MASTER

INELASTIC BENDING OF BEAMS UNDER

TIME-VARYING MOMENTS

- A STATE VARIABLE APPROACH

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ABSTRACT

The inelastic response of a beam to several time-varying moments is presented in this paper. The constitutive equations used to describe the beam material are due to Hart. This is one of several state variable theories of inelastic deformation that have been proposed recently. Hart's equations have been previously shown to accurately predict the response of uniaxial specimens to time-varying loads. It is seen from this paper that Hart's theory is able to qualitatively simulate various phenomena in creep and plasticity such as the effect of previous deformation history, yielding, strain recovery, material hardening and strain rate sensitivity in this case of bending of a beam. The computational scheme used to integrate the equations is very efficient.

INTRODUCTION

Nonelastic deformation of metallic structures under time varying loads has been an important subject of investigation in solid mechanics for many years. Recently, many investigators, such as Hart [1,2], Valanis [3,4], Onat [5], and Miller [6,7], to name only a few, have proposed their own theories for describing material constitutive properties. Among these theories, Hart's state variable theory for polycrystalline metals has attracted the authors' attention for some years.

In Hart's theory, two state variables, the hardness σ^* and the anelastic strain ϵ^a , are introduced to describe the effect of previous deformation history on subsequent mechanical behavior of metals and alloys. The hardness variable makes it possible to describe the phenomenon of yielding without an explicit yield surface. Many other experimentally observed phenomena in creep and plasticity, such as rate dependence, creep strain recovery and Bauschinger's effect are also included in this theory. Mukherjee, Kumar and Chang, [8-11], Waung and Lance [12-13] and Chang and Lance [14] have solved boundary value problems involving the inelastic deformation of metallic structures using Hart's theory. In all of the above, except [11], the authors assumed that under steady loading the anelastic strain components are negligible. In this paper, the role of the anelastic strain is included and arbitrarily time-varying loads are considered. Spheres, cylinders and discs were analysed in [11].

In the present paper, the problem of bending of a beam under a time-varying moment has been formulated using Hart's theory. As a numerical example, the pure bending of a rectangular nickel beam under various

time-varying moment histories has been analysed in detail. Many features observed in uniaxial experiments such as creep, creep strain recovery, yielding and strain rate sensitivity are qualitatively predicted from the present analysis.

Hart's uniaxial theory has a sound experimental basis. Recent comparison of theoretical predictions and experiments for various creep, tension and other uniaxial tests on nickel and 304 stainless steel specimens showed excellent correlation [15,16]. The present research should help the design of further experiments for the evaluation of Hart's theory in the presence of an inhomogeneous distribution of stresses.

HART'S CONSTITUTIVE RELATIONS

According to Hart [1,2], the total strain $\underline{\epsilon}$ at anytime consists of the elastic strain $\underline{\epsilon}^e$ and the nonelastic strain. Thus in terms of rates.

$$\dot{\underline{\epsilon}} = \dot{\underline{\epsilon}}^e + \dot{\underline{\epsilon}}^n \quad (1)$$

The non-elastic strain, deviatoric by definition, is time and history dependent and can be decomposed into two components

$$\dot{\underline{\epsilon}}^n = \dot{\underline{\epsilon}}^a + \dot{\underline{\epsilon}}^p \quad (2)$$

where $\underline{\epsilon}^a$, a state variable, is the anelastic strain, a stored strain which reflects the direction and magnitude of prior stress history, and $\underline{\epsilon}^p$ is the completely irrecoverable and path dependent permanent strain. In this theory, there is no yield surface and $\underline{\epsilon}^n$ is assumed to exist at all deformation states. In addition, Hart introduces a state variable σ^* , called hardness, which represents isotropic strain hardening.

The elastic strain is related to the stress through the usual Hooke's Law. Hart's theory for the nonelastic strain is represented by the rheological model shown in Fig. 1. The deviatoric part of the stress, $\underline{s}$, is divided into two auxilliary stresses $\underline{s}^a$ and $\underline{s}^f$, such that

$$\underline{s} = \underline{s}^a + \underline{s}^f \quad (3)$$

The relations between $\underline{s}^a$ and $\underline{s}^f$ and the strains $\underline{\epsilon}^n$, $\underline{\epsilon}^a$ and $\underline{\epsilon}^p$ are assumed to be flow rules of the type

$$\dot{\underline{\epsilon}}^a = \frac{3}{2} \frac{\underline{\epsilon}^a}{\sigma^a} \dot{\underline{s}}^a \quad (4)$$

$$\dot{\epsilon}^p = \frac{3}{2} \frac{\dot{\epsilon}^p}{\sigma^a} s^a \quad (5)$$

$$\dot{\epsilon}^n = \frac{3}{2} \frac{\dot{\epsilon}^n}{\sigma^f} s^f \quad (6)$$

where $\dot{\epsilon}^n, \dot{\epsilon}^p, \epsilon^a, \sigma^a$ and σ^f are invariant scalars defined as

$$\begin{aligned} \dot{\epsilon}^n &= \sqrt{\frac{2}{3} \dot{\epsilon}_{ij}^n \dot{\epsilon}_{ij}^n} & \dot{\epsilon}^p &= \sqrt{\frac{2}{3} \dot{\epsilon}_{ij}^p \dot{\epsilon}_{ij}^p} & \epsilon^a &= \sqrt{\frac{2}{3} \epsilon_{ij}^a \epsilon_{ij}^a} \\ \sigma^a &= \sqrt{\frac{3}{2} s_{ij}^a s_{ij}^a} & \sigma^f &= \sqrt{\frac{3}{2} s_{ij}^f s_{ij}^f} \end{aligned} \quad (7)$$

The equations relating the scalars and the equation for the hardness rate are

$$\sigma^a = M \epsilon^a \quad (8)$$

$$\dot{\epsilon}^n = a^* \left(\frac{\sigma^f}{M} \right)^m \quad (9)$$

$$\dot{\epsilon}^p = \dot{\epsilon}^* (\ln \sigma^* / \sigma^a)^{-\frac{1}{\lambda}} \quad (10)$$

$$\dot{\sigma}^* = \dot{\epsilon}^p \Gamma(\sigma^*, \sigma^a) \quad (11)$$

$$\text{where } \dot{\epsilon}^* = \dot{\epsilon}_s^* (\sigma^* / \sigma_s^*)^m \quad (12)$$

In the above, M, m, λ, a^* and $\dot{\epsilon}_s^*$ (for a standard hardness value σ_s^*) are flow parameters for a certain metal or alloy, some of which are temperature dependent. These, and the work hardening function Γ , are obtained from experiments. The reader is referred to references [1,2] for details of Hart's theory.

FORMULATION OF BEAM BENDING PROBLEM USING HART'S THEORY

The problem of bending of a beam under a general bending moment which may vary with time is formulated in this section using Hart's State Variable Theory. The formulation is developed for a beam whose cross-section is symmetric about the plane of bending as given in Fig 2.

Let the length of the beam be l , the width of the cross-section be $b(y)$, and the depth be $h_1 + h_2$ where the x axis is the neutral axis such that

$$\int_{-h_2}^{h_1} yb(y)dy = 0 \quad (13)$$

The moment of inertia of the cross section of the beam about the neutral axis is

$$I = \int_{-h_2}^{h_1} b(y)y^2 dy \quad (14)$$

The assumptions adopted in the present research are:

- i) strain is small,
- ii) plane sections remain plane after bending,
- iii) deflection and slope are small,
- iv) the inertia force of the beam during motion is small compared to the applied load.

From assumption (ii) it follows that

$$\dot{\epsilon}_{zz}^{(1)}(y,z,t) = y\dot{\kappa}(z,t) \quad (15)$$

where $\kappa(z,t)$ is the curvature of the beam defined under assumption (iii) as

$$\kappa(z,t) = - \frac{\partial^2 W(z,t)}{\partial z^2} \quad (16)$$

where $W(z,t)$ is the deflection of the beam. We denote the intensity of the distributed load at a typical cross-section, z , of the beam as $q(z,t)$, the shearing force as $V(z,t)$ and the bending moment as $M(z,t)$ (Fig. 2). Then under assumption (iv), equilibrium in the rate form requires

$$\frac{\partial \dot{V}(z,t)}{\partial z} = -\dot{q}(z,t) \quad (17)$$

$$\frac{\partial \dot{M}(z,t)}{\partial z} = \dot{V}(z,t) \quad (18)$$

$$\dot{M}(z,t) = \int_A \dot{\sigma}_{zz}(y,z,t) y \, dA \quad (19)$$

and

$$\int_{A''} \dot{\tau}_{yz}(y,z,t) dA'' = - \int_{A'} \Delta \dot{\sigma}_{zz}(y,z,t) dA' \quad (20)$$

where A is the area of cross section of the beam and A' and A'' are areas defined in the free body diagrams given in Fig. 2(c).

The stress σ_{yy} is assumed to be negligibly small so that the only nonzero stresses are σ_{zz} and $\tau_{yz} = \tau_{zy}$. Using Hooke's law, we can write the following equations for the nonzero strains

$$\epsilon_{zz}(y,z,t) = \frac{\sigma_{zz}(y,z,t)}{E} + \epsilon_{zz}^n(y,z,t) \quad (21)$$

$$\epsilon_{xx}(y,z,t) = \epsilon_{yy}(y,z,t) = -\frac{\nu}{E} \sigma_{zz}(y,z,t) + \epsilon_{xx}^n(y,z,t) \quad (22)$$

$$\epsilon_{yz}(y,z,t) = \epsilon_{zy}(y,z,t) = \frac{\tau_{yz}(y,z,t)}{2G} + \epsilon_{yz}^n(y,z,t) \quad (23)$$

where E , G and ν are the Young's modulus, shear modulus and Poisson's ratio, respectively.

Using equations (14), (15), (18), (19), (20) and (21) (in rate form), the following equations for the rates of curvature and the stresses are obtained

$$\dot{\kappa}(z,t) = \frac{\dot{M}(z,t)}{EI} + \frac{1}{I} \int_A [\dot{\epsilon}_{zz}^n(y,z,t)y] dA \quad (24)$$

$$\dot{\sigma}_{zz}(y,z,t) = E[y\dot{\kappa}(z,t) - \dot{\epsilon}_{zz}^n(y,z,t)] \quad (25)$$

$$\dot{\tau}_{yz}(y,z,t) = -\frac{\dot{V}(z,t)Q(y)}{I b(y)} - \frac{EQ(y)}{I b(y)} \int_A y \frac{\partial \dot{\epsilon}_{zz}^n}{\partial z}(y,z,t) dA + \frac{E}{b(y)} \int_{A'} \frac{\partial \dot{\epsilon}_{zz}^n}{\partial z}(y,z,t) dA' \quad (26)$$

where $Q(y) = \int_{A'} y dA'$

The integration scheme proceeds as follows. The initial values of ϵ^a and ϵ^p are assumed to be zero and the initial hardness is prescribed. The initial curvature and stresses are obtained from the solution of the corresponding elastic problem, i.e.

$$\begin{aligned} \kappa(z,0) &= \frac{M(z,0)}{EI}, \quad \sigma_{zz}(y,z,0) = \frac{M(z,0)y}{I} \\ \tau_{zy}(z,0) &= -\frac{V(z,0)Q(y)}{I b(y)} \end{aligned} \quad (27)$$

Hart's equations are used to find the initial rates of change with time of the nonelastic strain components and the state variables ϵ^a and σ^* . The deviatoric stresses, strains and strain rates, s^a , s^f , $\dot{\epsilon}^a$, $\dot{\epsilon}^p$ and $\dot{\epsilon}^n$, have the form

$$\underline{d} = \begin{pmatrix} d_{zz} & d_{zy} & 0 \\ d_{zy} & -\frac{1}{2} d_{zz} & 0 \\ 0 & 0 & -\frac{1}{2} d_{zz} \end{pmatrix} \quad (28)$$

in the present problem.

The curvature and stress rates are next obtained from equations (24-26). The quantities of interest after a small time interval Δt are now obtained by a suitable integration scheme and the process continued until the desired final time. A modified Euler predictor-corrector scheme with automatic time step control is used for the time-integration of the pure bending problem discussed in the next section.

PURE BENDING OF A BEAM OF RECTANGULAR CROSS SECTION

The problem of pure bending of a beam of rectangular cross section subjected to time-varying moments is chosen as an illustrative example. The beam is of length l , width b and depth h . In this example all the dependent variables are independent of z . Thus, there are no shear stresses and equation (26) is identically satisfied with

$$\tau_{yz} = \dot{\tau}_{yz} = 0$$

Pure nickel is chosen as the beam metal. For this metal, the work hardening function in equation (11) has the form [15,16]

$$\Gamma(\sigma^*, \sigma^a) = (\sigma^a / \sigma^*)^{\beta / \sigma^*} (\beta / \sigma^*)^{\delta} \quad (29)$$

The material constants for pure nickel at room temperature, used in the calculations, are [15,16]

$$M = 7.6, m = 5, \lambda = 0.15$$

$$\dot{a}^* = 5.249 \times 10^{27} \text{ sec}^{-1}, M = 15.5 \times 10^6 \text{ psi (106.9 GPa)}$$

$$\dot{\epsilon}_s^* = 1.52 \times 10^{-36} \text{ sec}^{-1}, \sigma_s^* = 23,505 \text{ psi (.162 GPa)}$$

$$\beta = 1.044 \times 10^5 \text{ psi (.72 GPa)}, \delta = 1.64, E = 29 \times 10^6 \text{ psi (200 GPa)}$$

$$\nu = 0.3$$

Calculations have been performed for a nickel beam of rectangular cross section with dimensions $l = 7$ in (17.78 cm), $b = 0.8$ in (2.032 cm), $h = 0.125$ in (.318 cm) under different types of moment histories such

as constant moment, step changes of moment and constant moment rate.

A modified Euler's predictor-corrector method with a prescribed error band to control the time step sizes, is used. In this scheme, predicted values are calculated by

$$u^P(t+dt) = u(t) + \dot{u}(t)dt,$$

and corrected values by

$$u^C(t+dt) = u(t) + 0.5 (\dot{u}(t) + \dot{u}^P(t))dt$$

and finally, if $\text{Min. Error} \leq \frac{|u^P(t+dt) - u^C(t+dt)|}{|u^P(t+dt)|} \leq \text{Max. Error}$, the new value $u(t+dt)$ is chosen as

$$u(t+dt) = 0.8 u^C(t+dt) + 0.2 u^P(t+dt).$$

Hart's equations are stiff and very small time steps ($\sim 10^{-17}$ sec) are sometimes required, especially under conditions of sudden load change. The region $\sigma^a \approx \sigma^*$ poses further problems since the logarithmic term in equation (10) becomes nearly singular here. This region, called the viscoplastic limit, is the onset of yield when the plastic strain rate increases from a very small value to an appreciable one. In this region, equation (10) is replaced by an alternative equation obtained by using $\dot{\sigma}^a = \dot{\sigma}^*$. Details of this procedure are available in references [11 and 15].

Fig. 3 shows creep curves for beams with different initial hardness values under the same applied moment. It is seen that beams with lower values of initial hardness creep at higher curvature rates. An initial hardness of 10,000 psi (.0689 GPa) represents nickel in the annealed

state. Under this loading condition, the maximum initial stress is 27,000 psi (.186 GPa), which is much higher than the initial hardness of the softer beams. Hence, for the softer beams, such as the beams with initial hardnesses of 10,000 and 15,000 psi (.0689 and .103 GPa), a large amount of plastic flow occurs in a very short period of time after loading. This, together with a higher creep rate, results in much more creep for these beams compared to the beams with higher initial hardness values. Figs. 4a and 4b show stress redistributions in a beam in creep for an initial hardness $\sigma_o^* = 15000$ psi (.103 GPa). Fig. 4a shows the redistribution of the bending stress σ_{zz} . It is seen that, after sudden application of the moment, plastic flow occurs in a very short period of time and the plastic front moves towards the center of the beam. The bending stress σ_{zz} then reaches a steady distribution during creep. Fig. 4b shows the redistribution σ , σ^a and σ^* during creep. The anelastic stress invariant increases very fast from an initial zero level. Yielding starts at the outermost fibers when $\sigma^a \approx \sigma^*$ at $y = \pm h/2$ and the yield front moves inwards towards the center of the beam. The invariant σ^a remains very close to σ^* in the viscoplastic region which exhibits substantial plastic flow. Work hardening causes increase of hardness in this region.

Fig. 5 shows the response of a beam to step changes in load. The response to complete unloading and reloading is shown for two values of initial hardness, 40 and 15 ksi (.2756 and .103 GPa). The harder beam exhibits essentially complete instantaneous recovery upon unloading and the curvature returns quickly upon reloading to a value close to that of a beam under a constant moment M_o . The situation is quite different however for the softer beam where, unlike the previous case, a considerable

amount of permanent deformation occurs under the moment

$M_0 = 56.25 \text{ lb in (6.355 N.m)}$ which gives a maximum initial stress of 27 ksi (.186 GPa). Delayed recovery is observed, followed by material hardening upon reloading. The effect of previous deformation history on subsequent behavior is evident from a comparison of the response of the softer beam upon initial loading with its response upon reloading at $t = 8000$ seconds. The letters A, B, C, B', C' and B'' denote the elastic responses of the beams to step changes in moments.

Next, the response of a beam to a steady moment of $1.25 M_0$ is compared with that of a beam subjected to a moment level M_0 for 4000 seconds and $1.25 M_0$ thereafter. It is seen that the curvature in the latter case increases suddenly upon the step increase of the moment and approaches that of the former case.

Fig. 6 shows moment curvature curves under linearly increasing applied moments. Results for three moment rates, 1125, 56.25 and 2.8125 lb. in/sec (127.1, 6.355 and .3178 N.m/sec) are presented. Strain rate sensitivity is observed. The higher the moment rate, the larger is the moment required to get the same accumulated curvature.

Fig. 7 shows the changing distributions of the bending stress σ_{zz} under a linearly increasing moment. The results are qualitatively similar to those obtained from classical plasticity. It is seen that σ_{zz} increases from zero in a linear fashion until yielding first occurs at the outermost fibers. The plastic front then moves towards the center of the beam. At the same time the metal work-hardens causing increase of σ_{zz} and σ^* with increasing moment. The redistribution of σ , σ^* and σ^a is shown in Fig. 8 for this case. The situation is similar to that in Fig. 4.

CONCLUSIONS

The problem of inelastic bending of a beam under a moment that varies in space and time is formulated in this paper. This formulation is capable of using any of a number of constitutive relations using state variables that have been recently proposed. A numerical example is presented for pure bending of a nickel beam at room temperature under various moment histories using Hart's constitutive relations. Hart's equations have been previously tested in a variety of uniaxial experiments under time varying loads in which the correlation between theory and experiment was excellent. The three-dimensional formulation used in this paper is, as yet, tentative. The numerical results presented here demonstrate that this multiaxial formulation of Hart's equations is capable of predicting various important phenomena such as yielding, creep, rate sensitivity, strain recovery and material hardening in this beam bending problem.

The differential equations are stiff and variable time steps are essential for integrating them. A rate formulation is used together with a modified Euler predictor-corrector scheme with automatic time step control. This strategy works very well in this problem.

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Figure 1. Schematic model representation of Hart's constitutive relations.

Figure 2

Figure 3. Creep of a beam under a constant moment $M = 56.25 \text{ lb-in}$ (6.355 N.m) for various values of initial hardness. ($1 \text{ in} = 2.54 \text{ cm}$, $1 \text{ psi} = 6.895 \text{ kPa}$)

Figure 4a. Stress redistribution during creep. $M = 52.25$ lb-in (5.903 N.m)
 $\sigma_o^* = 15$ ksi (0.103 GPa), (1 psi = 6.895 kPa)

Figure 4b. Redistribution of σ , σ^a and σ^* during creep. $M = 52.25$ lb-in (5.903 N.m), $\sigma_o^* = 15$ ksi (0.103 GPa), (1 psi = 6.895 kPa)

Figure 5. Curvature as function of time for step changes in moment.
 $M_0 = 56.25 \text{ lb-in}$ (6.355 N.m) ($1 \text{ in} = 2.54 \text{ cm}$, $1 \text{ psi} = 6.895 \text{ kPa}$)

Figure 6. Strain rate sensitivity for three values of constant moment rate. (1 lb-in/sec = 0.113 N.m/sec, 1 in = 2.54 cm)

Figure 7. Stress redistribution under a constant moment rate.
 $\dot{M} = 56.25$ lb-in/sec (6.355 N.m/sec) (1 psi = 6.895 kPa,
 1 lb-in = 0.113 N.m)

Figure 8. Redistribution of σ , σ^a and σ^* under a constant moment rate.
 $\dot{M} = 56.25 \text{ lb-in/sec}$ (6.355 N.m/sec) ($1 \text{ psi} = 6.895 \text{ kPa}$,
 $1 \text{ lb-in} = 0.113 \text{ N.m}$)

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