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KINETICS EQUATION FOR A CLUSTER OF ROVER REACTORS

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I. Introduction

A kinetics equation is a set of ordinary differential
equations governing the coarse time-dependence of effective
neutron populations in one or more regions of space. The present
interest is in obtaining a simplest form of kinetics equation
capable of giving the time-dependence of fission rates in individual
cores of a clustered array; moreover, it is desired that all
parameters introduced here be defined in terms of computable
functions of the basic nuclear properties of this system.

For the purpose of introducing notation and general rationale,
we first review the derivation of the "point-reactor" kinetics
equation. Secondly, we generalize to a "two-point" equation
wherein space is divided into two "points", one corresponding
to the reactor proper, and the other to the environment. The
final generalization to the kinetics equation for a cluster of
Rover reactors will then be essentially trivial.

II. "Point-Reactor" Kinetics Equation

We presume the system to be completely described, neutronically,
by the macroscopic collision cross section, $\sigma(E,\vec{\Omega},\vec{r})$, the scattering
kernel, $\sigma_s(E'\to E,\vec{\Omega}'\to\vec{\Omega})$, and the fission neutron kernels $\nu_p\sigma_f(E')\chi(E)$,
$\beta\nu\sigma_f(E')\chi_d(E,\tau)p(\tau)d\tau$ (i.e., the delayed neutron emission kernel).

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We presume the system to be finite and isolated from any environment, i.e., no neutrons enter the system from outside. The flux distribution, $\phi(E, \vec{\Omega}, \vec{r}, t)$, in this system is then given by the transport equation

$$\frac{1}{v}\frac{\partial\phi}{\partial t} + \Omega \cdot \nabla\phi + \sigma\phi = \int \phi(E', \vec{\Omega}', \vec{r}, t)\, [\sigma_s + \nu_p\sigma_f\chi]\,dE'd\Omega' \quad + S(E, \vec{\Omega}, \vec{r}, t)$$
$$+ \int \phi(E', \vec{\Omega}', \vec{r}, t-\tau)\beta\nu\sigma_f\chi_d p(\tau)\,d\tau dE'd\Omega' \qquad (1)$$

and the prompt neutron reproduction number, k, of this system is defined by the eigenvalue problem

$$-\Omega \cdot \nabla\phi^+ + \sigma\phi^+ = \int [\sigma_s + \frac{1}{k}\nu_p\sigma_f\chi](E\rightarrow E', \vec{\Omega}\rightarrow\vec{\Omega}')\,\phi^+(E', \vec{\Omega}, \vec{r})\,dEd\Omega' \qquad (2)$$

Because the system is neutronically isolated, Equations 1 and 2 are most conveniently solved for ϕ and ϕ^+ only within the system volume by applying the "isolation boundary conditions" $\varphi = 0$ for incoming directions $\varphi^+ = 0$ for outgoing directions $\vec{\Omega}$.

By subtracting Equation 2 multiplied by ϕ from Equation 1 multiplied by ϕ^+, and then integrating the result over the ranges of E and Ω and over the volume V, one formally obtains

$$\int_V \phi^+\frac{1}{v}\frac{\partial\phi}{\partial t}\,dEd\Omega dV = \left(1 - \frac{1}{k}\right)\int_V \phi\nu_p\sigma_f\chi\phi^+dE'd\Omega'dEd\Omega dV + \int_V \phi^+ S\,dEd\Omega dV$$

$$+ \int_V \phi(t-\tau)\beta\nu\sigma_f\chi_d p(\tau)\phi^+d\tau dEd\Omega dE'd\Omega'dV$$

$$- \oint_V \phi\phi^+\vec{\Omega}\cdot d\vec{S}\,dEd\Omega \qquad (3)$$

In this formal equation, the last, or surface integral, term represents the effective rate at which neutrons enter the region V from outside and this term is zero when the region V includes the system. In this section, we identify V with the volume of the system and omit the surface integral in Equation 3. This

equation is readily reduced to the "point-reactor" kinetics
equation by an artifice which renders ϕ^+ absolutely independent
of time: although reactivity variation is actually accomplished
by material displacements, i.e., changes in the macroscopic
cross sections, it is conveniently and essentially equivalently
accomplished, mathematically, by a change in the prompt neutron
emission factor, ν_p. From Equation 2, we see that when this
factor is $\nu_p(t)$, then the eigenvalue, $k(t)$, is such as to make
$\nu_p(t)/k(t)$ and thence the shape of ϕ^+ time independent. Since
ϕ^+ is governed by a linear homogeneous equation in the phase
space variables, it is defined only to within an arbitrarily
time-dependent factor. This factor we choose constant so that
the left-hand-side of Equation 3 may be rewritten as

$$\int_V \phi^+ \frac{1}{v} \frac{\partial \phi}{\partial t} \, dEd\Omega dV = \frac{d}{dt} \int_V \phi^+ \frac{1}{v} \phi dEd\Omega dV \equiv \frac{d}{dt} N \tag{4}$$

We now normalize ϕ^+ such that at some time, $t = t_o$,

$$\frac{N}{\ell} = \int_V \phi \nu_p \sigma_f \chi \phi^+ dE'd\Omega' dEd\Omega dV = \int_V \phi \nu_p \sigma_f dE'd\Omega' dV \tag{5}$$

If the time dependence of ϕ is factorable, Equation 5 will hold
at all times and the "prompt-neutron lifetime", ℓ, will be a
constant. Only for this case does N/ℓ continue to equal the
prompt neutron production rate and therefore be proportional to
reactor power. To the extent that the time-dependence of ϕ is
nearly factorable, ℓ is nearly constant and N/ℓ is nearly propor-
tional to reactor power. In words, we merely call N the "effective"
neutron population in the reactor, and N/ℓ the "effective prompt
neutron production rate". With the definition of these symbols

by means of Equations 4 and 5 and the additional definitions of "effective" delayed neutron fraction, β^*, and "effective" neutron source strength, S^*, as

$$\beta^* \int \frac{N}{l}(t-\tau)p(\tau)d\tau = \int_V \Phi(t-\tau)\nu\sigma_f\chi_d\Phi^+ p(\tau)d\tau dE'd\Omega'dEd\Omega dV \qquad (6)$$

$$S^* = \int_V S\Phi^+ dEd\Omega dV \qquad (7)$$

We may rewrite Equation 3 as the "point-reactor" kinetics equation

$$\frac{dN}{dt} + (\frac{1}{k} - 1)\frac{N}{l} = \beta^* \int \frac{N}{l}(t-\tau)p(\tau)d\tau + S^* \qquad (8)$$

Equation 8 is exact when the symbols are defined as above in Equations 2 to 7. It is not useful, however, unless $l \sim$ constant and N/l can be correlated with reactor power. To illustrate, consider a well-reflected reactor, perhaps even a Rover reactor, for which neutron residence time in the reflector is comparable to the decay time of prompt neutron linked fission chains. Here, the ratio of core neutron population to reflector neutron population depends rather sensitively on system reactivity (the so-called "time absorption" phenomenon) and the effective neutron population of the entire system is not a good index of core power. The time-dependence of flux shape can be partially accounted for by introducing two neutron population variables, say a core population and a reflector population, into a "two-point" kinetics equation. The particular choice is somewhat arbitrary but should lead to simple evaluation of reactor power. In the following development of a "two-point" kinetics equation, we shall designate the two regions of space simply as reactor and environment.

III. "Two-point" Kinetics Equation

We presume the reactor is identified with the volume V and that the flux distribution is given by Equation 1. When isolated from its environment, the reproduction number is defined by Equation 2 with "isolation boundary conditions" on the surface of V. We classify neutrons in V at time t into two groups: those neutrons which were born in V and remained there to time t, and, the remainder. The corresponding fluxes are ϕ_1 and ψ_1, with $\phi_1 + \psi_1$ equaling the total flux ϕ. The transport equations for ϕ_1 and ψ_1 in V are

$$\frac{1}{v}\frac{\partial \phi_1}{\partial t} + \Omega\cdot\nabla\phi_1 + \sigma\phi_1 = \int \phi_1 \sigma_s dE'd\Omega' + \int [\phi_1 + \psi_1]\nu_p\sigma_f\chi dE'd\Omega' \tag{1a}$$

$$+ S(E,\vec{\Omega},\vec{r},t) + \int [\phi_1 + \psi_1](t-\tau)\beta\nu\sigma_f\chi_d p(\tau)d\tau dE'd\Omega'$$

$$\frac{1}{v}\frac{\partial \psi_1}{\partial t} + \vec{\Omega}\cdot\nabla\psi_1 + \sigma\psi_1 = \int \psi_1 \sigma_s dE'd\Omega' \tag{1b}$$

with boundary conditions $\phi_1 = 0$ and $\psi_1 = \phi$ for incoming directions. For convenience, we shall label the solution of Equation 2 ϕ_1^+ and the eigenvalue k_1. Combining Equations 1a and 2 and also 1b and 2, we obtain the following analogues of Equation 3.

$$\int_V \phi_1^+ \frac{1}{v}\frac{\partial \phi_1}{\partial t} dEd\Omega dV = \left(1-\frac{1}{k_1}\right)\int_V \phi_1\nu_p\sigma_f\chi\phi_1^+ dE'd\Omega'dEd\Omega dV + \int_V \phi_1^+ S dEd\Omega dV \tag{3a}$$

$$+ \int_V \psi_1\nu_p\sigma_f\chi\phi_1^+ dE'd\Omega'dEd\Omega dV + \int_V [\phi_1 + \psi_1](t-\tau)\beta\nu\sigma_f\chi_d\phi_1^+ p(\tau)d\tau dE'd\Omega'dEd\Omega dV$$

$$\int_V \phi_1^+ \frac{1}{v}\frac{\partial \psi_1}{\partial t} dEd\Omega dV = -\frac{1}{k_1}\int_V \psi_1\nu_p\sigma_f\chi\phi_1^+ dE'd\Omega'dEd\Omega - \oint_V \phi\phi_1^+\vec{\Omega}\cdot\vec{ds}dEd\Omega \tag{3b}$$

The analogue of Equation 4 is

$$\int_V \phi_1^+ \frac{1}{v}\frac{\partial \phi_1}{\partial t} dEd\Omega dV = \frac{d}{dt}\int_V \phi_1^+ \frac{1}{v}\phi_1 dEd\Omega dV = \frac{d}{dt} N_1 \tag{4a}$$

$$\int_V \phi_1^+ \frac{1}{v} \frac{\partial \psi_1}{\partial t} \, dEd\Omega dV - \frac{d}{dt} \int_V \phi_1^+ \frac{1}{v} \psi_1 dEd\Omega dV \equiv \frac{d}{dt} N_1' \tag{4b}$$

and the analogue of Equation 5 is

$$\frac{N_1}{\mathcal{L}_1} = \int_V \phi_1 \nu_p \sigma_f \chi \phi_1^+ dE'd\Omega' dEd\Omega dV - \int_V \phi_1 \nu_p \sigma_f dE'd\Omega' dV \tag{5a}$$

$$\frac{N_1'}{\ell_1'} = \int_V \psi_1 \nu_p \sigma_f \chi \phi_1^+ dE'd\Omega' dEd\Omega dV \tag{5b}$$

Similarly Equations 6 and 7 generalize to

$$\beta_1^* \int \frac{N_1}{\mathcal{L}_1}(t-\tau) p(\tau) d\tau = \int_V \phi_1(t-\tau) \nu \sigma_f \chi_d \phi_1^+ p(\tau) d\tau dE'd\Omega' dEd\Omega dV \tag{6a}$$

$$\beta_1^* \int \frac{N_1'}{\mathcal{L}_1'}(t-\tau) p(\tau) d\tau = \int_V \psi_1(t-\tau) \nu \sigma_f \chi_d \phi_1^+ p(\tau) d\tau dE'd\Omega' dEd\Omega dV \tag{6b}$$

$$S_1^* = \int_V \phi_1^+ S dEd\Omega dV \tag{7a}$$

$$S_1'^* = - \oint_V \phi \phi_1^+ \vec{\Omega} \cdot \vec{dS} dEd\Omega \tag{7b}$$

Finally, the analogue of Equation 8 is the "two-point" kinetics equation

$$\frac{dN_1}{dt} + \frac{(1-k_1)}{k_1} \frac{N_1}{\mathcal{L}_1} = \beta_1^* \int \frac{N_1}{\mathcal{L}_1}(t-\tau) p(\tau) d\tau + S_1^* + \frac{N_1'}{\mathcal{L}_1'}$$
$$+ \beta_1'^* \int \frac{N_1'}{\mathcal{L}_1'}(t-\tau) p(\tau) d\tau \tag{8a}$$

$$\frac{dN_1'}{dt} + \frac{1}{k_1} \frac{N_1'}{\mathcal{L}_1'} = S_1'^* \tag{8b}$$

The only parameter contained in (8a) and (8b) not defined in terms of basic nuclear parameters is $S_1'^*$, i.e., the surface integral of (7b) which represents the effective neutron current into V from the environment. This current, of course, depends on the nuclear parameters of the environment and these, as yet, remain unspecified.

Formally, one may represent $S_1^{'*}$ as

$$S_1^{'*}(t) = <\phi_1^+> \int_r \left(\frac{N_1}{\ell_1} + \frac{N_1'}{\ell_1'}\right)(t-\tau)p_{11}(\tau)d\tau \tag{9}$$

with $p_{11}(\tau)d\tau$ being the probability that a fission neutron, born
at time t in V, leaves the reactor proper and then re-enters at
time $t+\tau$ in $d\tau$; $<\phi_1^+>$ is the mean effectiveness for such a neutron
coming into V; $(N_1/\ell_1 + N_1'/\ell_1')$ is the total effective fission
neutron production rate and is the index of reactor power. For
brevity, we have omitted from the right-hand-side of Equation 9
a contribution due to delayed neutrons. This contribution indeed
plays an important part in the kinetics of any system where $k_1 \to 0$,
such as a cavity-reactor with the cavity considered as the reactor
proper.

A feel for the various time delay parameters appearing in
(8a) and (8b) may be obtained from the following typical prompt
neutron regeneration loop: a neutron enters the reactor proper
(the source for Equation 8b), and, in the meantime $k_1\ell_1'$, diffuses
into the core and produces a fission; the neutrons from this
fission (the source for Equation 8a) produce a chain of fissions
of mean time duration $k_1\ell_1/(1-k_1)$; some neutrons from this chain
diffuse out of the reactor proper but eventually return to complete
the loop -- the mean time being $\bar{\tau} = \int \tau p_{11}(\tau)d\tau / \int p_{11}(\tau)d\tau$.

IV. Kinetics Equation for a Cluster of Rover Reactors

A cluster of reactors can still be described as one reactor
proper in V_1 plus an environment and the Equations 8a and 8b are
still applicable. The source term, $S_1^{'*}$, however, depends on the
power histories of each reactor of the cluster, and its evaluation

for an n-cluster array involves the solution of n such "two-point" kinetics equations. The whole set is generated from (8a) and (8b) by replacing the subscript 1 by $i = 1,2,\ldots,n$. The source term, $S_i'^*$, for the ith reactor proper is then formally

$$S_i'^* = < \psi_i > \sum_j \int \left(\frac{N_j}{\ell_j} + \frac{N_j^r}{\ell_j^r} \right)(t-\tau)p_{ji}(\tau)d\tau \tag{9a}$$

with $p_{ji}(\tau)d\tau$ the probability that a fission neutron born in j enters i a time τ in $d\tau$ later. The evaluation of p_{ji} may be quite complicated if there are important indirect paths from j to i such as perhaps ones which involve other elements of the cluster or the "environment" of the cluster-system. Nevertheless, once the macroscopic cross-sections of cluster-system and environment have been specified, a p_{ji} can, in principle, be computed by available Monte Carlo methods.

The set of 2n equations, described above, may be simplified somewhat in the application to a cluster of Rover reactors. Experimentally, a single Rover reactor has a well-defined Rossi-α and its time behavior thus is described by a point-reactor kinetics equation; we are thus permitted to choose, as the ith reactor proper, the ith Rover reactor and have $k_i \simeq 1$, i.e., each Rover reactor in the cluster is nearly critical. For this case, $N_i/\ell_i >> N_i'/\ell_i'$, and we may approximate $(N_i/\ell_i + N_i'/\ell_i') \simeq N_i/\ell_i$. With this approximation, Equation 8b is readily integrated to

$$\frac{N_i'}{\ell_i'} \simeq < \psi_i > \sum_j \int_0^\infty \int_0^\tau \frac{N_j}{\ell_j}(t-\tau)p_{ji}(\tau-\tau')e^{-\tau'/\ell_i'} \, d\tau d\tau'/\ell_i' \tag{8c}$$

Substitution of this expression for N_i'/ℓ_i' in (8a) reduces the set of 2n equations for the N_i and N_i' to a set of n equations for the N_i. Before making this substitution, it is convenient to introduce the symbols

$$\Delta k_{ji} = \langle \psi_i \rangle \iint_{0}^{\infty} {}_{0}^{\tau} p_{ji}(\tau-\tau')e^{-\tau'/\ell_i''} \, d\tau d\tau'/\ell_i' \tag{10}$$

$$W_{ji}(\tau) = \int_{0}^{\tau} p_{ji}(\tau-\tau')e^{-\tau'/\ell_i'} \, d\tau'/\ell_i' \Big/ \iint_{0}^{\infty} {}_{0}^{\tau} p_{ji}(\tau-\tau')e^{-\tau'/\ell_i'} d\tau d\tau'/\ell_i' \tag{11}$$

As seen from their definitions, Δk_{ji} represents the transfer probability of a fission neutron from j to i, and $W_{ji}(\tau)$ is the time distribution function for this transfer. Δk_{ii} is the transfer probability from i to i via its environment. With this symbolism, Equation 8a becomes

$$\frac{dN_i}{dt} + (1-k_i)\frac{N_i}{\ell_i} = \beta_i^* \int \frac{N_i}{\ell_i}(t-\tau)p(\tau)d\tau + \sum_j \Delta k_{ji} \int \frac{N_j}{\ell_j}(t-\tau)W_{ji}(\tau)d\tau + S_j^* \tag{12}$$

Equation 12 is the desired form of the kinetics equation for a cluster of Rover reactors. Computational procedures for evaluation of the parameters ℓ_i, ℓ_i', etc., have been indicated. Seale and Chezem have examined solutions of this form of equation with $\Delta k_{ii} = 0$, $W_{ji}(\tau) = \delta(\tau-\tau_{ji})$ where τ_{ji} equals the effective mean drift time of neutrons between the boundaries of j and i, and with $p(\tau)$ corresponding to the Keepin-Wimett six delayed neutron groups. This is a shrewd choice of parameters for antici- pated Rover reactor clusters although, strictly speaking, the choice $W_{ji}(\tau) = \delta(\tau-\tau_{ji})$ prevents Equation 12 from being represent- able as a finite set of ordinary differential equations: thus,

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(1), k_{ii} decreases approximately as the inverse fourth power of reactor separation distance whereas $k_{ji}(j \neq i)$ decreases only as the inverse square, and at anticipated separation distances $k_{ii} \ll k_{ji}(j \neq i)$; (2) the first moment of the distribution function $W_{ji}(\tau)$ is the sum of three mean times -- the time for a fission neutron to leak out of j, the drift time τ_{ji} from boundary of j to boundary of i, and the time from entrance to absorption in i -- and for the anticipated separation distances, the drift time term is dominant; furthermore the product of the Rossi-α by any τ_{ji} is small compared to unity and only the first moment of $W_{ji}(\tau)$ is of importance.

GEH:jf

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