > y_l$ may be written

$$\hat{f}_l = (\hat{r}/2iv_l) \partial_r f_0 \int_1^y dy' A_{4l}/A_1 \quad . \quad (68)$$

The lower limit "1" in the y -integral here imposes the usual boundary condition that $f_1(y + 1^-) = 0$. The lower limit y_l in the integral in Eq. (67) makes $\partial_y \hat{f}_l$ go to zero as y approaches the boundary $y = y_l$ where, as we recall from Fig. 3, $\hat{f}_l$ turns over.

Now we evaluate the radial flux Γ_{-1} . As in the nonresonant calculation, we use Eq. (15) specialized to the z_2 -coordinate set, finding

$$\Gamma_{-1} = \sum_l \int dK dy (4\pi M^{3/2} K^{1/2} \varepsilon^{1/2} A_1) g(\hat{r}_l^{**} \hat{f}_l + \text{c.c.}) \quad , \quad (69)$$

where "c.c." denotes the complex conjugate of the preceding term, and it is understood that the sum over l includes only those l values for which the resonant ordering is appropriate.

The y -integration in Eq. (69) may be broken into two parts, $\int_0^1 = \int_0^{y_l} + \int_{y_l}^1$, and Eqs. (65) and (68) used in the appropriate part. An integration by parts with respect to y brings the second of these contributions into a symmetric form. Because of the lower limits $y = y_l$ and $y = 1$ in Eqs. (67) and (68), no boundary term arises from this integration. Defining D_{-1} with the same normalization as in Eq. (56), one finds

$$D_{-1}^l(K) = D_{-1<}^l(K) + D_{-1>}^l(K) \quad , \quad (70)$$

where

$$D_{-1<}^2(K) \equiv \int_0^{y_\ell} dy A_1^{-3/2} \epsilon^{1/2} \frac{\hat{r}^2}{r^2} (2G_\ell/\ell)^2 / v_t, \quad (71)$$

and

$$D_{-1>}^2(K) \equiv \int_{y_\ell}^1 dy A_1^{-3/2} \epsilon^{1/2} \frac{\hat{r}^2}{r^2} (A_{4\ell}/A_1)^2 / v_t. \quad (72)$$

From Eq. (67), one has the approximate scaling

$$(A_{y\ell}/A_1) \sim G_\ell/(qN).$$

Using this in Eq. (72) in conjunction with Eqs. (70) and (71), one sees the origin of the heuristic result Eq. (6) given in Sec. II.

The derivation of the bounce-averaged result $\bar{D}_{-1}$, generalizing the collision-dominant result of Ref. 5 to arbitrary qN , is similar to, but a good deal simpler than, the one just given for D_{-1}^ℓ . It may be recovered by taking $\ell = 0$ in Eqs. (70) and (71):

$$\bar{D}_{-1} = D_{-1}^0 = \int_0^1 dy A_1^{-3/2} \epsilon^{1/2} \frac{\hat{r}^2}{r^2} (A_4/A_1)^2 / v_t. \quad (73)$$

IV. APPLICATION - ROTATIONAL EQUILIBRATION

Now we illustrate the theory developed in the preceding sections, while considering its implications for the equilibration rate of toroidal rotation in beam-heated tokamaks. Similar to previous work [14,15] studying rotational damping in rippled tokamaks, from the flux surface average of the toroidal component of the momentum equation, one obtains an evolution equation for the

plasma angular momentum [14-16]:

$$(d/dt)L_p(r) = T_{rp} + T_{xp} \quad (74)$$

Here, $L_p(r)dr$ is the angular momentum of a plasma shell at flux surface r of thickness dr :

$$L_p(r) = \sum_s \langle n_{s0} M_s R u_{\phi s} \rangle_f \approx \langle n_{i0} M_i R u_{\phi i} \rangle_f, \quad (75)$$

where $\langle \dots \rangle_f$ denotes an integral over a flux surface, and $n_{s0}(r)$ is the unperturbed density of species s . On the right side of Eq. (74) are the torques T_{ap} ($a = r, x$) exerted by other subsystems on the plasma, where $a = r$ designates the magnetic ripple and $a = x$ denotes external sources of torque that arise from beam heating, interactions with the wall, etc. If the ripple arises from a plasma-generated wave, Eq. (74) must be supplemented by an additional evolution equation for the total angular momentum $\bar{L}_r$ of the wave:

$$(d/dt)\bar{L}_r = \bar{T}_{pr} + \bar{T}_{xr} \quad (76)$$

The overbar in Eq. (76) denotes an integral over r of the quantity in question.

The torque $\bar{T}_{rp} = -\bar{T}_{pr}$ arises from the nonambipolar radial flux: the radial current $J_r = \sum_s e_s \Gamma_s = e_i \Gamma_i$ crossed with the poloidal magnetic field $B_p = b_p B$ produces a toroidal force giving T_{rp} :

$$T_{rp} = \sum_s \langle \Gamma_s M_s \Omega_{sp} R \rangle_f = \langle \Gamma_i M_i \Omega_{ip} R \rangle_f, \quad (77)$$

where $\Omega_{sp} \equiv b_p \Omega_s$ is the poloidal gyrofrequency. Thus, T_{rp} is essentially a constant times the radial flux, expressions for which are given in Sec. III.

The fluid rotation speed $u_{\phi s}$ is, as usual, fixed by assuming that the poloidal viscous resistance is large enough that the poloidal flow u_θ may be neglected. This should be valid for [16,17] $\varepsilon/r \delta \gg 1$, which is the case here. Then

$$u_{\phi s} = -b_p^{-1}(u_{*s} + u_E) \quad , \quad (78)$$

where $u_{*s} \equiv \partial_r p_s / M_s \Omega_s$ is the diamagnetic drift velocity, and u_E the $E \times B$ drift. For $e_r \equiv aeF_r/T_i$ large, u_E dominates u_{*s} in Eq. (78), and $u_{\phi s}$ is approximately the same as the particle drift velocity $R\Omega_\phi$.

A. Externally Induced Ripple

To make matters concrete, in Figs. 4 and 5 we plot $\tilde{T}_{rp}$ versus e_r for TFTR-like parameters ($R = 2.55$ m, $a = 0.83$ m, $q = 2$, $T_i = 2$ keV) for the values of density n_0 indicated on the figures, and for fixed ($\omega = 0$) perturbations, such as are induced by external currents. Figure 4 shows $\tilde{T}_{rp}(e_r, n_0)$ for a perturbation with $n = 20$, $m = 0$, and $\delta = 0.5 \times 10^{-3}$, taken to approximate the symmetry and strength of the field-coil-induced ripple around $r/a = 0.5$ in TFTR. Figure 5 is a plot of the same type, but for $n = 1$, $m = 0$, and the same δ as in Fig. 4. As discussed in Sec. II, a larger value of δ could occur for such perturbations, but the same value as in Fig. 4 is taken for purposes of comparison. For both plots, the ripple is assumed to have this amplitude over a fraction $\Delta V_p = 0.3 V_p$ of the plasma volume V_p .

The energy integral $\int dK$ in Eq. (56) is done as in Ref. 5 for the "mixed case," i.e., by taking, for any l, y and K the smaller of $D_{\pm 1}^2$ in the

integrand. We assume the usual Maxwellian dependence of f_0 on K , $f_0 = n_0 (2\pi v_T^2)^{-3/2} \exp(-K/T)$. For Γ_1 , the K -integral is then peaked around $K = T$, and so we approximate Ω_b in $(\omega - \ell \cdot Q)$ by its thermal value. Including the K -dependence of Ω_b will broaden the resonant peaks somewhat, but leave the situation qualitatively unchanged. The K -integration can then be done analytically, leaving only the y -integration (and the ℓ -sum) to be done numerically.

The densities taken in Fig. 4 straddle the transition from the banana drift ($n_0 = 0.3 \times 10^{19} \text{ m}^{-3}$) to the ripple-plateau ($n_0 = 3 \times 10^{19} \text{ m}^{-3}$) regime. For the low-density curve, the individual peaks are clearly distinguishable, separated by $\Delta e_r = 0.8$, corresponding to $\Delta \Omega_\phi = 4.3 \times 10^3 (\Delta e_r) = 3.44 \times 10^3 \text{ sec}^{-1}$, or to a change in rotation speed $\Delta u_\phi = 1.1 \times 10^4 (\Delta e_r) = 0.88 \times 10^4 \text{ m/sec}$. [For comparison, the sound speed is (for $T_e = 1 \text{ keV}$) $c_s = 2.1 \times 10^5 \text{ m/sec}$, so for $u_\phi = c_s$, one would need $e_r \approx 28.2$.] For the high-density case, there is a great deal of peak overlap, making $\bar{T}_{rp}$ vary fairly smoothly with e_r .

Too large to appear in Fig. 4 is the line indicating the strength of the driving beam torque, $-\bar{T}_{xp}$. For beam power $P_b = 4.5 \text{ MW}$ and average beam energy $E_b = 40 \text{ keV}$, one has $-\bar{T}_{xp} = 8.3 \text{ Nt-m}$. The net torque $\bar{T}_p$ on the plasma is the difference between the $-\bar{T}_{xp}$ line and $\bar{T}_{rp}$. Possible steady-state rotation speeds of the plasma thus occur for e_r such that $\bar{T}_{rp}(e_r) = -\bar{T}_{xp}$. Experimentally observed rotation speeds are in the range $1.5 \times 10^5 - 2 \times 10^5 \text{ m/sec}$ [8], corresponding to $e_r \sim 13 - 18$. From Fig. 4, one sees for the high-density case, which is more typical of TFTR parameters, that in this range $\bar{T}_{rp}$ is too small to compensate $-\bar{T}_{xp}$ by a factor of about 6, consistent with the earlier prevailing belief [18] that moderate- n ripple transport is inadequate to explain the rotational equilibration. However, the value of $\bar{T}_{xp}$ is

sensitive to parameters such as δ , $T_i(r)$, ΔV_p , etc., whose values are not precisely known. The value of δ used, for example, is the average value for TFTR at $r/a = 0.5$. However, the appropriate radius to have chosen is quite profile dependent, so that twice as large a δ would not be unrealistic, increasing $\bar{T}_{rp} \sim \delta^2$ by a factor of 4. Small changes in the assumed T_i , ΔV_p , or n_0 could easily make up the remainder of the factor of 6. Thus, within the substantial uncertainties involved, it must be concluded that moderate- n ripple transport may be adequate to explain the observed steady-state u_ϕ .

Similar remarks may be made for the time τ_ϕ required for the plasma to "spin up" to the steady-state u_ϕ . The experimentally measured value is $\tau_\phi^{\text{exp}} \approx 0.09$ sec [8]. From Eq. (74), the theoretical value is

$$\tau_\phi = \int_{.1}^f d\bar{L}_p(e_r) / \bar{T}_p(e_r) \quad , \quad (79)$$

where the symbols "i" and "f" in the limits of integration denote the initial and final values of $\bar{L}_p$ (or e_r) through which the spinnup occurs. A lower bound on τ_ϕ is the "free acceleration" time

$$\tau_{\phi 0} \equiv \bar{L}_p^f / \bar{T}_{xp} \quad , \quad (80)$$

i.e., the time it would take the plasma to reach the steady-state $\bar{L}_p$ or u_ϕ , if $\bar{T}_{rp}$ were set to zero in Eq. (79). It is also the e-folding time in the exponential approach to $\bar{L}_p^f$ which occurs if $\bar{T}_p(e_r)$ in Eq. (79) is taken to be strictly linear. Absorbing the factor of 6 in Fig. 4 into δ^2 [i.e., taking $\delta = \sqrt{6} (0.5 \times 10^{-3}) = 1.2 \times 10^{-3}$] to make the steady-state u_ϕ agree with experiment, one finds $\tau_{\phi 0} = 0.034$ sec assuming parabolic profiles for $n_0(r)$ and $u_\phi(r)$, and $\tau_{\phi 0} = 0.102$ sec assuming these profiles are flat. The spinnup

time and steady-state rotation speed are thus consistent.

While the torque generated by the moderate- n ripple appears marginally adequate to explain the experimental u_ϕ and τ_ϕ , that from low- n ripple is easily large enough. We refer to Fig. 5. For $n = 1$, the resonant peaks are twenty times as widely spaced ($\Delta e_r = 16$) as in the $n = 20$ case, so the peaks are well separated, the first peak ($l = 1$) occurring at $u_\phi = 1.1 \times 10^4$ (Δe_r) = 1.76×10^5 m/sec, approximately the experimentally observed value. At this value, the peak height is $\bar{T}_{rp} = 2.25$ Nt-m, slightly larger than the $n = 20$ ripple-plateau value of $n_0 = 3 \times 10^{19} \text{m}^{-3}$, which is comprised of the sum of about 10 overlapping peaks. (Put another way, the individual $n = 20$ peaks are about an order of magnitude smaller than the $n = 1$ peak, in accordance with the scaling expected from Eq. (6). This is apparent comparing the curves in Figs. 4 and 5 for $n_0 = 10^{19} \text{m}^{-3}$, where the individual peaks are visible for both values of n .) Because for $n = 1$ perturbations a realistic value for δ may be an order of magnitude larger than the value taken in Fig. 5, a perturbation of this type presents a large, abrupt barrier at $u_\phi = 1.76 \times 10^5$ m/sec to a plasma which accelerates essentially unimpeded at lower speeds. Thus, low- n perturbations also appear to provide approximately correct values for u_ϕ and τ_ϕ .

The scaling of u_ϕ with beam power which one expects for the two cases is quite different, of course. The smooth increase of $\bar{T}_{rp}$ with u_ϕ for the $n = 20$ perturbation implies a correspondingly smooth scaling of $u_\phi(P_b)$, while the localized spikes at successive harmonics for $n = 1$ implies a terminal u_ϕ which is almost independent of P_b , except for discrete jumps from one harmonic to the next, as $\bar{T}_{xp}$ becomes greater than the maximum $\bar{T}_{rp}$ for a given peak. (This analysis of the scaling is, of course, somewhat simplistic. For example, the position of the $l = 1$ peak in Fig. 5 depends upon Ω_b [cf. Eq.

(13)], which scales as $T_i^{1/2}$, which increases in some fashion with P_b .)

Experimentally, it has been found [8] that $u_\phi(P_b)$ is roughly linear for TFTR, while for ISX u_ϕ was insensitive to P_b . One possible explanation for this is thus that the low- n ripple is smaller than the moderate- n ripple in TFTR, but appreciably larger in ISX.

B. Internally Generated Ripple

We now consider in a more qualitative manner the implications of the theory for the interaction between the plasma and MHD modes within the plasma. In steady state one has, from Eq. (76) and the integral over r of Eq. (74),

$$0 = -\bar{T}_{xp} + \bar{T}_{xr} \quad , \quad (81)$$

and

$$0 = \bar{T}_{rp} + \bar{T}_{xp} \quad . \quad (82)$$

Adding these,

$$0 = \bar{T}_{xr} + \bar{T}_{xp} \quad . \quad (83)$$

$\bar{T}_{rp}$ here represents only the torque on the plasma from the MHD perturbation. The external torque $\bar{T}_{xp}$ now includes both that from the beam ($\bar{T}_{bp}$) as well as that from external coils ($\bar{T}_{cp}$). $\bar{T}_{rp}$ is a function of ω , which must adjust itself, shifting the positions of the resonant peaks in the plot of $\bar{T}_{rp}(e_r)$ so that Eq. (81) is satisfied. For simplicity, we set $\bar{T}_{xr} = 0$, though direct

interactions of the MHD mode with the wall are also possible [19].

An important point to notice is that $\bar{T}_{rp}$ does not enter into Eq. (83), so that, neglecting $\bar{T}_{xr}$, Eq. (83) has precisely the same content as in the case of the externally induced perturbation; the terminal rotation velocity occurs for u_ϕ such that the beam torque $\bar{T}_{bp}$ balances the torque $\bar{T}_{cp}$ from external ripple. Although $T_{rp}(r)$ may be very large, it does not affect the steady-state u_ϕ , because the internal mode has no direct way to exert a torque on the external world. Moreover, from Eq. (81), its radial integral $\bar{T}_{rp}$ must give zero. Thus, toward the plasma center, where the beam deposition is large, the plasma pulls the mode toward larger u_ϕ , and T_{rp} has one sign. Toward the plasma edge, neutrals and wall interactions tend to slow the rotation, so the mode pulls the plasma, and T_{rp} has opposite sign. Thus, while not directly affecting the average rotational equilibration time τ_ϕ , the mode ripple should cause the plasma to rotate more nearly as a rigid body. Because of the large ripple for such perturbations (cf. Sec. II), one expects the time t_ϕ for the plasma rotation at differing radii to equilibrate with the mode rotation to be small compared with τ_ϕ .

Because T_{rp} must integrate to zero, one expects T_{rp} to be localized about one of its zeros. These occur near the base of the resonant peaks, where the torque from the ions becomes smaller than that from the electrons (neglected in Figs. 4 and 5). Thus, one expects ω to adjust itself so that $\omega = l\Omega_b + n\Omega_\phi$, for some l . If $l = 0$ is selected, the mode rotation rate should be approximately equal to that of the plasma, in accordance with experimental observations [8].

V. CONCLUSION

We have developed a theory of banana-drift transport in tokamaks which generalizes previous theory in a number of directions. The theory is valid for ripple perturbations of arbitrary n , m , and ω , in contrast to earlier theory, which assumed $n \gtrsim 10$, $m = 0$, and $\omega = 0$ (time-independent perturbations). It has been shown that the radial transport which low- n perturbations produce can be much larger than that from moderate- n perturbations, the ripple-transport mechanism previously considered. The theory avoids bounce averaging, and so includes the contributions to transport from all harmonics l . This makes it applicable to rapidly rotating plasmas, and provides a continuous description of the transition from the banana-drift to the ripple-plateau regime as well.

The expressions for radial flux developed here correspond to a toroidal torque, and so the theory has been illustrated by considering its implications for toroidal rotation in beam-driven tokamaks. We find that observed terminal rotational velocities u_ϕ and equilibration times are consistent with torques arising from either moderate- n or low- n , externally imposed perturbations. Observed differences in scaling of u_ϕ with beam power in TFTR and ISX might be explained in terms of the different dependence on u_ϕ of the ripple transport from moderate- and low- n ripple.

Considering internally generated ripple, such as that arising from MHD modes, we conclude that unless these modes can strongly interact directly with the machine walls, they play little role in the gross rotational equilibration of the plasma. However, they should be very effective at transmitting torque radially, causing the plasma to rotate nearly rigidly over the radial range occupied by the mode, at a rate approximately the same as the mode rotation rate.

ACKNOWLEDGMENTS

I am grateful to R. J. Goldston, W. Park, S. D. Scott, R. B. White, and especially to A. H. Boozer for useful discussions.

This work supported by United State Department of Energy Contract No. DE-AC02-76-CHO-3073.

APPENDIX A

Here we show how the symmetric form Eq. (49) for the radial flux Γ is obtained from the definition of Γ in Eq. (15). First, because f_0 is independent of ϕ_0 and θ_b , and because from Eq. (46) and Eq. (48) the Fourier components of f_1^0 are $\pi/2$ out of phase with $\dot{r}$, f in Eq. (15) may be replaced with f_1^1 , given in Eq. (47).

We now symmetrize the resultant expression by integrating by parts twice, first with respect to a real-space variable parametrizing particle motion in a flux surface, and then with respect to a pitch-angle variable, making use of the self-adjoint structure of C . This is analogous to what is done in low- v calculations in axisymmetric [10] or bounce-averaged [20,21] theories. In those restricted situations, however, the first integration by parts is done in a way which appears to rely on special features of the expression for $\dot{r}$. The point we wish to stress here is that this manipulation can, in fact, be done rather generally.

We consider the integral in Eq. (15) in the z_2 -system. Then, we recall from Eq. (44) that J_{z2} is independent of the variables (ϕ_0, θ_b) parametrizing flux-surface position, as is $g(K)$, so these factors may be commuted outside the flux-surface integration $\oint d\phi_0 \oint d\theta_b$ in Γ . The remaining flux surface integral is thus

$$\Gamma \equiv \oint_f d\phi_0 \oint d\theta_b \dot{r} f_1^1, \quad (A1)$$

where $\oint_f$ denotes this flux-surface integration. We now make an additional change of variables, from (ϕ_0, θ_b) to (ζ_0, t_0) , given by the linear transformation

$$t_0 = \theta_b / \Omega_b, \quad \zeta_0 = \phi_0 - (\Omega_\phi / \Omega_b) \theta_b. \quad (A2)$$

As defined following Eq. (47), t_0 measures time elapsed along an unperturbed orbit, and ζ_0 is chosen to be constant along an unperturbed orbit, therefore serving as an orbit label. The limits of integration over ζ_0 and t_0 entering into γ may be chosen in more than one way, the only requirement being that the integration domain cover the (ϕ_0, θ_0) -domain $(-\pi \leq \phi_0 < \pi, -\pi \leq \theta_0 < \pi)$ exactly once. Then using Eq. (47) in (A1) and integrating by parts with respect to t_0 , one has

$$\begin{aligned} \gamma &= \Omega_b \oint_f d\zeta_0 dt_0 \dot{r} \int_0^{t_0} dt_0' C f_1^0 \\ &= -\Omega_b \oint_f d\zeta_0 dt_0 \delta r C f_1^0 \\ &= -\oint_f d\phi_0 d\theta_0 \delta r C f_1^0. \end{aligned} \quad (A3)$$

Because the edges $\phi_0 = \pm\pi$, $\theta_0 = \pm\pi$ in the (ϕ_0, θ_0) -domain are identified, no surface term arises in the integration by parts.

The remainder of the symmetrization procedure is standard. One replaces expression (A3) for (A1) in the full expression for Γ . Then using Eq. (23) for C , one integrates by parts once with respect to y , and using Eq. (46) for f_1^0 , one arrives at Eq. (49).

REFERENCES

- [1] STRINGER, T. E., Nucl. Fusion 12 (1972) 689.
- [2] CONNOR, J. W., HASTIE, R. J., Nucl. Fusion 13 (1973) 221.
- [3] BOOZER, A. H., Phys. Fluids 23, 2283 (1980).
- [4] GOLDSTON, R. J., WHITE, R. B., BOOZER, A. H., Phys. Rev. Lett. 47 (1981) 647.
- [5] LINSKER, R., BOOZER, A. H., Phys. Fluids 25 (1982), 143.
- [6] RECHESTER, A. B., ROSENBLUTH, M. N., Phys. Rev. Lett. 40 (1978) 38.
- [7] MYNICK, H. E., KROMMES, J. A., Phys. Fluids 23 (1980) 1229.
- [8] SCOTT, S. D., BITTER, M. L. (private communications).
- [9] YUSHMANOV, P. N., Nucl. Fusion 23 (1983) 1599.
- [10] HINTON, F. L., HAZELTINE, R. D., Reviews of Modern Physics 48, Part I (1976) 239.
- [11] LEWIS, H. R., SYMON, K. R., J. Math. Phys. 20 (1979) 413.
- [12] KAUFMAN, A. N., Phys. Fluids 15 (1972) 1063.
- [13] WHITE, R. B., GOLDSTON, R. J., MCGUIRE, K., BOOZER, A. H., MONTICELLO, D. A., PARK, W., Phys. Fluids 26 (1983) 2958.
- [14] MIKHAILOVSKII, A. B., TSYPIN, V. S., Sov. J. Plasma Phys. 10 (1984) 142.
- [15] TSANG, K. T., FRIEMAN, E. A., Phys. Fluids 19 (1976) 747.
- [16] SHAIN, K. C., CALLEN, J. D., Phys. Fluids 26 (1983) 3315.
- [17] MYNICK, H. E., Proc. Sherwood Theory Meeting, (Santa Fe, NM) paper 1-C-40 (1982).
- [18] EUBANK, H. P., et al., Tenth International Conference on Plasma Physics and Controlled Nuclear Fusion Research (London, 1984), paper IAEA-CN-44/A-V-3.
- [19] BOOZER, A. H., Phys. Fluids 24 (1981) 1387.

[20] HAZELTINE, R. D., CATTO, P. J., Phys. Fluids 24 (1981) 290.

[21] MYNICK, H. E., Phys. Fluids 26 (1983) 2609.

FIGURE CAPTIONS

FIG. 1. Depiction of the basic features of tokamak neoclassical transport. Axes are log-log. Figure 1(a) is the symmetric contribution, and 1(b) is the ripple-induced contribution.

FIG. 2. Sketch of diffusion coefficient D versus toroidal rotation frequency Ω_ϕ for a tokamak in the (generalized) banana-drift regime, showing the resonant peaks at successive harmonics l of the bounce frequency.

FIG. 3. Sketch of the amplitude $\hat{f}_l$ of the perturbed distribution function for a resonant harmonic l in the banana-drift regime. At small pitch angles [$y < y_l \equiv (l/2qN)^2$], the variation of $\hat{f}_l$ with bounce-phase produces the dominant collisional effect. For less deeply trapped particles ($y_l < y < 1$), the dominant collisional effect is the scattering of particles from one value of y to neighboring ones.

FIG. 4. Numerical evaluation of the toroidal torque (or, equivalently, of the radial flux) produced by a fixed ($\omega = 0$) $n = 20$, $m = 0$ perturbation, for TFTR parameters, versus $e_r \equiv aeE_r/T_i$ (or equivalently, Ω_ϕ). Shown are curves for three values of density n_0 , which show the transition from the lower collisionality banana-drift regime (where the resonant peaks are well separated) to the higher density ripple-plateau regime (where overlapping peaks produce a smooth curve).

FIG. 5. Plot of the toroidal torque versus e_r (or Ω_ϕ) as in Fig. 4, but for a low- n perturbation ($n = 1$, $m = 0 = \omega$), taken to have the same amplitude δ as in Fig. 4 for purposes of comparison. Shown is the interval in e_r in which the first resonant peak ($\ell = 1$) for the $n = 1$ perturbation appears, corresponding to a rotation velocity u_ϕ close to the value observed experimentally.

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Fig. 2

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Fig. 3

Fig. 4

Fig. 5

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