TWO-PHOTON PHYSICS AND THE COMING GENERATION OF HEAVY ION COLLIDERS*

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ABSTRACT

The possibilities for two-photon physics at the coming generation of heavy ion colliders is discussed. Particular attention is given to both $e^+e^-$ production and resonance production of the Higgs particle. For $e^+e^-$ production the inadequacy of traditional perturbation theory is outlined, and through the introduction of approximations valid for heavy ions it is shown how to sum a class of non-perturbative diagrams. The role of the nuclear form factor in suppressing the cross section for the heaviest resonances is also discussed. It is shown how this latter point affects the two-photon cross sections for $W^+,W^-$ and Higgs production at RHIC, LHC and SSC energies.

1. Introduction

The Relativistic Heavy Ion Collider (RHIC), currently under construction at Brookhaven National Laboratory, will accelerate fully stripped $^{197}$Au$^{79+}$ ions to a beam kinetic energy of 100 GeV/u. The first collider experiments, using up to four (six are actually available) distinct experimental areas, are planned to begin in the spring of 1997. The initial RHIC luminosity is expected to be $L \approx 2 \times 10^{26}$ cm$^{-2}$ sec$^{-1}$, with future upgrades to $L \approx 10^{27}$ considered feasible. The main purpose of RHIC is a study of the so-called quark-gluon plasma. This plasma is expected to be formed via the central collisions of the ions. In addition to these central collisions there is the possibility of studying peripheral electromagnetic interactions at RHIC, including electromagnetic production of resonances through the two photon mechanism. Electromagnetic processes at RHIC were the subject of a recent workshop$^1$ at Brookhaven.

It should be noted that RHIC will not be the world's only relativistic heavy ion collider; it is planned to have heavy ions at CERN's Large Hadron Collider (LHC) by the year 2000 ($^{208}$Pb$^{82+}$ beams at a kinetic energy of 3.8 Tev/u). For completeness we will also consider peripheral collisions of heavy ions at the SSC, i.e. $^{208}$Pb$^{82+}$ beams at a kinetic energy of 8 Tev/u.

Focussing on the peripheral or electromagnetic interactions at RHIC, the moving Coulomb field acts as an ensemble of virtual photons. In a semi-classical picture, the finite nuclear size $R$ imposes a cut-off in the energy spectra $\omega$ of the virtual photons.

given by \( \omega < \omega_0 \approx R^{-1} \gamma \), where \( \gamma \) is the Lorentz parameter of the beam \((\gamma = 108 \) for Au beams at RHIC). Within this crude argument, \( \omega_0 \) is a measure of the most massive pair that may be produced by the virtual photon spectra. For Au ions at RHIC it is found \( \omega_0 \approx 3 \) GeV, whereas for Pb at LHC \( \omega_0 \approx 100 \) GeV, and for the SSC \( \omega_0 \approx 250 \) GeV. In Tables 1 and 2 the magnitude of various resonance cross sections are shown for \(^{197}\)Au\(^{79+}\) ions at RHIC and \(^{208}\)Pb\(^{92+}\) ions at LHC and SSC. The calculations were performed in lowest order perturbation theory.

Table 1. \( \gamma\gamma \)-Physics Accessible at RHIC

<table>
<thead>
<tr>
<th>Final State</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(^+),e(^-)</td>
<td>HUGE</td>
</tr>
<tr>
<td>( \mu^+\mu^- )</td>
<td>360 mb</td>
</tr>
<tr>
<td>( \pi^+\pi^- )</td>
<td>5.3 ( \mu )b</td>
</tr>
<tr>
<td>( \pi^0 )</td>
<td>6.3 mb</td>
</tr>
<tr>
<td>( \eta^0 )</td>
<td>1.5 mb</td>
</tr>
<tr>
<td>f(1270)</td>
<td>1.3 mb</td>
</tr>
<tr>
<td>( \eta_c(2980) )</td>
<td>8 ( \mu )b</td>
</tr>
</tbody>
</table>

Table 2. \( \gamma\gamma \)-Physics Accessible at LHC/SSC

<table>
<thead>
<tr>
<th>Final State</th>
<th>( \sigma_{\text{LHC}} )</th>
<th>( \sigma_{\text{SSC}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^0 )</td>
<td>66 mb</td>
<td>90 mb</td>
</tr>
<tr>
<td>( \eta_c )</td>
<td>1.16 mb</td>
<td>1.95 mb</td>
</tr>
<tr>
<td>( \eta_b )</td>
<td>1 ( \mu )b</td>
<td>2 ( \mu )b</td>
</tr>
<tr>
<td>H(100 GeV)</td>
<td>0.15 nb</td>
<td>1 nb</td>
</tr>
<tr>
<td>W(^+)W(^-)</td>
<td>7.3 nb</td>
<td>146 nb</td>
</tr>
</tbody>
</table>

Of particular interest are the cross sections for e\(^+\),e\(^-\) production. As will be discussed in subsequent sections, these cross sections are so huge that conventional Q.E.D. perturbation theory is inadequate. Understanding e\(^+\),e\(^-\) production rates is essential in a heavy ion collider for these particles represent a potential large background in detectors. Furthermore, an electron may be captured in the K-orbit of a beam ion, thus changing the charge state and eventually causing the ion to be lost from the beam. This reaction is the largest beam intensity loss mechanism in a heavy ion collider. Also shown in Table 2 is the W\(^+\)W\(^-\) cross section for Pb peripheral collisions at both LHC and SSC energies. The value quoted for SSC energies is a factor of \( \sim 14 \) larger than \( \sqrt{s} = 40 \text{ TeV} \) protons \((gq \rightarrow \text{W}^+\text{W}^-) \) reaction. The possibility of indirectly producing light Higgs (mass \( \sim 100 \text{ GeV} \)) via peripheral heavy ion collisions in also shown in Table 2.

2. e\(^+\),e\(^-\) Reactions at RHIC

A Monte Carlo event generator named “ELVIRA” has been written for total, single and double differential cross sections of e\(^+\),e\(^-\) particles at RHIC. From perturbation
theory only, the total differential cross section is given as:

$$\sigma_{e^+e^-} = \frac{1}{(2\beta)^2} \sum_{\sigma_1\sigma_2} \int d^3k d^3q d^2p_\perp \frac{1}{(2\pi)^6} |B(k, q, p_\perp)|^2$$

(1)

where \( q \) is the positron momenta, \( k \) the electron momenta, \( p \) the so-called intermediate momenta, and \( B(k, q, p_\perp) \) the sum of a direct and exchange amplitude for pair production [equation (1) is written in the lab. or collider frame]. Schematically, this amplitude is a product of nuclear charge from factors and a tensor connecting the spinors of the interaction.\(^3,4\) For \(^{197}\text{Au}^{79+}\) ions at RHIC, \( \sigma_{e^+e^-} \) has the magnitude\(^3,4\) \( \sigma_{e^+e^-} = 33,000 \text{ b} \) in perturbation theory, i.e. for a luminosity value of \( L = 2 \times 10^{26} \text{cm}^{-2} \text{sec}^{-1} \), the production rate of pairs \( N \) is \( N \equiv \sigma_{e^+e^-} L \simeq 10^7 \text{ pairs/sec} \).

As shown in Fig. 2.1, most of the produced pairs travel in the forward direction or along the beam direction. Figure 2.2 shows the differential transverse momentum distribution \( p_\perp \) as a function of \( p_\perp \). For this variable, the nuclear form factor plays an important role and considerably reduces the moderately high energy tail of the distribution. In fact, for barrel shaped collider detectors at RHIC the important variable is the double differential cross section \( d^2N/dy dp_\perp \), where \( y \) is the rapidity variable.
In Fig. 2.3, the \( y-p_\perp \) contours are shown for Au beams at RHIC. Numerical studies have shown that the combination of small opening cycles of the large RHIC detectors, and the presence of solenoids to sweep the background \( e^+,e^- \) particles down the beam pipe, means that the potential high background notes from \( e^+,e^- \) production are not expected to be a serious problem for RHIC detectors. However, \( e^+,e^- \) background rates for so-called full acceptance detectors needs to be studied in detail.

Of considerable theoretical interest is the fact that \( e^+,e^- \) production, calculated via perturbation theory, violates the unitarity limit for both small \( (b \leq 2\lambda_e) \) impact parameters and RHIC energies. For instance, \( U^{92+} \) beams at an impact parameter of one electron Compton wavelength \( \lambda_e \) violates the unitary unit for a beam Lorentz parameter of \( \gamma \geq 31.6 \). This suggests for heavy ions at RHIC, LHC and SSC energies that a new domain of electrodynamics, i.e. multiple pair production from a single interaction, will be present. However, unlike \( e^+,e^- \) production from electron-positron collisions, the heavy ions momenta may be considered unperturbed by \( e^+,e^- \) pair production. With this assumption, together with the ansatz that any number of pairs may be created by an interaction but that each pair is independent of all other real and virtual pairs, it is possible to sum a whole class of higher order diagrams including pair creations followed by annihilations. With these assumptions alone, the \( S \)-matrix
Fig. 2.3: Contour plot of $d^2N/d|y|dp_\perp$ shown for a selected range of $p_\perp$ values.

The element for creation of particular pairs $\bar{i} j, \bar{i} j, \ldots$ is given by

$$S_{\bar{i} j\bar{i} j} = \left( \frac{1}{i\hbar} \int_{-\infty}^{\infty} V_{\bar{i} j}^{(+)} (t) \, dt \right) \left( \frac{1}{i\hbar} \int_{-\infty}^{\infty} V_{\bar{i} j}^{(+)} (t) \, dt \right) \ldots$$

$$\times \exp \left( -\frac{1}{\hbar^2} \sum_{\bar{i} j} \int_{-\infty}^{\infty} V_{\bar{i} j}^{(-)} (t'') \, dt'' \int_{-\infty}^{t''} V_{\bar{i} j}^{(+)} (t') \, dt' \right)$$

(2)

where

$$V_{\bar{i} j}^{\pm} (t = \infty) \equiv \frac{1}{i\hbar} \int_{-\infty}^{\infty} V_{\bar{i} j}^{\pm} (t) \, dt$$

(3)

are the creation ($+$) and destruction ($-$) perturbative matrix element of the $\bar{i} j$ pair. In this way, the probability for any $N$ pairs is

$$P(N) = \frac{1}{N!} \left[ \sum_{\bar{i} j} \left| V_{\bar{i} j}^{(+)} (\infty) \right|^2 \right]^N e^E$$

(4)
where

$$E = \Re \left( \frac{-2}{\hbar^2} \sum_{i,j} \int_{-\infty}^{\infty} V_{ij}^{(-)}(t'') dt'' \int_{-\infty}^{\infty} V_{ij}^{(+)}(t') dt' \right).$$  \hspace{1cm} (5)$$

Using properties of principal value integrals, and not making any approximations involving off-shell matrix elements, Eqs. (4) and (5) combine to become

$$P(N) = \frac{1}{N!} P_{\text{PT}}^N e^{-P_{\text{PT}}},$$  \hspace{1cm} (6)$$

where $P_{\text{PT}}$ is the probability at a given impact parameter for one pair production via perturbation theory. The Poisson form of Eq. (6) tells us how to interpret violation of unitarity in perturbation theory, for multiplying (6) by N and summing over N gives the average probability of the number of pairs is equal to $P_{\text{PT}}$. In this way an event generator like ELVIRA, though based on perturbation theory, will provide the all important average number of produced $e^+,e^-$ pairs. In Fig. 2.4 the $N$-pair cross section $\sigma_N$ is shown as a function of impact parameter for Pb beams at the LHC.\(^5\)

Obviously, the higher the multiple of produced pairs or measure of non-perturbative effects, the faster the fall off in impact parameter. It can be seen that multi-pair production has relatively large cross sections in heavy ion colliders.

In addition to free pair production it is possible to capture an electron into the bound orbit of a beam ion. This mechanism is the major loss of beam intensity at RHIC, and has been intensely studied within perturbation theory.\(^6\) Non-perturbative calculations of this process, via coupled channels theory, are under way at this time.\(^7\)

In perturbation theory the $Z$ and $\gamma$ scaling of free pair production or electron capture are given by\(^7\)

$$\sigma_{e^+,e^-} \propto Z^4(\ln \gamma)^3$$  \hspace{1cm} (7)$$

$$\sigma_{\text{CAP}} \propto Z^7 \ln \gamma.$$  \hspace{1cm} (8)

3. Production of Large Mass Resonances

For a given beam energy, as the mass of the produced pair increases, the cross section is found to be more sensitive to details of the nuclear charge form factor. In Table 3, the sensitivity of the coherent cross section is shown for different beam energies and different resonances masses. The quantity $q$ is defined by $q \sim M/2\gamma$ where $M$ is the invariant mass of the produced pair. $F^4(\sim q^2)$ reflects the dependence of the cross section on the nuclear form factor $F$, i.e. $d\sigma \sim F(-q^2)^4$.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$\gamma$</th>
<th>$q(\text{GeV})$</th>
<th>$F^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^+\mu^-$</td>
<td>RHIC</td>
<td>$10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>RHIC</td>
<td>$5 \times 10^{-2}$</td>
<td>0.5</td>
</tr>
<tr>
<td>$W^+W^-$</td>
<td>RHIC</td>
<td>0.8</td>
<td>$10^{-16}$</td>
</tr>
<tr>
<td>$W^+W^-$</td>
<td>LHC</td>
<td>$2 \times 10^{-2}$</td>
<td></td>
</tr>
</tbody>
</table>
The form factor dependence outlined in Table 3 clearly shows why Brookhaven's RHIC is too low on energy collider to produce exotic pairs (mass ≥ 3 GeV). In fact, Table 3 represents complementary result to the simple cut-off geometric argument ($\omega_0 \lesssim R^{-1}\gamma$) given in the introduction. In spite of the form factor suppression, the cross section magnitudes in Table 2 are quite impressive.

There has been considerable study on the possibility of using peripheral heavy ion collisions or two-photon mechanisms to generate Higgs particles. In particular, an intermediate mass Higgs ($m_Z < M_{HIGGS} < 2m_w$) appears the most promising mass range. Perhaps the most comprehensive study of Higgs production at the SSC is due to Bottcher et al. In Fig. 3.1 the Higgs production cross section is shown as a function of Higgs mass $M_{HIGGS}$. For 8 TeV/u $U$ beams and a machine luminosity value of $2 \times 10^{26}$cm$^{-2}$sec$^{-1}$ up to 25 mass 200 GeV Higgs particles per year may be expected. The form factor dependence of $\sigma$ is clearly illustrated in Fig. 3.1, for as the mass of the Higgs increases the cross section for its production falls rapidly.

Another vitally important problem is the consideration of backgrounds that may obscure the signal for Higgs production. It is now thought that photon-gluon fusion

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**Figure 2.4:** Probability distribution for $N$-pair production as a function of impact parameter $b$. $N$ is the number of pairs produced and $p_1(b)$ is measured in the text. Note that for this figure only, $\gamma$ is described in the text. Note that for this figure only, $\gamma$ is measured in the ion frame and $\gamma_{LAB}$ is the Lorentz parameter for the beam.
Fig. 3.1: Plot of Higgs production cross section as a function of Higgs mass. The curves correspond to three different U beam energies at the SSC.

is the main electromagnetic background. At the present time, calculations of this fusion reaction as a function of impact parameter completely dominate any signal from Higgs production. At this time, the possibility of using peripheral heavy ion collisions to generate and detect Higgs particles must be considered very difficult.

4. References

2. B. Müller, contribution to reference 1 above.
9. A. Schaefer, contribution to this conference.
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