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NONLINEAR SELF-SUSTAINMENT OF MAGNETIC ISLANDS

By

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NONLINEAR SELF-SUSTAINMENT OF MAGNETIC ISLANDS

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Abstract

A mechanism is proposed for a nonlinear self-sustainment of magnetic islands which relies on the presence of a chaotic region. Particles diffusing in the stochastic magnetic field produce a local increase of current which in turn sustains the original perturbation if the equilibrium profile is concave. It is shown that the new equilibrium is stable. In a tokamak the mechanism is operative in the region outside the $q=2$ surface, in agreement with the observation of strong electron confinement degradation in this region.

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To understand the mechanisms responsible for anomalous electron heat transport has been the goal of several theoretical investigations.¹⁻³ While the mechanisms operating in the good confinement region, between $q=1$ and $q=2$, seem well reproduced by microinstability-based models, to invoke any microinstability as the source of the strong confinement degradation observed in the electron channel beyond $q=2$ seems unreasonable in the light of present knowledge from linear stability analysis of short-scale modes.⁴⁻⁶ On the other hand, long wavelength modes, such as the $m=2$, $n=1$ tearing mode with $n(m)$ the toroidal (poloidal) number, while relevant in explaining the mechanism of major disruptions, turn out, both theoretically¹ and experimentally, to play a very minor role in affecting the steady state confinement properties of a discharge.

The issue of edge confinement degradation is particularly relevant in connection with the observation, on many experimental devices, of some sort of "profile consistency"⁷, i.e., a resiliency of the profile to the changes which might be expected during the application of heating power with very different deposition profiles. There is general agreement that such behavior is peculiar to tokamak devices, stellarators not exhibiting anything similar. It is, therefore, natural to associate profile consistency with the presence of a parallel electron current. However, as tearing modes are usually stable for toroidal number $n > 2$, a nonlinear destabilization mechanism has to be invoked in order to overcome the quasilinear stabilizing term proportional to Δ' .⁸

The model presented in this paper relies on the presence of a stochastic region due to the interaction of magnetic island chains. In this respect the model differs from nonlinear stability analyses of microtearing modes⁹, which do not rely on the presence of a macroscopic parallel current, and from similar analyses² which rely on the presence of an $E \times B$ drift due to the

radial ambipolar electric field.

Our argument is basically the following. Let us assume a toroidal magnetic field configuration consisting of an infinite set of island chains and characterized by the presence of good magnetic surfaces around the 'O' points imbedded in a chaotic zone (Fig. 1). Locally, at minor radius r the geometry is that of a slab. The magnetic perturbation can be represented as

$$\psi = \psi_0(r) + \sum_{m,n} \psi_{m,n}(r-r_{m,n}) \cos(m\theta - n\phi) , \quad (1)$$

where ψ_0 is the equilibrium flux, and $\psi_{m,n}$ is the linear eigenfunction associated with the island chain at $r = r_{m,n}$, where $q(r) = m/n$ and q is the safety factor. In order to measure the degree of stochasticity of the magnetic field given by Eq. (1), we introduce the parameter

$$\alpha = \frac{\sum_{m,n} w_n}{\Delta r} , \quad (2)$$

where $w_n = 4 [-\psi_{m,n}(0)/\psi_0''(r_{m,n})]$ has been assumed to be independent of m and the sum is taken over all modes with rational surface in Δr . The sum over m is thus restricted to the range $nq(r) < m < nq(r) + nq'\Delta r$ and, therefore,

$$\alpha = \sum_n nq'w_n . \quad (3)$$

Stochastic threshold occurs for $\alpha = \alpha_c \approx 1$. We will assume a mode spectrum with n bounded by N with the dependence of w_n on n to be determined in the analysis.

Now we want to evaluate the change in the parallel current in the vicinity of an 'X' point produced by the chaotic region. In the presence of

a stochastic zone, at any particular island chain particles arrive by diffusing radially from different island chains. If the time to complete this diffusion is short compared to the collision time and the correlation time, the particle will modify the local current distribution. Local modification will arise from particles which have diffused through a radial distance

$$L \approx (D \tau_{ac})^{1/2} . \quad (4)$$

In Eq. (4) τ_{ac} is the correlation time $\tau_{ac} = L_c/v_{te}$, with L_c being the parallel correlation length and D the radial diffusion coefficient in the collisionless limit.¹⁰ Both of these quantities are well-known in the limit of large amplitude fluctuations, when the motion is fully stochastic. In that case they are given by the quasilinear expressions

$$L_{cQL} = \kappa_{\parallel}^{-1} = 2\sqrt{2} \, qR/(w_n q' n) \quad (5)$$

and

$$D_{QL} = v_{te} \sum_{m,n} L_c \left(\frac{\delta B_r}{B} \right)^2 . \quad (6)$$

In the latter expressions v_{te} is the electron thermal velocity, R is the major radius, $\delta B_r/B = (nq'/q)w_n^2/16R$, and the sum is restricted to modes with $m = nq$. We are, however, interested in fluctuation levels only slightly above stochastic threshold. In this domain it is well-known^{11,12} that both diffusion and correlation lengths are significantly larger than the quasilinear expressions, owing to the persistence of correlated motion, and are typically two times the quasilinear values in the models in which they have been evaluated. We assume this to be a general phenomenon and use

$$D \approx 2D_{QL}, L_c = 2L_{cQL}.$$

The local modification in current will be $\delta J_{\parallel S} \approx L^2 J''_{\parallel 0}$. However, for the purpose of actually calculating the stability of a given m, n mode we need the component of $\delta J_{\parallel S}$ with the same pitch of the original perturbation, i.e.,

$$J_{\parallel m, n} = \frac{1}{\pi} \int_{-\pi}^{\pi} dy \delta J_{\parallel S} \cos y \approx L^2 J''_{\parallel 0} \mu(\alpha) \quad (7)$$

with $y = m\theta - n\phi$. The general behavior of $\mu(\alpha)$ as a function of the stochasticity parameter α is schematically presented in Fig. 2. For $\alpha < \alpha_c$, $\mu(\alpha) = 0$ because good magnetic surfaces are still present between different island chains. For $\alpha > \alpha_c$, $\mu(\alpha)$ first increases to become of order unity, and then, for $\alpha > \alpha_m$, decreases, finally vanishing at α_t . This is because the chaotic zone becomes so large that the good surfaces at the '0' point are also destroyed and the special character of the x-point domain is lost. Note that in Eq. (7) the quantity $\mu(\alpha)$, proportional to $J_{\parallel m, n}$, has been assumed to be independent of m and n which means that the '0' point disappears, for each m, n mode, for the same value of α . This must be regarded as being true in an average sense: the exact sequence in which the 0-points bifurcate depends on the spectrum, but they all vanish for α of order unity.

An equivalent derivation of Eq. (7) is the following. Upon dividing the parallel current into an equilibrium and a fluctuating part, the condition $\nabla \cdot \mathbf{J}_{\parallel} = 0$ yields

$$\tilde{\mathbf{B}}_0 \cdot \nabla \mathbf{J}_{\parallel} + \tilde{\mathbf{B}} \cdot \nabla J_{\parallel 0} + \tilde{\mathbf{B}} \cdot \nabla \mathbf{J}_{\parallel} = 0 \quad (8)$$

with the last term on the left-hand side of the same order as the other two. Taking the m, n component of Eq. (8), we obtain

$$J_{\parallel m, n} = -\frac{1}{k_{\parallel}} \frac{m}{r} \psi_{m, n} \frac{dJ_{\parallel 0}}{dr} - \frac{1}{\pi k_{\parallel}} \int_{-\pi}^{\pi} \cos y \, dy \, \tilde{B} \cdot \nabla J_{\parallel} \quad (9)$$

Using quasilinear theory, we obtain

$$J_{\parallel m, n} \approx -\frac{1}{k_{\parallel}} \frac{m}{r} \psi_{m, n} \frac{dJ_{\parallel 0}}{dr} - \frac{1}{\pi k_{\parallel}} \int_{-\pi}^{\pi} dy \cos y \sum_{m, n} \frac{m^2}{r^2} \frac{1}{k_{\parallel}} |\psi_{m, n}|^2 \frac{d^2 J_{\parallel 0}}{dr^2} \quad (10)$$

and, therefore, we obtain the same estimate as in Eq. (7). Using Eqs. (4) to (7), we find

$$J_{\parallel m, n} = \frac{\nu(\alpha)}{8\pi w_n} \sum_{n'} n' w_n^3 J_{\parallel 0}'' \quad (11)$$

Note that if $J_{\parallel 0}''/J_{\parallel 0} > 0$ the current at the 'X' point increases, and the island tends to be destabilized.

This destabilizing mechanism is balanced by the quasilinear stabilizing mechanism, i.e., by the fact that the production of any single helicity component of the field tends to increase the magnetic energy. The saturation condition, therefore, can be written as

$$J_{\parallel m, n} = -\frac{\Delta' \psi_1(0)}{w} = \frac{2\pi q}{r} \frac{w_n}{16} J_{\parallel 0} \quad (12)$$

where we have made the large m approximation $\Delta' \approx -2m/r$. The saturation condition Eq. (12) then becomes

$$\nu(\alpha) \sum_{n'} n' w_n^3 = \frac{q}{r} \frac{J_{\parallel 0}}{J_{\parallel 0}''} n^2 w_n^2 \quad (13)$$

Equation (11) can be satisfied for arbitrary n only if the spectrum falls off

as $w_n \sim 1/n^{1/2}$. Writing $w_n = w/n^{1/2}$, we can determine w in terms of α by using the definition of the stochasticity parameter Eq. (3), which yields

$$\alpha = wNq' \quad (14)$$

Inserting Eq. (14) into Eq. (13), we finally obtain

$$\alpha^3 \mu(\alpha) = p\alpha^2 \quad (15)$$

$$p = \frac{2 J_{\parallel 0} \hat{s} q^2 N}{\pi J_{\parallel 0}^2 r^2} \quad (16)$$

with $\hat{s} = q'r/q$ the local shear. Furthermore, $\alpha \approx 1$ gives $w_n = 1/(Nnq')$ or $\delta B/B = 1/(16qRq'N^2n)$. From Fig. 2 we see that Eq. (15) has only one solution corresponding to $\alpha=0$ if p is greater than some critical value p_c , while it has three solutions if $p < p_c$. In particular, the solutions $\alpha=0$ and $\alpha > \alpha_m$ are both stable because an increase in α decreases $J_{\parallel m,n}$ and a decrease in α increases $J_{\parallel m,n}$, while the solution $\alpha_c < \alpha < \alpha_m$ is unstable.

The parameter p can now be evaluated for typical tokamak parameters. For the flat and rounded current profiles¹³ p is significantly less than one, typically having a minimum near the $q = 2$ surface. For the peaked current profile p is only slightly less than one in a small region near $q = 2$. Upon assuming a Gaussian current density profile

$$J_{\parallel 0} \sim e^{-q_a r^2/a^2} \quad (17)$$

and $q_0=1$, we obtain $J_{\parallel 0}''/J_{\parallel 0} = q_a(4q_a r^2/a^2 - 2)/a^2$. Therefore, the self-sustainment mechanism is operative only for $q_a r^2/a^2 \gg 1/2$, with the value q_a

$r^2/a^2 = 1/2$ corresponding roughly with the good confinement region ($q \approx 4/3$). For $q \lesssim 1$ the perturbed current $J_{m,n}$ plays a stabilizing role, while in the good confinement region $1 \lesssim q \lesssim 2$, the current density profile has almost a zero second derivative. In these regions the temperature profile is determined, respectively, by the sawteeth and by microinstabilities.¹ The approximate values of p , as given by Eq. (16), are shown in Fig. 3 for different current profiles, all with $q_0 = 1$ and $q_a = 4$. Since p has to be of order one this mechanism works for $N \lesssim 5$. Obviously the exact values have to be determined taking into account all the factors of order unity which are neglected in the present calculation. It is interesting to note that conversely the condition $p = p_c$ yields exactly a Gaussian current density profile with a characteristic width which scales as $q_a^{-1/2}$ in agreement with several experimental findings. Note that for $\alpha \approx 1$ this model predicts a fluctuation amplitude for $q > 2$ of $\delta B_r/B \approx 5 \times 10^{-5}$ with $N = 3, 4$, $m \approx 6, 8$ which is not unreasonable. Similar fluctuations have been observed on Macrotor¹⁴ at a level of $\delta B/B \sim 10^{-5}, 10^{-4}$. The frequency spectrum is slowly decaying for $\omega < 25$ kHz with a tail decaying as ω^{-2} for $\omega > 25$ kHz. Relating the frequency to the mode number by noting that the $m = 2$ mode has a characteristic frequency of 7 kHz, we obtain $m \approx 6-8$ (or $n \approx 2-4$). Moreover, the signal decreases toward the edge of the discharge in agreement with our finding $\delta B/B \sim (qq')^{-1}$. Note that although the perturbed current has the tendency to decrease the concavity of the current profile, its magnitude is not great enough to modify it significantly.

Finally, we have to check if the typical values of the electron thermal conductivity beyond $q = 2$ are consistent with the quasilinear diffusion coefficient Eq. (6). Inserting the expression of $\delta B/B$ into Eq. (6), we obtain

$$D = 5 \times 10^{-2} \frac{v_{te} R}{q_s^2} \left(\frac{r}{R}\right)^2 \left(\frac{a}{N}\right)^3 .$$

At the $q = 2$ surface with $T_e = 250$ eV, $R = 1$ m, $q_a = 4$, we have $D = 50(a/N)^3$ m²/sec. Therefore, for $a \gtrsim 1$ and $N \sim 3-4$ we obtain $D \sim 2-6$ m²/sec, which is much larger than the value in the good confinement region - about 1 m²/sec.

Note also that the parameter p diverges toward $r=a$ in a divertor discharge because of the factor q_s^2 in Eq. (16). Therefore, the region around the separatrix tends to show better confinement. However, such an effect is localized very close to $r = a$. In order to make a global transition, as the L $\rightarrow$ H transition, the parameter p has to increase also in the interior. This, in turn, can be obtained by decreasing $J_{||0}''/J_{||0}$, e.g., by the formation of a temperature pedestal.

The results of the present analysis suggest that the control of the current density profile in the periphery of the discharge is a possible method for achieving better global electron confinement.

To conclude, we have found a nonlinear mechanism for the destabilization of magnetic perturbations, operative in the outside region of a tokamak discharge. The natural profile shape for the mechanism is Gaussian, and the fluctuation amplitude and spectrum seem reasonable.

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REFERENCES

- 1 F. Romanelli, W.M. Tang, and R.B. White, Nucl. Fusion 26 1515 (1986).
- 2 W.M. Tang, Nucl. Fusion 26, 1605 (1986).
- 3 P.H. Rebut, in Plasma Physics and Controlled Nuclear Fusion Research (Proceedings of the Eleventh International Conference, Kyoto, 1986), paper CN-47/E-III-4.
- 4 J.F. Drake, N.T. Gladd, C.S. Liu, and C.L. Chang, Phys. Rev. Lett. 44 994 (1980).
- 5 A.B. Hassam and J.F. Drake, Phys. Fluids 26 133 (1983).
- 6 W.M. Tang, F. Romanelli, and S. Briguglio, Fourteenth European Conference on Controlled Fusion and Plasma Physics (Madrid, 1987), Vol. III, p. 1154.
- 7 B. Coppi, Comments Plasma Phys. Controlled Nucl. Fusion 25 261 (1980).
- 8 R.B. White, D.A. Monticello, M.N. Rosenbluth, and B.V. Waddell, Phys. Fluids 20 800 (1977).
- 9 A. Samain, Plasma Phys. Controlled Fusion 26 5 (1984).
- 10 J.A. Krommes, C.R. Oberman, and R.G. Kleva, J. Plasma Phys. 30 11 (1983).
- 11 C.F.F. Karney, A.B. Rechester, and R.B. White, Physica 4D 425 (1982).
- 12 A.B. Rechester, M.N. Rosenbluth, and R.B. White, Phys. Rev. A23 2664 (1981).
- 13 H.P. Furth, P.H. Rutherford, and H. Selberg, Phys. Fluids 16 1054 (1971).
- 14 S.J. Zweben, C.R. Menyuk, and R.J. Taylor, Phys. Rev. Lett. 42 1270 (1979).

FIGURE CAPTIONS

- Fig. 1 Magnetic field topology. The shaded area is the stochastic region.
- Fig. 2 Normalized current $\mu(\alpha)$ vs. the stochasticity parameter. The self-sustainment condition is given by the intersection of $\alpha^3\mu(\alpha)$ with $\rho\alpha^2$.
- Fig. 3 The parameter $p(r)$, given by Eq. (16) for different current profiles, all with $q_0 = 1$, $q_a = 4$. Profiles used were peaked (P), Gaussian (G), rounded (R), and flat (F).

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Fig. 1

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Fig. 2

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Fig. 3

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