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The following were employed by and/or worked or cooperated on this project during the indicated reporting period.

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During this period, the Ph.D. degree in Applied Mathematics was awarded to D. Hobson, R. McLachlan, A. Morlet and T. Pham.

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Summary of work in progress and completed.

1. Vortex Dynamics and Turbulence.

The effort here has continued in a number of fundamental directions.

We have continued with our work on the Hamiltonian formulation of two dimensional vorticity distributions. Equations were developed by A. Rouhi describing the evolution of the moments of vortex patches. It was found that there is a closed system of lower-order moments when the shapes are elliptical, and that this provides an explanation of the exact solutions for elliptical vortices, discovered by Kirchhoff and generalised by Moore & Saffman and extended by Kida. These exact solutions are now of interest to planetary physicists and have been proposed as a description of the Jupiter red spot and similar structures. New methods are currently under investigation to study reentrant vortex patches (by Hamiltonian methods) in collaboration with Dr. R. MacKay.

The Hamiltonian structure of vortex patches suggests that methods of symplectic integration be employed. In collaboration with Dr. D. Pullin, an investigation into the accuracy of symplectic integrators was carried out by studying the particular configuration of four vortices. It was demonstrated that symplectic integrators are superior to conventional methods of integration in that they conserve many of the Poincare invariants and are more accurate at long times. We are working on the problem of constructing symplectic integrators for non-canonical systems like vortex patches.

Specific problems in two and three dimensional vortex motion are under study. Work is continuing on the properties of vortical states, in particular the existence of vortical states in plane Couette flow remains a topic of fundamental interest in connection with the problem of transition to turbulence. A new approach has been devised to deal with the difficulty that vortical states in plane Poiseuille flow (and also Poiseuille flow in a circular pipe) are the result of a bifurcation from infinity. A related study on the question of inviscid vortex breakdown is in progress. We are testing the hypothesis that axisymmetric breakdown can be identified with the non-existence of laminar flows without stagnation points or reverse flow.

The relation between vortex dynamics and transition to turbulence is a continuing field of study. Dr. Stephen Cowley carried out work while visiting our group on the nature of transition in boundary layers. He continued investigations on the use of Lagrangian variables for unsteady boundary layer separation, and the properties of large amplitude Tollmien-Schlichting waves which are believed to play an important role in the mechanism of transition to turbulence.

Two-dimensional turbulence at large Reynolds numbers has been studied by many workers, following the seminal work of Onsager, using the ideas and concepts of Statistical Mechanics. Recent work by Klimontovich and Haken has suggested that new ideas and methods in irreversible thermodynamics, employing a generalization of the concepts of entropy and temperature, may lead to an understanding of the properties of the turbulent flow and in particular the intermittent structure of the flows. We have started an investigation of these aspects and the first results are promising. We have verified that Klimontovich's prediction that increasing disorder is associated with a decrease of 'temperature' is consistent with calculated velocity fields.

2. Fingers and bubbles in Hele-Shaw cells and unbounded fluid.

In collaboration with Professors S. Tanveer and D.W. Moore, we have continued studies of interfaces at small to moderate Reynolds numbers. In particular, continuing studies of the motion, shape and stability of fingers and bubbles in Hele-Shaw cells are in progress. Attention has concentrated on the mathematics of the selection mechanism (which proves to be an example of the so-called expansions beyond all orders), and the physics of the boundary conditions at a retreating interface. The properties of large bubbles rising in unbounded fluid are being considered. Of interest here is the phenomenon of spherical cap and skirt formation.

3. Vortex Reconnection

Work is continuing on the numerical and theoretical analysis of the phenomenon of vortex reconnection from the point of view of its possible relation to the formation of flow singularities in the Euler equation. Using spectral methods over both uniform and stretched meshes, numerical simulations have been performed of two nearly parallel, counter-rotating vortex tubes over a range of Reynolds numbers. Substantial deformation of the vortex cores is observed and it is this process which seems to retard the stretching of the vortex tubes and apparently prevents the formation of a singularity (even in the absence of viscosity). While stretching does occur and the axial rate of strain (which drives the vorticity) does increase initially, it appears that insufficient axial strain is produced to create an unbounded vorticity in a finite time.

We have also shown that a model of reconnection proposed by Prof. P. G. Saffman explains qualitatively some of the behavior observed. For example, the model appears to capture the saturation of the axial strain as the Reynolds number increases. In addition, exact solutions have been found for this model. Using these solutions it has been shown that the time scale predicted by the model for reconnection (defined by a halving of the maximum vorticity) varies logarithmically with increasing Reynolds number. Comparisons with the results of our high resolution simulations show however, that this appears to be an overestimate and that the variation of the reconnection time increases very slowly with increasing Reynolds number. Further work is now in progress to investigate this discrepancy.

4. Pattern selection in solidifying systems

Work continues on several topics in the numerical and analytical study of dendritic growth. We have now extended the two dimensional implementation of our numerical method for simulation of solidification to those cases where the diffusion constants are unequal in the solid and liquid phases. Our numerical method tracks explicitly the solid-liquid interface while retaining the memory effects due to diffusion. This method can therefore now be applied to the dynamics of directional solidification with both constitutional and thermal effects included.

In collaboration with Dr. Kirk Brattkus we have applied this technique to the study of rapid solidification in which the solidifying sample is pulled from the melt at such high velocities that it becomes necessary to model the effects of nonequilibrium kinetics in addition to equilibrium effects due to surface tension. In this regime an intriguing instability is observed in which the advance of the interface is oscillatory. This has the effect of producing a banded structure in the solid sample. Using our numerical algorithm it has become possible to verify recently proposed asymptotic solutions of Davis and Merchant and to track these solutions well into the nonlinear regime.

5. Richtmyer-Meshkov Instability

Work has now been completed on a numerical study of Richtmyer-Meshkov instability in stratified fluid layers. The initial acceleration and subsequent instability of the fluid layer are induced by means of an impulsive pressure distribution. In collaboration with Thu Pham, who has recently received his thesis, it has been shown that such an initial condition is an adequate approximation of the effect of a weak shock impinging on a stratified fluid. The subsequent dynamics are then calculated numerically using the incompressible equations of motion. For single scale initial conditions, it has been found that the classical growth rates first calculated by Richtmyer are recovered only in the limit of a very thin interface. The effect of the finite thickness of the layer is shown to lead to significantly decreased growth rates.

For multi-scale initial disturbances the numerical solutions imply the possible existence of scaling behavior in the thickness of the layer as a function of time. The scaling exponent, however, appears to be very different from the value derived by Barenblatt on the basis of energy conservation and homogeneity of the ensuing turbulence. It is seen, however, for the calculations that the stratification leads to the formation of inhomogeneous structures in the layer (plumes) and it is these structures which determine the growth of the layer. Further investigation of the dynamics of these vortical plumes is currently in progress.

6. Wavy-Taylor Vortex Flows

The multigrid continuation code to compute wavy Taylor vortex flows has been modified to enable it to be suitable for automatic parallelization. Specifically the PCN procedures of Chandi and Taylor has been applied so that the main part of the code now runs on any of several multiprocessors. To accomplish this the alternating direction smoother had to be replaced by a more local procedure. We are presently using SOR with a multi-color ordering in modifications devised by David Harrar. The convergence is of course much slower than the previous version. To improve matters we have devised a new smoothing or iteration scheme which is a segment alternating direction method. More precisely we use ideas from Domain Decomposition to distribute the data into coordinate volumes and then use alternating directions in each volume. Testing of these new ideas will begin shortly. If they are successful we will employ a large multiprocessor to extend our previous calculations. The complete parallelization of the bulk of the code has been planned and should be easily accomplished when we are content with the iterative solver and smoothing procedures. Our goal is to be able to run production on the INTEL Δ -machine as soon as it is operable.

7. High Reynolds Number Laminar Flows

The calculations of flows over flat plates have been extended to much higher Reynolds numbers. A crucial step in this direction was to insure that accurate solutions of the linearized equations were obtained during the multigrid iteration cycles. Less well understood are improvements that result from increased accuracy in the difference scheme adjacent to the solid boundaries. With these improvements a clear agreement with the triple deck theory at the trailing edge is obtained. A general multigrid code for high Reynolds number laminar flows is being planned as a result of these successful calculations.

8. Numerical Analysis and Dynamical Systems

Past work in the theory of numerical methods for solving nonlinear two point boundary value problems has been found to be applicable to some deep open questions in the theory of dynamical systems. In particular Hilbert's sixteenth problem, which asks for a bound on the number of limit cycles for a plane polynomial system as a power of the degree, has been attacked using numerical analysis techniques. To relate the continuous and discrete versions of the problem a rather strong and surprising equivalence result has been obtained. Extensions of this work should enable us to use the results of calculations to insure the existence of relevant solutions to the continuous problem. Although a complete resolution of Hilbert's sixteenth problem is not expected from these methods they can rule out the occurrence of chaos in various numerical attempts to solve various problems in dynamics. This is a new effort in our program and it could grow significantly.

The following papers have been published, submitted for publication or prepared for publication during this period.

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Shelley, M.J., Meiron, D. I. and Orszag, S. A. Dynamical aspects of vortex reconnection, Journal of Fluid Mechanics (submitted).

Tanveer, S. & Saffman, P.G. Prediction of bubble velocity in a Hele-Shaw cell: thin film and contact angle effects. Phys Fluids A1 (1989) 219-223.

Theses titles

Dana Hobson: *Point vortex models for modon dynamics.*

Robert McLachlan: *Separated Viscous Flows via Multigrid.*

A. Morlet, *I. Numerical Experiments for the Computation of Invariant Curves in Dynamical Systems.*
II. Numerical Convergence Results for a One Dimensional Stefan Problem.

Thu Pham, *Numerical studies of incompressible Richtmyer-Meshkov instability in a stratified fluid.*

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