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THEORY FOR LASER-INDUCED BREAKDOWN OVER A VAPORIZING TARGET SURFACE

MASTER

Robert J. Harrach

December 1977

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CONTENTS

Abstract	1
Introduction	1
The Breakdown Model	1
Basic Rate Equations	1
Specification of Coefficients	3
Initial State of the Blowoff Vapor	4
Normalization of Coefficients and Variables	5
Numerical and Approximate Analytical Solutions	10
Comparison with Experiment and Conclusions	17
Notes and References	18
Appendix: List of Symbols	20

THEORY FOR LASER-INDUCED BREAKDOWN OVER A VAPORIZING TARGET SURFACE

ABSTRACT

In this report, a simple but accurate theoretical model is developed to describe the growth of avalanche ionization (breakdown) in the blowoff vapor from a laser-irradiated surface. This nonequilibrium model is applicable to laser intensities and wavelengths from $\sim 10^7$ to $10^{12} \mu\text{m}^2 \cdot \text{W}/\text{cm}^2$ for $\lambda_L^2 \cdot \mathcal{I}_0$, the square of the laser wavelength times its peak intensity. The model pertains only to the early stages of laser-target interaction, from the time surface vaporization begins to when first-degree ionization is achieved in the vapor. Approximate analytical solutions are found for the basic differential equations of the model, and such key quantities as breakdown time and threshold values of laser fluence and intensity are expressed in terms of simple formulas. With the model, we can thus easily predict general trends and specific parametric dependences. The analytical solutions are shown to agree well with accurate numerical solutions of the model's equations and with documented data on CO_2 laser-induced breakdown over aluminum targets.

INTRODUCTION

In this report, we describe a simple but accurate theoretical model for predicting electrical breakdown of the jet or plume of material vaporized from a laser-irradiated surface in a vacuum. The model applies to the early stage of electron avalanche formation, up to the time when a thin layer of plasma develops over the irradiated region. The subsequent processes of hydrodynamic motion, energy transport and exchange, and the laser-plasma interaction are beyond the scope of this model; this post-breakdown stage can be described separately with analytical or computer models.¹ Our model is similar to others describing cascade ionization in vaporized targets, e.g., those of Smith,² Thomas,³ and Barchukov et al.⁴

This early-stage nonequilibrium model includes two Maxwellian temperatures, one characterizing the electron energy distribution and the other the heavy-particle distribution; heavy vapor particles (atoms, ions, molecules) are assumed to be in mutual equilibrium but not in translational equilibrium with the much less-massive electrons.⁵ The laser intensities and wavelengths pertinent to the model range from 10^7 to $10^{12} \mu\text{m}^2 \cdot \text{W}/\text{cm}^2$ for $\lambda_L^2 \cdot \mathcal{I}_0$, where λ_L is the laser wavelength and $\mathcal{I}_0$ is the laser peak intensity. For the reader's convenience, a list of symbols used to describe the breakdown model is provided in the appendix.

Gas breakdown phenomena have been studied in depth, and much information has been published on this subject,^{6,7} especially since development of the laser made possible in 1962 the first observation of optical frequency breakdown.⁸ Compared with more general treatments of the gas breakdown problem, our model is restricted to describing the onset of breakdown at a surface in vacuum by a laser pulse with intensity and wavelength in the range specified above. Our goal is to obtain simple and approximate but useful results.

THE BREAKDOWN MODEL

Basic Rate Equations

Experiments have shown that breakdown over a laser-irradiated surface exhibits a sharp threshold in laser intensity. Below threshold, the gas vaporized from the surface remains largely transparent to the incoming laser radiation, but above threshold, an avalanche ionization (cascade) occurs, producing a shielding plasma cloud over the target. During avalanche ionization, the few naturally-occurring free electrons in the vapor

expelled from the surface are rapidly heated by the laser beam until they have sufficient energy to ionize heavy neutral particles in collisions. In turn, the newly-liberated electrons are heated along with their liberators in the laser field until they become energetic enough to produce further ionization from collisions with heavy particles. In this manner, an exponential growth of the free-electron population occurs, which rapidly increases the degree of vapor ionization from $\sim 10^{-4}$ to a value near 1; we define the degree of ionization as the ratio of the number density of ions to the total number density of heavy particles. As our description suggests, the relative ease with which the electron cascade develops depends on (1), the initial values of the electron temperature T_e and number density n_e in the blowoff vapor, and (2), the extent of imbalance between the rates of laser heating and electron energy loss from collisions with heavy particles.

To develop the basic equations, we first determine the initial values $T_e(0)$ and $n_e(0)$, where the time $t = 0$ is the instant at which the target surface begins to vaporize.⁹ We assume that $T_e(0) = T_v$, the target's boiling or vaporization temperature, and that $n_e(0)$ is determined by a condition of ionization equilibrium. The vapor density, temperature, and pressure are assumed to obey an ideal gas equation of state. The small amount of ionization in such an equilibrium vapor is determined by the Saha equation [Eq. (10)]. We further assume that T_v is related to the pressure exerted on the surface in reaction to material expulsion, as determined by the Clausius-Clapeyron equation [resulting in Eq. (16)]. The early stage of growth in T_e and n_e can be modeled by rate equations that account for only the most prominent interactions. We first consider the average energy density $\mathcal{E}$ for the Maxwell-distributed free electrons:

$$\mathcal{E} = \frac{3}{2} n_e k T_e + n_e k \Theta_1. \quad (1)$$

The first term is the average kinetic energy per cm^3 , while the second represents the potential energy relative to a zero-energy reference state in which the electron is in the atomic ground state. The term $k\Theta_1$ is the required ionization energy, where k is Boltzmann's constant and Θ_1 is in temperature units ($^{\circ}\text{K}$). We differentiate $\mathcal{E}$ with respect to time and set the result equal to source terms representing the rate of laser energy deposition less the rate of electron energy loss in heavy-particle elastic collisions:

$$\frac{d\mathcal{E}}{dt} = \frac{3}{2} n_e k \frac{dT_e}{dt} + \frac{3}{2} k \left(T_e + \frac{2}{3} \Theta_1 \right) \frac{dn_e}{dt} = k_L \mathcal{I} - \frac{3}{2} n_e k (T_e - T_a) \nu_{tr}, \quad (2)$$

where T_a ($^{\circ}\text{K}$) is the heavy-particle temperature, k_L (cm^{-1}) is the incremental absorption coefficient for the laser radiation of intensity $\mathcal{I}$ ($\text{ergs/s} \cdot \text{cm}^2$), and ν_{tr} (s^{-1}) is the effective collision frequency for kinetic energy transfer between electrons and heavy particles.¹⁰ Rearranging terms, the rate equation for the electron temperature is

$$\frac{dT_e}{dt} = \frac{2k_L \mathcal{I}}{3kn_e} - (T_e - T_a) \nu_{tr} - \left(\frac{1}{n_e} \frac{dn_e}{dt} \right) \left(T_e + \frac{2\Theta_1}{3} \right), \quad (3)$$

where the term in dn_e/dt represents energy loss from the electron distribution by inelastic ionizing collisions with heavy particles.¹¹ The rate of change of n_e from these collisions is given by an ionization-recombination reaction equation:

$$\frac{dn_e}{dt} = \alpha_1 n_a n_e - \beta_R n_i n_e^2, \quad (4)$$

where α_1 is the collisional ionization rate coefficient, β_R is the three-body recombination coefficient, and n_a and n_i are respectively, the number densities of heavy neutral particles and ions.¹² Before attempting to solve the coupled first-order differential Eqs. (3) and (4), we need to specify the coefficients k_L , ν_{tr} , α_1 , and β_R , which themselves are functions of the variables T_e and n_e .¹³

Specification of Coefficients

We first consider the laser beam absorption coefficient k_L . The electrons in the weakly ionized blowoff vapor absorb laser energy principally by electron-ion (e-i) and electron-atom (e-a) inverse bremsstrahlung. A free electron cannot absorb electromagnetic radiation except when it encounters an atom or ion, which allows energy and momentum to be simultaneously conserved during the interaction. The interparticle force field is the long-range or screened Coulomb interaction for e-i bremsstrahlung; for e-a, it is a short-range, induced-dipole (or higher-order) electromagnetic force field. The coefficient k_L , which appears in the equation for laser beam attenuation in the form $d\mathcal{I}(x)/dx = -k_L \mathcal{I}(x)$, is the sum of k_{e-i} and k_{e-a} where

$$k_{e-i} = \frac{3.7 \times 10^8}{\sqrt{T_e} \nu_L^3} Z^2 [\exp(\hbar \nu_L / k T_e) - 1] n_e n_i \quad (5)$$

and

$$k_{e-a} = \frac{e^2 \nu_c n_e}{\pi m c \nu_L^2} \quad (6)$$

Here ν_L is the laser frequency in Hz and Z is the ionic charge ($n_i = Z n_D$). The electron-atom collision frequency in Eq. (6) is

$$\nu_c = n_a \frac{v_{e-a} \sigma_c}{v_c} \quad (7)$$

We will treat the cross section σ_c as a given constant and will use for the average relative velocity of electrons and atoms the relation $\bar{v}_{e-a} \approx \sqrt{8kT_e/\pi m}$, where m is the electron mass.

The ionization and recombination coefficients are assumed to be given functions of temperature for different gases of interest. The term α_i generally goes as $\nu_e[(O_i/T_e) + 2] \exp(-O_i/T_e)$, and the relation between β_R and α_i can be found by using the principle of detailed balance. For most target materials of our interest, O_i is large compared with the electron temperature at which breakdown is established; thus we can approximate α_i in the form

$$\alpha_i \approx C_i (kT_e)^{3/2} \exp(-O_i/T_e), \quad (8)$$

where C_i is a constant inferred from experimental data. In equilibrium, $dn_e/dt = 0$; therefore, Eq. (4) gives

$$\beta_R / \alpha_i = \bar{n}_a / \bar{n}_i \bar{n}_e, \quad (9)$$

where the bars denote equilibrium values of the atom, ion, and electron population densities. The right side of Eq. (9) is given by the Saha equation

$$\bar{n}_i \bar{n}_e / \bar{n}_a = (g_i/g_a) 2 (2\pi m k T_e / h^2)^{3/2} \exp(-O_i/T_e), \quad (10)$$

where g_i and g_a are the degeneracies of the ion and atom ground states and h is Planck's constant. By using Eqs. (8) and (10) in Eq. (9), we determine β_R as a function of T_e :

$$\beta_R = C_i [2(2\pi m/h^2)^{3/2} (g_i/g_a) k T_e]^{-1}.$$

Initial State of the Blowoff Vapor

The state of the vapor layer as it emerges from the irradiated surface can be estimated by considering the gasdynamic conservation equations that apply at the target surface before the vapor is hot or dense enough to strongly interact with the laser beam. The laser energy is deposited in a surface layer on the solid, which we consider to be infinitesimally thin. Density, pressure, and particle velocity vary discontinuously across this absorption layer. Denoting the rate that mass is evaporated from the target by $dM/dt = \dot{M}$ (g/s cm^2), the conservation equations for mass, momentum, and energy are

$$\dot{M} \cong \rho_{g,0} u_{g,0} \quad (11)$$

$$p_{s,0} \cong p_{g,0} + \dot{M} u_{g,0} \quad (12)$$

and

$$\dot{M} \left[\mathcal{Q}_v + \frac{1}{2} u_{g,0}^2 \right] \cong \alpha \mathcal{I}_0 \quad (13)$$

The index 0 refers to conditions at the absorption layer, s refers to the solid side (more generally, the condensed phase side), and g to the gas side of this layer. Equation (13) balances the rate that fluence $\alpha \mathcal{I}_0$ (energy/cm²) is coupled through the surface with the rate fluence is consumed in vaporizing the absorption layer and imparting a velocity $u_{g,0}$ to the blowoff vapor. $\mathcal{I}_0$ (erg/s·cm²) is the laser intensity, for simplicity assumed to be a constant, and α is the surface absorptivity, which is equal to one minus the surface reflectivity. The parameter $\mathcal{Q}_v$ is an effective heat of ablation.

In addition to these conservation equations, we assume that the thermodynamic variables of the vapor are related by an ideal gas equation of state:

$$p_{g,0} = \rho_{g,0} \hat{R} T_{g,0} \quad (14)$$

where $\hat{R} = 8.32 \times 10^7$ (ergs/mol°K)/ $\mathcal{A}$ (g/mol) is the ratio of the universal gas constant and the atomic weight of the vapor.

There are six unknown quantities in these four equations (Eqs. 11-14), so two more equations are required. One comes from the Chapman-Jouguet condition, which for our approximation, amounts to setting the gas velocity $u_{g,0}$ equal to the speed of sound in the vapor:

$$u_{g,0} \cong (\gamma p_{g,0} / \rho_{g,0})^{1/2} = (\gamma \hat{R} T_{g,0})^{1/2} \quad (15)$$

where $\gamma = C_p/C_v$, the ratio of specific heats. Another equation comes from the Clausius-Clapeyron relation¹⁴ for the rate of vapor pressure change with temperature; integrating this relation we find that the vaporization (boiling) temperature $T_{g,0}$ is given in terms of the surface pressure $p_{g,0}$ by

$$T_{g,0} = T_v^{\text{ref}} [1 - b \ln(p_{g,0}/p_{\text{ref}})]^{-1} \quad (16)$$

in which T_v^{ref} is the known boiling temperature of the solid at some convenient pressure p^{ref} . The constant b is approximately given by

$$b = \hat{R} T_v^{\text{ref}} / \mathcal{Q}_v \quad (17)$$

Equations (11) through (16) determine the early-time equilibrium values of the thermodynamic variables for a specified target and input laser intensity. A convenient way to present the solutions is to express all the

unknowns in terms of $T_{g,0}$, which is given in terms of $\mathcal{J}_0$ by a transcendental equation. Introducing a dimensionless parameter Δ by the definition

$$T_{g,0} = T_v^{\text{ref}} \Delta, \quad (18)$$

then in terms of Δ , the gas dynamic variables are

$$p_{g,0} = p^{\text{ref}} \exp \left(\frac{\Delta - 1}{b\Delta} \right), \quad (19)$$

$$p_{s,0} = (1 + \gamma)p_{g,0}, \quad (20)$$

$$u_{g,0} = [\gamma \hat{R} T_v^{\text{ref}} \Delta]^{1/2}, \quad (21)$$

$$\dot{M} = (\alpha \mathcal{J}_0 \mathcal{Q}_v) \left(1 + \frac{\gamma \hat{R} T_v^{\text{ref}} \Delta}{2 \mathcal{Q}_v} \right)^{-1}, \quad (22)$$

and

$$\rho_{g,0} = \dot{M}/u_{g,0} = \frac{p^{\text{ref}}}{\hat{R} T_v^{\text{ref}} \Delta} \exp \left(\frac{\Delta - 1}{b\Delta} \right), \quad (23)$$

where Δ is determined by the transcendental equation

$$\left(1 + \frac{\gamma \hat{R} T_v^{\text{ref}} \Delta}{2 \mathcal{Q}_v} \right) \Delta^{-1/2} \exp \left(\frac{\Delta - 1}{b\Delta} \right) = \left(\frac{\alpha \mathcal{J}_0}{\mathcal{Q}_v p^{\text{ref}}} \right) \left(\frac{\hat{R} T_v^{\text{ref}}}{\gamma} \right)^{1/2}. \quad (24)$$

As an example, consider an aluminum-slab target. The boiling temperature T_v^{ref} at a pressure p^{ref} of 10^6 dyn/cm² (1 bar) is = 2750°K. The effective heat of ablation $\mathcal{Q}_v$ can be approximated as the sum of the latent heats of melting and vaporization plus the heat required to raise the target temperature from an ambient value T_{amb} to T_v^{ref} . Thus,

$$\mathcal{Q}_v \cong L_m + L_v + C_p (T_v^{\text{ref}} - T_{\text{amb}}), \quad (25)$$

which for aluminum gives $\mathcal{Q}_v = (0.4 + 11.0 + 2.2) \times 10^{10}$ (ergs/g), or 13.6 (kJ/g). We use the monatomic gas value γ of 5/3, and the gas constant $\hat{R}$ of $8.32 \times 10^7/27$ (erg/g°K) or 0.31 (J/g°K). The Δ -equation [Eq. (24)] for an aluminum target then reads

$$\Delta^{-1/2} (1 + a_1 \Delta) \exp \left[\frac{a_2 (\Delta - 1)}{\Delta} \right] = a_3 \alpha \mathcal{J}_0, \quad (26)$$

with $a_1 = 0.0519$, $a_2 = 16.057$, and $a_3 = 5241.8$ cm²/GW. By specifying values for the absorbed laser intensity $\alpha \mathcal{J}_0$ in GW/cm², we can readily calculate the corresponding Δ -values, and from them the gas parameters from Eqs. (18)-(23). The results are shown in Figs. 1-3. Here, we are interested in the $\mathcal{J}_0$ -dependence of the vapor temperature $T_{g,0}$ and the vapor particle density $n_g(0) = \rho_{g,0} N_A / \mathcal{A}$, where N_A is Avogadro's number and $\mathcal{A}$ is the atomic weight.

Normalization of Coefficients and Variables

It is convenient to normalize all the time- and temperature-dependent quantities to their values at the time $t = 0$ when surface vaporization commences. In the previous section we saw how the initial temperatures

Fig. 1. Relationship between the initial temperature $\Delta = T_{g,0}/T_v^{\text{ref}}$ of the blowoff vapor and the absorbed laser power per unit area αI_0 (left ordinates) and between Δ and the initial density $n_a(0)$ of the vapor (right ordinates) for an Al target. Two different ranges of variables are shown in (a) and (b).

Fig. 2. Dependence of aluminum blowoff vapor density $n_a(0)$ on absorbed laser power per unit area αI_0 . Approximately, $n_a(0)$ goes as $(\alpha I_0)^{0.95}$.

Fig. 3. Dependence of gas dynamic variables on the level of absorbed laser power per unit area, for an aluminum target. $\dot{M}$ = mass vaporization rate, $u_{g,0}$ = velocity of blowoff vapor, and $p_{g,0}$ = pressure in blowoff vapor.

$T_e(0) = T_a(0) = T_{g,0}$ and the vapor density $n_a(0)$ are determined for a specified target material and a given laser intensity. In terms of these, the average electron velocity is initially

$$\overline{v_e(0)} \cong \left[\frac{8kT_e(0)}{\pi m} \right]^{1/2} = 6.21 \times 10^5 \sqrt{T_e(0)},$$

with $T_e(0)$ in $^{\circ}\text{K}$ and $\overline{v_e(0)}$ in cm/s. The electron/heavy-particle collision frequency is $\nu_c(0) = n_a(0)\overline{v_e(0)}\sigma_c$ and the initial value of the effective collision frequency $\nu_{tr}(0)$ for kinetic energy transfer in elastic collisions is given in terms of $\nu_c(0)$ and the atomic weight of the vapor by ¹⁰

$$\nu_{tr}(0) = \left(\frac{2}{1836A} \right) \nu_c(0) = \frac{2n_a(0)\overline{v_e(0)}\sigma_c}{1836A}.$$

The initial concentration of free electrons, assuming at most first-degree ionization in the blowoff vapor, is given by Eq. (10):

Fig. 4. Initial free-electron density $n_e(0)$ in aluminum blowoff vapor as a function of absorbed laser power per unit area, αI_0 .

$$\frac{n_e(0)n_i(0)}{n_a(0)} \cong \frac{n_e^2(0)}{n_a(0)} \text{ (cm}^{-3}\text{)} = 2.415 \times 10^{15} \left(\frac{2g_i}{g_a} \right) [T_e(0)^\circ\text{K}]^{3/2} \exp \left[- \frac{\Theta_i}{T_e(0)} \right]. \quad (27)$$

For the ratio of degeneracy factors of ion and atom ground states g_i/g_a , we use an approximate expression valid at high temperatures:

$$g_i/g_a = q/(2n^2 - q + 1),$$

where n is the principal quantum number for the atom and q is the number of electrons in its partially filled shell. In this relation, excited states are neglected and full degeneracy within a principal quantum number n is assumed. For hydrogen, $n = 1 = q$, giving $g_i/g_a = 1/2$, while for aluminum, with electronic configuration (2; 2, 6; 2, 1), we have $n = 3 = q$ and $g_i/g_a = 3/16$. Figure 4 shows the predicted dependence of $n_e(0)$ on absorbed laser intensity for an aluminum target.

The initial values of the coefficients for electron-ion and electron-atom inverse bremsstrahlung absorption are found from Eqs. (5) and (6) to be

$$k_{e-i}(0) \text{ (cm}^{-1}\text{)} = 1.37 \times 10^{-23} Z^3 \lambda_L^3 n_e^2(0) [T_e(0)]^{-1/2} \left\{ \exp \left[\frac{1.44}{\lambda_L T_e(0)} \right] - 1 \right\}, \quad (28)$$

$$k_{e-a}(0) \text{ (cm}^{-1}\text{)} = 1.86 \times 10^{-18} \sqrt{T_e(0)} \lambda_L^2 \sigma_c n_a(0) n_e(0), \quad (29)$$

respectively, with λ_L in cm and $T_e(0)$ in $^\circ\text{K}$. Finally, the ionization rate coefficient at $t = 0$ has the value

$$\alpha_1(0) \text{ (cm}^3/\text{s)} = C_1 [kT_e(0)]^{1/2} \exp \left[-\Theta_1/T_e(0) \right],$$

and the corresponding recombination rate coefficient is

$$\beta_R(0) \text{ (cm}^6/\text{s)} = \frac{n_a(0)\alpha_1(0)}{n_e^2(0)} = 4.87 \times 10^{-24} C_1 (2g_i/g_a)^{-1} [T_e(0)]^{-1}.$$

In writing these quantities, we can now easily factor out their dependence on electron temperature and electron (and heavy particle) densities. For example,

$$\begin{aligned} \nu_c &= \nu_c(0) [n_a/n_a(0)] [T_e/T_e(0)]^{1/2}, \quad k_{e-a} = k_{e-a}(0) [\nu_c/\nu_c(0)] [n_e/n_e(0)] \\ &= k_{e-a}(0) [n_a/n_a(0)] [n_e/n_e(0)] [T_e/T_e(0)]^{1/2}, \end{aligned}$$

etc. In such relations, the dependent variables always appear as dimensionless ratios, so it is convenient to introduce new variables $\mathcal{T}$ and $\mathcal{N}$ to represent, respectively, the electron temperature and electron number density:

$$\mathcal{T} = T_e(t)/T_e(0), \quad (30)$$

$$\mathcal{N} = n_e(t)/n_e(0), \quad (31)$$

where Eq. (27) fixes $n_e(0)$ and Eqs. (18) and (24) determine $T_e(0) = T_{e,0}$. The basic rate Eqs. (3) and (4) then take the forms

$$\frac{d\mathcal{T}}{dt} = \mathcal{T}^{1/2} \left[\gamma_{\text{eff}} - \left(\mathcal{T} + \frac{2\theta}{3} \right) \xi \right], \quad (32)$$

and

$$\frac{d\mathcal{N}}{dt} = \mathcal{T}^{1/2} \xi \mathcal{N}. \quad (33)$$

The term θ represents the ratio of the atom's ionization temperature and the initial kinetic temperature of the free electrons. The term θ is defined as

$$\theta = \Theta_1/T_e(0), \quad (34)$$

and the coefficients γ_{eff} and ξ are given by

$$\begin{aligned} \gamma_{\text{eff}} = \gamma_{\text{eff}}(\mathcal{N}, \mathcal{T}) &= \frac{2k_{e-a}(0)\mathcal{T}_0\mathcal{N}}{3kT_e(0)n_e(0)\mathcal{T}} \left\{ \frac{\exp [1.44/\lambda_L T_e(0)\mathcal{T}] - 1}{\exp [1.44/\lambda_L T_e(0)] - 1} \right\} \\ &+ \frac{2k_{e-a}(0)\mathcal{T}_0n_a}{3kT_e(0)n_e(0)n_a(0)} - \nu_{\text{tr}}(0)[n_a/n_a(0)] \left[\mathcal{T} - \frac{T_a}{T_e(0)} \right] \end{aligned} \quad (35)$$

and

$$\xi = \xi(\mathcal{N}, \mathcal{T}) = \alpha_1(0)n_a \left\{ \exp \left[\frac{\theta(\mathcal{T} - 1)}{\mathcal{T}} \right] - \mathcal{N}^2 \mathcal{T}^{-3/2} [n_a(0)/n_a] \right\}. \quad (36)$$

The nonequilibrium growth of free electron temperature and density in the early stage of the laser/target interaction is therefore described by these nonlinear differential Eqs. (32) and (33), to be solved subject to the initial conditions $\mathcal{T}(0) = 1 = \mathcal{N}(0)$. A third equation of interest is obtained by dividing Eq. (33) by Eq. (32) to eliminate the explicit time-dependence:

$$\frac{d\mathcal{N}}{d\mathcal{T}} = \frac{\xi \mathcal{N}}{\left[\gamma_{\text{eff}} - \left(\mathcal{T} + \frac{2\theta}{3} \right) \xi \right]} \quad (37)$$

NUMERICAL AND APPROXIMATE ANALYTICAL SOLUTIONS

Analysis of the nonlinear rate Eqs. (32) and (33) reveals two distinct stages in the evolution of the normalized electron temperature $\mathcal{T}$ and number density $\mathcal{N}$. These can be readily identified by focusing our attention on the widely varying magnitudes of the temperature- and density-dependent coefficient ξ and the combination $[\gamma_{\text{eff}} - (\mathcal{T} + 2\theta/3)\xi]$. In this way we can find coarse but useful approximate analytical solutions for $\mathcal{T}$ and $\mathcal{N}$, as well as formulas for several other quantities that characterize the breakdown process. After deriving these results by a heuristic method, we will compare them to accurate numerical results in a specific situation.

As the plume of vaporized material is expelled from the target surface, the initial free-electron population remains small but is heated to a high temperature. To see this, we observe that at time $t = 0$ (when $\mathcal{T} = 1 = \mathcal{N}$) the coefficient ξ has its minimum value, zero, whereas γ_{eff} is large and positive, mainly because of electron-atom inverse bremsstrahlung absorption. In this circumstance, Eq. (32) predicts that $\mathcal{T}$ rises almost linearly with time, while according to Eq. (33), $\mathcal{N}$ rises hardly at all. However, after $\mathcal{T}$ has grown to a high enough value, the magnitude of ξ has increased to such an extent that the value of $[\gamma_{\text{eff}} - (\mathcal{T} + 2\theta/3)\xi]$ sharply declines toward zero, thereby abruptly stopping the rise in $\mathcal{T}$. We can therefore define a characteristic heating time t^* and electron temperature $\mathcal{T}^*$ achieved in this stage by setting $d\mathcal{T}/dt \approx 0$ at $t = t^*$ and using the condition that the electron density has not changed appreciably up to this time ($\mathcal{N} \approx \mathcal{N}(0) = 1$). This gives the relation

$$\gamma_{\text{eff}}(1, \mathcal{T}^*) \approx \left(\mathcal{T}^* + \frac{2\theta}{3} \right) \xi(1, \mathcal{T}^*), \quad (38)$$

which after using Eqs. (35) and (36) becomes an implicit equation for $\mathcal{T}^*$:

$$\exp[\theta(\mathcal{T}^* - 1)/\mathcal{T}^*] \approx \frac{3 \left\{ [2k_{e-a}(0)\mathcal{T}_0/3kTe(0)n_e(0)] - \nu_{tr}(0)(\mathcal{T}^* - 1) \right\}}{2\theta\alpha_i(0)n_i(0) [1 + (3\mathcal{T}^*/2\theta)]} \quad (39)$$

In this heating stage, we have neglected any change in temperature and density of the heavy particles by setting

$$n_i/n_i(0) \approx 1, \quad (40a)$$

and

$$T_s/T_s(0) \approx 1, \quad (40b)$$

and in Eq. (39) the e-i absorption term has been neglected relative to the e-a term. By expanding the right side of Eq. (39) in powers of $\mathcal{T}^*$, we can evaluate $\mathcal{T}^* \approx \mathcal{T}_m^*$ to any desired order m of accuracy, where m is the highest power of $\mathcal{T}^*$ retained on the right side of the equation. Thus, to zero-order accuracy, we calculate

$$\mathcal{T}^* \approx \mathcal{T}_{(0)}^* = \left[1 - \frac{\ln \Gamma}{\theta} \right]^{-1}, \quad (41)$$

to first-order,

Table 1. Intensity-dependence of the electron temperature $T_e^* = T_e(0) \cdot \mathcal{T}^*$ in aluminum vapor at end of heating stage, and comparison of accurate and approximate expressions for $\mathcal{T}^*$.

$\lambda_L, \mu\text{m}$	$\mathcal{J}_0, \text{GW/cm}^2$	$\mathcal{T}_0^*$	$\mathcal{T}_1^*$	$\mathcal{T}_2^*$	$\mathcal{T}^*$	$T_e^*, ^\circ\text{K}$
1.06	0.1	1.727	1.332	1.571	1.529	5612
1.06	1	1.646	1.577	1.592	1.590	7300
1.06	10	1.623	1.562	1.578	1.575	9659
1.06	100	1.591	1.468	1.514	1.504	14024
10.6	0.001	2.473	2.032	—	2.124	5368
10.6	0.01	2.577	2.467	2.491	2.488	7266
10.6	0.1	2.871	2.762	2.790	2.785	9650
10.6	1	3.478	3.209	3.311	3.289	14021

$$\mathcal{T}^* \approx \mathcal{T}_{(1)}^* = \left\{ 1 - \frac{1}{\theta} \ln \left[\Gamma - \frac{3\mathcal{T}(0)^*}{2\theta} \left(\Gamma + \frac{\nu_{tr}(0)}{\alpha_1(0)n_a(0)} \right) \right] \right\}^{-1}, \quad (42)$$

to second-order,

$$\mathcal{T}^* \approx \mathcal{T}_{(2)}^* = \left\{ 1 - \frac{1}{\theta} \ln \left[\Gamma - \frac{3\mathcal{T}_{(1)}^*}{2\theta} \left(1 - \frac{3\mathcal{T}_{(1)}^*}{2\theta} \right) \left(\Gamma + \frac{\nu_{tr}(0)}{\alpha_1(0)n_a(0)} \right) \right] \right\}^{-1}, \quad (43)$$

etc., where

$$\Gamma \equiv \frac{3 \left\{ [2k_{e-a}(0)\mathcal{J}_0/3kT_e(0)n_e(0)] + \nu_{tr}(0) \right\}}{2\theta\alpha_1(0)n_a(0)}. \quad (44)$$

For an aluminum target, Table 1 compares the approximations represented by Eqs. (41) through (43) with the exact solution for $\mathcal{T}^*$ vs $\mathcal{J}_0$ based on Eq. (39); also shown in Table 1 and Fig. 5 is the dependence of the electron temperature $T_e^* = \mathcal{T}^* T_e(0)$ on $\mathcal{J}_0$. In these calculations, the parameter values used for aluminum are

$$\theta_1 = 69360^\circ\text{K (or } k\theta_1 = 5.98 \text{ eV)},$$

$$\sigma_e = 1.8 \times 10^{-15} \text{ cm}^2,$$

$$g_1/g_a = 3/16,$$

and

$$C_1 = 0.036 \text{ cm}^3 \text{ s}^{-1} \text{ erg}^{-1/2}.$$

An estimate of the time t^* required to reach the temperature $\mathcal{T}^*$ can be derived by approximating the rate of temperature increase throughout the entire interval $0 \leq t \leq t^*$ with a constant equal to its initial rate $(d\mathcal{T}/dt)_{t=0} = \gamma_{eff}(1,1)$. This gives

$$(\mathcal{T}^* - 1)/(t^* - 0) \approx \gamma_{eff}(1,1),$$

and using Eq. (35) we have

$$t^* = \frac{3(\mathcal{T}^* - 1)kT_e(0)n_e(0)}{2\mathcal{J}_0[k_{e-a}(0) + k_{e-i}(0)]}. \quad (45)$$

Fig. 5. Electron temperature T_e^* (K) achieved in Al vapor during the heating stage as a function of laser peak intensity S_0 (GW/cm²). Results for laser wavelength $\lambda_L = 1.06 \mu\text{m}$ are given by ordinate values on the left side, and those for $\lambda_L = 10.6 \mu\text{m}$ on the right. Near the bottom of the figure, where the curves diverge, the dashed curve refers to laser wavelength $\lambda_L = 1.06 \mu\text{m}$.

Table 2 shows some values of t^* vs S_0 for the problem involving an aluminum target irradiated by Nd- and CO₂-laser beams.

As the heating stage comes to its end, an ionization-production stage begins. Throughout the electron heating stage, we have approximated $\mathcal{F}$ by a linearly increasing function of time and have neglected any change in the size of the free-electron population; i.e., we have used the functional forms

$$\mathcal{F} \approx \frac{2\mathcal{G}[k_{e-a}(0) + k_{e-i}(0)]t}{3kT_e(0)n_e(0)} + 1, \quad (46a)$$

and

$$\mathcal{N} \approx \mathcal{N}(0) = 1, \quad (46b)$$

for $0 < t < t^*$. By contrast, in the ionization-production interval beyond t^* , the electron population density rapidly increases from atoms undergoing ionizing collisions with hot ($\mathcal{F} = \mathcal{F}^*$) electrons. These same collisions drive the electron energy, preventing the average electron temperature from rising much above $\mathcal{F}^*$ in this stage.

The rate of ionization growth can be estimated in Eq. (33) by using the temperature

$$\mathcal{F} = \text{constant} = \mathcal{F}^*, \quad (47a)$$

for $t^* < t < t_{BD}$ along with the expression

$$\xi \approx \alpha_i(0)n_i(0) \exp [\theta(\mathcal{F}^* - 1)/\mathcal{F}^*].$$

Then upon integrating Eq. (33) we find

$$\mathcal{N} \approx \exp \left(\frac{t - t^*}{t_G} \right), \quad (47b)$$

for $t^* < t < t_{BD}$, where the ionization growth time t_G is defined by

Table 2. Variation of characteristic times t^* , t_G , and t_{BD} vs peak laser intensity $\mathcal{I}_0$ for aluminum target problem.

$\lambda_L, \mu m$	$\mathcal{I}_0, GW/cm^2$	t^*, s	t_G, s	t_{BD}, s
1.06	0.1	1.06×10^{-9}	6.57×10^{-8}	6.03×10^{-7}
1.06	1	1.50×10^{-11}	3.74×10^{-10}	3.07×10^{-9}
1.06	10	2.09×10^{-13}	3.90×10^{-12}	2.80×10^{-11}
1.06	100	2.99×10^{-15}	4.46×10^{-14}	2.66×10^{-13}
10.6	0.001	4.37×10^{-7}	1.88×10^{-5}	2.11×10^{-4}
10.6	0.01	6.53×10^{-9}	5.87×10^{-8}	6.13×10^{-7}
10.6	0.1	9.52×10^{-11}	5.49×10^{-10}	5.27×10^{-9}
10.6	1	1.48×10^{-12}	5.46×10^{-12}	4.78×10^{-11}

$$t_G = \frac{\exp[-\theta(\mathcal{I}^* - 1)/\mathcal{I}^*]}{\alpha_1(n_A(0)\sqrt{\mathcal{I}^*}} \quad (48)$$

The upper time limit $t = t_{BD}$ for Eqs. (47a) and (47b) to be valid is the nominal gas breakdown time, defined here as the time at which a state of approximately full first-degree ionization [$n_e \cong n_e(0) + n_a(0)$] is achieved. Numerical solutions of our basic rate equations show that as this state is approached, the effects of neutral atom depletion and (to a lesser extent) the heating of the heavy particles, become increasingly important, violating the convenient approximations represented by Eqs. (40a) and (40b). Nevertheless an adequate estimate of the breakdown time results by setting $\mathcal{N} \cong n_a(0)/n_e(0)$ [i.e., $n_e(t_{BD}) \cong n_a(0)$] in Eq. (47b) to get

$$t_{BD} \cong t^* + t_G \ln [n_a(0)/n_e(0)] \quad (49)$$

This equation is fairly insensitive to the value chosen for $n_e(t_{BD})$ because it enters only in the logarithmic term. Generally, the time required for growth of ionization t_G and the nominal breakdown time t_{BD} are long compared with the heating time t^* . For the study of an aluminum target, these times are compared in Table 2, and Fig. 6 shows the predicted variation of t_{BD} with incident (peak) intensity over a wide range of values. As indicated in the axes of Fig. 6, an alternative interpretation of this graph [and Eq. (49)] gives the intensity threshold level $\mathcal{I}_0^{thr}$ required to produce breakdown as a function of laser pulse duration t_p . Notice that to a close approximation, t_{BD} goes as $\mathcal{I}_0^{-2}$, or $\mathcal{I}_0^{thr}$ goes as $t_p^{-1/2}$.

When breakdown occurs, two quantities of interest are the spatial extent x_v of the blowoff vapor and the amount E_{BD} of laser fluence (energy/cm²) that has been delivered to the target up to this time. The length x_v can be estimated by $x_v \cong u_{g,0} t_{BD}$, with $u_{g,0}$ given by Eq. (21) and Fig. 3; E_{BD} is given by $E_{BD} = \mathcal{I}_0 t_{BD} = \mathcal{I}_0^{thr} t_p$ for a constant intensity (square wave) laser pulse (Figs. 7 and 8).

In our simplified analysis of the ionization-production stage, we have considered $\mathcal{I}$ to be approximately constant at the value $\mathcal{I}^*$ in Eq. (47a). To estimate how much the electron temperature actually increases during this interval, we observe that as $\mathcal{N}$ gets very large in the presence of only slightly rising $\mathcal{I}$, the coefficient ξ will pass through a maximum value and then begin to fall toward its initial level of $\xi = 0$. While this happens, $d\mathcal{N}/dt$ again approaches zero and $d\mathcal{I}/dt$ approaches the large value $\sqrt{\mathcal{I}}\gamma_{eff}$. Therefore, near the end of the ionization production stage $\mathcal{N}$ must level off from its rapid growth rate and $\mathcal{I}$ must once again begin to rise substantially,¹⁵ this time predominantly from e-i rather than from e-a bremsstrahlung absorption. A rough upper limit for the electron temperature near the nominal time t_{BD} of first-degree breakdown follows by setting $\mathcal{N} \cong f n_a(0)/n_e(0)$ [i.e., $n_e \cong f n_a(0)$] and $n_a \cong (1-f)n_a(0)$ in the condition

$$\xi(\mathcal{N}, \mathcal{I}; n_a) \cong 0, \quad (50)$$

where f is some specified fraction (say $0.1 < f < 1$) indicating the extent of first-degree ionization. This leads to

Fig. 6. Breakdown time t_{BD} vs peak value S_0 of incident laser intensity for an aluminum target irradiated by (a) a 1.06- μm - and (b) a 10.6- μm -wavelength laser. Alternatively, this figure shows the breakdown threshold intensity S_0^{thr} as a function of laser pulse duration t_p .

Fig. 7. Approximate thickness x_v of the blowoff vapor cloud at the breakdown time t_{BD} , for an aluminum target. Results for a laser wavelength $\lambda_L = 1.06 \mu\text{m}$ refer to the left ordinate; those for $\lambda_L = 10.6 \mu\text{m}$ refer to the right ordinate.

Table 3. Comparison of dimensionless temperatures $\mathcal{T}_f$ and $\mathcal{T}^*$ for the problem of an aluminum target exposed to Nd-laser radiation.

$\mathcal{I}_0$ GW/cm ²	$\mathcal{T}^*$	$\mathcal{T}_f$ for $t =$			
		0.1	0.5	0.8	0.9
0.1	1.53	2.84	5.08	7.64	9.83
1	1.59	3.07	6.84	11.8	16.9

Fig. 8. Fluence level E_{BD} required for a laser square wave of intensity $\mathcal{I}_0$ to produce breakdown in the blowoff vapor from an Al target.

$$\exp \left[\frac{\theta(\mathcal{T}_f - 1)}{\mathcal{T}_f} \right] \cong \frac{[f n_a(0)/n_e(0)]^2}{(1-f) \mathcal{T}_f^{3/2}}, \quad (51)$$

in which $\mathcal{T}_f$ is the upper-limit estimate for the normalized electron temperature corresponding to a fractional ionization f . Table 3 compares some $\mathcal{T}_f$ values to $\mathcal{T}^*$ for the study of Nd-laser/aluminum-target interaction.

Another formula we derive pertains to the minimum threshold laser intensity required to initiate breakdown in the blowoff vapor by a continuous wave (cw) laser. In Fig. 6, we have seen how the threshold intensity $\mathcal{I}_0^{thr}$ varies with laser pulse duration t_p for an aluminum target irradiated by Nd- and CO₂-laser beams; we now consider the minimum $\mathcal{I}_0^{thr}$ value, which is approached in the limit as t_p is increased indefinitely. As Figs. 6 and 8 illustrate, the present theory predicts that $\mathcal{I}_0^{thr}(t_p)$, $t_{BD}(\mathcal{I}_0)$, and $E_{BD}(\mathcal{I}_0)$ are all smoothly varying, monotonic functions, so we cannot find an $\mathcal{I}_0$ level below which breakdown is impossible within the assumptions of our model. However, from Figs. 6 and 8, below an intensity level of $\sim 10^{-1}$ GW/cm² (for 1.06 μ m radiation) and 10^{-3} GW/cm² (for 10.6 μ m radiation), both the required laser fluence and breakdown time tend toward very large values. More generally, inspection of the electron temperature-rate Eq. (32) shows that the cw threshold intensity is fairly well-represented by the formula

$$\mathcal{I}_{0,cw}^{thr} \text{ (GW/cm}^2\text{)} \cong \frac{T_v \text{ (K}^2\text{)}}{\mathcal{A} \text{ (g/mol)} \lambda_L^2 \text{ (}\mu\text{m)}}, \quad (52)$$

where T_v is the normal (atmospheric pressure) boiling point for the target material; e.g., $T_v = 2.75 \text{ k}^\circ\text{K}$ for Al. Notice that this threshold intensity varies inversely as the square of laser wavelength and linearly with the initial temperature of the blowoff vapor [approximated in Eq. (52) by T_v]; on the other hand it is independent of the initial vapor density $n_a(0)$.

Our emphasis on the derivation of analytical approximations for $\mathcal{F}$, $\mathcal{M}$ and other key quantities in the analysis, as opposed to accurate numerical solutions of the breakdown model equations, is justified for two reasons. First, the analytical approximations agree closely with the numerical solutions, well within the expected degree of correspondence between the breakdown model and experimental results. Second, while requiring some numerical evaluation such as solving Eq. (24) to find the initial state variables for the blowoff vapor, the analytical formulas are easier to deal with and readily reveal trends and parametric dependences.

Fig. 9. Comparison of numerical (solid curves) and approximate analytical (dashed curves) solutions for time development of normalized electron density $N-1$ and temperature T (Al target, $1.06 \mu\text{m}$ wavelength, $1 \text{ GW}/\text{cm}^2$ incident intensity).

Fig. 10. Numerical solution for electron density $N-1$ as a function of electron temperature T (Al target, $\lambda_L = 1.06 \mu\text{m}$, $I_0 = 1 \text{ GW}/\text{cm}^2$).

To compare with numerical results, we show in Fig. 9 the predicted evolution of $\mathcal{F}$ and $\mathcal{N}$ based on Eqs. (46) and (47) juxtaposed with an accurate numerical solution of the basic rate Eqs. (32) and (33) for an aluminum target irradiated by a Nd-laser beam ($\lambda_L = 1.06 \mu\text{m}$) with incident intensity $\mathcal{S}_0 = 1 \text{ GW/cm}^2$. The general forms of Eqs. (46a) and (47a) for $\mathcal{F}(t)$ and Eqs. (46b) and (47b) for $\mathcal{N}(t)$, and the values of the characteristic quantities t^* , $\mathcal{F}^*$, and t_G are satisfactory approximations. It is worth noting that the time step size in the numerical solution must be kept small ($\Delta t \approx t_G/500$) because of the tight coupling of $\mathcal{N}$ and $\mathcal{F}$. This feature is illustrated in Fig. 10, which shows the solution of Eq. (37) for $\mathcal{M}\mathcal{F}$ in this case; one $\mathcal{F}$ rises to a value of about $\mathcal{F}^*$, the incremental change of $\mathcal{N}$ with further increases in $\mathcal{F}$ becomes very large.

COMPARISON WITH EXPERIMENT AND CONCLUSIONS

A sampling of the extent of agreement between predictions of the breakdown model and experimental results is shown in Fig. 11. The isolated points represent diverse measurements of threshold intensity vs laser pulse duration that have been reported for aluminum targets irradiated by $10.6 \mu\text{m}$ CO_2 -laser beams. The data from Fig. 11 have been cited and discussed by Smith.⁶ No effort has been made to fit the breakdown model prediction (solid curve) to this data; e.g., we have used $\alpha = 0.05$ to represent the average absorptivity of aluminum at $10.6 \mu\text{m}$, rather than attempting to optimize the choice of α in any way. Nevertheless, the agreement is accurate enough. The three data points (3, 5, and 6) representing laser-beam interactions with large aluminum plates are within a factor of about two of the theory over the broad range of pulse durations (1 μs to cw) and beam spot sizes (300 μm to 1.5 cm). Also, the theory is not designed for small-particle targets; for example, we have made no allowance for the possible dependence of α on particle size and have neglected an electron diffusion loss term in Eq. (2), which is important for very small targets. However, the data points 1, 2, and 4 for breakdown of aluminum particles or small-surface inclusions falls well within order-of-magnitude agreement with the theory from 20 ns to 20 μs in pulse duration and from 2 to 50 μm in particle size.

In conclusion, our breakdown model provides a simple and reasonably accurate description of the growth of avalanche ionization in the blowoff vapor over a laser-irradiated surface. The approximate analytical formulas for the breakdown time, intensity threshold, etc., are particularly useful for quantitative estimates and for showing parametric dependences.

Fig. 11. Comparison of theory with experimental results on breakdown of aluminum targets by $10.6 \mu\text{m}$ laser beams. Solid curve is a prediction of the breakdown model (same as part (b), Fig. 6). The sources and conditions for the data points labeled 1-6 are: (1) Al plate, 20 ns pulse, breakdown site is a $2\text{-}\mu\text{m}$ -diam inclusion¹⁶; (2) 50 μm -diam Al particles suspended into beam, 200 ns pulse¹⁷; (3) Al plate, 1 μs pulse, 0.03 cm beam spot size¹⁸; (4) 30 μm -diam alumina particles dropped into beam, 20 μs pulse¹⁹; (5) Al plate, 1.5 cm beam spot size, 1.5 ms pulse²⁰; (6) Al plate, cw beam, 0.2 cm spot size.²¹

NOTES AND REFERENCES

1. Once a high degree of ionization and high temperature have been established, the continued laser/target interaction under some conditions can be approximated as a steady-state ablation process. For example, see O. N. Krokhin in *Physics of High Energy Density*, P. Caldirola and H. Knoepfel, Eds. (Academic, N. Y., 1971), pp. 278-305. Alternatively, we might need to perform a detailed numerical modeling of the myriad physical processes, such as comprised by G. Zimmerman's LASNEX computer program at Lawrence Livermore Laboratory.
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5. The representation of light and heavy particles by distinct, coupled Maxwellian temperatures is discussed by Zel'dovich and Raizer in Ref. 6, Ch. 6. In a plasma with electron and ion densities n_e and n_i and temperatures T_e and T_i , the free electrons equilibrate among themselves in a relaxation time given by $\tau_{ee}(s) \approx [T_e(^{\circ}K)]^{3/2}/38 n_e(\text{cm}^{-3})$, whereas the mutual equilibration time for ions is longer by a factor $(T_i/T_e)^{3/2} (n_e/n_i)(M/m)^{1/2}$, where M/m is the ratio of ion and electron masses. The time required for the electrons and ions to come into equilibrium with each other is $\tau_{ei}(s) \approx 25 [T_e(^{\circ}K)]^{3/2}/n_i(\text{cm}^{-3}) Z^2$, where Z is the ionic charge and $\mathcal{A}$ is the atomic weight of the ions.
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9. Formulas specifying the exposure time and amount of laser fluence required to bring the target surface to its vaporization temperature are derived in R. J. Harrach, *J. Appl. Phys.* **48**, 2370 (1977). For a high-intensity step-function pulse irradiating a thermally thick target, the vaporization time is most simply approximated as $(0.79/\kappa)[K(T_v - T_0)/\alpha_0 \mathcal{J}_0]^2$, where K and κ are respectively the thermal conductivity and thermal diffusivity of the target, T_v is the vaporization temperature, T_0 is the initial temperature of the target, and $\alpha_0 \mathcal{J}_0$ is the absorbed laser power per unit area. Using K , κ , and T_v values appropriate for an aluminum target, as discussed in the reference cited above, this expression gives $t_v(\text{ns}) \approx 19 \times 10^{-2} [\alpha_0 \mathcal{J}_0 (\text{GW}/\text{cm}^2)]^{-2}$.
10. The relation between ν_{tr} and the gas kinetic collision rate ν_c for electrons and heavy particles is $\nu_{tr} = (2m/M)\nu_c$, where m/M is the mass ratio for the particles; ν_c is given in terms of the heavy-particle density n_a , electron average velocity $\bar{v}_e$, and collision cross section σ_c by $\nu_c = n_a \bar{v}_e \sigma_c$. The form of the elastic collisional energy exchange term in Eq. (2), being proportional to the difference $(T_e - T_a)$ in kinetic temperatures of the two groups of particles, is taken from L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience, N. Y., 1962), 2nd Ed., Ch. 5, Sec. 3.
11. The effects in particular, of atomic excitations, hydrodynamic expansion, and thermal radiation losses, are neglected in Eqs. (2) and (3).
12. The contributions of photoionization and photorecombination are assumed to be negligibly small compared with those of collisional ionization and three-body recombination.
13. In addition to Eqs. (3) and (4) for T_e and n_e , for completeness, it is also necessary to have a pair of rate equations for the heavy particle temperature and number density; e.g., $\dot{T}_a \approx (T_e - T_a)\nu_{tr}$ and $\dot{n}_a = -n_a$ (for $Z=1$). However, compared with T_e and n_e , T_a and n_a change only slightly during the formation of the electron avalanche.
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15. The rise in electron temperature T_e then arrests the decline in ξ , so that ξ never reaches zero. In the presence of increasingly high temperature, the second- and higher-order ionization processes begin to be important; our interest is limited to the earlier times ($t < t_{BD}$) when first-degree ionization dominates.

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APPENDIX: LIST OF SYMBOLS

c	Speed of light.
C_1	Constant in formula for ionization rate coefficient, $\text{cm}^3 \cdot \text{s}^{-1} \cdot \text{erg}^{-1/2}$.
C_p, C_v	Specific heats at constant pressure and volume.
E_{BD}	Laser fluence required for breakdown, J/cm^2 .
f	Ratio of singly-charged ions to neutral atoms.
g_i, g_a	Degeneracy factors of ion and atom ground states.
h	Planck's constant.
K	Thermal conductivity $\text{W}/\text{cm} \cdot ^\circ\text{K}$.
k	Boltzmann's constant.
$k_{e-a}, k_{e-a}(0)$	Absorption coefficient for laser radiation due to electron-atom inverse bremsstrahlung. The argument (0) in this and several of the following entries refers to the time $t = 0$ when surface vaporization commences.
$k_{e-i}, k_{e-i}(0)$	Absorption coefficient for laser radiation due to electron-ion inverse bremsstrahlung.
k_L	Total absorption coefficient; $k_L = k_{e-a} + k_{e-i}$.
L_m, L_v	Latent heats of fusion and vaporization.
M	Heavy particle mass.
$\dot{M}$	Mass vaporization rate from the target surface.
m	Mass of electron.
n	Principal quantum number of atom.
$n_a, n_a(0), \bar{n}_a$	Number of atoms per unit volume; bar in this and following entries refers to the thermal equilibrium value.
$n_i, \bar{n}_i$	Number of ions per unit volume.
$n_e, n_e(0), \bar{n}_e$	Number of electrons per unit volume.

$\dot{n}_e, \dot{n}_a$	Time rates of change of number densities.
N_A	Avogadro's number.
$p_{g,0}$	Pressure in vapor phase at position of absorption layer.
p^{ref}	Reference pressure level for defining the boiling point.
$p_{s,0}$	Pressure in condensed phase at position of absorption layer.
q	Number of electrons in partially filled atom shell.
$\hat{R}$	Gas constant per atom, equal to universal gas constant divided by atomic weight of vapor, $\text{erg/g} \cdot ^\circ\text{K}$.
$T_a, T_a(0)$	Maxwellian temperature of heavy particles (atoms, ions), $^\circ\text{K}$.
T_{amb}	Ambient temperature, $^\circ\text{K}$.
$T_e, T_e(0)$	Maxwellian temperature of electrons, $^\circ\text{K}$.
T_e^*	Electron temperature at end of heating stage, $^\circ\text{K}$.
$T_{g,0}$	Temperature in vapor phase at position of absorption layer, $^\circ\text{K}$.
T_v	Boiling or vaporization temperature, $^\circ\text{K}$.
$T_v^{(ref)}$	Reference value of boiling temperature at pressure level p^{ref} , $^\circ\text{K}$.
$\dot{T}_e, \dot{T}_a$	Time rates of change of temperatures.
t	Time variable.
$t = 0$	Time at which surface vaporization commences.
$t = t^*$	End of heating stage for initial population of free electrons.
$t = t_{BD}$	Time when full first-degree ionization is achieved in blowoff vapor.
t_G	Time interval required for growth of ionization.
t_v	Illumination time interval required for surface to reach its vaporization temperature.
t_p	Laser pulse duration.

Δt	Time step for numerical calculation.
$u_{e,0}$	Blowoff vapor velocity at position of absorption layer.
$\bar{v}_{e-s}$	Average relative velocity of electrons and heavy particles.
$\overline{v_e(0)}$	Average electron velocity at time of onset of surface vaporization.
x_v	Spatial extent of blowoff vapor at time of breakdown.
Z	Average charge of an ion.
A	Atomic weight of vapor, g/mol.
$\mathcal{E}$	Average energy density of Maxwell-distributed free electrons, J/cm ³ .
$\mathcal{I}$	Laser beam intensity, W/cm ² .
$\mathcal{I}_0$	Peak value of laser beam intensity.
$\mathcal{I}_0^{(thr)}$	Threshold value of peak laser intensity required for breakdown.
$\mathcal{I}_{0,cw}^{thr}$	Threshold value for a continuous wave (cw) laser beam.
$\mathcal{L}_v$	Effective heat of ablation, J/g.
$\mathcal{N}, \mathcal{N}(0)$	Dimensionless free electron density.
$\mathcal{T}, \mathcal{T}(0)$	Dimensionless free electron temperature.
$\mathcal{T}^*$	Value of $\mathcal{T}$ at end of heating stage.
$\mathcal{T}_m^*$	Approximate value of $\mathcal{T}^*$, to order m of accuracy.
$\mathcal{T}_f$	Value of $\mathcal{T}$ at time when a fraction f of the atoms has been singly ionized.
α	Surface absorptivity for laser radiation, equal to one minus the surface reflectivity.
α_0	Constant value representing the surface absorptivity.
$\alpha_1, \alpha_I(0)$	Collisional ionization rate coefficient, cm ³ /s.
$\beta_R, \beta_R(0)$	Three-body recombination coefficient, cm ⁶ /s.
γ	Ratio of specific heats (C_p/C_v).

γ_{eff}	Dimensionless heating coefficient in $\mathcal{F}$ -equation.
Γ	Dimensionless factor occurring in $\mathcal{F}_m^*$ expressions.
Δ	Normalized value of temperature in vapor phase at position of absorption layer.
O_1	Atomic ionization energy, °K.
θ	Dimensionless ratio of atomic ionization energy and initial kinetic energy of free electrons.
κ	Thermal diffusivity, cm ² /s.
λ_L	Laser wavelength.
$\nu_c, \nu_c(0)$	Collision frequency between electrons and heavy particles, s ⁻¹ .
$\nu_{tr}, \nu_{tr}(0)$	Effective collision frequency for transfer of kinetic energy in collisions between electrons and heavy particles, s ⁻¹ .
ξ	Dimensionless ionization-production coefficient in $\mathcal{N}$ -equation.
$\rho_{g,0}$	Density in vapor phase at position of absorption layer.
σ_c	Cross section for electron-heavy particle collisions, cm ² .
τ_{e-e}	Relaxation time for electron-electron energy exchange.
τ_{e-i}	Relaxation time for electron-ion energy exchange.