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OPTIMAL DIGITAL COMPUTER CONTROL OF NUCLEAR REACTORS

by

Walter C. Lipinski

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OPTIMAL DIGITAL COMPUTER CONTROL OF NUCLEAR REACTORS

by

Walter C. Lipinski

Reactor Engineering Division

Reproduction of a thesis submitted to the
Illinois Institute of Technology
in partial fulfillment of the requirements
for the degree of
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ABSTRACT

Lipinski, Walter Charles; *Optimal Digital Computer Control of Nuclear Reactors*; Ph.D., Electrical Engineering Department; Illinois Institute of Technology; January, 1969. Adviser: Professor Andre G. Vacroux.

Prefaced by a literature survey of earlier applications of modern control theory and presentation of pertinent kinetics equations, the dissertation describes the sequential analytical investigation of a digital computer control system to implement nuclear reactor control and estimation functions.

First, nonlinear plant and measurement equations are derived for a deterministic one-group prompt-jump point model, using rate of reactivity change as control input. Next, state-space concepts are introduced, resultant equations are expressed in vector-matrix notation, linearized by a first-order Taylor series expansion, and solved for a discrete-time input.

Dynamic programming yields an optimal stationary feedback control law which minimizes a quadratic performance index for a discrete-time system. An index consisting of the sum squares of the neutron density derivations is defined and augmented to include terms in reactivity and control input. With the aid of an iterative digital computer program the stationary feedback matrix is calculated for selected values of weighting coefficients. Corresponding transient behavioral plots of the nonlinear system show that for the performance index as defined, the

neutron density deviation is decreased to zero in one sample interval after a step disturbance in reactivity.

In order to satisfy the optimal control law requirement that all state variables be available, a nonlinear estimator is used to generate estimates of nonmeasurable system state variables. Estimator equations, based on a set of finite-difference equations, are derived by minimizing a performance index consisting of the sum squares of errors in the previous estimate and in the current measurement. The resulting nonlinear equations are solved iteratively on a digital computer. Since the system is described by differential equations, integration is used to obtain the numerical values required by the estimator during the iteration sequence.

Finally, the cascade combination of an optimal estimator and optimal controller yields a control system whose performance is unequal to a system without an estimator. Estimates generated for the nonlinear system necessitate a large control input at the first sampling following a reactivity disturbance. Inclusion of a computation time delay results in further degraded performance. If an integrator is incorporated into the nonlinear estimator, the integration step size must be reduced when a control input is present. Since the computer programs used to solve the estimator equations and to compute the control input are not compiled for minimum time execution, no conclusion can be made with regard to real-time control capability.

The dissertation includes a comprehensive literature survey of earlier applications of modern control theory to nuclear reactors, a detailed review of pertinent reactor kinetics equations, and a wealth of selected nuclear and control engineering bibliographies.

CHAPTER 1

INTRODUCTION

1.1 Growth of nuclear power plants

Achievement of the first self-sustaining nuclear fission chain reaction in 1942 was recognized by Enrico Fermi and his colleagues as the initial objective toward creation of a destructive weapon. However, each scientist also recognized the constructive potential of controlling and converting the heat of fission into useful mechanical and electrical energy. In fact, one of the earliest concepts of converting nuclear energy into useful electrical energy — the Daniels Experimental Power Pile at Oak Ridge National Laboratory — was based on studies initiated in 1944 by Dr. Farrington Daniels, a member of this historic group. Unfortunately, national security prevailed and the application of controlled nuclear power was directed toward military logistics.

In 1947, Congress authorized the development of a nuclear reactor for submarine propulsion. Work initiated at Argonne National Laboratory near Lemont, Illinois, led to the construction and operation, on March 30, 1953, of the first nuclear propulsion system in a section of a submarine hull at the National Reactor Testing Station in Idaho. This land-based installation was the forerunner of the pressurized water system used in the submarine Nautilus, which was launched the following year. This launching represented the first milestone of the Naval Reactors Program which has since revolutionized naval strategy.

New reactor concepts for municipal power systems also were pioneered by Argonne scientists and engineers through the design, development, construction, and operation of simplified experiments or small-scale prototype systems at the Argonne test site in Idaho. Such was the case in 1951, when Experimental Breeder Reactor-I became the first nuclear reactor to generate electricity (170 kilowatts), thereby demonstrating the technical feasibility of: using unmoderated reactors for generation of useful power, employing sodium and sodium-potassium alloy as coolants, and breeding plutonium fuel. This experiment led the way to subsequent construction and operation, in 1963, of: EBR-II, a prototype fast power breeder central station plant; and the Enrico Fermi Atomic Power Plant, the world's first large fast breeder nuclear power plant.

In 1953, a series of Boiling Reactor Experiments (BORAX-I, -II, -III) were started at the Idaho test site. These experiments ultimately demonstrated the inherent power stability of the boiling water reactor concept. On July 17, 1955, the town of Arco, Idaho, was temporarily serviced with electricity generated by the BORAX-III power plant.

The technology gained from the BORAX experiments was applied in the construction of the Experimental Boiling Water Reactor (EBWR) at Argonne. On December 29, 1956, EBWR achieved its rated electrical output of 5,000 kilowatts, and thus became the first of a series of prototype central station power reactors to go into operation in the USAEC Civilian Power Reactor Development Program.

Two years later (May, 1958), the Shippingport Atomic Power Station in Pittsburgh, Pa., was dedicated as the first large-scale, nuclear power-generating plant (60,000 electrical kilowatts) in the

United States. Built by Westinghouse Electric Corporation as part of the same Civilian Power Reactor Development Program, the Shippingport plant design is based on the pressurized, light-water reactor concept.

Since 1958, the growth of nuclear powered central station plants in the United States has exceeded early predictions. This growth has been achieved by making nuclear plants economically competitive with conventional fossil-fueled plants. The most recent survey [1]* lists 13 operable, 31 being built, and 40 planned. Of these plants, 81 are based on the boiling and pressurized light-water reactor concepts.

As a consequence of the ever-increasing demand for uranium to fuel the light-water-cooled reactor power plants, the U.S. Atomic Energy Commission (USAEC) has given the highest priority to development of liquid-metal-cooled fast breeder reactors. In August, 1968, a Liquid-Metal Fast Breeder Reactor (LMFBR) program plan was issued. The overall objective is to achieve, through research and development, the technology required to design, construct, and safely, reliably, and economically operate fast breeder reactors for use in central station nuclear power plants. Volume 4 of that plan specifies the instrumentation and control developments essential to reliable and safe operation of an LMFBR plant [2].

1.2 Outline of dissertation

The research described in this dissertation was undertaken with the objective of applying modern control theory to the analysis and design of an optimal control system for a liquid metal fast breeder reactor. The fundamental problems of finding the optimal regulator control law and of estimating the states of the nonlinear deterministic system model

*Numbers in brackets pertain to references cited on pages 164 to 173.

have been solved. A natural consequence of applying dynamic programming to obtain the feedback regulator solution and iteration to the estimation problem is the requirement that a digital computer be used to implement the control and estimation functions.

Chapter 2 is devoted to a review of earlier applications of optimal control theory to nuclear reactor control problems. Since it was not feasible to discuss the specific applications in detail, appropriate references are cited. In addition, extensive selected bibliographies of nuclear and control engineering literature have been compiled for those who wish to specialize in this area.

Chapter 3 contains the equations which describe the reactor system. A one-group delayed neutron model is used as an approximation to the six-group system. A further simplification of the system equations is achieved by using a prompt-jump approximation.

In Chapter 4, the system differential equations are defined in terms of state variables and matrices. Nonlinear system equations are linearized using nominal values and the resulting set of equations is solved with discrete-time inputs.

Chapter 5 treats the solution of the closed loop regulator problem by applying dynamic programming to obtain the minimum of a specified performance index and the resulting transient response is discussed.

The closed loop solutions of Chapter 5 idealistically assume that all state variables are measurable; therefore, the solution of a deterministic estimator is derived in Chapter 6. Chapter 7 considers the combined problem of estimation and control.

Finally, the work is summarized, along with conclusions and recommendations for future research, in Chapter 8.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

From 1942 to 1960, analysis and design of control systems for nuclear reactors was based on classical methods.

Modern reactor control theory, which is concerned with optimal processes, emerged from Wiener's [6] theory in 1942, Bellman's [7] dynamic programming techniques in 1954, and Pontryagin's [8] Maximum Principle in 1956. Although several papers on off-line optimization of nuclear fuel management and xenon shutdown programs were published, Kallay [3], in 1960, was the first to relate modern control theory to nuclear reactors.

Early application of digital computer techniques to power reactors was limited primarily to data handling and on-line computations. In 1962, an issue of Nucleonics [4] was devoted to a special report on on-line computers for power reactors. At the 1964 Geneva Conference, Schultz and Legler [5] presented a status report on the application of digital computer techniques to reactor operation. Today, computer control systems are installed on several nuclear reactors, but these installations are on critical facilities or limited only to process control on power reactors. Literature describing these systems are listed in the general nuclear bibliography.

2.2 Previous investigations

Kallay [3] suggested four applications of dynamic programming techniques to nuclear reactors: optimization of poison distribution, optimization of over-all plant efficiency with respect to component cost, design of optimal control programs, and determination of flow distribution through a heat exchanger. Under control applications, Kallay outlined the optimal solution to a minimum energy start-up problem.

Foureau [9] used Pontryagin's maximum principle, a single group of delayed neutrons, and a constraint on the rate of change of reactivity, to determine the switching boundaries for a reactor start-up program.

Shen and Haag [10, 11, 12, 14] and Haag [13] used Pontryagin's maximum principle to solve an optimum start-up problem using a one-group delayed neutron model and a prompt-jump approximation. In the resulting control scheme, the switching conditions on the input were determined by nonlinear functions of time.

Mulcahey [15, 16] analyzed the time optimal control of nuclear reactors with velocity-limited control devices. His model consisted of a fast reactor with one group of delayed neutrons and a reactivity feedback, which was a function of the power level. The prompt-jump approximation was employed, and the resulting set of equations was solved analytically. System behavior was studied with analog and digital computers. He concluded that a power-level-based switching controller should be adopted.

Rosztoczy [17, 18] used the maximum principle and analyzed three optimization problems: a shutdown program for minimum xenon buildup, flux state changes in nuclear reactors, and minimum fuel loading. The model consisted of a single group of delayed neutrons and a reactivity feedback proportional to the power level. An integral performance index equal to reactivity squared was minimized by solving the resulting two-point boundary value problem on an analog computer. A suboptimal minimum-time solution was investigated by decreasing the time to execute a change in power level. Power level changes with minimum control energy were investigated by assuming a performance index equal to the integral of the reactivity rate squared. The solutions presented were open loop, and the control input was generated as a function of the adjoint variables.

Ruiz [19] used Pontryagin's maximum principle to minimize an integral performance index consisting of the sum of power deviation squared and square of the product of reactivity and power. One group of delayed neutrons was assumed. A closed loop control law was derived which required pre-programmed time variable coefficients.

Ash [20] used dynamic programming to derive a functional equation which would cause a boiling reactor to be driven back to its equilibrium condition in minimum time by continuously moving control rods.

Hermsen [21] used Wiener's theory and a linearized model of the reactor to design a closed loop control system based on minimization of an integral squared error index. Also, Z transform theory was used to design a control system which would be suitable for computer control. Pontryagin's maximum principle was applied to a system consisting of six groups of delayed neutrons and a model based on Newton's law of cooling. A set of 2(m + 7) equations resulted, where m was the number of temperature nodes. Dimensionality of the problem was reduced by going to a one-group linearized model, and a closed-loop control law was derived. The maximum principle also was used to solve the minimum-time problem with and without a constraint on the reactivity rate. In view of the difficulties encountered in obtaining solutions, it was suggested that dynamic programming be applied to the problem in future research.

Kliger [22, 42] used Holder's inequality to solve the minimum-time control problem subject to a constraint consisting of the product of reactivity and flux. One group of delayed neutrons was assumed. He derived a closed loop switching function, and proposed that a state estimator be used to generate the non-measurable state variables.

Mohler [22, 24, 25] used the maximum principle to analyze the minimum-time control of neutron density subject to a magnitude constraint on reactivity. A bang-bang control law was derived. In order to maintain constant power level, an additional input was required, after the last switching, to offset the effect of delayed neutrons. For the case of a six-group delayed neutron model, a feedback reactivity proportional to the sum of the rate of change of precursors was required to hold power level constant. A dither control was proposed as an alternate solution.

Weaver et al. [26] investigated: suboptimal closed-loop control employing the second method of Lyapunov, nonlinear stability of coupled core reactors described by a set of differential-difference equations, synthesis of optimal closed-loop control of nuclear reactor systems, and limits of validity for some approximations in reactor dynamics.

Secker and Weaver [27, 28] investigated optimal closed-loop control using a set of equations linearized around a nominal trajectory, and a quadratic performance index. Application of Pontryagin's maximum principle led to a matrix Riccati equation. The optimal filter for state-variable estimation was derived using Kalman's method for differential systems, and a matrix Riccati equation was solved for the optimal gain. The resulting closed-loop control system required storage of the preprogrammed control variable and nominal state trajectory.

Melsa [29, 30] extended the work reported previously by Weaver et al. [26]. Suboptimal control with a singular control matrix was investigated and applied to the control of a nuclear rocket.

Kliger [31] defined a control variable which was equal to the product of neutron flux density and reactivity and made the neutron kinetics equations linear. Reactivity was recovered as a true input control quantity by dividing the control variable by the measured flux. He applied the maximum principle to the problem using an integral performance index, and obtained the optimal control function in terms of the state and adjoint variables. Using back substitution, he then solved for the control function in terms of the state variables. An estimator was designed to generate the delayed neutron states from neutron flux measurements.

Duncombe [32, 33, 34] used the same linearizing approximation as Kliger to investigate on-line optimization of nuclear reactor load control in the presence of nonlinearities. To carry through this simplification, the performance index included a term of reactivity times flux squared. Based on this approximation, the results obtained by Duncombe must be judged accordingly. The optimal closed loop solution was obtained by using the maximum principle and deriving a matrix Riccati equation. The solution of the matrix Riccati equation varied with the varying load demand. To apply the correct feedback at each instant, it was necessary to calculate the parameters of the feedback network in effectively zero time. An analog computer was used to solve the matrix Riccati equation in 0.1 real time and to simulate the reactor plant. All of the state variables were obtained from the simulation. In his conclusions, Duncombe pointed out that in an actual application, the reactor plant simulation would be replaced by the reactor itself, however, he did not state that a state estimator would be necessary to generate non-measurable variables.

Monta and Lennox [35] investigated time-optimal digital computer control for the NRU reactor by applying the method of Desoer and Wing [36].

Kliger [37] extended his work [31] to analysis of an optimal control system for nuclear reactors with a generalized temperature feedback. The problem was subdivided such that a specific controller yielded the coolant flow and neutron density to minimize a performance index, and a universal controller forced the reactor neutron density to follow the desired neutron density. The maximum principle was applied, and the resulting set of equations was solved to obtain the optimal control law. The control law required all state variables, so an estimator was designed to generate delayed neutron estimates from neutron flux measurements.

Sokolova [38] analyzed the problem of determining an optimum control law for a nuclear power plant. A set of 29 differential equations, bilinear in the state variables and in the state and two control variables, was used to describe the plant, which consisted of a reactor, a regenerator, a cooler, and a turbocompressor. A quadratic performance index was used, and dynamic programming was applied. Two control equations were derived: one linear in the state variables and the other nonlinear. Lyapunov's method was applied to guarantee stability of the control system. Implementation of the control scheme required that all state variables be measurable.

Weaver et al. [39] investigated: optimal feedback control of nuclear reactor systems, modeling with Lyapunov functions, and linear system design using state variable feedback. The optimal control investigation used the linearizing substitution of Kliger [31]. A quadratic error index and prompt reactor model were used and a time-varying gain was obtained for the optimal feedback control by means of Bellman's equation. The analysis was repeated on reactor models using prompt nonlinear, linear delayed, and nonlinear delayed neutrons, with and without feedback. The developed methods were then used to analyze the start-up of a nuclear rocket.

Higgins [40] and Higgins and Schultz [41] investigated the stability of certain nonlinear time-varying systems of automatic control. They used the second method of Lyapunov, the Popov frequency criterion, and the matrix inequality method. As an example, the stability theory was applied to the simplified nuclear rocket propulsion system considered by Mohler (1962).

Monta [43, 44, 45] investigated the time-optimal control of nuclear reactors. One group of delayed neutrons and a prompt jump approximation were assumed. The maximum principle was used to derive the switching trajectories in state space, with and without constraints. The discrete version of the maximum principle was used to analyze a system with a pulse-width-modulated-reactivity input. An experiment was performed on the Toshiba Training and Research Reactor using a digital control computer. Computing time delay, control rod motor time constant, one-group approximation, and reactivity estimates had to be taken into account for practical reasons.

Humphries [46, 47] used a parameter adjustment model to investigate adaptive control of a nuclear rocket engine. The proportional control gain for the control poison was the parameter adaptively adjusted and the maximum core surface temperature was the variable adaptively controlled. The performance index consisted of the integral squared response error, which was formed by comparing the system output with that of the reference model. To evaluate the performance index, the nuclear rocket engine equations were linearized, the prompt neutron lifetime was set equal to zero, and the effects of delayed neutrons were neglected. Parseval's theorem was used to evaluate the performance index as a function of gain. It was shown that propellant savings of up to 20,000 pounds per transition from idle to full power are possible with adaptive control.

Saluja [48], and Saluja, Sage, and Uhrig [49] analyzed open and closed-loop control of nuclear systems. Three performance indices were considered: integral of reactivity squared, integral of reactivity squared and neutron density deviation squared, and the previous index with reactivity set equal to a proportional flux integral function of neutron density error. The maximum principle was applied, and quasi-linearization was used to solve the resulting two-point boundary value problem. Convergence was obtained in no more than four iterations for all problems. The suboptimal closed-loop control law yielded poorer performance than the open-loop control law. It was suggested that an adaptive-type control be considered to improve performance.

Ellis [50], and Sage and Ellis [51] presented a sequential suboptimal adaptive control philosophy which encompassed both identification and control. A general nonlinear differential system was modeled by a linear time varying system of assumed form. The system was assumed stationary over subintervals of time. This allowed a controller to generate a sequential control law which minimized an integral of time weighted quadratic form of error and control effort. The method was used to generate an optimum closed-loop control for the start-up dynamics of a nuclear reactor system.

Masters [52], and Sage and Masters [52] derived a sequential method for on-line estimation of the state variables and parameters of discrete, nonlinear, dynamic systems. The discrete version of the maximum principle was employed to obtain the canonic equations of the least-squares optimal estimator. Also, a discretized invariant imbedding technique was applied to solve the resulting two-point boundary value problem. A system of sequential equations was then obtained by application of variational methods to the optimal trajectory. The estimation procedure provided the best least-squares estimate of the state vector, given noisy measurements at discrete intervals of time. The method was applied to a nuclear reactor, with a single group of delayed neutrons, and the system state and one parameter were estimated.

Ogawa, Kaji, and Ozawa [54] analyzed the time-optimal control of nuclear reactors with two kinds of internal feedback: a prompt feedback generated by variations of fuel temperature and coolant density, and a delayed feedback governed by variations of moderator temperature. System stability was examined by investigating the behavior of the linearized system near an equilibrium point. The maximum principle was applied to the quasilinear system to obtain the optimum control law.

Rasetti and Vallauri [55] discussed the maximum principle and dynamic programming. A nuclear propulsion plant for a commercial ship with four steam generators and one pressurizer was analyzed for time-optimal control using the maximum principle. The canonical equations were compared to the results obtained by applying Bellman's equation.

Tataru, Bajenescu, and Ghetaru [56] considered the closed-loop regulator problem of a nuclear reactor. The small signal transfer function of a reactor was used. A scheme was derived to keep the loop gain constant by using a perturbing signal and a computing device to offset gain changes caused by power level changes.

Partain [57], and Partain and Bailey [58, 59] studied the application of Z transforms to linearized kinetics equations. Digital simulation was used to investigate system behavior.

Herring [60], Herring et al. [61], Weaver [62] and Weaver and Vanasse [68] developed a method for designing control systems by using state variable feedback. This method was applied to a two-temperature-region reactor and to a coupled-core reactor. Linearized transfer functions were used for the reactor systems. A method also was outlined for generating non-measurable state variables by placing frequency dependent elements in the feedback path.

Miyazaki [63] applied Wiener's theory [6] of least-squares optimization with quadratic constraint to the design of reactor control systems. The deterministic case was investigated by taking the integral square error for the criterion function and the integral square of reactivity rate for the control function. The stochastic case was studied by substituting the mean-square error and mean-square reactivity rate, respectively. Transfer functions for various step sizes and ramp inputs were derived.

Habegger [64], and Habegger, Bailey, and Kadavanich [65] applied quasilinearization and Kalman filter techniques to estimate nuclear parameters in the EBWR, PUR-I, and EBR-II reactors.

Melsa et al. [66] investigated: system identification using a random search method, data reconstruction using non-resetting integrators, and sub-optimal closed-loop control using invariant imbedding.

Mohler [67] analyzed the fuel-optimal control of a nuclear propulsion system by means of the maximum principle, Lagrange multipliers and computers. Practical problems were shown to be complicated by state constraints and high dimensionality. A minimum-time, prompt-neutron control process with reactivity rate and amplitude constraint was analyzed.

Mohler and Price [69, 70, 102] investigated application of linear programming procedures to optimal control of nuclear rocket reactors which had inequality magnitude constraints imposed on the control and state. Nonlinear equations were transformed into a form suitable for linear programming by using a first-order Taylor series expansion.

Marciniak [71, 101] studied the time-optimal digital control of zero power nuclear reactors. Sampled-data control system theory, including Z-transforms and discrete state variables, was used to design a control system which would: increase power level while maintaining a minimum period, and reach demand power level with little, or no, overshoot. Of the various data-holds investigated, the zero-order hold was the most stable. A time optimal study was made of a one-group delayed neutron reactor using the maximum principle, and the switching equation was derived. This switching equation and the zero-order hold were used to derive a control program, which was applied to noise-free reactor models simulated on a digital computer. A modified version of the control program was used on the Argonne Thermal Source Reactor.

CHAPTER 3

REACTOR DYNAMICS

3.1 Introduction

The derivation of the nuclear reactor kinetics equations, starting from neutron physics fundamentals, is well documented. These include treatments of the subject by: Glasstone and Edlund [72, Weinberg and Wigner [73], Meghreblian and Holmes [74], Isbin [75], or Ash [76], and a handbook presentation by Radkowsky [77]. An excellent treatment on general reactor dynamics is given by Gyftopolous [78], and the specific subject of fast reactor kinetics is treated by McCarthy and Okrent [79]. A discussion of the general subject of reactor dynamics and control is given by: Ash [76], Harrer [80], Keepin [81], Schultz [82], and Weaver [83, 84].

3.2 Six-group delayed neutron model

The point-model kinetics equations for a nuclear reactor are:

$$\frac{dn(t)}{dt} = \frac{\delta k(t) - \beta}{\ell} n(t) + \sum_i \lambda_i c_i(t) \tag{3.1}$$

and

$$\frac{dc_i(t)}{dt} = \frac{\beta_i}{\ell} n(t) - \lambda_i c_i(t) \qquad i = 1,\ldots,6 \tag{3.2}$$

where

$n(t)$ = neutron density

$\delta k(t)$ = reactivity

β = total delayed neutron fraction

ℓ = neutron lifetime

λ_i = decay constant of the *i*th neutron precursor

$c_i(t)$ = concentration of delayed neutrons of group i

β_i = delayed neutron fraction of group i

Reactor power level is proportional to neutron density. At low power levels, reactivity is not a function of the neutron density; therefore Eqs. (3.1) and (3.2) are commonly referred to as the zero power kinetics equations.

In Eq. (3.1) reactivity is a function of time, and for this condition, Eqs. (3.1) and (3.2) are linear with time varying coefficients. At high power levels, reactivity is a function of the neutron density, and the equations become nonlinear.

The values of λ_i and β_i for U-235 fueled fast reactors [85, p. 18] are listed in Table 3.1.

TABLE 3.1

DELAYED NEUTRON YIELD FROM FAST FISSION IN U-235

λ_i	β_i	a_i
0.0127	0.000247	0.038
0.0317	0.00138	0.213
0.115	0.00122	0.188
0.311	0.00265	0.407
1.40	0.000832	0.128
3.87	0.000169	0.026

The relative abundance is given by $a_i = \beta_i/\beta$. The total delayed neutron fraction is obtained from $\beta = \Sigma\beta_i$, and for the values of β_i in Table 3.1, $\beta = 0.0065$. Typically, $\ell = 10^{-7}$ sec for a fast reactor.

If the following variables are defined

$$\alpha = \beta/\ell \tag{3.3}$$

$$\alpha_i = \beta_i/\ell \tag{3.4}$$

$$\rho(t) = \delta k(t)/\beta \tag{3.5}$$

and substituted into Eqs. (3.1) and (3.2), then

$$\dot{n}(t) = \alpha\rho(t)n(t) - \alpha n(t) + \sum_i \lambda_i c_i(t) \tag{3.6}$$

$$\dot{c}_i(t) = \alpha_i n(t) - \lambda_i c_i(t) \qquad i = 1,\ldots,6 \tag{3.7}$$

where the dot notation designates the derivative with respect to time, and ρ is reactivity in dollars. Typically, $|\rho|<1$.

At equilibrium, the time derivatives are equal to zero, which on solving Eq. (3.7) gives

$$c_i(0) = \alpha_i n(0)/\lambda_i \tag{3.8}$$

The delayed neutron concentration can be normalized by defining

$$z_i(t) = (\lambda_i/\alpha)\, c_i(t) \tag{3.9}$$

Substitution of Eq. (3.9) into Eqs. (3.6) and (3.7) results in a set of normalized equations

$$\dot{n}(t) = \alpha\rho(t)n(t) - \alpha n(t) + \alpha \sum_i z_i(t) \tag{3.10}$$

$$\dot{z}_i(t) = \lambda_i\,[a_i n(t) - z_i(t)] \qquad i = 1,\ldots,6 \tag{3.11}$$

where the equilibrium solution requires that $z_i(0) = a_i n(0)$ and $\sum_i z_i(0) = n(0)$ because $\sum_i a_i = 1$.

3.3 Transient response of six-group model

For a step input of reactivity, the kinetics equations can be solved by application of the Laplace transform. Under the conditions of a step input $\rho(t) = \rho$, a constant. This constant value of reactivity is substituted into the equation before transformation. The initial conditions of $n(0)$ and $z_i(0)$ are the values of $n(t)$ and $z_i(t)$ which exist just prior to the step addition of reactivity.

With ρ set equal to a constant, taking the Laplace transform of Eqs. (3.10) and (3.11) results in

$$sN(s) - n(0) = \alpha\rho N(s) - \alpha N(s) + \alpha \sum_i Z_i(s) \tag{3.12}$$

$$sZ_i(s) - z_i(0) = \lambda_i[a_i N(s) - Z_i(s)] \qquad i = 1,\ldots,6 \tag{3.13}$$

Equation (3.13) is solved for $Z_i(s)$ to give

$$Z_i(s) = \frac{a_i\lambda_i}{s + \lambda_i} N(s) + \frac{z_i(0)}{s + \lambda_i} \qquad i = 1,\ldots,6 \tag{3.14}$$

Equation (3.14) is then substituted into Eq. (3.12) to obtain an equation for $N(s)$. Thus

$$N(s) = \frac{n(0) + \alpha \sum_{i=1}^{6} \frac{z_i(0)}{s + \lambda_i}}{s + \alpha(1 - \rho) - \alpha \sum_{i=1}^{6} \frac{\lambda_i a_i}{s + \lambda_i}} \tag{3.15}$$

Remembering that $\sum_i a_i = 1$, the denominator of Eq. (3.15) can be rearranged to yield:

$$N(s) = \frac{n(0) + \alpha \sum_{i=1}^{6} \frac{z_i(0)}{s + \lambda_i}}{s - \alpha\rho + \alpha \sum_{i=1}^{6} \frac{a_i s}{s + \lambda_i}} \tag{3.16}$$

Equation (3.16) is valid for any arbitrary initial conditions of $n(0)$ and $z_i(0)$. If the system is at equilibrium before the reactivity addition, then

$$z_i(0) = a_i n(0) \tag{3.17}$$

and substitution of Eq. (3.17) into Eq. (3.16) results in an expression of N(s) as a function of the initial neutron density. Thus

$$N(s) = \frac{1 + \alpha \sum_{i=1}^{6} \frac{a_i}{s + \lambda_i}}{s - \alpha\rho + \alpha \sum_{i=1}^{6} \frac{a_i s}{s + \lambda_i}} n(0) \tag{3.18}$$

In order to find the inverse Laplace transform of Eq. (3.18), the roots of the denominator must be known. If the numerator and denominator of Eq. (2.18) are multiplied by the factors $s + \lambda_i$, a seventh-order polynomial in s is obtained for the denominator, with coefficients consisting of complicated combinations of products and sums of the λ_i [82, pp. 110-111]. This polynomial is then factored for the roots.

An alternate method is to apply iteration to the denominator of Eq. (3.18) by means of the Newton-Raphson algorithm [86, p. 78] as follows:

$$s_{n+1} = s_n - \frac{F(s_n)}{F'(s_n)} \tag{3.19}$$

which converges quadratically to yield the solution of $F(s_{n+1}) = 0$ with

$$F(s) = s - \alpha\rho + \alpha \sum_i \frac{a_i s}{s + \lambda_i} \tag{3.20}$$

$$F'(s) = 1 + \alpha \sum_i \frac{a_i \lambda_i}{(s + \lambda_i)^2} \tag{3.21}$$

where F(s) is the denominator of Eq. (3.18) and F'(s) is the derivative of F(s) with respect to s.

Substitution of Eqs. (3.20) and (3.21) into Eq. (3.19) results in

$$s_{n+1} = s_n - \frac{s_n - \alpha\rho + \alpha \sum_i \frac{a_i s_n}{s_n + \lambda_i}}{1 + \alpha \sum_i \frac{a_i \lambda_i}{(s_n + \lambda_i)^2}} \tag{3.22}$$

which can be rearranged as follows:

$$s_{n+1} = \frac{\rho - \sum_i \frac{a_i s_n^2}{(s_n + \lambda_i)^2}}{\frac{1}{\alpha} + \sum_i \frac{a_i \lambda_i}{(s_n + \lambda_i)^2}} \tag{3.23}$$

In order for Eq. (3.23) to converge, suitable initial values must be chosen for the various roots. For positive ρ, one root is positive and all others are negative and range between the λ_i values [76, p. 32]. For ρ negative, all seven roots are negative. The most negative root is approximately equal to $\alpha(\rho - 1)$.

Equation (3.18) can be expressed as a partial fraction expansion. That is,

$$N(s) = \sum_{i=1}^{7} \frac{B_i}{s - s_i} n(0) \tag{3.24}$$

Since the poles of Eq. (3.18) are simple, the coefficients B_i of Eq. (3.24) can be obtained from:

$$B_i = \left. \frac{1 + \alpha \sum_i \frac{a_i}{s + \lambda_i}}{F'(s)} n(0) \right|_{s = s_i} \tag{3.25}$$

where $F'(s)$ is given by Eq. (3.21).

The Roots of Prompt Jump Equation computer program which finds the s_i and calculates the corresponding B_i is listed in Appendix G. Table 3.2 lists the s_i and B_i for a step input of $\rho = 0.1$.

TABLE 3.2

ROOTS OF KINETICS EQUATIONS AND TRANSIENT RESPONSE COEFFICIENTS FOR ρ= 0.1

s_i	B_i
.01046741	1.2924847
-.01438199	-0.03533592
-.06525568	-0.08955314
-.19093692	-0.04046886
-1.2253240	-0.01346368
-3.7713468	-0.00255375
-58,500.482	-0.11110930

The solution for the neutron density as a function of time, obtained by taking the inverse transform of Eq. (3.24), is

$$n(t) = \sum_{i=1}^{7} B_i e^{s_i t} \qquad (3.26)$$

The time constant corresponding to the most negative root in Table 3.2 is 17 μsec. If Eq. (3.26) is evaluated at t = 0.001 sec, using the values in Table 3.2, n(0) = 1.0, and the Reactor Response to Step Delta K computer program listed in Appendix G, then n(0.001) = 1.111. The flux has jumped 11.1% in 1 msec, and remains at this level until the terms in Eq. (3.26) with longer time constants began to exert their influence.

3.4 Prompt-jump approximation

In the analysis which follows, detailed reactor transient behavior at times less than 1 msec will not be of interest. Transient behavior in this case can be adequately described by employing the prompt-jump approximation. Setting $\dot{n}(t) = 0$ in Eq. (3.10) results in

$$0 = \alpha\rho(t)n(t) - \alpha n(t) + \alpha \sum_{i=1}^{6} z_i(t) \tag{3.27}$$

which is then solved for n(t):

$$n(t) = \frac{\sum_{i=1}^{6} z_i(t)}{1 - \rho(t)} \tag{3.28}$$

The neutron density is eliminated from Eq. (3.11) by substituting Eq. (3.28) for n(t) to obtain

$$\dot{z}_i(t) = \frac{\lambda_i a_i \sum_{i=1}^{6} z_i(t)}{1 - \rho(t)} - \lambda_i z_i(t) \tag{3.29}$$

Reactor response to a step input can be determined by means of Eqs. (3.28) and (3.29). For the case of equilibrium conditions prior to the step, $\rho = 0$ and $\sum z_i(0-) = n(0-)$. Immediately after the step

$$n(0+) = \frac{1}{1 - \rho} n(0-) \tag{3.30}$$

and n(t) has increased by the factor $1/(1 - \rho)$. If $\rho = 0.1$,

$$\frac{1}{1 - \rho} = 1.111 \tag{3.31}$$

which is the same as the transient response calculated previously for $t = 0.001$ sec and $n(0) = 1.0$.

3.5 One-group delayed neutron model

A further reduction in system dimensionality can be achieved by considering a single group of delayed neutrons. With this assumption, Eqs. (3.10) and (3.11) become

$$\dot{n}(t) = \alpha\rho(t)n(t) - \alpha n(t) + \alpha z(t) \quad (3.32)$$

$$\dot{z}(t) = \lambda[n(t) - z(t)] \quad (3.33)$$

The single-group decay constant λ must be suitably chosen if the one-group approximation is to provide useable results. In previous applications of the approximation, λ has been selected on the basis of best asymptotic behavior as $t\to\infty$. This method of selection is not the best for studying transient behavior at times of the order of one second; therefore an alternate method based on a matching of the transient response is proposed.

3.6 Transient response of one-group model

The transient response of the one-group model to a step input of reactivity can be determined by taking the Laplace transform of Eqs. (3.32) and (3.33) or equivalently modifying the six-group result of Eq. (3.16) to give

$$N(s) = \frac{n(0) + \dfrac{\alpha z(0)}{s + \lambda}}{s - \alpha\rho + \dfrac{\alpha s}{s + \lambda}} \quad (3.34)$$

which alternately can be written

$$N(s) = \frac{(s + \lambda)n(0) + \alpha z(0)}{s^2 + (\lambda + \alpha - \alpha\rho)s - \alpha\rho\lambda} \quad (3.35)$$

Given the numerical values of λ, α, and ρ, the roots of Eq. (3.35) may be calculated directly. These roots may be approximated by using the quadratic formula and the product relationship of the roots to obtain

$$s_1 \approx \lambda\rho/(1 - \rho) \tag{3.36}$$

$$s_2 \approx -\alpha(1 - \rho) - \lambda/(1 - \rho) \tag{3.37}$$

assuming that $\lambda \ll \alpha$.

The partial fraction expansion and inverse transformation of Eq. (3.35), using the roots given by Eqs. (3.36) and (3.37), results in

$$n(t) = \frac{\lambda e^{s_1 t} + [\alpha(1 - \rho)^2 + \lambda\rho]e^{s_2 t}}{\alpha(1 - \rho)^2 + \lambda(1 + \rho)}\, n(0) + \frac{\alpha(1 - \rho)[e^{s_1 t} - e^{s_2 t}]}{\alpha(1 - \rho)^2 + \lambda(1 + \rho)}\, z(0) \tag{3.38}$$

3.7 Transient response of one-group prompt-jump model

The prompt-jump approximation can be applied to Eq. (3.32) by setting $\dot{n}(t) = 0$ and solving for $n(t)$. Then

$$n(t) = \frac{z(t)}{1 - \rho(t)} \tag{3.39}$$

This solution for $n(t)$ is substituted into Eq. (3.33) to obtain an equation in $z(t)$ and $\rho(t)$. Thus

$$\dot{z}(t) = \frac{\lambda\rho(t)z(t)}{1 - \rho(t)} \tag{3.40}$$

The solution of Eq. (3.40) is

$$z(t) = z(0) \exp \int_0^t \frac{\lambda\rho(t)}{1 - \rho(t)}\, dt \tag{3.41}$$

and the flux density solution is obtained by substituting Eq. (3.41) into Eq. (3.39) to obtain

$$n(t) = \frac{z(0)}{1 - \rho(t)} \exp \int_0^t \frac{\lambda\rho(t)}{1 - \rho(t)} dt \tag{3.42}$$

If $\rho = 0$ for $t<0$, then $z(0) = n(0)$, and Eq. (3.42) becomes

$$n(t) = \frac{n(0)}{1 - \rho(t)} \exp \int_0^t \frac{\lambda\rho(t)}{1 - \rho(t)} dt \tag{3.43}$$

If reactivity is constant, then $\rho(t) = \rho$, and Eq. (3.43) becomes

$$n(t) = \frac{n(0)}{1 - \rho} \exp [\lambda\rho t/(1 - \rho)] \tag{3.44}$$

The same result is obtained from Eq. (3.38) for $t>0.001$ sec because the contribution from the second exponential term is then negligible.

3.8 Selection of one-group decay constant

In later analyses, reactor transient behavior will be examined in response to input signals occurring at one second intervals. It is therefore desirable to select a λ which will provide the best approximate transient response at the end of one second. For the case of $\rho = 0.1$, $n(0) = 1.0$, and $t = 1$ sec, Eq. (3.43) is set equal to Eq. (3.26) using the values in Table 3.2. This results in $\lambda = 0.312$. This value of λ will be used in subsequent calculations which utilize the single-group model. Note that, within accuracy limits, this particular value of λ coincides with one of the intermediate values of λ listed in Table 3.1.

3.9 Reactivity input

Reactivity changes in an actual system are effected by a control rod mechanism. Figure 3.1 shows a block diagram of a reactivity input system.

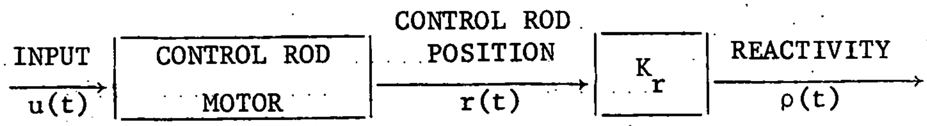

Fig. 3.1 Reactivity input system

The gain K_r has been included in Fig. 3.1 to account for the control rod calibration in terms of units of reactivity for units of position.

The control rod motor transfer function is given by

$$\frac{R(s)}{U(s)} = \frac{K_m}{s(1 + sT_m)} \tag{3.45}$$

which can be expressed as a differential equation as follows:

$$\dot{r}(t) + T_m\ddot{r}(t) = K_m u(t) \tag{3.46}$$

If it is assumed that the motor time constant is negligible, then Eq. (3.46) reduces to

$$\dot{r}(t) = K_m u(t) \tag{3.47}$$

Reactivity is related to control rod position by

$$\rho(t) = K_r r(t) \tag{3.48}$$

which upon substitution into Eq. (3.47) yields

$$\dot{\rho}(t) = K_m K_r u(t) \tag{3.49}$$

If $K_m K_r$ is set equal to one, then the units of u(t) are given directly in dollars per second, and Eq. (3.49) becomes

$$\dot{\rho}(t) = u(t) \tag{3.50}$$

Equation (3.50) shall be used in subsequent analysis to express the functional dependence of reactivity on an input.

CHAPTER 4

STATE SPACE REPRESENTATION OF REACTOR DYNAMICS

4.1 Introduction

The classical methods of control system analysis and design are based on input-output relationships of systems generally represented by one nth order differential equation. Modern control theory utilizes the concepts of state space and state variables, and an nth order system is represented by a set of n first-order differential equations.

The selection of a set of state variables to represent a system described by one nth order differential equation is not unique. In the case of reactor kinetics, formulation of system equations from physical considerations has led to a natural selection of state variables, and the system is initially described by n first-order differential equations.

It is convenient to first apply the concept of state space to a reactor with one group of delayed neutrons and then extend it to a reactor with six groups. For the one-group reactor, the neutron density n(t) and delayed neutron precursor density c(t) are the two variables which uniquely describe the state of the reactor at any time t. The state space for the reactor is two dimensional, a plane, and its coordinates are n(t) and c(t). The two coordinates are specified by a pair of ordered numbers, a vector. The state of the reactor at any time t can be associated with a point in a plane. Given $n(t_0)$ and $c(t_0)$, which determine the reactor state at any time t_0, and the reactivity $\rho(t)$ for $t > t_0$, the future behavior of the reactor can be predicted by solving the system differential equations, and the change in system state is traced as a line in the state plane. If the system is simulated on an analog computer, the neutron density and delayed neutron concentration can be individually displayed on digital meters, individually recorded as a function of time, and plotted on an X-Y recorder. The readings from the two digital meters provide information on the instantaneous state, and the X-Y recorder traces a line in the state plane. The individual recordings provide a parametric display as a function of time.

If two groups of delayed neutrons are used to describe the reactor, then the state space is three dimensional and has the coordinates n, c_1, and c_2. Specifying the values of n, c_1, and c_2 at any time t locates a point in the three dimensional space which describes the state of the reactor. If the reactivity $\rho(t)$ is given, the future behavior of the reactor is traced as a line in the three dimensional state space. The values of n, c_1, and c_2 at any instant are represented by an ordered set of numbers, a vector. The term *vector* is applied to the unique description of a point by an ordered set of numbers and is not intended to imply a directed line segment from the origin. An analog computer simulation will require three digital meters and three recorders. Since three-dimensional X-Y-Z plotters are not available, projections on the X-Y, X-Z, and Y-Z planes may be recorded to afford an indirect visualization of system behavior in the state space. The readings from the three digital meters provide information on the instantaneous state, and the individual recordings provide a parametric display as a function of time.

With six-groups of delayed neutrons, the state space is seven-dimensional, and seven differential equations are used to describe the system. An ordered set of seven numbers, a vector, describes the system state at any instant of time. An analog computer simulation requires seven digital meters and seven recorders. Twenty-one X-Y plotters would be required to plot all paired combinations of variables if the display method of the three dimensional case was to be extended. In this case, the change in system state cannot be visualized in three dimensional space, but the readings from the seven digital meters specify the instantaneous state and the individual recordings provide the parametric display as a function of time. The ordered set of meter readings gives the numerical value of the system state vector at any instant.

The above discussion may be summarized as follows: n state variables x_1, x_2, $x_3,\ldots,$ x_n are needed to describe completely the behavior of a system described by a set of n first-order differential equations. The set of n state variables can be considered as n components of a vector $\underline{x}$, called the *state vector*. A *state space* is an n-dimensional space in which x_1, $x_2,\ldots,$ x_n are the coordinates. The state of the system at time t can then be represented by a point in an n-dimensional state space. The locus of points in the state space is called a *trajectory*.

Vector-matrix notation is convenient for the representation of system differential equations in state-space analysis. The solution of vector-matrix differential equations is discussed briefly in Appendix A. Detailed treatments of state-space analysis and vector-matrix equations have been published by: Zadeh and Desoer [87], DeRusso, Roy, and Close [88], Gupta [89], Ogata [90], Timothy and Bona [91], and Chen and Haas [92].

4.2 Six-group representation

Using vector-matrix notation, Eqs. (3.1) and (3.2) can be written:

$$\begin{bmatrix} \dot{c}_1 \\ \dot{c}_2 \\ \dot{c}_3 \\ \dot{c}_4 \\ \dot{c}_5 \\ \dot{c}_6 \\ \dot{n} \end{bmatrix} = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 & \beta_1/\ell \\ 0 & -\lambda_2 & 0 & 0 & 0 & 0 & \beta_2/\ell \\ 0 & 0 & -\lambda_3 & 0 & 0 & 0 & \beta_3/\ell \\ 0 & 0 & 0 & -\lambda_4 & 0 & 0 & \beta_4/\ell \\ 0 & 0 & 0 & 0 & -\lambda_5 & 0 & \beta_5/\ell \\ 0 & 0 & 0 & 0 & 0 & -\lambda_6 & \beta_6/\ell \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & [\delta k(t)-\beta]/\ell \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ n \end{bmatrix} \quad (4.1)$$

On defining the generalized state vector:

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ n \end{bmatrix} \quad (4.2)$$

Eq. (4.1) can be rewritten in the form

$$\dot{\underline{x}}(t) = A(t)\underline{x}(t) \quad (4.3)$$

where

$$A(t) = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 & \beta_1/\ell \\ 0 & -\lambda_2 & 0 & 0 & 0 & 0 & \beta_2/\ell \\ 0 & 0 & -\lambda_3 & 0 & 0 & 0 & \beta_3/\ell \\ 0 & 0 & 0 & -\lambda_4 & 0 & 0 & \beta_4/\ell \\ 0 & 0 & 0 & 0 & -\lambda_5 & 0 & \beta_5/\ell \\ 0 & 0 & 0 & 0 & 0 & -\lambda_6 & \beta_6/\ell \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \lambda_5 & \lambda_6 & [\delta k(t)-\beta]/\ell \end{bmatrix} \qquad (4.4)$$

As shown in Appendix A, the solution of Eq. (4.3) is given by

$$\underline{x}(t) = \Phi(t,t_0)\underline{x}(t_0) \qquad (4.5)$$

where $\Phi(t,t_0)$ is the state transition matrix.

Similarly, Eqs. (3.10) and (3.11) can be written as Eq. (4.3) with

$$\underline{x}(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \\ z_6(t) \\ n(t) \end{bmatrix} \qquad (4.6)$$

and

$$A(t) = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 & \lambda_1 a_1 \\ 0 & -\lambda_2 & 0 & 0 & 0 & 0 & \lambda_2 a_2 \\ 0 & 0 & -\lambda_3 & 0 & 0 & 0 & \lambda_3 a_3 \\ 0 & 0 & 0 & -\lambda_4 & 0 & 0 & \lambda_4 a_4 \\ 0 & 0 & 0 & 0 & -\lambda_5 & 0 & \lambda_5 a_5 \\ 0 & 0 & 0 & 0 & 0 & -\lambda_6 & \lambda_6 a_6 \\ \alpha & \alpha & \alpha & \alpha & \alpha & \alpha & \alpha[\rho(t)-1] \end{bmatrix} \qquad (4.7)$$

4.3 Six-group prompt-jump representation

The matrix equation corresponding to Eq. (3.29) is

$$\dot{z}(t) = A(t)\underline{z}(t) \tag{4.8}$$

with

$$\underline{z}(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \\ z_6(t) \end{bmatrix} \tag{4.9}$$

and

$$A(t) = \frac{1}{1-\rho(t)} \cdot \begin{bmatrix} \lambda_1[\rho(t)+a_1-1] & \lambda_1 a_1 & \lambda_1 a_1 & \lambda_1 a_1 & \lambda_1 a_1 & \lambda_1 a_1 \\ \lambda_2 a_2 & \lambda_2[\rho(t)+a_2-1] & \lambda_2 a_2 & \lambda_2 a_2 & \lambda_2 a_2 & \lambda_2 a_2 \\ \lambda_3 a_3 & \lambda_3 a_3 & \lambda_3[\rho(t)+a_3-1] & \lambda_3 a_3 & \lambda_3 a_3 & \lambda_3 a_3 \\ \lambda_4 a_4 & \lambda_4 a_4 & \lambda_4 a_4 & \lambda_4[\rho(t)+a_4-1] & \lambda_4 a_4 & \lambda_4 a_4 \\ \lambda_5 a_5 & \lambda_5 a_5 & \lambda_5 a_5 & \lambda_5 a_5 & \lambda_5[\rho(t)+a_5-1] & \lambda_5 a_5 \\ \lambda_6 a_6 & \lambda_6 a_6 & \lambda_6 a_6 & \lambda_6 a_6 & \lambda_6 a_6 & \lambda_6[\rho(t)+a_6-1] \end{bmatrix} \tag{4.10}$$

4.4 One-group representation

The kinetics equations with one group of delayed neutrons, Eqs. (3.32) and (3.33), can be written in matrix notation as Eq. (4.3) with

$$\underline{x}(t) = \begin{bmatrix} z(t) \\ n(t) \end{bmatrix} \tag{4.11}$$

and

$$A(t) = \begin{bmatrix} -\lambda & \lambda \\ \alpha & \alpha[\rho(t) - 1] \end{bmatrix} \tag{4.12}$$

If reactivity is constant with $\rho(t)=\rho$, then the system equation is

$$\dot{\underline{x}}(t) = A\underline{x} \tag{4.13}$$

where

$$A = \begin{bmatrix} -\lambda & \lambda \\ \alpha & \alpha(\rho - 1) \end{bmatrix} \tag{4.14}$$

The solution of Eq. (4.13) can be obtained, as shown in Appendix A, by taking the Laplace transform of Eq. (4.13) to obtain

$$s\underline{X}(s) - \underline{x}(0) = A\underline{X}(s) \tag{4.15}$$

which can be solved for $\underline{X}(s)$:

$$\underline{X}(s) = [sI - A]^{-1}\underline{x}(0) \tag{4.16}$$

where I is the unit matrix. Equation (4.16) can be written in terms of the Laplace transform of the state transition matrix $\Phi(t)$ as

$$\underline{X}(s) = \Phi(s)\underline{x}(0) \tag{4.17}$$

where $\Phi(s)$, the resolvent matrix, is given by

$$\Phi(s) = [sI - A]^{-1} \tag{4.18}$$

Taking the inverse Laplace transform of Eq. (4.18) results in

$$\Phi(t) = \mathcal{L}^{-1}[sI - A]^{-1} \tag{4.19}$$

where $\Phi(t)$ is the state transition matrix. Using Eq. (4.19), the inverse transform of Eq. (4.17) can be written as

$$\underline{x}(t) = \Phi(t)\underline{x}(0) \tag{4.20}$$

For the matrix defined by Eq. (4.14)

$$sI-A = \begin{bmatrix} s+\lambda & -\lambda \\ -\alpha & s+\alpha(1-\rho) \end{bmatrix} \tag{4.21}$$

and $\Phi(s)$ is given by

$$\Phi(s) = \begin{bmatrix} \dfrac{s+\alpha(1-\rho)}{s^2+(\lambda+\alpha-\alpha\rho)s-\lambda\alpha\rho} & \dfrac{\lambda}{s^2+(\lambda+\alpha-\alpha\rho)s-\lambda\alpha\rho} \\ \dfrac{\alpha}{s^2+(\lambda+\alpha-\alpha\rho)s-\lambda\alpha\rho} & \dfrac{s+\lambda}{s^2+(\lambda+\alpha-\alpha\rho)s-\lambda\alpha\rho} \end{bmatrix} \tag{4.22}$$

If the root approximations given in Eqs. (3.36) and (3.37) are substituted into Eq. (4.22), then

$$\Phi(s) = \begin{bmatrix} \dfrac{s+\alpha(1-\rho)}{(s-s_1)(s-s_2)} & \dfrac{\lambda}{(s-s_1)(s-s_2)} \\ \dfrac{\alpha}{(s-s_1)(s-s_2)} & \dfrac{s+\lambda}{(s-s_1)(s-s_2)} \end{bmatrix} \tag{4.23}$$

where

$$s_1 = \lambda\rho/(1 - \rho) \tag{4.24}$$

$$s_2 = -\alpha(1 - \rho) - \lambda/(1 - \rho) \tag{4.25}$$

The state transition matrix is obtained by taking the inverse Laplace transform of Eq. (4.23):

$$\Phi(t) = \begin{bmatrix} \phi_{11}(t) & \phi_{12}(t) \\ \phi_{21}(t) & \phi_{22}(t) \end{bmatrix} \tag{4.26}$$

where

$$\phi_{11}(t) = \frac{[\alpha(1 - \rho)^2 + \lambda\rho]e^{s_1 t} + \lambda e^{s_2 t}}{\alpha(1 - \rho)^2 + \lambda(1 + \rho)} \tag{4.27}$$

$$\phi_{12}(t) = \frac{\lambda(1 - \rho)(e^{s_1 t} - e^{s_2 t})}{\alpha(1 - \rho)^2 + \lambda(1 + \rho)} \tag{4.28}$$

$$\phi_{21}(t) = \frac{\alpha(1-\rho)(e^{s_1 t} - e^{s_2 t})}{\alpha(1-\rho)^2 + \lambda(1+\rho)} \tag{4.29}$$

$$\phi_{22}(t) = \frac{\lambda e^{s_1 t} + [\alpha(1-\rho)^2 + \lambda\rho]e^{s_2 t}}{\alpha(1-\rho)^2 + \lambda(1+\rho)} \tag{4.30}$$

For t = 0, Eq. (4.26) becomes

$$\Phi(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \tag{4.31}$$

which is one of the properties of the state transition matrix.

The solution for n(t) given by Eq. (4.20), with $\Phi(t)$ given by Eq. (4.26), is identical to the result obtained previously in Eq. (3.38), except that Eq. (4.20) gives, in addition, the solution for the second state variable z(t).

4.5 One-group prompt-jump representation

The system based on the prompt-jump approximation is described by Eqs. (3.39) and (3.40). If the reactivity input is considered, the system equations are augmented by including Eq. (3.50) as follows:

$$\dot{z} = \frac{\lambda\rho z}{1-\rho} \tag{4.32}$$

$$\dot{\rho} = u \tag{4.33}$$

$$n = \frac{z}{1-\rho} \tag{4.34}$$

These equations are expressed in matrix notation as:

$$\underline{\dot{x}} = \underline{f}(\underline{x}, u) \tag{4.35}$$

$$y = h(\underline{x}) \tag{4.36}$$

where

$$\underline{x} = \begin{bmatrix} z \\ \rho \end{bmatrix} \tag{4.37}$$

$$\underline{f} = \begin{bmatrix} f_1(z, \rho) \\ f_2(u) \end{bmatrix} \tag{4.38}$$

$$f_1(z, \rho) = \frac{\lambda \rho z}{1 - \rho} \tag{4.39}$$

$$f_2(u) = u \tag{4.40}$$

$$y = n \tag{4.41}$$

and

$$h(\underline{x}) = h(z, \rho) = \frac{z}{1 - \rho} \tag{4.42}$$

Equation (4.35) is the system nonlinear vector-matrix differential equation, and Eq. (4.36) is the scalar nonlinear measurement equation. The system has a single input u and a single output y.

4.6 Linearization of the system and measurement equations

The system and measurement equations are linearized by considering small perturbations about nominal values of the neutron density n*, normalized precursor level z*, and control input u*. To find the differential equations relating the deviations, expand Eq. (4.35) in a Taylor series

$$\underline{x} = \underline{f}(\underline{x}^*, u^*) + \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{x^*} (\underline{x} - \underline{x}^*) + \left.\frac{\partial \underline{f}}{\partial u}\right|_{u^*} (u - u^*) + \cdots \tag{4.43}$$

Define

$$\delta\underline{x} = \underline{x} - \underline{x}^* \tag{4.44}$$

$$\delta u = u - u^* \tag{4.45}$$

note that

$$\dot{\underline{x}}^* = \underline{f}(\underline{x}^*, u^*) \tag{4.46}$$

then

$$\delta\dot{\underline{x}} = \dot{\underline{x}} - \dot{\underline{x}}^* \tag{4.47}$$

Finally, substitute Eqs. (4.44), (4.45), and (4.47) into Eq. (4.43), retaining only first-order terms, to obtain

$$\delta\dot{\underline{x}} = \left.\frac{\partial \underline{f}}{\partial \underline{x}}\right|_{x^*} \delta\underline{x} + \left.\frac{\partial \underline{f}}{\partial u}\right|_{u^*} \delta u \tag{4.48}$$

where

$$\frac{\partial \underline{f}}{\partial \underline{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \tag{4.49}$$

and

$$\frac{\partial \underline{f}}{\partial u} = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \vdots \\ \frac{\partial f_n}{\partial u} \end{bmatrix} \tag{4.50}$$

The measurement equation (4.36) is similarly expanded to obtain

$$y = h(\underline{x}^*) + \left.\frac{\partial h}{\partial \underline{x}}\right|_{x^*} (\underline{x} - \underline{x}^*) + \cdots \tag{4.51}$$

which can be written

$$\delta y = \left.\frac{\partial h}{\partial \underline{x}}\right|_{x^*} \delta\underline{x} \tag{4.52}$$

where

$$\delta y = y - h(\underline{x}^*) \tag{4.53}$$

and

$$\frac{\partial h}{\partial \underline{x}} = \left[\frac{\partial h}{\partial x_1} \cdots \frac{\partial h}{\partial x_n}\right] \tag{4.54}$$

By defining

$$A = \frac{\partial \underline{f}}{\partial \underline{x}} \tag{4.55}$$

$$D = \frac{\partial \underline{f}}{\partial u} \tag{4.56}$$

and

$$H = \frac{\partial h}{\partial \underline{x}} \tag{4.57}$$

Eqs. (4.48) and (4.52) can be written

$$\delta \dot{\underline{x}} = A\delta\underline{x} + D\delta u \tag{4.58}$$

$$\delta y = H\delta\underline{x} \tag{4.59}$$

The matrices A, D, and H corresponding to Eqs. (4.32), (4.33) and (4.34) are

$$A = \begin{bmatrix} \dfrac{\lambda\rho^*}{1 - \rho^*} & \dfrac{\lambda z^*}{(1 - \rho^*)^2} \\ 0 & 0 \end{bmatrix} \tag{4.60}$$

$$D = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{4.61}$$

$$H = \begin{bmatrix} \dfrac{1}{1 - \rho^*} & \dfrac{z^*}{(1 - \rho^*)^2} \end{bmatrix} \tag{4.62}$$

For the particular case in which the reactor is at equilibrium, the nominal values are: $z^* = 1.0$, $\rho^* = 0$, and $u^* = 0$, and the system and measurement equations become

$$\delta\dot{\underline{x}} = \begin{bmatrix} 0 & \lambda \\ 0 & 0 \end{bmatrix} \delta\underline{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \delta u \tag{4.63}$$

$$\delta y = \begin{bmatrix} 1 & 1 \end{bmatrix} \delta\underline{x} \tag{4.64}$$

where

$$\delta\underline{x} = \begin{bmatrix} z - 1.0 \\ \rho \end{bmatrix} = \begin{bmatrix} \delta z \\ \rho \end{bmatrix} \tag{4.65}$$

$$\delta u = u \tag{4.66}$$

and

$$\delta y = n - 1.0 = \delta n \tag{4.67}$$

4.7 Solution of the state-space equations with discrete-time inputs

For a discrete-time input, u is constant for T seconds which can be expressed as

$$u(t) = u_k \qquad kT<t\leq(k + 1)T \tag{4.68}$$

After substituting Eq. (4.68) into Eq. (4.33) and integrating,

$$\rho(t) = \rho(t_k) + u_k(t - t_k) \tag{4.69}$$

which can be written

$$\rho(t) = \rho_k + u_k(t - t_k) \tag{4.70}$$

where ρ_k is the reactivity at the beginning of the interval. Equation (4.70) is substituted into Eq. (4.32) to obtain

$$\dot{z} = \frac{\lambda z[\rho_k + u_k(t - t_k)]}{1 - \rho_k - u_k(t - t_k)} \tag{4.71}$$

which when integrated yields

$$\ell n\left[\frac{z(t)}{z_k}\right] = \frac{\lambda}{u_k} \ell n\left[\frac{1 - \rho_k}{1 - \rho_k - u_k(t - t_k)}\right] - \lambda(t - t_k) \tag{4.72}$$

At $t = t_{k+1}$, Eq. (4.72) is solved for z_{k+1} to obtain

$$z_{k+1} = z_k \exp\left[\frac{\lambda}{u_k} \ell n\left(\frac{1 - \rho_k}{1 - \rho_k - u_k T}\right) - \lambda T\right] \tag{4.73}$$

If $u_k = 0$, then integration of Eq. (4.71) results in

$$\ell n\left[\frac{\dot{z}(t)}{z_k}\right] = \frac{\lambda \rho_k}{1 - \rho_k} (t - t_k) \tag{4.74}$$

which for $t = t_{k+1}$ yields

$$z_{k+1} = z_k \exp\left[\frac{\lambda \rho_k T}{1 - \rho_k}\right] \tag{4.75}$$

Similarly, ρ_{k+1} is obtained from Eq. (4.70) with $t = t_{k+1}$. Thus

$$\rho_{k+1} = \rho_k + u_k T \tag{4.76}$$

Equations (4.73) and (4.76) provide the finite difference solutions of the system equations at the sampling instants kT. These solutions are exact and do not involve any approximation of the derivative. If $u_k = 0$, Eq. (4.75) is used in place of Eq. (4.73). The corresponding finite difference measurement equation is

$$n_k = \frac{z_k}{1 - \rho_k} \tag{4.77}$$

These finite difference equations may be expressed in matrix notation as follows:

$$\underline{x}_{k+1} = \underline{g}(\underline{x}_k, u_k) \tag{4.78}$$

and

$$y_k = h(\underline{x}_k) \tag{4.79}$$

where

$$\underline{x}_k = \begin{bmatrix} z_k \\ \rho_k \end{bmatrix} \tag{4.80}$$

$$g_1 = z_k \exp\left[\frac{\lambda}{u_k} \ell n\left(\frac{1 - \rho_k}{1 - \rho_k - u_k T}\right) - \lambda T\right] \tag{4.81}$$

$$g_2 = \rho_k + u_k T \tag{4.82}$$

and

$$h(\underline{x}_k) = \frac{z_k}{1 - \rho_k} \tag{4.83}$$

4.8 Solution of the linearized equations with discrete-time input

If the delta notation of variable deviation is omitted, Eq. (4.58) can be written

$$\dot{\underline{x}} = A\underline{x} + Du \tag{4.84}$$

When $u = u_k$ for $t_k < t \le t_{k+1}$, the Laplace transformation of Eq. (4.84) yields

$$s\underline{X}(s) - \underline{x}_k = A\underline{X}(s) + D\frac{u_k}{s} \tag{4.85}$$

which is solved for $\underline{X}(s)$ as follows:

$$\underline{X}(s) = [sI - A]^{-1} \underline{x}_k + \frac{1}{s}[sI - A]^{-1} Du_k \tag{4.86}$$

The solution for $\underline{x}(t)$ is obtained from the inverse transformation of Eq. (4.86) as

$$\underline{x}(t) = \Phi(t - t_k)\underline{x}_k + u_k \int_{t_k}^{t} \Phi(t - \tau)Dd\tau \tag{4.87}$$

or

$$\underline{x}(t) = \Phi(t - t_k)\underline{x}_k + u_k \int_{0}^{t-t_k} \Phi(\tau)Dd\tau \tag{4.88}$$

At $t = t_{k+1}$

$$\underline{x}_{k+1} = \Phi(T)\underline{x}_k + u_k \int_{0}^{T} \Phi(\tau)Dd\tau \tag{4.89}$$

Equation (4.89) can be written

$$\underline{x}_{k+1} = \Phi\underline{x}_k + Gu_k \tag{4.90}$$

where the control distribution matrix

$$G = \int_{0}^{T} \Phi(\tau)Dd\tau \tag{4.91}$$

As shown in Appendix A

$$\int_{0}^{T} \Phi(\tau)d\tau = A^{-1}[\Phi(T) - I] \tag{4.92}$$

therefore

$$G = A^{-1}[\Phi - I]D \tag{4.93}$$

when A^{-1} exists.

On applying the above procedure to Eq. (4.58), and using the A and D matrices of Eqs. (4.60) and (4.61)

$$\Phi = \begin{bmatrix} \exp\left(\frac{\lambda T\rho^*}{1 - \rho^*}\right) & \frac{-z^*\left[1 - \exp\left(\frac{\lambda T\rho^*}{1 - \rho^*}\right)\right]}{\rho^*(1 - \rho^*)} \\ 0 & 1 \end{bmatrix} \tag{4.94}$$

$$G = \begin{bmatrix} \frac{-z^*T}{\rho^*(1 - \rho^*)} - \frac{z^*}{\lambda\rho^{*2}}\left[1 - \exp\left(\frac{\lambda T\rho^*}{1 - \rho^*}\right)\right] \\ T \end{bmatrix} \tag{4.95}$$

If the nominal values correspond to equilibrium conditions, $z^* = 1.0$ and $\rho^* = 0$, and Eqs. (4.94) and (4.95) reduce to

$$\Phi = \begin{bmatrix} 1 & \lambda T \\ 0 & 1 \end{bmatrix} \tag{4.96}$$

$$G = \begin{bmatrix} \frac{\lambda T^2}{2} \\ T \end{bmatrix} \tag{4.97}$$

Substituting Eqs. (4.96) and (4.97) into Eq. (4.90) results in the discrete system equation

$$\underline{x}_{k+1} = \begin{bmatrix} 1 & \lambda T \\ 0 & 1 \end{bmatrix} \underline{x}_k + \begin{bmatrix} \lambda T^2/2 \\ T \end{bmatrix} u_k \tag{4.98}$$

and the discrete output measurement equation obtained using Eq. (4.62) is

$$y_k = [1 \quad 1]\, \underline{x}_k \tag{4.99}$$

Equations (4.98) and (4.99) will be used in deriving the optimal closed loop control law for the regulator problem.

CHAPTER 5

OPTIMAL CONTROL OF NUCLEAR SYSTEMS BY STATE VARIABLE FEEDBACK

5.1 Introduction

Regulation of neutron density in a reactor requires a feedback control law which will compensate for disturbances that occur infrequently and randomly anywhere in time from zero to infinity. If attention is focused on a single disturbance and system noise is neglected, a deterministic regulator problem is formulated.

Dynamic programming is readily applied to linear discrete-time systems, and in the case of a quadratic performance index, leads to the direct calculation of the optimal linear feedback control law. If the performance index is to be minimized over a finite time interval, the feedback control law is a function of time; for an infinite time interval, the feedback control law is stationary and all state variables are fed back through fixed gains. Thus, discrete dynamic programming yields the solution to the reactor regulator problem, if the continuous system is sampled at discrete time intervals.

For a general discussion of dynamic programming, see Bellman [93], Bellman and Kalaba [94], and Dreyfus [95]; and for the dynamic programming solution of discrete-time systems with a quadratic performance index, see Tou [96, p. 45; 97, p. 345] and Lapidus and Luus [98, p. 155].

5.2 Dynamic programming solution of the linear regulator problem

For the discrete-time linear system described by

$$\underline{x}_{k-1} = \Phi\underline{x}_k + Gu_k \tag{5.1}$$

and a quadratic performance index of the form

$$I_N = \sum_{k=1}^{N} (\underline{x}_k^T Q \underline{x}_k + cu_{k-1}^2) \tag{5.2}$$

where $\underline{x}_k^T$ is the transpose of $\underline{x}_k$, Q is an n×n positive-definite or semi-definite symmetrical matrix, and c is a positive constant, the optimal control law which minimizes I_N, as shown in Appendix B, is given by

$$u_k = B_{N-k}\underline{x}_k \tag{5.3}$$

where

$$B_j = -\frac{G^T[Q + P_{j-1}]\Phi}{G^T[Q + P_{j-1}]G + c} \tag{5.4}$$

and

$$P_j = [\Phi + GB_j]^T[Q + P_{j-1}][\Phi + GB_j] + cB_j^T B_j \tag{5.5}$$

In Eq. (5.3), the feedback matrix B_{N-k}, a row matrix, is obtained from the iterative solution of Eqs. (5.4) and (5.5). The matrix P_j defined by Eq. (5.5) is n×n and symmetrical. Starting with $P_o = 0$, Eqs. (5.4) and (5.5) yield B_1, P_1, B_2, P_2, ... If the upper limit of summation in Eq. (5.2) is allowed to approach infinity, then B_j converges to a stationary matrix B and Eq. (5.3) reduces to

$$u_k = B\underline{x}_k \tag{5.6}$$

The product of the row matrix B and the state vector $\underline{x}_k$ yields the optimal feedback u_k as indicated in Eq. (5.6).

5.3 Performance indices and constraints

Using Eq. (4.65), the general performance index given by Eq. (5.2) can be written in expanded form as a function of the delayed neutron deviation, reactivity, and reactivity rate:

$$I_N = \sum_{k=1}^{N} (Q_{11}\delta z_k^2 + 2Q_{12}\delta z_k \rho_k + Q_{22}\rho_k^2 + cu_{k-1}^2) \tag{5.7}$$

where

$$\delta z_k = z_k - 1.0 \tag{5.8}$$

and

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix} \tag{5.9}$$

To regulate the neutron density, a performance index which is a function of the neutron density deviation is defined by:

$$I_N = \sum_{k=1}^{N} \delta n_k^2 \tag{5.10}$$

where

$$\delta n_k = n_k - 1.0 \tag{5.11}$$

Equation (4.99) written in expanded form yields

$$\delta n_k = \delta z_k + \rho_k \tag{5.12}$$

Substitution of Eq. (5.12) into Eq. (5.10) gives

$$I_N = \sum_{k=1}^{N} (\delta z_k^2 + 2\delta z_k + \rho_k^2) \tag{5.13}$$

Comparison of Eq. (5.13) with Eq. (5.7) results in

$$Q = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \tag{5.14}$$

$$R = 0 \tag{5.15}$$

The Q matrix defined by Eq. (5.14) satisfies the performance index of Eq. (5.10). The optimal control law obtained using this matrix will minimize the sum of the squares of the neutron density deviations at sampling instants.

To reduce the magnitude of the reactivity rate which is applied to correct a disturbance, a penalty term which weights u_{k-1} can be added to Eq. (5.10). Similarly, reactivity can be returned to zero more quickly after a disturbance by adding a penalty term which weights ρ_k. With these additional terms, Eq. (5.10) becomes

$$I_N = \sum_{k=1}^{N} (\delta n_k^2 + a\rho_k^2 + cu_{k-1}^2) \tag{5.16}$$

where a and c are the weighting coefficients. If Eq. (5.12) is substituted into Eq. (5.16), the corresponding matrix

$$Q = \begin{bmatrix} 1 & 1 \\ 1 & 1+a \end{bmatrix} \tag{5.17}$$

will result in the minimization of the sum of the squares of the neutron density deviation and the reactivity at the sampling instants.

5.4 Reactor transient response and the performance index

The optimal control law given by Eq. (5.6) is for a linear system as described by Eq. (5.1). Thus, in order to apply the method to the control of a nuclear reactor, the linearized discrete-time Eqs. (4.98) and (4.99) are used, and the Φ and G matrices are substituted into Eqs. (5.4) and (5.5) with $\lambda = 0.31$ and $T = 1$. Arbitrary values are assigned to the a and c weighting coefficients of Eq. (5.16), and the Q matrix of Eq. (5.17) and the coefficient c are substituted into Eqs. (5.4) and (5.5). Equations (5.4) and (5.5) are solved iteratively with $N \to \infty$ to obtain the stationary control law. The Calculation of Feedback Matrix computer program listed in Appendix G iteratively evaluates the B matrix until the difference between successive iterations diminishes to 10^{-7}. Table 5.1 lists the B matrices calculated for nine combinations of a and c.

TABLE 5.1

FEEDBACK MATRIX COEFFICIENTS

a	c	b_1	b_2
0	0	-0.8658008	-1.1341991
0	1	-0.5403229	-0.7918012
0	10	-0.2411739	-0.4557331
1	0	-0.6372618	-1.0987756
1	1	-0.4680735	-0.8534584
1	10	-0.2313746	-0.5005205
10	0	-0.2880492	-1.0446476
10	1	-0.2658030	-0.9705480
10	10	-0.1829017	-0.6938182

The transient response of the reactor is calculated using the nonlinear system Eq. (4.78), the nonlinear measurement Eq. (4.79), and the linear feedback Eq. (5.6). The Calculation of Transient Response computer program listed in Appendix G solves these equations and plots are generated by the Plot Program for Transient Response computer program. Equation (4.81) is unsatisfactory for numerical evaluation with small values of u_k; therefore, a series expansion for Eq. (4.81), derived in Appendix C, is used in the computer program.

Although 1 sec was selected for the control law sampling interval, the system response is evaluated at intermediate sampling instants of 0.1 sec to demonstrate that there is no inter sample ripple.

Figure 5.1 shows the reactor transient response with an initial disturbance of $\rho(0+) = 0.1$ and performance index weighting coefficients $a = 0$ and $c = 0$. At $t = (0-)$, the system is at equilibrium, which corresponds to $\rho(0-) = 0$, $\delta z(0-) = 0$, and $\delta n(0-) = 0$. At $t = (0+)$, a step change of reactivity occurs which gives rise to the prompt jump in neutron density. The control law minimizes the performance index given in Eq. (5.10) by driving the neutron density deviation to essentially zero in 1 sec. The control input at time zero is determined from the product of ρ_0 and b_2 from Table 5.1 or

$$u_0 = -0.1134 \text{ \$/sec} \qquad (5.18)$$

The initial control effort is proportional to the reactivity disturbance and inversely proportional to the sampling interval. If the sample interval is doubled, the neutron density deviation is driven to zero in 2 sec and the initial control effort is halved. Similarly, if the sample interval is halved, the initial control effort is doubled.

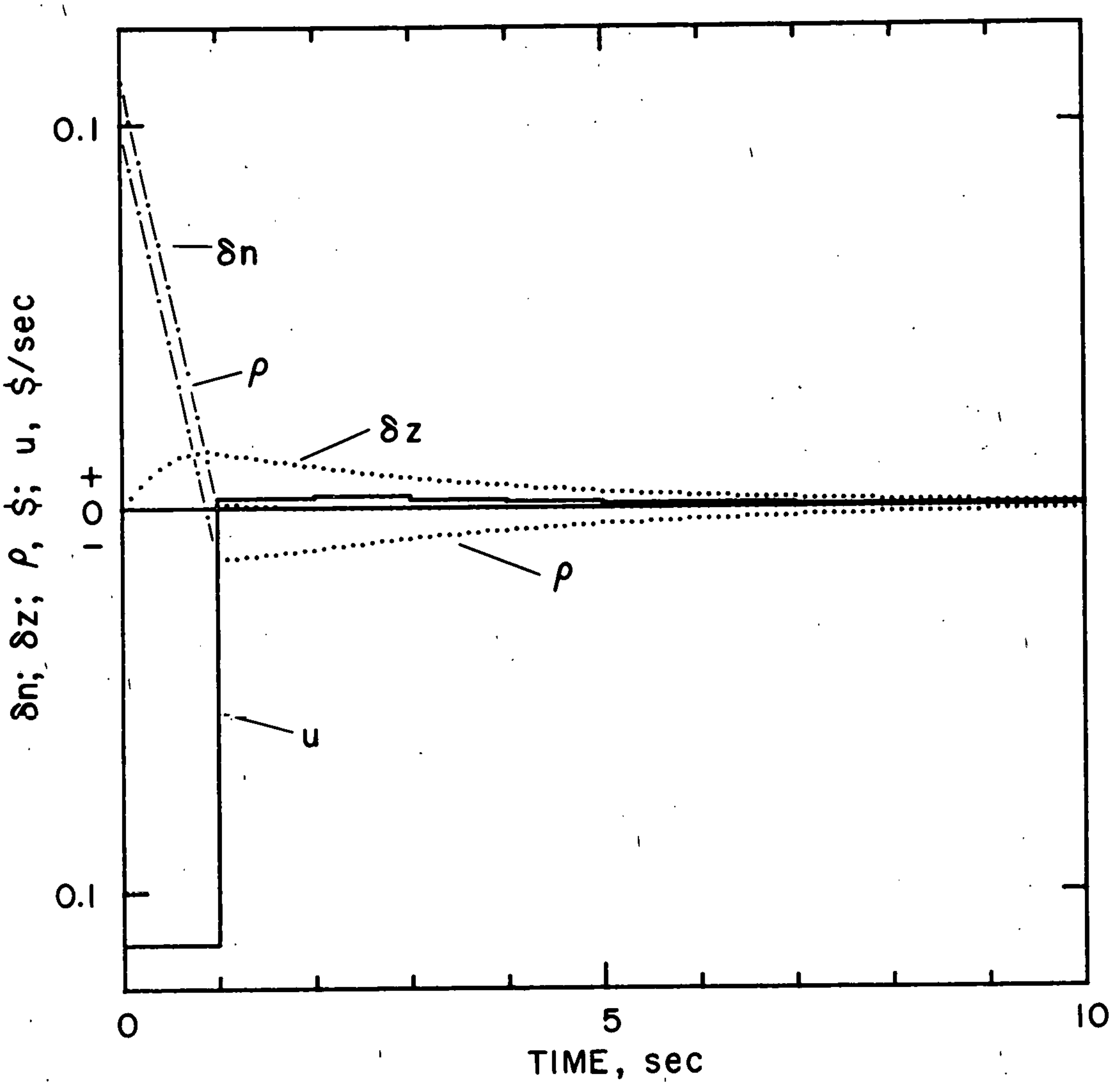

Fig. 5.1. Transient response for a=0, c=0, ρ_0=0.1, δz_0=0.

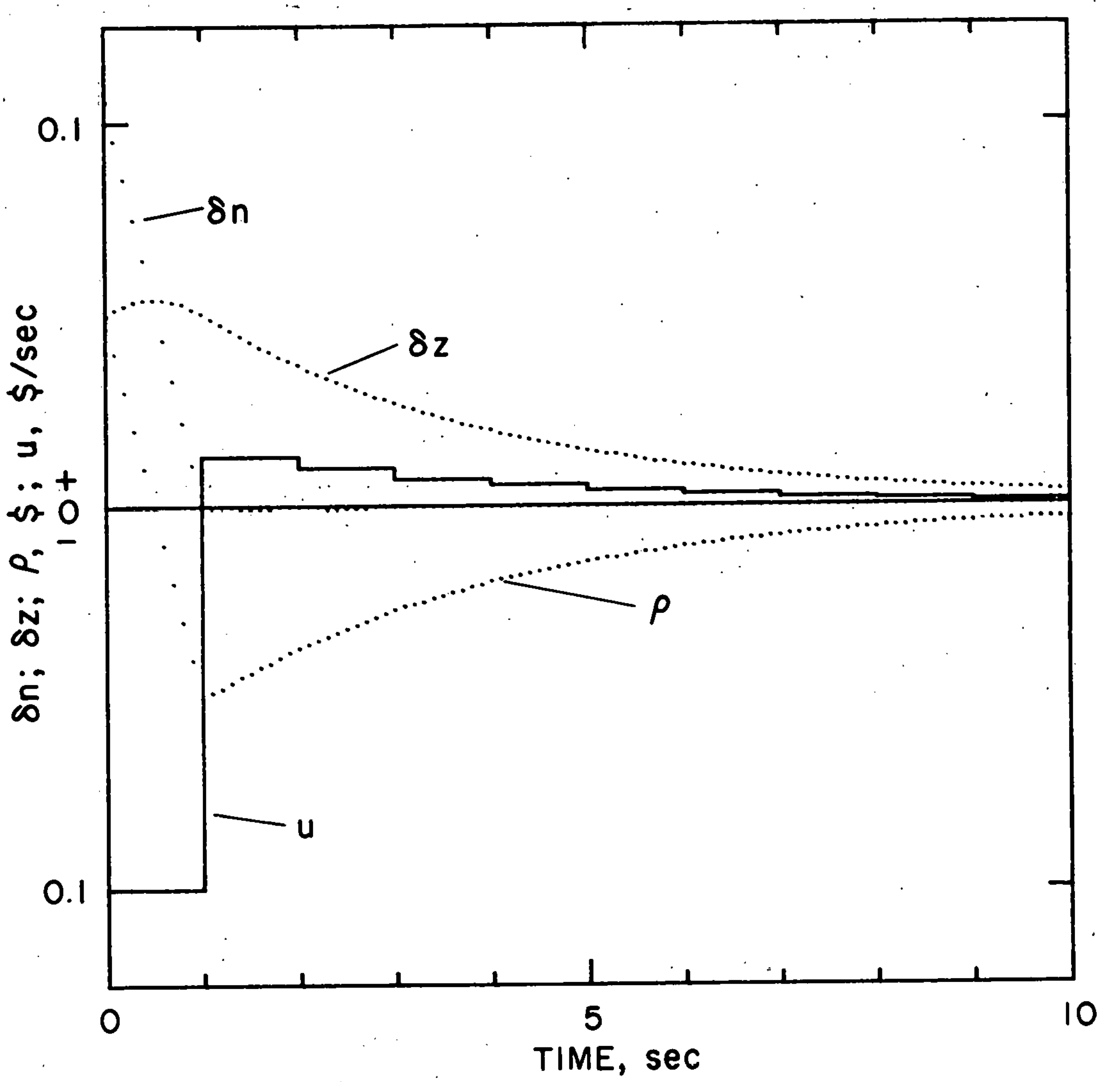

Fig. 5.2. Transient response for a=0, c=0, ρ_0=0.05, δz_0=0.05.

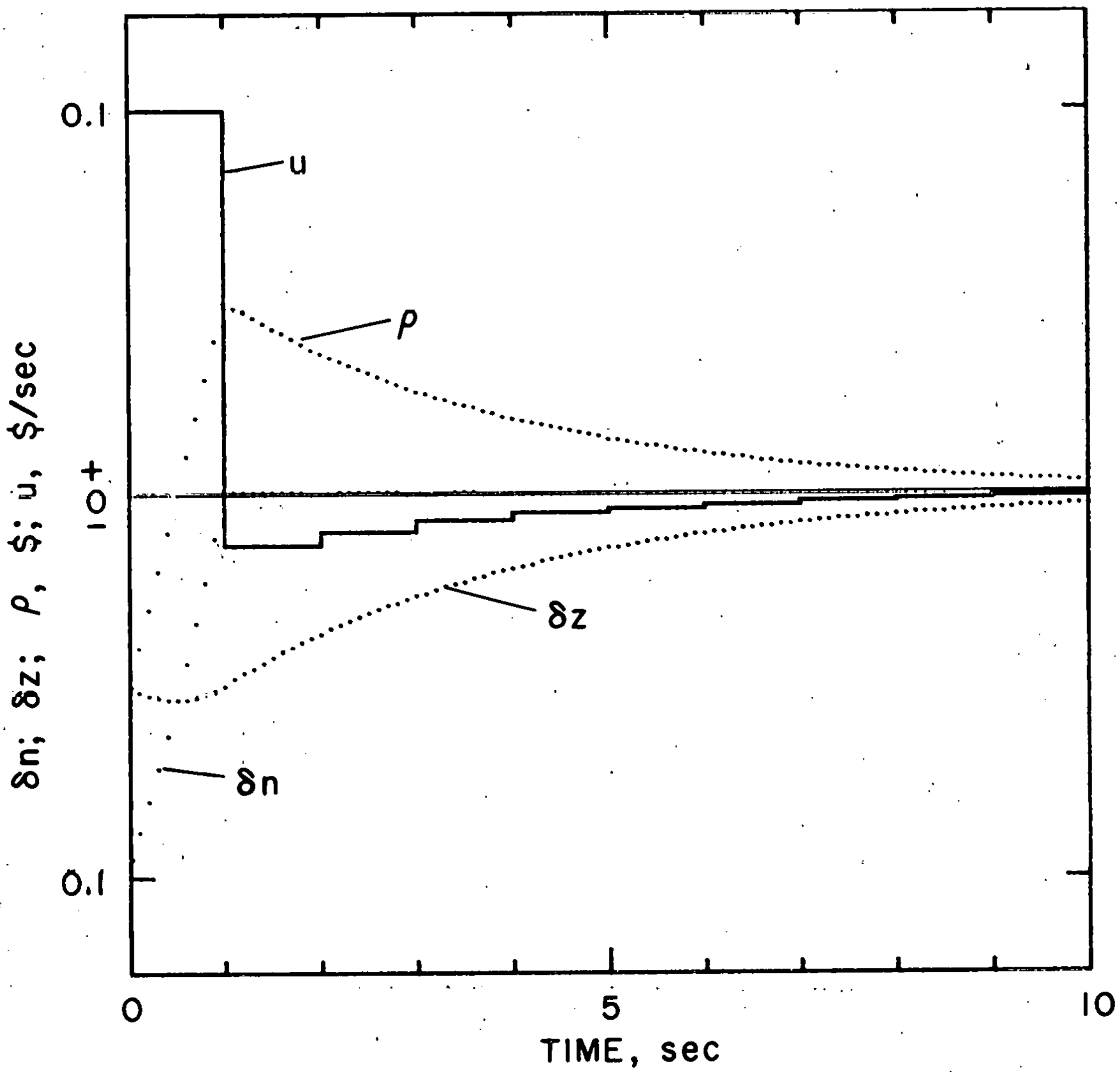

Fig. 5.3. Transient response for a=0, c=0, ρ_0=-0.05, δz_0=-0.05.

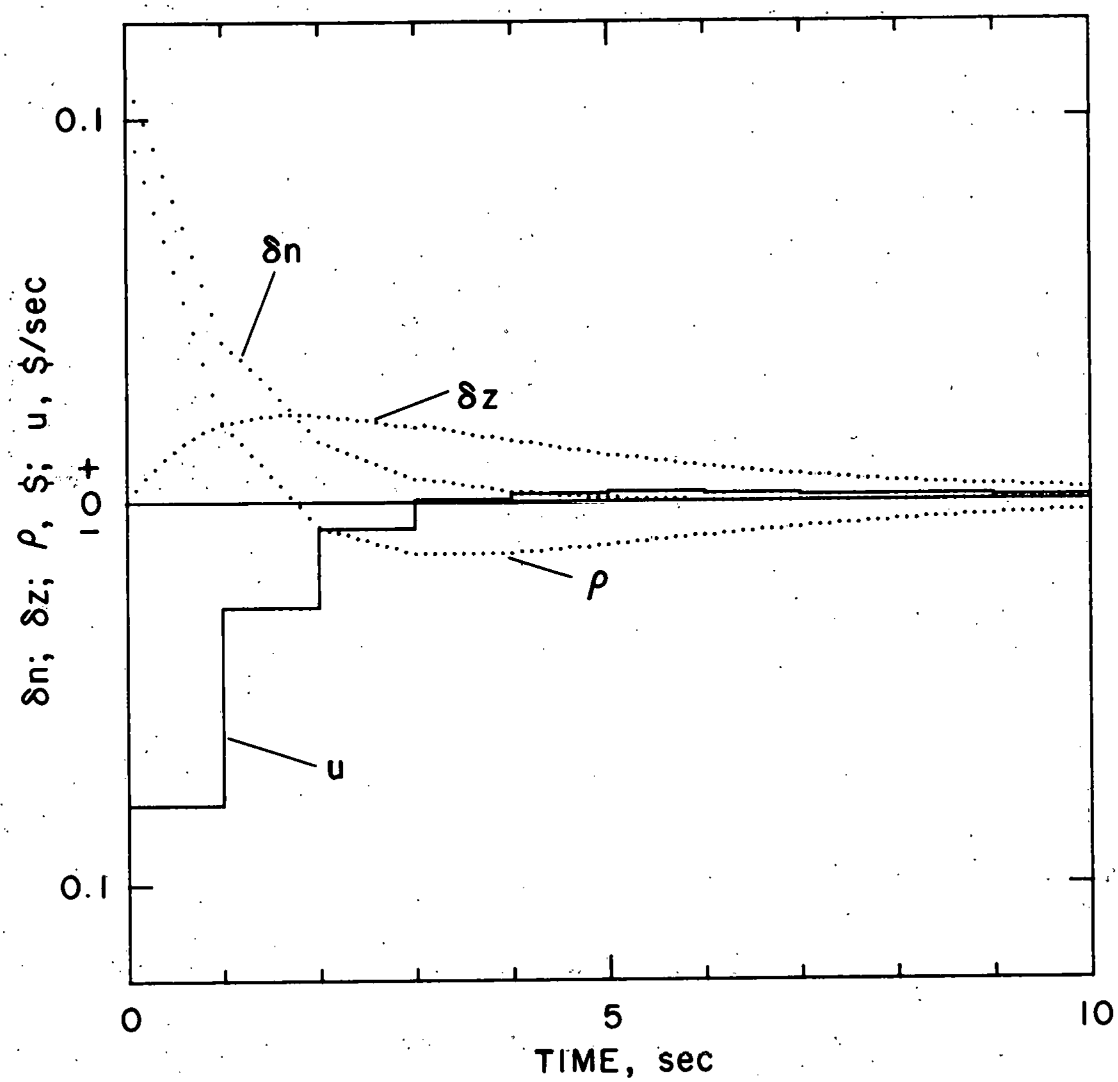

Fig. 5.4. Transient response for a=0, c=1, ρ_0=0.1, δz_0=0.

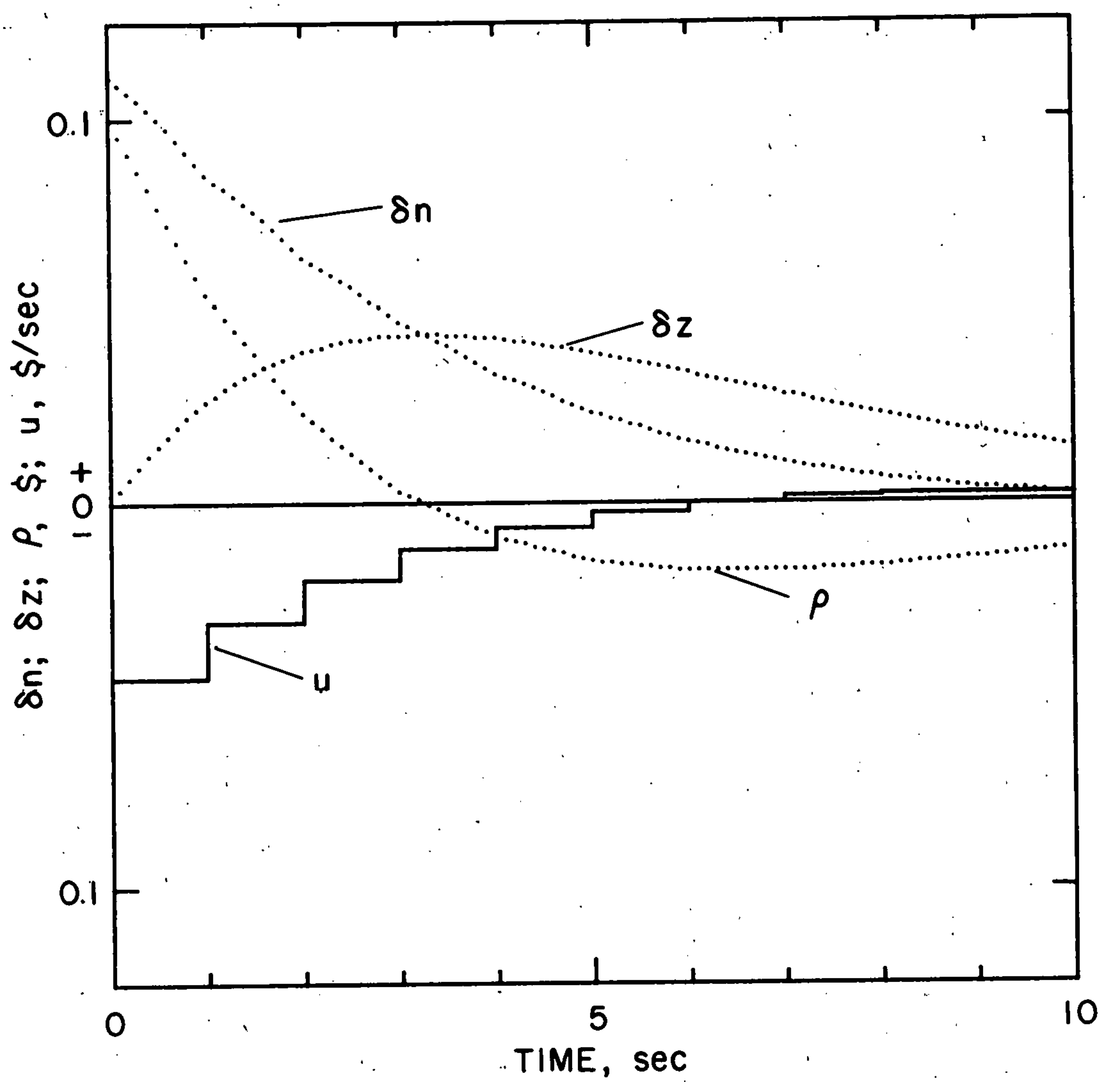

Fig. 5.5. Transient response for a=0, c=10, ρ_0=0.1, δz_0=0.

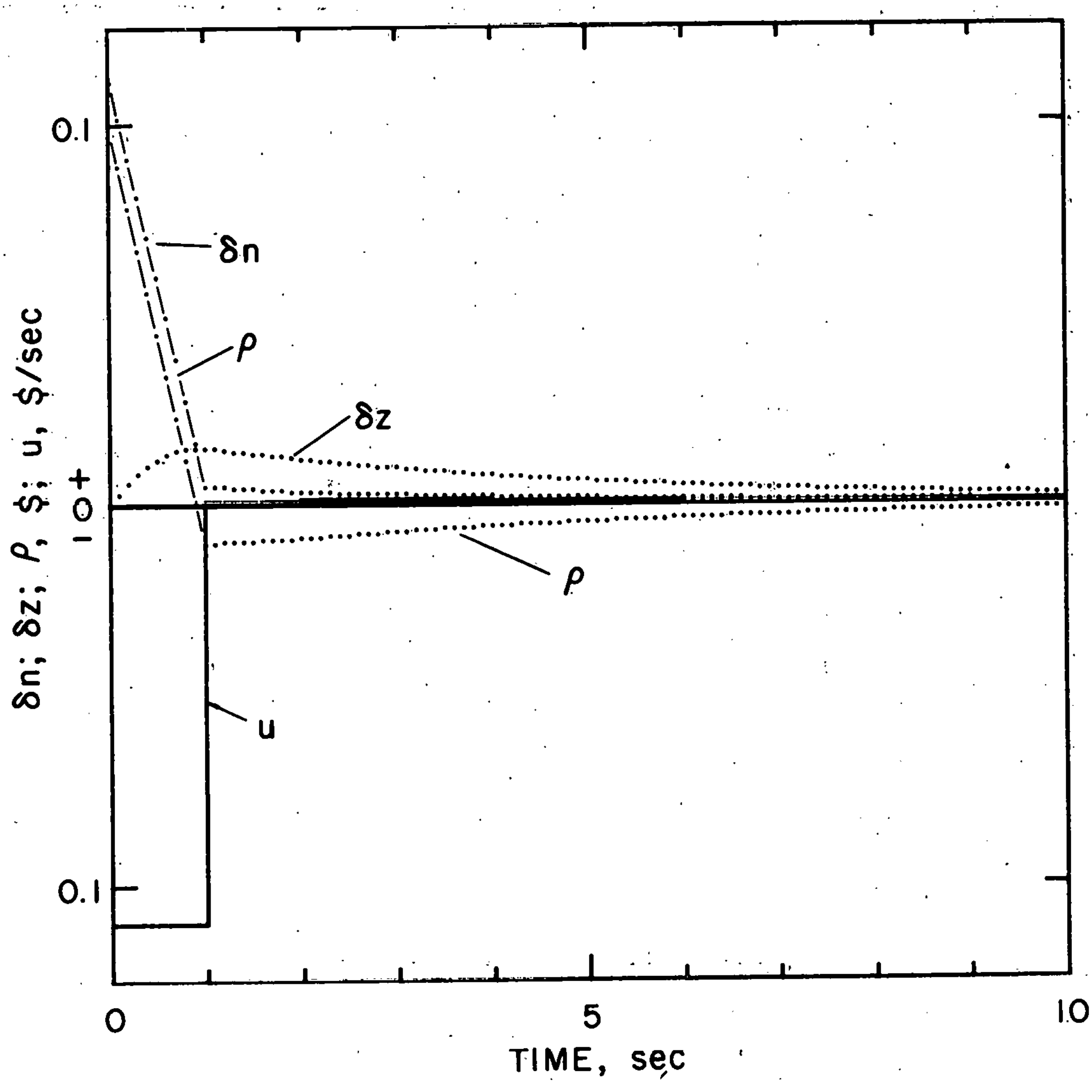

Fig. 5.6. Transient response for a=1, c=0, ρ_0=0.1, δz_0=0.

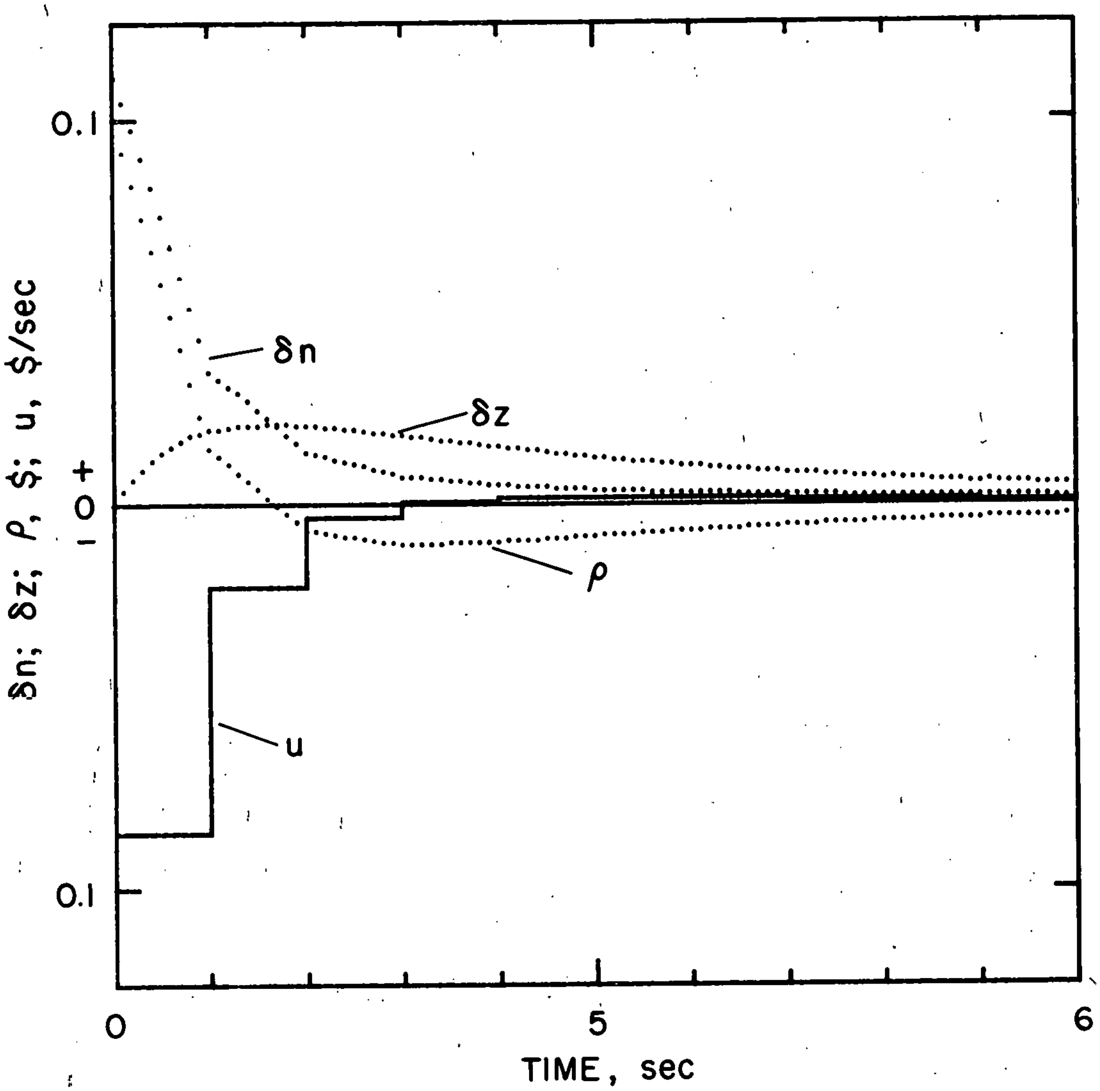

Fig. 5.7. Transient response for a=1, c=1, ρ_0=0.1, δz_0=0.

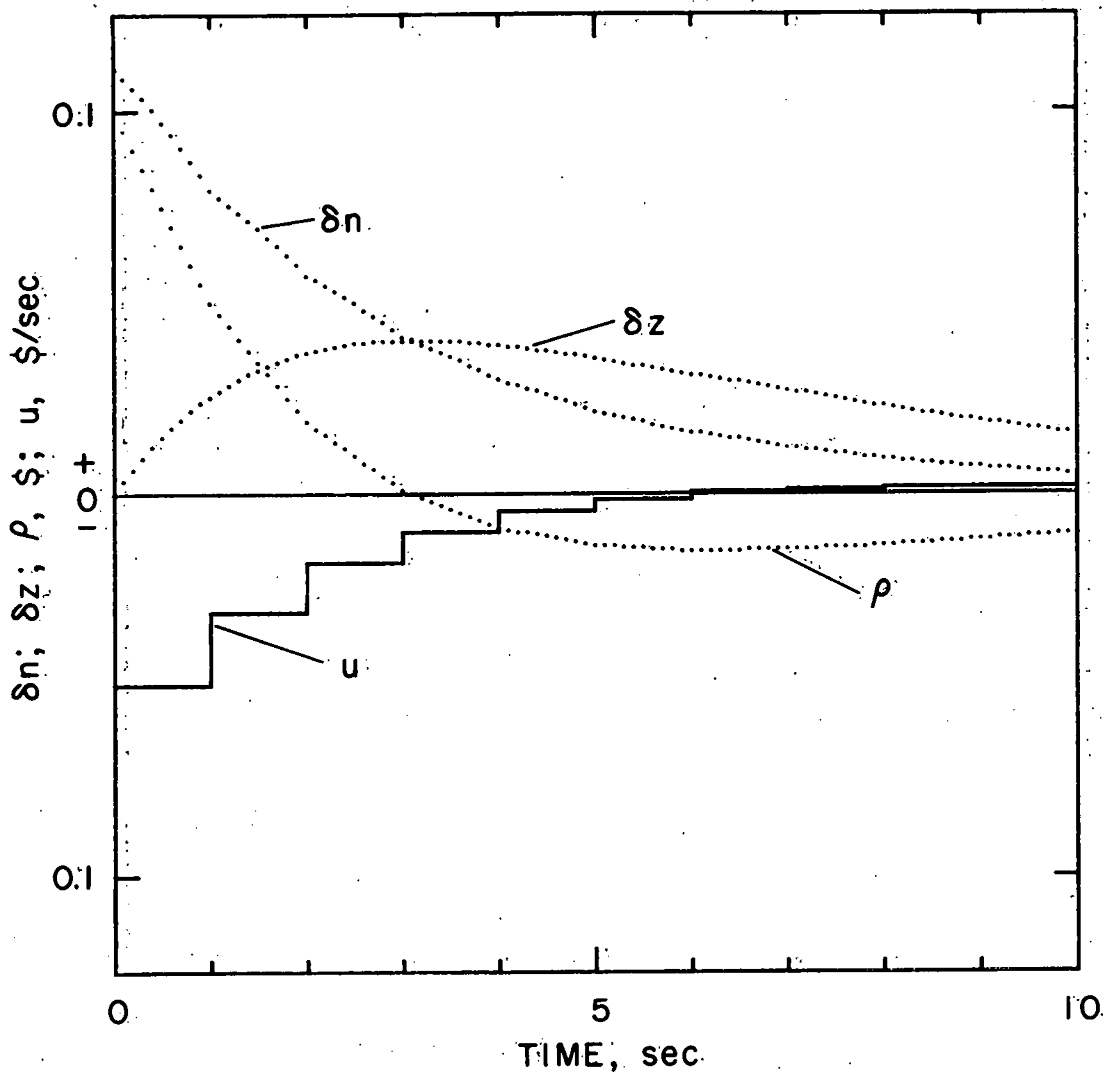

Fig. 5.8. Transient response for a=1, c=10, ρ_0=0.1, δz_0=0.

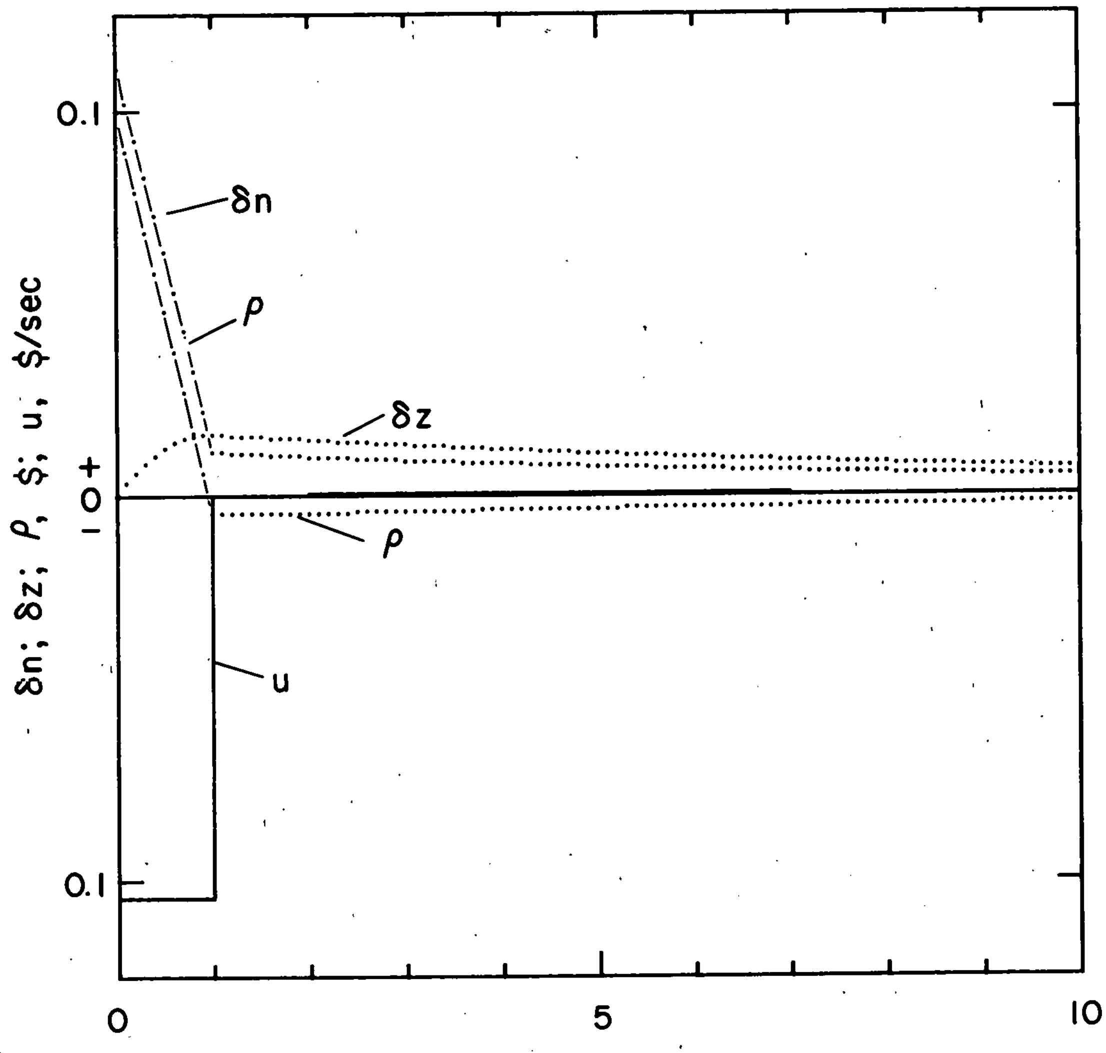

Fig. 5.9. Transient response for a=10, c=0, ρ_0=0.1, δz_0=0.

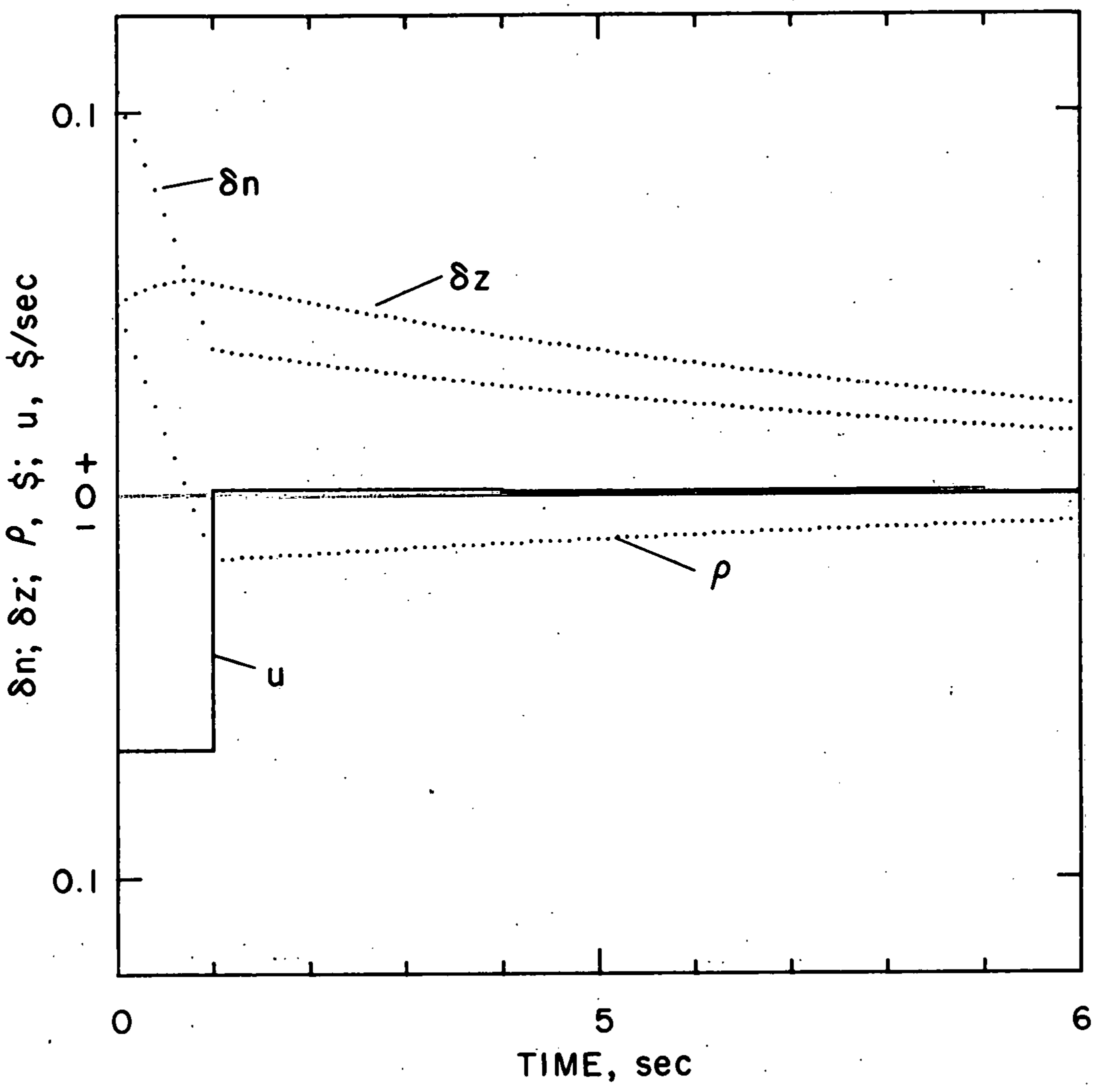

Fig. 5.10. Transient response for a=10, c=0, ρ_0=0.05, δz_0=0.05.

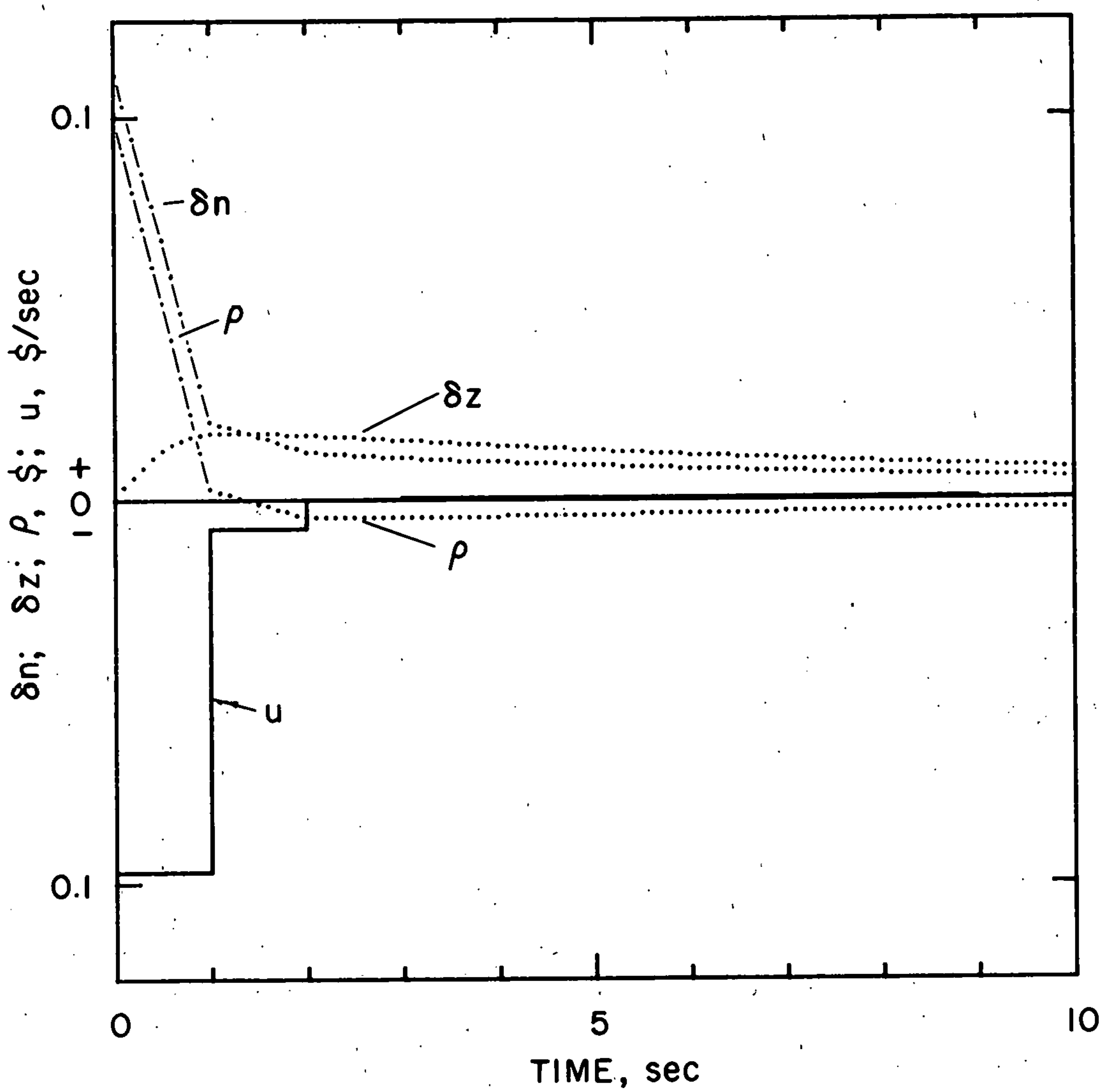

Fig. 5.11. Transient response for a=10, c=1, ρ_0=0.1, δz_0=0.

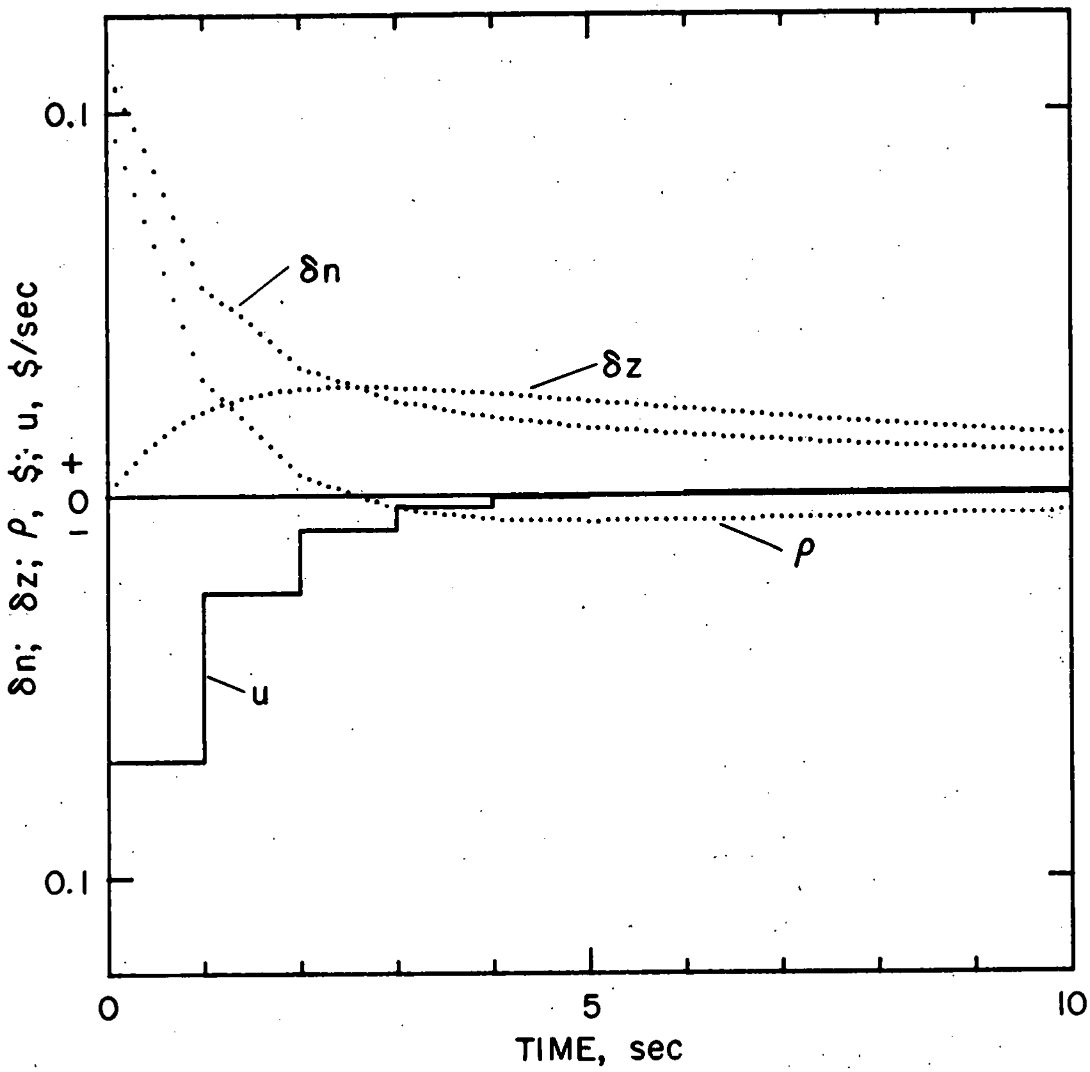

Fig. 5.12. Transient response for a=10, c=10, ρ_0=0.1, δz_0=0.

Figures 5.2 and 5.3 describe system behavior for the same performance index as above, except the initial conditions are different. For Fig. 5.2: $\rho(0-) = 0$, $\delta z(0-) = 0.05$, and $\delta n(0-) = 0.05$; for Fig. 5.3: $\rho(0-) = 0$, $\delta z(0-) = -0.05$ and $\delta n(0-) = -0.05$. These initial conditions correspond to a system which has not recovered from a prior disturbance and consequently is not at equilibrium at $t = (0-)$. The disturbance for Fig. 5.2 is $\rho(0+) = 0.05$, and $\rho(0+) = -0.05$ for Fig. 5.3. In both cases, the neutron density deviation is driven to zero in 1 sec, and the reactivity and delayed neutron deviation asymptotically approach zero.

Comparison of Figs. 5.1, 5.4, and 5.5 shows the effect of adding a control penalty term to the performance index with $c = 0$, $c = 1$, and $c = 10$, respectively. Here, the magnitude of the initial control effort is reduced at the expense of the neutron density deviation not being returned to zero in 1 sec. In Fig. 5.4, the neutron density returns to 1% in 2.6 sec and for Fig. 5.5 in 7.1 sec.

The effect of adding a reactivity term to the performance index can be seen by comparing Figs. 5.1, 5.6, and 5.9, and Figs. 5.2 and 5.10. In Fig. 5.10, the area under the reactivity curve has been reduced at the expense of the neutron density deviation remaining off-normal for a longer period.

Figure 5.7 shows the system behavior with uniform weight assigned to the neutron density deviation, reactivity, and control effort. Figure 12 shows the effect of reducing the weight assigned to the neutron density deviation. Comparison of Fig. 5.7 with Figs. 5.8 and 5.11 shows the effect of increased weight on control effort and reactivity, respectively.

Implementation of the optimal control law given by Eq. (5.6) requires that the system state be known at each sampling instant. In a nuclear reactor, the delayed neutron precursor density and reactivity cannot be measured; consequently, they must be estimated from measurements of the neutron density. An optimal estimator which performs this function is derived in the following chapter.

CHAPTER 6

ESTIMATION OF NUCLEAR SYSTEM STATE VARIABLES

6.1 Introduction

In 1806, Legendre [103] established estimation theory as a mathematical technique with the first publication on least-squares estimation.

In 1960, Kalman [104] solved the Wiener problem for discrete-time systems using state-transition analysis and orthogonal projections, and presented the principle of duality which showed the relationship between stochastic estimation and deterministic control. In a paper on the general theory of control systems [105], he introduced the concepts of controllability and observability. At the joint automatic control conference, Kalman and Bucy [106] extended the method to continuous systems. In a fourth paper, Kalman [107] summarized the contributions of the earlier papers and added a number of theorems and examples.

Ho [108] demonstrated the correspondence between the well-known method of least squares [109] and the optimal-filtering theory of Kalman. He showed that most of the results in linear filtering and prediction theory can be easily derived via a simple lemma on matrix inversion.

Lee [110] in his chapter on optimal estimation discussed: the Wiener filter, the continuous and discrete Wiener-Kalman filter, least-squares estimation, maximum-likelihood estimation, and the Bayesian approach to estimation.

Ohap and Stubberbud [111] developed a technique for estimating the state of a nonlinear system which combines Kalman's procedure with quasi-linearization. Their technique is not optimal in the strict sense since the linearized dynamic equations are approximations to the nonlinear equations. One advantage of the method is that unlike perturbation equations no *a priori* state of the system must be assumed.

Cox [112] surveyed the methods available for resolving discrete-time estimation problems: Bayesian and weighted least-squares estimation. Least-squares estimation was applied to nonlinear plant and measurement-vector-difference equations. A cost function was formulated which consisted of a linear combination of quadratic forms in errors of an *a priori* estimate, present observation, and plant noise. The constraint due to the plant equation was included by using a Lagrange multiplier, and minimization of the cost function resulted in a pair of nonlinear equations. The latter were solved iteratively to obtain the optimal estimate. Linearized Kalman filtering was indicated as being equivalent to a single iteration.

An alternate method of solving a cost function also was described. This method results in a two-point boundary value problem which is solved by successive approximations. M-step smoothing was introduced as a method to alleviate the difficulty of computer memory requirements increasing linearly with the number of observations. It was pointed out that for systems with no plant noise, the linearized Kalman filter is asymptotically open loop because the filter gain approaches zero.

Mowery [113] presented an optimal filter solution for a plant described by a nonlinear-vector-differential equation and a nonlinear-vector-measurement equation. The nonlinear plant equations were linearized about a nominal solution and a set of difference equations was obtained. The nonlinear measurement equation was similarly linearized. A criterion function was formulated which consisted of a linear combination of quadratic forms in errors of an *a priori* estimate and present observation. Minimizing the criterion function with respect to the new estimate resulted in a set of nonlinear normal equations. The solution of the linearized plant equation was used to derive the relationship between the *a priori* and *a posteriori* error weighting matrices. An iteration scheme was proposed to reduce the disparity between the nominal state vector and the true value.

Deutsch [114] in a chapter on differential equation techniques for linear filtering and prediction included the Kalman-Bucy method, discrete-time estimation, nonstationary estimation, and Bayes'-estimation formulation.

Sridhar and Pearson [115] presented an approximate solution to the problem of digital sequential, least-squares estimation of states and parameters in nonlinear processes. Observations were assumed to be linear, and a cost function was formulated which consisted of the sum of a linear combination of quadratic forms in errors of the state vector estimates and observations. A Lagrange multiplier vector was used to add the plant constraint to the cost function. Minimization of the cost function resulted in a nonlinear two-point boundary value problem which was solved by invariant imbedding to obtain the filter equations. An example was presented for the solution of a system represented by a nonlinear differential equation. Integration was used to obtain the solution of the nonlinear plant equation at discrete time intervals. Similarly, the plant variational equation was integrated to obtain the value of the derivative of the plant nonlinear difference equation with respect to the state vector.

Peschon, et al., [116, p. 70; 117, p. 6-8] derived an extended Kalman filter by linearizing the process and measurement nonlinear finite difference equations around the last estimate.

Phillips [118] used least-squares theory to formulate a cost function for a discrete-time nonlinear plant and nonlinear measurement system. A Lagrange multiplier was used to include the plant equation constraint. The two-point boundary value problem which results from the minimization of the cost function was solved by invariant imbedding to obtain the filter equations. The resulting filter equations extend the earlier work of Sridhar and Pearson [115] by considering a nonlinear measurement equation.

Sorenson [119] investigated optimal estimation and control policies for discrete-time, stochastic, dynamic systems. Perturbation techniques were applied, terms higher than first order were retained, and the estimation and control policies were determined using the Bayesian approach. In Reference 120 he summarized Kalman filtering techniques. A system consisting of a nonlinear plant and nonlinear measurement equation was analyzed by using linear perturbation equations with the coefficients evaluated at nominal values.

Sage and Masters [121] showed the relationship between least-squares-curve fitting and optimum filtering for linear systems. The Kalman-Bucy solution to the Wiener filtering problem was presented using least-squares techniques and the Bayesian rule. Relationships between least-squares, minimum-variance, and minimum-mean-squared-error estimates also were described.

Irwin [122] investigated estimation for discrete-time systems. The Bayesian, maximum likelihood, conditional expectation, dynamic programming, orthogonal projection, and two-point boundary value problem approaches were used to derive the Kalman filter equations. The solutions for nonlinear systems consisted of: the Kalman filter linearized about the present estimate; iterative solution of the equations resulting from the dynamic programming approach; and the two-point boundary value problem approach. A new approach was presented for the nonlinear estimator which utilized a performance index consisting of the logarithm of the conditional probability of the present estimate based on a set of measurements. Minimization of the performance index resulted in a set of nonlinear algebraic equations whose solution yields the optimal estimate.

Pearson [123] extended the the work of Sridhar and Pearson [115] to include nonlinear measurements. His result was the same as that of Phillips [118].

Liebeldt [124] included a chapter on linear discrete dynamic estimation and derived the Kalman filter.

Sage [125] devoted chapters to optimum state estimation in linear stationary systems, optimum filtering for nonstationary continuous systems, and least-squares curve fitting and state estimation in discrete linear systems.

Of the estimation methods outlined above, the iterative procedure presented by Cox comes closest to providing the solution for the deterministic nuclear system state estimator. The filter gain for a deterministic system with the fastest observation scheme is different from the filter gain derived for a stochastic system, so a sequential development of a nuclear system state estimator is presented starting with discrete-time equations and a linear Kalman estimator. Although the estimator derivation is based on discrete-time difference equations, integration is introduced into the estimator to make the method directly applicable to a plant described by a nonlinear vector differential equation and nonlinear measurement equation.

6.2 Kalman filter

For the discrete-time linear system described by

$$\underline{x}_{k+1} = \Phi \underline{x}_k \tag{6.1}$$

and

$$y_k = H\underline{x}_k \tag{6.2}$$

the fastest observation scheme is uniquely determined by

$$\underline{\hat{x}}_{k+1} = \Phi\underline{\hat{x}}_k + \underline{f}_1(y_k - \hat{y}_k) \tag{6.3}$$

where $\underline{\hat{x}}_k$ is the estimate of the system state at instant k; $\underline{f}_1$ is the first element of the dual basis of $\underline{f}_1^*, \ldots, f_n^*$, where

$$\underline{f}_i^* = (\Phi^T)^{-i}H^T \tag{6.4}$$

and

$$\hat{y}_k = H\underline{\hat{x}}_k \tag{6.5}$$

If the dual basis of F is

$$F^* = [\underline{f}_1^*, \ldots, \underline{f}_n^*] \tag{6.6}$$

then

$$F = (F^{*T})^{-1} \tag{6.7}$$

or

$$F = [\underline{f}_1, \ldots, \underline{f}_n] \tag{6.8}$$

For the discrete-time linear reactor Eqs. (4.98) and (4.99),

$$\underline{f}_1^* = \begin{bmatrix} 1 & 0 \\ \lambda T & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 - \lambda T \end{bmatrix} \tag{6.9}$$

$$\underline{f}_2^* = \begin{bmatrix} 1 & 0 \\ \lambda T & 2 \end{bmatrix}^{-2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 - 2\lambda T \end{bmatrix} \tag{6.10}$$

$$F^* = \begin{bmatrix} 1 & 1 \\ 1 - \lambda T & 1 - 2\lambda T \end{bmatrix} \tag{6.11}$$

$$F = \begin{bmatrix} (2\lambda T - 1)/\lambda T & 1 - 1/\lambda T \\ 1/\lambda T & 1/\lambda T \end{bmatrix} \tag{6.12}$$

and

$$\underline{f}_1 = \begin{bmatrix} (2\lambda T - 1)/\lambda T \\ 1/\lambda T \end{bmatrix} \tag{6.13}$$

Thus the estimator described by Eq. (6.3) with the $\underline{f}_1$ of Eq. (6.13) will generate an optimal estimate of the system state, after a disturbance, using a maximum of two output measurements. In general, for an nth-order system, the optimal estimate is obtained using a maximum of n output measurements.

As shown in Figs. 5.1 through 5.12, the reactivity and delayed neutron deviation do not correspond to the nominal values of $z^* = 1.0$

and $\rho^* = 0$ which were assumed in deriving Eqs. (4.98) and (4.99); therefore, it would be better to use Eqs. (4.62) and (4.94) to evaluate the H and Φ matrices, except the nominal values must be known. The extended Kalman filter method uses the last estimate as the nominal value, which is satisfactory if successive values do not change rapidly. As will be shown later, there is a very large change in nominal values after a reactivity disturbance; thus the extended Kalman filter fails to provide the correct estimates of the reactor state. The question of unknown nominal values is resolved by using the iteration method proposed by Cox [112].

6.3 Linear estimation by matrix inversion

For the dynamic system described by

$$\underline{x}_{k+1} = \Phi(k+1, k)\underline{x}_k \tag{6.14}$$

and

$$y_k = H_k\underline{x}_k \tag{6.15}$$

assume k output measurements have been made which are related as follows:

$$\begin{aligned} y_1 &= H_1\underline{x}_1 \\ y_2 &= H_2\underline{x}_2 \\ &\vdots \\ y_k &= H_k\underline{x}_k \end{aligned} \tag{6.16}$$

These measurements can be referred to $\underline{x}_k$ by using Eq. (6.14) with $\underline{x}_j = \phi(j,k)\underline{x}_k$, and written in composite form. Thus

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_k \end{bmatrix} = \begin{bmatrix} H_1 \Phi(1,k)\underline{x}_k \\ H_2 \Phi(2,k)\underline{x}_k \\ \cdot \\ \cdot \\ \cdot \\ H_k \Phi(k,k)\underline{x}_k \end{bmatrix} \tag{6.17}$$

Equation (6.17) can be partitioned to yield

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ y_k \end{bmatrix} = \begin{bmatrix} H_1\Phi(1,k) \\ \hline H_2\Phi(2,k) \\ \hline \cdot \\ \cdot \\ \cdot \\ \hline H_k\Phi(k,k) \end{bmatrix} \underline{x}_k \tag{6.18}$$

and written more compactly as

$$\underline{y}_k = \underline{H}_k\underline{x}_k \tag{6.19}$$

where $\underline{y}_k$ is the vector of output measurements, and $\underline{H}_k$ is the composite matrix shown in Eq. (6.18).

The fastest observation scheme is obtained when the number of output measurements is equal to the order of the system. With k = n, Eq. (6.19) can be solved for $\underline{x}_k$ by left-multiplying by $\underline{H}_k^T$,

$$\underline{H}_k^T\underline{y}_k = \underline{H}_k^T\underline{H}_k\underline{x}_k \tag{6.20}$$

and by $[\underline{H}_k^T H_k]^{-1}$, to finally obtain

$$\underline{x}_k = [\underline{H}_k^T\underline{H}_k]^{-1}\underline{H}_k^T\underline{y}_k \tag{6.21}$$

which gives the optimal estimate of the state at instant k for a set of k measurements.

A sequential form for estimation can be obtained by writing Eq. (6.18) as follows:

$$\begin{bmatrix} \underline{y}_{k-1} \\ \hline y_k \end{bmatrix} = \begin{bmatrix} \underline{H}_{k-1}\underline{x}_{k-1} \\ \hline H_k\underline{x}_k \end{bmatrix} \tag{6.22}$$

where $\underline{y}_{k-1}$ is a vector of k-1 output measurements, and $\underline{H}_{k-1}$ is a composite matrix defined by the first k-1 elements in Eq. (6.18). The vector $\underline{x}_{k-1}$ can be written in terms of $\underline{x}_k$. With simplified notation

$$\underline{x}_{k-1} = \Phi^{-1}(k,k-1)\underline{x}_k = \Phi_{k-1}^{-1}\underline{x}_k \tag{6.23}$$

and substitution of Eq. (6.23) into (6.22) yields

$$\begin{bmatrix} \underline{y}_{k-1} \\ \hline y_k \end{bmatrix} = \begin{bmatrix} \underline{H}_{k-1}\Phi_{k-1}^{-1} \\ \hline H_k \end{bmatrix} \underline{x}_k \tag{6.24}$$

Solution of Eq. (6.24) for $\underline{x}_k$ is obtained by multiplication by the inverse matrix:

$$\underline{x}_k = \begin{bmatrix} \underline{H}_{k-1}\Phi_{k-1}^{-1} \\ \hline H_k \end{bmatrix}^{-1} \begin{bmatrix} \underline{y}_{k-1} \\ \hline y_k \end{bmatrix} \tag{6.25}$$

and comparison with Eq. (6.21) shows that

$$\begin{bmatrix} \underline{H}_{k-1}\Phi_{k-1}^{-1} \\ \hline H_k \end{bmatrix}^{-1} = [\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1}[\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T} \mid H_k^T] \tag{6.26}$$

where $(\Phi_{k-1}^{-1})^T = \Phi_{k-1}^{-T}$. Therefore

$$\underline{x}_k = [\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1}[\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{y}_{k-1} + H_k^T y_k] \quad (6.27)$$

Equation (6.27) can be written in terms of $\underline{x}_{k-1}$ by substituting for $\underline{y}_{k-1}$ from Eq. (6.22) to obtain

$$\underline{x}_k = [\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1}[\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\underline{x}_{k-1} + H_k^T y_k] \quad (6.28)$$

Equation (6.28) can be rearranged into a form containing an error correction term by multiplying both sides of the equation with the result that

$$[\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]\underline{x}_k = \Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\underline{x}_{k-1} + H_k^T y_k \quad (6.29)$$

If the term $H_k^T H_k \Phi_{k-1}\underline{x}_{k-1}$ is added and subtracted to the right hand side, Eq. (6.29) can be written as follows:

$$[\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]\underline{x}_k = \Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\Phi_{k-1}^{-1}\Phi_{k-1}\underline{x}_{k-1} + H_k^T H_k \Phi_{k-1}\underline{x}_{k-1} - H_k^T H_k \Phi_{k-1}\underline{x}_{k-1} + H_k^T y_k \quad (6.30)$$

Multiplication of both sides of Eq. (6.30) finally yields

$$\underline{x}_k = \Phi_{k-1}\underline{x}_{k-1} + [\Phi_{k-1}^{-T}\underline{H}_{k-1}^{T}\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1}H_k^T[y_k - H_k\Phi_{k-1}\underline{x}_{k-1}] \quad (6.31)$$

which is in the form of Kalman's Eq. (6.3), except that $\underline{x}_k$ is generated with the y_k output sample. This is the filtering equation.

If $\hat{\underline{x}}_{k-1}$ and $\hat{\underline{x}}_k$ are defined as the optimal filter outputs, then an optimal estimate is predicted by using the transition matrix to yield

$$\bar{\underline{x}}_k = \Phi_{k-1}\hat{\underline{x}}_{k-1} \tag{6.32}$$

where $\bar{\underline{x}}_k$ is the predicted value of $\hat{\underline{x}}_k$ obtained using the y_{k-1} measurement. Equation (6.32) is the prediction equation.

If in Eq. (6.31), $\underline{x}_{k-1}$ and $\underline{x}_k$ are replaced by $\hat{\underline{x}}_{k-1}$ and $\hat{\underline{x}}_k$, respectively, and Eq. (6.32) is used, then Eq. (6.31) becomes

$$\hat{\underline{x}}_k = \bar{\underline{x}}_k + [\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1} H_k^T[y_k - H_k\bar{\underline{x}}_k] \tag{6.33}$$

The system state at t = (k + 1)T is predicted from

$$\bar{\underline{x}}_{k+1} = \Phi_k\hat{\underline{x}}_k \tag{6.34}$$

If both sides of Eq. (6.33) are multiplied by Φ_k and Eq. (6.34) is substituted for the left side

$$\bar{\underline{x}}_{k+1} = \Phi_k\bar{\underline{x}}_k + \Phi_k[\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1} H_k^T[y_k - H_k\bar{\underline{x}}_k] \tag{6.35}$$

which is Kalman's formula with

$$\underline{f}_1 = \Phi_k[\Phi_{k-1}^{-T}\underline{H}_{k-1}^T\underline{H}_{k-1}\Phi_{k-1}^{-1} + H_k^T H_k]^{-1} H_k^T \tag{6.36}$$

Equation (6.33) yields the optimal estimate of the system at instant k using an *a priori* estimate $\bar{\underline{x}}_k$ and an error correction term based on measurement y_k. $\hat{\underline{x}}_k$ is the *a posteriori* estimate. A new *a priori* estimate is generated using Eq. (6.34).

Equation (6.35) generates a new *a priori* estimate from the old *a priori* estimate with an error correction term based on the current measurement.

6.4 Linear estimation by least-squares minimization

The least-squares estimate of $\underline{x}_k$ is obtained by minimizing the following cost function:

$$J = (y_1 - H_1\underline{x}_1)^2 + (y_2 - H_2\underline{x}_2)^2 + \ldots + (y_k - H_k\underline{x}_k)^2 \tag{6.37}$$

subject to

$$\underline{x}_k = \Phi_{k-1}\underline{x}_{k-1} \tag{6.38}$$

Equation (6.37) can be written using Eq. (6.22) as follows:

$$J = [\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}]^T[\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}] + (y_k - H_k\underline{x}_k)^2 \tag{6.39}$$

The constraint defined by Eq. (6.38) can be included by defining a vector Lagrangian multiplier λ and augmenting Eq. (6.39). The new cost function is

$$J = [\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}]^T[\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}] + (y_k - H_k\underline{x}_k)^2 + \lambda^T[\underline{x}_k - \Phi_{k-1}\underline{x}_{k-1}] \tag{6.40}$$

Setting the gradient of J with respect to $\underline{x}_{k-1}$, $\underline{x}_k$, and λ, respectively equal to zero yields

$$\frac{\partial J}{\partial \underline{x}_{k-1}} = -2[\underline{y}_{k-1} - \underline{H}_{k-1}\underline{x}_{k-1}]^T H_{k-1} - \lambda^T\Phi_{k-1} = 0 \tag{6.41}$$

and

$$\frac{\partial J}{\partial \underline{x}_k} = -2[y_k - H_k\underline{x}_k]^T H_k + \lambda^T = 0 \tag{6.42}$$

and

$$\frac{\partial J}{\partial \lambda} = [\underline{x}_k - \Phi_{k-1}\underline{x}_{k-1}]^T = 0 \tag{6.43}$$

Equation (6.43) is the original system Eq. (6.38). Equation (6.42) is solved for λ^T to obtain

$$\lambda^T = -2[y_k - H_k \underline{x}_k]H_k \tag{6.44}$$

and λ^T is eliminated from Eq. (6.41) with the result that

$$\underline{H}_{k-1}^T \underline{y}_{k-1} - \underline{H}_{k-1}^T \underline{H}_{k-1} \underline{x}_{k-1} = \Phi_{k-1}^T H_k^T y_k - \Phi_{k-1}^T H_k^T H_k \underline{x}_k \tag{6.45}$$

which on multiplication by Φ_{k-1}^{-T} yields

$$\Phi_{k-1}^{-T} \underline{H}_{k-1}^T \underline{y}_{k-1} - \Phi_{k-1}^{-T} \underline{H}_{k-1}^T \underline{H}_{k-1} \underline{x}_{k-1} = H_k^T y_k - H_k^T H_k \underline{x}_k \tag{6.46}$$

If $\underline{x}_{k-1}$ is replaced by using Eq. (6.38), then Eq. (6.46) can be written

$$\Phi_{k-1}^{-T} \underline{H}_{k-1}^T \underline{y}_{k-1} - \Phi_{k-1}^{-T} \underline{H}_{k-1}^T \underline{H}_{k-1} \Phi_{k-1}^{-1} \underline{x}_k = H_k^T y_k - H_k^T H_k \underline{x}_k \tag{6.47}$$

which, in turn, can be rearranged in the form of Eq. (6.27) by using the matrix inverse.

An alternate cost function can be defined [112, 113, 122]:

$$J = [\underline{H}_\alpha (\underline{x}_{k-1} - \underline{\alpha})]^2 + (y_k - H_k \underline{x}_k)^2 + \lambda^T [\underline{x}_k - \Phi_{k-1} \underline{x}_{k-1}] \tag{6.48}$$

where $\underline{\alpha}$ is the previous estimate. Setting the gradients of J with respect to $\underline{x}_{k-1}$ and $\underline{x}_k$, respectively equal to zero yields

$$\frac{\partial J}{\partial \underline{x}_{k-1}} = 2[\underline{H}_\alpha (\underline{x}_{k-1} - \underline{\alpha})]^T \underline{H}_\alpha - \lambda^T \Phi_{k-1} = 0 \tag{6.49}$$

$$\frac{\partial J}{\partial \underline{x}_k} = -2[y_k - H_k \underline{x}_k]^T H_k + \lambda^T = 0 \tag{6.50}$$

and elimination of λ results in

$$[\underline{H}_\alpha(\underline{x}_{k-1} - \underline{\alpha})]^T\underline{H}_\alpha = [y_k - H_k\underline{x}_k]^T H_k \Phi_{k-1} \tag{6.51}$$

Transposing Eq. (6.51) and multiplying by Φ_{k-1}^{-1} leads to

$$\Phi_{k-1}^{-1}\underline{H}_\alpha^T\underline{H}_\alpha\underline{x}_{k-1} - \Phi_{k-1}^{-1}\underline{H}_\alpha^T\underline{H}_\alpha\underline{\alpha} = H_k^T y_k - H_k^T H_k \underline{x}_k \tag{6.52}$$

Equation (6.38) is used to eliminate $\underline{x}_{k-1}$ with the result that

$$\Phi_{k-1}^{-1}\underline{H}_\alpha^T\underline{H}_\alpha\Phi_{k-1}^{-1}\underline{x}_k - \Phi_{k-1}^{-1}\underline{H}_\alpha^T\underline{H}_\alpha\underline{\alpha} = H_k^T y_k - H_k^T H_k \underline{x}_k \tag{6.53}$$

Equation (6.53) can be rearranged in the form of Eq. (6.29) which is obtained by the matrix inverse.

6.5 <u>Nonlinear estimation by least-squares minimization and iteration</u>

For the nonlinear plant defined by

$$\underline{x}_k = \underline{f}(\underline{x}_{k-1}) \tag{6.54}$$

and the nonlinear measurement equation

$$y_k = h(\underline{x}_k) \tag{6.55}$$

an optimal estimate of the system state can be obtained by minimizing the following cost function:

$$J = [\underline{H}_\alpha(\underline{x}_{k-1} - \underline{\alpha})]^2 + [y_k - h(\underline{x}_k)]^2 + \lambda^T[\underline{x}_k - \underline{f}(\underline{x}_{k-1})] \tag{6.56}$$

where $\underline{\alpha}$ is the previous estimate. Setting the gradients of J with respect to $\underline{x}_{k-1}$ and $\underline{x}_k$, respectively equal to zero yields

$$\frac{\partial J}{\partial \underline{x}_{k-1}} = 2[\underline{H}_\alpha(\underline{x}_{k-1} - \underline{\alpha})]^T\underline{H}_\alpha - \lambda^T F_{k-1} = 0 \tag{6.57}$$

and

$$\frac{\partial J}{\partial \underline{x}_k} = -2[y_k - h(\underline{x}_k)]^T H_k + \lambda^T = 0 \tag{6.58}$$

where

$$F_{k-1} = \frac{\partial \underline{f}(\underline{x}_{k-1})}{\partial \underline{x}_{k-1}} \tag{6.59}$$

and

$$H_k = \frac{\partial h(\underline{x}_k)}{\partial \underline{x}_k} \tag{6.60}$$

Eliminating λ^T from Eqs. (6.57) and (6.58) results in

$$[\underline{H}_\alpha(\underline{x}_{k-1} - \underline{\alpha})]^T \underline{H}_\alpha = [y_k - h(x_k)]^T H_k F_{k-1} \tag{6.61}$$

and after transposing, Eq. (6.61) becomes

$$\underline{H}_\alpha^T \underline{H}_\alpha(\underline{x}_{k-1} - \underline{\alpha}) = F_{k-1}^T H_k^T [y_k - h(x_k)] \tag{6.62}$$

Equations (6.54) and (6.62) must be satisfied for J to be a minimum.

The estimation process may be interpreted as follows. Given the last estimate $\underline{\alpha}$ based on a measurement y_{k-1}, a revised estimate $\underline{x}_{k-1}$ is made which must satisfy

$$\underline{H}_\alpha(\underline{x}_{k-1} - \underline{\alpha}) = 0 \tag{6.63}$$

This revised estimate is used in Eq. (6.54) to obtain an estimate of $\underline{x}_k$, which, in turn, must satisfy

$$y_k - h(\underline{x}_k) = 0 \tag{6.64}$$

Nonlinear Eqs. (6.54) and (6.62) can be solved by iteration by using a first-order Taylor expansion:

$$\underline{x}_k^{i+1} = \underline{f}(\underline{x}_{k-1}^{i+1}) \approx \underline{f}(x_{k-1}^{i}) + F_{k-1}(x_{k-1}^{i+1} - x_{k-1}^{i}) \tag{6.65}$$

$$h(\underline{x}_k^{i+1}) \approx h(\underline{x}_k) + H_k(x_k^{i+1} - x_k^{i}) \tag{6.66}$$

where the superscripts identify the iteration sequence. Equation (6.62) at the i+1 iteration is

$$\underline{H}_\alpha^T \underline{H}_\alpha (\underline{x}_{k-1}^{i+1} - \underline{\alpha}) = F_{k-1}^T H_k^T [y_k - h(x_k^{i+1})] \tag{6.67}$$

and substitution of Eqs. (6.65) and (6.66) into Eq. (6.67) results in

$$\underline{H}_\alpha^T \underline{H}_\alpha (\underline{x}_{k-1}^{i+1} - \underline{\alpha}) = F_{k-1}^T H_k^T [y_k - h(x_k^i) + H_k \underline{x}_k^i - H_k \underline{f}(\underline{x}_{k-1}^i)$$
$$- H_k F_{k-1} \underline{x}_{k-1}^{i+1} + H_k F_{k-1} \underline{x}_{k-1}^i] = 0 \tag{6.68}$$

The term $\underline{H}_\alpha^T \underline{H}_\alpha \underline{x}_{k-1}^i$ is added and subtracted to Eq. (6.68) to obtain

$$[\underline{H}_\alpha^T \underline{H}_\alpha + F_{k-1}^T H_k^T H_k F_{k-1}] \underline{x}_{k-1}^{i+1} = [\underline{H}_\alpha^T \underline{H}_\alpha + F_{k-1}^T H_k^T H_k F_{k-1}] \underline{x}_{k-1}^i$$
$$+ F_{k-1}^T H_k^T \{y_k - h(\underline{x}_k^i) + H_k [\underline{x}_k^i - \underline{f}(\underline{x}_{k-1}^i)]\}$$
$$+ \underline{H}_\alpha^T \underline{H}_\alpha (\underline{\alpha} - \underline{x}_{k-1}^i) \tag{6.69}$$

Multiplication of Eq. (6.69) by the inverse matrix yields

$$\underline{x}_{k-1}^{i+1} = \underline{x}_{k-1}^i + [\underline{H}_\alpha^T \underline{H}_\alpha + F_{k-1}^T H_k^T H_k F_{k-1}]^{-1} \{F_{k-1}^T H_k^T [y_k - h(\underline{x}_k^i)$$
$$+ H_k (\underline{x}_k^i - \underline{f}(\underline{x}_{k-1}^i))] + \underline{H}_\alpha^T \underline{H}_\alpha (\underline{\alpha} - \underline{x}_{k-1}^i)\} \tag{6.70}$$

and the i+1 estimate for $\underline{x}_k$ is obtained from

$$\underline{x}_k^{i+1} = \underline{f}(x_{k-1}^i) + F_{k-1}(x_{k-1}^{i+1} - x_{k-1}^i) \tag{6.71}$$

Equations (6.70) and (6.71) are the estimator equations for a system consisting of a nonlinear plant with a nonlinear measurement. The iteration sequence is started by selecting

$$\underline{x}_{k-1}^1 = \underline{\alpha} \tag{6.72}$$

and

$$\underline{x}_k^1 = \underline{f}(\underline{\alpha}) \tag{6.73}$$

With each iteration, the H_k and F_{k-1} matrices are re-evaluated and a new matrix inverse is calculated. A matrix inversion lemma applied to stochastic systems to eliminate the inversion is not applicable to Eq. (6.70) [125, p. 276].

The term $\underline{H}_\alpha(\underline{\alpha} - \underline{x}_{k-1}^i)$ which appears on the right hand side of Eq. (6.70) is identically equal to zero throughout the iteration sequence. A proof that

$$\underline{H}_\alpha(\underline{\alpha} - \underline{x}_{k-1}^i) = \underline{0} \tag{6.74}$$

is given in Appendix D.

6.6 Nonlinear estimation of continuous systems with discrete time measurements

The nonlinear estimator defined by Eqs. (6.70) and (6.71) was derived for a system described by nonlinear difference Eqs. (6.54) and (6.55).

For a plant described by

$$\dot{\underline{x}} = \underline{g}(\underline{x}) \tag{6.75}$$

the value of $\underline{x}_k$ is obtained by integration:

$$\underline{x}_k = \underline{f}(\underline{x}_{k-1}) = \underline{x}_{k-1} + \int_0^T \underline{g}(\underline{x})dt \tag{6.76}$$

The estimator requires $\partial\underline{f}(x_{k-1})/\partial\underline{x}_{k-1}$ which is obtained by integrating the solution of the plant variational equation. The variational equation is given by

$$\delta\dot{\underline{x}} = G\delta\underline{x} \tag{6.77}$$

where

$$G = \frac{\partial \underline{g}}{\partial \underline{x}} \tag{6.78}$$

Equation (6.77) is a linear equation and has the solution

$$\delta \underline{x}_k = \Phi(T)\delta \underline{x}_{k-1} \tag{6.79}$$

where the state-transition matrix Φ satisfies the matrix differential equation

$$\dot{\Phi}(t) = G\Phi(t) \tag{6.80}$$

with $\Phi(0) = I$. As indicated by Eq. (6.79), the transition matrix of the linearized system measures the change in $\underline{x}_k$ per unit change in $\underline{x}_{k-1}$; therefore

$$F_{k-1} = \frac{\partial \underline{f}(\underline{x}_{k-1})}{\partial \underline{x}_{k-1}} = F((k-1)T) = \int_0^T GF dt \tag{6.81}$$

with $F(0) = I$. Thus simultaneous integration of Eqs. (6.75) and (6.81) provides the information required by the estimator, and the analytic solution of the nonlinear plant differential equation is not required. An analytic comparison of Eq. (6.81) for the reactor equations is presented in Appendix E.

6.7 Performance of nuclear system state estimator

For the nuclear system nonlinear discrete-time Eqs. (4.75), (4.76) and (4.83), the performance index is defined:

$$J = [y_{k-1} - h(\underline{x}_{k-1})]^2 + [y_k - h(\underline{x}_k)]^2 \tag{6.82}$$

subject to

$$z_k = z_{k-1}\exp[\lambda\rho_{k-1}T/(1 - \rho_{k-1})] \tag{6.83}$$

$$\rho_k = \rho_{k-1} \tag{6.84}$$

The performance index is a minimum when

$$y_{k-1} = h(\underline{x}_{k-1}) = z_{k-1}/(1 - \rho_{k-1}) \tag{6.85}$$

$$y_k = h(\underline{x}_k) = z_k/(1 - \rho_k) \tag{6.86}$$

Using Eqs. (6.83), (6.84), (6.85), and (6.86) and two successive output samples, the solution for reactivity is

$$\hat{\rho}_{k-1} = \hat{\rho}_k = \frac{\ln(y_k/y_{k-1})}{\lambda T + \ln(y_k/y_{k-1})} \tag{6.87}$$

and for the delayed neutron precursor density

$$\hat{z}_{k-1} = (1 - \hat{\rho}_{k-1})y_{k-1} \tag{6.88}$$

The solution for $\hat{z}_k$ is obtained from Eq. (6.83) using Eqs. (6.87) and (6.88). Numerical values for the analytic solution of the estimator equations are obtained by using the Analytic Estimator Solutions computer program listed in Appendix G. The programmed value of y_0 is unity, and y_1 is calculated in response to a step change in reactivity occurring at t = (0+). Table 6.1 lists the analytic estimator solutions for different values of reactivity disturbances. These values are used to determine whether the estimator with iteration, programmed to solve Eqs. (6.70) and (6.71), generates the correct estimate in one sample after a disturbance.

The Finite Difference System with Estimator and Control computer program (listed in Appendix G), with the control loop opened by setting u = 0, generates samples of the output measurement by solving the plant finite difference Eqs. (6.83) and (6.84), the measurement Eq. (4.77),

TABLE 6.1

ANALYTIC ESTIMATOR SOLUTIONS

$\rho(0+)$	$\hat{\rho}_1$	$\hat{z}_1$	$\hat{n}_1$
0.25	0.55778428	0.65380766	1.4784811
0.20	0.49233880	0.68571204	1.3507277
0.10	0.31081242	0.79259985	1.1500494
-0.10	-0.66212729	1.4690358	0.88382870
-0.20	-3.0783149	3.2274608	0.79137116
-0.25	-11.471604	9.3774779	0.75190634

and the estimator Eqs. (6.70) and (6.71). Consecutive iterations of the estimator equations are performed until the performance index is equal to or less than a specified value, which can be expressed as

$$J \leq \varepsilon_1 \tag{6.89}$$

Thus by changing ε_1, the accuracy and number of iterations can be controlled.

The matrices F_{k-1} and H_k are obtained by differentiating Eqs. (6.83), (6.84), and (6.86), respectively, to obtain

$$F_{k-1} = \begin{bmatrix} \exp\left(\frac{\lambda T \rho_{k-1}}{1-\rho_{k-1}}\right) & \frac{\lambda T z_{k-1}}{(1-\rho_{k-1})^2} \exp\left(\frac{\lambda T \rho_{k-1}}{1-\rho_{k-1}}\right) \\ 0 & 1 \end{bmatrix} \tag{6.90}$$

$$H_k = [1/(1-\rho_k) \quad z_k/(1-\rho_k)^2] \tag{6.91}$$

A worst-case analysis is used to investigate the performance of the estimator. Since a disturbance can occur anywhere within one sample interval, the worst case is when it occurs immediately after the measurement. The estimator is initialized by assuming the system to be in equilibrium up to t = 0. Thus

$$\underline{\alpha} = \hat{\underline{x}}_0 = \underline{x}(0-) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{6.92}$$

$$H_\alpha = [1 \; 1] \tag{6.93}$$

and

$$\underline{x}(0+) = \begin{bmatrix} 1 \\ \rho(0+) \end{bmatrix} \tag{6.94}$$

For large reactivity disturbances, the first iteration produces an estimate of ρ_{k-1} which exceeds unity. If this happens, a discontinuity is crossed and the estimator is not able to converge. The computer program contains an arbitrary hard limit on ρ_{k-1} of 0.8. With this limit, the estimator produces correct estimates for step changes in reactivity up to +0.56\$. Similarly, a discontinuity exists at −0.27\$. Thus the useable range of the estimator for step disturbances is from −0.27\$ to +5.6\$.

Tables 6.2 and 6.3 show, respectively, estimator performance for $\hat{\rho}_k$, $\hat{z}_k$, and $\hat{n}_k$ in response to step reactivity disturbances of +0.1\$ and −0.1\$ with an iteration accuracy of $\varepsilon_1 = 10^{-4}$. The number of iterations is indicated in column I, and the estimated values are given beneath the true values. For $\rho = 0.1$, the estimate is generated in four iterations and agrees up to the fifth decimal place with the values in Table 6.1. At the end of the second sample interval, the system state

TABLE 6.2

ESTIMATOR PERFORMANCE WITH FINITE-DIFFERENCE SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-4}$, AND $\rho(0+) = 0.1$

k	ρ_k	z_k	n_k	I
0	.10000000	1.00000005	1.11111114	
	.00000000*	1.00000005*	1.00000005*	
1	.10000000	1.03504454	1.15004947	
	.31082521*	.79259022*	1.15005689*	4
2	.10000000	1.07131714	1.19035235	
	.09907985*	1.07241802*	1.19035855*	3
3	.10000000	1.10886090	1.23206765	
	.09999066*	1.10887280*	1.23206808*	1
4	.10000000	1.14772035	1.27524482	
	.09999905*	1.14772158*	1.27524482*	1
5	.10000000	1.18794163	1.31993512	
	.09999996*	1.18794169*	1.31993512*	1
6	.10000000	1.22957243	1.36619157	
	.09999996*	1.22957249*	1.36619157*	1
7	.10000000	1.27266217	1.41406904	
	.09999996*	1.27266222*	1.41406904*	1
8	.10000000	1.31726195	1.46362438	
	.09999999*	1.31726200*	1.46362439*	1
9	.10000000	1.36342472	1.51491633	
	.09999993*	1.36342481*	1.51491633*	1
10	.10000000	1.41120523	1.56800579	
	.09999998*	1.41120525*	1.56800579*	1

*Estimate

TABLE 6.3

ESTIMATOR PERFORMANCE WITH FINITE-DIFFERENCE SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-4}$, AND $\rho(0+) = -0.1$

k	ρ_k	z_k	n_k	I
0	-.10000000	1.00000005	.90909093	
	.00000000*	1.00000005*	1.00000005*	
1	-.10000000	.97221162	.88382875	
	-.66060630*	1.46791877*	.88396555*	3
2	-.10000000	.94519543	.85926857	
	-.09012711*	.93673362*	.85928843*	4
3	-.10000000	.91892997	.83539089	
	-.10006901*	.91900903*	.83541036*	1
4	-.10000000	.89339439	.81217672	
	-.10009105*	.89346834*	.81217672*	1
5	-.10000000	.86856840	.78960765	
	-.09999991*	.86856834*	.78960766*	1
6	-.10000000	.84443229	.76766572	
	-.09999999*	.84443226*	.76766571*	1
7	-.10000000	.82096686	.74633353	
	-.09999995*	.82096683*	.74633353*	1
8	-.10000000	.79815353	.72559412	
	-.09999991*	.79815348*	.72559413*	1
9	-.10000000	.77597413	.70543103	
	-.09999999*	.77597411*	.70543101*	1
10	-.10000000	.75441106	.68582824	
	-.09999995*	.75441103*	.68582824*	1

*Estimate

TABLE 6.4

ESTIMATOR PERFORMANCE WITH FINITE-DIFFERENCE SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-6}$, AND $\rho(0+) = 0.25$

k	ρ_k	z_k	n_k	I
0	.25000001 .00000000*	1.00000005 1.00000005*	1.33333338 1.00000005*	
1	.25000001 .55778435*	1.10886100 .65380763*	1.47848131 1.47848132*	6
2	.25000001 .24997180*	1.22957266 1.22962016*	1.63943018 1.63943187*	4
3	.25000001 .24999822*	1.36342509 1.36342834*	1.81790011 1.81790009*	1
4	.25000001 .25000005*	1.51184884 1.51184879*	2.01579849 2.01579849*	1
5	.25000001 .25000005*	1.67643014 1.67643008*	2.23524020 2.23524020*	1
6	.25000001 .25000003*	1.85892791 1.85892788*	2.47857056 2.47857056*	1
7	.25000001 .25000003*	2.06129260 2.06129257*	2.74839009 2.74839009*	1
8	.25000001 .25000001*	2.28568686 2.28568683*	3.04758244 3.04758241*	1
9	.25000001 .25000004*	2.53450889 2.53450883*	3.37934517 3.37934520*	1
10	.25000001 .25000002*	2.81041793 2.81041790*	3.74722387 3.74722384*	1

*Estimate

TABLE 6.5

ESTIMATOR PERFORMANCE WITH FINITE-DIFFERENCE SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-6}$, AND $\rho(0+) = -0.25$

k	ρ_k	z_k	n_k	I
0	-.25000001 .00000000*	1.00000005 1.00000005*	.80000001 1.00000005*	
1	-.25000001 -11.47042594*	.93988293 9.37661416*	.75190635 .75190811*	7
2	-.25000001 -.24271586*	.88337989 .87823433*	.70670392 .70670567*	7
3	-.25000001 -.24998595*	.83027365 .83027136*	.66421894 .66422455*	1
4	-.25000001 -.25004289*	.78036003 .78038681*	.62428803 .62428803*	1
5	-.25000001 -.24999987*	.73344706 .73344699*	.58675765 .58675765*	1
6	-.25000001 -.25000008*	.68935435 .68935439*	.55148348 .55148349*	1
7	-.25000001 -.24999993*	.64791237 .64791234*	.51832990 .51832991*	1
8	-.25000001 -.24999997*	.60896177 .60896174*	.48716940 .48716940*	1
9	-.25000001 -.24999992*	.57235275 .57235272*	.45788219 .45788220*	1
10	-.25000001 -.24999997*	.53794457 .53794455*	.43035565 .43035564*	1

*Estimate

TABLE 6.6

ESTIMATOR PERFORMANCE WITH INTEGRATED SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, AND $\rho(0+) = 0.1$

k	ρ_k	z_k	n_k	Δt	I
0	.10000000	1.00000005	1.11111114		
	.00000000*	1.00000005*	1.00000005*		
1	.10000000	1.03504454	1.15004947		
	.31418966*	.78872010*	1.15005569*	1.00000	4
2	.10000000	1.07131714	1.19035235		
	.09946450*	1.07196009*	1.19035851*	1.00000	3
3	.10000000	1.10886090	1.23206765		
	.10033641*	1.10844578*	1.23206806*	1.00000	1
4	.10000000	1.14772035	1.27524482		
	.10034495*	1.14728047*	1.27524485*	1.00000	1
5	.10000000	1.18794163	1.31993512		
	.10034578*	1.18748521*	1.31993510*	1.00000	1
6	.10000000	1.22957243	1.36619157		
	.10034587*	1.22909993*	1.36619160*	1.00000	1
7	.10000000	1.27266217	1.41406904		
	.10034579*	1.27217317*	1.41406901*	1.00000	1
8	.10000000	1.31726195	1.46362438		
	.10034589*	1.31675571*	1.46362441*	1.00000	1
9	.10000000	1.36342472	1.51491633		
	.10034579*	1.36290086*	1.51491632*	1.00000	1
10	.10000000	1.41120523	1.56800579		
	.10034587*	1.41066292*	1.56800580*	1.00000	1

*Estimate

TABLE 6.7

ESTIMATOR PERFORMANCE WITH INTEGRATED SYSTEM EQUATIONS, $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, AND $\rho(0+) = 0.1$

k	ρ_k	z_k	n_k	Δt	I
0	.10000000	1.00000005	1.11111114		
	.00000000*	1.00000005*	1.00000005*		
1	.10000000	1.03504454	1.15004947		
	.31165936*	.79163070*	1.15005660*	.25000	4
2	.10000000	1.07131714	1.19035235		
	.09925989*	1.07220364*	1.19035848*	.50000	3
3	.10000000	1.10886090	1.23206765		
	.10016345*	1.10865993*	1.23206809*	.50000	1
4	.10000000	1.14772035	1.27524482		
	.10017159*	1.14750156*	1.27524486*	.50000	1
5	.10000000	1.18794163	1.31993512		
	.10017248*	1.18771397*	1.31993513*	.50000	1
6	.10000000	1.22957243	1.36619157		
	.10017259*	1.22933665*	1.36619160*	.50000	1
7	.10000000	1.27266217	1.41406904		
	.10017251*	1.27241819*	1.41406901*	.50000	1
8	.10000000	1.31726195	1.46362438		
	.10017267*	1.31700924*	1.46362439*	.50000	1
9	.10000000	1.36342472	1.51491633		
	.10017254*	1.36316331*	1.51491630*	.50000	1
10	.10000000	1.41120523	1.56800579		
	.10017259*	1.41093461*	1.56800582*	.50000	1

*Estimate

is estimated to within 0.1%, requiring three iterations. Thereafter, a single iteration is used to track the system. The estimation sequence after k = 2, corresponds to an extended Kalman filter in which the previous estimate is used to evaluate the F and H matrices.

Tables 6.4 and 6.5 show, respectively, the estimator performance in response to reactivity disturbances of +0.25$ and -0.25$ with an iteration accuracy of $\varepsilon_1 = 10^{-6}$. The increase in iteration accuracy is required to obtain a good estimate for $\hat{\beta}_2$. For $\varepsilon_1 = 10^{-5}$, $\hat{\beta}_2 =$ -0.15607480; and for $\varepsilon_1 = 10^{-4}$, $\hat{\rho}_2 = 0.08458834$. For $\varepsilon_1 = 10^{-4}$, $\hat{\rho}_3 =$ -0.25450338 and one additional sample is required to obtain an accurate estimate of the system state. As indicated in Table 6.5, seven iterations are required for $\hat{\underline{x}}_1$ and $\hat{\underline{x}}_2$, and one iteration is used thereafter.

The estimates in Tables 6.2 to 6.5 are for a system described by finite-difference equations. The performance of an estimator which uses integration of the system equations is investigated by using the Differential System With Estimator and Control computer program (listed in Appendix G) with the feedback control loop opened by setting u = 0. The plant differential equations are given by Eqs. (4.32) and (4.33); and the variational equation used to calculate F_{k-1} is given by Eq. (4.60). The matrix differential equation to be integrated is

$$\begin{bmatrix} \dot{F}_{11} & \dot{F}_{12} \\ \dot{F}_{21} & \dot{F}_{22} \end{bmatrix} = \begin{bmatrix} \lambda\rho(t)/[1-\rho(t)] & z(t)/[1-\rho(t)]^2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \qquad (6.95)$$

and after multiplication yields

$$\dot{F}_{11} = \frac{\lambda\rho(t)}{1-\rho(t)} F_{11} + \frac{\lambda z(t)}{[1-\rho(t)]^2} F_{21} \qquad (6.96)$$

$$\dot{F}_{12} = \frac{\lambda\rho(t)}{1-\rho(t)} F_{12} + \frac{\lambda z(t)}{[1-\rho(t)]^2} F_{22} \tag{6.97}$$

and

$$\dot{F}_{21} = \dot{F}_{22} = 0 \tag{6.98}$$

Solution of Eq. (6.98) requires that F_{21} = constant and F_{22} = constant, but the initial conditions require $F(0) = I$. Therefore, $F_{21} = 0$ and $F_{22} = 1$. After substitution of F_{21} and F_{22}, Eqs. (6.96) and (6.97) reduce to

$$\dot{F}_{11} = \frac{\lambda\rho(t)}{1-\rho(t)} F_{11} \tag{6.99}$$

$$\dot{F}_{12} = \frac{\lambda\rho(t)}{1-\rho(t)} F_{12} + \frac{\lambda z(t)}{[1-\rho(t)]^2} \tag{6.100}$$

The initial conditions are: $F_{11}(0) = 1$ and $F_{12}(0) = 0$. Simultaneous integration of Eqs. (4.32), (4.33), (6.99), and (6.100) yield the solutions for $\underline{x}_k$ and F_{k-1}.

The integration is performed numerically, therefore the accuracy of integration is dependent upon the step size. The Kutta-Merson method [126; 127, p. 24] given in Appendix F is used because of its one-step starting feature and error computation. The integration step size Δt is automatically adjusted to meet a specified accuracy requirement. The parameter ε_2 in the computer program, specifies the integration accuracy.

Tables 6.6 and 6.7 show, respectively, the estimator performance for a step change in reactivity of 0.1 with integration accuracies ε_2 of 10^{-2} and 10^{-3} and an iteration accuracy ε_1 of 10^{-4}. Comparison of Table 6.6 with Table 6.1 shows 1% accuracy of $\underline{x}_1$ and 0.5% accuracy for $\underline{x}_2$. For $\underline{x}_3$ and subsequent estimates, a steady error of approximately

0.34% is obtained for $\hat{\rho}$. The integration step size Δt, automatically selected by the integration subroutine, is shown to be 1 sec for each sample interval with the number of iterations remaining the same as in Table 6.2. Table 6.7 shows 0.3% accuracy for $\underline{x}_1$ with $\Delta t = 0.25$, and 0.74% accuracy for $\underline{x}_2$ with $\Delta t = 0.5$. For $\hat{x}_3$ and subsequent estimates, the steady error is 0.17% and $\Delta t = 0.5$. When $\Delta t = 0.25$, the equations of the integration subroutine are solved four times for each iteration, or 16 times for four iterations.

CHAPTER 7

COMBINED ESTIMATION AND CONTROL OF NUCLEAR SYSTEMS

7.1 Introduction

The problem of combined estimation and control has been investigated elsewhere [96, 97, 110, 125 and 128] with a resulting separation theorem. This theorem states that for linear systems subject to Gaussian noise with a quadratic cost function, the optimum stochastic controller is realized by cascading an optimal estimator with a deterministic optimum controller. The separation theorem does not apply to nonlinear systems with optimality guaranteed.

In Chapter 5, optimal control of a nuclear reactor was investigated using a control law which is a linear function of the state variables. The state variables: reactivity and delayed neutron precursor density, are not measureable. Therefore, in Chapter 6, an investigation was made of an optimal estimator which generates estimates of reactivity and delayed neutron precursor density from measurements of the prompt neutron density. In this chapter, the transient performance of the system is investigated with combined estimation and control.

7.2 Combined estimation and control

In Chapter 6, the estimator equations were derived with the assumption that the plant was not under control. With the plant under control, the linear prediction Eq. (6.32) is modified as follows:

$$\underline{\bar{x}}_k = \Phi_{k-1}\underline{\hat{x}}_{k-1} + Gu_{k-1} \tag{7.1}$$

and the linear filter Eq. (6.33) generates $\underline{\hat{x}}_k$ using $\underline{\bar{x}}_k$ of Eq. (7.1) and the y_k measurement. The control variable u_k is computed from

$$u_k = B\underline{\hat{x}}_k \tag{7.2}$$

and the predicted estimate of $\underline{\bar{x}}_{k+1}$ is obtained by using Eq. (7.1).

With control, the nonlinear plant Eq. (6.54) becomes

$$\underline{x}_k = \underline{f}(\underline{x}_{k-1}, u_{k-1}) \tag{7.3}$$

and Eq. (6.59) is written:

$$F_{k-1} = \frac{\partial \underline{f}(\underline{x}_{k-1}, u_{k-1})}{\partial \underline{x}_{k-1}} \tag{7.4}$$

Equation (6.70) for the nonlinear filter remains unchanged, except that F_{k-1} is computed using Eq. (7.4), and the new form for Eq. (6.71) is

$$\underline{x}_k^{i+1} = \underline{f}(\underline{x}_{k-1}^{i}, u_{k-1}) + F_{k-1}(\underline{x}_{k-1}^{i+1} - \underline{x}_{k-1}^{i}) \tag{7.5}$$

At the end of the iteration sequence

$$\underline{\alpha}_k = \underline{x}_k^{i+1} \tag{7.6}$$

and the control variable u_k is computed from

$$u_k = B\underline{\alpha}_k \tag{7.7}$$

With control, the nonlinear plant Eq. (6.75) is

$$\dot{\underline{x}} = \underline{g}(\underline{x}, u) \tag{7.8}$$

and the value of $\underline{x}_k$ is obtained by integration:

$$\underline{x}_k = \underline{x}_{k-1} + \int_0^T \underline{g}(\underline{x}, u_{k-1})dt \tag{7.9}$$

The variational Eq. (6.77) remains unchanged, except that G defined by

Eq. (6.78), is replaced by

$$G = \frac{\partial \underline{g}(\underline{x}, u_{k-1})}{\partial \underline{x}} \tag{7.10}$$

7.3 Combined estimation and control with delay

The preceding calculation of the control variable assumed that a measurement is made at t = kT, the estimator equations are solved iteratively for a new estimate, the new control input is calculated, and the control is applied at t = kT. A more realistic control analysis should consider that a finite time is required to compute a new estimate and control input. The fastest sampling rate is determined by the time T required to execute the calculations outlined above.

The estimation equations remain valid, except that the control input must be delayed by one sample interval. Instead of using Eq. (7.7) to calculate the control at u_k, Eq. (7.3) with $\underline{\alpha}_{k-1}$ is used to predict the system state at t = kT:

$$\overline{\underline{x}}_k = \underline{f}(\underline{\alpha}_{k-1}, u_{k-1}) \tag{7.10}$$

Finally, the control input to be applied at t = kT is obtained using Eq. (7.10), with the result that

$$u_k = B\overline{\underline{x}}_k \tag{7.11}$$

If the calculations are completed in less than T seconds, u_k is stored, until t = kT, and then applied as an input after the measurement is made.

The new sequence is:

1. Obtain a measurement y_k.
2. Apply the previously calculated control input u_k.

3. Solve the estimator equations to obtain $\hat{\underline{x}}_k$.
4. Use the estimator output $\hat{\underline{x}}_k$ and control input u_k to predict the state of the plant at $t = (k+1)T$.
5. Use the predicted estimate $\bar{\underline{x}}_{k+1}$ to calculate a new control input u_{k+1}.
6. Store the control input u_{k+1} until the next measurement at $t = (k+1)T$.
7. Repeat the sequence.

If the total time to execute the above sequence is equal to the sampling period T, then the storage time is zero.

7.4 Nuclear control system performance

The performance of the control system, consisting of an estimator cascaded with the linear control law, is investigated with the plant described first by a difference equation and second by a differential equation.

The difference equation description of the plant is given by Eq. (4.78), and the measurement is given by Eq. (4.83).

The matrix F_{k-1} is obtained by differentiating Eq. (4.78) with the result that

$$F_{k-1} = \begin{bmatrix} \exp\left(\frac{\lambda}{u_{k-1}} \ell n \frac{1-\rho_{k-1}}{1-\rho_{k-1}-u_{k-1}T} - \lambda T\right) & \frac{\lambda T z_{k-1}}{(1-\rho_{k-1})(1-\rho_{k-1}-u_{k-1}T)} \exp\left(\frac{\lambda}{u_{k-1}} \ell n \frac{1-\rho_{k-1}}{1-\rho_{k-1}-u_{k-1}T} - \lambda T\right) \\ 0 & 1 \end{bmatrix} \quad (7.12)$$

and if $u_k = 0$, Eq. (6.90) is used. The matrix H_k is given by Eq. (6.91).

The nonlinear differential equation of the plant is given by Eq. (4.35), and the differential equations for F are given by Eqs. (6.99) and (6.100), and $F_{21} = 0$ and $F_{22} = 1$. In the finite-difference description, F_{k-1} is an explicit function of u_{k-1}, but in the differential description, F is not a direct function of u_{k-1}. The influence of control on F arises through the simultaneous integration of Eqs. (4.35), (6.99), and (6.100), as shown in Appendix E.

Figures 7.1 through 7.4 show the transient response for the system described by the finite-difference equations. These equations are solved by the Finite Difference System with Estimator and Control computer program. In Fig. 7.1, the response is for a step disturbance of $\rho = 0.1\$$ with no delay required for estimation and calculation of control effort. Since the disturbance occurs immediately after the measurement, the control for u_0 is zero. At the end of the first sample, the estimator generates an optimal estimate $\hat{n}_1$, which is the same as the value given in Table 6.2, and the control $u_1 = 0.173\$/\text{sec}$. After the second sample, the estimator generates the correct estimate of the system state, the control input is computed, and the neutron density deviation is driven to zero. For samples at t = 3 sec and greater, the neutron density deviation is zero, and the delayed neutron deviation and reactivity approach zero asymptotically.

The transient response plotted in Fig. 7.2 is obtained by calculating the control input using Eqs. (7.10) and (7.11). The estimate generated from the measurement made at t = 1 sec, is used with $u_1 = 0$ to obtain a predicted estimate $\overline{x}_2$. This estimate is used to calculate u_2. The estimate generated from the measurement made at t = 2 sec gives the true state of the plant. The estimate $\hat{x}_2$ is used

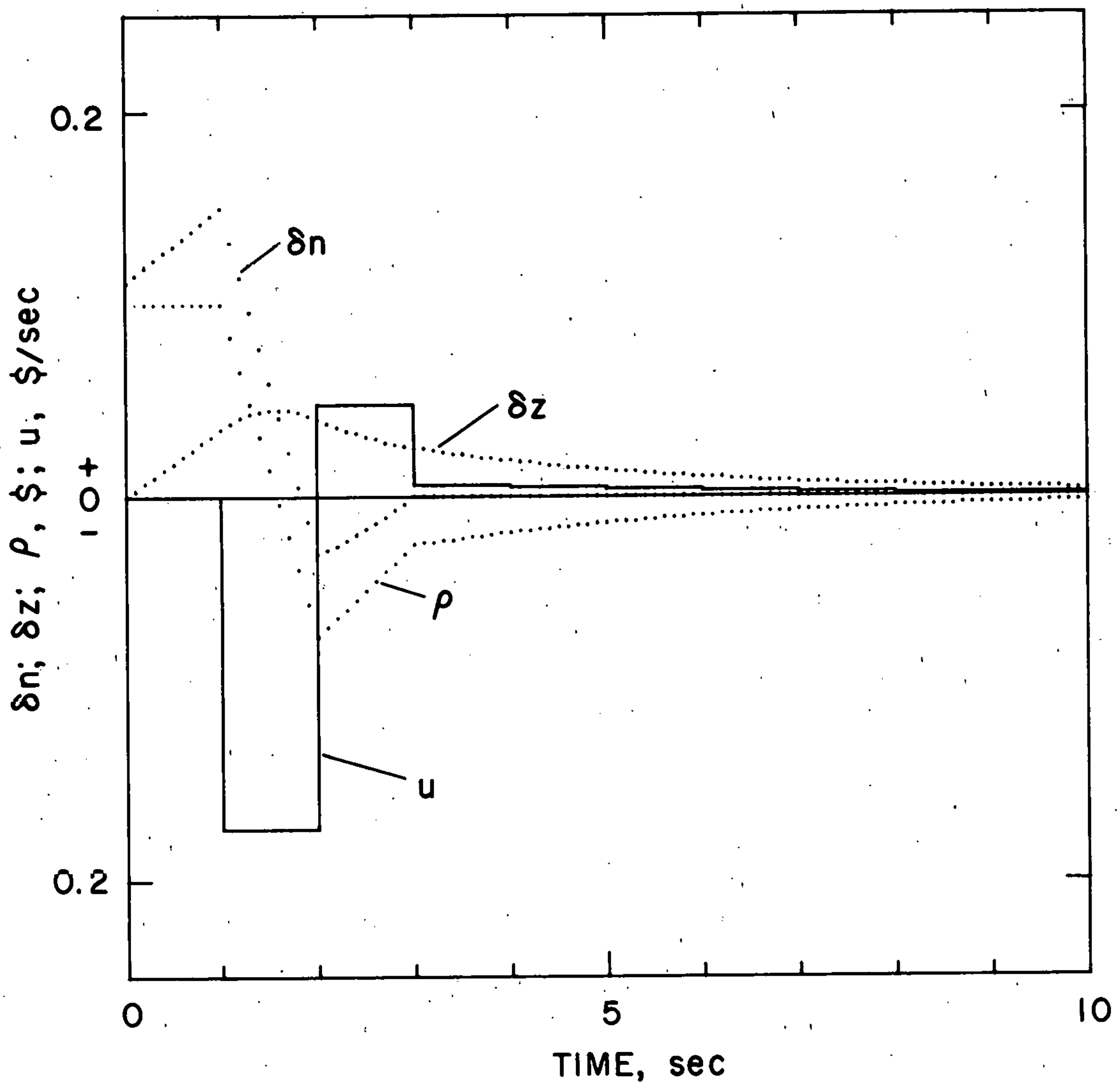

Fig. 7.1 Transient response of system described by finite-difference equations for $\varepsilon_1 = 10^{-6}$ and $\rho_0 = 0.1$, without control delay.

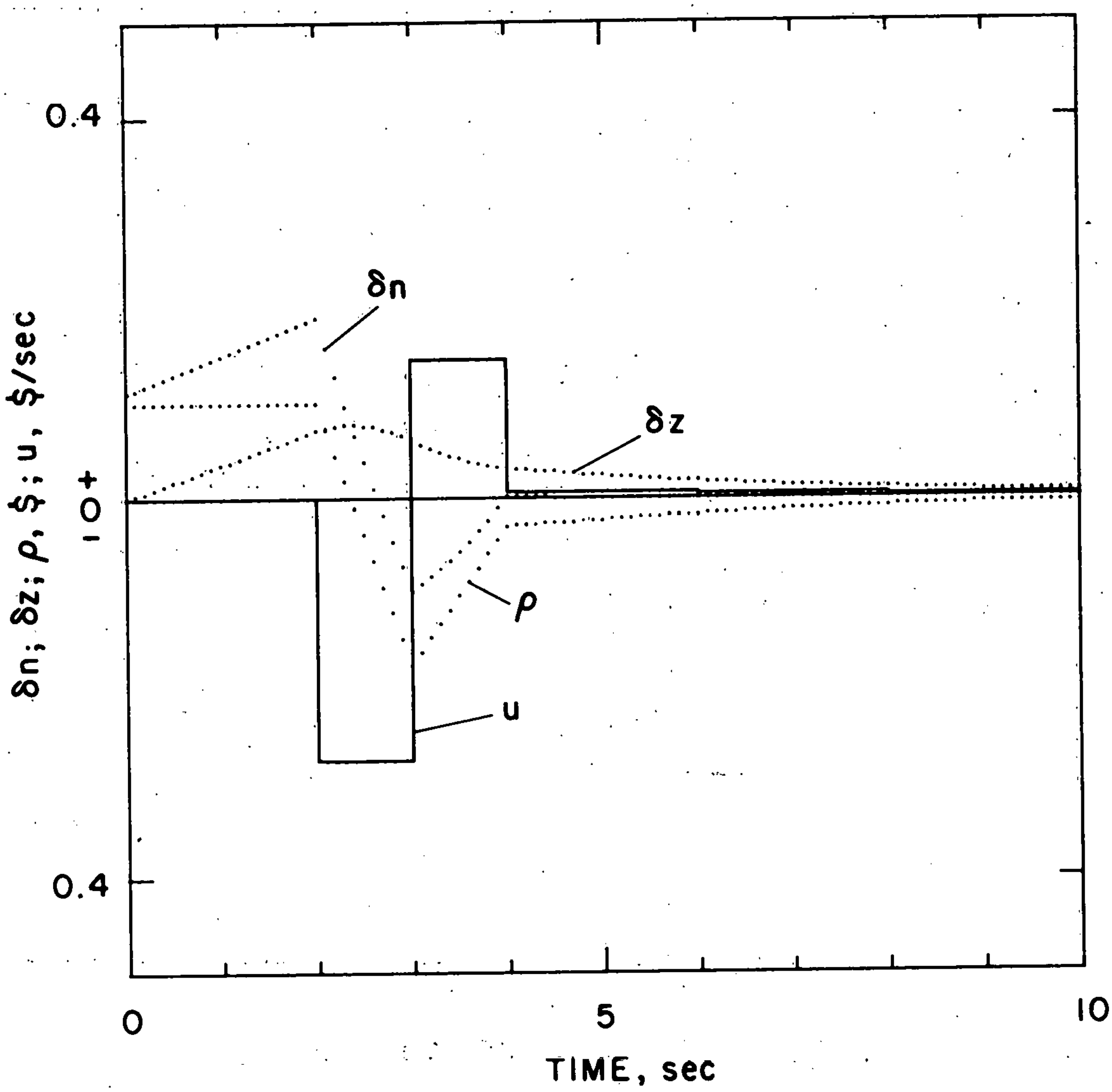

Fig. 7.2 Transient response of system described by finite-difference equations for $\epsilon_1 = 10^{-6}$ and $\rho_0 = 0.1$, with control delay.

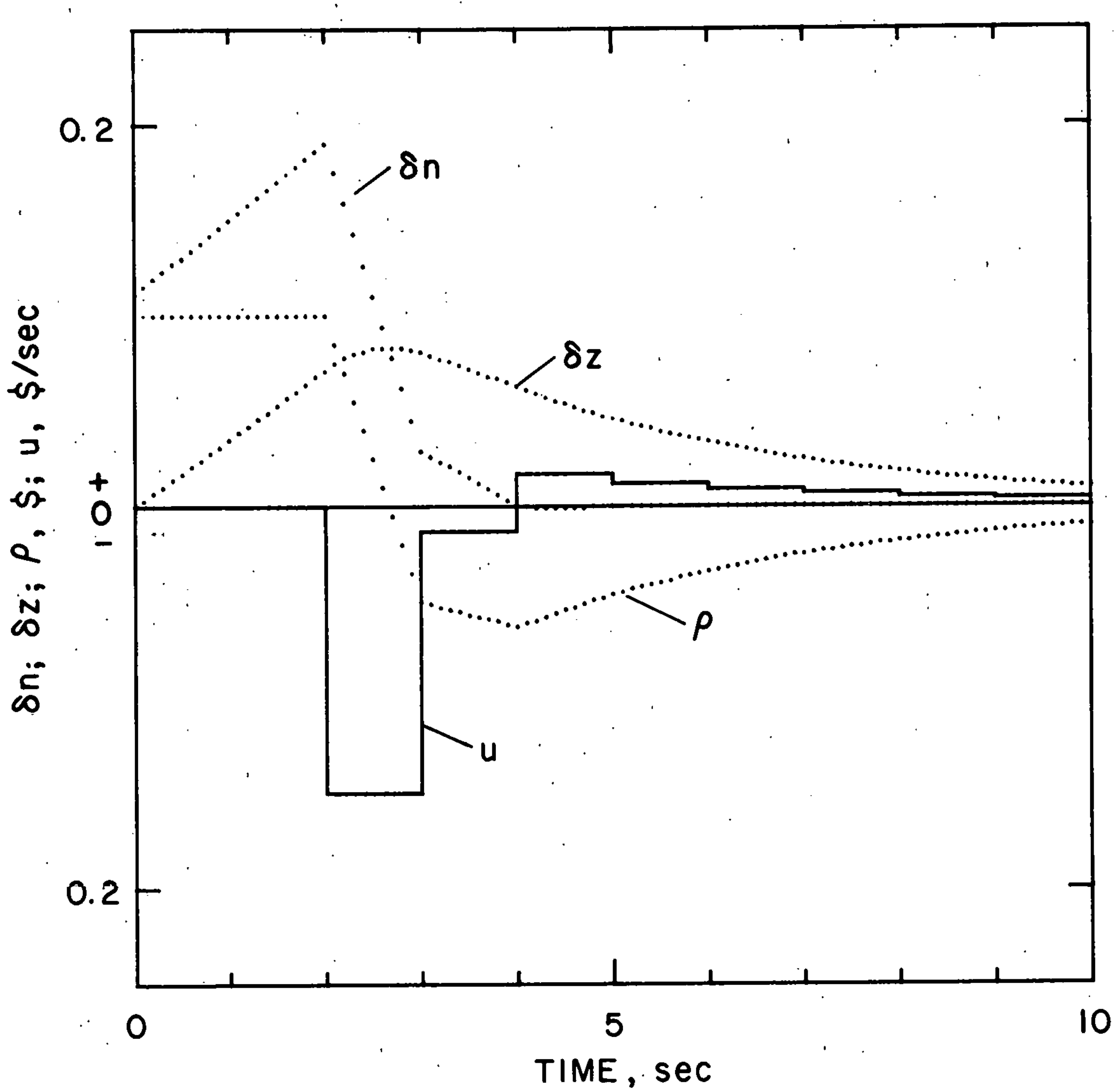

Fig. 7.3 Transient response of system described by finite-difference equations for $\varepsilon_1 = 10^{-6}$ and $\rho_0 = 0.1$, with delayed and bounded control.

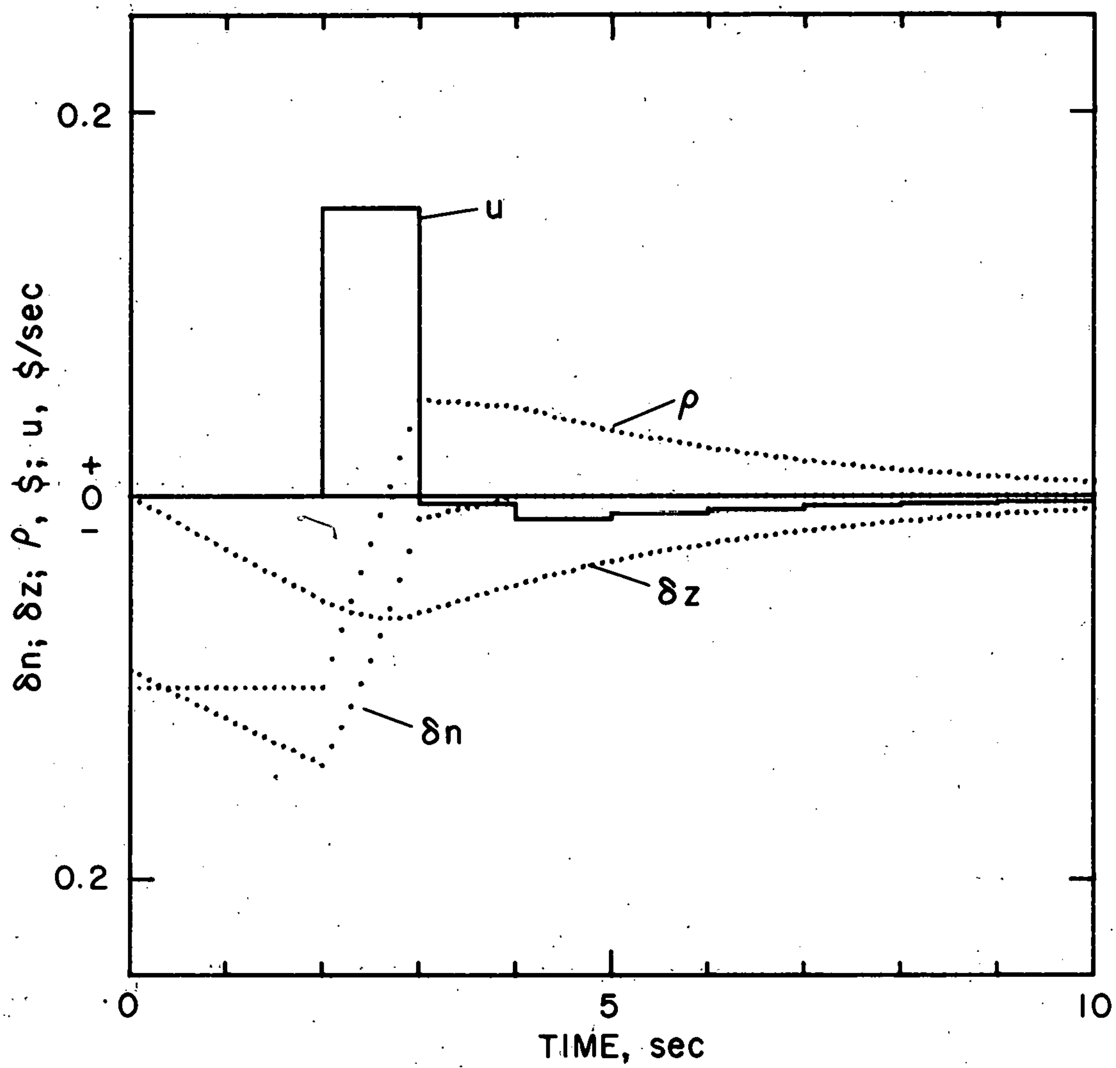

Fig. 7.4 Transient response of system described by finite-difference equations for $\varepsilon_1 = 10^{-6}$ and $\rho_0 = -0.1$, with delayed and bounded control.

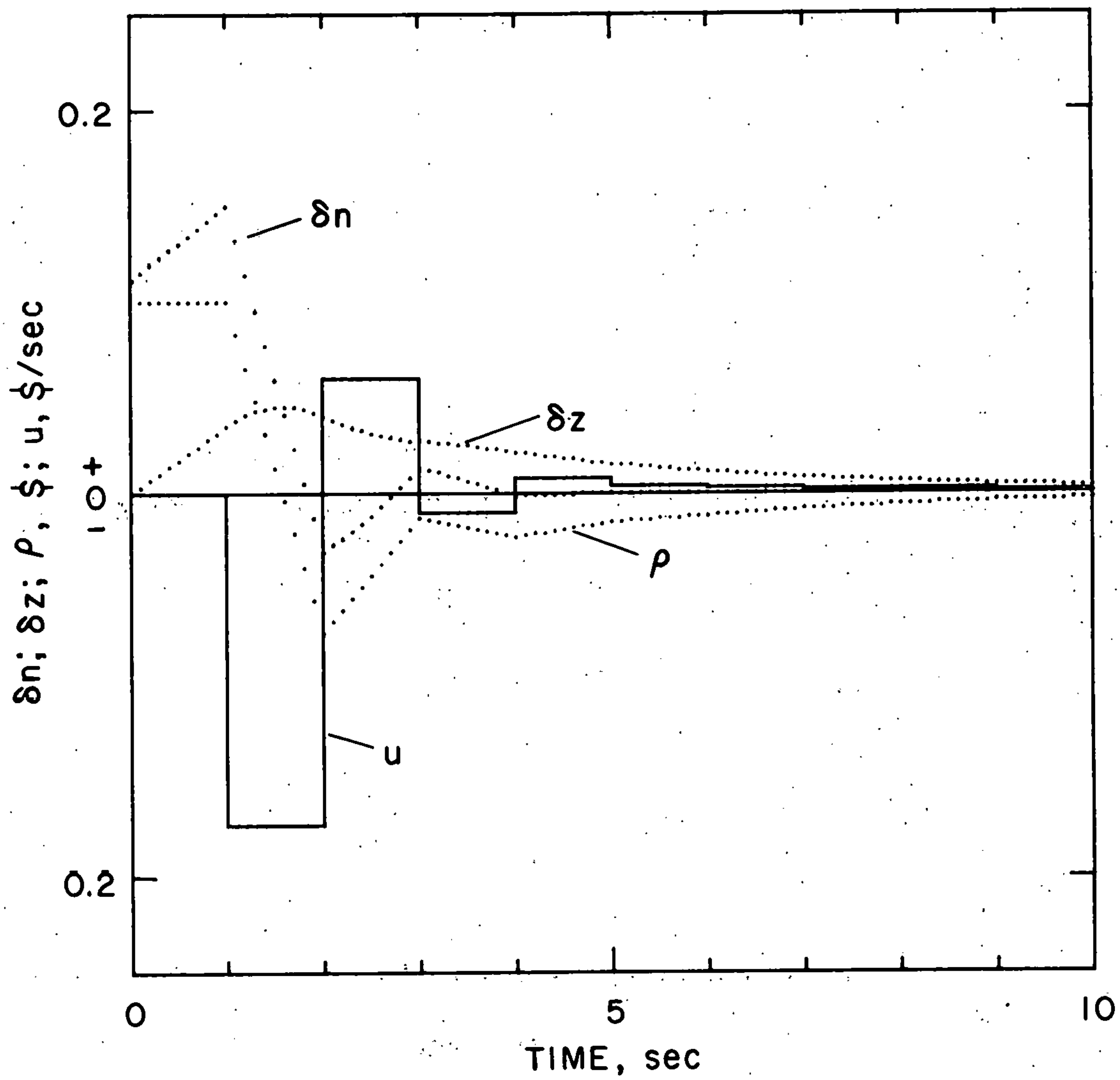

Fig. 7.5 **Transient** response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, and $\rho_0 = 0.1$, without control delay.

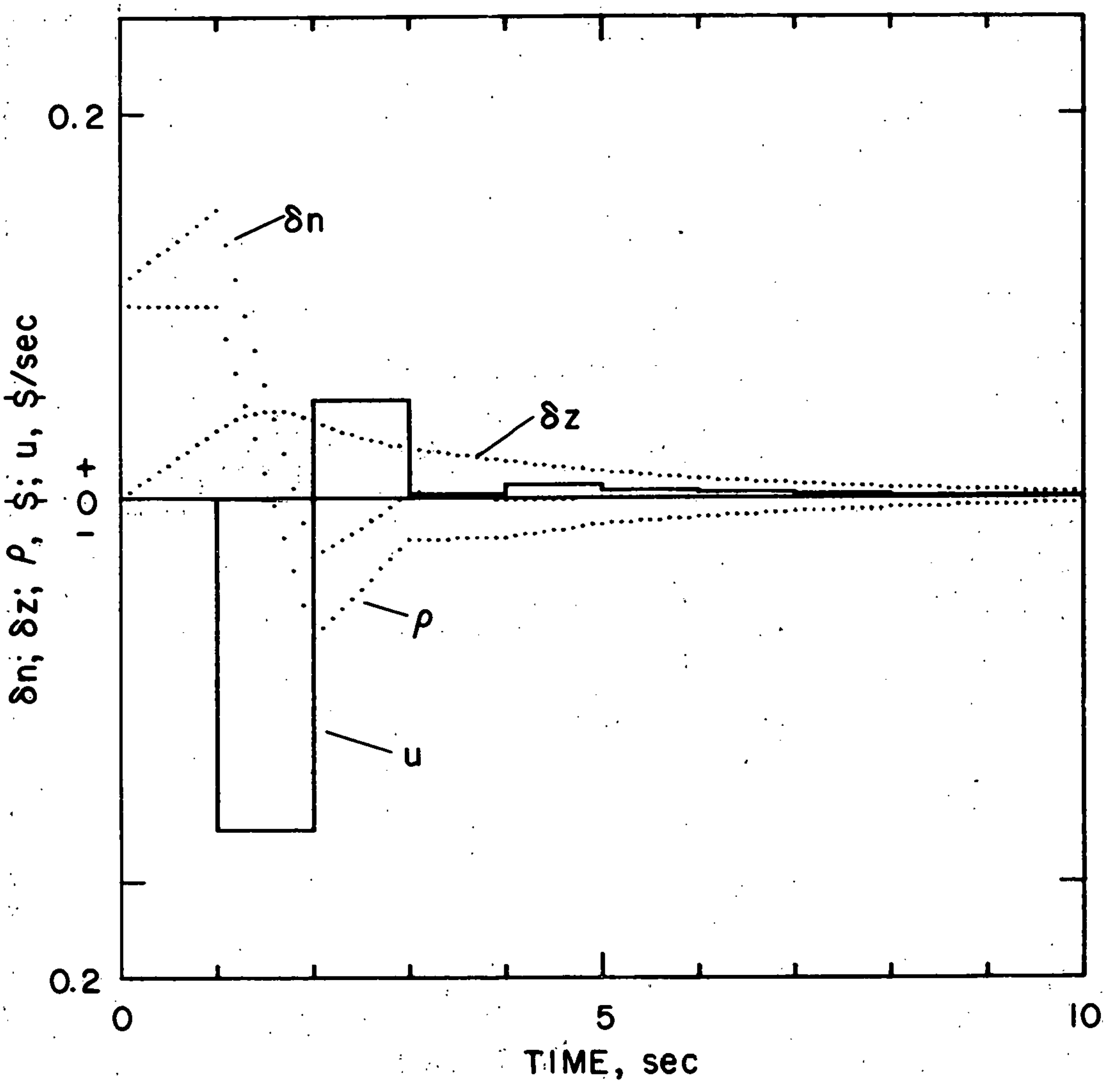

Fig. 7.6 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, and $\rho_0 = 0.1$, without control delay.

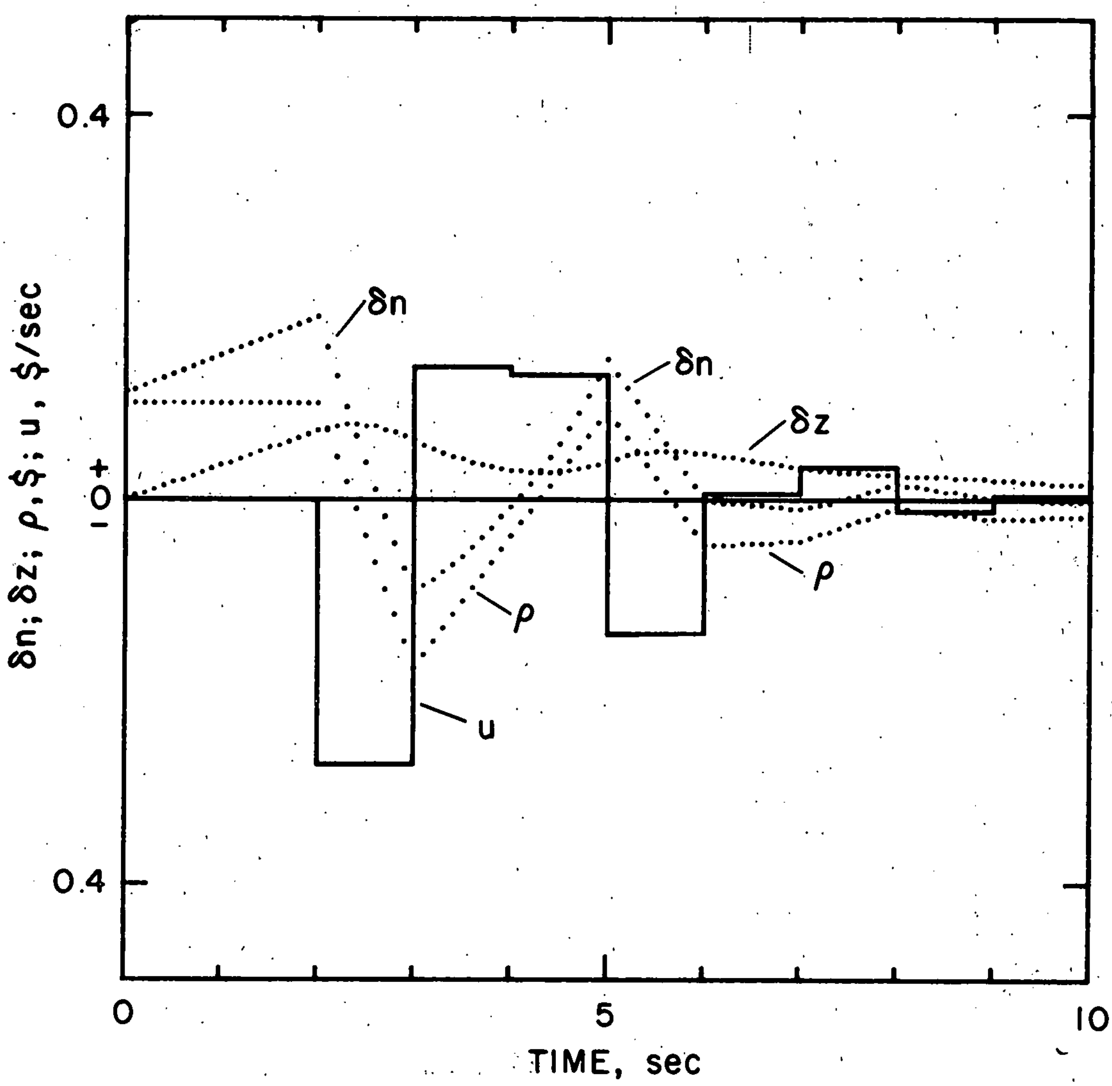

Fig. 7.7 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, and $\rho_0 = 0.1$, with control delay.

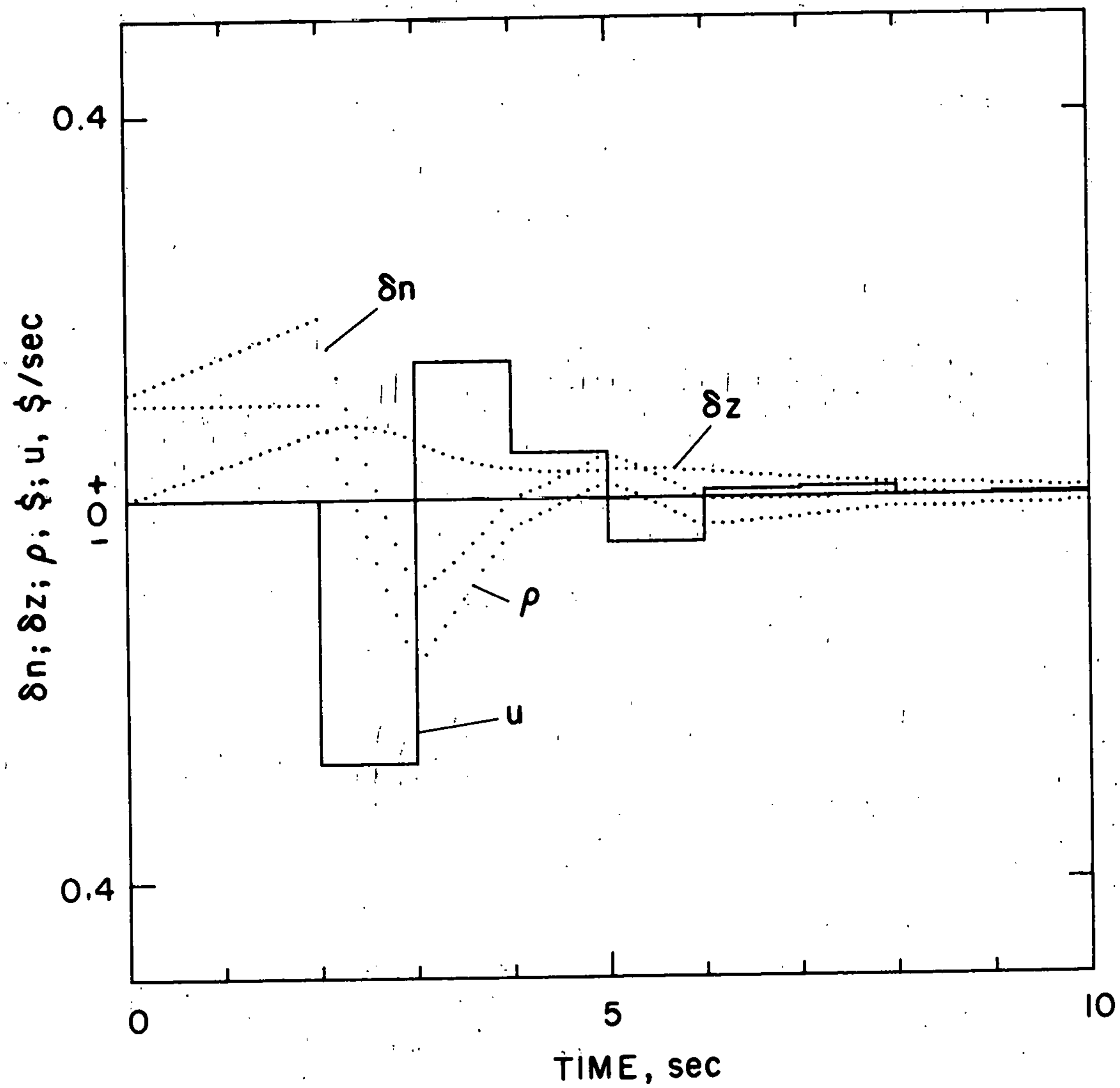

Fig. 7.8 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, and $\rho_0 = 0.1$, with control delay.

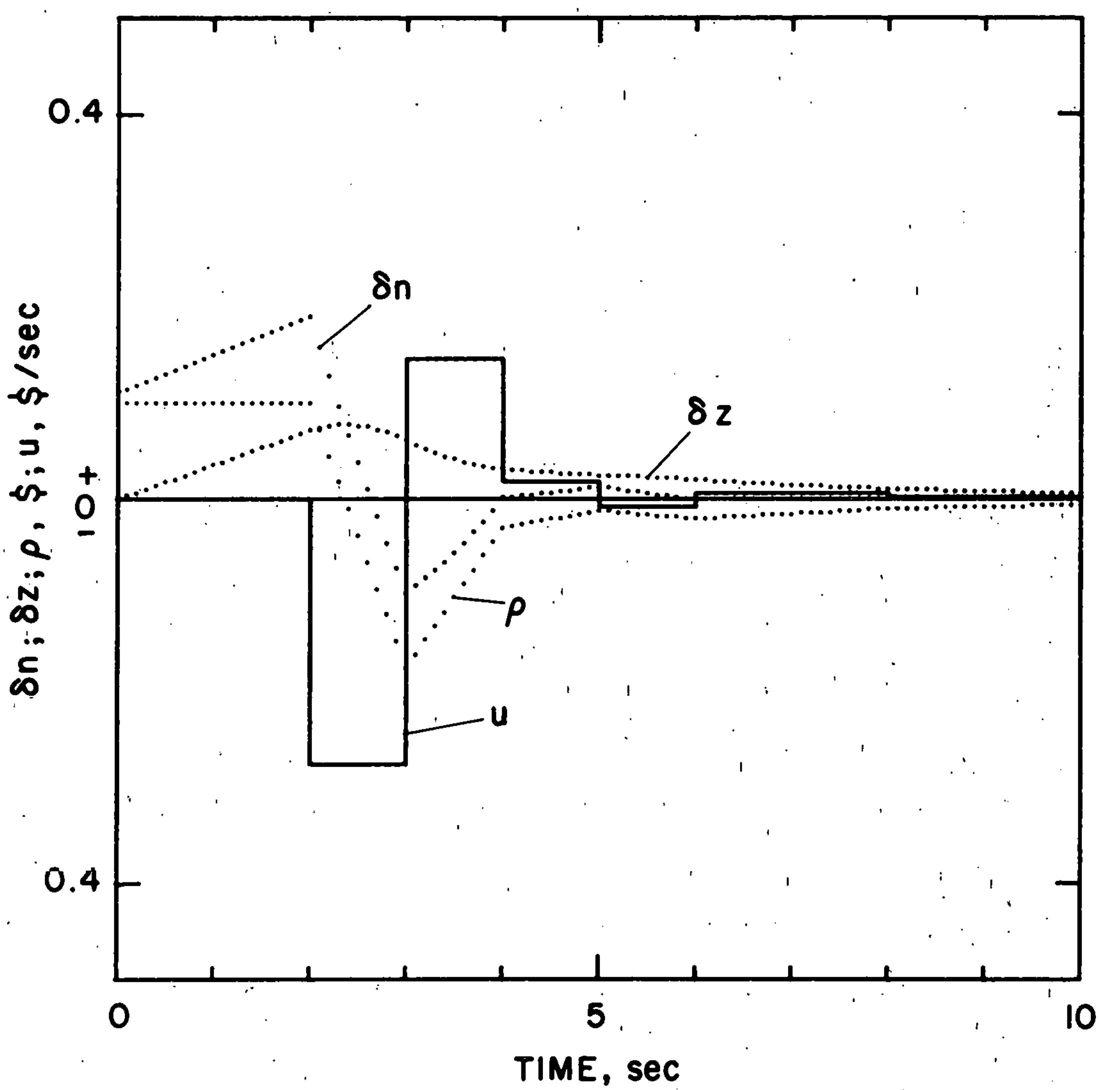

Fig. 7.9 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-5}$, $\varepsilon_2 = 10^{-4}$, and $\rho_0 = 0.1$, with control delay.

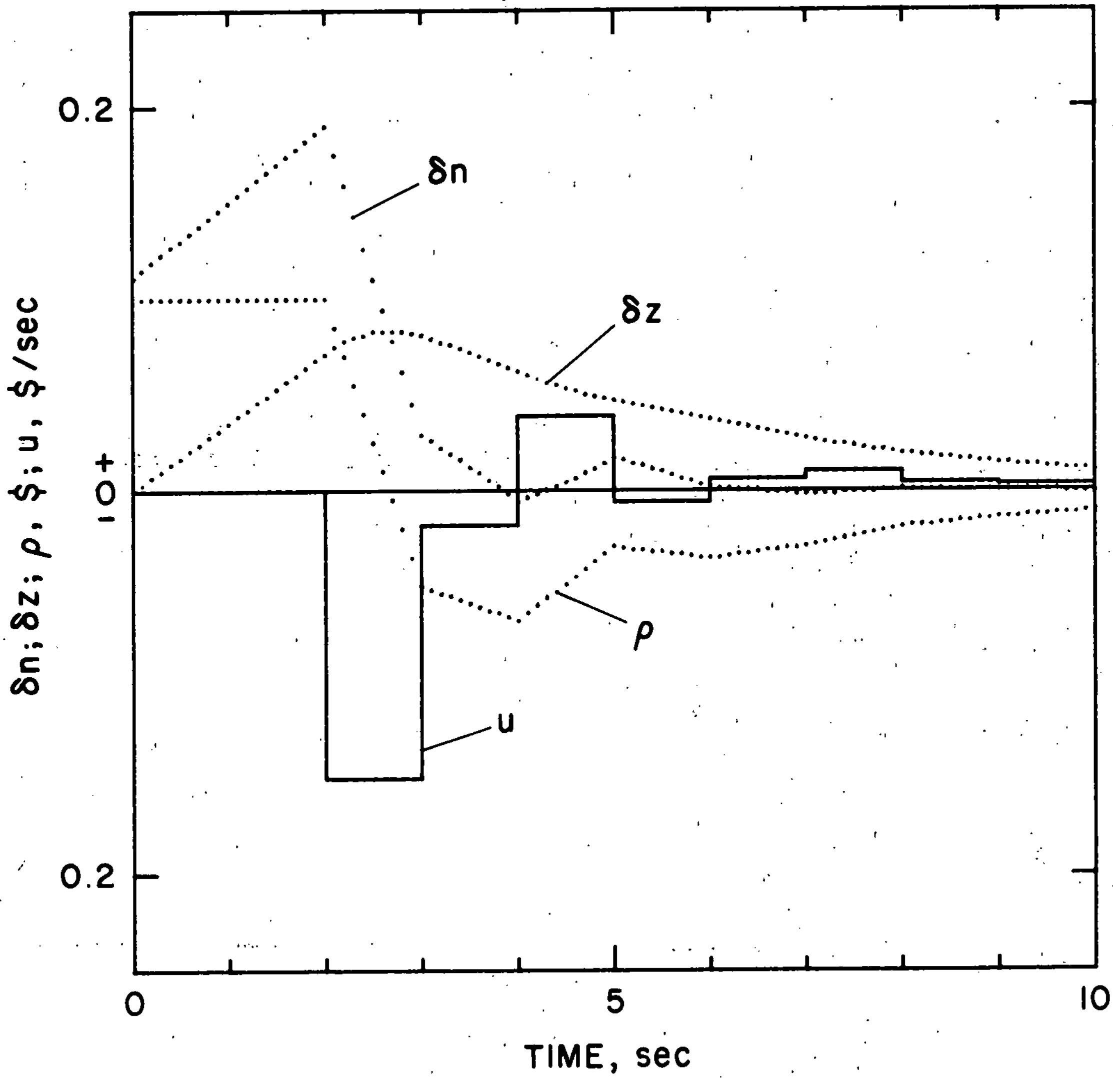

Fig. 7.10 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, and $\rho_0 = 0.1$, with delayed and bounded control.

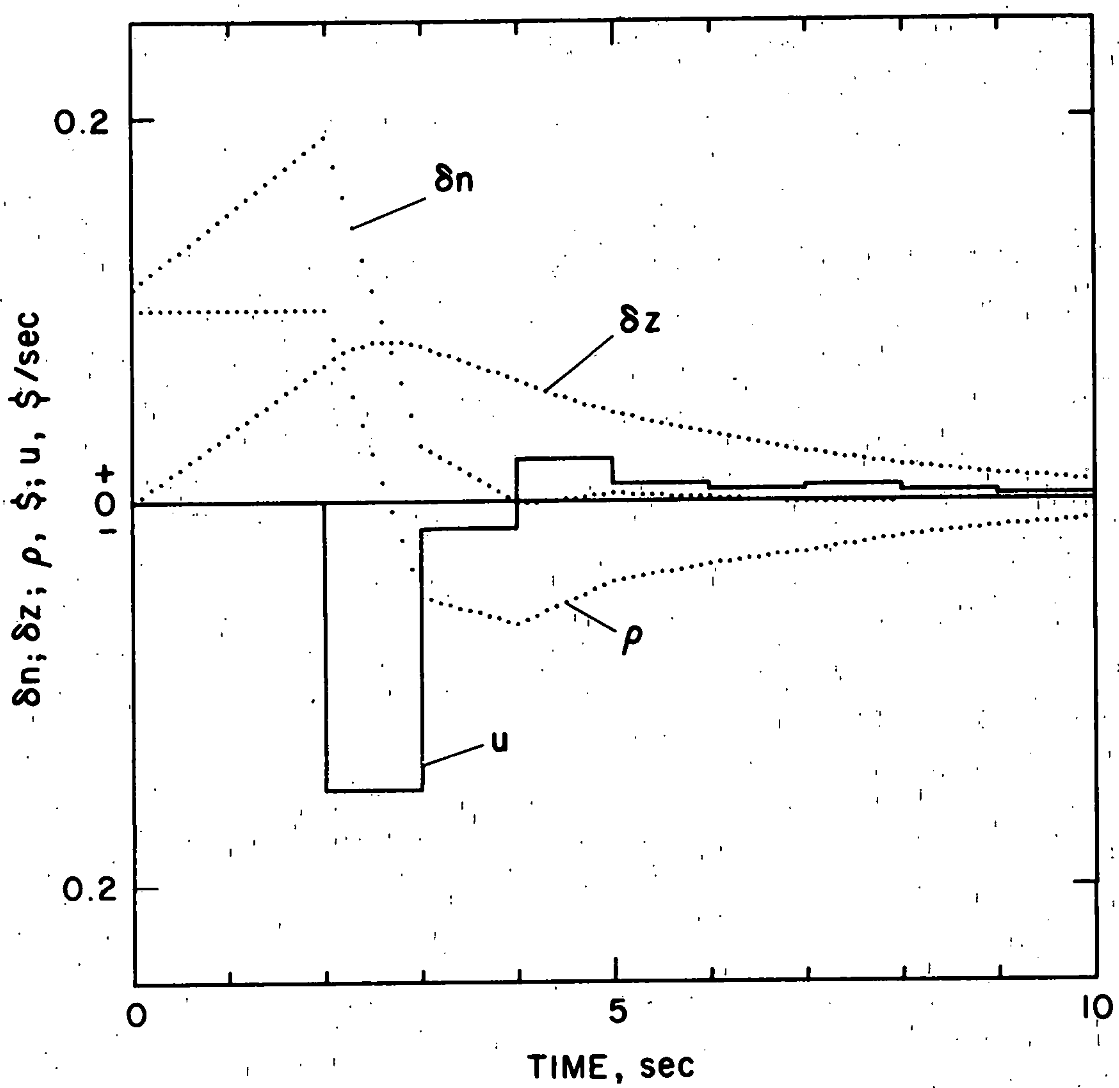

Fig. 7.11 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, and $\rho_0 = 0.1$, with delayed and bounded control.

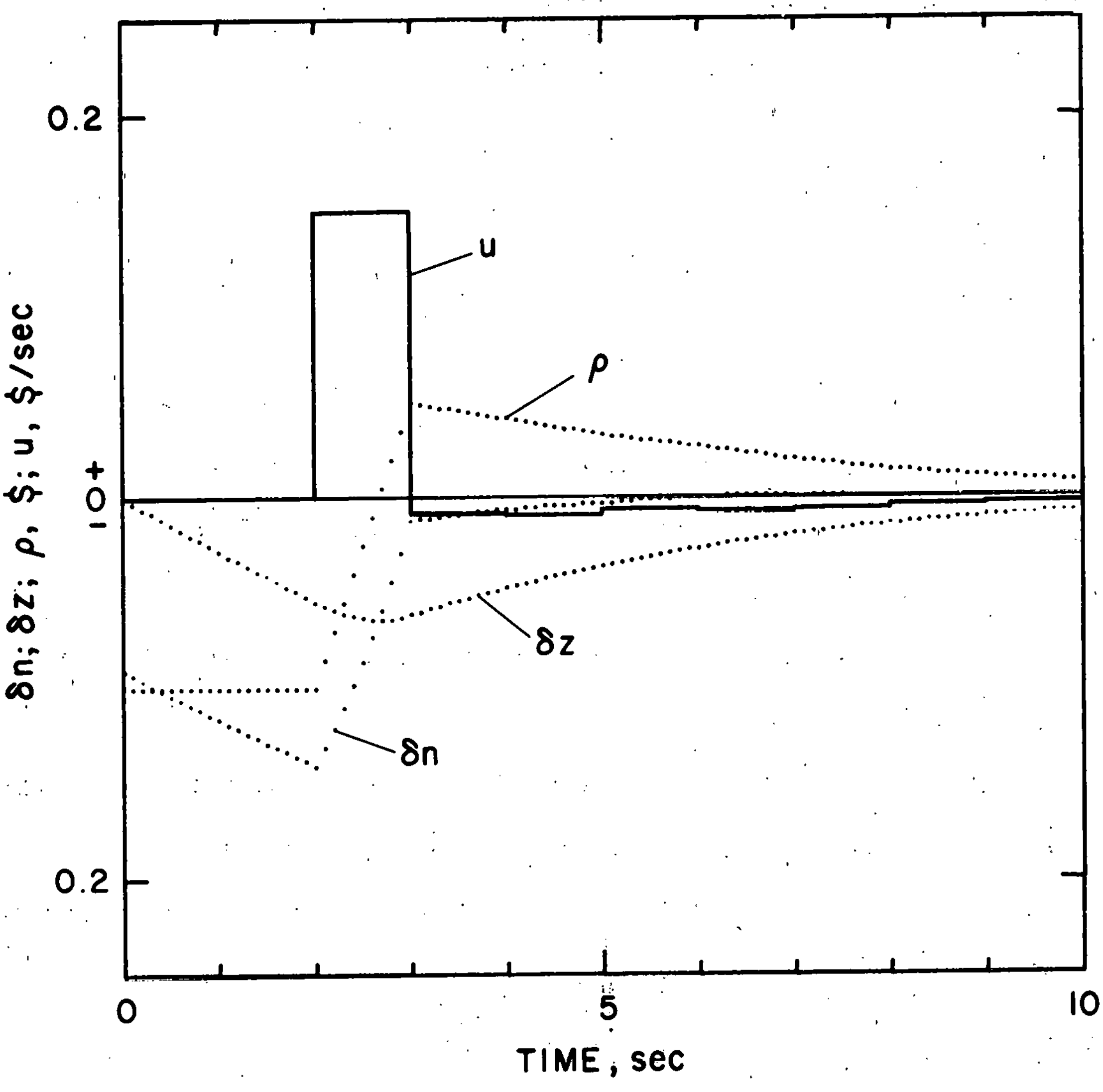

Fig. 7.12 Transient response of system described by differential equations for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, and $\rho_0 = -0.1$, with delayed and bounded control.

with u_2 to obtain a predicted estimate $\overline{\underline{x}}_3$. The control u_3, calculated using $\overline{\underline{x}}_3$, drives the neutron density deviation to zero, and the delayed neutron deviation and reactivity approach zero asymptotically.

The control input u_2 applied at t = 2 in Fig. 7.2 does not correspond to an optimal control because it is not generated from estimates which correspond to the system state. The control is constrained [129, 130] by programming a hard limit of 0.15 \$/sec on u_k:

$$|u_k| \leq UL \tag{7.13}$$

where

$$UL = 0.15 \text{ \$/sec} \tag{7.14}$$

Figure 7.3 shows the transient response with u_k constrained. The neutron density deviation at t = 3 is closer to normal, and u_3 drives the deviation to zero.

Figure 7.4 shows the system response to a reactivity disturbance of -0.1\$ with u_k constrained.

Figures 7.5 through 7.12 show the transient response for the system described by differential equations. These equations are solved by the Differential System with Estimator and Control computer program using specified values for the iteration accuracy ε_1 and for the integration accuracy ε_2.

The transient response to a step disturbance of ρ = 0.1\$ with no control delay is shown in Fig. 7.5 for $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-2}$. Comparison with Fig. 7.1 shows that the control u_2 does not drive the neutron density deviation to zero at t = 3 sec. This is due to an error in $\hat{x}_2$. However, the control u_3 drives the neutron density deviation to zero, and the reactivity and delayed neutron precursor density approach zero asymptotically. In Fig. 7.6, where $\varepsilon_2 = 10^{-3}$, the estimate $\hat{x}_2$ is closer to the true state. This results in a u_2 which drives the neutron density to approximately zero. For the estimate $\underline{\hat{x}}_1$, $\Delta t = 0.125$ sec for the first iteration and $\Delta t = 0.25$ sec for the next three iterations. For the estimate $\underline{\hat{x}}_2$, $\Delta t = 0.25$ sec for two iterations. Thereafter, $\Delta t = 1$ sec.

The transient response with control delay is shown in Fig. 7.7 for $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-2}$. Here, the deviation in neutron density is 19% at $t = 2$ sec and $u_2 = -0.276$ \$/sec. The integration increment is 1 sec for all iterations, which results in large errors in the state estimates. The oscillations in the neutron density and reactivity are damped for this particular initial condition and set of parameters. Figure 7.8 shows the response with $\varepsilon_1 = 10^{-4}$ and $\varepsilon_2 = 10^{-3}$. The peak in the neutron density deviation at $t = 5$ sec is reduced, and greater damping is shown in the oscillatory behavior. In Fig. 7.9 for $\varepsilon_1 = 10^{-5}$ and $\varepsilon_2 = 10^{-4}$, the estimate $\underline{\hat{x}}_4$ results in u_5 which drives the neutron density deviation to zero. Except for a neutron density deviation of 1.26% at $t = 5$ sec, the response is similar to that plotted in Fig. 7.2. Four iterations are required for $\underline{\hat{x}}_1$ with an integration increment of $\Delta t = 0.0625$ sec. The three iterations for $\underline{\hat{x}}_2$ use integration increments of 0.0625, 0.125, and 0.25 sec, consecutively. Estimate $\underline{\hat{x}}_3$ is obtained in one iteration with $\Delta t = 0.0625$; $\underline{\hat{x}}_4$ is obtained in two iterations with $\Delta t = 0.125$ sec. The next two estimates, $\underline{\hat{x}}_5$ and $\underline{\hat{x}}_6$, are obtained in one iteration with $\Delta t = 0.5$ sec. Estimates for $t = 7$ sec and greater are obtained in one iteration with $\Delta t = 1$ sec. Thus $\underline{\hat{x}}_1$ requires the greatest number of calculations with 64 solutions of the integrator equations.

Figure 7.10 shows the transient response for $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-2}$, with delay and a control bound of 0.15 \$/sec. The integration increment is 0.5 sec for the first iteration and is 1 sec thereafter. In comparison with Fig. 7.3, the neutron density deviation has an error of 1.7% at t = 5 sec.

In Fig. 7.11 where $\varepsilon_2 = 10^{-3}$, the neutron density deviation at t = 5 sec is 0.4%. The first iteration requires a $\Delta t = 0.125$ sec, and the next three iterations are with $\Delta t = 0.25$ sec to obtain $\hat{x}_1$. For $\hat{x}_2$, three iterations are required with $\Delta t = 0.25$ sec, whereas, one iteration is required with $\Delta t = 1$ sec for succeeding estimates. Figure 7.12 shows the system response for a reactivity disturbance of -0.1\$.

CHAPTER 8

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

8.1 Summary

The six-group point-model kinetics equations for a nuclear reactor were normalized, solved with a step change in reactivity, and compared to the transient response obtained using a prompt-jump approximation. This demonstrated that for the control system investigation, it is satisfactory to use the prompt-jump approximation with a resulting reduction in order of the system. A further approximation was introduced by using a single group of delayed neutrons. The decay constant for the one-group model was selected by making a comparison with the transient response of the six-group model at 1 sec. The rate of change of reactivity was chosen as a control input by neglecting the control rod motor time constant.

State-space concepts were introduced and vector matrix notation was used to express: the six-group point model kinetics equation, the normalized six-group kinetics equation, the six-group prompt-jump model, the one-group kinetics equation, and the one-group prompt-jump model. A first-order Taylor series expansion was used to linearize the one-group prompt-jump equation. The one-group prompt-jump equation and the linearized equations were solved with a discrete-time input.

An optimal stationary feedback control law was used to minimize a quadratic performance index for a discrete-time system. A performance index was defined which consisted of the sum of the squares of the neutron density deviation. This index was augmented to include terms in reactivity and control. For selected values of the weighting coefficients, the stationary feedback matrix was calculated using an iterative digital computer program. System transient behavior was plotted to demonstrate the influence of the weighting coefficients. For the performance index as defined, the neutron density deviation is driven to zero in one sample interval after a step disturbance in reactivity. The control law assumes that all state variables are available, but the specific variables reactivity and delayed neutron precursor density cannot be measured.

Kalman's filter was derived for a linear deterministic system by a matrix inversion lemma and by minimization of a least-squares cost function. The resulting filter equations showed the relationship of the optimal filter gain to the state transition and measurement matrices. A nonlinear estimator was derived by minimizing a least-squares performance index and iteration was used to solve the resulting nonlinear equations. The filter derivations were based on the assumption that the system was described by finite-difference equations. Therefore, the plant and variational equations were integrated to obtain the necessary numerical values required by the estimator.

An algebraic solution of the reactor equations was derived to obtain the estimated system state after a step disturbance in reactivity. This solution was compared to the solution obtained by iteration to measure the performance of the nonlinear estimator. A digital computer program was used to solve the estimator equations and iterations were performed automatically until the estimator performance index was reduced to a specified value. The performance of the estimator for a nuclear system described by finite-difference equations was investigated with different iteration accuracies. Because of its one-step starting feature and error estimation, the Kutta-Merson algorithm was used to integrate the plant and variational equations. The error estimate was used to automatically adjust the integration step size to meet a specified accuracy requirement. The performance of the estimator using integration was investigated as a function of iteration accuracy and integration accuracy.

Control of a nuclear reactor was investigated by cascading the optimal estimator with the optimal controller. After a reactivity disturbance, the optimal estimator requires two samples to estimate the true state of the plant. After the second sample, the optimal controller drives the neutron density deviation to zero in one sample. If it is assumed that one sample interval is required to perform the estimation and control calculations, then the delayed neutron deviation is driven to zero in one sample after the third measurement is made. A constraint on the control variable was introduced to reduce the magnitude of the control input applied after the second estimate is made. The performance of the cascaded control system with an estimator using integration was investigated as a function of iteration accuracy and integration accuracy. With a small integration step size, system performance with integration is equal to that of the system described by finite-difference equations. The penalty for increased accuracy is an increase in computation time.

8.2 Conclusions

The optimal control law derived for a discrete-time linear system with a quadratic cost function demonstrated that a deviation in neutron density could be reduced to zero in one sample interval. The stationary feedback control law for the reactor was derived by linearizing the reactor equations around the desired nominal values. The plotted responses (Figs. 5.1 - 5.12) are idealistic because the optimal control requires knowledge of the reactivity and delayed neutron precursor density at each sampling instant. From a process standpoint, this is a physical impossibility, because these variables are not measureable and therefore must be estimated.

The nonlinear estimator using iteration works very well for a system described by nonlinear plant and measurement difference equations. If integration is used to estimate the state of a system described by a nonlinear differential equation, the integration step size must be reduced to maintain estimation accuracy; as a consequence, the computation time is increased. For higher-order systems, the combination of iteration and sequential integration can easily result in an estimation time exceeding one second. Integration of a set of simultaneous equations can be more profitably assigned to an analog computer with a factor of ten applied to the problem time scale. Thus, an integration over one sample interval in problem time can be obtained in one-tenth of a sample interval in real time. The number of equations to be integrated will not change the integration time, since all equations are integrated simultaneously. Thus, the nonlinear estimator becomes a hybrid system, with a digital computer solving the estimator difference equations and an analog computer solving the system differential equations.

The cascade combination of an estimator and controller results in a control system whose performance is no longer equal to that of a system without an estimator. Whereas, the linearized reactor equations result in a linear stationary control law which controls the nonlinear system satisfactorily under the assumption that all state variables are measureable, the performance of the cascaded system demonstrates that the estimates generated for the nonlinear system result in a large control input at the first sampling instant after a disturbance. Inclusion of computation time delay results in further degraded performance. A bound on the control variable can be used to limit the control inputs until the estimator establishes the true state of the system. If an integrator is included as part of the nonlinear estimator, the integration step size must be reduced to even smaller values when a control input is present.

The computer programs used to solve the estimator equations and to compute the control input are not compiled for minimum time execution; therefore, no conclusions can be made as to real-time control capability.

8.3 Recommendations for future research

A hybrid computer system should be used to establish feasibility of real-time control. An analog computer should be used to simulate the reactor system, and a digital computer should be used for the estimation and control calculations. The reactor equations should be expanded to include six groups of delayed neutrons. Use of the six-group model will encourage inclusion of an integrator in the estimator, because an analytic description by finite-difference equations will be difficult.

Starting with the one-group model, the regulator problem should be investigated with noise added to the plant and measurement equations. The stochastic system should be expanded to include the six-group model.

The deterministic and stochastic one-group and six-group models should be used to investigate control of demand changes in reactor power level from source range to power operation, with and without reactivity feedback.

At very low power levels, a nuclear reaction is a multiplicative Poisson process. Optimal estimation theory should be applied to the design of a reactivity meter.

The methods of estimation and control applied to the kinetics equations should be expanded to include the primary system, the secondary system, and the turbine-generator system, with automatic start-up, operation, and shutdown.

Optimal control theory should be used to establish ultimate system performance without regard to cost. Since total optimization of the control of a nuclear plant includes the performance of the controller and its cost, an investigation should be made to determine whether a significant savings in equipment cost is possible by accepting slightly less than optimal performance.

APPENDIX A

VECTOR-MATRIX DIFFERENTIAL EQUATIONS

The homogeneous differential equation for a linear time-invariant system is given in vector-matrix form by

$$\dot{\underline{x}}(t) = A\underline{x}(t), \quad \underline{x}(t_0) = \underline{x}_0 \tag{A.1}$$

The solution to Eq. (A.1) is

$$\underline{x}(t) = \Phi(t - t_0)\underline{x}(t_0) \tag{A.2}$$

where the state transition matrix is defined by

$$\Phi(t - t_0) = \exp[A(t - t_0)] \tag{A.3}$$

The matrix exp(At) is defined by the infinite series

$$\exp(At) = I + At + A^2t^2/2! + A^3t^3/3! + \cdots, \tag{A.4}$$

Substitution of Eq. (A.2) into Eq. (A.1) yields

$$\dot{\Phi}(t - t_0) = A\Phi(t - t_0) \tag{A.5}$$

Use of Eq. (A.4) in Eq. (A.5) verifies that Eq. (A.2) is a solution of Eq. (A.1). Note that when $t = t_0$,

$$\Phi(0) = I \tag{A.6}$$

and the boundary conditions of Eq. (A.2) are satisfied.

The state transition matrix $\Phi(t)$ can be calculated by using Eq. (A.4)

$$\Phi(t) = I + At + A^2t^2/2! + A^3t^3/3! + \cdots \tag{A.7}$$

or by taking the Laplace transform of both sides of Eq. (A.1) to obtain

$$s\underline{X}(s) - \underline{x}(0) = A\underline{X}(s) \tag{A.8}$$

Rearrangement of Eq. (A.8) leads to

$$\underline{X}(s) = [sI - A]^{-1}\underline{x}(0) \tag{A.9}$$

which alternately can be written as

$$\underline{X}(s) = \Phi(s)\underline{x}(0) \tag{A.10}$$

where $\Phi(s)$, the resolvent of matrix A, is given by

$$\Phi(s) = [sI - A]^{-1} \tag{A.11}$$

The state transition matrix $\Phi(t)$ is obtained by taking the inverse Laplace transform of both sides of Eq. (A.11) which can be expressed:

$$\Phi(t) = \mathcal{L}^{-1}[sI - A]^{-1} \tag{A.12}$$

The solution to the nonhomogeneous equation

$$\dot{\underline{x}}(t) = A\underline{x}(t) + Bu(t) \tag{A.13}$$

is obtained by first taking the Laplace transform of both sides to obtain

$$s\underline{X}(s) - \underline{x}(0) = A\underline{X}(s) + BU(s) \tag{A.14}$$

rearranging

$$\underline{X}(s) = [sI - A]^{-1}\underline{x}(0) + [sI - A]^{-1}BU(s) \tag{A.15}$$

and then taking the inverse Laplace transform of both sides with the result that

$$\underline{x}(t) = \Phi(t)\underline{x}(0) + \int_0^t \Phi(t - \tau)Bu(\tau)d\tau \tag{A.16}$$

where the convolution theorem is used to obtain the integral term. If the initial time is given as t_0 instead of zero, then

$$\underline{x}(t) = \Phi(t - t_0)\underline{x}(t_0) + \int_{t_0}^{t} \Phi(t - \tau)Bu(\tau)d\tau \tag{A.17}$$

For a discrete-time input u_k where

$$u(t) = u_k \qquad kT < t \leq (k+1)T \tag{A.18}$$

Eq. (A.17) is written

$$\underline{x}(t) = \phi(t - t_k)\underline{x}_k + u_k \int_{t_k}^{t} \phi(t - \tau)Bd\tau \tag{A.19}$$

or

$$\underline{x}(t) = \Phi(t - t_k)\underline{x}_k + u_k \int_{0}^{t-t_k} \Phi(\tau)Bd\tau \tag{A.20}$$

The integral term of Eq. (A.20) can be evaluated by integrating Eq. (A.7) from zero to T:

$$\int_0^T \Phi(\tau)d\tau = IT + AT^2/2 + A^2T^3/3! + \cdots \tag{A.21}$$

Multiplication by A of both sides of Eq. (A.21) yields

$$A\int_0^T \Phi(\tau)d\tau = AT + A^2T^2/2 + A^3T^3/3! + \cdots \tag{A.22}$$

The unit matrix can be added to both sides of Eq. (A.22) as follows:

$$I + A\int_0^T \Phi(\tau)d\tau = I + AT + A^2T^2/2 + A^3T^3/3! + \cdots \tag{A.23}$$

but the right hand side of Eq. (A.23) is $\Phi(T)$. Therefore,

$$I + A\int_0^T \Phi(\tau)d\tau = \Phi(T) \qquad (A.24)$$

which can be rewritten

$$\int_0^{t-t_k} \Phi(\tau)d\tau = A^{-1}[\Phi(t - t_k) - I] \qquad (A.25)$$

if A^{-1} exists. Substitution of Eq. (A.25) into Eq. (A.20) yields

$$\underline{x}(t) = \Phi(t - t_k)\underline{x}_k + A^{-1}[\Phi(t - t_k) - I]Bu_k \qquad (A.26)$$

and at $t = (k+1)T$

$$\underline{x}_{k+1} = \Phi(T)\underline{x}_k + A^{-1}[\Phi(T) - I]Bu_k \qquad (A.27)$$

The homogeneous matrix differential equation of a time-varying linear system is

$$\dot{\underline{x}}(t) = A(t)\underline{x}(t), \quad \underline{x}(t_0) = x_0 \qquad (A.28)$$

Any solution of Eq. (A.28) is given by

$$\underline{x}(t) = \Phi(t, t_0)\underline{x}(t_0) \qquad (A.29)$$

This is verified by substituting Eq. (A.29) into Eq. (A.28) with the result that

$$\dot{\Phi}(t, t_0) = A(t)\Phi(t, t_0) \qquad (A.30)$$

and

$$\dot{\underline{x}}(t) = \frac{d}{dt}[\Phi(t, t_0)\underline{x}(t_0)]$$

$$= A(t)\Phi(t, t_0)\underline{x}(t_0)$$

$$= A(t)\underline{x}(t) \qquad (A.31)$$

Also

$$\Phi(t_0, t_0) = I \tag{A.32}$$

and the boundary conditions are satisfied. Integration of Eq. (A.28) yields

$$\underline{x}(t) = \underline{x}(t_0) + \int_{t_0}^{t} A(\tau)\underline{x}(\tau)d\tau \tag{A.33}$$

which can be solved by repeated substitution of the right side into the integral for $\underline{x}$. The first substitution yields

$$\underline{x}(t) = \underline{x}(t_0) + \int_{t_0}^{t} A(\tau)[\underline{x}(t_0) + \int_{t_0}^{\tau} A(\nu)\underline{x}(\nu)d\nu]d\tau \tag{A.34}$$

Define the operator

$$Q(\ ) = \int_{t_0}^{t} (\ )d\tau \tag{A.35}$$

which leads to the following series as a solution of Eq. (A.22):

$$\underline{x}(t) = [I + Q(A) + Q(AQ(A)) + Q(AQ(AQ(A))) + \cdots]\underline{x}(t_0) \tag{A.36}$$

Comparison of Eq. (A.36) with Eq. (A.29) shows that the state transition matrix for a time-varying system is given by:

$$\Phi(t, t_0) = I + Q(A) + Q(AQ(A)) + Q(AQ(AQ(A))) + \cdots \tag{A.37}$$

If A is constant matrix, then

$$\Phi(t, t_0) = I + A(t - t_0) + A^2(t - t_0)^2/2! + A^3(t - t_0)^3/3! + \cdots \tag{A.38}$$

which is the same as Eq. (A.7) with the argument replaced by $t - t_0$.

Assume that the solution of the nonhomogeneous differential equation

$$\dot{\underline{x}}(t) = A(t)\underline{x}(t) + B(t)u(t), \quad \underline{x}(t_0) = \underline{x}_0 \tag{A.39}$$

is given by

$$\underline{x}(t) = \Phi(t, t_0)\underline{y}(t) \tag{A.40}$$

Then

$$\dot{\underline{x}}(t) = \Phi(t, t_0)\dot{\underline{y}}(t) + \dot{\Phi}(t, t_0)\underline{y}(t) \tag{A.41}$$

and Eq. (A.30) is substituted into Eq. (A.41) to eliminate $\dot{\Phi}$. Thus

$$\dot{\underline{x}}(t) = \Phi(t, t_0)\dot{\underline{y}}(t) + A(t)\Phi(t, t_0)\underline{y}(t) \tag{A.42}$$

Substitution of Eq. (A.39) into Eq. (A.38) results in

$$\dot{\underline{x}}(t) = A(t)\Phi(t, t_0)\underline{y}(t) + B(t)u(t) \tag{A.43}$$

which on comparison with Eq. (A.42) results in

$$\Phi(t, t_0)\dot{\underline{y}}(t) = B(t)u(t) \tag{A.44}$$

and y(t) is obtained by integration. Thus

$$\underline{y}(t) = \underline{y}(t_0) + \int_{t_0}^{t} \Phi^{-1}(\tau, t_0)B(\tau)u(\tau)d\tau \tag{A.45}$$

At $t = t_0$, Eqs. (A.32) and (A.40) result in

$$\underline{y}(t_0) = \underline{x}(t_0) \tag{A.46}$$

Equation (A.40) is solved for $\underline{y}(t)$ and substituted with Eq. (A.46) into Eq. (A.45) to yield

$$\Phi^{-1}(t, t_0)\underline{x}(t) = \underline{x}(t_0) + \int_{t_0}^{t} \Phi^{-1}(\tau, t_0)B(\tau)u(\tau)d\tau \tag{A.47}$$

The solution for $\underline{x}(t)$ is

$$\underline{x}(t) = \Phi(t, t_0)\underline{x}(t_0) + \Phi(t, t_0)\int_{t_0}^{t} \Phi^{-1}(\tau, t_0)B(\tau)u(\tau)d\tau \tag{A.48}$$

Using the properties of the state transition matrix

$$\Phi^{-1}(\tau,\ t_0) = \Phi(t_0,\ \tau) \tag{A.49}$$

and

$$\Phi(t,\ t_0)\Phi(t_0,\ \tau) = \Phi(t,\ \tau) \tag{A.50}$$

Eq. (A.48) can be written

$$\underline{x}(t) = \Phi(t,\ t_0)\underline{x}(t_0) + \int_{t_0}^{t} \Phi(t,\tau)B(\tau)u(\tau)d\tau \tag{A.51}$$

For the discrete-time input defined by Eq. (A.18), Eq. (A.51) becomes

$$\underline{x}(t_{k+1}) = \Phi(t_{k+1},\ t_k)\underline{x}(t_k) + u_k \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},\tau)B(\tau)d\tau \tag{A.52}$$

which can be written

$$\underline{x}_{k+1} = \Phi(k+1,\ k)\underline{x}_k + u_k \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},\tau)B(\tau)d\tau \tag{A.53}$$

APPENDIX B

OPTIMAL CONTROL LAW FOR A DISCRETE-TIME LINEAR SYSTEM WITH A QUADRATIC PERFORMANCE INDEX

For a discrete-time linear system described by

$$\underline{x}_{k+1} = \Phi\underline{x}_k + Gu_k \tag{B.1}$$

and a quadratic performance index of the form

$$I_N = \sum_{k=1}^{N} (\underline{x}_k^T Q\underline{x}_k + cu_{k-1}^2) \tag{B.2}$$

the optimal control law can be found by the method of dynamic programming.

There is a sequence: $u_0, u_1, u_2, \cdots, u_{N-1}$ which will make I_N a minimum. Let the minimum value of I_N be denoted by

$$f_N[\underline{x}(0)] = \min_{\substack{u_0\\u_1\\\cdots\\u_{N-1}}} \sum_{k=1}^{N} [\underline{x}_k^T Q\underline{x}_k + cu_{k-1}^2] \tag{B.3}$$

For the last N-j stages of an N-stage process

$$f_{N-j}[\underline{x}(j)] = \min_{\substack{u_j\\u_{j+1}\\\cdots\\u_{N-1}}} \sum_{k=j+1}^{N} [\underline{x}_k^T Q\underline{x}_k + cu_{k-1}^2] \tag{B.4}$$

The principle of optimality [132, p. 57] may be used to interpret the selection of $u_0, u_1, u_2, \cdots, u_{N-1}$ as a sequence of decision processes. The principle of optimality states: "An optimal policy has the property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

Then, by the principle of optimality, Eq. (B.4) reduces to

$$f_{N-j}(\underline{x}_j) = \min_{u_j} [\underline{x}_{j+1}^T Q \underline{x}_{j+1} + cu_j^2 + f_{N-(j+1)}(\underline{x}_{j+1})] \quad (B.5)$$

Starting with j = 0,

$$f_N(\underline{x}_0) = \min_{u_0} [\underline{x}_1^T Q \underline{x}_1 + cu_0^2 + f_{N-1}(\underline{x}_1)] \quad (B.6)$$

$$f_{N-1}(\underline{x}_1) = \min_{u_1} [\underline{x}_2^T Q \underline{x}_2 + cu_1^2 + f_{N-2}(\underline{x}_2)] \quad (B.7)$$

$$\vdots$$

$$f_1(\underline{x}_{N-1}) = \min_{u_{N-1}} [\underline{x}_N^T Q \underline{x}_N + cu_{N-1}^2 + f_0(x_N)] \quad (B.8)$$

Define

$$f_0(\underline{x}_N) = 0 \quad (B.9)$$

Since the functional f is quadratic in $\underline{x}$, both f_{N-j} and $f_{N-(j+1)}$ can be expressed in quadratic forms. Let

$$f_{N-j}(\underline{x}_j) = \underline{x}_j^T P_{N-j} \underline{x}_j \quad (B.10)$$

and

$$f_{N-(j+1)}(\underline{x}_{j+1}) = \underline{x}_{j+1}^T P_{N-(j+1)} \underline{x}_{j+1} \quad (B.11)$$

where the P matrices are n×n and symmetrical.

On substitution of Eq. (B.11) into Eq. (B.5)

$$f_{N-j}(\underline{x}_j) = \min_{u_j} [\underline{x}_{j+1}^T Q \underline{x}_{j+1} + cu_j^2 + \underline{x}_{j+1}^T P_{N-(j+1)} \underline{x}_{j+1}] \quad (B.12)$$

Define

$$S_{N-(j+1)} = Q + P_{N-(j+1)} \tag{B.13}$$

Then

$$f_{N-j}(\underline{x}_j) = \min_{u_j} [\underline{x}_{j+1}^T S_{N-(j+1)} x_{j+1} + cu_j^2] \tag{B.14}$$

but x_{j+1} is a function of u_j. Then, after substitution of Eq. (B.1), Eq. (B.14) becomes

$$f_{N-j}(\underline{x}_j) = \min_{u_j} [(\Phi\underline{x}_j + Gu_j)^T S_{N-(j+1)} (\Phi\underline{x}_j + Gu_j) + cu_j^2] \tag{B.15}$$

The minimum of Eq. (B.15) may be found by taking the derivative with respect to u_j and equating the result to zero. Thus

$$2[\Phi\underline{x}_j + Gu_j]^T S_{N-(j+1)} G + 2cu_j = 0 \tag{B.16}$$

which can be expanded to give

$$\underline{x}_j^T \Phi S_{N-(j+1)} G + G^T S_{N-(j+1)} Gu_j + cu_j = 0 \tag{B.17}$$

Taking the transpose of Eq. (B.17) and solving for u_j results in

$$u_j = - \frac{G^T S_{N-(j+1)} \Phi}{G^T S_{N-(j+1)} G + c} \underline{x}_j \tag{B.18}$$

which may be expressed in linear form by

$$u_j = B_{N-j} \underline{x}_j \tag{B.19}$$

where

$$B_{N-j} = - \frac{G^T S_{N-(j+1)} \Phi}{G^T S_{N-(j+1)} G + c} \tag{B.20}$$

or

$$B_{N-j} = \frac{G^T[Q + P_{N-(j+1)}]\Phi}{G^T[Q + P_{N-(j+1)}]G + c} \tag{B.21}$$

The recurrence relationship for the P matrices is obtained by substituting Eqs. (B.10), (B.13), and (B.19) into Eq. (B.15) to obtain

$$\underline{x}_j^T P_{N-j} \underline{x}_j = \underline{x}_j^T (\Phi + GB_{N-j})^T (Q + P_{N-(j+1)})(\Phi + GB_{N-j})\underline{x}_j$$
$$+ c\underline{x}_j^T B_{N-j}^T B_{N-j} \underline{x}_j \tag{B.22}$$

Comparing both sides of Eq. (B.22) leads to

$$P_{N-j} = (\Phi + GB_{N-j})^T (Q + P_{N-(j+1)})(\Phi + GB_{N-j}) + cB_{N-j}^T B_{N-j} \tag{B.23}$$

Equations (B.21) and (B.23) give the desired recurrence relationship for the B and P matrices. Starting with j = N-1, and $P_N = 0$, the sequence is: B_1, P_1, B_2, P_2, $\cdots$, P_{N-1}, B_N.

When $N \to \infty$ in Eq. (B.2), the control process becomes an infinite stage process, and the feedback control law given by Eq. (B.19) becomes time invariant.

APPENDIX C

SERIES EXPANSION OF DISCRETE-TIME REACTOR EQUATION

Integration of the reactor kinetics equations results in the following discrete-time solution for the normalized delayed neutron precursor density:

$$z_{k+1} = z_k \exp\left[\frac{\lambda}{u_k} \ln \frac{1 - \rho_k}{1 - \rho_k - u_k T} - \lambda T\right] \tag{C.1}$$

which is unsatisfactory for numerical computation as $u_k \to 0$.

Equation (C.1) may be expanded in a Taylor series by defining

$$f(u_k) = \ln \frac{1 - \rho_k}{1 - \rho_k - u_k T} \tag{C.2}$$

Then

$$f'(u_k) = \frac{T}{1 - \rho_k - u_k T} \tag{C.3}$$

$$f''(u_k) = \frac{T^2}{(1 - \rho_k - u_k T)^2} \tag{C.4}$$

$$f'''(u_k) = \frac{2T^3}{(1 - \rho_k - u_k T)^3} \tag{C.5}$$

and

$$f^n(u_k) = \frac{(n - 1)!\ T^n}{(1 - \rho_k - u_k T)^n} \tag{C.6}$$

The Taylor series expansion for Eq. (C.2) is

$$f(u_k) = 0 + \frac{T}{1-\rho_k} u_k + \frac{T^2}{2(1-\rho_k)^2} u_k^2 + \cdots + \frac{T^n}{n(1-\rho_k)^n} u_k^n + \cdots \quad (C.7)$$

Substitution of Eq. (C.7) into Eq. (C.1) results in

$$z_{k+1} = z_k \exp\left[\frac{\lambda T}{1-\rho_k} + \frac{\lambda T^2}{2(1-\rho_k)^2} u_k + \cdots + \frac{\lambda T^n}{n(1-\rho_k)^n} u_k^{n-1} + \cdots - \lambda T\right] \quad (C.8)$$

The first and last terms inside of the bracket may be combined with the result that

$$z_{k+1} = z_k \exp\left[\frac{\lambda T \rho_k}{1-\rho_k} + \frac{\lambda T^2}{2(1-\rho_k)^2} u_k + \cdots + \frac{\lambda T^n}{n(1-\rho_k)^n} u_k^{n-1} + \cdots\right] \quad (C.9)$$

Let

$$x = u_k T/(1 - \rho_k) \quad (C.10)$$

Then Eq. (C.9) may be expressed as follows

$$z_{k+1} = z_k \exp\left[\frac{\lambda T}{1-\rho_k} \left(\rho_k + \frac{x}{2} + \frac{x^2}{3} + \cdots + \frac{x^{n-1}}{n} + \cdots\right)\right] \quad (C.11)$$

When $u_k = 0$, $x = 0$, and Eq. (C.11) reduces to Eq. (4.75).

APPENDIX D

PROOF THAT ONE ERROR TERM OF OPTIMAL ESTIMATOR IS ZERO

For the nonlinear estimator with iteration, the estimate $\underline{x}_{k-1}^{i+1}$ is given by

$$\underline{x}_{k-1}^{i+1} = \underline{x}_{k-1}^{i} + [\underline{H}_\alpha^T\underline{H}_\alpha + F_{k-1}^T H_k^T H_k F_{k-1}]^{-1} \{F_{k-1}^T H_k^T [y_k - h(\underline{x}_k^i) + H_k(\underline{x}_k^i - \underline{f}(\underline{x}_{k-1}^i))] + \underline{H}_\alpha^T \underline{H}_\alpha(\underline{\alpha} - \underline{x}_{k-1}^i)\} \tag{D.1}$$

It can be demonstrated that

$$\underline{H}_\alpha(\underline{\alpha} - \underline{x}_{k-1}^i) = 0 \tag{D.2}$$

throughout the iteration sequence by first rearranging Eq. (D.1) and multiplying both sides to obtain

$$[\underline{H}_\alpha^T\underline{H}_\alpha + F_{k-1}^T H_k^T H_k F_{k-1}]\underline{x}_{k-1}^{i+1} = F_{k-1}^T H_k^T H_k F_{k-1}\underline{x}_{k-1}^i + F_{k-1}^T H_k^T [y_k - h(\underline{x}_k^i) + H_k(\underline{x}_k^i - f(\underline{x}_{k-1}^i))] + \underline{H}_\alpha^T \underline{H}_\alpha \underline{\alpha} \tag{D.3}$$

Then, if the term $F_{k-1}^T H_k^T H_k F_{k-1}\underline{\alpha}$ is added and subtracted to the right side

$$[\underline{H}_\alpha^T\underline{H}_\alpha + F_{k-1}^T H_k^T H_k F_{k-1}]\underline{x}_{k-1}^{i+1} = F_{k-1}^T H_k^T H_k F_{k-1}\underline{\alpha} + \underline{H}_\alpha^T \underline{H}_\alpha \underline{\alpha} + F_{k-1}^T H_k^T [y_k - h(\underline{x}_k^i) + H_k\underline{x}_k^i - H_k f(\underline{x}_{k-1}^i) - H_k F_{k-1}\underline{\alpha} + H_k F_{k-1}\underline{x}_{k-1}^i] \tag{D.4}$$

If β^i is defined

$$\underline{\beta}^i = f(\underline{x}^i_{k-1}) + F_{k-1}[\underline{\alpha} - \underline{x}^i_{k-1}] \qquad (D.5)$$

substituted into Eq. (D.4)

$$[\underline{H}^T_\alpha \underline{H}_\alpha + F^T_{k-1}H^T_k H_k F_{k-1}]\underline{x}^{i+1}_{k-1} = [\underline{H}^T_\alpha \underline{H}_\alpha + F^T_{k-1}H^T_k H_k F_{k-1}]\underline{\alpha} + F^T_{k-1}H^T_k[y_k - h(\underline{x}^i_k) + H_k(\underline{x}^i_k - \underline{\beta}^i)] \qquad (D.6)$$

and both sides are multiplied

$$\underline{x}^{i+1}_{k-1} = \underline{\alpha} + [\underline{H}^T_\alpha \underline{H}_\alpha + F^T_{k-1}H^T_k H_k F_{k-1}]^{-1} F^T_{k-1}H^T_k[y_k - h(\underline{x}^i_k) + H_k(\underline{x}^i_k - \underline{\beta}^i)] \qquad (D.7)$$

If the term $\underline{\alpha}$ is subtracted from both sides of Eq. (D.7) and the resulting equation is multiplied by $\underline{H}_\alpha$

$$\underline{H}_\alpha(\underline{x}^{i+1}_{k-1} - \underline{\alpha}) = \underline{H}_\alpha[\underline{H}^T_\alpha \underline{H}_\alpha + F^T_{k-1}H^T_k H_k F_{k-1}]^{-1} F^T_{k-1}H^T_k[y_k - h(\underline{x}^i_k) + H_k(\underline{x}^i_k - \underline{\beta}^i)] \qquad (D.8)$$

If the left side of Eq. (D.8) is equal to a zero column vector and the error terms in the bracket on the right side are not zero, then

$$\underline{H}_\alpha[\underline{H}^T_\alpha \underline{H}_\alpha + F^T_{k-1}H^T_k H_k F_{k-1}]^{-1} F^T_{k-1}H^T_k = \underline{0} \qquad (D.9)$$

The equality of Eq. (D.9) can be demonstrated by using the matrix inversion

$$\underline{x}_{k-1} = \begin{bmatrix} \underline{H}_\alpha \\ \hline H_k F_{k-1} \end{bmatrix}^{-1} \begin{bmatrix} \underline{y}_\alpha \\ \hline y_k \end{bmatrix} \qquad (D.10)$$

or

$$\underline{x}_{k-1} = M^{-1}[\underline{H}^T_\alpha \mid F^T_{k-1}H^T_k] \begin{bmatrix} \underline{y}_\alpha \\ \hline y_k \end{bmatrix} \qquad (D.11)$$

where

$$M = \underline{H}^T_\alpha \underline{H}_\alpha + F^T_{k-1}H^T_k H_k F_{k-1} \qquad (D.12)$$

Multiplying both sides of Eq. (D.11) by the composite H matrix yields

$$\begin{bmatrix} \underline{H}_\alpha \\ \hline F_{k-1}H_k \end{bmatrix} \underline{x}_{k-1} = \begin{bmatrix} \underline{H}_\alpha \\ \hline F_{k-1}H_k \end{bmatrix} M^{-1} [\underline{H}_\alpha^T \mid F_{k-1}^T H_k^T] \begin{bmatrix} \underline{y}_\alpha \\ \hline y_k \end{bmatrix} \tag{D.13}$$

The left side is equal to the measurement vector, therefore Eq. (D.13) can be written

$$\begin{bmatrix} \underline{y}_\alpha \\ \hline y_k \end{bmatrix} = \left[\begin{array}{c|c} \underline{H}_\alpha M^{-1} \underline{H}_\alpha^T & \underline{H}_\alpha M^{-1} F_{k-1}^T H_k^T \\ \hline F_{k-1} H_k M^{-1} \underline{H}_\alpha^T & F_{k-1} H_k M^{-1} F_{k-1}^T H_k^T \end{array}\right] \begin{bmatrix} \underline{y}_\alpha \\ \hline y_k \end{bmatrix} \tag{D.14}$$

Since $\underline{y}_\alpha$ and y_k are independent, the partioned matrix of Eq. (D.14) is an n×n unit matrix, and

$$\underline{H}_\alpha M^{-1} \underline{H}_\alpha^T = I \qquad (n-1 \times n-1) \tag{D.15}$$

$$\underline{H}_\alpha M^{-1} F_{k-1}^T H_k^T = \underline{0} \qquad (n-1 \times 1) \tag{D.16}$$

$$F_{k-1} H_k M^{-1} \underline{H}_\alpha^T = \underline{0} \qquad (1 \times n-1) \tag{D.17}$$

$$F_{k-1} H_k M^{-1} F_{k-1}^T H_k^T = 1 \tag{D.18}$$

Equation (D.9) is verified by Eq. (D.16).

Since the iteration sequence is started with $\underline{x}_{k-1}^1 = \underline{\alpha}$, the first error correction term contributed by $\underline{H}_\alpha(\underline{\alpha} - \underline{x}_{k-1}^1)$ is zero, and all subsequent values are zero. Therefore, the term $\underline{H}_\alpha(\underline{\alpha} - \underline{x}_{k-1}^i)$ may be omitted from the estimator.

APPENDIX E

DIFFERENCE SOLUTIONS BY INTEGRATION

Although numerical integration is used in the digital computer calculation of the reactor state estimates, the reactor equations can be integrated analytically to demonstrate that F_{k-1} obtained by integration is equal to F_{k-1} obtained by differentiation of the plant difference equation. The following equations are integrated simultaneously from zero to t:

$$\dot{z}(t) = \lambda z(t)\rho(t)/[1 - \rho(t)] \qquad z(0) = z_{k-1} \tag{E.1}$$

$$\dot{\rho}(t) = u_{k-1} \qquad \rho(0) = \rho_{k-1} \tag{E.2}$$

$$\dot{\phi}_{11}(t) = \lambda\rho(t)\phi_{11}(t)/[1 - \rho(t)] \qquad \phi_{11}(0) = 1 \tag{E.3}$$

$$\dot{\phi}_{12}(t) = \lambda\rho(t)\phi_{11}(t)/[1 - \rho(t)] + \lambda z(t)/[1 - \rho(t)]^2 \qquad \phi_{12}(0) = 0 \tag{E.4}$$

First, Eq. (E.2) is integrated to obtain

$$\rho(t) = \rho_{k-1} + u_{k-1}t \tag{E.5}$$

which is substituted into Eq. (E.1), yielding

$$z(t) = z_{k-1}\exp\left[\frac{\lambda}{u_{k-1}}\,\ell n\left(\frac{1-\rho_{k-1}}{1-\rho_{k-1}-u_{k-1}t}\right) - \lambda t\right] \tag{E.6}$$

Next Eq. (E.5) is substituted into Eq. (E.3) and integrated

$$\phi_{11}(t) = \exp\left[\frac{\lambda}{u_{k-1}} \ln\left(\frac{1-\rho_{k-1}}{1-\rho_{k-1}-u_{k-1}t}\right) - \lambda t\right] \tag{E.7}$$

Finally, Eqs. (E.5), (E.6), and (E.7) are substituted into Eq. (E.4) and integrated, with the result

$$\phi_{12}(t) = \frac{\lambda t z_{k-1}}{(1-\rho_{k-1})(1-\rho_{k-1}-u_{k-1}t)} \exp\left[\frac{\lambda}{u_{k-1}} \ln\left(\frac{1-\rho_{k-1}}{1-\rho_{k-1}-u_{k-1}t}\right) - \lambda t\right] \tag{E.8}$$

At t = T, Eqs. (E.7) and (E.8) agree with matrix Eq. (7.12), which is obtained by differentiating the plant finite-difference equations.

APPENDIX F

KUTTA-MERSON INTEGRATION ALGORITHM

Merson [126] proposed an integration method which does not require a special starting feature and which can be used with automatic interval adjustment. The Kutta-Merson process uses the equations

$$y_1 = y_0 + \frac{1}{3}hf(x_0, y_0) \tag{F.1}$$

$$y_2 = y_0 + \frac{1}{6}hf(x_0, y_0) + \frac{1}{6}hf(x_0 + \frac{1}{3}h, y_1) \tag{F.2}$$

$$y_3 = y_0 + \frac{1}{8}hf(x_0, y_0) + \frac{3}{8}hf(x_0 + \frac{1}{3}h, y_2) \tag{F.3}$$

$$y_4 = y_0 + \frac{1}{2}hf(x_0, y_0) - \frac{3}{2}hf(x_0 + \frac{1}{3}h, y_2) + 2hf(x_0 + \frac{1}{2}h, y_3) \tag{F.4}$$

$$y_5 = y_0 + \frac{1}{6}hf(x_0, y_0) + \frac{2}{3}hf(x_0 + \frac{1}{2}h, y_3) + \frac{1}{6}hf(x_0 + h, y_4) \tag{F.5}$$

Merson showed that the error in y_4 is $-h^5y^{(v)}/120$, and in y_5 is $-h^5y^{(v)}/720$; and that a good estimate of the error in the computed y_5 is $0.2(y_4 - y_5)$.

Automatic interval adjustment is accomplished by specifying the integration accuracy ε_2 and adjusting h. If

$$|0.2(y_4 - y_5)| > \varepsilon_2 \tag{F.6}$$

h is halved. If

$$64\ |0.2(y_4 - y_5)| < \varepsilon_2 \tag{F.7}$$

then h is doubled.

The advantage of the Kutta-Merson method is that it facilitates rapid interval selection for exploratory calculations requiring specified accuracy; however it does require additional computation time in comparison to other methods.

APPENDIX G

DIGITAL COMPUTER PROGRAMS

1. ROOTS OF PROMPT JUMP EQUATION

```
      DIMENSION S(7), B(7), A(7) , D(7),ST(7)
    1 FORMAT (1H1)
    2 FORMAT (F12.8)
    3 FORMAT (9X, 1HB, 16X, 1HS)
    4 FORMAT (E14.8, 4X, E14.8)
    5 FORMAT (1H )
      A(1) = 0.038
      A(2) = 0.213
      A(3) = 0.188
      A(4) = 0.407
      A(5) = 0.128
      A(6) = 0.026
      D(1) = 0.0127
      D(2) = 0.0317
      D(3) = 0.115
      D(4) = 0.311
      D(5) = 1.40
      D(6) = 3.87
      S(1) = 0.01
      S(2) = -0.014
      S(3)= -0.065
      S(4) = -0.19
      S(5) = -1.25
      S(6) = -3.75
      S(7) = -65000.0
      ALPHA = 65000.0
      X = 1.0
      RHO = 0.1
      ERR = 1.0E-7
   10 DO 15 I = 1,7
      ST(I) = 0.0
   11 SUM 1 = 0.0
      SUM 2 = 0.0
      DO 12 J = 1,6
      SUM1 = SUM1 + A(J) *(S(I)/(S(I)+D(J)))**2
      SUM2 = SUM2 + A(J)*D(J)/(S(I)+D(J))**2
   12 CONTINUE
      S(I) = (RHO - SUM1)/(SUM2 + X/ALPHA)
      DIFF = S(I) - ST(I)
      ST(I) = S(I)
      WRITE TYPE 2, S(I)
      IF (DIFF) 16, 17, 17
   16 DIFF = - DIFF
   17 IF(DIFF - ERR) 13, 13, 11
   13 SUM 3 = 0.0
      DO 14 K = 1,6
      SUM3 = SUM3 + A(K)/(S(I) + D(K))
   14 CONTINUE
```

```
      B(I) =(SUM3 + X/ALPHA)/(SUM2 + X/ALPHA)
      WRITE TYPE 5
   15 CONTINUE
      PRINT 1
      PRINT 3
      PRINT 4, (B(I), S(I), I=1,7)
      PUNCH 4, (B(I), S(I), I=1,7)
      END
```

2. REACTOR RESPONSE TO STEP DELTA K

```
      DIMENSION S(7), B(7)
    1 FORMAT (1H1)
    2 FORMAT (2F16.8)
    3 FORMAT (6X, 4HTIME, 12X, 4HFLUX, 12X, 6HFLUX 1,/)
      READ 2, (B(I), S(I), I=1,7)
      PRINT 1
      PRINT 3
      DELT = 0.0001
      DO 11 N = 1,101
      T = (N - 1) * DELT
      FLUX = 0.0
      DO 10 I = 1,7
   10 FLUX = FLUX + B(I) * EXPF(S(I)*T)
      PRINT 2, T, FLUX
   11 CONTINUE
      END
```

3. CALCULATION OF FEEDBACK MATRIX

```
      DIMENSION PHI(7,7),H(7),Q(7,7),P(7,7),S(7,7),HTSPHI(7),
     1B(7),PSI(7,7),PSIPSI(7,7),BTB(7,7)
   10 FORMAT (F16.8)
   21 FORMAT (4F16.8)
   11 FORMAT (I1)
      READ 11, N
      READ 10,  ALAMBD,T
   20 READ 10, A, C
   12 FORMAT (1H1//40X51HCALCULATION OF FEEDBACK MATRIX WITH
     1CONTROL PENALTY)
   13 FORMAT (1H0,9X,6HLAMBDA,20X,1HT,23X,1HA,23X,1HC,23X,1HN)
      PRINT 12
      PRINT 13
   14 FORMAT (1H ,4(4X,F16.8,4X),11X,I1)
      PRINT 14, ALAMBD,T,A,C,N
   15 FORMAT (1H0,24X,3HPHI,32X,1HH,33X,1HQ)
      PRINT 15
   16 FORMAT (1H0,10X,2F16.8,10X,F16,8,10X,2F16,8)
   17 FORMAT (1H0,14X,1HS,29X,1HB,28X,3HPSI,28X,1HP)
   18 FORMAT (1H0,1X,3(2F14.8,2X),2F14.8)
   19 FORMAT (1H0,1X,2F14.8,32X,2F14,8,2X,2F14.8)
      PHI(1,1) = 1.
      PHI(1,2) = T * ALAMBD
      PHI(2,1) = 0.
      PHI(2,2) = 1.
      H(1) = 0.5 * T ** 2 * ALAMBD
      H(2) = T
      Q(1,1) = 1,
      Q(1,2) = 1.
      Q(2,1) = 1.
      Q(2,2) = 1, + A
      PRINT 16, PHI(1,1),PHI(1,2),H(1),Q(1,1),Q(1,2)
      PRINT 16, PHI(2,1),PHI(2,2),H(2),Q(2,1),Q(2,2)
      PRINT 17
      DO 100 I = 1,N
      DO 100 J = 1,N
  100 P(I,J) = 0.
      B1TEMP = 0.0
      B2TEMP = 0,0
   99 DO 101 I = 1,N
      DO 101 J = 1,N
  101 S(I,J) = Q(I,J) + P(I,J)
      HTSH = 0.
      DO 102 I = 1,N
      DO 102 J = 1,N
  102 HTSH = HTSH + H(I) * S(I,J) * H(J)
      DEN = HTSH + C
      DO 105 K = 1,N
```

```
      HTSPHI (K) = 0.
      DO 104 I =1,N
      DO 104 J = 1,N
  104 HTSPHI (K) = HTSPHI (K) + H(I) * S(I,J) * PHI(J,K)
  105 B(K) = -HTSPHI(K)/DEN
      DO 106 I = 1,N
      DO 106 J = 1,N
  106 PSI(I,J) = PHI(I,J) + H(I) * B(J)
      DO 107 I = 1,N
      DO 107 J = 1,N
      PSIPSI(I,J) = 0.
      DO 107 K = 1,N
      DO 107 L = 1,N
  107 PSIPSI(I,J) = PSIPSI(I,J) + PSI(K,I) * S(K,L) * PSI(L,J)
      DO 108 I = 1,N
      DO 108 J = 1,N
  108 BTB(I,J) = B(I) * B(J)
      DO 109 I = 1,N
      DO 109 J = 1,N
  109 P(I,J) = PSIPSI(I,J) + C * BTB(I,J)
      DIFF1 = B1TEMP - B(1)
      IF (DIFF1) 30,31,31
   30 DIFF1 = -DIFF1
   31 IF(DIFF1 - 0.0000001) 33,33,32
   32 B1TEMP = B(1)
      GO TO 99
   33 DIFF2 = B2TEMP - B(2)
      IF (DIFF2) 34,35,35
   34 DIFF2 = -DIFF2
   35 IF(DIFF2 - 0.0000001) 37,37,36
   36 B2TEMP = B(2)
      GO TO 99
   37 PRINT 18, S(1,1),S(1,2),B(1),B(2),PSI(1,1),PSI(1,2),
     1P(1,1),P(1,2)
      PRINT 19, S(2,1),S(2,2),PSI(2,1),PSI(2,2),P(2,1),P(2,2)
      PUNCH 21, B(1), B(2), A, C
      GO TO 20
  111 END
```

4. CALCULATION OF TRANSIENT RESPONSE

```
10 FORMAT (1H1,28X,18HTRANSIENT RESPONSE)
11 FORMAT (1H0,20X,2HB1,27X,2HB2)
12 FORMAT (1H ,14X,F14.8,15X,F14.8,59X,F4,0)
13 FORMAT (F14.8)
14 FORMAT (1H0,3X,1HN,14X,1HU,17X,3HRHO,17X,1HZ,16X,4HFLUX,
  116X,2HPI/)
15 FORMAT (1H ,I4,4(3X,F16.8))
16 FORMAT (I5,4F14,8)
17 FORMAT (I5,F14.8)
18 FORMAT (I5)
19 FORMAT (4F16.8)
25 FORMAT (84X,F16,8)
   READ 18, M
   READ 13, T
   READ 13, ALAMBD
   READ 13, RHO0
   READ 13, Z0
20 READ 13, RUNNO
   READ 19, B1, B2, A,C
   PRINT 10
   PRINT 11
   PRINT 12, B1, B2, RUNNO
   PRINT 14
   N = 0
   PI = 0.
   FLUX = Z0/(1.0 - RHO0)
   U = B1 * (Z0 - 1.0) + B2 * RHO0
   RHO = RHO0
   DELTAN = FLUX - 1.0
   DELTAZ = Z0 - 1,0
   PRINT 15, N, U, RHO, DELTAZ, DELTAN
   PUNCH 16, N, U, RHO, DELTAZ, DELTAN
   ZK = Z0
   RHOK = RHO0
21 DO 22 K = 1,10
   X = U*K*T/(1.0-RHOK)
   SER = (ALAMBD*K*T/(1.0-RHOK))*(RHOK+X*(1,0/2,0+X*(1.0/3.0
  1+X*(1.0/4.0+X*(1.0/5.0+X*(1.0/6.0+X*(1,0/7.0+X*(1.0/8,0+
  2X*(1.0/9.0+X/10,0))))))))
   Z = ZK*EXPF(SER)
   RHO = RHOK + U*K*T
   FLUX = Z/(1.0-RHO)
   DELTAN = FLUX - 1.0
   DELTAZ = Z - 1.0
   N = N+1
   PRINT 15, N, U, RHO, DELTAZ, DELTAN
   PUNCH 16, N, U, RHO, DELTAZ, DELTAN
22 CONTINUE
```

```
      ZK = Z
      RHOK = RHO
      PI = PI + (FLUX - 1.0)**2 + A*RHO**2 + C*U**2
      U = B1*(ZK-1.0)+B2*RHOK
      PUNCH 17,N,U
      PRINT 25, PI
      IF (N-M) 21,23,23
   23 IF(SENSE SWITCH 1) 24,20
   24 PAUSE 1
      GO TO 20
      END
```

5. PLOT PROGRAM FOR TRANSIENT RESPONSE

```
      DIMENSION N(111),U(111),RHO(111),Z(111),FLUX(111),A(111)
    1 FORMAT (F6.0)
    2 FORMAT (5F16.8)
    3 FORMAT (I5,4F14.8)
    6 FORMAT (1H0, 5F16.8,4X,F6.0)
      SN = 20.0
      S = 0.2
      SU = S
      SRHO = S
      SZ = S
      SFLUX = S
    9 READ 1, RUN NO
   12 READ 3, (N(I),U(I),RHO(I),Z(I),FLUX(I),I=1,111)
      IRUNNO = RUN NO
      DIGIT1 = IRUNNO/10
      DIGIT2 = RUN NO - DIGIT1 * 10.
      DO 27 J = 1,3
      X = PLOTF (2.0,2.0,1)
      X = PLOTF (0.0,0.0,2)
      X = PLOTF (0.0,0.02,3)
      X = PLOTF (DIGIT1,0.0,4)
      X = PLOTF (10.0,0.0,3)
      X = PLOTF(0.0,0.0,2)
      X = PLOTF(DIGIT2,0.0,4)
      X = PLOTF (0.0,-11.0,3)
      X = PLOTF (0.0,0.0,2)
      X = PLOTF (1.0,1.0,1)
      X = PLOTF (0.0,2.5,3)
      X = PLOTF (0.0,-2.5,4)
      X = PLOTF (0.0,2.0,3)
      X = PLOTF (0.12,2.0,4)
      X = PLOTF (0.0,2.0,3)
      X = PLOTF (0.0,0.0,3)
      X = PLOTF (5.0,0.0,4)
      X = PLOTF (0.0,0.0,3)
      X = PLOTF (0.0,-2.0,3)
      X = PLOTF (0.12,-2.0,4)
      X = PLOTF (0.0,-2.0,3)
      X = PLOTF (0.0,-2.5,3)
      DO 34 I = 1,9
      T = 0.5 * I
      X = PLOTF (T,-2.5,4)
      IF(I - 5) 31,32,31
   31 X = PLOTF (T,-2.44,4)
      GO TO 33
   32 X = PLOTF (T,-2.38,4)
   33 X = PLOTF (T,-2.5,3)
   34 CONTINUE
```

```
      X = PLOTF (5.0,-2.5,4)
      X = PLOTF (5.0,-2.0,4)
      X = PLOTF (4.88,-2.0,4)
      X = PLOTF (5.0,-2.0,3)
      X = PLOTF (5.0,2.0,4)
      X = PLOTF (4.88,2.0,4)
      X = PLOTF (5.0,2.0,3)
      X = PLOTF (5.0,2.5,4)
      DO 38 I = 1,9
      T = 5.0 - 0.5 * I
      X = PLOTF (T,2.5,4)
      IF(I - 5) 35,36,35
   35 X = PLOTF (T,2.44,4)
      GO TO 37
   36 X = PLOTF (T,2.38,4)
   37 X = PLOTF (T,2.5,3)
   38 CONTINUE
      X = PLOTF (0.0,2.5,4)
      X = PLOTF (0.0,0.0,3)
      DO 14 I = 1,111
   14 A(I)=N(I)
      A(1) = 0.3
      X = PLOTF (SN,SU,1)
      X = PLOTF (0.0,0.0,2)
      X = PLOTF(A(1),U(1),3)
   13 DO 15 I=2,110
      X=PLOTF(A(I),U(I),4)
   15 CONTINUE
      X = PLOTF(0.0,0.0,3)
      X = PLOTF(SN,SRHO,1)
      X = PLOTF(0.0,0.0,2)
      K = 0
   18 I1 = 2 + 11*K
      I2 = 9 + I1
   16 DO 17 I = I1,I2
      X = PLOTF(A(I),RHO(I),3)
   17 CONTINUE
      K = K + 1
      IF (K - 10) 18,19,19
   19 REF = 0.0
      X = PLOTF(0.0,0.0,3)
      X = PLOTF(SN,SZ,1)
   20 X = PLOTF (0.0,REF,2)
      X = PLOTF(A(1),Z(1),3)
      K = 0
   21 I1 = 2 + 11*K
      I2 = 9 + I1
      DO 22 I = I1,I2
      X = PLOTF (A(I),Z(I),3)
   22 CONTINUE
      K = K + 1
      IF (K - 10) 21,23,23
   23 X = PLOTF (0.0,REF,3)
      X=PLOTF(SN,SFLUX,1)
```

```
      X = PLOTF(0.0,REF,2)
      X = PLOTF (A(1), FLUX(1),3)
      K = 0
   24 I1 = 2 + 11*K
      I2 = 9 + I1
      DO 25 I = I1,I2
      X = PLOTF(A(I),FLUX(I),3)
   25 CONTINUE
      K = K + 1
      IF(K - 10) 24,26,26
   26 X=PLOTF(0.0,REF,3)
      X = PLOTF(1.0,1.0,1)
      X = PLOTF(0.0,0.0,2)
      X = PLOTF (-5.0,17.5,3)
   27 CONTINUE
      GO TO 9
      END
```

6. ANALYTIC ESTIMATOR SOLUTIONS

```
 1 FORMAT (6F16.8)
 2 FORMAT (1H1)
   D = 0.31
   PRINT 2
   DO 10 N = 1,200
   RHO = -1.01 + 0.01 * N
   Z = EXPF(D * RHO/(1.0 - RHO))
   Y = Z/(1.0 - RHO)
   RHO1 = LOGF(Y)/(D+LOGF(Y))
   Z1 = 1.0 - RHO1
   Z2 = Z1 * EXPF(D*RHO1/(1.0 - RHO1))
10 PRINT 1, RHO, Z, Y, RHO1, Z1, Z2
   END
```

7. FINITE DIFFERENCE SYSTEM WITH ESTIMATOR AND CONTROL

```
      DIMENSION ALPHA(2),           X1(4), X2(4),  HTH1(2,2),
     1F(2,2), H2(2), HTH2(2,2), C2INV(2,2),
     2X1ERR(2),X2ERR(2), XBAR(4),H1(2), C2(2,2),
     3H2F(2),HFTHF(2,2),H2FTYE(2),HTH1DX(2),VECTOR(2),DELTX1(2)
     4,X1TEMP(2),X1DIFF(2),FX1DIF(2), Y2(4)
   10 FORMAT(1H1,28X,18HTRANSIENT RESPONSE,30X,4HB1 =,F14.8,5X,
     14HB2 =, F14.8)
   11 FORMAT (1H )
   12 FORMAT (5X,4(3X,F16.8),27X,I3)
   13 FORMAT (F14.8)
   14 FORMAT (1H0,3X,1HN,14X,1HU,17X,3HRHO,17X,1HZ,16X,4HFLUX/)
   15 FORMAT (1H ,I4,4(3X,F16.8))
   16 FORMAT (I5,4F14,8)
   17 FORMAT (I5,F14.8)
   18 FORMAT (I5)
   19 FORMAT (4F16.8)
      READ 18, M
      READ 13, T
      READ 13, ALAMBD
      READ 13, Z0
      READ 19, B1, B2, A,C
   20 READ 13, RHO0
      PRINT 10, B1, B2
      PRINT 11
      PRINT 14
      EPS1 = 1.0E-6
      N = 0
      PI = 0.
      FLUX = Z0/(1.0 - RHO0)
      U = 0.0
      UT = 0.0
      UL = 1.0
      YK = FLUX
      D = ALAMBD
      ALPHA(1) = 1.0
      ALPHA(2) = 0.0
      PRINT 15, N, RHO0, Z0, FLUX
      DELTAZ = Z0 - 1.0
      DELTAN = FLUX - 1.0
      PUNCH 16, N, U, RHO0, DELTAZ, DELTAN
      RHOK = RHO0
      ZK = Z0
      DELT = 0.1
      H1(1) = 1.0
      H1(2) = 1.0
      HTH1(1,1) = 1.0
      HTH1(1,2) = 1.0
      HTH1(2,1) = 1.0
```

```
      HTH1(2,2) = 1.0
   21 DO 22 K = 1,10
      IF (U) 25,24,25
   24 Z = ZK * EXPF((ALAMBD * RHOK * K * 0.1 * T)/(1.0 - RHOK))
      GO TO 26
   25 Z=ZK*EXPF((ALAMBD/U)*LOGF((1.0-RHOK)/(1.0-RHOK-U*K*0.1*T)
     1)-ALAMBD*K*0.1*T)
   26 RHO = RHOK + U*K*0.1*T
      FLUX = Z/(1.0-RHO)
      N = N+1
      PRINT 15, N, RHO, Z, FLUX
      DELTAZ = Z - 1.0
      DELTAN = FLUX - 1.0
      PUNCH 16, N, U, RHO, DELTAZ, DELTAN
   22 CONTINUE
      ZK = Z
      RHOK = RHO
      YK = FLUX
      L = 1
      X1(1) = ALPHA(1)
      X1(2) = ALPHA(2)
      IF (U) 28,23,28
   23 EXPO = EXPF(D*T*X1(2)/(1.0 - X1(2)))
      X2(1) = X1(1) * EXPO
      X2(2) = X1(2)
      GO TO 27
   28 EXPC = EXPF((D/U)*LOGF((1.0-X1(2))/(1.0-X1(2)-U*T))-D*T)
      X2(1) = X1(1) * EXPC
      X2(2) = X1(2) + U * T
      GO TO 29
   27 EXPO = EXPF(D*T*X1(2)/(1.0 - X1(2)))
      XBAR(1) = X1(1) * EXPO
      XBAR(2) = X1(2)
      F(1,1) = EXPO
      F(1,2) = (D*T*X1(1)/(1.0 - X1(2))**2)*EXPO
      GO TO 30
   29 EXPC = EXPF((D/U)*LOGF((1.0-X1(2))/(1.0-X1(2)-U*T))-D*T)
      XBAR(1) = X1(1) * EXPC
      XBAR(2) = X1(2) + U * T
      F(1,1) = EXPC
      F(1,2) = D*T*X1(1)*EXPC/((1.0-X1(2))*(1.0-X1(2)-U*T))
   30 F(2,1) = 0.0
      F(2,2) = 1.0
      H2(1) = 1.0/(1.0 - X2(2))
      H2(2) = X2(1)/(1.0 - X2(2))**2
      DO 31 I = 1,2
      DO 31 J = 1,2
   31 HTH2(I,J) = H2(I) * H2(J)
      DO 32 I = 1,2
      H2F(I) = 0.0
      DO 32 J = 1,2
   32 H2F(I) = H2F(I) + H2(J) * F(J,I)
      DO 33 I=1,2
      DO 33 J = 1,2
```

```
33 HFTHF(I,J) = H2F(I) * H2F(J)
   DO 34 I = 1,2
   DO 34 J = 1,2
34 C2INV(I,J) = HTH1(I,J) + HFTHF(I,J)
   DEN2 = C2INV(1,1)*C2INV(2,2) - C2INV(1,2)*C2INV(2,1)
   C2(1,1) = C2INV(2,2)/DEN2
   C2(1,2) = -C2INV(1,2)/DEN2
   C2(2,1) = -C2INV(2,1)/DEN2
   C2(2,2) = C2INV(1,1)/DEN2
   DO 35 I = 1,2
35 X1ERR(I) = ALPHA(I) - X1(I)
   HX1ERR = 0.0
   DO 36 I = 1,2
36 HX1ERR = HX1ERR + H1(I) * X1ERR(I)
   PI1 = HX1ERR**2
   DO 37 I = 1,2
37 X2ERR(I) = X2(I) - XBAR(I)
   YH = X2(1)/ (1.0 - X2(2))
   YERR1 = YK - YH
   PI2 = YERR1 **2
   HX2ERR = 0.0
   DO 38 I = 1,2
38 HX2ERR = HX2ERR + H2(I) * X2ERR(I)
   YERR = YERR1 + HX2ERR
   DO 39, I = 1,2
39 H2FTYE(I) = H2F(I) * YERR
   DO 40 I = 1,2
40 HTH1DX(I) = H1(I) * HX1ERR
   DO 41 I = 1,2
41 VECTOR (I) = H2FTYE(I) + HTH1DX(I)
   DO 42 I = 1,2
   DELTX1(I) = 0.0
   DO 42 J = 1,2
42 DELTX1(I) = DELTX1(I) + C2(I,J) * VECTOR(J)
   DO 43, I = 1,2
43 X1TEMP(I) = X1(I)
   DO 44, I = 1,2
44 X1(I) = X1(I) + DELTX1(I)
   IF(X1(2) - 0.8) 46,46,45
45 X1(1) = X1TEMP(1) + DELTX1(1) * (0.8 - U*T - X1TEMP(2))/
  1DELTX1(2)
   X1(2) = 0.8 - U * T
46 CONTINUE
   DO 47, I = 1,2
47 X1DIFF(I) = X1(I) - X1TEMP(I)
   DO 48 I = 1,2
   FX1DIF(I) = 0.0
   DO 48 J = 1,2
48 FX1DIF(I) = FX1DIF(I) + F(I,J) * X1DIFF(J)
   DO 49 I = 1,2
49 X2(I) = XBAR(I) + FX1DIF(I)
   PI = PI1 + PI2
   IF (PI - EPS1  ) 51,51,50
50 L = L + 1
```

```
      IF (U) 29,27,29
   51 YH = X2(1)/ (1.0 - X2(2))
      U  = B1 * (X2(1) - 1.0) + B2 * X2(2)
      PRINT 12, U,   X2(2), X2(1), YH, L
      PUNCH 17,N,U
      DO 56 I = 1,2
      DO 56 J = 1,2
   56 HTH1(I,J) = HTH2(I,J)
      DO 57 I = 1,2
   57 ALPHA(I) = X2(I)
      DO 58 I = 1,2
      H1(I) = H2(I)
   58 X1(I) = X2(I)
      IF (U) 61,60,61
   60 EXPO = EXPF(D*T*X1(2) /(1.0 - X1(2)))
      X2(1) = X1(1) * EXPO
      X2(2) = X1(2)
      GO TO 62
   61 EXPO = EXPF((D/U)*LOGF((1.0-X1(2))/(1.0-X1(2)-U*T))-D*T)
      X2(1) = X1(1) * EXPO
      X2(2) = X1(2) + U * T
   62 UT = B1 * (X2(1) - 1.0) + B2 * X2(2)
      IF(UT - UL) 53,52,52
   52 UT = UL
      GO TO 55
   53 IF(UT + UL) 54,55,55
   54 UT = -UL
   55 IF (N-M) 21,59,59
   59 CONTINUE
      GO TO 20
      END
```

8. DIFFERENTIAL SYSTEM WITH ESTIMATOR AND CONTROL

```
      COMMON U, H, EPS2
      DIMENSION ALPHA(2),           X1(4), X2(4),  HTH1(2,2),
     1F(2,2), H2(2), HTH2(2,2), C2INV(2,2),
     2X1ERR(2),X2ERR(2), XBAR(4),H1(2), C2(2,2),
     3H2F(2),HFTHF(2,2),H2FTYE(2),HTH1DX(2),VECTOR(2),DELTX1(2)
     4,X1TEMP(2),X1DIFF(2),FX1DIF(2), Y2(4)
   10 FORMAT(1H1,28X,18HTRANSIENT RESPONSE,30X,4HB1 =,F14.8,5X,
     14HB2 =, F14.8)
   11 FORMAT (1H )
   12 FORMAT (5X,4(3X,F16.8),27X,I3)
   13 FORMAT (F14.8)
   14 FORMAT (1H0,3X,1HN,14X,1HU,17X,3HRHO,17X,1HZ,16X,4HFLUX/)
   15 FORMAT (1H ,I4,4(3X,F16.8))
   16 FORMAT (I5,4F14,8)
   17 FORMAT (I5,F14.8)
   18 FORMAT (I5)
   19 FORMAT (4F16.8)
      READ 18, M
      READ 13, T
      READ 13, ALAMBD
      READ 13, Z0
      READ 19, B1, B2, A,C
   20 READ 13, RHO0
      PRINT 10, B1, B2
      PRINT 11
      PRINT 14
      EPS1 = 1.0E-6
      EPS2 = 1.0E-4
      N = 0
      PI = 0.
      FLUX = Z0/(1.0 - RHO0)
      U = 0.0
      UT = 0.0
      UL = 1.0
      YK = FLUX
      D = ALAMBD
      ALPHA(1) = 1.0
      ALPHA(2) = 0.0
      PRINT 15, N, U, RHO0, Z0, FLUX
      DELTAZ = Z0 - 1.0
      DELTAN = FLUX - 1.0
      PUNCH 16, N, U, RHO0, DELTAZ, DELTAN
      RHOK = RHO0
      ZK = Z0
      DELT = 0.1
      H1(1) = 1.0
      H1(2) = 1.0
      HTH1(1,1) = 1.0
```

```
      HTH1(1,2) = 1.0
      HTH1(2,1) = 1.0
      HTH1(2,2) = 1.0
   21 DO 22 K = 1,10
      IF (U) 25,24,25
   24 Z = ZK * EXPF((ALAMBD * RHOK * K * 0.1 * T)/(1.0 - RHOK))
      GO TO 26
   25 Z=ZK*EXPF((ALAMBD/U)*LOGF((1.0-RHOK)/(1.0-RHOK-U*K*0.1*T)
     1)-ALAMBD*K*0.1*T)
   26 RHO = RHOK + U*K*0.1*T
      FLUX = Z/(1.0-RHO)
      N = N+1
      PRINT 15,N,U,RHO,Z,FLUX
      DELTAZ = Z - 1.0
      DELTAN = FLUX - 1.0
      PUNCH 16, N, U, RHO, DELTAZ, DELTAN
   22 CONTINUE
      WRITE TYPE 11
      ZK = Z
      RHOK = RHO
      YK = FLUX
      L = 1
      X1(1) = ALPHA(1)
      X1(2) = ALPHA(2)
      X1(3) = 1.0
      X1(4) = 0.0
      CALL INTEGR (X1, X2)
   29 X1(3) = 1.0
      X1(4) = 0.0
      CALL INTEGR (X1, XBAR)
      F(1,1) = XBAR(3)
      F(1,2) = XBAR(4)
   30 F(2,1) = 0.0
      F(2,2) = 1.0
      H2(1) = 1.0/(1.0 - X2(2))
      H2(2) = X2(1)/(1.0 - X2(2))**2
      DO 31 I = 1,2
      DO 31 J = 1,2
   31 HTH2(I,J) = H2(I) * H2(J)
      DO 32 I = 1,2
      H2F(I) = 0.0
      DO 32 J = 1,2
   32 H2F(I) = H2F(I) + H2(J) * F(J,I)
      DO 33 I=1,2
      DO 33 J = 1,2
   33 HFTHF(I,J) = H2F(I) * H2F(J)
      DO 34 I = 1,2
      DO 34 J = 1,2
   34 C2INV(I,J) = HTH1(I,J) + HFTHF(I,J)
      DEN2 = C2INV(1,1)*C2INV(2,2) - C2INV(1,2)*C2INV(2,1)
      C2(1,1) = C2INV(2,2)/DEN2
      C2(1,2) = -C2INV(1,2)/DEN2
      C2(2,1) = -C2INV(2,1)/DEN2
      C2(2,2) = C2INV(1,1)/DEN2
```

```
      DO 35 I = 1,2
   35 X1ERR(I) = ALPHA(I) - X1(I)
      HX1ERR = 0.0
      DO 36 I = 1,2
   36 HX1ERR = HX1ERR + H1(I) * X1ERR(I)
      PI1 = HX1ERR**2
      DO 37 I = 1,2
   37 X2ERR(I) = X2(I) - XBAR(I)
      YH = X2(1)/ (1.0 - X2(2))
      YERR1 = YK - YH
      PI2 = YERR1 **2
      HX2ERR = 0.0
      DO 38 I = 1,2
   38 HX2ERR = HX2ERR + H2(I) * X2ERR(I)
      YERR = YERR1 + HX2ERR
      DO 39, I = 1,2
   39 H2FTYE(I) = H2F(I) * YERR
      DO 40 I = 1,2
   40 HTH1DX(I) = H1(I) * HX1ERR
      DO 41 I = 1,2
   41 VECTOR (I) = H2FTYE(I) + HTH1DX(I)
      DO 42 I = 1,2
      DELTX1(I) = 0.0
      DO 42 J = 1,2
   42 DELTX1(I) = DELTX1(I) + C2(I,J) * VECTOR(J)
      DO 43, I = 1,2
   43 X1TEMP(I) = X1(I)
      DO 44, I = 1,2
   44 X1(I) = X1(I) + DELTX1(I)
      IF(X1(2) - 0.8) 46,46,45
   45 X1(1) = X1TEMP(1) + DELTX1(1) * (0.8 - U*T - X1TEMP(2))/
     1DELTX1(2)
      X1(2) = 0.8 - U * T
   46 CONTINUE
      DO 47, I = 1,2
   47 X1DIFF(I) = X1(I) - X1TEMP(I)
      DO 48 I = 1,2
      FX1DIF(I) = 0.0
      DO 48 J = 1,2
   48 FX1DIF(I) = FX1DIF(I) + F(I,J) * X1DIFF(J)
      DO 49 I = 1,2
   49 X2(I) = XBAR(I) + FX1DIF(I)
      PI = PI1 + PI2
      IF (PI - EPS1  ) 51,51,50
   50 L = L + 1
      WRITE TYPE 18, L
      GO TO 29
   51 YH = X2(1)/ (1.0 - X2(2))
      U = UT
      PRINT 12, U, X2(2), X2(1), YH, L
      PUNCH 17,N,U
      DO 56 I = 1,2
      DO 56 J = 1,2
   56 HTH1(I,J) = HTH2(I,J)
```

```
      DO 57 I = 1,2
   57 ALPHA(I) = X2(I)
      DO 58 I = 1,2
      H1(I) = H2(I)
   58 X1(I) = X2(I)
      CALL INTEGR(X1, X2)
   62 UT = B1 * (X2(1) - 1.0) + B2 * X2(2)
      IF(UT - UL) 53,52,52
   52 UT = UL
      GO TO 55
   53 IF(UT + UL) 54,55,55
   54 UT = -UL
   55 IF (N-M) 21,59,59
   59 GO TO 20
      END

      SUBROUTINE INTEGR (X1, Y2)
      COMMON U, H, EPS2
      DIMENSION Y0(4), Y1(4), Y2(4), F0(4), F1(4), F2(4),
     1ERROR(4), F(4), Y(4), X1(4)
    1 FORMAT (F14.8)
      N = 4
      H = 1.0
      DO 30 I=1,N
   30 Y0(I) = X1(I)
   39  LOC = 0
      MLOC = 1
   38 HA = .33333333*H
      HB = .16666667*H
      HC = .125*H
      HD = .375*H
      HE = .5*H
      HF = 1.5*H
      HG = 2.*H
      HH = .66666667*H
      WRITE TYPE 1, H
   48  CALL FCT (Y0, F0)
      DO 41 I=1,N
   41  Y1(I) = Y0(I) + HA * F0(I)
      CALL FCT (Y1, F1)
      DO 42 I=1,N
   42  Y1(I) = Y0(I) + HB*F0(I) + HB*F1(I)
      CALL FCT (Y1, F2)
      DO 44 I=1,N
   44  Y1(I) = Y0(I) + HE*F0(I) - HF*F1(I) + HG*F2(I)
      CALL FCT (Y1, F1)
      DO 45 I=1,N
   45 Y2(I) = Y0(I) + HB*F0(I) + HH*F2(I) + HB*F1(I)
      DO 34 I=1,N
      ERROR(I) = .2 * ABSF(Y1(I) - Y2(I))
      IF (EPS2- ERROR(I)) 35, 34, 34
   34 CONTINUE
      DO 32 I=1,N
   32 Y0(I) = Y2(I)
```

```
      LOC = LOC + 1
   33 IF (LOC - MLOC) 37, 99, 99
8  37 IF (LOC * 1) 48, 47, 48
   47 IF (MLOC - 2) 48, 49, 49
   49 DO 31 I=1,N
      IF (EPS2- ERROR(I) * 64.) 48, 48, 31
   31 CONTINUE
   24 H = HG
      LOC = LOC / 2
      MLOC = MLOC / 2
      GO TO 38
   35 H = HE
      MLOC = MLOC * 2
      LOC = LOC * 2
      GO TO 38
   99 RETURN
      END

      SUBROUTINE FCT (Y, F)
      COMMON U
      DIMENSION Y(4), F(4)
      F(1) = 0.31  * Y(1) * Y(2) /(1.0 - Y(2))
      F(2) = U
      F(3) = 0.31  * Y(3) * Y(2)/(1.0 - Y(2))
      F(4) = 0.31 *(Y(2)*Y(4)+Y(1)/(1.0-Y(2)))/(1.0-Y(2))
      RETURN
      END
```

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