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REACTOR LABORATORY EXPERIMENTS

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PREFACE

This document is in part a record of the proceedings of the 1961 Reactor Laboratory Institute, held for American university professors of nuclear engineering at Argonne National Laboratory. It was sponsored jointly by the United States Atomic Energy Commission and The American Society for Engineering Education.

It is at the same time an exposition of a basic set of experiments in nuclear reactor physics, engineering and technology developed over the course of several years by the staff of the International Institute of Nuclear Science and Engineering. As such, the experiments have each been evolved primarily for instructional purposes; each has been presented to many students by the teaching staff and has proven to be eminently satisfactory for this purpose.

As presented here, the experimental sections begin with a simple description of the experiment in its theoretical and practical aspects. In the majority of cases, additional appendices are added to include discussion questions, extended treatment of some aspect of the exercise, and short notes on related subjects prepared by the students or staff. A final section occasionally appears showing the results actually obtained by the participants in the 1961 summer Institute.

INTRODUCTION

1. The 1961 ASEE-AEC Advanced Reactor Laboratory Institute (June 19 - July 14, 1961)

The 1961 R.L.I. was tailored to meet the needs of the university professor in teaching experimental reactor physics and engineering. A fully formalized curriculum was presented, weighted in such a way that each professor spent eighty percent of his time in the laboratory and the remainder in lectures and review discussion sessions. Previous attendance at a basic institute was assumed; thus familiarity with elementary techniques as well as exponential systems was taken for granted. Approximately eighty-five percent of the experiments presented used the critical reactors, and a total of twenty-two experiments were made available to the students. As a secondary goal, the laboratory experimental material was organized in such a way that each attendee was presented with a Laboratory Manual of writeups for twenty of the experiments presented.

The General Program

Fifteen ASEE professors were registered in this Institute (four of the original applicants were redirected to the Stanford University advanced institute), and sixteen days were available to do experiments. Each experiment was designed to occupy one day of the professors' time. By dividing the attendees into three groups of five, on the average three concurrent experiments were in progress at all times. Experiments, therefore, fell into either of three main headings:

- I Experiments Associated With the Argonaut Reactor
- II Experiments Associated With the AGN-201 Reactor
- III Experiments Associated With Other Reactors or Auxiliary Equipment

Experimental writeups for twenty of the experiments were available and were distributed to the professors as the basis for the Experiment Manual. Experiments were so correlated that, although three or four reactors were used, no experiment was repeated; in addition, experiments were so chosen that, as much as possible, repetitive techniques such as flux mapping, routine foil counting, etc. were minimized.

Lectures and discussion components were divided as follows:

- a. A question session proceeded most of the reactor experiments.

- b. A summary of the results of experiments were presented several days after the experiment by one of the attendees.
- c. A topical report on a related subject was presented by each professor in attendance.
- d. Several correlated lectures by outside persons were presented to extend and complement the other lectures and the laboratory experiments.

In general a formal lecture or discussion period occupied the first hour of each day.

The Laboratory Experiments

The following list presents the twenty-two experiments made available to the professors (twenty-one were performed):

I The Argonaut Reactor Series

1. Introduction and Rod Calibration*. The students were given an opportunity to understand the reactor and to operate it individually. Kinetic behavior of the reactor on period measurement and rod drops was used to calibrate the reactor control rod.
2. Total Neutron Cross Section*. A beam of neutrons from the reactor was used to measure total cross section of several materials.
3. Resonance Escape Probability Measurement*. In which by use of cadmium covered depleted uranium and activation methods, the value of p for the Argonaut reactor was measured.
4. Neutron Age in Water. By measurement of the distribution of epi thermal neutrons in water and correction for leakage, the value of the Fermi age, τ , was determined.
5. Neutron Diffusion Length in H_2O . In a well-defined geometry, the distribution of thermal neutrons was measured and from this the value of the diffusion length, L , was calculated.
6. Fast Fission Factor Measurement. Using uranium of two enrichments and "catcher" foil techniques, the relative fission rate in U^{235} and U^{238} is determined.

7. Void Coefficient of Reactivity. A quantitative determination of a void coefficient for the Argonaut reactor was the object of this measurement.
8. Argonaut Reactor Transfer Function*. The transfer function was evaluated by oscillating the reactivity of the Argonaut through a range of frequencies.
9. Argonaut Reactor Controller. With the information from the previous experiment, a controller is specified and the transfer function of the closed loop measured.
10. Neutron Flux Measurement by Induced Activation Methods. Thermal, resonance and fast flux are measured by various techniques including threshold detectors.

II The AGN-201 Reactor Series

11. Introduction and Approach to Critical*. Measurement of reactor multiplication and projection to infinite multiplication at critical was carried out.
12. Danger Coefficient of Reactivity*. Effective reactor cross section of several absorbers was determined by comparison methods.
13. Importance Function for Neutron Absorption*. The spatial distribution of reactor sensitivity (statistical weight) to absorbers was measured.
14. Activation Cross Section. The absorption cross section for indium was determined by irradiation in known flux and absolute counting was performed.
15. Reactor Power Calibration. From the measured average neutron flux the fission rate and consequent power output was calculated.
16. Neutron Temperature Measurement. The effective temperature of the Maxwellian distribution was determined by both absorption and transmission techniques.

III Experiments Associated With Other Reactors and Auxiliary Equipment

17. Standard Pile Experiment. The Argonne standard source is used to calibrate standard counters and to measure the efficiency of a BF_3 counter.
18. Temperature Coefficient of Reactivity. (Experimental writeup for the Argonaut reactor, experiment performed on CP-5 reactor.) Reactor is heated and cooled, and the change of reactivity observed on calibrated control rod.
19. EBWR Simulator. An analogue device in which reactor startup, xenon poisoning, etc. is synthesized.
20. Analogue Computer Experiment. Actually a series of two experiments in which the kinetic equation is set up and various problems in the reactor behavior solved.
21. Measurement of β/ℓ by Pile Noise Technique. This experiment was scheduled for the Experimental Boiling Water Reactor; a late change in its operation schedule due to a turbine failure prevented its performance, however.
22. Isotope Production and Half Life Determination. A sample of sodium, irradiated in the CP-5 reactor, was used to demonstrate the principle of qualitative isotopic analysis using its decay period and radiation for identification.

The Formal Lecture Series

The following subjects were discussed by speakers from Argonne National Laboratory.

1. A series of lectures on "The Nature and Application of Research Reactors"
2. "Critical Facilities as Used to Make Reactor Physics Measurements"
3. "Reactor Transfer Function Measurement"
4. "Radiation Shielding"

The Topical Reports

A five-page writeup of assigned topical material was prepared by each professor and was mimeographed for inclusion in the Laboratory Manual of the Institute. The same material was presented orally at one of the morning lecture periods. The subjects for these reports were selected to extend the experimental and lecture material; a list of titles follows:

1. Neutron Velocity Selectors
2. An Analytic Experimental Technique for Predicting Criticality
3. Principles of Reactor Control
4. Cross Section Measurement Using the Pile Oscillator Technique
5. Resonance Integral Evaluation
6. Measurement of Temperature Coefficient of the CP-5 Reactor
7. Neutron Detection by Activation Methods
8. Considerations Involved in Reactor Moderator Selection
9. Measurement of Diffusion Length by Pulsed Neutron Source Technique
10. Neutron Spectra in Reactors
11. Calculation of Xenon Build-up Using an Analogue Computer
12. Calculation of Reactor Kinetic Behavior Using an Analogue Computer
13. Boiling Water Power Reactors
14. Relation Between Temperature and Void Coefficients of Reactivity
15. Application of Transfer Function Measurements to Reactor Design & Operations

In general the first nine experiments (marked with asterisks) were performed by each of the professors in the Reactor Laboratory Institute; the experiments were scheduled for them in advance, and were performed during the first two weeks of the Institute. The schedule for the last period of the Institute was made to suit the specified interests of the attendees.

By careful scheduling and by holding classes on Saturdays, all but three of the 140 expressed requests were met.

2. Student List

University Professors in attendance June 19 - July 14

Abele, Ernests Dept. of Physics Ohio Northern University Ada, Ohio	Rothschild, Kurt Dept. of Physics Montana State College Bozeman, Montana
Burnham, George H. Dept. of Physics Norwich University Northfield, Vermont	Shadduck, Hugh A. Dept. of Chemical Engineering Youngstown University Youngstown, Ohio
Clack, Robert W. Dept. of Nuclear Engineering Kansas State University Manhattan, Kansas	Stewart, Brother Joseph Dept. of Electrical Engineering Manhattan College Riverdale, New York 71, New York
Herbst, John J. Dept. of Nuclear Engineering New York University University Heights Bronx 53, New York, New York	Wilbur, Leslie C. Dept. of Mechanical Engineering Worcester Polytechnic Institute Worcester, Massachusetts
Liverhant, Salomon E. Dept. of Science Maritime College State University of New York Fort Schuyler, New York 65, N. Y.	Williamson, Thomas G. Dept. of Nuclear Engineering University of Virginia Charlottesville, Virginia
Rehm, Thomas R. Dept. of Chemical Engineering University of Denver Denver 10, Colorado	Wilson, John A. Dept. of Physics Air Force Institute of Technology Wright-Patterson Air Force Base Dayton, Ohio
Reiser, Castle O. Dept. of Chemical Engineering Arizona State University Tempe, Arizona	Wilson, William E., Jr. Dept. of Chemical Engineering University of Washington Seattle 5, Washington
Roberts, Carlyle J. Dept. of Applied Biology Georgia Institute of Technology Atlanta 13, Georgia	

3. Staff Participation List

The following persons were associated with the 1961 Institute at Argonne National Laboratory. The nature of their participation is briefly described.

William J. Sturm, Technical Director - Responsible for the development of many of the reactor experiments in the Institute; selection, organization of all experiments, lectures and moderator of discussion sessions.

Jerome E. Baird, Program Coordinator - Responsible for integration of the Reactor Laboratory Institute into the International Institute and Argonne National Laboratory. Scheduled experimental and recreational programs for the participating professors.

H. Casson - Presented experiment on absolute flux measurement in Standard Pile.

W. H. McCorkle - Delivered a series of lectures on research reactors and their applications.

G. S. Pawlicki - Presented experiments on Argonaut controller and delivered a lecture and experiment on Argonaut reactor transfer function.

W. C. Redman - Lectured on reactor physics measurements as made in critical facilities.

D. A. Daavettila - Presented experiments on resonance escape probability and fast fission factor measurements.

W. F. Gilbert - Presented experiments on total neutron cross section, danger coefficient, and activation cross section.

M. Grotenhuis, Associate Director of International Institute Division - Presented a lecture on reactor shielding.

J. M. Paul - Assisted in the Argonaut transfer function measurement and controller experiment.

K. C. Ruzich - Presented experiments on approach to critical with the AGN-201 reactor, importance function measurement and power calibration, and experiment on measurement of fast flux by threshold detectors.

- J. H. Talboy, Jr. - Presented an experiment on the EBWR simulator.
- J. J. Becker - Assisted in experiments at the Argonaut reactor.
- J. A. Beidelman - Presented temperature coefficient experiment at the CP-5 reactor.
- L. T. Bryant - Presented basic principles of and kinetic equation solutions at the analogue computer.
- A. J. Cassel - Presented an experiment on isotope production and activation analysis measurement.
- J. W. Concagh - Assisted in experiments at the AGN-201 reactor.
- R. R. Farrell - Assisted in measurements at the Standard Pile.
- A. G. Januska - Presented a demonstration of radiation safety instrumentation and instruction in its use by personnel working about reactors.
- D. A. Jones - Assisted in experiments on Argonaut transfer function and reactor controller.
- L. C. Just - Aided in the presentation of kinetic equation solutions at the analogue computer.
- L. L. Lawyer - Presented experiments on rod calibration, neutron age in water, and neutron diffusion length.
- V. H. Shoemaker - Assisted in all experiments at the AGN-201 reactor.
- J. Spalek - Presented experiments on neutron temperature measurement.
- R. Springer - Assisted in experiments at the Argonaut reactor.
- E. R. Stansberry - Assisted in the presentation of the CP-5 temperature coefficient experiment.

REACTOR LABORATORY EXPERIMENTS

Proceedings of 1961 ASEE-AEC Institute

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R.L.I. EXPERIMENT NUMBER 1

ARGONAUT INTRODUCTION AND APPROACH TO CRITICAL

INSTRUCTOR: REACTOR STAFF

ABSTRACT

The Argonaut reactor was first brought to criticality early in 1957, and since that time, operational experience has proven it to be of good design and well suited for both training and research purposes. There are now about twenty Argonaut-type reactors in the world which are either complete or under construction, and more are being planned. The reactor design and safety features are discussed in the Appendix.

The Argonaut is introduced by tour and lecture; then the students perform a complete reactor checkout and the approach to critical experiment using a control rod. This is followed by a brief study of some reactor kinetics, and an experimental determination of some delayed neutron parameters.

ARGONAUT INTRODUCTION AND APPROACH TO CRITICAL

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - 1. Reactor Checkout
 - 2. Approach to Critical
 - 3. Supercritical Operations
 - 4. Delayed Neutrons
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - 1. Discussion Questions
 - 2. Nucleonics Reprint on the Argonaut Reactor

I. INTRODUCTION

The aim of the first part of the experiment day is to acquaint the student with some of the details of the reactor design through a tour and lecture, and to introduce the operation and safety aspects of the reactor by means of a complete reactor checkout. The second part of the day is devoted to the approach to critical experiment using a control rod, and to neutron density behavior studies.

The approach to critical experiment using a control rod, although of lesser practical significance, duplicates in essence the principal concepts and procedures followed during an approach using fuel, which is an important reactor operation discussed in the next paragraph. The main difference in procedure of the two approaches is the removal of poison versus the addition of fuel, but in either case, the net result on the reactor is the same because both add positive reactivity primarily by increasing the thermal utilization. During the approach to critical experiment, it is interesting to observe the neutron static and kinetic behavior during subcritical operation, and how the behavior changes as criticality is approached and later passed.

Whenever the critical mass is uncertain or some change has been made in core geometry that might possibly affect significantly the critical mass to an unknown degree, an approach to critical experiment is made to determine critical mass. Starting usually with no fuel in the core, the neutron population due to the source is measured. Then a small fuel addition is made and the population re-measured. This is followed by another fuel addition, but now from the change in population, the critical mass can be

predicted according to the theory developed in the next section. More fuel additions are made, and after each addition the critical mass can be predicted with increased accuracy. The caution and accuracy of the approach, which depend on the particular problem and conditions, are reflected in the size and number of the fuel additions. As the critical mass is very closely approached, the neutron population increases tremendously over source level and becomes difficult to measure, however, by this time the critical mass is well known.

The initial approach with a new reactor is a check on the accuracy of the reactor calculations, and insures that more fuel (reactivity) is not loaded in than can be safely controlled. One can conclude that the approach to critical experiment is an important experiment in that it is the first operational step with all reactors, and it is a necessary procedure from time to time thereafter. About 5 to 10 per year are made at the Argonaut.

II THEORY

A schematic view of the reactor is shown below:

Let C_0 , C_1 , and C_2 (neutron count rates) be respectively related to Y_0 , Y_1 and Y_2 (control rod positions), and let C_0 and Y_0 be the initial values. The absolute value of C_0 is not significant as it depends on detector location and associated electronics. All count rates are normalized to C_0 so that the increase in count rate is given by $M = C_i / C_0$, where M is the multiplication.

If the reactor is in the condition that only the negative reactivity of a control rod is holding it sub-critical, the reactor will approach critical as the control rod is removed and at some control rod position the reactor will reach critical. In order to duplicate the fuel approach the aim here is to predict the critical control rod position -- starting with the rod in the down position. Each removal step will introduce positive reactivity just as if a few grams of U^{235} were added because both act primarily through the

thermal utilization factor "f" to increase k_{eff} . The relationship of k_{eff} is shown by the fundamental reactor equation:

$$k_{eff} = \eta \epsilon p f \times \text{non leakage probabilities.} \quad (1)$$

When $k_{eff} = 1$ the reactor is critical.

The equation relating k_{eff} and multiplication is readily derived from the sketch below where time unit 1 is the present and all others precede it. At each time unit Q neutrons were introduced.

The population at time unit 1 is given by

$$MQ = Q + kQ_2 + k^2 Q_3 + k^3 Q_4, \quad (2)$$

or rewritten as below after cancelling out the Q

$$M = \frac{1}{1 - k_{eff}} \quad \text{for } k_{eff} < 1 \quad (3)$$

Note from equation (3) that as k_{eff} approaches one, M approaches infinity, and $1/M$ would approach zero. If a plot of $1/M$ versus rod position (reactivity) is made and the curve extrapolated to a $1/M$ value of zero, that control rod position must be such that $k_{eff} = 1$, hence the critical rod position. Or if fuel was being added, it would be the critical mass since the abscissa would be in grams of U^{235} or in some other fuel weight units.

For this experiment, 4 neutron detectors will be used, two of which are cadmium covered to decrease their sensitivity. During the course of the experiment, the new rod position will be determined based on data from these four detectors. The emphasis is on a slow conservative approach to critical just as it would be for an actual approach.

III. APPARATUS

Reactor and associated instrumentation for neutron counting. At the Argonaut BF₃, proportional counters are used.

IV. PROCEDURE

1. Reactor Checkout: To assure safe operation of the reactor, a formal sequence of operation to follow to start the reactor up is demanded. We will follow a sequence which assures the operating personnel that all safety considerations have been properly provided for.

2. Approach to Critical: When the checkout has been completed, take the fine and coarse control rods to the positions specified by the instructor. After waiting an appropriate length of time, record rod positions, temperature, and initial count rates of the four neutron detectors on the provided data sheets in Section V. The four neutron detectors are called A, B, C, and D, and their count rates are C_a , C_b , C_c , and C_d .

Withdraw one of the control rods a fraction of the way out while maintaining extreme vigilance to preclude the possibility of the reactor becoming supercritical without your knowledge. Wait for steady state, then record rod position, count rate on counter A (C_a), inverse count rate relative to the initial count rate, (C_{a0}/C_a), similar count rates and inverse count rates for each of the counters B, C, and D, and the most conservative estimated critical rod position as indicated by the data.

An attempt will be made to complete the experiment with enough time remaining to discuss any questions which you have not had answered and to solidify the results of the experiment.

3. Supercritical Operation: After attaining criticality at as low a power level as possible, remove one of the control rods to a position which will yield about a 60 second period. Then plot power level as a function of time for about five minutes. How long does it take after the rod movement for the period to become asymptotic?

4. Delayed Neutrons: A short study of delayed neutron parameters. Make the reactor critical at approximately 100 watts. When you are satisfied that all transients have died out, scram the reactor. Make a semi-log plot of the resulting flux as a function of time until the flux is as indicated by the lowest range on Linear #1 is within about 50% of chart zero. Then analyze the resultant curve to determine the delayed neutron parameters, i.e., periods and approximate relative abundances. Only the three longest periods can be resolved.

V. RESULTSInitial Conditions:

Rod Positions: Course _____ Fine _____ Shim _____

Water Temperature: _____

 $\underline{C_{ao}}$ $\underline{C_{bo}}$ $\underline{C_{co}}$ $\underline{C_{do}}$

Count Rates: _____

Approach to Critical:

Position	$\underline{C_a}$	$\frac{C_{ao}}{C_a}$	$\underline{C_b}$	$\frac{C_{bo}}{C_b}$	$\underline{C_c}$	$\frac{C_{co}}{C_o}$	$\underline{C_d}$	$\frac{C_{do}}{C_d}$	Estimated critical rod position

VI. ANALYSIS AND CONCLUSIONS:

This is left as an exercise for the students, but you should include your curves of $1/M$ versus rod position.

VII. BIBLIOGRAPHY:

1. "Elements of Nuclear Reactor Theory," S. Glasstone and M. C. Edlund, (D. Van Nostrand Co., Inc., 1952), pp. 194-198, 220-222, and 356 and 361.

Experiment Written By: D. Daavettila

VIII. APPENDICES

1. DISCUSSION QUESTIONS

The following prepositions should be considered prior to your arrival at the Argonaut. An effort will be made to discuss them just prior to and/or during your checkout of the reactor, as time permits.

1. What is the importance to a neutron chain reacting system of:
 - a) fission
 - b) fission in U^{235} yielding greater than 1 n
 - c) the (n, 2n) reaction
 - d) Carbon (graphite)
 - e) heat
 - f) the (γ , n) reaction
 - g) U^{238}
 - h) neutron induced fission in U^{235}
 - i) spontaneous fission
 - j) H_2O
2. What is the importance of delayed neutrons? Are they necessary to a neutron chain reaction? What is their practical import? Assume that there are no delayed neutrons, how would you start the reactor?
3. Assuming a subcritical assembly containing a source of thermal neutrons of strength Q, establish the relationship between source multiplication of the system and its k_{eff} in steady state.
4. Conventionally, the reciprocal of counting rate at a fixed position in the assembly is plotted versus amount of fuel or versus withdrawal of poison. Relate these variables to those in the multiplication equation.
5. When plotting inverse count rate as a function of % of rod withdrawal, for an initial approach to critical, will you use a linear, semi-log or log-log plot?
6. Do you expect the above curve to be a straight line, concave or convex?
7. What do we mean by "critical" and what criterion may we use to determine that this condition exists in our assembly?
9. Assume a small step insertion of reactivity. What will be the reactor response after the initial transient?
10. Among the products of fission, there exist particles which are larger, and some which are more highly ionizing than neutrons, also gamma rays and heat. In view of this, why do we select to measure the neutron population?

11. If boron trifluoride counters are being used for an initial approach to criticality, would you expect their gamma sensitivity to appreciably affect the results? Their location? Their sensitivity? Why?
12. Of what importance is the artificial neutron source to reactor startup and operation?
13. In addition to massive concrete shielding, what other precautions are taken to prevent radiation exposure?
14. What action should be taken if, during the course of this experiment, you feel the detectors are not indicating the true situation in the reactor?
15. For the initial approach to criticality, in the case of a high power reactor designed to operate at elevated temperatures, what precautions must be taken that are not of concern to us at the Argonaut?
16. What difficulties do you foresee with this experiment?

2. ARGONAUT--ARGONNE'S REACTOR FOR UNIVERSITY TRAINING

Nucleonics 15(3), 62-65 (1957)

FIG. 1. Model of completed reactor shows light-concrete shielding A, heavy-concrete shield above core B, shield-test facility C, core D, thermal column E, and lines leading to dump and storage tank F

FIG. 2. Reactor configuration consists of external and internal graphite reflectors and annular core containing fuel, graphite, and water

Argonaut—Argonne's Reactor

Annular core provides greatest flexibility for experimenters.

Dump system and gas sparge increase the safety of this reactor, which is inherently safe due to its temperature characteristics

By R. H. ARMSTRONG and C. N. KELBER *Argonne National Laboratory, Lemont, Illinois*

INSTRUCTION AND RESEARCH in reactor physics involve studies of (a) spatial, temporal, and energetic distributions of neutrons in chain-reacting systems and (b) the interactions of neutrons with matter. For studies of the first kind the usual high-flux research reactor is not suitable. It is generally encumbered with massive heat-removal systems and permanent experimental ports that preclude use of the reactor as a measuring instrument or a flexible demonstration assembly.

On the other hand, zero-power reactors have proved to be vital facilities for experiments pertaining not only to design problems, but also to general

problems in reactor physics. They have also been, almost without notice, the most effective instruments for training reactor physicists. It is important that the distinctions just made be considered, since there are fundamental differences in application between high-flux and low-flux reactors, even when cost is not considered.

In designing a system for university application in reactor physics, ease and safety of operation, versatility, and economy of construction have been considered paramount. The primary criteria for choosing among competitive systems are that an experiment performed by a student must yield results that are reasonably accurate if the student has done his work reasonably well, and that a student wishing to do original work should not be thwarted by an inflexible reactor.

The major requirement of an experiment is that it be sensitive to changes in the parameter examined and insensitive to changes in other quantities. In

this respect a reactor is not usually an ideal system, since it is almost indiscriminately sensitive. Nevertheless a reactor with an annular geometry offers, inside the annulus, an ideal spot for testing components. Such a geometry is also excellent for conducting internal exponential experiments—a technique that promises much in the way of aid and comfort for the reactor engineer.

Over-All Description

Argonaut is a thermal, heterogeneous, and water-cooled reactor. The design is based on an MTR-type loading of an annular core reflected internally and externally by graphite. Safety is inherent in the core type by virtue of the temperature characteristics. Additional safety has been introduced by several design features. In unrestricted locations the reactor will operate safely at powers up to 10 kw and thermal-neutron fluxes up to $\sim 10^{11}$ n/cm²/sec. A reliable cost esti-

Read Also

- A. T. Biehl, et al. Compact, low-cost reactor emphasizes safety. *Nucleonics* 14, No. 9, 100 (1956)
- Two firms to sell Argonauts. *Nucleonics* 15, No. 1, R8 (1957)
- Small 5-watt reactor is for teaching, research. *Nucleonics* 14, No. 10, 98 (1956)

TABLE 1—Characteristics of the Argonaut

Maximum power	10 kw
Maximum thermal flux	10^{11} n/cm ² /sec
Critical mass	3.7 kg of U (20% enriched)
Type of core	Annular, heterogeneous
Core size	36 in. o.d., 24 in. i.d., 24 in. high
Moderator	90 gal H ₂ O + 476 lb graphite
Internal reflector (graphite)	24 in. diam × 46 in. high
External reflector (graphite)	62 in. square × 48 in. high
Main shield	72 in. ordinary concrete (150 lb/ft ³)
Shield over lattice	12 in. barytes concrete (220 lb/ft ³)
Over-all width (across flats of hexagon)	17 ft
Over-all height	~7 ft
Temperature coefficient of reactivity	$-1.065 \times 10^{-4}/^{\circ}\text{C}$
Water circulation rate (max)	~6.5 gpm
Storage-tank volume	275 gal
Utility requirements	
Electricity	40 kw, 110-v a-c, single phase
Water	2 gpm, 75° F

TABLE 2—What Does It Cost?

Building additions	\$ 9,041
Graphite	26,771
Startup source (Sb-Bc)	250
Heat exchanger	175
Pump	135
General piping, valves, tank	3,591
Reactor vessels	1,359
Shielding	4,845
Control units	1,876
Structure	1,953
Fuel assemblies	2,830
Shield-test facility	1,200
Electronics and instruments	15,000
Loading mechanisms	5,650
Miscellaneous	2,011
Total	\$76,693
Argonaut building, slab, and foundation	\$20,670

for University Training

mate including building is \$100,000. Table 1 lists the essential characteristics. Figure 1 shows a model of the reactor, and Table 2 is a cost breakdown.

An annular geometry offers more volume in which to conduct experiments than other types. This is at the cost of a large critical mass since the shape is fundamentally a leaky one. However, since 20%-enriched uranium is being released in generous quantities for research purposes, it is felt that critical mass is important only in the way it affects safety and operation, not as a prime consideration.

The design has been developed on a do-it-yourself philosophy. It requires a minimum of skilled personnel for erection and tryout. Low cost figures result from designing wherever possible to use standard, off-the-shelf items. Basic design of Argonaut stemmed from the KAPL TTR, now NTR (1).

Mechanical Design

The Argonaut assembly is a graphite cube with a central annular core. The annulus is formed by two concentric aluminum tanks. In its center is a cylindrical graphite reflector that can be removed and replaced by a portion of a lattice assembly for internal exponential experiments. The top of the reactor is kept free of mechanisms so as

to be accessible for loading fuel assemblies, insertion of experiments in the central reflector, and possible installation of external subcritical assemblies.

A total of 22 experimental ports are available. Five are in the central reflector as shown in Fig. 2. Against one face of the graphite cube is a graphite thermal column 5 ft wide, 5 ft long, and 4 ft high (Fig. 1). This has 15 horizontal experimental ports. Ordinary concrete blocks are stacked against the two adjacent faces, each of which has one experimental port.

Opposite the thermal column is a mobile flat-bed truck, on which is mounted a removable water tank that is 4 ft wide, 6 ft long, and 3½ ft high. It is designed for shielding studies with an aqueous medium. When it is removed, solid materials can be stacked on the truck bed for testing. A hand jack is provided to move the truck toward the lattice and away.

Core

The core annulus contains fuel elements, graphite, and water. The 204 fuel plates are arranged in 12 clusters of 17 plates per cluster. Moderation is provided by graphite segments between clusters and water between fuel plates in each cluster. The graphite segments—waterproofed with Krylon aluminum paint—are hung with alumi-

num mounting brackets from the edge of the aluminum outer reactor vessel.

Fuel plates are as shown in Fig. 3. They are made of 20%-enriched U₃O₈ suspended in an aluminum matrix. By the use of a box jig they are easily assembled in clusters and secured by aluminum screws.

An antimony-beryllium source provides neutrons for startup. It consists of an antimony plug in a beryllium cube that is moved into the reflector zone by an electric drive when it is to be used. It produces 10⁸ neutrons/sec.

Water System

A water-circulation system serves for heat removal and also, by means of a quick-dump arrangement and a gas sparge, for safety shutdown.

Figure 4 is a circuit diagram of the water system, and Fig. 5 shows its components in their actual locations. Tap water flows into a 5-gal/hr laboratory mixed-bed ion exchanger and then to the aluminum dump tank to supply cooling water for the reactor. This cooling water flows through the tube section of the heat exchanger. A separate circuit provides cooling water for the shell of the heat exchanger. Flow rate through the shell is controlled by a solenoid valve actuated by thermal switches in the reactor outlet line.

FIG. 4. Flow diagram of water and gas-sparging system

FIG. 3. Fuel assembly is cluster of 17 aluminum plates with dispersed U_2O_3

Only about 125 gal of makeup water is necessary to operate the reactor when graphite is used in the center of the annulus. Extra tank capacity is available in case the experimenter wants to remove the central graphite column and the inner aluminum tank and substitute an entirely different core.

The tank has an overflow line and an opening in the top for insertion of apparatus when this is desired. Its outlet feeds through a strainer into the suction side of a $\frac{1}{4}$ -hp stainless-steel centrifugal pump* rated at 20 gpm against a 14-ft head. Discharge of the pump is connected through an adjustable flow-control valve and a check valve to the tube side of a stainless-steel shell-and-tube heat exchanger† with 4.3 ft² of surface. From the heat exchanger the stream passes through a heater to the inlet of the reactor tank. The heater provides moderator-temperature control. It is energized by a series of thermal switches and a snap-action controller in the reactor outlet line.

Coolant is pumped upward through the reactor, over a weir, and back into the main dump tank. Its level is monitored by two safety float switches,

one to indicate when the water has reached the operating level and one to stop the pump if the level becomes too high.

Cooling by natural convection is possible at low power. The normally closed solenoid valve between tank and heat exchanger opens, dropping the water level and permitting circulation through the shorter circuit.

Emergency shutdown. Two completely independent systems are provided for emergency shutdowns. A dump line empties the coolant-moderator into the dump tank. Simultaneously a nitrogen sparging system introduces bubbles to reduce moderator density.

Welded to the bottom of the main reactor tank is a 6-in. aluminum dump line that leads to the dump-and-storage tank. The line passes through a 6-in. rubber-lined, electrically operated butterfly valve that is normally held closed by a magnetic clutch. If the clutch is de-energized, a weighted lever arm opens the valve and empties the water into the dump tank. Torque required to open the valve is ~ 78 in.-lb.

The supplementary safety system introduces nitrogen through a series of 35-micron porosity spargers. The gas passes through a quick-opening valve

to produce a rapid decrease of moderator density. The gas system is made fail-safe by using the nitrogen pressure to hold closed a 1-in. moderator drain.

Shielding

The area directly above the cubic lattice is shielded by a $5 \times 5 \times 1$ -ft concrete-filled steel box. A 3-ft-diameter stepped plug is located directly over the core, and in this plug are smaller plugs for the insertion of fuel assemblies and test materials into the central column. All plugs contain heavy concrete. In high-power experiments additional bricks of ordinary concrete will supplement the heavy-concrete shield as shown in Fig. 1. For moving shielding components a 3-ton jib crane with a 15-ft boom are is provided.

Shielding above the lattice and the mobile truck is supported by a structural assembly made by bolting together 6-in. flanged aluminum beams. In addition a 5-ft $\times$ 5-ft $\times$ $\frac{1}{2}$ -in. aluminum plate is bolted over the lattice, and several 6 $\times$ 6-in. wooden beams are laid side by side above the shield-test tank. These pieces form an adequate structure that can be dismantled and reassembled with simple tools if it is desirable to move or modify

* Eastern Industries Model WA.

† Ross Heater Div., American Standard, no. 301-2, 34,000-Btu/hr rating.

the reactor. On the 4 inner columns the loading is 10,000 lb/column, and on the 2 outer ones it is 4,700 lb/column. Maximum bending stress on the horizontal beams is 7,000 psi. Aside from the column loads the floor loading is 1,500 lb/ft².

Instrumentation

The reactor is operated from a console that contains the following standard instrumentation: 2 strip-chart recorders; 1 logarithmic-amplifier period and power trip; 2 d-c linear-amplifier high-level trips; 2 counting channels, one with rate meter, both with scalers, linear amplifiers, and pulse-height discriminators; 2 area gamma monitors; 1 master rotary switch for startup.

Because the reactor is inherently safe, the major control problem has been to design cheap, effective secondary control mechanisms. This is complicated by our desire to leave the top face free for external subcritical assemblies. Several satisfactory control systems were proposed and tested before a final selection was made.

The control system initially used is a mechanism consisting of a small rotary solenoid operating through a magnetic clutch and brake to a steel cable. The cable moves a 7 × 7-in. plate assembly that contains cadmium as poison.

For a training reactor a startup pattern that does not require unusual skill or lengthy training for mastery is highly desirable. For Argonaut a sequential system of interlocks has been designed so that the operator must bring the reactor to critical in a specified manner. Each operation is controlled through one central selector switch. If a step is omitted, or an interlock not satisfied, the operator cannot go on to the next step. The rotary startup switch passes to the following 9 positions in the order listed: source, safety rod 1, safety rod 2, safety rod 3, dump valve, water level, shim rod, coarse control rod, fine control rod.

Safety

In addition to the previously discussed safety features that have been built into the water system, Argonaut has inherent safety in the temperature characteristics of its core. The group of such inherently safe reactors includes the Materials Testing Reactor and Borax at Arco and the Low-Intensity Testing Reactor and the Bulk-Shield

Experiments with Argonaut

With the standard Argonaut system one can perform the following kinds of experiments:

PILE-OSCILLATOR MEASUREMENTS and danger-coefficient studies in the central column.

EXTERNAL EXPONENTIAL EXPERIMENTS on the top or side of the system (10⁶ thermal neutrons/cm²/sec).

CRITICAL MASS DETERMINATIONS on the annular core with various geometries and concentrations.

FUEL STANDARDIZATIONS and instrument calibrations in the various irradiation spaces.

SHIELDING STUDIES in the shielding tank.

IRRADIATIONS of fuel elements to get internal flux distributions, etc., in the thermal column.

Test Facility at Oak Ridge. Borax tests have demonstrated that in reactors of this type it is difficult to cause a nuclear runaway even when safety circuits are inoperative (2).

With normal reactor loadings multiple failure of safety devices could permit an excursion, but it would be terminated automatically by the increase of moderator volume and void formation, without any release of fission products to the environment.

The only dangerous situation that we can conceive would occur if a large mass of reactive material were dropped into the internal thermal column with fuel in the annulus and water up. This accident could stem only from premeditated folly, since standard procedure of unloading the annulus and dropping the water while making an internal change is enforced by multiple interlocks and made convenient by the system design. Thus, the nuclear system of Argonaut is safe if the user takes only minimal precautions against grossly unsafe practices.

Operating Personnel

Competent personnel are required to supervise Argonaut operation. Initially, of course, only people with previous training in existing installations would be competent supervisors, although such training ought to be an important phase of student activity. Experience at Argonne has been that an experimental physicist can be checked out as a pile operator after about four months' training; students working with an Argonaut can probably be trained in about four to six months depending on their skills. Further experience at the International School of Nuclear Science and Engineering should yield more information on this.

The operator or supervisor has following responsibilities:

1. Monitors, trips, and safety devices must be checked operationally for proper function. If these devices are not working properly, the responsible person must have authority to shut down the reactor until necessary repairs are made.

2. All experimental proposals must be reviewed with regard to their safety, and all experimenters must be taught safe operation.

3. Checks must be made that experiments are being safely performed.

An internal exponential experiment converts Argonaut into a much different reactor. Hence, such experiments must be subjected to a separate safeguard review unless they differ in only minor respects from previously tested systems.

Part of the safety program is the provision that the excess reactivity built into the lattice be limited, and that all fuel be under control of one responsible person. The interlock and trip circuits are of conventional design for research reactors with a few novel additions. Chief among them is a measurement of the source-flux multiplication prior to raising the shim rods for criticality. This prevents introducing too much excess reactivity into the system at the beginning of an experiment. A burglar alarm prevents personnel from working on the top of the reactor when it is operating.

* * *

This article is a compilation of work done by the Argonaut group: D. H. Lennox, project engineer, R. H. Armstrong, F. Bewersdorf, C. N. Kelber, W. H. Kolb, A. Selep, and B. I. Spinrad, with the generous assistance of the other divisions of Argonne National Laboratory. The help of Rudi Yu Tung Yang is particularly appreciated.

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R.L.I. EXPERIMENT NUMBER 2

AN INTRODUCTION TO THE AGN-201 REACTOR AND
AN APPROACH TO CRITICAL

INSTRUCTOR: REACTOR STAFF

ABSTRACT

This experiment is designed to familiarize the participant with the AGN-201 reactor and its operation. The experiment includes a detailed description of the reactor, performance of the daily reactor checkout and an approach to critical on the fine control rod. The participant will acquire an adeptness for safe reactor operating techniques in steady state and transient situations both for a subcritical and critical reactor.

AN INTRODUCTION TO THE AGN-201 REACTOR AND

AN APPROACH TO CRITICAL

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - 1. Reactor Checkout
 - 2. Approach to Critical
 - 3. Period Measurements
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - 1. Discussion Questions
 - 2. AGN Characteristics
 - 3. Nucleonics Reprint, "Compact, Low Cost Reactor Emphasizes Safety"

I. INTRODUCTION

Since the use of a nuclear reactor as an experimental machine requires a thorough knowledge of its physical makeup, the first part of this experiment will be devoted to a description of the reactor components and equipment and a visual inspection of the facility. A daily reactor checkout will then be performed to introduce the operation and safety aspects of the reactor. The reactor checkout includes a step by step check of the reactor conditions, the neutron monitoring instruments and the scram circuits.

The concepts involved in an approach to critical with the fine control rod will be brought out in the latter part of the experiment. The approach to critical will be performed with the intention of showing the participant what is occurring in the reactor core during startup. The experiment will actually simulate the last steps of a criticality experiment, which is the basic experiment performed to determine the critical mass of a new system. The final result of the experiment will yield the position of the fine control rod at which the reactor becomes a self sustaining neutron chain reacting system.

II. THEORY

When a neutron source is placed near a subcritical reactor, the fissions induced in the fuel multiply the source neutrons entering the core. The multiplication is defined as the ratio of the total thermal neutron flux, due to source neutrons and fission neutrons to the flux due to the source alone. The degree of multiplication is dependent upon the magnitude of the reproduction factor of the assembly. The equation relating k_{eff} and multiplication is readily derived from the sketch below where Q neutrons were introduced each second.

The population at the present time (1) is given by,

$$Q + k_{\text{eff}}Q + k_{\text{eff}}^2Q + k_{\text{eff}}^3Q + k_{\text{eff}}^4Q + \dots$$

The multiplication, M , of the source neutrons as previously defined equals

$$M = \frac{Q + k_{\text{eff}}Q + k_{\text{eff}}^2Q + k_{\text{eff}}^3Q + \dots}{Q}$$

Since this expression has been derived for a subcritical reactor, $k_{\text{eff}} < 1$ the expression for the multiplication may be rewritten after cancelling out the Q

$$M = \frac{1}{1 - k_{\text{eff}}}$$

It can be noted from this equation that as k_{eff} approaches one, the multiplication approaches infinity and $1/M$ would approach zero.

The experimental arrangement for the approach to critical on the fine control rod can be seen in the diagram below,

The measurement of the neutron population at the detector location can be assumed to be proportional to the thermal neutron flux in the reactor core. With the fine control rod at the edge of the core, the reproduction factor equals $(k_{\text{eff}})_0$. Let C_0 equal the measurement of the neutron population by the detector. If the rod is moved into the core to a desired position, the reproduction factor becomes $(k_{\text{eff}})_i$ and the new neutron population measurement C_i . With all neutron measurements normalized to C_0 , the multiplication at any rod position is given by $M_i = C_i/C_0$. A linear plot of the inverse of the multiplication verses rod position can be extrapolated to a $1/M$ value of zero where the fine control rod is in its critical position. The linear extrapolation is based on the assumption that the change in the reproduction factor per centimeter movement of the fine rod is constant. This assumption has been found to be true only after the rod has been moved past 8 cm of its total 23.1 cm. travel.

When a reactor is made supercritical, $k_{\text{eff}} > 1$, the neutron population will begin to rise to higher levels. This rise in neutron level can be described as a summation of exponentials

$$\phi(t) = \sum_{i=0}^m A_i e^{\omega_i t}$$

Where the A_i 's are constants determined by the initial conditions of the reactor and the ω_i 's are the $m + 1$ roots of the fundamental inhour equation.

The rate at which the flux rises is dependent upon the magnitude of k_{eff} , the amount that k_{eff} is above unity. All the terms in the expression above, except the first, have negative exponents when positive reactivity is added, and their contribution to the flux decreases rapidly to zero after a short time has elapsed. This summation then reduces to,

$$\phi(t) = \phi_0 e^{\omega_0 t}$$

where ϕ_0 = flux at $t = 0$

ω_0 = the inverse of the reactor period

FIG. 1 - REACTOR ASSEMBLY

The positive asymptotic period $T = 1/\omega_0$, can easily be determined from a plot of neutron population versus time. The period is defined as the time it takes the neutron level to increase by a factor of e , after the transient terms have died out. Another important parameter is the doubling time. This is defined as the time it takes the neutron level to double. It is related to the period by the following expression:

$$T = \frac{t_2}{\ln 2} \text{ where } t_2 \text{ is the doubling time}$$

It is possible to determine the reactivity $(k_{\text{eff}} - 1)/k_{\text{eff}}$ of the assembly which corresponding to a certain measured positive period through the used of the basic inhour equation. This equation, derived from the time dependent diffusion equation, involves the delayed neutron parameters and the thermal neutron lifetime of the reactor in question.

III. APPARATUS

- A. Reactor and associated instrumentation for neutron counting
- B. External BF_3 count system to be placed in access port number four.

IV. PROCEDURE

1. Reactor Checkout - The daily reactor checkout will be performed. The checkout will follow a sequence of checks to assure that the reactor equipment is functioning properly.

2. Approach to Critical - After the reactor is taken to its subcritical position (safety rods in), the fine and coarse control rods are inserted to the position specified by the instructor. After waiting an appropriate length of time record rod positions, temperature and the initial readings for the following instruments:

- Channel 1. Count rate meter
- Channel 2. Logarithmic micro-microammeter
- Channel 3. Linear micro-microammeter
- Channel 4. BF_3 counter (placed in access port #4)

Insert the fine control rod a fraction of the way in while maintaining extreme vigilance to preclude the possibility of the reactor becoming supercritical without your knowledge. Wait for steady state, then record rod

FIG. 2 - CORE ASSEMBLY

position, temperature, and instrument readings. Determine the inverse of the multiplication relative to the initial instrument readings. Plot the inverse of the multiplication versus rod position for each instrument. Estimate the critical rod position as indicated by this plot.

Select the next rod position on the basis of the previous data. Proceed in this fashion, approaching criticality in a decreasing incremental fashion. Record your final estimated critical rod positions.

3. Positive Period Measurement - After obtaining criticality insert one of the control rods to a position which will yield a period of about 100 seconds. Determine this period by measuring the doubling time of the power level on Channels 1 and 3. After rising to about 10 milliwatts return rods to their critical positions and assure that the power remains constant. Are the critical rod settings the same as you estimated? From the chart recorder which has monitored the power rise, plot the power level versus time on semilog paper. Does the period remain constant with time? Why?

V. RESULTS

The student is to make his own data sheet. Include with your results the plot of inverse multiplication versus rod position and of power level versus time.

VI. ANALYSIS AND CONCLUSIONS

A brief analysis of the experiment together with significant conclusions is left as an exercise for the student.

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Experiment Written By: K. Ruzich

VIII. APPENDICES

1. DISCUSSION QUESTIONS

The following propositions should be considered prior to your arrival at the AGN-201. An effort will be made to discuss them prior to and/or during your checkout of the reactor, as time permits.

1. What is the importance to a neutron chain reacting system of:
 - a) fission
 - b) the $(n, 2n)$ reaction
 - c) the (γ, n) reaction
 - d) spontaneous fission
2. What is the importance to a thermal nuclear reactor of:
 - a) fission in U^{235} yielding greater than 1 n
 - b) neutron induced fission in U^{235}
 - c) a moderator
 - d) a reflector
 - e) U^{238}
 - f) heat
3. What are delayed neutrons? Are they necessary to a neutron chain reaction? What is their practical import? Assume that there are no delayed neutrons - what control problems exist?
4. If we have a subcritical reactor containing a neutron source, what will its response be to a small positive step insertion of reactivity?
5. Assuming a subcritical assembly containing a source of thermal neutrons of strength Q , establish the relationship between source multiplication of the system and its k_{eff} in steady state?
6. Conventionally, the reciprocal of counting rate at a fixed position in the assembly is plotted versus amount of fuel or versus withdrawal of poison. Relate these variables to those in the multiplication equation.
7. How will the reciprocal of the counting rate vary with the position of a control rod containing fuel? What shape do you expect this curve to have?
8. Why are the positions of the neutron detectors important relative to the source and core?
9. What do we mean by "critical" and what criterion may we use to determine that this condition exists in our assembly?
10. How does the critical control rod position vary with power level?

11. Among the products of fission there exist particles which are larger, and some which are more highly ionizing than neutrons, also gamma rays and heat. In view of this, why do we select to measure the neutron population?
12. Of what importance is the artificial neutron source to reactor startup and operation? What types of sources can be used?
13. In the AGN-201 reactor, positive reactivity is added by inserting fuel control rods in the upward direction from a position below the core. In the Argonaut reactor, positive reactivity is added by removal of absorbing control rods in the upward direction to a position above the core. In both cases, why is it desirable to add reactivity in the manner described?
14. What action should be taken, if during the course of this experiment, you feel the detectors are not indicating the true situation in the reactor?
15. In addition to the reactor shielding, what other precautions are taken to prevent radiation exposure?

2. AGN CHARACTERISTICS

The AGN-201 is a useful low-power reactor that has the distinct advantage of being of small overall size, which enables it to be readily located in existing buildings. The reactor is designed for use in education, research, medical diagnosis and industrial control. The main designing objectives were to make the reactor as safe and foolproof as possible, to use a minimum critical mass and to provide a high analytical sensitivity.

The AGN-201 is a 100 milliwatt, homogeneous, thermal reactor. The core is made up of a series of circular polyethylene discs 25 centimeters in diameter. It contains 20% enriched UO_2 and has a critical mass of approximately 650 grams of U^{235} . The cylindrical core is surrounded by a graphite reflector 20 centimeters thick and is shielded by 10 centimeters of lead and 55 centimeters of water. The core and part of the reflector are contained in an aluminum tank which has re-entrant thimbles in its base, into which four control and safety rods are inserted. Each rod contains fuel sealed in an aluminum capsule, so that reactivity increases as rods are inserted. Both safety rods and one control rod are held in position by electro-magnets for scram purposes. Each of the four rods is driven by a lead screw. The core is divided into half sections with a plane of separation occurring at the level of a one inch diameter through "glory" hole. The lower section is held in place by a thermal fuse which is designed to soften at 100°C , and when it functions as specified, permits the section to drop two inches to the bottom of the core tank. The excess reactivity is limited to a maximum of 0.25 percent or a minimum period of about 15 seconds by the amount of fuel available. The two safety

rods and the coarse control rod are each worth approximately 1.3 percent reactivity. The fine control rod, fuel loaded, is worth about 0.305 percent. The neutron flux is monitored by three detectors, two BF_3 ionization chambers and one BF_3 proportional counter, all of which are located in the water tank just outside the lead shield. These detectors are connected respectively to a logarithmic micro-microammeter, a linear micro-microammeter and a pulse amplifier and count rate meter located on the reactor console. Each indicator is connected through a sensitrol relay to a scram circuit. Additional safety interlocks provide for reactor shutdown if the level of the shield water drops, if the reactor temperature falls below 16°C , or if an earthquake occurs. Sequential interlocks are also present to ensure that the proper operational method is followed.

3. COMPACT, LOW-COST REACTOR EMPHASIZES SAFETY

Nucleonics 14(9), 100-103 (1956)

Compact, Low-Cost Reactor

For use in education, medicine, and industry, this 100-mw system consists of a 9½-ft-high reactor unit and remote control console. Installation in existing buildings requires no extra shielding

By A. T. BIEHL, R. FAYRAM, R. HENLEY, G. A. LINENBERGER, H. MONTGOMERY, R. MAINHARDT, G. NEWBY, J. RANDALL, D. SAWLE, and J. SHERWIN
Aerojet-General Nucleonics, Subsidiary of Aerojet-General Corporation, San Ramon, California

A SMALL RESEARCH reactor, the AGN-201, is designed for use in education, research, medical diagnosis, and industrial process control. It is especially intended for applications in which high thermal-neutron flux is not necessary but low cost, maximum safety, portability, and high sensitivity are required. Its basic characteristics are shown in the table on p. 101.

The reactor and control units are shown in Fig. 1. The two are connected together by a cable. Shielding

is designed to prevent overexposure of personnel even in the unlikely event of an accidental power surge. Additional safety features include power and reactivity monitors to operate a scram mechanism and a fuse that will cause the core to separate in case of a power surge.

Purposes

In designing the AGN-201, the objectives were to make the reactor as safe and foolproof as possible, to use a

minimum of uranium no more than 20% enriched (for export purposes), to provide adequate in-pile exposure volume with a thermal-neutron flux as high as possible per unit of power, to make it as small, light, and inexpensive as possible, to provide an effectively infinite life, and to provide a high analytical sensitivity for danger-coefficient tests.

ENGINEERING DESIGN

Within certain constraints, the AGN-

FIG. 2. Core tank contains uranium-polyethylene core, safety fuse, and portions of graphite reflector

FIG. 3. Reactor unit contains core, reflector, lead shield, and borated-water shield. Control and safety rods are inserted from the bottom

Emphasizes Safety

201 is a minimum-critical-mass reactor. Its critical mass is estimated as 600 gm of U^{235} in the form of 20%-enriched UO_2 . It is contained in the form of a powder dispersed in a solid polyethyl-

ene moderator. No sacrifice in safety has been made to lower the amount of U^{235} required.

The theoretically smallest critical mass of U^{235} is estimated to be under

300 gm. Such a design would use a polyethylene moderator and a thick, high-density BeO reflector. Containers, control rods, etc., would increase the critical mass. A design of this type was considered, but it was discarded because of health hazards and high cost associated with handling of BeO . A thin high-density graphite moderator replaces the BeO .

Characteristics of the AGN-201 Reactor

Power, licensed	100 mw
Power, with special provisions	3 watts
Maximum thermal flux (100 mw)	4.5×10^6 n/cm ² /sec
Critical mass	3,000 gm of U (20% enriched)
Core size (cylinder)	10 in. dia, 9½ in. high
Moderator	11 kgm polyethylene
Impregnation density (U^{235})	50 mg/cm ³
Reflector (graphite)	20 cm thick (1.70 gm/cm ³)
Shield (lead)	10 cm thick (7,500 lb)
(water with 3% HBO_3)	55 cm thick (8,700 lb)
Over-all diameter	6½ ft
Over-all height	9 ft
Over-all weight of reactor unit	22,000 lb
—with water shielding drained	13,300 lb
Temperature coefficient of reactivity	$-3.6 \times 10^{-4}/^{\circ}C$

Thermal Neutron Flux

Since the thermal neutron flux of any reactor is nearly proportional to power divided by critical mass, minimizing the critical mass for safety reasons also provides the maximal neutron flux for any given power and fission-product accumulation.

Size and Weight

The problem of size and weight of the AGN-201 depends almost entirely on the shielding to prevent radiation overexposure to personnel in a power

surge. Thus more than enough is provided for normal operation at 100 mw. Shield design is determined by estimates of the maximum conceivable accident. This is estimated as 2×10^6 joules for a 2% step change in reactivity. The maximal dose limit for personnel just outside the shield during such an accident has been chosen as 1 rem, the equivalent of three weeks at the "maximal permissible level" as established by the USAEC. With the establishment of these parameters it is then possible to design a minimal-weight shield.

Core Life

Sufficient evidence exists on the effects of ionizing radiation on polyethylene to allow a fair estimate of core life. Experiments show there should not be noticeable dimensional changes or foaming at doses less than 1,000 Mrep (10^{11} ergs/gm). Assuming that 10% of the fission energy appears as ionizing radiation in the polyethylene, the dose rate at 100 mw (10^6 ergs/sec) is about 3 Mrep/yr. Thus the core has an estimated minimal life of 300 yr at a 100-mw power level (24 hours a day, 7 days a week). The fuel burnup is almost inconsequential, amounting to some 30 μ gm/yr. Thus there ap-

pears to be no fundamental reason why a very long life can not be expected.

Core Design

To facilitate core construction and assembly, the core is made in nine sections of various thicknesses (Fig. 2). The thinner sections allow varying the amount of U^{235} in the core in small enough increments for adequate control of the critical mass for assembly and experimental purposes. Each of the four bottom disks has three 5-cm-dia holes and one 2.5-cm hole. Two are for safety rods; one is for the coarse control rod; and the small one is for the fine control rod. A horizontal "glory hole," about 1 in. in diameter, passes through the center of the reactor.

The core disks are formed from a mixture of polyethylene and UO_2 in the form of 10–20-micron-dia UO_2 particles. The total density of UO_2 is then 284 mg/cm³, yielding a weight ratio of 1 gm of UO_2 to 3.16 gm of polyethylene.

Neutron source. A 10-mc Ra-Be source is located in the graphite reflector to supply neutrons on startup. With safety rods cocked and control rods all the way out, the multiplication of the reactor is sufficient to generate 30 μ w of power and give indications on all instruments. Thus the reactor power can be safely varied by a factor of 3,000 (0.030–100 mw) by use of the control rods.

Reflector, lead shield. The reflector consists of 20 cm of high-density graphite (1.70 gm/cm³) on all sides of the core (Fig. 3). Appropriate holes are provided for the glory hole, safety and control rods, and access ports. Both core and graphite reflector are surrounded by 10 cm of lead shielding. The reflector weighs about 750 lb, and the lead shield weighs about 6,300 lb.

Access ports. A total of four access ports plus the glory hole and a thermal column are provided for experimental work. The four access ports (4-in. dia) pass completely through the reflector. The glory hole (1 in. dia) passes completely through the core. Under normal operating conditions all ports are filled with appropriate plugs or stringers to match surrounding materials.

Fluid-Tight Tanks

As shown in Fig. 3, three concentric fluid-tight tanks surround the core. They are designated, respectively, the core tank, the reactor tank, and the water tank.

Core tank. The core tank has been designed to contain any fission gases given off by the core. Commercial alloyed aluminum is the structural material throughout. The tank has upper and lower sections, separated by an aluminum disk passing through the main cylinder in the same plane as the glory hole. This disk separates the core into two parts and is part of the safety-fuse system. Detachable top and bottom cover plates, as well as control- and safety-rod thimbles, form an integral part of the tank.

The lower section of the core tank contains half of the core material as well as a two-piece cylindrical section of graphite reflector. These pieces of core material and the inner piece of graphite reflector are supported by an aluminum rod projecting downward from a fuse link supported from the bottom cover plate. Ample space at the bottom of the cylinder is provided to insure free fall of the bottom half of the core-plus-reflector section when the fuse melts in any accidental power surge. This fall is further facilitated by a graphite-to-graphite taper joint that insures against binding.

The upper section of the core tank is filled with six sections of core. A space for core expansion and gas accumulation is provided in the top of the tank.

Reactor tank. The lead shielding, reflector, and core are enclosed in and supported by a thick steel tank of 43.4-cm radius. It has a removable top cover to permit access to the core tank. This tank acts as a secondary container for the core-tank assembly and, with the glory hole and access ports closed, is completely fluid-tight.

Water tank. The water tank is the third and outermost of the fluid-tight containers. This main structural member is 6½ ft in diameter and constructed of steel. Access can be gained to the top of the core assembly, even with water in the water tank, by removing the top cover plate and thermal-column tank. The fast-neutron water shield in the water tank consists of about 1,000 gal of water containing 250 lb of dissolved boric acid.

The upper or "thermal-column" tank serves as a shield when it is filled with water and lead or as a thermal column when it is filled with graphite. It is removable to allow taking out either the cover of the core tank or the core tank itself.

FIG. 4. Control rod is driven by reversible motor through lead-screw coupling. Active length is 15 cm. Fuel is enclosed in two aluminum containers to provide double fluid-tight seal

FIG. 5. Control system has two boron ion chambers and BF_3 proportional counter for power-level monitors. Micromicroammeter amplifiers connect ion chambers to relay-and-recorder system. BF_3 counter is connected through counting-rate meter to relay system. Relays can be set to scram reactor at any predetermined power level

Safety and Control Rods

The AGN-201 reactor has two safety rods and two control rods. The two safety rods and the coarse control rod are identical. The fine control rod is similar but has half the diameter of the others. Each rod contains 0–20 gm of U^{235} and operates so that reactivity is increased as the rod is inserted. The amount of reactivity each rod controls increases as the amount of active material contained increases. With the maximum of U^{235} in the coarse rods (13.3 gm at 50 mg/cm³) each rod controls about 1.6% reactivity. The fine control rod has been adjusted to control about 0.2% reactivity.

Figure 4 shows the design of the control rod. The active fuel in the rod is enclosed in two aluminum containers. The outermost container provides the fluid seal with the core tank, and the innermost one seals the active fuel in the rod. By this design a double fluid-tight seal is maintained for the core and for control and safety rods.

Control and safety rods also serve another purpose. They provide a means of obtaining a fine adjustment of the uranium in the core, offering a convenient method of adding or removing small amounts of fuel. They are designed for use in critical experiments during startup as well as for small adjustments in available reactivity under normal operating conditions.

Safety-rod operation. The safety rods are in the safe or subcritical posi-

tion when they are out. In this position the active fuel in the rod is just inside the lead shield. The rods are inserted one at a time by a drive mechanism requiring about 1 min to travel the full 25-cm stroke. The release mechanism is constructed so that if a scram signal is received at any time, the three large rods are driven to their outermost positions.

The safety system is a fail-safe design. The scram signal opens the direct-pull holding magnets allowing the rods to be accelerated outward by both gravity and compressed springs. The spring constant is such that the rods are initially accelerated with a force of 5 g, requiring a total withdrawal time of 140 msec. The reactivity change of both safety rods is 0.9% during the first 50 msec. The rods are decelerated by a hydraulic dashpot in the last 12.5 cm of travel.

Control-rod operation. Reversible d-c motors drive both coarse and fine control rods through lead-screw assemblies. The motors are controlled by switches at the control console. At maximum speed of travel the coarse rod causes a reactivity change of about $3 \times 10^{-4} \text{ sec}^{-1}$. Positions of both control rods are indicated remotely at the control console. Interlocks prevent control-rod movement unless the safety rods are "cocked." The safety rods cannot be cocked until the control rods have reached their safe or starting positions.

Control Console

The control system is very similar to that used for other low-power reactors. It is shown schematically in Fig. 5. Scrams are initiated by a power failure, an earthquake detector, a water-shield-level detector, low-temperature detectors, a manual scram button, optional access-port interlocks, and the power-level monitor.

It can be seen from Fig. 5 that the recorder can be switched to monitor any of the neutron-detection devices. Both ion chambers are located just outside the lead shield in the borated-water neutron shield. The BF_3 counter is just outside the lead shield in the region occupied by control and safety rods.

In the event of a scram, any one of the three detectors or other safety devices actuates the scram system causing automatic and rapid ejection of both the safety rods and the control rods, and operation of both warning light and bell.

Safety

The system is designed to minimize the possibility of a nuclear runaway, and, in the event of such an excursion, to protect personnel from excessive radiation. Time constants of the scram system may not be such as to stop a fast excursion before thermal expansion of the core would catch it. On the other hand, careful analysis shows that such self-limiting does prevent liberation of a dangerous amount of energy. The action of the core fuse (made of polystyrene containing 100 mgm/cm³ of U^{235}) and the safety rods then insures against a repetition of the same accident. The core fuse will heat faster than the core and melt at 100° C.

Fission products will be contained for the most part in the core. Since this is made of polyethylene cross-linked by radiation, it will not melt at anticipated temperatures. If all the fission products created in the greatest conceivable accident were liberated in a room of 10⁶-liter capacity, estimates indicate that the level of the most dangerous isotope, I^{131} , would be just barely at a dangerous level.

* * *

The authors acknowledge the early conceptual contributions of North American Aviation, Inc., and the Applied Radiation Corp. for their evaluations of similar low-power research reactors.

4. 1961 ASEE-AEC Reactor Laboratory Institute Writeup

June, 1961

AN ANALYTICAL-EXPERIMENTAL TECHNIQUE
FOR PREDICTING CRITICALITY

Excerpted from a Paper Presented by

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Entitled

Radiation Safety Consideration for Subcritical Reactors

Presented at the 1961 Annual Meeting of the Health Physics Society

The multiplication experiment is described in References (1), (2). The curves obtained in the multiplication experiment are a function of the location of the detector and geometry of the system.⁽²⁾ The multiplication factor k_{eff} , which is the ratio of the neutrons in one generation to the neutrons in the previous generation, can be obtained from experimental measurements on one subcritical configuration. These data permit determination of k_{eff} versus size of subcritical up to and including the critical configuration according to the following analysis:

Solution of the well known one group diffusion equation (1) for a subcritical assembly delineated in Figure (1) results in

$$\phi = \sum_m \sum_n A_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sinh \gamma_{mn} (c-z) \quad (1)$$

where m and n are odd and

$$-\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 + \gamma_{mn}^2 + B_{mat.}^2 = 0 \quad (2)$$

Assuming a point source of thermal neutrons at the origin, equation (1) can be cast into the form⁽⁷⁾

$$\phi = C_E C_H e^{-\gamma_{11} z} \quad (3)$$

$$\text{where } C_E = \frac{1 - e^{-2\gamma_{11}(c-z)}}{1 + e^{-2\gamma_{11}c}} \quad (4)$$

$$\text{and } C_H = 1 + \left[\frac{\gamma_{11} \cos \frac{p\pi x}{a} \cos \frac{q\pi y}{b} \frac{1 - e^{-2\gamma_{pq}(c-z)}}{1 + e^{-2\gamma_{pq}c}}}{\gamma_{pq} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} C_E} \right] \quad (5)$$

In equation (5), p and q are odd and cannot be unity simultaneously.

Experimentally determined flux data obtained at different values of z in the center of the assembly can be used to determine γ_{11} according to the following iteration scheme. First, an approximate value of γ_{11} , $(\gamma_{11})_1$, is obtained from equation (3) by assuming $C_E = C_H = 1$; that is $(\gamma_{11})_1$ is the negative slope of the logarithm of relative flux versus z . $(\gamma_{11})_1$ is then used to calculate initial trial values of C_E and C_H , $(C_E)_1$ and $(C_H)_1$ respectively, which are then used to compute $\left(\frac{\phi}{C_E C_H}\right)_1$ from equation (3). A second, more accurate value of γ_{11} , $(\gamma_{11})_2$ is the negative slope of the logarithm of $\left(\frac{\phi}{C_E C_H}\right)_1$ versus z . This iteration procedure is continued until any desired precision of γ_{11} is obtained. The material buckling, B_{mat}^2 , is then determined from equation (2) with $m = n = 1$ using the subcritical dimensions a , b , and $c + \delta$. From the critical equation

$$1 = \frac{k_{\infty} e^{-B_{geom}^2 \tau}}{1 + L^2 B_{geom}^2} = \frac{k_{\infty}}{1 + M^2 B^2} \quad (6)$$

where $M^2 = L^2 + \tau$, k_{∞} can be determined if the assembly constants, L^2 , τ and B_{mat}^2 , are known, since $B^2 = B_{mat}^2 = B_{geom}^2$ at criticality. L^2 and τ may be determined experimentally according to References (3) and (4). The multiplication factor, k_{eff} , for the Kansas State University graphite subcritical assembly is determined from critical equation

$$k_{eff} = \frac{k_{\infty} e^{-B_{geom}^2 \tau}}{1 + L^2 B_{geom}^2} \quad (7)$$

where the factor B_{geom}^2 is determined from the equation

$$B_{geom}^2 = 2 \frac{\pi^2}{a^2} + \frac{\pi^2}{(c + \delta)^2} = (2.55) \frac{\pi^2}{a^2} \quad (8)$$

By choosing different dimensions of a up to and including criticality, it is possible to determine k_{eff} versus a and thus to anticipate the nature of the change of k_{eff} as the size of the subcritical assembly is increased. Such a curve for the Kansas State University assembly, Figure (2), is shown in Figure (3). A logic diagram, Figure (4), for the IBM 650 digital computer for the analysis of data to determine k_{eff} as herein described is included.

A prime tenet of any adequate radiation safety program requires that calibrated survey meters sensitive to each type of ionizing radiation which

might be encountered shall be available for use. Since subcritical reactors involve the escape of gamma, fast and thermal neutron radiation, instruments of the appropriate type were calibrated to determine the contributions to the total dose rate from each type radiation.

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Figure 1 Sketch of a Subcritical Assembly

Figure 2 Graphite moderated subcritical reactor designed and built at Kansas State University.

Figure 3 Effect of size, a , on multiplication factor k_{eff} .

Figure 4 Logic diagram for IBM 650 program for material buckling, B_{mat}^2 , and multiplication factor, k_{eff} .

R.L.I. EXPERIMENT NUMBER 3

CONTROL ROD CALIBRATION

INSTRUCTOR: REACTOR STAFF

ABSTRACT

It is the purpose of this experiment to calibrate an Argonaut control rod in terms of reactivity versus rod position by means of the reactor kinetic equations and to observe other examples of kinetic reactor behavior. Both the "positive period" and "rod drop methods" are utilized and an intercomparison of the two methods made.

CONTROL ROD CALIBRATION

OUTLINE

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I. INTRODUCTION

When attempting to quantitatively determine the relative worth of various reactor parameters, it is convenient, if not absolutely necessary, that the effect of a variation in those parameters be assigned some standard unit of effect on the reactor. The effect most commonly used is that on reactivity which can be expressed in any of several units. An additional convenience is obtained by using a control element as a calibrated secondary standard.

The reactivity equivalent of a control rod may be obtained, at least roughly by calculation, but ultimately the values must be checked by direct measurement. In an earlier experiment we studied the subcritical operation of the reactor and determined criteria for criticality. Using this knowledge, we want in this experiment to investigate the reactor's kinetic behavior as it deviates from criticality, both for positive reactivities ($k > 1$) and negative reactivities ($k < 1$) and, by means of the reactor's response to a control rod's movements, to calibrate the control rod in terms of reactivity.

II. THEORY

A. General:

In the following section, it will be our purpose to establish those relationships which govern the reactor flux response to simple transients in reactivity in order that we may measure the magnitude of those transients. We wish to find a quantitative relationship between flux and reactivity. From our knowledge of the fission process and of reactors, we would imagine that this relationship would also be time and space dependent, but we can essentially eliminate these dependences by fixing time and space; i.e., by considering only one value of time and space.

The theory necessary to this experiment is more thoroughly treated in reference (6). The following comments are designed to present these derivations in an abbreviated and less rigorous fashion.

Our approach will be to assume a very general relationship between flux and reactivity and then to find the limitations on and the exact function of the reactivity. Then, in order to find these limitations, we begin by establishing the diffusion equation for thermal neutrons in a non equilibrium system. We next modify that diffusion equation for the effect of delayed neutrons and simplify. This gives us a differential equation involving flux and delayed neutron precursor density as space and time variables. Next, we write an equation for the net rate of formation of the delayed neutron precursors. This gives us two linear first order differential equations so that we may restrict the reactivity and time functions which we originally assumed. Then by imposing initial conditions, we can eliminate the delayed neutron precursor density from the modified diffusion equation. Finally we simplify the modified diffusion equation and arrange it in the standard "characteristic inhour equation" form. This equation relates our original function of reactivity to some of the nuclear properties of the materials present in the reactor.

B. Derivation;

As mentioned above, we begin by assuming our flux versus reactivity equation in a general way. We would expect this to be an exponential function involving reactivity and time in addition to the reactor's nuclear properties.

Let us assume it to be of the form

$$\phi = f_0(\rho) e^{g_0(\rho) h_0(t)} + f_1(\rho) e^{g_1(\rho) h_1(t)} + \dots + f_i(\rho) e^{g_i(\rho) h_i(t)} + \dots + f_n(\rho) e^{g_n(\rho) h_n(t)}. \quad (1)$$

It then remains for us to find the functions f_i , g_i , and h_i .

To determine these functions, we begin by writing the diffusion equation for thermal neutrons in a non-equilibrium system:

$$D \nabla^2 \phi - \Sigma_a \phi + S = \frac{1}{v} \frac{\partial \phi}{\partial t} \quad (2)$$

Here $D \nabla^2 \phi$ is the neutron leakage per cm^3 per second, $\Sigma_a \phi$ is the number of thermal neutrons per cm^3 per second being absorbed; and

$$\frac{1}{v} \frac{\partial \phi}{\partial t} = \frac{\partial n}{\partial t}$$

where $\phi = nv$.

S is the thermal neutron source term due to fissions, or

$$S = k \Sigma_a \phi e^{-B^2 \tau}, \quad (3)$$

where k = the infinite multiplication constant and $e^{-B^2 \tau}$ is the non leakage probability during slowing down.

Since we are considering a critical reactor, we may assume that there is no primary source present. However, since we are interested in the effects of delayed neutrons, we must consider our source term to be composed of two parts -- one the source of prompt fission neutrons and one the source of delayed fission neutrons. If we let β represent the fraction of the total number of fission neutrons that are delayed and β_i the fraction that are associated with a single type of delay (a single precursor) then $(1-\beta)$ is the fraction of fission neutrons that are emitted promptly. We can then write our prompt source term as before, except that it must be decreased by a factor of $(1-\beta)$. An additional source term must be included for the delay neutron source. The rate of formation of delayed neutrons of the i th kind is equal to the rate of decay of its precursor which is $\lambda_i C_i$ where λ_i is its decay constant and C_i is its density. Thus, the total rate of formation of delayed neutrons will be $\sum_{i=1}^m \lambda_i C_i$. The thermal neutron source term due to delayed neutrons is just this rate of formation times the resonance escape probability times the non leakage probability. The two source terms are then

$$\begin{aligned} S &= \text{prompt source term} + \text{delayed source term} \\ &= (1-\beta) k \Sigma_a \phi e^{-B^2 \tau} + p e^{-B^2 \tau} \sum_{i=1}^m \lambda_i C_i. \end{aligned} \quad (4)$$

Thus our modified diffusion equation is

$$D \nabla^2 \phi - \Sigma_a \phi + [(1-\beta) k \Sigma_a \phi e^{-B^2 \tau} + p e^{-B^2 \tau} \sum_{i=1}^m \lambda_i C_i] = \frac{1}{v} \frac{\partial \phi}{\partial t} \quad (5)$$

where ϕ and C_i are functions of both space and time.

Assuming space and time variables separable (which requires that reactivity be changed in a step fashion) $\nabla^2 \phi$ may be replaced by $-B^2 \phi$ in equation (5). The resulting equation will then simplify to the differential equation

$$[k_{\text{eff}}(1-\beta) - 1] \phi + \frac{\rho k_{\text{eff}}}{k \sum_a} \sum_{i=1}^m \lambda_i C_i = \ell \frac{d\phi}{dt} \quad (6)$$

by dividing through by $\sum_a$, substituting L^2 for $\frac{D}{\sum_a}$, dividing through by $1 + L^2 B^2$, and introducing the definitions

$$\text{and} \quad k_{\text{eff}} = \frac{k_e - B^2 \tau}{1 + L^2 B^2}$$

$$\ell = \frac{\ell_a}{1 + L^2 B^2} .$$

where ℓ_a = neutron lifetime in an infinite medium

ℓ = neutron lifetime in a finite medium

Equation (6) is a differential equation involving flux and delayed neutron precursor density as space and time variables. Our next step will be to eliminate the delayed neutron precursor density by writing an additional differential equation for the net rate of formation of the delayed neutron precursors and simultaneously imposing initial conditions on these two differential equations.

Since the overall rate of production of fission neutrons, including both prompt and delayed, is $\frac{k}{\rho} \sum_a \phi$, then the rate of formation in the fission process of the precursors of the i^{th} group is $\beta_i \left(\frac{k}{\rho}\right) \sum_a \phi$. Also, since C_i is the concentration of these precursors, their rate of radioactive decay is $\lambda_i C_i$ ($-\lambda_i C_i$ rate of formation). Then the net rate of formation of the delayed neutron precursors of the i^{th} group is

$$\frac{\partial C_i}{\partial t} = \beta_i \frac{k}{\rho} \sum_a \phi - \lambda_i C_i \quad (7)$$

where again C_i and ϕ are functions of both time and space.

We now wish to solve the differential equations (6) and (7). Their solution will be of the form

$$\phi = \phi_0 e^{\omega t}$$

and

$$C_i = C_{i0} e^{\omega t} , \quad (8)$$

which restricts the functional relationship originally assumed in (1) in that

$$h_i(t) = t \quad .$$

By substituting the relationships (8) into equation (7) we may evaluate C_{i0} in terms of ϕ_0 , ω , and the reactor constants. Substituting this result together with equations (8) into (6) the result is

$$k_{\text{eff}}(1-\beta) - 1 + k_{\text{eff}} \sum_{i=1}^m \frac{\lambda_i \beta_i}{\omega + \lambda_i} = \ell \omega \quad (9)$$

Using the definitions

$$k_{\text{ex}} = k_{\text{eff}} - 1$$

and

$$\rho = \frac{k_{\text{ex}}}{k_{\text{eff}}} = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}}$$

(9) reduces to the "characteristic inhour" equation

$$\rho = \frac{\ell \omega}{\ell \omega + 1} + \frac{1}{\ell \omega + 1} \sum_{i=1}^m \frac{\omega \beta_i}{\omega + \lambda_i} \quad (10)$$

which is seen to be an algebraic equation of degree $m + 1$ in ω , so that there are $m + 1$ values of ω corresponding to each value of the reactivity ρ . Thus, we now see that in equation (1) $g_i(\rho)$ is just the parameter ω_i and that m refers to the number of delayed neutron precursor groups. A plot of equation (10) is given as figure 2 in Appendix II. Equation (1) may now be rewritten in the form given in Appendix II, i.e.,

$$\frac{\phi(t)}{\phi(0)} = \sum_{j=1}^7 A_j e^{\omega_j t} \quad (11)$$

where

$$A_j = (1-\rho) \frac{\frac{\beta_j}{(\omega_j + \beta_j)}}{1 + \sum_{i=1}^6 \frac{\beta_i}{(\omega_j + \beta_i)}} = \frac{f_i(\rho)}{\phi_0} \quad (12)$$

Here ϕ_0 is the flux just before a step change in reactivity and $\phi(t)$ is the time dependent flux after the step change.

C. Positive Reactivities:

If the reactivity is positive, i.e., $k_{\text{eff}} > 1$, Figure 2, Appendix II shows that of the $m + 1$ roots of equation 10, m of them are negative and one is positive. We also note that each of the negative ω 's numerically is of the same order as one of the decay constants, λ_i . Therefore, as the time, t , after

the step change increases, the contributions of all terms beyond the first on the right hand side of equation (11) decrease rapidly to zero, because of the negative ω values, the first term increases with time because of its positive ω value. Hence, after a very short time, equation (11) reduces to

$$\frac{\phi(t)}{\phi(0)} = A_0 e^{\omega_0 t}$$

We may now define the stable reactor period or the asymptotic reactor period, T , as

$$T \equiv \frac{1}{\omega_0}$$

where T is the reactor period after the lapse of sufficient time for the contributions of the transient terms to damp out. A plot of this period versus the reactivity required to produce it for the Argonaut is given as Figure 3 in Appendix II.

D. Negative Reactivities:

Again referring to Figure 2, Appendix II, if ρ is negative, all $m + 1$ values of ω are negative. Equation (11) will then consist of $m + 1$ terms all of which have negative exponents. The flux ratio will thus decrease with time. The first term, with the smallest numerical value of ω , (ω_0) will decrease at a slower rate than the others. Therefore, for any input of reactivity (negative), the reactor will finally yield a negative stable period equal to $\frac{1}{\omega_0}$. There is presented (as Figure 5 in Appendix II) the numerical solution in graphical form of equation (11) for negative reactivities.

III. APPARATUS

The control element to be calibrated is one of the Argonaut control rods normally used for control of the reactor. Data sheets are provided as part of Section V. Working models of the curves presented as Figures 3 and 5 in Appendix II will be provided. No additional equipment is necessary.

IV. PROCEDURE

Figure 5 of Appendix II can be used to determine the negative reactivity input required to produce a given flux response as a function of time. Use of these curves presupposes that the reactor is in a "one slab loading" condition and that the assembly is critical. If these conditions are fulfilled, one need merely cause the control rod being calibrated to be inserted fully and rapidly into the assembly and to read the flux response of the system from detection instruments and strip chart recorders provided. This procedure will be iterated with successively larger reactivity input, noting the portion of the rod's travel over which it moves, until finally the entire rod travel is used as an input. The reactivity values thus obtained can be plotted versus the

position from which the rod began to fall to obtain an integral calibration or "worth" curve.

An alternate method of calibrating the rod is as follows:

The reactor is made critical with the rod to be calibrated in its most fully inserted position. The rod is then withdrawn a known amount and after waiting for the flux to be represented by the dominant term (~ 90 seconds for most practical values of reactivity) the e-folding time of the reactor flux is measured. A control rod other than the one being calibrated is then used to return the reactor to criticality at a low flux. The rod being calibrated is then again withdrawn a known amount and the above procedure repeated. The reactivity equivalents of the periods can then be found from the computed curve (Fig. 3) in Appendix II.

It should be noted that, while calibrating the control rod under consideration, the control rod being used to compensate is also in effect being calibrated.

Using the available systems for insertions of positive and negative reactivities, the reactor's response to various types of transients may be observed and qualitatively analyzed. A discussion of a typical transient will be conducted prior to the experiment in response to Question 5 of Appendix I.

V. RESULTS

The following sample portion of a data sheet will serve as a guide to assist in the collecting of the data. Data sheets can be made available at the time of the experiment.

CONTROL ROD CALIBRATION

ROD DROP DATA

Chart zero _____ Shim rod _____ Coarse rod _____ Fine rod _____ to _____

Time after drop (sec)	ϕ gross	ϕ net	$\phi t / \phi_0$	ρ (\$)	ρ av. _____ \$ = _____ %
0					
10					
20					
30					
40					
50					

VI. ANALYSIS AND CONCLUSIONS

The integral control rod calibration curve should be qualitatively compared with the curve predicted in Question 4 of Appendix I. In addition, the values of reactivity obtained from the "period measurement" method should be quantitatively compared with those obtained by the "rod drop" method.

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Experiment Written By: L. L. Lawyer

VIII. APENDICES1. DISCUSSION QUESTIONS

The following propositions should be considered prior to your arrival at the Argonaut. An effort will be made to discuss them just prior to and/or during your checkout of the reactor as time permits.

1. What are the basic variables involved in control rod calibration?
2. Define the following:
 - a. multiplication factor
 - b. reactivity
 - c. inhour
 - d. $\% \Delta k/k$
 - e. \$ (dollars)
3. Of what use is a control rod calibration curve?
4. The change in reactivity due to the insertion of an absorber at any point in a reactor may be taken to be proportional to the square of the thermal neutron flux at that point. On this basis, derive the general shape of a predicted integral control rod calibration curve of reactivity versus the extent of withdrawal of the control rod. You may assume the flux to have a cosine distribution in the direction of rod motion and that the flux perturbation due to the rod is not severe.
5. Assume the Argonaut to be critical with the source out. At time zero, a step insertion of $-.1\% \Delta k/k$ is made and at time t the reactor is returned to critical by inserting $+.1\% \Delta k/k$. Make a rough sketch of ϕ as a function of time.
6. Would you expect the control rod calibration to be a function of the power level at which it is measured?
7. Explain the discrepancy between Figure 6.3 in Reference 3 and Figure 11.10 in Reference 6.
8. By the terms "shadowing" or "interaction" between control rods, we mean that the worth of the fine rod over a given interval of its travel is a function of the coarse rod position and vice versa. Would you expect the fine rod worth over this interval to be directly or inversely related to the coarse rod position (in $\%$ of withdrawal)? Why?

9. What considerations dictate the design of control rods? Of safety rods? Shim rods?
10. Name as many methods as you can conceive of controlling a reactor.

2. DETERMINATION OF TIME BEHAVIOR OF NEUTRON DENSITY AND
OF REACTIVITY ON THE ARGONAUT REACTOR

Nuclear Science and Engineering 6(3), 233-237 (1959)

Determination of Time Behavior of Neutron Density and of Reactivity on the Argonaut Reactor¹

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Received April 10, 1959

Theoretical and experimental results of the time behavior of neutron density as a function of both positive and negative step changes in reactivity are presented. The theoretical results are obtained from solutions of the space-independent kinetic equations of a bare thermal reactor based on the Fermi continuous slowing down model and using six groups of delayed neutrons. Theoretical results are given as a function of both positive and negative step changes in reactivity. Experimental results of reactivity worth and of rod calibrations based on pedagogical experiments with the Argonaut Reactor and verifying the theoretical data are presented together with the details of the pedagogical experiment. An analytically constructed thermal flux function obtained from results of reactivity measurements in the reactor is compared with the actual recorded flux from the reactor. Experimental results obtained with the Argonaut Reactor indicate that the theoretical kinetic behavior predicted in this paper are applicable to the actual Argonaut Reactor.

THEORETICAL DEVELOPMENT

The kinetics discussed in this paper refer to a "one-slab" loading (Fig. 1) of a bare thermal reactor, and in general are not applicable to a "two-slab" loading where the cores are kinetically coupled (1).

Solution (2, 3, 4) of the space-independent kinetic equations

$$\begin{aligned} \frac{dn(t)}{dt} &= \frac{\rho - \beta}{l(1 - \rho)} n(t) + \sum_{i=1}^6 \lambda_i C_i(t), \\ \frac{dC_i(t)}{dt} &= -\lambda_i C_i(t) + \frac{\beta_i}{l(1 - \rho)} n(t) \end{aligned} \quad (1)$$

for the neutron density n and the delayed neutron precursor density C_i for step changes in reactivity

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¹ Based on work done under the auspices of the U.S. Atomic Energy Commission. This paper is one of a series of seven which is now being published as an internal report at Argonne National Laboratory. The original report is entitled "Pedagogical Applications of an Argonaut Reactor." ANL 5989.

$\rho(= (k_{eff} - 1)/k_{eff})$ lead to the characteristic "in-hour" equation

$$\rho = \frac{\omega l}{1 + \omega l} + \frac{1}{1 + \omega l} \sum_{i=1}^6 \frac{\omega \beta_i}{\omega + \lambda_i}, \quad (2)$$

where l is the prompt neutron lifetime, β_i is the fraction of neutrons in the i th delayed group and λ_i is the corresponding precursor decay constant. Eq. (2) is a 7th degree polynomial equation in ω . That is, for particular neutron delay parameters β_i and λ_i , and for a particular prompt neutron lifetime l , Eq. (2) gives 7 distinct roots for ω for each numerical value

TABLE I
DELAYED NEUTRON PARAMETERS
(Prompt Neutron Lifetime = 2.00×10^{-4} Sec)

Group, i	Fraction, β_i	Decay constants, λ_i seconds ⁻¹
1	2.50×10^{-4}	1.246×10^{-2}
2	1.60×10^{-3}	3.150×10^{-2}
3	2.13×10^{-3}	1.535×10^{-1}
4	2.40×10^{-3}	4.560×10^{-1}
5	8.50×10^{-4}	1.612
6	2.50×10^{-4}	14.300

FIG. 1. Schematic representation of Argonaut Reactor core, safety and control rods, fuel loading, and internal thermal column.

FIG. 2. Negative step changes in reactivity, ρ dollars, versus negative roots, ω , seconds⁻¹, of characteristic equation.

of the reactivity ρ . An expression for the flux ratio $\phi(t)/\phi(0)$, where $\phi(t)$ is the flux at time t after the step change in reactivity and $\phi(0)$ is the steady-state flux level at criticality just prior to a step change, derived from Eqs. (1) and (2) is

$$\phi(t)/\phi(0) = \sum_{j=1}^7 A_j e^{\omega_j t}, \quad (3)$$

where

$$A_j = (1 - \rho) \frac{l + \sum_{i=1}^6 \beta_i / (\omega_j + \lambda_i)}{l(1 - \rho) + \sum_{i=1}^6 \beta_i \lambda_i / (\omega_j + \lambda_i)^2}. \quad (4)$$

The numerical data were calculated using the Hughes delayed neutron parameters given in Table I

from the equations as follows²: Figures 2 and 3—Eq. (2), Fig. 4—Eq. (4), and Fig. 5—Eq. (3).

USE OF ANALYTICAL CURVES

For a given numerical value of a negative step change in reactivity ρ , Figs. 2 and 4 can be used to determine numerical values of roots ω_j of the inhour equation and coefficients A_j of the flux ratio equation from which one can analytically construct the flux ratio versus time function using Eq. (3).

Positive reactivity can be measured, after transients have become negligible, by measuring the reactor period T (equal to $1/\omega_1$) from data obtained

² An IBM 650 code for solving these equations using other parameters is available from the Department of Nuclear Engineering, Kansas State University, Manhattan, Kansas.

from the flux level chart recorder of the reactor and by reading the reactivity from Fig. 3. In measuring reactivity by positive period measurement, it is necessary to wait until the flux is essentially represented by the dominant term $A_1 e^{\omega_1 t}$ of the flux ratio Eq. (3).

Fig. 3. Positive reactivity, ρ dollars, versus positive asymptotic period T seconds.

Fig. 4. Coefficients A_j of flux equation versus negative step changes in reactivity ρ dollars.

Fig. 5. Flux ratio $\phi(t)/\phi(0)$ versus negative reactivity ρ dollars at time t seconds after step insertion of reactivity.

Fig. 6. Flux ratio $\phi(t)/\phi(0)$ versus time t seconds after rod drop (Step change in reactivity of -0.27 dollars).

Figure 5 can be used to determine the magnitude of a negative reactivity step from data obtained from the flux level chart recorder without waiting for the transients to die out. Values of the flux ratio, $\phi(t)/\phi(0)$ determined from the flux level chart of the reactor for particular times t after the step change in reactivity, are used in Fig. 5 to determine the magnitude of the step. It is evident that the reactivity read from these curves for each value of the flux ratio after a particular step must be essentially the same if the experimental data and the theory are in agreement.

REACTIVITY MEASUREMENTS

A flux ratio $\phi(t)/\phi(0)$ versus time t curve obtained from the Argonaut reactor for a step³ change in reactivity equal to -0.27 dollars is shown in Fig. 6. This reactivity was determined using Figure 5. From this known reactivity, Figs. 2 and 4 were used to determine the following numerical values of ω_j and A_j :

j	A_j	ω_j
1	0.295	-0.101
2	0.260	-0.019
3	0.153	-0.100
4	0.058	-0.348
5	0.017	-1.480
6	0.011	-13.800
7	0.206	-48.200

Numerical values calculated from Eq. (3) were plotted on Fig. 6 to show the excellent consistency of the analytical with the experimental result. The individual terms of Eq. (3) were also plotted on

³ The time required for a complete "rod drop" on the Argonaut Reactor is 0.28 sec.

TABLE II
ROD DROP DATA

Rod dropped	Rod position ^a before drop (% of total travel)	Rod position ^a after drop (% of total travel)	Reactivity worth (\$) (\$)
Fine control rod	100	0	0.568 ± 0.011
Fine control rod	18	0	0.0465 ± 0.001
Fine control rod	26.5	0	0.113 ± 0.002
Fine control rod	34	0	0.164 ± 0.002
Fine control rod	42	26.5	0.143 ± 0.002
Fine control rod	63.5	42	0.194 ± 0.003
Fine control rod	75	63.5	0.0672 ± 0.002
Fine control rod	90	75	0.0392 ± 0.000
Fine control rod	100	90	0.0076 ± 0.001
Coarse control rod	100	0	1.30 ± 0.05
Safety rod #3	100	0	1.29 ± 0.05

^a "0" indicates position of maximum insertion of control element and "100" indicates position of maximum withdrawal.

Fig. 6 to illustrate their transient behavior. The two terms $A_6e^{w_6t}$ and $A_7e^{w_7t}$ are not shown since their contribution cannot be plotted on this scale. Figure 6 also illustrates that measurements of step changes in reactivity using negative asymptotic period determinations are usually not practical because of the relatively long time required for the transients to disappear. Representative negative step changes in reactivity accomplished by rod drops³ on the Argonaut reactor are presented in Table II. At least five observations of flux ratio were taken to determine reactivity using Fig. 5 for each step insertion of reactivity shown in Table II.

CONTROL ROD CALIBRATION

It is possible to calculate theoretically the effectiveness of a control rod, but the calculations are quite difficult in all except a few idealized lattice configurations. Hence, the effectiveness of a control rod is usually determined empirically. Since it is desirable to know the change in reactivity effected by a particular change in control rod position, the control rods are usually calibrated in terms of reactivity.

There are two methods of control rod calibration most often used at present. One method is to partially withdraw a control rod and then to determine the reactivity change by measuring the resulting "stable period." The other method is to rapidly in-

sert, either partially or completely, a control rod into the core and follow the change in flux level as a function of time. This is the "rod drop" method.

A calibration of the fine control rod of the Argonaut has been performed by both methods and is shown in this paper. Although the results given in the preceding section show that the total reactivity worth of a control rod can be measured with a single drop, the reactivity worth distribution over the entire travel of the control rod, and the shape of its integral worth curve, can only be determined by successive period or rod drop measurements of relatively small magnitude.

For obtaining an integral worth curve by the rod drop method, the following procedure is most generally used. The rod to be calibrated, in this case the fine control rod, is withdrawn completely. The reactor is then made critical with one of the other control elements. The flux level at criticality, $\phi(0)$, is noted and the fine control rod is then dropped part way into the core. The flux level $\phi(t)$ as a function of time after the rod drop, is noted for several minutes after the drop. The negative reactivity of this drop can then be found from Fig. 5. The reactor is again brought to criticality with another control element and the procedure is repeated until the fine control rod is completely inserted and thus completely calibrated.

FIG. 7. Fine control rod calibration curve: Integral reactivity worth ρ dollars of fine control rod versus fine control rod position.

The integral worth curve can also be obtained by positive period measurements. In this case, the fine control rod is fully inserted at the beginning and the reactor is made critical with one of the other control elements. The fine control rod is then withdrawn a few per cent of its total travel. After waiting a sufficient time (see explanation of curves) for the transient terms to become negligible with respect to the dominant or "stable" one, the period can be read from the flux level chart recorder. Since the circuit feeding the recorder is a linear circuit, it is more convenient to read the doubling time (i.e., the time necessary for the flux level to increase by a factor of two) and then divide this quantity by $\ln 2$ to calculate the period. After calculating the period, the reactivity of the step is then determined from Fig. 3 (the stable period, $T = 1/\omega_1$). The reactor is again brought to critical with one of the other control elements, the fine control rod is again withdrawn a few per cent of its total travel, and the reactivity is again measured. This procedure is repeated until the fine rod is fully withdrawn and hence fully calibrated.

Figure 7 is a calibration curve for the fine control rod, constructed by using both rod drops using Fig. 5 and positive period measurements using Fig. 3.

Total reactivity worth of the fine control rod of the Argonaut reactor as measured by three different methods was as follows:

Reactivity worth of fine control rod by single rod drop measurement..... \$0.568 $\pm$ 0.011

Reactivity worth of fine control rod by successive rod drop measurements..... \$0.564 $\pm$ 0.010

Reactivity worth of fine control rod by successive period measurements..... \$0.550 $\pm$ 0.014

The experimental results shown in Fig. 7 demonstrate that the "shadowing effect" of different coarse control rod settings which are necessary to bring the reactor back to critical after each of the successive fine rod drops does not affect the integral worth of the fine rod. This is demonstrated by noting that the total fine rod reactivity worth is essentially the same as determined by either a total rod drop or by successive rod drops. Consistency of the theoretical results is also demonstrated in Fig. 7 from the fact that the fine rod calibration obtained by successive asymptotic positive period measurements is in good agreement with the rod calibration obtained by successive negative step insertions of reactivity.

Data sheets used to facilitate the taking of data discussed in this paper are presented in Ref. (4).

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PRINCIPLES OF REACTOR CONTROL

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The purpose of a control system is to provide for smooth and steady operation of the reactor under the operating conditions for which the reactor was designed. In addition the control system must provide a means for safe start-ups, for safe rise to power operating level and maintaining of that level, and for shut-down of reactor both under normal use and under emergency. In general the emergency shut-down should be such that it prevents damage to the reactor.

Since the power level of a reactor is proportional to the neutron flux, the obvious basis for control of the reactor is to vary the reactivity or the effective multiplication.

In general the methods used to vary the reactivity are: the insertion, or removal of (1) fuel, (2) moderator, (3) reflector, (4) a neutron absorber, (5) use of burnable poisons (6) introduction of voids which might be considered under (1), (2) or (3) and (7) for safety reasons a negative thermal coefficient of reactivity. A combination of the above methods is generally used in the control of most reactors.

The AGN-201 is a good example of a low-power small experimental reactor using the insertion or removal of fuel to control its reactivity.

The Argonaut uses a neutron absorber as control rods, and provides means of dumping the moderator, and introducing voids by nitrogen sparging as additional control means.

Reactors designed to operate for long periods of time obtain some control by means of burnable poisons.

Most reactors use a number of control rods or plates for start up and power level control. The rods are classified as (1) safety rods (kept in reserve for emergency shut down) (2) shim rods (for coarse control to provide adjustment for processes having a long time cycle) (3) regulating rods (for adjustment of power level and to compensate for short-term changes in reactivity).

The safety rods in general are arranged so that they may drop by gravitational means in event of drive mechanism failure, for this reason fuel rods are generally introduced into the reactor from below and absorbing rods from above.

In deciding the location of the control rods in a reactor it is important to consider the effect of the rod on the flux distribution since the overall effectiveness of two or more rods is not necessarily equal to the sum of the effectiveness of the individual rods. The coarse rods should be located symmetrically in order to maintain a uniform flux distribution while the location of regulating rods is less important since they change reactivity and hence flux by only a small amount.

The calibration of the control rod is best determined by direct measurement, as was illustrated in the experiment to determine the worth of the Argonaut control rod.

Solutions for reactivity equivalent of control rods on the basis of reactor theory have been solved for a number of special cases (see Glasstone & Edlund - Elements of Nuclear Reactor Theory Chap. XI or Murray - Nuclear Reactor Physics chap. 8.)

In general the calibration of a rod shows that at the start of insertion or when nearing complete insertion the control rod exhibits only relatively small changes in reactivity, whereas in the intermediate region the changes are about five times as great and vary linearly or almost linearly. This is due to the fact that neutron flux near the edges of the reactor is lower than that near the center of the reactor.

To augment the rods and provide a means of measuring their effectiveness various types of instruments are used in the control of reactors. Since instruments can be made to respond to changes in the neutron flux very rapidly and since flux variations must be known over a very large range (flux may vary by a factor of 10^{10} between start-up and operating level) many different types of instruments may be necessary.

In the start-up range ionization chambers and proportional counters along with log count rate meters and period meters are used. For higher fluxes fission chambers and compensated ion chambers are used in addition to log n and period meters. For power operation u-ammeters reading the ion current directly may be used to measure neutron flux or reactor power. For automatic control parallel plate ionization chambers are used because of their extremely rapid response. Boron thermopiles are often used for power indication at operating level.

Rod position indicators and means for driving rods in and out of the reactor are also essential for control of the reactor.

A general block diagram of a reactor control system might be as illustrated below. Arrow heads indicate the direction of information flow.

The operator loop affects certain instruments which pass information to the operator who in turn exerts action on control block. The automatic loop by-passes the operator and affects the control block directly.

The human operator is a valuable part of the control system because of the flexibility and variety of his response to different situations. In general his activities fall into two main groups, routine and emergency.

The routine activities include reactor check-out and start-up, change in power level or operation at fixed level, and shut-down.

Emergency activities could involve instrument failure, power failure, failure of limit controls and so forth.

The final control considerations deal with the safety system. This system begins with basic interlock circuits which require checkout, to the emergency actions required to prevent excessively high flux levels, excessively short periods of flux increase, diminished coolant flow instrument or equipment failure, and accidents.

As an integral part of the safety system the operator, when it comes to safety action, must be able to override the automatic circuits.

A safety action may be more or less severe and in high power reactors may take on several different phases of action such as fast scram, slow scram, reverse, and fast or slow set back to a predetermined level.

In small low power reactors there may be only one type of scram action for all emergencies which completely shuts down the reactor.

In summary the principles of reactor control must deal essentially with changes in reactivity of the reactor. The means of controlling these changes and instruments for measuring the amount of change are essential to smooth and steady operation of the system and certain safety precautions both automatic and operator controlled should be inherent in all nuclear reactor systems.

A good control system design should satisfy the following conditions:

- (1) In routine operation the role of the operator can be minor and straight forward with only cost limiting automatic controls.
- (2) For emergency operation the operator should be given as much scope as is consistent with safety.
- (3) At all phases of operation positive safe shut-down action should be accessible to both the operator and to the automatic safety circuit.

References:

- Glasstone - Principles of Nuclear Reactor Engineering
Chap. 6.
Murray - Nuclear Reactor Physics Chap. 8
Glasstone & Edlund - The Elements of Nuclear Reactor Theory
Chap. 11

STUDENT RESULTS

Experiment Performed By E. Abele, G. H. Burnham, S. E. Liverhant,
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CONTROL ROD CALIBRATION

ROD DROP DATA

Chart zero 4.5 Shim rod 0 Coarse rod 46 Fine rod 70 to 0

Time after drop (sec.)	ϕ gross	ϕ net	$\phi t/\phi o$	ρ (\$)	
0	94.4	89.9			ρ av. <u>0.424\$</u> = <u>0.318%</u>
10	43.2	38.5	0.428	.42	
20	33.7	29.2	0.323	.43	
30	28.0	23.5	0.262	.43	
40	24.6	19.5	0.267	.43	
50	20.8	16.3	0.180	.41	

Chart zero 4.5 Shim rod 0 Coarse rod 43.2 Fine rod 80 to 0

Time after drop (sec.)	ϕ gross	ϕ net	$\phi t/\phi o$	ρ (\$)	
0	94.5	90	0.400		ρ av. <u>0.474\$</u> = <u>0.355%</u>
10	40.5	36	0.300	0.47	
20	31.5	27		0.47	
30	26.8	22	0.244	0.47	
40	22.0	17.5	0.194	0.47	
50	19.4	14.9	0.166	0.49	

Chart zero 4.5 Shim rod 0 Coarse rod 41 Fine rod 100 to 0

Time after drop (sec.)	ϕ gross	ϕ net	$\phi t/\phi o$	ρ (\$)	
0	95.5	90.5			ρ av. <u>0.488\$</u> = <u>0.366%</u>
10	40.5	35.5	0.392	0.48	
20	30.5	26.0	0.288	0.48	
30	25.5	21.0	0.232	0.49	
40	21.5	17.0	0.188	0.48	
50	18.5	14.0	0.155	0.51	

Chart zero 4.5 Shim rod 0 Fine rod 100 Coarse rod 41 to 0

Time after drop (sec.)	ϕ gross	ϕ net	$\phi t/\phi o$	ρ (\$)	
0	96.5	92.0			ρ av. <u>0.338\$</u> = <u>0.253%</u>
10	49.5	45.0	0.490	0.332	
20	39.5	35.0	0.381	0.337	
30	33.5	29.0	0.316	0.338	
40	29.0	24.5	0.266	0.337	
50	25.5	21.0	0.228	0.346	

CONTROL ROD CALIBRATION

Critical at Coarse 100
Fine 0Coolant Temp. 79° F.
25° C.ROD DROP DATAChart zero 4.5 Shim rod 0 Coarse rod 78.1 Fine rod 20 to 0

Time after drop (sec.)	ϕ gross	ϕ net	$\phi t/\phi o$	ρ (\$)	
0	92.5	88.0			ρ av. <u>0.05\$</u> = <u>.038%</u>
10	81.0	76.5	0.870	0.05	
20	77.0	72.5	0.825	0.048	
30	74.5	70.0	0.795	0.049	
40	70.5	66.0	0.750	0.052	
50	67.5	63.0	0.715	0.052	

Chart zero 4.5 Shim rod 0 Coarse rod 65 Fine rod 35 to 0

Time after drop (sec.)	ϕ gross	ϕ net	$\phi t/\phi o$	ρ (\$)	
0	94.0	89.5			ϕ av. <u>0.160\$</u> = <u>0.12%</u>
10	65.0	60.5	0.675	0.155	
20	57.0	52.5	0.585	0.157	
30	51.2	46.7	0.520	0.160	
40	46.2	42.0	0.470	0.162	
50	42.2	37.7	0.420	0.165	

Chart zero 4.5 Shim rod 0 Coarse rod 54.6 Fine rod 50 to 0

Time after drop (sec.)	ϕ gross	ϕ net	$\phi t/\phi o$	ρ (\$)	
0	95.0	90.5			ρ av. <u>0.295\$</u> = <u>0.221%</u>
10	52.5	48.0	0.530	0.282	
20	43.2	38.7	0.423	0.298	
30	36.5	32.0	0.352	0.295	
40	32.0	27.5	0.304	0.292	
50	28.5	24.0	0.266	0.297	

Chart zero 4.5 Shim rod 0 Coarse rod 49.0 Fine rod 60 to 0

Time after drop (sec.)	ϕ gross	ϕ net	$\phi t/\phi o$	ρ (\$)	
0	94.3	89.8			ρ av. <u>0.370\$</u> = <u>0.277%</u>
10	46.0	41.5	0.464	0.365	
20	36.5	32.0	0.358	0.370	
30	31.0	26.5	0.286	0.385	
40	26.5	22.0	0.246	0.370	
50	23.5	19.0	0.212	0.360

R.L.I. EXPERIMENT NUMBER 4

ABSORPTION CROSS SECTION MEASUREMENTS BY THE
DANGER COEFFICIENT METHOD

INSTRUCTOR: K. C. RUZICH

ABSTRACT

This experiment is designed to acquaint the participant with the use of the danger coefficient method to measure the absorption cross sections of certain elements. The experiment consists of a measurement of the sensitivity of the AGN-201 reactor at the core center with a standard $1/v$ absorber and subsequent cross section measurements of a selected group of materials in terms of the reactor's sensitivity. Reactivity will be determined by measurement of positive periods with and without the material in the reactor core. The excess reactivity of the supercritical reactor is related directly to the positive period measurement through the basic inhour equation. The results of the experiment will be compared with the known thermal cross sections and a complete analysis of any discrepancies will be made.

ABSORPTION CROSS SECTION MEASUREMENTS BY THE DANGER COEFFICIENT METHOD

OUTLINE

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 - 1. Daily Reactor Checkout
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 - 2. Plot of Inhour Equation

I. INTRODUCTION

The use of a nuclear reactor to measure the absorption cross sections of certain elements was formulated early in the history of the reactor field. During the late forties, many elements with unknown or doubtful thermal neutron cross sections were being studied for possible use in nuclear reactors. There was also a definite need to measure the purity of materials which were to be used as moderators, reflectors, etc.

A critical reactor itself, in some cases, is highly sensitive to the loss of neutrons when an absorbing material is placed in its core. The material is "dangerous" pertaining to the sustention of the neutron chain reaction. The neutron absorbing material reduces the value of the reproduction factor, k , by decreasing the thermal utilization. If the material's effect on the moderating and scattering properties of the reactor can be assumed small, a measurement of the change in k , reactivity, will indicate the material's neutron absorption ability. The sensitivity of a reactor, which is defined as the change in the reproduction factor occurring when one square centimeter of absorber cross sectional area is placed at a certain position in the reactor core, can be estimated by theoretical considerations. The use of the basic reactor diffusion equations to describe the change in k caused by small changes in reactor parameters is formulated in perturbation theory. Experimentally; the sensitivity of the reactor can be measured in terms of a standard absorber. Boron, whose absorption cross section has a $1/v$

dependence, will be used for this measurement. Its cross section will be taken as the value at 2200 m/sec. The subsequent cross section measurements of $1/v$ absorbers by the danger coefficient method will be their respective values at the same velocity. For an absorber which does not have a $1/v$ dependence, the measured cross section will be for that of an equivalent $1/v$ absorber which produces the same effect in the reactor.

Many factors are involved in this experiment which will effect its results. For each material, every atom should have the same chance of absorbing a neutron. No self shielding or flux depression should be present. Materials whose cross sections exhibit low level absorption peaks in the resonance region cannot be used unless a correction can be made for resonance absorption. The effect of scattering and moderation on the reproduction factor must be assumed negligible. Any change in reactivity due to temperature or pressure fluctuation during the course of the experiment is of course undesirable.

II. THEORY

When an absorber is placed in a reactor at a certain location, the number of neutrons absorbed depends upon the magnitude of the material's cross section over the neutron energy range and the neutron flux energy distribution. The effect of the loss of neutrons on the reproduction factor, the reactivity change, is dependent on the fractional number of neutrons lost and the importance of these neutrons relative to keeping the reactor in a critical state. An absorber placed at the edge of the reactor core will not have as great an effect on k as it would if it were placed in the core center. This is true because the absorption rate would be lower, and the daughters of the neutrons on the core edge have a smaller non-leakage probability than those produced by neutrons at the core center. The maximum sensitivity of a reactor for an absorbing material can be expected to be located at the core center where the neutron flux is the highest.

The danger coefficient method for absorption cross section measurements is actually a comparison of the absorption abilities of a standard $1/v$ absorber, boron, and a material whose cross section is to be measured. By making the comparison of these two materials at a single location in the reactor's core, it is possible to ensure that the neutrons absorbed by the materials are of the same importance to the neutron chain reaction. Assuming the simplified model of a bare thermal reactor and one group diffusion theory, the following expression given by perturbation theory describes the reactivity change due to the insertion of an absorber:

$$\rho = \frac{\Sigma_a \phi^2 \Delta V}{\int \nu \Sigma_F \phi^2 dV} = \frac{\Sigma_a \Delta V \phi^2}{\nu \Sigma_F \int \phi^2 dV} \quad (1)$$

where, Σ_a = absorber macroscopic absorption cross section

ΔV = volume of the absorber

ϕ = thermal neutron flux at any instant

Σ_F = core's macroscopic fission cross section

ν = number of neutrons per fission

and, the denominator is integrated over the core volume. The ϕ^2 weighting term involves the reaction rates and the spatial importance of the neutrons in question. From this expression it can be seen that the sensitivity of a reactor is inversely proportional to the reactor volume. A reactivity comparison of a standard absorber and a material of unknown cross section will yield a ratio of the total absorption area of the absorbers ($\Sigma_a \Delta V$) providing the neutron flux spatial distribution remains constant. Flux depression in the vicinity of the absorber could be undesirable since this might change the distribution.

The neutron flux expressed in equation (1) as the thermal flux is actually a much more complicated function, with a strong energy dependence. Neutrons are produced at fission energies and then slowed down into the thermal region. A material which exhibits a $1/v$ cross section actually absorbs not only thermal neutrons but neutrons in the resonance region as well. The amount of resonance absorption will depend on the magnitude of the resonance flux which fortunately is much less than the thermal flux in a thermal reactor. The ratio of the resonance absorption to the total absorption should be less than 5% for a $1/v$ absorber. A large number of elements exhibit cross sections with a $1/v$ dependence through the thermal region and low level resonance absorption peaks in the resonance region. Materials of this type are capable of absorbing a great number of resonance neutrons. A danger coefficient measurement comparing this type of a material with boron necessitates the use of a correction factor to eliminate the resonance absorption. If the material can be assumed to be a $1/v$ absorber through the neutron energy range, no corrections are necessary since the neutrons in the resonance region affect both the sample and the standard boron identically. The cross section for boron is taken as the value at 2200 m/sec. The result of the danger coefficient measurement of a $1/v$ material will be the absorption cross section at the same velocity. If the sample does not have a $1/v$ dependence, the cross section obtained will be that of an equivalent $1/v$ absorber which produces the same effect in the reactor.

The effect of the absorbing material on the moderating and scattering properties of the reactor must be taken into consideration if a material does not have a large absorption cross section. According to perturbation theory, the scattering effect of a sample, which changes the leakage properties of the

core, is weighed against the gradient of the flux in the reactor core. The gradient of the flux at the core center is equal to zero, hence a danger coefficient measurement at this location eliminates the influence of scattering on reactivity. The moderating ability of the sample will depend on its slowing down power ($\xi \Sigma_s$). The reproduction factor will be altered through a change in the resonance escape probability of the reactor. If the slowing down power of the sample is small compared to its absorption cross section, the sample's moderating effect on reactivity can be assumed small for a danger coefficient measurement.

The reactivity changes that occur in the reactor when an absorber is placed at its center will be measured in terms of asymptotic positive periods. The stable period of a supercritical reactor is related to the reactivity through the basic inhour equation. This expression involves the prompt neutron lifetime and the delayed neutron parameters. (See Appendix 2.)

III. APPARATUS

- A. AGN-201 Reactor and console
- B. External BF_3 proportional counting system
 - a. BF_3 cylindrical counter (one inch diameter)
 - b. Preamplifier
 - c. AID amplifier
 - d. MPC scaling unit
- C. Sample materials which are homogeneously distributed in 3/4" diameter cylindrical lucite discs
 - a. Boron (H_3BO_3)
 - b. Copper
 - c. Cobalt
 - d. Cadmium (CdCO_3)
 - e. Sodium Chloride (Na Cl)
 - f. Lucite

IV. PROCEDURE

1. A daily reactor checkout will be performed. The checkout assures the operator that the reactor equipment is functioning properly.
2. The standard boron sample A is placed at the core center in the glory hole. Following the proper procedures, the reactor is brought to a near critical position at about 5 milliwatts. The source is then driven out of the reactor. By inserting the control rods, the reactor is made supercritical. The rods should be positioned to yield a positive asymptotic period

of about 1500 seconds. This can be estimated from the chart recorder monitoring channel three. The exact stable period of the reactor is determined by the external BF_3 counting system. The BF_3 counter is placed at a desired location in access port number four. The counts per fraction of a minute are then taken periodically. A plot of the counting rate versus time is made on semilog paper. The plotting should commence when the first count is taken, this will enable the participant to determine whether or not the reactor is rising on a stable period. The stable reactor period will be determined from the semilog plot and recorded along with the control rod position and temperature on the data sheet. The coarse control rod is then lowered five centimeters to a position which renders the reactor very subcritical. After the power level has dropped below 5 milliwatts, the boron standard is removed. Its removal must be performed cautiously to lessen the chance of scrambling the reactor. The reactor operator should pay close attention to the console instruments and inform the participants changing the sample of any sudden changes in the instrument readings. A lucite blank is then placed at the center of the glory hole in place of the boron sample A. Caution is again stressed in this operation, since positive reactivity is being inserted. The coarse control rod is then raised to its previously recorded position. Once the reactor has reached its stable period, a period measurement should be made with the external BF_3 counter as described above. The reactor's period should of course be shorter since the boron absorber has been removed from the core. Record control rod settings, period and temperatures. The reactor is once again brought to a subcritical position and boron sample B is inserted into the glory hole. The procedure for measuring the period is then the same as it was for the lucite blank and boron sample A. The reactivity of the reactor for each run can be evaluated from the basic equation, which relates the reactor period to the reactivity. A plot of this equation can be found in Appendix 2. The reactivity worth of the boron samples is determined by subtracting the reactivity for a sample run from that of a blank run. Assuming the cross section of boron at 2200 m-sec, the sensitivity of the reactor, $\Delta k/k/\text{cm}^2$, at the core center can be calculated.

3. The materials chosen for the cross section measurement are cobalt, sodium chloride, cadmium carbonate and copper. Each lucite sample contains a known amount of material. The experimental procedure for measuring the worth of the samples is identical to that of the boron standard. The reactor period with the samples on the core center line are measured in the following order:

a. Cadmium Carbonate	d. Sodium Chloride
b. Cobalt	e. Copper
c. Lucite Blank	f. Lucite Blank

The excess reactivity for each run is determined from the plot of the basic inhour equation. The reactivity worth of each sample is found by subtracting the reactivity of the sample's run from that of the blank lucite. Two blank

runs are made to ensure that no changes have occurred which might effect the core's excess reactivity. The control rods for each of the six runs must of course be at the same positions. Once the worth of each sample is determined, its cross section can easily be calculated knowing the amount of material present in the sample and the sensitivity of the reactor.

DATA SHEET

Sample	Sample Wt.	Lucite Wt.	Control Rod Positions		Temp.	Period
			Fr	Cr		
Boron A						
Lucite						
Boron B						
CdCO ₃						
Cobalt						
Lucite						
NaCl						
Copper						
Lucite						

V. RESULTS

The results of the experiment can be placed in the tables below:

Reactor Sensitivity (Measured With 1/v Boron Standard)

$$\text{Boron } \sigma_a = 755 \times 10^{-24} \text{cm}^2$$

Boron Standard	$N_T \sigma$	Period	Reactor $\Delta k/k$ ($\mu\rho$)	Standard Worth ($\mu\rho$)	Reactor Sensitivity $\mu\rho/\text{cm}^2$
Standard A					
Lucite					
Standard B					

Average Sensitivity _____

Absorption Cross Section Measurements

Sample	No. Atoms or Molecules	Period	Reactor $\Delta k/k$ ($\mu\rho$)	Sample Worth $\mu\rho$	cm^2 of Absorber	σ_{2200}
Cadmium						
Cobalt						
Lucite						
NaCl						
Copper						
Lucite						

VI. ANALYSIS OF RESULTS AND CONCLUSIONS

This section is to be completed by the participant in his experimental writeup. The cross sections obtained for the samples should be compared with known values. Any differences should be analyzed thoroughly and some conclusions should be stated as to why the errors existed.

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Experiment Written By: K. Ruzich

VIII. APPENDICES

1. DISCUSSION QUESTIONS

The following questions, which are directly related to the experiment on hand, should be considered prior to your arrival at the AGN-201. An effort will be made to discuss them before the performance of the experiment.

1. Describe the danger coefficient method for absorption cross section measurements. What basic variables are involved in the experiment?
2. What is meant by the sensitivity of the reactor? Why is it a function of the core volume?
3. Which factor in the four factor formulas is changed when an absorber is placed in a critical reactor and why?
4. How and why will the location of the absorber in the reactor core influence the magnitude of the change in the reproduction factor?
5. Besides absorption, what other properties of the sample will change the reactivity? How can these effects be kept small?
6. How will the absorber's reactivity worth be measured in this experiment? What basic equation will be utilized in its measurement?
7. The standard chosen for the sensitivity was boron. Why was this choice made? What is its absorption reaction rate independent of?
8. How will the absorption of neutrons in the resonance region effect a cross section measurement? When will a correction for this absorption have to be made? How can this correction be determined?
9. How will the following affect the results of the experiment?
 - a) Temperature changes
 - b) Flux depression and self shielding of the sample
 - c) Mounting the samples in a lucite disc.How can their effects on reactivity be kept small?
10. What other methods can be used for absorption across section measurements?

2. PLOT OF INHOUR EQUATION

3. 1961 ASEE-AEC Reactor Laboratory Institute Writeup

ABSORPTION CROSS SECTION BY PILE OSCILLATOR TECHNIQUE

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The pile oscillator technique for measuring absorption cross sections was developed in 1945 when it was discovered that measurements by danger coefficient methods, using a graphite pile, were limited by short period fluctuations in barometric pressure with attendant reactivity charges associated with the pile nitrogen content. Since that time considerable work has been done at national laboratories in the United States, England and France to develop pile oscillator techniques.

The usefulness of the pile oscillator stems from the fact that when a sample is moved from one location to another in a critical reactor, or when the sample is moved into and out of the reactor core, the amplitude of the pile power variation is proportional to the neutron absorption cross section of the sample. The oscillator may be used in any portion of the reactor, or

it may be used in the reflector or thermal column. Oscillations in locations where the neutron importance is small will produce small reactivity effects but the effect in each case is still strictly proportional to the absorption cross section. When a sample is oscillated in the thermal column, the "local power" flux amplitude oscillation may be measured by using a suitable detector placed near the point of oscillation.

Since the location of the sample may be in any region of the reactor, the oscillator technique is useful for measuring either the resonance absorption integral or the thermal absorption of the sample. For example, a cadmium tube may be provided in the fuel region and the sample oscillated in the tube. Direct comparison can then be made of the power amplitude variation produced by oscillation of the sample with that produced by oscillation of a standard resonance absorber such as gold. The samples must be geometrically similar, oscillated in the same location with the same frequency, and the results normalized to the same mean power level.

To measure the thermal absorption cross section the sample may be oscillated in the thermal column. In this case the neutron importance to core reactivity is very small and the core power level is essentially unaffected. A local detector is then used to measure the local depression. At Oak Ridge, for example, samples were oscillated in a hollow ionization chamber located inside the thermal column. Again comparison must be made with a standard absorber.

Hughes' cites the limit of detectability for the Argonne oscillator at about 0.0005 ih, the equivalent of about 25μ gm of boron. This sensitivity is accomplished by the use of electronic circuits designed to detect and record only the oscillating component of the detector signal.

Sample scattering effects on reactivity vary with the sample location and fuel loading. A sample with an appreciable scattering cross section located at the edge of the core may improve the reflector characteristics of that location and cause a gain in reactivity. If the fuel has a high U-238 content, and if the sample is located in the core center near the U-238, Hughes' points out that inelastic scattering may lower the neutron energy and reduce the fast fission factor ϵ . This would produce a falsely high value for the absorption coefficient, since negative reactivity would be caused by scattering rather than absorption, but the effect is quite small. In general these scattering effects would be significant primarily in a graphite moderated or reflected pile with relatively low U-235 enrichment. The effect of scattering can be minimized by oscillating at a point where the flux gradient is very small, such as at the center of the core, or conceivably, at the top of the reflector peak. It is interesting to note that it has been found experimentally that water reflectors may in some cases have a positive temperature or void coefficient. In this case the void introduced in the reflector by the sample holder can cause a positive charge in reactivity.

Oscillation of the empty holder provides a means of correcting for this phenomenon.

Theoretical justification for the assumption that pile power amplitude changes are directly proportional to the absorption cross section of an oscillating sample may be indicated by a simplified or group treatment. Rigorous analysis for multigroup theory including the effects of delayed neutrons is available in the literature.

The derivation that follows is based on the outline in Murray.²

If we make the usual one group theory assumptions the reactor equation may be written as

$$D \nabla^2 \phi + \phi \Sigma_a (k-1) = \frac{1}{v} \frac{\partial \phi}{\partial t} \quad \text{Eq. A}$$

If we introduce a small neutron absorber at any point in the reactor, Σ_a becomes $\Sigma'_a = \Sigma_a + \Delta \Sigma_a$ and ϕ becomes ϕ' . Assume the perturbed flux varies exponentially with time. Then $\phi' = \phi e^{\lambda t}$ where λ is the reciprocal of the period.

$$\frac{\partial \phi'}{\partial t} = \phi \lambda e^{\lambda t} = \phi' \lambda$$

$$\text{Substituting in Eq. A, } \frac{\phi' \lambda}{v} = D \nabla^2 \phi' + \phi' \Sigma'_a (k' - 1) \quad \text{Eq. B}$$

If the perturbation is small and the reactor not far from critical we may assume that the shape of the flux is constant with time so that $\nabla^2 \phi' = -B^2 \phi'$.

From one group theory we can write the following relationships:

$$\frac{D}{\Sigma'_a} = L'^2 \text{ and } k_e = \frac{k'}{1 + B^2 L'^2}$$

Substituting in Eq. B and simplifying we obtain the following:

$$-D B^2 \phi' + (k' - 1) \Sigma'_a \phi' = \frac{\lambda \phi'}{v}$$

$$-B^2 \phi' + \frac{(k' - 1)}{L'^2} \phi' = \frac{\lambda \phi'}{D v}$$

$$[-B^2 L'^2 + k' - 1] \phi' = \frac{\lambda \phi'}{v \Sigma'_a}$$

$$\left[k' - \frac{k'}{k_e} \right] \phi' = \frac{\lambda \phi'}{v \Sigma_a'} \\ k' \frac{\Delta k_e}{k_e} \Sigma_a' \phi' = \frac{\lambda \phi'}{v}$$

Next multiply each side of the above equation by ϕ and integrate over the reactor volume, noting that λ , v , and $\frac{\Delta k_e}{k_e}$ are not space dependent.

$$\int_V \frac{\phi \phi' \lambda dV}{v} = \int_V \Sigma_a' k' \frac{\Delta k_e}{k_e} \phi' \phi dV$$

Therefore

$$\frac{\Delta k_e}{k_e} = \rho = \frac{\lambda/v \int_V \phi' \phi dV}{\int_V k' \Sigma_a' \phi \phi' dV} \quad \text{Eq. C}$$

The following steps are inserted to eliminate λ/v .

For the one group critical reactor, $D \nabla^2 \phi + (k-1) \Sigma_a \phi = 0$

Multiply by ϕ' and integrate over the reactor volume to obtain

$$\int_V [D \phi' \nabla^2 \phi + (k-1) \Sigma_a' \phi \phi'] dV = 0$$

Multiply equation B by ϕ and integrate over the reactor volume to obtain

$$\int_V \frac{\phi \phi' \lambda}{v} dV = \int_V [D \phi \nabla^2 \phi' + \phi' \phi \Sigma_a' (k' - 1)] dV$$

Subtracting the last equation from the preceding we get:

$$\int_V D \phi' \nabla^2 \phi dV - \int_V D \phi \nabla^2 \phi' dV + \int_V (k-1) \Sigma_a \phi \phi' dV$$

$$- \int_V \phi' \phi \Sigma_a (k' - 1) dV - \int_V \phi' \phi \Delta \Sigma_a (k' - 1) dV = \int_V \frac{\phi \phi' \lambda}{v} dV$$

Since ϕ and ϕ' are self-adjoint,¹² the first two terms disappear. For small perturbations the next two terms also cancel and we have left

$$\int_V \frac{\phi \phi' \lambda}{v} dV = \int_V (k' - 1) \Delta \Sigma_a \phi' \phi dV$$

Since we assumed that $\Delta \Sigma_a$ was an absorber introduced at one point in the reactor it follows that the right hand side of this equation must be zero except in the immediate region R of the absorber. Solving for λ/v we get

$$\frac{\lambda}{v} = \frac{\int_R \Delta \Sigma_a (k' - 1) \phi \phi' dR}{\int_V \phi \phi' dV}$$

Substituting in Eq. C we have

$$\rho = \frac{\int_R \Delta \Sigma_a (k' - 1) \phi \phi' dR}{\int_V \phi \phi' dV} \times \frac{\int_V \phi \phi' dV}{\int_V k' \Sigma_a' \phi \phi dV}$$

with $\phi \approx \phi'$ we have

$$\rho = \frac{\int_R \Delta \Sigma_a (k' - 1) \phi^2 dR}{\int_V k' \Sigma_a' \phi^2 dV} = \frac{\Delta k_e}{k_e} \quad \text{Eq. D}$$

For small changes in reactivity the time dependent reactor equations show that

$$n = n_0 e^{-\frac{\Delta k_e t}{\ell}}$$

Expanding this equation we get $n = n_0 [1 + \frac{\Delta k_e t}{\ell} + (\frac{\Delta k_e t}{\ell})^2 + \dots]$

If Δk_e is small and if the absorber is inserted and withdrawn quickly,

$$n - n_o = n_o \frac{\Delta k_e t}{\ell} \therefore \Delta k_e = \frac{(n - n_o) \ell}{n_o t}$$

Substituting in equation D and solving for $n - n_o$ we obtain

$$n - n_o = \frac{n_o k_e t}{\ell} = \frac{\int_R \Delta \Sigma_a (k' - 1) \phi^2 dR}{\int_V k' \Sigma'_a \phi^2 dV}$$

If $\Delta \Sigma_a$ is small and homogeneous it may be assumed a constant over the region R. Since $\Delta \Sigma_a$ was rapidly inserted to its final location and essentially sees only the energy spectrum of its final location, it is independent of time. Furthermore k' and Σ'_a may be assumed constant over the reactor. Let

$$\frac{k_e (k' - 1)}{\ell k' \Sigma'_a} = C_1 \text{ a constant.}$$

Replace t by $f(wt)$ to indicate a periodic time dependence.

Then

$$n - n_o = C_1 n_o \Delta \Sigma_a \frac{\int_R \phi^2 dR}{\int_V \phi^2 dV} f(wt)$$

It can be seen from this result that the amplitude of pile power, n , is directly proportional to the cross section of the inserted absorber $\Delta \Sigma_a$ and is also proportional to the power.

Physically the oscillator consists of a device which will quickly and accurately position a sample in a given core location, allow it to dwell there for a short time, and then quickly remove it, repeating this cycle over and over. Typical times might be 1 second travel time for insertion and withdrawal and 10 to 15 second dwell period. Commonly the motion is reciprocating. Care must be taken to insure that both timing and sample positioning are accurately reproducible, and that the reactor power level is accurately known.

In operation the oscillator is set in motion and the reactor controls adjusted so that the envelope of reactor power from "sample inserted" to "sample withdrawn" stays constant with time. A comparison of the amplitude of the power oscillations is then made for the known reference samples and the unknown absorber to be measured.

A tabulation of some of the more interesting references on pile oscillators are compiled by Mr. J. W. Little, a WPI graduate student, is included in the bibliography, although most of the references were not readily available or used during the preparation of this discussion.

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STUDENT RESULTS

Experiment Performed By: T. R. Rehm, K. Rothschild, H. A. Shadduck, L. C. Wilbur, W. E. Wilson, Jr. June 26, 1961. Submitted by L. C. Wilbur

Tabulation of results

Reactor Sensitivity :

Group A	1423 $\mu\text{p}/\text{cm}^2$
Group B	1348 $\mu\text{p}/\text{cm}^2$
Group C	1390 $\mu\text{p}/\text{cm}^2$

Material	Sample Worth μp	σ_{2200} (measured)	σ_{2200} test	% Diff
Cadmium				
Group A	276	2620		
Group B	258.5	2605		
Group C	296	2890		
Average		2705	2550	+6.1%
Cobalt				
Group A	260	390		
Group B	265.5	42.5		
Group C	271	42.1		
Average		40.9	37	+10.5%
Na Cl				
Group A	208	33.2		
Group B	172.5	28.9		
Group C	214	35.4		
Average		32.5	34.1	-4.7%
Copper				
Group A	179	3.75		
Group B	165.5	3.66		
Group C	188	4.11		
Average		3.84	3.69	+4.1%

Neutron Absorption by Danger Coefficient Method

Outline and Sample Calculations

26 June 61
Group C

Step 1 Insert Boron impregnated plastic sample #6-S A into AGN 201 glory hole sandwiched between two Aluminum push rods.

Step 2 Adjust regulating rod until reactor is supercritical on a long period. Wait until period is stable.

Step 3 Take counts for about .3 minutes every minute and plot counts per unit time vs time on semi log graph paper.

Since $\Phi = \Phi_0 e^{t/T}$

$$\ln \frac{\Phi}{\Phi_0} = t/T \quad \text{and} \quad T = \frac{t}{\ln \frac{\Phi}{\Phi_0}} = \frac{t}{\ln \frac{C}{C_0}}$$

Data for Boron Sample

Counts taken every min in 0.3 min counting intervals

15447
16477
17530
18604
19668
20627
21681
23152
24494
25833
27504

Since data plotted as a straight line the period was stable.

$t = 10 \text{ min.}$

$C_0 = 15447 \text{ counts/unit time}$

$C_t = 27504 \text{ counts/unit time}$

$$T = \frac{(10)(60)}{\ln \frac{27500}{15500}} = \frac{600}{.572} = 1050 \text{ sec.}$$

Step 4

From a plot of the inhour equation (furnished) read reactivity in μp .

For $T = 1050 \text{ sec.}$ $\rho = 89 \mu\text{p}$

Step 5

Bring reactor subcritical. Remove Boron sample and replace with a plastic dummy. Return regulating rod to same position it held while boron sample was in place. Wait for stable period.

Step 6

Take and plot data in same manner as for boron, above.

Results: $t = 8 \text{ min}$
 $C_0 = 2380$
 $C_t = 20600$

$$T = \frac{8(60)}{\ln \frac{20600}{2380}} = 222 \text{ sec}$$

From inhour plot $\rho = 389 \mu\text{p}$

Step 7 Calculate Sensitivity

Sample weight (furnished) 0.0310 gms

Sample molecular weight 61.84
(H_3BO_3)

$$\text{Total atoms of Boron} = \text{total molecules of Compound} = \frac{wt}{A} N_{\text{Avog}} \\ = \frac{(0.031)(602)10^{24}}{61.84} = 302 \times 10^{19} = N_T \text{ atoms of B in sample}$$

 σ_a Boron = 755 b (furnished)

$$\text{Total cm}^2 \text{ of sample} = N_T \sigma = (755)(10^{-24})(302)(10^{19}) = 0.228$$

$$\text{Net reactivity worth of sample} = \rho_{\text{Dummy}} - \rho_{\text{sample}} = 389 - 89 = 300 \mu\rho$$

$$\text{By definition Reactor Sensitivity} \equiv \frac{\rho_{\text{net}}}{C_{\text{m}^2}}$$

$$\text{Reactor sensitivity} = \frac{300}{.228} = 1315 \mu\rho/\text{cm}^2$$

Step 8 Repeat above procedure for a boron sample of different weight

Results

Wt of new sample #6-1 B = 0.0155

 $C_{\text{m}^2} = .114$ From semilog plot of counts $T = 405$ secFrom inhour eq plot $\rho = 222 \mu\rho$ Net reactivity = $389 - 222 = 167$

$$\text{Reactor sensitivity} = \frac{167}{.114} = 1465 \mu\rho/\text{cm}^2$$

Note There should have been close agreement between the two values of sensitivity. The average will be used

$$\frac{1315 + 1465}{2} = 1390 = \text{avg reactor sensitivity}$$

Step 9 Insert a sample containing Cadmium while reactor is subcritical

Again set regulating blade to original supercritical position and wait for a stable period. Take and plot data as before

Results Cadmium sample 7-1

 $t = 10$ min $C_0 = 12,500$ $C_t = 22,800$

$$T = \frac{600}{\ln \frac{228}{125}} = 998 \text{ Sec}$$

$$\rho = 93 \mu\rho$$

$$\text{Net worth} = 389 - 93 = 296$$

$$\frac{\mu\rho_{\text{Boron}}}{C_{\text{m}^2 \text{ Boron}}} = \text{Sensitivity} = \frac{\mu\rho_{\text{sample}}}{C_{\text{m}^2 \text{ sample}}}$$

$$\therefore C_{\text{m}^2 \text{ sample}} = \frac{\text{net worth of sample}}{\text{Reactor Sensitivity}} = \frac{296}{1390} = .213$$

$$N_T \sigma = C_{\text{m}^2}$$

$$\sigma = \frac{C_{\text{m}^2}}{N_T}$$

$$\sigma = \frac{.213}{568 \times 10^{18}} = 3750$$

For a $\frac{1}{x}$ absorber this would be the final result. Since Cd is not a $\frac{1}{x}$ absorber a correction factor of 1.3 (furnished) must be used to relate the σ above to σ_{2200}

$$\frac{3750}{1.3} = 2890 = \sigma_{2200}$$

From the literature σ_{2200} should be 2550

V. RESULTS

The results of the experiment can be placed in the tables below:

Reactor Sensitivity (Measured With 1/v Boron Standard)

$$\text{Boron } \sigma_a = 755 \times 10^{-24} \text{ cm}^2$$

Boron Standard	H_{10}^{-1} (cm ⁻¹)	Period sec	Reactor $\Delta k/k$ (pp)	Standard Worth (pp)	Reactor Sensitivity $\mu\text{d/cm}^2$
Standard A	.228	1045	89	300	-1315
Lucite		222	389	167	-1465
Standard B	.114	405	222		

Average Sensitivity 1390

Absorption Cross Section Measurements

Sample	No. Atoms or Molecules	Period	Reactor $\Delta k/k$ (pp)	Sample Worth (pp)	cm ² of Absorber	σ_{2200}	Text σ_{2200}
Cadmium	56.8×10^{18}	998	93	296	.213	2890	2550
Cobalt	46.3×10^{20}	780	118	271	.195	42.1	37.0
Lucite		222	389				
NaCl	43.3×10^{20}	521	175	214	.154	35.4	34.1
Copper	32.9×10^{21}	448	201	188	.135	4.11	3.69
Lucite							

A H_2BO_3 $CdCO_3$ Co $NaCl$ Cu
 61.84 172.42 58.94 58.45 63.54

Neutron Absorption by Danger Coefficient Method 26 Jun 61

Bkpd 90 CPM

Run	Sample	CTS/taken every min for 3 min					
6-5 A	Boron	15447 16477 17520 18604 19668 20627 21681 23152 24494 25833 27504	$t = 10 \text{ min}$ $C_0 = 15500$ $C_2 = 27500$ $T = \frac{(10)(60)}{\ln \frac{27500}{15500}} = \frac{600}{.574} = 1045 \text{ sec}$ $\rho = \frac{105 \mu p}{89} \text{ uncorrected}$				
Lucite 3-4		2731 3100 3611 4060 4593 5264 6239 7135 8192 9173 10535 12069 13834 15850 18433 20762	for 2 min over 5 min $t = 8 \text{ min}$ $C_0 = 2380$ $C_8 = 20600$ $T = \frac{480}{\ln \frac{20600}{2380}} = \frac{480}{\ln 8.66} = \frac{480}{2.16} = 222 \text{ sec}$ $\rho = 389 \mu p$	Source out			
6-1 B	Boron	8809 10247 12131 14022 16239 18969 21869	for 3 min every 1 min $t = 6 \text{ min}$ $C_0 = 14000$ $C_6 = 34000$ $T = \frac{360}{\ln \frac{34000}{14000}} = \frac{360}{\ln 2.43} = \frac{360}{.888} = 405 \text{ sec}$ $\rho = 222 \mu p$

R.L.I. EXPERIMENT NUMBER 5

TEMPERATURE COEFFICIENT OF REACTIVITY

INSTRUCTOR: D. A. DAAVETILA

ABSTRACT

Reactors are subject to temperature fluctuations for a variety of reasons which include changes in ambient temperature and reactor power. These temperature fluctuations cause reactivity disturbances through changes in the physical and nuclear properties of the core. The relationship between temperature and reactivity (temperature coefficient) is usually stated as the magnitude and sign of the reactivity due to a unit change in temperature.

In this experiment the overall temperature coefficient of the Argonaut reactor is measured. The basic procedure is to change the temperature of the water moderator, and from the movements of a calibrated control rod required to maintain criticality, the reactivity worth of the temperature change is determined.

TEMPERATURE COEFFICIENT OF REACTIVITY

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - 1. General
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 - 3. Nuclear Coefficient
- III. APPARATUS
- IV. PROCEDURE
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- VIII. APPENDICES
 - 1. Discussion Questions
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I. INTRODUCTION

In nuclear reactors, and especially those primarily concerned with energy production in the form of heat, temperature transients are inevitable. For this reason, the operation of a reactor in the practical sense requires that the temperature coefficient of reactivity be small, so that a steady state can be maintained by means of the control system. The reactor will then remain essentially stable in spite of moderate fluctuations in temperature. If, in addition to a low magnitude, the sense of this reactivity coefficient is such as to decrease the reactivity of the system, the safety of the reactor will be improved.

In this experiment the overall temperature coefficient is measured, but one could measure various local coefficients due to a temperature change in any region or component of the core or reflector.

II. THEORY

1. General:

The qualitative relationship between temperature and reactivity will be developed from a consideration of the effect of temperature on the individual factors of the two group criticality equation.

$$k_{eff} = \eta \epsilon p f \left\{ \frac{1}{1 + L^2 B^2} \right\} e^{-\tau B^2} \quad (1)$$

where

η = number of fast neutrons born per thermal neutron captured in the fuel

ε = (fast fission factor) number of fast neutrons slowing down past the U^{238} fast fission threshold per fast neutron from thermal fission

p = (resonance escape probability) probability of a fast neutron escaping non fission capture while slowing down to thermal energies

f = (thermal utilization) fraction of thermal neutrons captured in the fuel to those captured in the core.

$\varepsilon^{-\tau B^2}$ = fast non leakage probability (L_1) where τ is the neutron age and B^2 the geometric buckling

$\frac{1}{1 + L^2 B^2}$ = thermal non leakage probability (L_2) where L is the thermal diffusion length.

The reactor is critical when $k_{eff} = 1$, that is on the average one of the neutrons born in a thermal fission causes another fission. Then the neutron population is independent of time and is said to be self sustaining. If $k_{eff} = 1$ the population will continuously change, increasing if $k_{eff} > 1$ and otherwise decreasing. The rate of change of population is a function of the reactivity (ρ) defined as,

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \quad (2)$$

The temperature coefficient of reactivity is given by

$$\frac{d\rho}{dT} \quad (3)$$

where T is usually in $^{\circ}C$.

Usually as a safety feature and for stability reasons, the overall reactivity temperature coefficient is designed to be negative. If the temperature of a critical reactor increases, k_{eff} becomes less than one, and the reactor is then subcritical at the new temperature. On the other hand if $d\rho/dT$ were positive, and as the fission process releases heat, the reactor would be unstable and become increasingly more supercritical as the temperature increased.

The effect of temperature on the factors of equation (1) arise mainly from the neutron energy dependence of the microscopic absorption and to a lesser extent scattering cross sections, and from the variation of the macroscopic cross sections due to variations in core density. Also the effect of volume changes on the geometrical buckling is an important effect. With increasing temperature some factors increase, some decrease, and some hardly change, so that overall coefficient depends on which factors dominate.

The temperature coefficient is not easily calculated, and it usually confirmed or determined experimentally. In the following discussion the coefficient is considered separable into a density and nuclear coefficient as is usually done for large uranium fueled thermal reactors.

2. Density Coefficient:

As the temperature increases, the core material densities decrease while the core volume increases, and the reactivity associated with these physical changes is of opposite sign.

A decrease in density decreases the macroscopic cross sections and increases the mean free paths for absorption and scattering. The result is that the thermal diffusion length (L) increases. But partially if not completely compensating for the increase in L^2 , is the decrease in B^2 due to increased core volume. Generally the net result of the density decrease is a decrease in thermal non leakage probability. The identical situation to a lesser degree occurs for τ and B^2 which reduce the fast non leakage probability.

Another effect of a density decrease which can be very important in over-moderated liquid or gas cooled heterogeneous reactors, is the decrease of poison in the core due to the expansion of the coolant-moderator.

3. Nuclear Coefficient:

As the core temperature increases the neutron energy increases as does the vibratory energy of the core atoms. With increasing neutron energy, the microscopic absorption cross section (excluding resonances) and the scattering cross section decrease. Generally for neutron energies less than that of the first resonance, the absorption cross sections vary as $1/v$ for most materials.

In the following discussion, the temperature dependence of τ and ϵ will be considered, as usual, to be negligible, because they are functions of high energy cross sections which are hardly temperature dependent.

With increasing neutron energy beyond 0.025 eV, η decreases for natural or slightly enriched uranium reactors because whereas U^{238} is a $1/v$ absorber, U^{235} is less than $1/v$. Hence, as the temperature increases, U^{238} will non-fission capture an increasingly larger fraction of thermal

neutrons. However, the change in η is small, and it usually is considered to be temperature independent.

Because of Doppler broadening of the U^{238} resonance peaks with increasing temperature, ρ decreases. The Doppler broadening arises from the thermal vibration of the U^{238} atoms.

The thermal utilization is often considered to be temperature independent because all cross sections would vary as $1/v$, but U^{235} is not $1/v$, so a slight decrease of f with increasing temperature is expected. In the case of heterogeneous reactors, the decrease in uranium absorption with increasing temperature will decrease the disadvantage factors. This effect can be quite large and can even result in a positive fuel temperature coefficient.

The decrease in microscopic cross sections will increase L^2 and decrease the thermal non leakage probability.

III. APPARATUS

1. Argonaut reactor
2. Heaters to heat moderator (See Fig. 5, Appendix II)
3. Temperature sensing devices (See Fig. 6, Appendix II)

The temperature sensing devices consist of plexiglass frames in which is strung a piece of nickel wire 0.0089 inches in diameter and approximately twice the length of a fuel box. One of these devices will be placed in each of the fuel boxes. The individual nickel wires will then be connected in series by means of a piece of large diameter, low resistance wire. These elements will then indicate a rather good average core temperature. An impedance bridge will be used to measure the resistance of the entire assembly. This resistance can then be converted to a temperature by means of a calibration curve. If a calibration curve is not available, the students will utilize equipment available at the Argonaut and calibrate the resistance of these devices against a Hg standard thermometer. One calibration curve is given in Fig. 3 of Appendix II.

IV. PROCEDURE

The chronological order of events will be somewhat as follows:

1. Question-discussion period

TEMPERATURE COEFFICIENT APPARATUS

2. Placement of the experimental apparatus
3. Reactor checkout and determination of criticality
4. Adjustment of reactor temperature and the determination of the resultant reactivity changes
5. Experiment summary and computation of the reactivity temperature coefficient

The reactor will be checked out and taken critical by the students. During this time the heat exchanger may be run to stabilize the temperature at some value below ambient, thereby providing a wider range of temperature. Final criticality should be at about 1/2 watt since this flux level is high enough to give relatively noise-free operation of the linear chassis and yet produce a reasonably small increase in the residual flux after shutdown.

The reactor should be maintained at criticality all during the experiment. This may be accomplished by adjustment of the control rods. The emphasis here is on the critical situation rather than on the maintenance of a particular power level.

Two 2KW heaters provided in the "pit" will be used to increase the temperature of the reactor coolant. Water is circulated through the heaters and the dump tank. This water is also circulated between the dump tank and the reactor vessel. The heaters operate automatically to maintain a constant temperature. This temperature is determined by a control system located in the temperature control rack near the pit. By this method the temperature of the reactor coolant will be raised to and maintained for a period of time at a few points between the temperature of the tap water (starting temperature) and 120° F, the upper limit being set by the necessity of preventing, as far as possible, the formation of condensate on control rods and other electrical appliances.

After selecting a particular temperature at the temperature control rack, record data until the temperature stabilizes at the newly selected value. This data will consist of reactor core average temperature, reactor overflow temperature, control rod position, $\Delta\rho$, and time. The transient states will take of the order of 1-1/2 hours, changes in average reactor temperatures becoming evident about twenty minutes after the heaters come on. This step will then be repeated beginning with a change in demand temperature at the temperature control rack until reaching the upper limit temperature.

The reactivity temperature coefficient can then be calculated from the foregoing data for each of the several increments and for the overall change.

V. RESULTS

Each student should make a table to record the basic data of temperature, control rod position, and time. From this data, make a plot of reactivity versus temperature, and include the rod calibration curve used to calculate.

VI. ANALYSIS AND CONCLUSIONS

This is left as an exercise for the student. Include a discussion of the shape of the reactivity-temperature curve shape.

VII. BIBLIOGRAPHY

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2. "The Elements of Nuclear Reactor Theory," S. Glasstone and M. C. Edlund, (D. Van Nostrand Co., Inc., 1952) pp. 339-344.

Experiment Written By: D. Daavettila

VIII. APPENDICES

1. DISCUSSION QUESTIONS

1. What is "nuclear coefficient", "density temperature coefficient"?
2. Discuss each of these temperature coefficients (nuclear and density) in the following regions in the Argonaut reactor:
 - a. Moderator
 - b. Reflector
 - c. Fuel
 - d. Control Rods
3. Do the fission product poisons affect either of the above coefficients? If so, how? Might this effect be appreciable?
4. In considering the density temperature coefficient, what is the qualitative relationship between L^2 , τ and density?
5. In general, would the reactivity temperature coefficient be of greater magnitude for a homogeneous or heterogeneous reactor?
6. In reactors in which light water acts almost exclusively as coolant while the neutron moderation takes place in another medium, the temperature coefficient may be positive. Explain.
7. For the initial approach to criticality, in the case of a high power reactor designed to operate at elevated temperatures, what precautions must be taken that are not of concern to us at the Argonaut?
8. Explain neutron Doppler effect in a heterogeneous, water moderated, slightly enriched system, and its relationship to temperature coefficient of reactivity.
9. In a solid homogeneous fuel moderator reactor such as Triga, would you expect the reactivity temperature coefficient to be more or less negative than that of the Argonaut? Why?
10. Qualitatively discuss the effect of temperature changes (in the region of room temperature) on the factors of the four factor formula.

2. EXPERIMENT REPORT ON TEMPERATURE COEFFICIENT OF REACTIVITY OF ARGONAUT REACTOR

P. Guenther, R. Jain, M. Kirshenbaum, P. Larsen,
Q. Nyugen, D. Smith, and S. Wang

Introduction

The temperature change in any region or component of a reactor can have an appreciable effect upon the reactivity of the reactor. The knowledge of this effect therefore becomes essential for a specific reactor. A term, temperature coefficient, is thus introduced to describe this phenomenon.

The temperature coefficient, denoted by α , is thus defined as the rate of change of the value of multiplication factor with respect to temperature of any component or region of the reactor. We can also define an overall temperature coefficient which gives the total change in reactivity of the reactor with respect to the overall average change of the temperature of the reactor. Putting it mathematically,

$$\alpha = \frac{d k_{\text{eff}}}{dt}$$

$$\text{or} \quad \alpha = \frac{d\rho}{dt}, \text{ when } k_{\text{eff}} \approx 1.$$

where ρ is the reactivity of the reactor and t the temperature.

In our experiment α has been estimated by measuring the total change in the reactivity of the reactor when heated water is circulated through the core. The change in reactivity is measured on the calibrated fine control rod and the temperature is measured as the weighted average temperature of the moderator, i.e., water, in the core. The experiment is performed in both the conditions of temperature, the dynamic and the static. In dynamic conditions the temperature is not in equilibrium and it rises with time. Under static conditions the temperature is kept steady for sufficiently long time.

The variation of reactivity with time is given in Figure 1. Figure 1 also contains the variation of temperature with time. Figure 2 gives the reactivity change as a function of temperature. The static points are shown distinctly in Figure 2. The slope of the curve passing through the static points gives the value of α . The suitable fit for these four points is a straight line.

Figure 3 gives the calibration of resistance for temperature measurement. The calibration of the control rod is given in Figure 4. Figure 5 shows the flow diagram and heater arrangement. Figure 6 shows arrangement of resistance wire and plexiglass.

The description of apparatus and the method is given subsequently.

Apparatus

Heating

The secondary pump forces water from the dump tank through heater #1 and #2 (21 kilowatts each) and back into the dump tank. The primary pump causes this heated water to flow into the annular region of the reactor vessel where it mixes with the moderator water, thus effectively changing the moderator temperature. Water leaves the core by an overflow pipe and returns to the dump tank. (See Figure 4.)

Temperature Measurement:

A length of 0.0089" diameter nickel wire, approximately 60 inches long was used as a temperature-sensing element in each of the fuel boxes. This length of wire was strung on a plexiglass holder (see Figure 6) which was placed between two fuel plates in the fuel box. After insertion, the resistance elements were connected in series by short nickel wires connecting the resistance element of each box to the adjacent box. Insulated nickel wires led from the resistance elements to an impedance bridge a few feet from the control console.

The series resistance of the temperature-sensing resistance elements was measured by a Wheatstone bridge. The bridge used was an ESI Model 250-C1 Impedance Bridge by Electro Measurements, Inc.

Data and Calculation

Data were recorded as follows:

1. Time (min.)

Time was observed from the clock on the right rack beside the control console. Then, the time for each reading of data and the time of putting on heater were recorded.

2. R (ohms)

The resistance of the nickel wire and the leads was read off from the impedance bridge. The resistance of the leads was found to be $.14 \Omega$ before the experiment began and was subtracted from the observed total resistance. The net resistance was then recorded in this column.

3. Moderator temperature in core ($^{\circ}\text{C}$)

The moderator temperature was obtained from the net resistance of the nickel wire (column 2) and a calibration curve (Figure 3) of temperature vs. resistance of the nickel wire was plotted. The calibration had previously been made with the same nickel wires interconnecting the six individual elements as when mounted in the reactor.

4. Outlet temperature T ($^{\circ}\text{F}$)

This was taken from the indicator on the control console, and was recorded as a matter of interest. This is the temperature of the water leaving the reactor vessel. This will be seen to differ somewhat from the temperature obtained from the resistance bridge.

5. Fine control rod position

The reactor was initially made critical by positioning the coarse rod, leaving the fine rod to be manipulated during the experiment. Each fine rod position was recorded.

6. Reactivity (in dollars) ρ (\$)

This was obtained for a particular control rod position from the most recent fine rod calibration curve that was made for this specific one-slab loading. The calibration of the control rod is given in Figure 4.

Procedure

With the necessary apparatus in place inside the reactor vessel, and the top shielding in place, the reactor was started up and made critical at a power level of .4 watt, approximately, or 1×10^{-8} on the log meter.

When criticality was reached at about 0.4 watts, it was then maintained at the same flux by manipulating the fine control rod. The secondary pump was set in operation to insure an isothermal state of water temperature. Data was taken at every 5 minutes (later on, ten minutes) and entered into data sheets. When equilibrium state of temperature was observed at 13:10, heater condition No. 1 was put on to increase moderator temperature. Similarly heater condition No. 2 was put on at 14:20.

Final Calculation

1. A curve of temperature ($^{\circ}\text{C}$) vs. time was plotted in Figure 1. This temperature-time curve showed steady or equilibrium states at temperatures of 19.3°C , 21.5°C , 26.3°C and the first point of equilibrium at 14.3°C . The four points were selected for calculating the temperature coefficient, α . Figure 1 also contains the plot of reactivity vs. time.

2. On Figure 2 the Δk in \$ and % vs. moderator temperature were plotted. The four equilibrium points were so chosen that the highest reactivity value for the same temperature should represent equilibrium state the best. Each of them is marked in a square box. The four equilibrium points form almost a straight line. Hence the slope of the straight line, i.e., the value of α , is found as: (see Figure 2)

$$\begin{aligned}\alpha &= \frac{.155 - .028}{27 - 14} \\ &= -0.00977 \% \Delta k / ^\circ \text{C}.\end{aligned}$$

Discussion

Figure 1 shows the general behavior of the increase of reactivity and temperature with time. The flat portion on Curve A, showing temperature vs. time, indicates the steady temperature conditions in the reactor. There are four such regions marked 1 to 4. Curve B, giving reactivity vs. time, shows sharp gradients except in the vicinity of the steady region of the temperature. The behavior of the reactivity in these regions is important because the shape of this curve in these regions indicates whether the reactivity of the system is lagging or leading with respect to the temperature measured experimentally. In regions 3 and 4, Curve B has a small positive gradient. This indicates the reactivity of the system is lagging behind the temperature. This behavior will be reflected by a sharp gradient of the reactivity curve below the equilibrium points in Figure 2.

The lagging behavior would also be indicated by the fact that if we join the four steady points by almost a straight line in the reactivity vs. temperature graph then the measured (reactivity) points on that curve would lie below this line. We actually observe this behavior as shown in Figure 2 except near the region 2. In this region, the dynamic curve, passing through the observed points, does not approach the steady point with a sharp gradient. Moreover, in this region below the steady point the observed points lie above the straight line passing through the steady points. This situation suggests as if the reactivity of the system is leading with respect to the measured average temperature of moderator. This means that the temperature of other components, say fuel and reflector, etc., rises more rapidly than that of the moderator, and therefore leads the measured temperature.

A suggestive explanation of the fact that a considerable increase in temperature of the moderator is observed (in Figure 2) without a consequent increase in reactivity in the approach to point 2 would be that some other reactivity change (of opposite sign) took place simultaneously. As such, a cooling of the graphite reflector while the moderator was being heated could be imagined. That this was probably the case is indicated by the fact that the

reactor moderator was cooled well below room temperature before the experiment began and that the average reactor temperature was above the moderator temperature at points 1 and 2 before the experiment was commenced. Notice also that, in approaching point 2, the rate of change in temperature with respect to time was comparatively very small and consequently allowed components like the graphite reflector to exercise their temperature (cooling) effect. The room temperature was 25.6°C .

Figure 1

ARGONAUT TEMPERATURE

Figure 2
TEMPERATURE COEFFICIENT
OF REACTIVITY
 Argonaut

Figure 3
CALIBRATION CURVE OF THE
TEMPERATURE-SENSING
NICKEL WIRES

Figure 3a

CALIBRATION CURVE OF THE
TEMPERATURE-SENSING
NICKEL WIRES

Figure 4

CONTROL ROD
CALIBRATION CURVES

Figure 5

FLOW DIAGRAM HEATER ARRANGEMENT

For heating: Open valves
A, B, F, G, H, I and J

Figure 6

PLEXIGLASS HOLDER FOR RESISTANCE WIRES

3. 1961 ASEE-AEC Reactor Laboratory Institute Writeup

CP-5 TEMPERATURE COEFFICIENT MEASUREMENT

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A. Introduction

The basic method, described in the body of this experiment, for measurement of the Argonaut temperature coefficient of reactivity may be applied equally as well to many other reactors. In particular, the normal controls and instrumentation of the Argonne Research Reactor (CP-5) permit an estimation of the temperature coefficient to be made quite simply. In addition, this exercise affords the student an opportunity to become more familiar with the CP-5 reactor.

A very brief description of CP-5 will be given here. Prior to conducting the experiment more detailed information will be provided by the instructor. CP-5 is a heavy water (D_2O) cooled and moderated machine rated at a thermal power level of 10,000 KW. It is used primarily as a neutron source for research purposes and, therefore, operates almost continuously. The core proper approximates a 2 ft. right cylinder. Surrounding this is a reflector of 2 ft. of D_2O followed on the side and bottom by 2 ft of graphite. Physically this takes the form of a 6 ft diameter cylindrical aluminum tank supported within a 2 ft thick graphite cup and filled to a depth of about 6 ft with heavy water. The highly enriched uranium fuel is alloyed and clad with aluminum. The original fuel assemblies were of the flat plate, MTR type. However, those currently in use consist of three concentric fuel bearing tubes between which cooling water flows. A typical loading consists of 16 assemblies containing a total of approximately 2.4 kg of U-235.

The core region is about 8.4% aluminum and 91.6% D_2O by volume. Neutron economy is excellent due to the absence of significant parasitic absorbers. The D_2O coolant enters the reactor vessel at the bottom and is distributed to the individual fuel assemblies by a plenum. It then flows up through the channels between the fuel tubes and exits through ports in the assemblies at some distance below the water level. It mixes with the bulk water in the reactor vessel and eventually passes out of the vessel through the heat exchanger and back to the inlet plenum. The water level in the tank is maintained constant by continuous overflow through a fixed pipe within the reactor vessel. Because the water level is constant, the geometrical size and shape of the core and reflector do not vary with temperature (neglecting

the very small change in dimensions of the aluminum tank). As a result, the buckling (B^2) is constant over the temperature range of interest.

The linear coefficients of expansion of the core materials are as follows:

aluminum $2.38 \times 10^{-5}/^{\circ}\text{C}$

fuel alloy $1.94 \times 10^{-5}/^{\circ}\text{C}$

D_2O $9.88 \times 10^{-5}/^{\circ}\text{C}$.

From the relative magnitude of these values it is apparent that the temperature coefficient of reactivity will depend primarily on the thermal expansion and density change of the D_2O . As noted above, expansion of the D_2O will not effect the geometric buckling. The effect of temperature change on the other parameters in the two group criticality equation (1) is briefly considered below.

The value of η for CP-5 is 2.08 and is essentially independent of temperature. Because of the high enrichment and very low parasitic absorption the values of p and ϵ are very close to unity. The fast fission factor has negligible temperature dependence while the change in resonance escape probability makes a small positive contribution to the total reactivity coefficient. This contribution is actually derived from the $1/v$ portion of the microscopic fuel absorption cross-section rather than from the resonance region. Since both fuel and non-fuel core components are close to $1/v$, the variation in f will be due to density changes alone. The relatively greater coefficient of expansion of the non-fuel components results in a small, positive temperature coefficient of thermal utilization.

The thermal non-leakage probability varies inversely with the diffusion area. L^2 in turn increases with increased temperature due to the change in both the microscopic cross section and density of the material. The result is a rather large negative contribution to the reactivity coefficient. The coefficient associated with the fast non-leakage probability will also be negative due to the change in Fermi age with moderator density and, therefore, temperature. The net temperature coefficient of reactivity, consisting primarily of two small positive contributions due to p and f , and two large negative components associated with the fast and thermal non-leakage probabilities, is strongly negative in CP-5.

B. Experimental Procedure

The change in reactivity produced by a change in reactor temperature will be measured by observing the movement of the regulating rod required to

keep the reactor just critical. The change in reactor temperature can be produced by causing an imbalance between rate of production and rate of removal of heat. As a matter of convenience, the measurements will be made by first raising the temperature from the normal level of $\sim 50^{\circ}\text{C}$ to $\sim 60^{\circ}\text{C}$. After CP-5 has stabilized at this new temperature, the flow rate in the secondary, light water system will be increased. This will result in an increase in the rate of heat rejection to the atmosphere via the cooling tower and a decrease in the temperature of the primary, D_2O system. The decrease in moderator temperature must be slow enough that the automatic control system can maintain the reactor in a quasi-equilibrium condition at all times. The control system accomplishes this by inserting the regulating rod to just compensate for the gain in reactivity caused by the drop in temperature. The position of the regulating rod at any time is assumed to correspond to the "just critical" position. From the rod position data and the appropriate calibration curve, the reactivity change as a function of temperature is determined. Since the calibration of the regulating rod is dependent on the position of the four shim-safety blades care must be taken in selecting the appropriate curve from those which the instructor will supply.

The reactor temperature will be estimated from control room instruments which record the D_2O inlet temperature (T_{in}) as it enters the plenum chamber in the bottom of the core tank and the temperature rise (ΔT) across the fuel element. The average reactor temperature (T_{av}) will be assumed equal to the inlet temperature plus one-half the temperature rise across the fuel element, or

$$T_{\text{av}} = T_{\text{in}} + \frac{1}{2} \Delta T$$

The value of T_{av} calculated from the equation provides a reasonable estimate of the average temperature of the D_2O within the fuel assemblies. However, because of the loose thermal coupling between the cooling water within the assemblies and the bulk moderator outside of and surrounding the assemblies, the relation between T_{av} and the actual bulk moderator temperature will depend on the rate at which the inlet temperature is decreased. As a result, it is important that the change in temperature occur at a constant rate. In this case, assuming that the temperature coefficient is independent of temperature over the range of interest, the method will give a good approximation to the true temperature coefficient which could be measured by a more sophisticated experiment in which the temperature throughout the core and reflector was made uniform prior to each reading. The values of T_{av} should be plotted against rod worth. The degree of linearity of the resulting curve will give some indication of the validity of the result. From the slope of this curve the temperature coefficient of reactivity may be found.

C. Results

Each student should make a table to record the basic data of T_{in} , ΔT , regulating rod position, and time. The shim-safety rod position, and time. The shim-safety rod positions should also be noted. T_{av} should be calculated from the above equation and the measured values of T_{in} and ΔT . Rod worth for each regulating rod position should be extracted from the appropriate rod calibration curve. Plot rod worth as a function of T_{av} .

D. Analysis and Conclusions

This is left as an exercise for the student. The validity of the results should be considered.

R.L.I. EXPERIMENT NUMBER 6

VOID COEFFICIENT OF REACTIVITY

INSTRUCTOR: REACTOR STAFF

ABSTRACT

A critical reactor will most likely become either sub or super critical when void is introduced into the core. The magnitude and sign of the reactivity effect due to the void (void coefficient) is a complex function of core design, void location and void volume. The void coefficient is an important parameter in reactor safety studies and reactor operation.

In this experiment, a void coefficient of the Argonaut reactor is measured by a determination of the difference in critical control rod positions when the lucite void holders located in the fuel boxes are empty or full of water.

VOID COEFFICIENT OF REACTIVITY

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
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- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
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- VIII. APPENDICES
 - 1. Discussion Questions
 - 2. The Effect of Fuel Plate Spacing on the Void Coefficient of Reactivity in Argonaut-Type Reactors

I. INTRODUCTION

The void coefficient is usually defined as the magnitude and sign of the reactivity introduced by a void volume of one per cent of the moderator volume. If the core contains two moderators, whether both or only one is being considered in the coefficient must be stated. Also as the void coefficient is strongly position dependent, it is important to consider and report whether, and to what degree, the average or a particular void coefficient has been measured.

Detailed knowledge of the void coefficient is obviously of primary importance in boiling water power reactor design. In hazard evaluation of liquid moderated reactors, the coefficient when negative is considered to be a safety aspect of the design because an excursion which would cause the moderator to boil would reduce the reactivity. In most reactors the temperature coefficient would also be acting to limit the excursion.

The void coefficient experiment at the Argonaut is a measure of the static reactivity worth of void located at the center of each fuel box. Lack of time prohibits a more extensive investigation in one day; although more extensive studies have been made, such as the report included in the appendix.

II. THEORY

The qualitative relationship of reactivity to void in the moderator can be developed from a consideration of the effect of void on the parameters of the two group criticality equation

of the two group criticality equation

$$k_{eff} = \frac{\eta \epsilon p f e^{-\tau B^2}}{1 + L^2 B^2} \quad (1)$$

where η = neutrons born per thermal neutron captured in the uranium, ϵ = fast fission factor; p = resonance escape probability, f = thermal utilization, τ = neutron age, L^2 = thermal diffusion area, and B^2 is the geometric buckling. The quantity $e^{-\tau B^2}$ is fast non leakage probability, and $1/(1 + L^2 B^2)$ is the thermal non leakage probability. When the reactor is critical ($k_{eff} = 1$), introducing void will probably make $k_{eff} \neq 1$, and the reactor power will begin to change with time. This rate of change of power level is a function of the reactivity, $k_{eff} - 1/k_{eff}$. The void coefficient is a measure of the reactor's sensitivity to void, and it is written as follows

$$\text{Void Coefficient} = \frac{k_{eff} - 1}{k_{eff}} \bigg/ \text{per cent moderator void} \quad (2)$$

One effect of the void in equation (1) will be a reduction of the non leakage probabilities because the reduced moderator density decreases the macroscopic cross sections for scattering and absorption which increases both τ and L^2 . They vary with density as

$$\tau + L^2 = \tau_0 \left(\frac{d_0}{d} \right)^2 + L_0^2 \left(\frac{d_0}{d} \right)^2 \quad (3)$$

where d is the density. If the decrease in moderator density results in an increased reactor volume, the decrease of B^2 will at least in part compensate for the increase in τ and L^2 .

Because flux peaking in the moderator is reduced when void is present, both moderator neutron capture and the disadvantage factor are reduced which results in an increase in f .

$$f = \frac{\Sigma_f}{\Sigma_f + \frac{\bar{\phi}_m}{\bar{\phi}_f} \Sigma_m} \quad (4)$$

where $\bar{\phi}_m/\bar{\phi}_f$ is the disadvantage factor and subscripts m and f refer to moderator and fuel. If the reactor is overmoderated, the removal of moderator will be to the reactor a removal of poison which is a positive reactivity contribution.

In general, ρ decreases when void is introduced because of the increased resonance flux, as the neutrons have increased probability of reaching another fuel plate while still having resonance energy.

As U^{238} only fissions for neutrons with energy in excess of about 1 mev, after only a few moderator collisions the neutrons will be below the threshold. A void between fuel plates increases the probability of a neutron colliding with a U^{238} atom while it yet is above the threshold, hence ϵ would tend to increase.

The effect on η is negligible. With hardening of the neutron spectrum, ν increases but the ratio of U^{238} to U^{235} captures also increases.

The overall effect of void will depend on which factors dominate, and in most cases the overall effect of void is to reduce k_{eff} .

III. APPARATUS

1. Argonaut Reactor
2. Lucite void holders - see photograph on the following page.

IV. PROCEDURE

After completing the reactor check out, the reactor is brought to criticality at low power with the void holders in place and empty. When the critical rod positions have been determined, the fine rod is dropped to its "in" position for a reason developed later. After a two minute wait, the reactor is scrammed. The void holders are filled with water. Then the reactor is brought back to criticality with the control rods in their first run positions, except for the fine rod. The actual adjustment for criticality is by means of the fine control rod.

The second position of the fine rod will be less than the first, and the difference in rod position is due to the void. Prior to the void experiment, the fine control rod was calibrated in terms of reactivity versus rod position (rod calibration experiment), and from the calibration curve the reactivity worth of the rod position difference -- hence void worth -- can be directly determined.

Two alternate methods of determining the void worth are to be performed. The first method is to remove the fine control rod from position 2 to position 1. The reactor will now be on a positive period. After waiting 90 seconds for the asymptotic period to be established, measure the period by determining the time it takes the power to increase by a factor of 2.7. This period determination can be readily obtained from the linear power level recorder chart. A calculated curve is available that gives the positive reactivity associated with a given period.

VOID COEFFICIENT APPARATUS

The second alternate method is to drop the fine rod to its "in" position from critical position 2. This step was also performed from position 1. From the linear power level recorder chart following a rod drop, the flux ratios of ϕ_t/ϕ_o are determined for various times. Calculated curves are available that give the negative reactivity associated with a certain ϕ_t/ϕ_o ratio at time t . Subtracting the reactivity worth of position 2 from position 1 yields the negative reactivity worth of the void.

From the known void volume, water moderator volume, and average reactivity worth, calculate the void coefficient.

V. RESULTS

RUN #1 Temperature: _____
 Rod positions: Fine _____ Coarse _____ Shim _____

Void Positions	Void Volume
	cm ³
	cm ³
	cm ³
	cm ³
	cm ³
	cm ³
Total Void Volume:	cm ³

RUN #2 Temperature: _____
 Rod positions: Fine _____ Coarse _____ Shim _____
 Reactor period seconds = \$ _____ = _____ %

Calculations:

$$\begin{aligned} \text{Volume of one fuel box} &= \text{_____ cm} \times \text{_____ cm} \times \text{_____ cm} = \text{_____ cm}^3 \\ \text{Volume of one fuel plate} &= \text{_____ cm} \times \text{_____ cm} \times \text{_____ cm} = \text{_____ cm}^3 \\ \text{Volume of fuel plates in 1 box} &= 17 \times \text{_____ cm}^3 = \text{_____ cm}^3 \end{aligned}$$

Effective volume of moderator

$$\text{in one fuel box} = \text{_____ cm}^3 \text{ minus } \text{_____ cm}^3 = \text{_____ cm}^3$$

$$\text{Total effective moderator volume} = 6 \times \text{_____ cm}^3 = \text{_____ cm}^3$$

$$\% \text{ void volume} = \frac{\text{cm}^3}{\text{cm}^3} = \text{_____ \%}$$

$$\Delta K (\text{rod calib. curve}) = \text{_____ \%} \quad \Delta K (\text{average}) = \text{_____ \%}$$

$$\Delta K (\text{period meas.}) = \text{_____ \%} \quad \Delta K (\text{temp.}) = \text{_____ \%}$$

$$\Delta K (k_{\text{ex}} \text{ meas.}) = \text{_____ \%}$$

$$\Delta K \text{ corrected} = \Delta K(\text{av}) - \Delta K(\text{temp}) = \text{_____ \%} \text{ _____ \%} = \text{_____ \%}$$

$$\text{Void Coefficient} = \frac{\text{_____ \%}}{\text{_____ \%}} = \frac{\text{_____ \%}}{\text{_____ \%}} \frac{\Delta K}{\text{Void}}$$

VI. ANALYSIS AND CONCLUSIONS

This is an exercise for the student. Include in your discussion the nature or relationship of the compensation for void by the removal of a control rod. Which factor of equation (1) is most directly a function of rod position?

Discuss how the magnitude of the void coefficient that you have measured is related to the average coefficient.

VII. BIBLIOGRAPHY

1. S. Glasstone and M. C. Edlund, "The Elements of Nuclear Reactor Theory," (D. Van Nostrand Co., Inc., New York, 1952).

Experiment Written by: D. A. Daavettila and L. Lawyer

VIII. APPENDICES1. Discussion Questions

The following propositions should be considered prior to your arrival at the Argonaut. An effort will be made to discuss them just prior to and/or during your checkout of the reactor, as time permits.

1. What is a reactor void coefficient and of what use is a knowledge of the void coefficient of reactivity?
2. Discuss the effect of the void on resonance escape, thermal utilization, fast fission factor η , neutron age and diffusion length.
3. Compare the microscopic cross sections for absorption and scattering for hydrogen and for oxygen. Which is the predominant absorber? Discuss the moderating efficacy of hydrogen and oxygen.
4. In measuring the void coefficient of reactivity for the Argonaut, we will insert a void of the order of 25 cm^3 and thus measure a heterogeneous "bulk" void coefficient. For a reactor run at constant temperature, what other types of void coefficient might be of interest in reactors in general?
5. Discuss the necessary relationship between temperature coefficient and void coefficient in a water moderated system, i.e., what physical variable do they have in common?
6. Explain neutron Doppler effect in a heterogeneous, water moderated, slightly enriched system, and its relationship to temperature and void coefficients of reactivity.
7. Might a reactor design incorporate overmoderation? Why?
8. How will you determine the overall reactivity effect of the insertion of the voids in this experiment. What objections might there be to using the control rod calibration curve obtained during the control rod calibration experiment.
9. A commercially available resinous compound named "plexiglass" is used in this experiment as the device which contains the void (air space). Strictly from a consideration of their nuclear properties, is this material to be preferred to a metal void container?
10. Does the simulation of void by air space introduce a significant error in this experiment? If so, estimate the magnitude of such an error.

2. The Effect of Fuel Plate Spacing on the Void Coefficient of Reactivity

in Argonaut-Type Reactors

P. Ameglio, W. Grant, P. Guenther, R. Jain,
M. Kirshenbaum, Q. Nyugen, D. Smith and S. Wang

Introduction

The presence of voids in a reactor has a marked effect on its reactivity. The voids can be produced in the reactor inherently, as in the boiling water reactors, or they may be introduced in the reactor purposely, as, for example, using the voids as a tool to control the reactivity of the reactor. As such, the voids can be quite uniformly distributed in the reactor or they may be introduced at certain specific positions in specific sizes.

The effect of voids on reactivity of the reactor will depend upon their location in the reactor. In the case of a heterogeneous reactor, in particular, the effect of voids on the reactivity will also depend, rather strongly, upon the volume ratio of the moderator to fuel and their location with respect to the fuel elements. In fact, as the results of present study will show, the void coefficient of reactivity changes from negative in the case of undermoderated systems to positive for overmoderated systems. This behavior in the heterogeneous systems can be explained easily in the following way:

The reactivity factor, k_{∞} , is given by the well known relation

$$k_{\infty} = \eta \epsilon p f.$$

The thermal utilization factor, f , is given by the relation

$$f = \frac{\sum_{af} \phi_f}{\sum_{af} \phi_f + \sum_{am} \phi_m + \dots} \quad (1)$$

The subscript "f" denotes fuel and "m" denotes moderator. ϕ is the average flux.

$$\sum_{ai} = \frac{V_i N_i \sigma_{ai}}{V_i} \quad (1a)$$

where V_i is the volume of i th material; N_i is the number of atoms per cm^3 and σ_{ai} is its absorption cross section. The factor p , the resonance escape probability, is given by the relation:

$$\ln p = - \frac{V_f N_f \int_{res} \sigma_{eff} \bar{\phi}_f}{V_m \sum_m N_m \sigma_{s_m} \bar{\phi}_m} \quad (2)$$

where $\frac{\bar{\phi}_f}{\bar{\phi}_m}$ is resonance flux ratio in fuel and the moderator.

Now, the introduction of voids reduces the effective volume of moderator, V_m , by the amount V_{void} . V_{void} is the volume of voids. Then, as can be seen from equations (1) and (2), the decrease of V_{void} tends to increase f and decrease p . Therefore, depending upon the volume ratio of fuel to moderator and the location of voids with respect to the fuel elements, the net effect can be positive or negative.

Void Coefficient of Reactivity

In order to express the effect of voids on the reactivity quantitatively, one has to define a coefficient of reactivity associated with the voids. But in the light of the discussion in our Introduction, it is difficult to give a clear cut definition of this quantity. Although it may be better to define it as the ratio of percent change in reactivity per unit volume of void, i.e., $\% \Delta k$ -volume of void, a more simple definition of void coefficient of reactivity can be the ratio of the percent change in reactivity to percent ratio of the volume of voids to the volume of moderator. Let us call this α . The limitation of this definition can be seen immediately. The value of α will change with the position of voids in the reactor and the volume ratio of fuel to moderator. The situation of voids with respect to one another and their location with respect to fuel element also markedly effects the value of α . Thus, in order to have a full information of α , one has to study the behavior of α as a function of all these factors.

In our present study, we have measured the variation of the value of α with the change of volume ratio of fuel to moderator, keeping all other factors constant as far as was possible. The experiment was done on the Argonaut Reactor. In this reactor the fuel is in the form of 1/8" thick plates separated normally by 1/4" thick layer of water. Because of this simplicity in the geometry of the core the variation in the moderator to fuel volume ratio α can be expressed in terms of the variation in fuel plate spacing. The various moderator to fuel volume ratios V_m/V_f , and the effect of the introduction of voids for these values of V_m/V_f can be simulated as the local perturbation in the otherwise normal core of the Argonaut reactor by varying the spacing between these two plates only between which the void is to be kept. In our experiment, α has been measured as a function of this fuel plate spacing.

A similar study was made previously by Gilbert and Springer (Ref. 1). The results of their study are discussed along with our results. The difference between their study and the present one is in the location of the voids in the core. This is described in detail in the next section.

Description of Experiment

The experiment was performed on the standard one-slab, six box fuel loading consisting of boxes 13 through 18. The voids were introduced by using the void holders made of plexiglass. This material has approximately the same nuclear properties as water. The void holders were 26 inches long and 1 inch wide. The thickness of void holders was kept always equal to the fuel plate spacing being investigated. The voids were made by milling a slot of 17 inches long and 3/4 inch wide with bottom and top plexiglass sheets of thicknesses 0.090 inch and 0.065 inch respectively.

Two voids were used, one each in box 14 and box 16. They were placed in the middle of the boxes. This arrangement was chosen for two reasons: firstly, to avoid any interference between the voids; and secondly, to locate the voids symmetrically in the positions of comparatively high fluxes in the core. These positions were kept fixed as far as was possible throughout the experiment in order to avoid any effect of variation of relative position of voids and of their location in the core. The location of voids with respect to fuel plates was such that there were only thin sheets of plexiglass (the top and bottom covers) between the voids and the adjacent fuel plates. There was no water layer in between. This arrangement approximates the situation when voids are produced in the reactor touching the fuel elements. Thus, the thickness of voids increases with the increase of fuel plate spacing. This arrangement differs from the one used by Gilbert and Springer (Ref. 1). In their experiment, the two voids were kept in the same fuel box with only a fuel plate between them. The voids were kept in box 16.

In order to disturb the core least, the spacing between only those two plates which were adjacent to the voids was changed. The resulting small fuel shuffling in the two boxes is not expected to introduce any significant error in the measurement. Also, any effect of increase of void size with the increase of plate spacing on the void coefficient of reactivity, α , has been neglected.

Procedure

For each run the voids were introduced at the specified positions. The reactor was brought to criticality at a low power and the control rod position was noted. The reactor was then taken critical at a somewhat higher power and then the rod drop method was used to find the reactivity worth. The reactor was then shut down and the voids were flooded with distilled water. The

control rod was drawn out to the previous critical position and the resulting positive period was measured. From the charts available, the excess reactivity was measured. The reactor was then made critical at somewhat higher power and again the rod drop method was used to find the reactivity worth. The difference in reactivity measured by two drops was calculated with the help of a set of charts available. It was found that the two methods give very consistent results. An average value of two results was taken. Temperature correction was made by adding 0.005% $k/^{\circ}\text{F}$ to the results.

Before performing the experiment, it was checked that the reactor had sufficient excess reactivity.

Results

The experiment was done by varying fuel plate spacing from 1/4" to 1-1/2". The results of the experiment are given by Curve A in Figure 1. Curve B in Figure 1 shows the results of the study made by Gilbert and Springer (Ref. 1). There is one more point denoted by C in Figure 1. This point was also studied by Gilbert and Springer (Ref. 1) by putting together two voids, each of one inch overall thickness.

Both the curves in Figure 1 show in common that the void coefficient of reactivity, α , has a large negative value at 1/4" plate spacing. It becomes positive at about 1 inch plate spacing and remains positive for the rest of the fuel spacings studied. Both the curves show an inward curvature having a tendency to become flat in the positive region corresponding to the larger plate spacing. It is interesting to note that these curves have a good resemblance in shape to the negative differential plot of the reactivity curve, k_{∞} against V_m/V_f , of the Argonaut type of systems. A typical plot of the differential curve is given in Figure 2.

The two curves in Figure 1 differ from each other in the fact that Curve B gives in general a large value of α for the various plate spacings. This shows that the neighboring voids interfere constructively. Point C in Figure 1 indicates that the presence of a thin layer of plexiglass, or to that effect water, in a large void does not effect the results seriously.

Discussion

The results show that a qualitative study of the effect of voids in the Argonaut Type of systems of various moderator to fuel volume ratios can be made by simulating them as a local perturbation in the standard core of the Argonaut Reactor. It is very difficult to get a quantitative inference out of this study. One of the reasons is the difficulty of defining the exact region of perturbation and therefore the effective moderator to fuel volume ratio V_m/V_f in

Figure 1
VOID COEFFICIENT α AS A FUNCTION OF FUEL PLATE SPACING

Figure 2
DIFFERENTIAL OF REACTIVITY CURVE AS A FUNCTION OF
MODERATOR TO FUEL VOLUME RATIO

III. 3/4 inch Fuel Plate Spacing

IV. 1 inch Fuel Plate Spacing

V. 1-1/2 inch Fuel Plate Spacing

that region. It therefore forbids to make a good correlation with the reactivity curve, k_{∞} against V_m/V_f .

The authors wish to acknowledge the collaboration of R. Springer and L. Lawyer in the conduct of the experiment.

Reference 1 - "The Effect of Fuel Plate Spacing on the Void Coefficient of Reactivity of the Argonaut" by William S. Gilbert and Robert Springer, August 7, 1959.

3. 1961 ASEE-AEC Reactor Laboratory Institute Writeup

VOID COEFFICIENT OF REACTIVITY

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Voids may occur in the reactor in the form of bubbles either in the cooling liquid or, in the case of a boiling water reactor, in the moderator, furthermore, whenever samples for irradiation are brought into or near the core the containers may only be partly filled with material the rest representing a void. The effect of such a void is a reduction in the overall density of the moderator. The mean free path of the neutrons depends on the density of the medium; also N , the number of atoms per cc is directly proportional to the density. Hence the macroscopic cross sections will be affected by the presence of voids and through the macroscopic cross sections the resonance escape probability and the thermal utilization. Secondary effects on the fast fission factor and on η tend to change the quantities also but these changes are small compared to the others and may be negligible under certain conditions. In addition to these factors B^2 , the buckling, L^2 , the thermal diffusion area and τ , the age are density dependent so that the overall effect of introducing voids is a change of k_{eff} and therefore of the reactivity. Moreover the effect is influenced by the position of void.

In order to estimate the changes produced by the void a coefficient is introduced which measures the change in reactivity as a function of the volume ratio of void to moderator. One such definition is

$$\frac{d\rho}{dV} = \frac{\text{change in reactivity in } \%}{100 \times \left(\frac{\text{change in moderator volume}}{\text{total moderator volume}} \right)}$$

For boiling water reactors it is more convenient to express the coefficient in terms of density. J. A. Thie² defines the void coefficient by:

$$\frac{d\rho}{d\alpha} \quad \text{where } \alpha, \text{ the steam void fraction}$$

$$\alpha \equiv 1 - \frac{\text{density in coolant channel}}{\text{density of water at saturation}}$$

The effect of a void on various parameters

1. The thermal utilization is defined as

$$f \equiv \frac{\Sigma_{af} \phi_f}{\Sigma_{af} \phi_f + \Sigma_{am} \phi_m + \Sigma_i \Sigma_{ai} \phi_i}$$

where the subscripts f and m stand for fuel and moderator and i for any other material present. A void in the moderator reduces Σ_{am} and consequently increases f.

2. The resonance escape probability

$$p \equiv \exp \left(-\frac{N_f V_f}{\xi N_m \Sigma_m} \frac{\bar{\phi}_f}{\bar{\phi}_m} \int_{res} \sigma_a^{eff} \frac{dE}{E} \right)$$

where the average fluxes

$$\bar{\phi}_f = \frac{1}{V_f} \int_f \phi_f dV$$

$$\bar{\phi}_m = \frac{1}{V_m} \int_m \phi_m dV$$

contains both Σ_m and N_m in the denominator of the exponent, hence a reduction of both factors due to a reduction of scattering nuclei will increase the negative fraction and consequently reduce p.

3. ϵ , the fast fission factor has a tendency to increase with an increase in void volume especially if the void is located between fuel plates since the probability of fast neutron collisions with U-238 increases.

4. The effect on η is very small. The slight increase of fast fission neutrons is balanced by an increase in the ratio of U-238/U-235 captures.

5. The diffusion area L^2 and the Fermi age τ are defined;

$$L^2 \equiv \frac{D}{\Sigma_a} = \frac{1}{3\Sigma_a \Sigma_t}$$

$$\tau = \int_{E_{\text{thermal}}}^{E_0} \frac{dE}{3\xi \Sigma_s \Sigma_t}$$

where D = diffusion coefficient.

Σ_a = macroscopic absorption coefficient

Σ_t = inverse of transport mean free path

ξ = average logarithmic energy decrement.

Both quantities are inversely proportional to the square of the macroscopic cross sections, hence inversely proportional to the square of the density. Introduction of a void therefore increases L^2 as well as τ . In homogeneous systems one may assume

$$L^2 + \tau = M^2 = L_o^2 \left(\frac{d_o}{d}\right)^2 + \tau_o \left(\frac{d_o}{d}\right)^2 = M_o \left(\frac{d_o}{d}\right)^2$$

Here the zero subscript indicates the value of the parameters in the absence of voids.

6. The change in the buckling B^2 due to a void may be seen by considering a simple case: the spherical reactor where

$$B^2 = \left(\frac{\pi}{R}\right)^2$$

$$\text{Since } R = \left(\frac{3}{4\pi} V\right)^{1/3}$$

$$B = \frac{C_1}{(V)^{1/2}}$$

$$\text{where } C_1 = \frac{\pi}{\left(\frac{3}{4\pi}\right)^{1/3}}$$

$$\frac{dB}{dV} = -\frac{C_2}{(V)^{2/3}}$$

$$\text{where } C_2 = \left(\frac{\pi}{3}\right)^{4/3} \frac{1}{(4)^{1/3}}$$

The overall effect of a void in terms of the steam void fraction α can be determined in the following fashion:

$$1. \quad \frac{d\rho}{d\alpha} = \frac{\partial \rho}{\partial L^2} \frac{dL^2}{d\alpha} + \frac{\partial \rho}{\partial \tau} \frac{d\tau}{d\alpha} + \frac{\partial \rho}{\partial B^2} \frac{dB^2}{d\alpha} + \frac{\partial \rho}{\partial \epsilon} \frac{d\epsilon}{d\alpha} + \frac{\partial \rho}{\partial \eta} \frac{d\eta}{d\alpha} + \frac{\partial \rho}{\partial p} \frac{dp}{d\alpha} + \frac{\partial \rho}{\partial f} \frac{df}{d\alpha}$$

$$2. \quad \rho = \frac{K_{eff} - 1}{K_{eff}} = 1 - \frac{1}{K_{eff}}$$

$$3. \quad K_{eff} = \frac{K e^{-B^2 \tau}}{1 + L^2 B^2}$$

hence

$$4. \quad \rho = 1 - \frac{(1 + L^2 B^2) e^{B^2 \tau}}{K}$$

furthermore,

$$5. \quad \frac{e^{B^2 \tau}}{K} = \frac{1 - \rho}{1 + L^2 B^2}$$

$$6. \quad \frac{\partial \rho}{\partial B^2} = - \frac{B^2 e^{B^2 \tau}}{K} = - \frac{(1 - \rho) B^2}{1 + L^2 B^2}$$

$$7. \quad \frac{\partial \rho}{\partial B^2} = \frac{-L^2 - \tau(1 + B^2 L^2) e^{B^2 \tau}}{K} = - \frac{[L^2 + \tau(1 + B^2 L^2)] (1 - \rho)}{1 + B^2 L^2} = - \frac{L^2}{1 + L^2 B^2} + \tau (1 - \rho)$$

$$8. \quad \frac{\partial \rho}{\partial \epsilon} = \frac{\partial}{\partial K} \left[1 - \left(\frac{1 + L^2 B^2}{K} \right) e^{B^2 \tau} \right] \frac{\partial K}{\partial \epsilon}$$

but $\frac{\partial K}{\partial \epsilon} = \eta^* p f = K/\epsilon$ hence

$$9. \quad \frac{\partial \rho}{\partial \epsilon} = \left(\frac{(1 + L^2 B^2) e^{B^2 \tau}}{K^2} \right) \left(\frac{K}{\epsilon} \right) = (1 - \rho) \frac{1}{\epsilon} \quad \text{and similarly}$$

$$10. \quad \frac{\partial \rho}{\partial \eta} = (1 - \rho) \frac{1}{\eta}$$

$$11. \quad \frac{\partial \rho}{\partial f} = (1 - \rho) \frac{1}{f}$$

$$12. \quad \frac{\partial \rho}{\partial p} = (1 - \rho) \frac{1}{p} \quad \text{so that}$$

$$13. \quad \frac{d\rho}{d\alpha} = (1 - \rho) \left[\frac{-B^2}{1 + L^2 B^2} \frac{dL^2}{d\alpha} - \frac{B^2 d\tau}{d\alpha} - \frac{L^2}{1 + B^2 L^2} \frac{dB^2}{d\alpha} + \frac{1}{\epsilon} \frac{d\epsilon}{d\alpha} + \frac{1}{\eta} \frac{d\eta}{d\alpha} + \frac{1}{p} \frac{dp}{d\alpha} + \frac{1}{f} \frac{df}{d\alpha} \right]$$

Since ρ is small compared to 1 it may be neglected. The terms in the brackets have been computed by Thie for a particular loading of the EBWR

$$-\frac{B^2}{1+L^2B^2} \frac{dL^2}{d\alpha} = -.004$$

$$-B^2 \frac{d\tau}{d\alpha} = -.110$$

$$\frac{-L^2}{1+L^2B^2} \frac{dB^2}{d\alpha} = +.011$$

$$\frac{1}{\varepsilon} \frac{d\varepsilon}{d\alpha} = +.016$$

$$\frac{1}{\eta} \frac{d\eta}{d\alpha} = 0$$

$$\frac{1}{f} \frac{df}{d\alpha} = +.208$$

$$\frac{1}{p} \frac{dp}{d\alpha} = -.181$$

$$\frac{d\rho}{d\alpha} = -.060$$

Experimental Values of the Void Coefficient - The void coefficient as a function of fractional volume change.

Experiments were performed by J. A. Thie¹ on the EBWR to obtain a relation between the change in reactivity as a function of the fractional volume change of the moderator. Styrofoam, which is inflated polyethylene^{5,7}, was used in the form of sheets to simulate changes in density. Uniformly distributed voids ("temperature voids") were simulated by uniform perforations corresponding to the water density loss as a consequence of temperature changes. "Boiling void" sheets with non-uniform perforations simulated a void volume proportional to the non-uniform water density loss at full power. The results of his measurements are shown in Figure 2 and 3 for a particular loading and particular rod positions. It was found that the void coefficient was sensitive to the control rod position, an effect not well understood.

In an experiment done by Ameglio et al⁹ and by W. S. Gilbert and R. Springer the change in the volume ratio of fuel to moderator was achieved by a variation of fuel plate spacing. The 1/8" thick fuel plates are normally separated by a 1/4" thick layer of water. Various ratios V_m/V_f and the effect of the introduction of voids for these values can be simulated as local perturbations in the otherwise normal core by varying the spacing between the two plates containing the void. Figure 1, Appendix 2 shows the results with

the void coefficient as a function of fuel spacing. The reactivity changes were measured in terms of rod length inserted or withdrawn. In Gilbert's experiment the two voids were placed in the same fuel box while Ameglio distributed them in two fuel boxes. Curve A reflects Ameglio's experiment, curve B that of Gilbert. The latter curve contains a point C which was obtained by bringing the two voids together. These curves show a negative void coefficient which gradually decreases and becomes zero at a spacing of about 1". Further increase of the spacing has the effect of making the void coefficient positive. This may be explained by assuming that the reactor was overmoderated. A void removes a part of the moderator which is equivalent to the removal of poison and therefore to the introduction of positive reactivity. However, if voids are disregarded altogether and the change of K_{eff} plotted as a function of V_m/V_f a similar curve is obtained as seen in Figure 2, Appendix 2. It is therefore questionable how much of the measured void coefficient can be attributed to the presence of the void.

The void coefficient as a function of position

Experiments performed by Thie show a strong dependence of the void coefficient on the position of the void. Again the voids were simulated by perforated styrofoam sheets and radial traverses of these sheets were made through distances of 6 to 23 inches from the center of the core. It is seen in the diagram (Figure 1) that the negative void coefficient increases to a maximum value of $-.2\%$ at 11" and decreases for larger distances to $-.1$ at 21" from the center.

Relation of the void coefficient to the temperature coefficient

The change of reactivity due to voids can be considered as an effect due to an equivalent temperature change causing the same effective change in density. If an average value of the void coefficient is known the temperature coefficient due to density change can then be easily obtained since

$$\frac{\Delta \rho}{\Delta T} = \frac{\Delta \rho}{\Delta V} \cdot \frac{\Delta V}{\Delta T} \quad \text{where} \quad \frac{\Delta \rho}{\Delta T} = \text{void coefficient}$$

$$\text{and} \quad \frac{\Delta V}{\Delta T} = \alpha V \quad \alpha \text{ being the volume coefficient of expansion.}$$

FIG. 1
RADIAL VARIATION OF VOID COEFFICIENT - CORE
LOADING NO. 26 CONTROL RODS IN UPPER 66% OF CORE

FIG. 2
TEMPERATURE VOID ADDITION
TO CORE LOADING NO. 33

FIG. 3
BOILING VOID REMOVAL FROM
CORE LOADING NO. 33

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R.L.I. EXPERIMENT NUMBER 7

REACTIVITY IMPORTANCE FUNCTION OF NEUTRON ABSORBERS IN
A NUCLEAR REACTOR

INSTRUCTOR: W. F. GILBERT

ABSTRACT

The effect of a thermal neutron absorber on the reactivity of a nuclear reactor is found experimentally as a function of the absorber position in the reactor. The relative thermal flux at each location of the absorber is determined by counting a wire made of dysprosium dispersed in aluminum which is irradiated in the reactor. The experimentally determined absorber importance function is compared with the importance function based on one group perturbation theory and a neutron importance in the reactor as a function of position is determined by comparison with the results of a two group perturbation theory.

REACTIVITY IMPORTANCE FUNCTION OF NEUTRON ABSORBERS IN A NUCLEAR REACTOR

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - 1. Discussion Questions

I. INTRODUCTION

The effect of various materials on the reactivity of a nuclear reactor is dependent not only upon the nuclear properties of the material but is also a function of the position of the material in the reactor. A compilation of the relative reactivity effect of a given material at various positions in a reactor is known as the reactivity importance function for that material. A general knowledge of importance functions is essential to reactor designers and operators as they allow the prediction of reactivity effects of various materials placed at given positions in a reactor.

A knowledge of the importance functions for materials having the nuclear properties of neutron absorption, scattering and moderation is basic in working with nuclear reactors. Only the importance function for a neutron absorbing material is considered in this experiment, and the experimental results obtained will be compared with those predicted by perturbation theory.

II. THEORY

The change in reactivity produced when a neutron absorbing sample is added to a critical reactor system is derived in reactor theory as

$$\Delta \rho = \frac{\Delta k}{k} = \frac{\int_{\text{sample}} \phi \Sigma_a \phi^+ dV}{k \int_{\text{reactor}} \phi \Sigma_a \phi^+ dV} = \frac{\int_{\text{sample}} \phi \Sigma_a \phi^+ dV}{\int_{\text{reactor}} \phi \nu \Sigma_f \phi^+ dV} \quad (1)$$

where $\phi \Sigma_a$ is the neutron absorption rate per unit volume.

ϕ^+ is the importance of neutrons at each specific location.

$\phi \Sigma_f \nu$ is the neutron production rate per unit volume. If the neutron importance could be considered constant throughout the reactor, this expression for the reactivity change becomes simply the fraction of total neutron absorptions in the reactor system occurring in the sample.

The neutron importance in the reactor is, however, a strong function of position in the reactor usually being maximum near the center of the reactor and approaching zero at some point near the edge of the reactor. This is largely because of neutron leakage from the reactor and, since neutrons nearer the edge have a greater chance of leaking from the system, their importance to the system is reduced.

If a physically small sample is used, the neutron flux and its importance throughout the sample can be considered constant and removed from the integral in the numerator of equation (1). If the sample's effect on k can be assumed small and the neutron distribution remains nearly constant, the denominator can be considered unchanged by the addition of the sample and therefore constant. Equation (1) then becomes

$$\Delta \rho = \frac{\phi_s \phi_s^+ \int_{\text{sample}} \Sigma_a dv}{K} \quad (2)$$

where the subscript s refers to values of the parameters at the location of the sample and K is a constant. For a given sample the integral $\int \Sigma_a dv$ is constant and therefore the reactivity change due to the sample is found to be

$$\Delta \rho = C \phi_s \phi_s^+ \quad (3)$$

where C is a constant.

The reactivity effect of the sample is therefore found to be proportional to the product of the neutron flux and the neutron importance at the location of the sample. This product is known as the importance function for an absorber placed in the reactor.

Equation (3) used out of context would lead us to believe that changing the neutron flux at the sample by any means, such as changing the power of the reactor, would cause the reactivity effect of the sample to change also. Reference to equation (1), however, indicates that the numerator and denominator increase equally when the power changes since the shape of the flux distribution is independent of power level. Therefore, the flux in

equation (3) must be considered as a function of position only and not as an absolute value.

In reactor perturbation theory, the neutron importance, ϕ_+ , is the adjoint flux obtained by interchanging rows and columns in the neutron flux matrix operator and solving. The resulting solution is orthogonal to the flux. In one group perturbation theory the neutron flux is self adjoint or the neutron importance has the same variation with position in the reactor as does the flux. For this case, the importance function becomes the square of the flux or, for an absorber placed in the reactor.

$$\Delta \rho = c' \phi^2 \quad (4)$$

Where c' is a constant.

This approximation is often used to obtain a rough estimate of the worth of an absorber placed at different position in a reactor since the shape of the flux is usually reasonably well-known. Two group perturbation theory predicts a neutron importance having a somewhat different variation with position than does the neutron flux. In most cases the two group neutron importance, or adjoint flux, is flatter near the center of the reactor and drops off more sharply near the edges than does the flux. This is especially true for a reflected reactor.

III. APPARATUS

1. Reactor
2. End window wire counter for beta counting.
3. Instrumentation for period measurement if the reactor instrumentation is not used.
4. Dysprosium in aluminum wire for thermal flux plot.
5. Suitable absorber sample.

IV. PROCEDURE

The dysprosium in aluminum wire is placed in the glory hole through the core and irradiated to produce sufficient activity for counting. The wire is counted along its length in an end window wire counter to determine the thermal flux distribution through the core region. Correction for the decay time must be made, but since only a relative flux plot is desired, no correction for the irradiation time need be made. For convenience, the resulting thermal flux plot may be normalized to unity at the point of maximum flux.

The absorber sample is placed at the center of the glory hole and the reactor made critical. The sample is then moved to various positions measured from the center of the glory hole and the period of the reactor measured at each position the control rod position being held fixed. The reactivity for each period measurement is determined and plotted as a function of the sample position. A normalized importance function is prepared by considering the reactivity to be zero when the sample is removed from the reactor and the maximum reactivity effect to be unity.

The normalized thermal flux, normalized thermal flux squared, and the normalized importance function should be plotted on the same graph for the purpose of comparison. From the importance function and thermal flux values obtain the neutron importance as a function of position and plot on the same sheet.

V. RESULTS

Results will consist of the normalized plots of thermal flux, thermal flux squared, importance function and neutron importance as a function of position in the reactor.

VI. ANALYSIS AND CONCLUSIONS

The experimental importance function should be compared with that predicted by one group theory. The importance of errors in the various measurements made should be considered.

VII. BIBLIOGRAPHY

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Experiment written by: William F. Gilbert

VII. APPENDIX

1. Discussion Questions

The following should be carefully considered by the student prior to the time when the experiment is scheduled to be performed.

1. The effect of inserting an absorber into a reactor is to change k_{∞} . Which factor in the four factor formula is changed and why?
2. What factors determine the amount of absorber which must be inserted at a particular position in a reactor to obtain a given change in reactivity?
3. Why can the activity induced in dysprosium be assumed to be due to only the thermal neutron flux in the reactor?
4. Why is the center of the reactor chosen as the initial position of the absorber in this experiment?
5. Would a temperature change in the reactor during the course of the experiment have any effect on the results?
6. One method of mounting the small amount of absorber needed for this experiment would be to mix the absorber with lucite powder and mold the mixture into a solid sample. What effect would this have on the results obtained? How might we eliminate this effect?
7. Must flux depression and self shielding in the sample be considered in evaluating the results of this experiment?
8. In this experiment, would you expect to obtain the same results with a $1/v$ absorber such as boron as you would with a non $1/v$ absorber such as cadmium?
9. What methods could be used to evaluate the reactivity changes produced by the absorber? Why is the removal of absorber to produce a positive period more desirable than the insertion of absorber to produce a negative period?
10. For a reactor rising in power on an asymptotic period the flux variation with time is given as $\phi_2 = \phi_1 e^{t/\tau}$ where τ is the period of the reactor. Derive an expression for the period in terms of the flux levels measured at two times separated by an interval Δt .

STUDENT RESULTS

Experiment Performed By R. W. Clack, J. J. Herbst, C. J. Roberts, B. J. Stewart, T. G. Williamson, M. J. Oestmann. July 6, 1961 Submitted By M. J. Oestmann.

19. J. OESTMANN
JULY 6, 1961

REACTIVITY IMPORTANCE OF NEUTRON ABSORBERS

IN A NUCLEAR REACTOR

IMPORTANCE FUNCTION (I.F.)
DEFINITION

I.F. RELATES THE RELATIVE REACTIVITY EFFECT
OF A GIVEN NEUTRON ABSORBING MATERIAL
AT VARIOUS POSITIONS IN THE REACTOR

$$\Delta\rho = \frac{\Delta k}{k} = K \sum_a V \rho_n \phi_n^*$$

WHERE $\sum_a V \phi_n^*$ = TOTAL ABSORPTION RATE OF NEUTRONS
 ρ_n^* = NEUTRON IMPORTANCE OR
ADJOINT FLUX

$$I.F. = \Delta\rho = K \sum_a V \phi_n^* \quad (\text{ONE-GROUP THEORY})$$

$$\Delta\rho = K' \phi_n^* \quad (\text{TWO-GROUP THEORY})$$

WHERE $K' = K \sum_a V$ = CONST FOR A GIVEN SAMPLE

PROCEDURE

1. MEASURE THERMAL FLUX WITH DYSPROSIUM-
ALUMINUM WIRE IN THE GLORY HOLE OF
AGN-201. COUNT ALONG LENGTH (9") OF WIRE
IN BF₃ WIRE COUNTER. $\phi = K C_0^{BF_3}$
 $\phi/\phi_0 = C_0^{BF_3}/C_0$; $(\phi/\phi_0)^2 = (C_0^{BF_3}/C_0)^2$
2. REPLACE ABOVE WIRE WITH ABSORBER SAMPLE
OF BUL (KRYLON SPRAY) ON ALUMINUM ROD
AND MEASURE REACTIVITY FROM PERIOD
MEASUREMENTS USING BF₃ PROPORTIONAL
COUNTER AND THE INHOUR EQUATION
CURVES OF PERIOD VS. REACTIVITY.
CONTROL ROD POSITION IS HELD CONSTANT
CALCULATE THE IMPORTANCE FUNCTION
AND NEUTRON IMPORTANCE FROM THE
REACTIVITY DETERMINATIONS

EXPERIMENTAL DATA RECORDED ON AGN-201

COARSE ROD 22.97 cm.
FINE ROD 22.97 cm.
RECORDING-41 (3X10⁻⁹)
CHANNEL -1 190,200
DYSPROSIUM $T_{1/2} = 2.4$ hr.
 $C_0^{BF_3} = 10006$ barn

FLUX TRAVERSE OF AGN-201 - GLORY HOLE									
WAIT TIME	2-DE	TOTAL COUNTS	BKGD COUNTS	NET COUNTS	$\frac{C}{2-DE}$	DISTANCE FROM CENTER INCHES	ϕ/ϕ_0	$(\phi/\phi_0)^2$	
0	0	11383	43	16340	16340	0	1.00	1.00	
3	1.015	15856	43	15813	16058	1/2	0.982	0.964	
6	1.030	15404	43	15361	15820	1	0.968	0.936	
9	1.046	15021	43	14978	15670	1 1/2	0.958	0.918	
12	1.062	14290	43	14247	15140	2	0.927	0.860	
15	1.077	13627	43	13584	14610	2 1/2	0.895	0.801	
18	1.094	12228	43	12185	13320	3	0.815	0.664	
21	1.110	10835	43	10792	11970	3 1/2	0.733	0.537	
24	1.127	9499	43	9456	10880	4	0.665	0.442	
27	1.144	8352	43	8309	9500	4 1/2	0.581	0.338	
30	1.161	6885	43	6842	7950	5	0.486	0.236	
33	1.178	6052	43	6009	7070	5 1/2	0.433	0.187	
36	1.196	5440	43	5397	6460	6	0.395	0.156	
39	1.214	4811	43	4768	5800	6 1/2	0.354	0.126	
42	1.233	4914	43	4871	6400	7	0.367	0.135	
45	1.251	3917	43	3874	4850	7 1/2	0.297	0.088	
48	1.270	3619	43	3576	4540	8	0.278	0.078	
51	1.289		43			8 1/2			
54	1.309		43			9			
CHECK ON OTHER SIDE OF WIRE									
52	1.295	12998	43	12955	16760	0	1.03	1.062	
55	1.315	12889	43	12846	16880	1	1.03	1.062	
58	1.335	11921	43	11878	15850	2	0.969	0.939	
61	1.355	5147	22	5125	6940	3	0.850	0.723	
63	1.368	4312	22	4290	5865	4	0.722	0.521	
65	1.382	3076	22	3054	4220	5	0.578	0.268	
67	1.396	2496	22	2474	3450	6	0.422	0.178	
69	1.410	2296	22	2274	3210	6 1/2	0.392	0.154	
71	1.424	2138	22	2116	3008	7	0.369	0.136	
73	1.431	1874	22	1852	2650	7 1/2	0.329	0.108	

B ₂ C NEUTRON ABSORBER PLACED IN CENTER OF GLOOMY HOLE, AND REACTOR MADE CRITICAL			B ₂ C NEUTRON ABSORBER PLACED IN CENTER OF GLOOMY HOLE, AND REACTOR MADE CRITICAL			B ₂ C NEUTRON ABSORBER PLACED IN CENTER OF GLOOMY HOLE, AND REACTOR MADE CRITICAL		
COARSE ROD = 23.50 cm			COARSE ROD = 23.50 cm			COARSE ROD = 23.50 cm		
FINE ROD = 23.82 cm			FINE ROD = 23.82 cm			FINE ROD = 23.82 cm		
DISTANCE FROM CENTER, INCHES	TIME, MIN.	COUNTS	DISTANCE FROM CENTER, INCHES	TIME, MIN.	COUNTS	DISTANCE FROM CENTER, INCHES	TIME, MIN.	COUNTS
1	1	10,986	1 1/2	1	11,314	2	1	10948
	2	10,919		2	12089		2	12023
	3	11,001		3	12545		3	12828
	4	11355		4	12549		4	13725
	5	11510		5	13258		5	14792
	6	11658		6	13260		6	15537
	7	11782		7	13919		7	16494
	8	11752		8	14361		8	17616
	9	12142		9	14624		9	18957
	10	12342		10	15215		10	20535
	11	12353		11	15869		11	21358
	12	12633		12	16345		12	23144
	13	12713		13	17069		13	24647
	14	12770		14	17251		14	26436
	15	13289		15	18167		15	27899
2 1/2	1	11720	3	1	12668	3 1/2	1	7158
	2	12918		2	14785		2	9211
	3	14535		3	17479		3	11235
	4	14017		4	20239		4	139—
	5	17815		5	23661		5	17563
	6	19728					6	21312
	7	22120						
	8	24835						
4	1	5636	4 1/2	1	6545	5	1	4910
	2	7675		2	9387		2	7741
	3	10278		3	13388		3	11571
	4	13172		4	18492		4	17278
	5	17729		5	26197		5	25376
	6	23725		6	36408			

DISTANCE FROM CENTER INCHES	TIME, MIN	COUNTS	DISTANCE FROM CENTER, INCHES	TIME, MIN	COUNTS
6	1	4631	8	1	4547
	2	7338		2	7670
	3	11423		3	12107
	4	18184		4	19861
	5	28379		5	31937

IMPORTANCE FUNCTION OF B₄C NEUTRON ABSORBER

DISTANCE FROM CENTER, INCHES	$\gamma = \frac{(t_2 - t_1) 60}{\ln C_2/C_1}$				Δt SEC	γ	μp	$\rho_0 - p$	IMPORTANCE FUNCTION $\frac{\rho_0 - p}{\rho_0}$
	C_1	C_2	C_2/C_1	$\ln C_2/C_1$					
0									1.0
1	10550	13180	1.24	0.216	840	3890	249	595.1	0.959
1 1/2	11400	18125	1.57	0.464	840	1810	570	578	0.932
2	11250	28100	2.50	0.917	840	915	1020	578	0.835
2 1/2	11700	24400	2.08	0.733	420	573	1580	462	0.745
3	12700	37750	2.97	1.09	420	385	2325	388	0.625
3 1/2	7300	32500	4.45	1.49	420	282	3100	310	0.570
4	5800	41000	7.07	1.956	420	215	3250	225	0.563
4 1/2	6600	73000	11.05	2.40	420	175	480	140	0.226
5	5250	83500	15.9	2.76	420	152.5	525	95	0.153
6	4630	28400	6.14	1.815	240	132.0	600	20	0.0322
8	4700	51000	10.85	2.38	300	126.0	620	0	0.0
NEUTRON IMPORTANCE									
DISTANCE FROM CENTER, INCHES	IMPORTANCE $\frac{\rho_0 - p}{\rho_0}$	FLUX ϕ/ϕ_0	NEUTRON IMPORTANCE $\frac{\rho_0 - p}{\rho_0} / \frac{\phi}{\phi_0}$						
0	0.959	0.968	0.990						
1 1/2	0.932	0.958	0.973						
2	0.835	0.927	0.900						
2 1/2	0.745	0.825	0.833						
3	0.625	0.815	0.768						
3 1/2	0.570	0.733	0.695						
4	0.563	0.665	0.546						
4 1/2	0.226	0.581	0.389						
5	0.153	0.486	0.315						
6	0.0322	0.395	0.0815						
8	0.0	0.278	0.0						

EXPERIMENTAL RESULTS PLOTTED

ANALYSIS + CONCLUSIONS

1. COMPARE EXPERIMENTAL IMPORTANCE FUNCTION WITH THAT PREDICTED BY ONE GROUP THEORY

$$\text{IMPORTANCE FCN} = \text{I.F.} = \phi = K \Sigma V \rho_n \phi_n^+ \\ = K' \rho_n \phi_n^+$$

2. COMPARE EXPTAL CURVE $(\rho_{\infty} - \rho/\rho_{\infty})$ WITH THAT OBTAINED FOR $(\phi/\phi_0)^2$

EXCEPT WITHIN 2" FROM CORE THE I.F. CURVE IS LESS OR SHIFTED TO THE LEFT OF THAT OBTAINED FOR $(\phi/\phi_0)^2$

$(\phi/\phi_0)^2$ - IS A ROUGH ESTIMATE OF THE WIDTH OF THE ABSORBER AT DIFFERENT POSITIONS IN A REACTOR
- IS ABOUT 5-7% HIGHER IN RELATIVE WIDTH THAN IS $\frac{(\rho_{\infty} - \rho)}{\rho_{\infty}}$

IT IS EXPECTED THAT THE 2-GROUP NEUTRON IMPORTANCE BE FLATTER NEAR THE CENTER OF THE REACTOR + DROPS OFF MORE SHARPLY NEAR THE EDGES THAN DOES THE FLUX. (TRUE IN A REFLECTED REACTOR)

2. ERRORS

ANY PARAMETER SUCH AS TEMP THAT WOULD CHANGE OR AFFECT THE REACTIVITY + WILL CAUSE POOR RESULTS

NO CORRECTIONS MADE FOR FLUX DEPRESSION OR SELF-SHIELDING BY ABSORBER SINCE VALUES ARE RELATIVE.

R.L.I. EXPERIMENT NUMBER 8

NEUTRON FLUX MEASUREMENTS BY INDUCED ACTIVATION METHODS

INSTRUCTOR: K. C. RUZICH

ABSTRACT

In this experiment, measurements are made of the fast, resonance, and thermal neutron flux at the same location in the Argonaut core using induced activation detectors. Thoroughly discussed are; the selection and preparation of the foil materials (detectors), the calibration of the counting apparatus, the determination of the induced activity from the foil counting data, and finally the calculation of the particular neutron flux from the activity data.

NEUTRON FLUX MEASUREMENTS BY INDUCED ACTIVATION METHODS

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
 - 1. Selection and Preparation of the Activation Detectors to be Used for the Flux Measurements
 - a. Thermal Flux
 - b. Resonance Flux
 - c. Fast Flux
 - 2. Calibration of the Counting Systems for Determining the Induced Activity of the Detectors
 - a. Geiger-Muller End Window Counter
 - b. Scintillation Counter
 - 3. Irradiation of the Selected Detectors in the Core of the Argonaut Reactor
 - 4. Determination of the Induced Activity by Counting on the Calibrated Counters
 - 5. Calculation of the Neutron Flux
 - a. Thermal Flux
 - b. Resonance Flux
 - c. Fast Flux
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - 1. Discussion Questions
 - 2. Extended Discussion of Neutron Flux Measurements by Induced Activation Methods

I. INTRODUCTION

Neutron induced transmutations occur in certain stable elements when they are placed in a reactor, and the activity of the radioactive products can be measured with an appropriate counting system. If the response (cross section variation with energy) of the detector is known, the neutron flux can then be determined. The neutrons in a reactor are spread over a large energy range - from fission energies down to the thermal region. Many detectors have responses which are well suited for measuring the neutron flux in certain parts of this energy range. In this experiment measurements will be made of the neutron flux at a point in the core of the Argonaut using induced activation detectors with desired responses throughout the neutron

energy spectrum. The results which will be obtained will be strongly dependent on the theory involved in the experiment, since the detectors response and the shape of the neutron energy spectrum are assumed to be known.

II. THEORY

A detailed writeup of the theory involved in this experiment can be found in Appendix II.

III. APPARATUS

- A. Argonaut Reactor
 - B. Induced Activation Detectors
 - 1. Gold foil discs
 - 2. Indium foil discs
 - 3. Manganese foil discs
 - 4. Standard gold foils
 - 5. Sodium carbonate pellets
 - 6. Phosphorous pellets
 - 7. Sulfur pellets
 - C. Aluminum and cadmium foil covers
 - D. Counting Systems
 - 1. G-M end window counter
 - 2. NaI scintillation counter
 - E. Detector holders for reactor irradiation
- Pictures of 1. Foil discs and pellets
- 2. Holder
 - 3. Counting equipment

IV. PROCEDURE

1. Selection and Preparation of the Activation Detectors to be Used for the Flux Measurements.

a. Thermal Flux

Gold and indium will be used to measure the thermal flux in the Argonaut reactor. The thermal activation cross sections for these materials are well-known. In the thermal region of the neutron flux spectrum, both materials can be assumed to be $1/v$ detectors. In order to insure that no self shielding or flux depression occurs during the irradiation, the detector material was made extremely thin. A known mass of each detector material was irradiated in a reactor to a desired activity. The material was then counted on a Geiger-Mueller counter,

using a beta absorber and only counting gammas. The material was then placed in an evacuated bell jar and evaporated onto polystyrene discs, 15/16 inch in diameter. The discs were then removed from the bell jar and counted using the same Geiger-Mueller system. The counting rate at this time was extrapolated back to the initial counting period. The ratio of the counting rates would then determine the ratio of the mass of the material on the polystyrene disc to the original mass. This method requires that the efficiency of the Geiger-Mueller system remains the same for both the disc and the original material. Gamma counting on the Geiger-Mueller system is advantageous in this case, since the efficiency of the system will depend primarily on the geometry factor. Self absorption and forward scattering can be considered the same in both cases (negligible). Most of the error in the mass determination will come directly from the Geiger-Mueller system itself. Since the initial counting rate is fairly high, it becomes necessary to consider the dead time of the counting system. However, this determination should not make the error involved in the mass measurement greater than about 10 per cent.

b. Resonance Flux

Gold and indium discs of the same type as in (1) will be used to measure the slowing down density at specific energies. In this region of interest, these detector cross sections are fairly well known in terms of resonance activation integrals (based on a $1/E$ neutron flux spectrum being present). However, since each material exhibits a main resonance peak which makes up most of the activation integral, an indication of the neutron flux at these resonance energies can be made. The importance of having thin detectors is most prominent in this region. The peak cross section at these resonance energies rises to 30,000 barns, hence self shielding inside a thick detector will be an extremely large factor.

Cadmium covers of 20 mil thickness have been chosen to separate the thermal activation from the resonance activation. The cadmium difference method will be used with manganese and sodium detectors, along with indium and gold detectors, to determine whether or not the neutron spectrum has a $1/E$ dependence. The manganese detectors are made up as foil discs in the same way as the gold and indium. It will be necessary only to determine the relative weights of these detectors, and this can be done by irradiating two discs in the same flux and counting the activation produced. The sodium detectors are one inch in diameter pressed pellets in the form of Na_2CO_3 . These pellets are approximately 1/8 inch thick, and are sprayed with Krylon to prevent flaking. Each pellet is weighed to determine the number of sodium atoms in the pellet.

VACUUM SYSTEM AND BELL JAR

c. Fast Flux

The fast flux shall be measured with a sulfur detector. The threshold reaction $S^{32}(n,p)P^{32}$ will be utilized. The sulfur detectors are made up in pellet form, one inch in diameter and one-quarter inch thick. The sulfur was easily pressed into pellets, no binder was needed. The pellets were sprayed with Krylon for protection. Precipitated sulfur, USP grade, was used. A phosphorus pellet was used for the calibration of the end-window Geiger-Mueller counter. The pressed pellet made of red phosphorous is the same size as the sulfur pellet. The amorphous red phosphorous powder was mixed with a small amount of binder ($\sim 4^0/s$ polyvinyl alcohol powder) before being pressed. The pressed pellet was also sprayed with Krylon.

The thickness of the pellets are purposely made larger than the range of P^{32} , beta particle ($\beta = 1.70$ MEV, range ~ 800 mg/cm²). This insures that the active weight of the pellets that the counter "sees" is the same.

A standard gold foil (49.0 mg/cm², one square cm in area) is used to measure the thermal neutron flux in which the phosphorous pellet is irradiated.

2. Calibration of the Counting Systems for Determining the Induced Activity of the Detectors.

a. Geiger-Mueller End Window Counter

This system is used for counting the P^{32} beta particles (1.7 MEV). There are five shelf positions at varying distances from the tube. The shelf positions were calibrated relative to adjacent shelves and then to a particular shelf which is absolutely calibrated. To make this absolute calibration, a phosphorous pellet is irradiated in a well thermalized flux along with a standard gold foil. The gold foil is counted on a calibrated Geiger-Mueller counter which has been calibrated previously using a standard pile for the known flux. From this measurement, the thermal flux in which the phosphorous was irradiated can be determined. The phosphorous pellet is then counted on the end-window counter. As previously stated the range of the beta particle determines the active weight which is counted. The counting rate equals,

$$C_p^k = \phi_{th} \sigma_p N_p V_p (1 - e^{-\lambda_{32} T_p})$$

C_p = corrected counts per sec at end of irradiation

λ_{32} = P^{32} decay constant (min⁻¹)

T_p = irradiation time (min)

- N_p = number of P^{31} atoms per cm^3
 σ_p = P^{31} activation cross section cm^2
 Q_{th} = thermal flux neutrons/ cm^2 sec
 V_p = active volume of the pellet
 K = inverse of the counter efficiency
 N_o = Avagadro's number
 A_p = phosphorous atomic weight
 ρ_p = density of phosphorous pellet
 $(V\rho)_p$ = active weight of p
 S_p = content of P^{32} in the pellet $O/W \times 100$

The efficiency can be expressed,

$$\frac{K}{(V\rho)_p} = \frac{\phi_{th} \sigma_p N_o S_p (1 - e^{-\lambda_{32} T_p})}{C_p A_p}$$

This is an accurate expression for the efficiency if V_p does not differ appreciably from V_s , the active volume of the sulfur pellet. If these volumes are quite different, the overall geometry factors of each pellet would differ also. Fortunately, this is not the case, for the densities of the two types of pellets are almost equivalent.

The activation for the sulfur pellet can be expressed:

$$KC_s = \phi_f \sigma_o N_s V_s (1 - e^{-\lambda_{32} T_s})$$

Where the factors involved are the same as the previous case, but with the proper subscripts, and

$$\begin{aligned} \phi_f &= \text{the integrated flux above 2.9 MEV} \\ \sigma_o &= \text{microscopic cross section for } S^{32}(n,p)P^{32} \text{ reaction} \\ k_{32} &= \text{isotopic content of } S^{32} \\ \phi_f &= \frac{KC_s A_s}{\sigma_o \rho_s V_s N_o k_{32} (1 - e^{-\lambda_{32} T_s})} \end{aligned}$$

Since the half life of P^{32} is 14.3 days, $\lambda_{32} t$ is small for a one hour irradiation.

$$(1 - e^{-\lambda_{32} T_p}) = \lambda_{32} T$$

Substituting for $\frac{K}{(V\rho)_s}$ which equals $\frac{K}{(V\rho)_p}$ and cancelling terms

$$\phi_f = \frac{C_s \sigma_p A_s T_p \phi_{th} S_p}{C_p \sigma_o A_p T_s k_{32}}$$

To eliminate an S^{35} beta produced by thermal activation in the sulfur pellet, a thin 40 mg/cm² absorber should be kept over the window when beta counting.

This absorber must also be present for the counter calibration with the phosphorous pellets.

b. Scintillation Counter

A 2 x 2 inch NaI scintillation counter is used to count gamma rays. The efficiency of this system for counting gamma rays is much higher than that of a Geiger-Mueller counter. The counting system is calibrated for the gold and indium detectors. The method used to calibrate the counter consists of the following:

- (a) Indium and gold foils are made which are of the same size as the foil discs but much thicker (Au ~ 55 mg/cm², In ~ 93 mg/cm²).
- (b) These foils are irradiated in the standard pile where the thermal flux and the cadmium ratios are known.
- (c) The activity of each foil is calculated, making the proper corrections for self shielding and resonance activation.
- (d) The foils are counted on the five shelves of the scintillation counter.
- (e) The corrected counting rates are extrapolated back to the removal time. The efficiency of each shelf position is determined by the ratio of the activity to the counting rate at the time of removal.

The efficiency of the counter measured in this manner will be somewhat in error since the calibration foils are much thicker than the foil discs. This error should be small compared to the error involved in the standard pile flux (10 per cent and possibly greater). The self absorption of gamma rays in the thick foils will only be about 1 per cent. This is the primary reason for choosing to count gamma rays instead of beta particles. In order to calibrate a counter a thick foil had to be used to obtain enough activation in the standard pile. If the beta particles were counted the self absorption loss would be as high as 20 per cent for the gold foil used in the calibration.

GAMMA RAY SPECTROMETER

The manganese foil discs and the sodium pellets do not require an absolute measurement of their activity since only their cadmium ratio is necessary for the experiment. The manganese will be counted on an end-window Geiger-Mueller counter, and the sodium on the NaI scintillation counter.

3. Irradiation of the Selected Detectors in the Core of the Argonaut Reactor.

Ideally all the detectors should be irradiated at a certain position in the reactor. This would require numerous irradiation runs at a certain power level. To eliminate some of these runs the detectors, in pairs, will be irradiated at different locations along the vertical center line of the core. A special aluminum detector holder has been made for this experiment. Since the spacing of the fuel plates is only 1/4 inch the center fuel plate of fuel box 16 (or 4) will be removed and the holder inserted in its place. The diagram below describes the detector placement on the holder. Two separate runs will be made, the first with the detectors covered with aluminum

(bare) and the second with cadmium covers. A gold foil disc will be irradiated with the sulfur pellet in both irradiations. These two detectors will monitor the flux to insure that the power level is the same for both irradiations. They will also serve to compare the bare activations at positions (1) and (2). From previous irradiations it has been found that the cadmium ratio is constant throughout the region where the detectors are placed. This indicates that the thermal to resonance flux ratio remains constant. The thermal flux at positions (1) and (3) should be approximately the same since these locations are symmetrical about the horizontal centerline near the center of the core.

The irradiation time for each run will be one hour at 100 watts ($\sim 1 \times 10^9$ thermal flux) for a one slab loading. The reactor will be brought to the

desired power level on a period less than 20 seconds. This will eliminate any correction needed for the activation of detectors during start-up.

During the first irradiation a phosphorous pellet and standard gold foil will be placed in the J-10 stringer inside the external thermal column. This is the pellet that will be used to calibrate the end-window counter for P^{32} betas. The counter has also been standardized for the standard gold foil, hence it can be used to determine the thermal flux in which the phosphorous is irradiated.

4. Determination of Induced Activity by Counting on the Calibrated Counters.

After the detectors are removed from the reactor they will be taken to the staff counting room in building 24 to be counted.

NaI Scintillation Counter

- (a) Gold foil discs
- (b) Indium foil discs
- (c) Sodium carbonate pellets

The calibration of this system for gold and indium is listed in the table below:

<u>Gold Calibration</u>		<u>Indium Calibration</u>	
H.V. 960 Window level 10.0 Integrate Coarse gain #4 Amplifier gain 1500		H.V. 960 Window level 10.0 Integrate Coarse gain #3 Amplifier gain 1500	
SHELF	dPM/CPM	SHELF	dPM/CPM
1	7.23	1	5.12
2	14.4	2	9.89
		3	15.9
		4	22.8
		5	29.4

Geiger-Mueller End-Window Counter

- (a) Sulfur pellets
- (b) Phosphorous pellets
- (c) Standard gold foil
- (d) Manganese foil discs

The standardization of the end-window Geiger-Mueller counter using standard gold foil ($\sim 50 \text{ mg/cm}^2$) follows below. The listed values are for the standard foil holder in all five shelf positions, no absorber was used.

G-M end-window Standard Gold Foil

Calibration High Voltage 600 V Discriminator setting 90	
SHELF	dPM/CPM
1	5.88
2	11.5
3	20.8
4	34.8
5	53.7

5. Calculation of the Neutron Flux.

The table below lists the constants of each detector material needed for the flux calculations. The thermal activation cross sections given are those for a neutron velocity of 2200 m/sec. The activation reaction and the products most prominent decay particles are listed. The resonance activation integrals (taken from reference 3) have a lower cut-off energy of 0.5 ev. Alpha, the ratio of the resonance component of the resonance activation to the $1/v$ component, is listed for those detectors used in the cadmium difference method.

DETECTOR	REACTION	THERMAL σ BARNs	$\int \alpha \frac{dE}{E}$	T 1/2	λ^{-1} MIN	α
Au	$n + \text{Au}^{197} \rightarrow \text{Au}^{198}$ $\beta^- \text{ MEV} = .960(98.6\%)$ $\gamma \text{ MEV} = .412(100\%)$	96 98	1558	2.7 day	1.78×10^{-4}	34
In	$n + \text{In}^{115} \rightarrow \text{In}^{116}$ $\beta^- \text{ MEV} = 1.00(51\%)$.87(28%) .60(21%) $\gamma \text{ MEV} = 1.09(54\%)$ 1.27(75%) 0.41, 1.49, 2.09(25% each)	145	2640	54.2 min	1.28×10^{-2}	40
Mn	$n + \text{Mn}^{55} \rightarrow \text{Mn}^{56}$ $\beta^- \text{ MEV} = 2.81(50\%)$ 1.04(30%) 0.65(20%) $\gamma \text{ MEV} = 0.845(100\%)$ 1.81(30%) 2.13(20%)	13.4	11.8	2.58 hr	4.48×10^{-3}	0.97
Na	$n + \text{Na}^{23} \rightarrow \text{Na}^{24}$ $\beta^- \text{ MEV} = 1.34(100\%)$ $\gamma \text{ MEV} = 1.38(100\%)$ 2.76(100%)	0.56	0.24	15.0 hr	7.70×10^{-4}	0
P	$n + \text{P}^{31} \rightarrow \text{P}^{32}$ $\beta^- \text{ MEV} = 1.70(100\%)$	0.23		14.3 day	3.37×10^{-5}	
S	$\text{S}^{32} (n, p) \text{P}^{32}$	Reaction Cross Section 0.30	Effec- tive Cut-Off Energy 2.9 MEV	14.3 day	3.37×10^{-5}	

The counting rates obtained on part 4 for each detector should first be extrapolated back to the time at which the detector was removed from the reactor. This counting rate divided by the detector materials weight will be denoted c_o .

a. Thermal Flux

From identical bare and cadmium covered detector irradiations the thermal activation can be determined:

$$(c_o)_{th} = (c_o)_{bare} - x(c_o)_{cd}$$

where x is the correction factor dealing with that portion of the resonance activation lost in the cadmium covered irradiation. For all the detectors except indium x equals one; for indium it is slightly greater than one (1.035). The thermal activity is determined for the gold and indium detectors since the counter efficiency is known for these materials.

$$A_{th} = KC_o W_b = \sigma_p n_p v_p N_T (1 - e^{-\lambda T})$$

v_p = most probable neutron velocity

W_b = weight of bare detectors

n_o = neutron density

N_T = total number of detector atoms

The thermal flux, the total neutron density times the velocity of 2200 m/sec is determined from this fundamental relationship. This quantity should be transformed to the more correct representation of the thermal neutron flux as a monoenergetic neutron (neutron density $\times$ the average velocity). A temperature correction assuming the neutron temperature is that of the moderator should also be made.

b. Resonance Flux

The cadmium difference method for the determination of the $1/E$ flux dependence requires the measurement of the cadmium ratio of the gold, indium, manganese and sodium detectors. The cadmium ratio is expressed:

$$R_{cd}^{-1} = \frac{(c_o)_{th}}{(c_o)_{cd}}$$

When the value of $(R_{cd}-1)$ for each detector is substituted into the following equation:

$$\frac{\phi_r}{\phi_{th}} = \frac{2.22}{(R_{cd} - 1)(1 + \alpha)}$$

The ratio of ϕ_r / ϕ_{th} should be approximately the same. If this is found to be true, the differential flux spectrum within the energy response region of the detectors will be:

$$\phi(E) = \frac{\phi_r}{E}$$

The calculation of the flux-per-lne interval (ϕ_r) can be made with each of the detector materials since the thermal flux is known.

Separate calculations to measure the slowing down density with the gold and indium detectors will also be made. The saturated activity of the detector can be expressed:

$$A_{\infty} = \frac{K C_o W_{cd}}{(1 - e^{-\lambda T})} = \frac{q(E_n)}{\xi \Sigma_s} N_T \int_{0.5}^{\infty} \sigma \frac{dE}{E}$$

W_{cd} = weight of cadmium covered detector

K = inverse counter efficiency

E_n = resonance peak energy

The slowing down density at the detector's resonance energy will be determined from this expression. The slowing down power in the Argonaut reactor at the resonance energy of the detectors can easily be determined:

$$(\Sigma_s \xi)_{core} = (\Sigma_s \xi)_{H_2O} + (\Sigma_s \xi)_{graphite}.$$

The molecular and atomic density of H_2O and graphite is taken from reference 4, page 9, and the scattering cross sections from BNL 325.

$$\Sigma_s \xi = 0.01945 (43.7) (.92) + 0.016944 (4.8) (.158) = 0.795 \text{ cm}^{-1}$$

c. Fast Flux

The sulfur threshold detector is used to measure the integrated fast flux above 2.9 MEV. In part C, the equation used to determine the fast flux is derived in conjunction with a phosphorous pellet calibration of the Geiger-Mueller counter.

$$\phi_f = \frac{C_s \sigma_p A_s T_p S_p}{C_p \sigma_s A_p T_s k_{32}} \phi_{th}$$

The thermal flux (ϕ_{th}) in which the phosphorous pellet was irradiated is measured with a standard gold foil. The equation used for this calculation is:

$$A = K C_o W_{STD} = \sigma_p \phi_{th} N_T (1 - e^{-\lambda T})$$

where K = inverse efficiency of the standard end-window counter and

W_{STD} = weight of standard foil

V. RESULTS

The results of the experiment, calculated from the previously derived equations, consist of the following:

- A. Thermal neutron flux (neutron density x the average neutron velocity).
- B. Resonance neutron flux per $\ln e$ energy interval.
- C. Slowing down density at Indium and Gold resonance energies.
- D. Integrated fast neutron flux above 2.9 MEV.

A table should be made up containing the result obtained with each detector in its respective neutron flux region.

VI. ANALYSIS AND CONCLUSIONS

This section should contain a discussion of the results obtained in the experiment. Since a number of detectors were used to measure both the thermal and the resonance flux, a comparison of their results can be made. Measurement of the resonance flux is dependent upon a $1/E$ neutron spectrum being present in the reactor core. The cadmium difference method was used to determine if this dependency exists. What explanation can be given if it is found, that a $1/E$ spectrum is not present? Many errors can arise in the procedure used for this experiment. The measured weight of the foil disc detectors could involve a large error. The technique used to calibrate the counters for measuring the detector's activity could be refined to give more accurate results. Careful analysis of the results would, of course, be a lengthy task in itself; it is suggested that each student try to examine the

more prominent errors that could have arisen and why there are any deviations from the expected theoretical results.

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VIII. APPENDICES

1. DISCUSSION QUESTIONS

The following is a list of topics and questions which will be discussed prior to the performance of the experiment. Students are requested to go over the available material and be prepared to join in the discussion.

1. The relationship between neutron flux, activation rate and detector activity.
 - a. What is the fundamental equation describing the activation rate of a detector?
 - b. After a detector has been activated in a reactor for time t , what is the simple relationship describing its activity?
What is the saturated activity?
2. Determination of detector activity by counting methods.
 - a. If a detector's activity is measured during an interval time between t_2 and t_3 , what will the expression for the saturated activity be? Assume that the counting system has an efficiency of one. What assumptions can be made about this equation?
 - b. What methods can be used to calibrate a counting system to be used for determining the detector's activity?
3. Measurement of the thermal flux is performed by using a $1/v$ detector and a cadmium ratio measurement to eliminate any activation occurring above the thermal region.
 - a. What is meant by a $1/v$ detector? Why is it advantageous to use such a detector to measure the thermal flux? What is actually measured by the detector?
 - b. What is meant by hardening of the flux spectrum in the detector in question? How can this be eliminated?
 - c. What is the energy distribution of neutrons in a weakly absorbing medium where the neutrons are in thermal equilibrium with the moderating atoms?
 - d. What is the most probable velocity of the neutrons in the Maxwell-Boltzmann distribution? The average velocity?
 - e. How can the thermal flux be represented as a monoenergetic flux?
 - f. How does the temperature of the moderator compare with that of the neutrons? If the neutron temperature is assumed equal to the moderator temperature, what correction must be made to the monoenergetic flux representation?
 - g. Since the $1/v$ detector is placed in the core of the reactor, neutrons of higher energies than the thermal neutrons are also present. How can these neutrons be eliminated? What factors are involved in this method?
 - h. From a cadmium ratio measurement it is possible to determine the activation in the detector due to neutrons below some effective cut-off

energy. Under what circumstances does this represent only the thermal activation?

4. Measurement of the neutron flux in the slowing down region necessitates the use of resonance activation detectors. The thermal activation is eliminated by a cadmium ratio measurement.
 - a. What type of an energy distribution will the neutron flux per unit energy exhibit in this region? What conditions must prevail?
 - b. What is the slowing down density? Is it constant in this region? If absorption is present what factor is involved?
 - c. Where does the $1/E$ neutron spectrum join the thermal neutron distribution?
 - d. What is the activation resonance integral? What two components make it up? By what methods can it be determined? Will the integral be dependent upon the cadmium cover thickness which is needed to eliminate thermal activation?
 - e. Explain self-shielding and flux depression in detector foils. How will a resonance detector be effected by these factors?
 - f. What is the cadmium difference method for determining a $1/E$ neutron flux dependence?
5. Measurement of the integrated fast neutron flux with threshold detectors.
 - a. How are the fast neutrons produced in a reactor? What type of an energy spectrum will these fast neutrons have in a U^{235} reactor?
 - b. What is the energy dependence of a threshold reaction initiated by neutrons? Are these reactions exothermic or endothermic? List three threshold reactions which can be initiated by fast neutrons.
 - c. For the following reaction, $S^{32}(n,p)P^{32}$, determine the threshold energy by mass considerations.
 - d. At the threshold energy what is the kinetic energy of the products? If the energy of the neutron is greater than the E_T , what happens to the excess energy?
 - e. When one of the products is an alpha particle or proton, the electrostatic potential barrier of compound nucleus must be considered, what effect is this on the reaction's cross section? How does the energy of the neutron above E_T effect the penetratability?
 - f. If the cross section for a threshold reaction is known, explain how the integrated fast flux above some effective E can be measured. What assumptions must be made?

2. EXTENDED DISCUSSION OF NEUTRON FLUX MEASUREMENTS BY INDUCED ACTIVATION METHODS

DETECTOR ACTIVITY

The rate at which a detector will be activated when placed in a neutron flux, ϕ , can be expressed:

$$dr = N(x,y,z) \sigma_{\text{act}}(E) \phi(E,x,y,z) dE dV$$

where $N(x,y,z)$ = target atoms/unit volume at x,y,z

$\sigma_{\text{act}}(E)$ = microscopic activation cross section

$\phi(E,x,y,z)$ = neutron flux/unit energy at x,y,z

If the neutron flux is assumed constant within the detector and N_T is equal to the total number of detector atoms, the total activation rate is:

$$r = N_T \int_0^{\infty} \sigma(E) \phi(E) dE \quad (1)$$

or
$$R = \frac{r}{N_T} = \int_0^{\infty} \sigma(E) v n(E) dE$$

R = activation rate/target atom

$n(E)$ = neutron density/unit energy

v = neutron velocity

The activation rate will remain constant with time if the neutron flux remains constant and the number of target atoms destroyed by activation is very small compared with the total number of target atoms.

The activity of the detector at any time during its irradiation is determined from the following.

Rate of change of radioactive atoms = rate of production - rate of decay

$$\frac{dA}{dt} = r - \lambda A \quad A = \text{no. of radioactive atoms at time } t$$

λ = decay constant for A

Solving this differential equation, with A equal to zero at time $t = 0$

$$A = \frac{r}{\lambda} (1 - e^{-\lambda T})$$

Therefore, the activity of the detector at time T is:

$$A \lambda = r(1 - e^{-\lambda T}) \quad (2)$$

At saturation, where $\frac{dA}{dt} = 0$, $\lambda T \cong \infty$

$$A_{\infty} \lambda = r = N_T \int_0^{\infty} \sigma(E) \phi(E) dE$$

The number of radioactive atoms and the activity of these atoms can therefore easily be determined at the time the detector ceases to be irradiated. Following the irradiation the number of radioactive atoms and the activity of these atoms will decrease according to the well known decay law.

$$A = A_0 e^{-\lambda t}$$

$$A \lambda = A_0 \lambda e^{-\lambda t}$$

A_0 = no. of radioactive atoms at $t = 0$ (end of irradiation)

t = time after the irradiation

λ = decay constant

If a measurement of this activity is taken between t_2 and t_3 the total number of counts will be:

$$kC_T = \int_{t_2}^{t_3} A \lambda dt = \int_{t_2}^{t_3} A_0 \lambda e^{-\lambda t} dt \quad k = \text{inverse efficiency of the counter}$$

$$kC_T = A_0 (e^{-\lambda t_2} - e^{-\lambda t_3})$$

$$= A_0 e^{-\lambda t_2} (1 - e^{-\lambda (t_3 - t_2)})$$

From this measurement A_0 can be determined

$$\lambda A_0 = \frac{kC_T \lambda}{e^{-\lambda t_2} (1 - e^{-\lambda t_c})} \quad (3)$$

$$\text{If } t_c \ll t_{1/2}$$

$$t_c = t_3 - t_2$$

$$\lambda t_c \ll .693$$

$$1 - e^{-\lambda t_c} = \lambda t_c$$

$$A_o = \frac{kC_T}{e^{-\lambda t_2} t_c \lambda}$$

$$\lambda A_o = \frac{kC_T}{t_c e^{-\lambda t_2}}$$

or for a more exact relationship:

$$A_o \lambda = \frac{kC_T}{t_c e^{-\lambda t'_2}}$$

$$t'_2 = t_2 + \frac{t_c}{2}$$

Relating this activity measurement to the neutron flux by equations (1) and (2):

$$\frac{A_o \lambda}{(1 - e^{-\lambda T})} = r = N_T \int_0^\infty \sigma(E) \phi(E) dE \quad (4)$$

T = irradiation time

NEUTRON FLUX MEASUREMENTS

The neutrons in a thermal nuclear reactor are distributed over a large energy range. Neutrons are continuously being created by the fission process at energies ranging from .05 to 15 MEV (U^{235}). These neutrons are then quickly slowed down by making elastic collisions with a moderating material. The slowing down process continues until the neutrons are in thermal equilibrium with the molecules of the medium. This is the thermal neutron region where the net energy exchange between the medium and the neutrons is zero.

Figure 1

The neutron spectrum can hence be divided in three regions (Figure 1). In each region certain detector materials have a desired activation response which can lead to a measurement of the flux.

THERMAL REGION

In the thermal region of the neutron energy spectrum, the kinetic energies of the neutrons will be distributed statistically according to the Maxwell-Boltzmann distribution law. This distribution derived from the kinetic theory of gases is applicable to the neutron gas only if the medium in which the neutrons are present is weakly absorbing. The velocity distribution is:

$$dn = n(v)dv = \frac{4\pi n_0}{(2\pi kT/m)^{3/2}} v^2 \exp\left(-\frac{1/2 mv^2}{kT}\right) dv \quad (5)$$

where n_0 = total neutron density

k = Boltzmann constant

T = neutron temperature (absolute)

With $E = 1/2 mv^2$ the energy distribution can be written:

$$dn = n(E)dE = \frac{2\pi n_0}{(\pi kT)^{3/2}} E^{1/2} \exp\left(-\frac{E}{kT}\right) dE \quad (5a)$$

The most probable velocity, where $n(v)$ is a maximum, is found by differentiating the neutron velocity distribution and the result equal to zero.

$$v_p = \left(\frac{2kT}{m}\right)^{1/2} \quad E_p = kT$$

$$v_p = (1.648 \times 10^8 T)^{1/2} \quad (6)$$

At room temperature 20°C, v_p will be 2200 m/sec, corresponding to an energy of 0.025 ev. The neutrons average velocity can be obtained from:

$$\bar{v} = \frac{\int_0^\infty n(v) v dv}{\int_0^\infty n(v) dv}$$

$$\bar{v} = \frac{2}{\sqrt{\pi}} v_p \quad (7)$$

The type of detector used in the thermal region normally exhibits an activation cross section which varies as $1/v$. The activation rate of $1/v$ detector is independent of the velocity of the neutrons.

$$\begin{aligned}
 r &= N_T \int_0^{\infty} \sigma(E) \phi(E) dE \\
 &= N_T \int_0^{\infty} \sigma(E) n(E) v dE \\
 &= k N_T \int_0^{\infty} n(E) dE \qquad \sigma \sim \frac{1}{v} \\
 &\qquad \qquad \qquad \sigma = \frac{k}{v} \\
 r &= k N_T n_o \qquad \qquad \qquad (8)
 \end{aligned}$$

It is obvious that the activation rate is a measurement of neutron density, not neutron flux. Hence, a change in neutron velocity (temperature) would not be detected because the change in flux would be cancelled by the change in cross section. To determine the neutron flux from the measured activation rate it is customary to use the cross section at the most probable velocity.

$$\frac{r}{N_T \sigma_P} = v_p n_o \qquad (9)$$

This is why the thermal cross sections tables are those for 2200 m/sec. For materials that do not follow the $1/v$ law in the thermal region a correction can be calculated to determine its equivalent $1/v$ cross section. A ratio of the actual activation to that of a $1/v$ material of the same 2200 m/sec. cross section value can be calculated from the known cross section curve of the material.

$$F = \frac{\int n(v) v \sigma dv}{\int n(v) v \frac{k}{v} dv} = \frac{1}{n_o} \int n(v) \frac{\sigma}{k/v} dv$$

F will vary slowly with the temperature of the neutron distribution.

A more strict representation of the neutron flux would be the use of the average velocity times the neutron density (no. of neutrons crossing unit area per sec). This is determined by dividing the reaction rate by the cross section at the average velocity of the Maxwellian neutron distribution:

$$\frac{r}{N_T \sigma_o} = \bar{v} n_o \qquad (10)$$

The flux is now the neutron density multiplied by the average velocity of the Maxwellian neutron distribution. This is still not an absolute determination of the neutron flux since the temperature of the neutron was not taken in account. The assumption was made that the Maxwellian neutron distribution was that corresponding to a temperature of 20° C. If it is assumed that the neutron temperature is the same as that of the moderator the average velocity of the Maxwellian distribution at this temperature can easily be evaluated.

$$\begin{aligned} \text{Since, } \bar{v} &= \frac{2}{(\pi)^{1/2}} v_p = \frac{2}{(\pi)^{1/2}} \left(\frac{2kT_N}{m} \right)^{1/2} \\ \bar{v} &= \left(\frac{8kT_N}{\pi m} \right)^{1/2} & T_N = T_M \\ \frac{\bar{v}_{293^\circ}}{\bar{v}_{T_M}} &= \left(\frac{293}{T_N} \right)^{1/2} \end{aligned} \quad (11)$$

It must be noted that the energy distribution of neutron is not exactly Maxwellian insofar as it corresponds to a neutron temperature which is slightly higher than that of the moderator. This shift in the Maxwellian peak is caused by the preferential absorption of lower energy neutrons (spectrum hardening) and by the steady addition of neutrons at higher energies which have been slowed down from their fission energies.

SLOWING DOWN REGION

In this region fast neutrons are being slowed down from their fission energies by making elastic collisions with the atoms of the moderator. If it is assumed that Q fast neutrons are being created per cm^3 per second in a large volume of moderator, the number of neutrons per cm^3 slowing down past E per sec can be expressed:

$$q(E) = \frac{N(E) \Delta E}{\Delta t} \quad N(E) = \text{neutron density per unit energy at } E$$

$$q(E) = N(E) \Delta E v \Sigma_s \quad \Delta t = \text{the time it takes the neutrons to transit through } \Delta E$$

If ΔE is small

$$\frac{\Delta E}{E} = \Delta \ln E = \xi$$

$$\Delta t = \frac{\lambda_s}{v} = \frac{\text{scattering mean free path}}{\text{velocity corresponding to } E}$$

$$\lambda_s = 1/\Sigma_s$$

$$q(E) = N(E) v \xi E \Sigma_s$$

ξ = average logarithmic energy decrement per collision

In absence of absorption $q(E) = Q$

Rearranging

$$N(E)v \sum_s \sigma_s dE = \frac{Q}{E \xi} dE$$

number of scattering collision = influx gain

$$\phi(E)dE = \frac{Q}{\sum_s \xi} \frac{dE}{E} \quad (12)$$

Hence, neglecting neutron absorption and leakage, the neutron flux in the slowing down region is proportional to $1/E$. The energy distribution will actually be a sum over many $1/E$ distributions, each one extending up to a particular energy in the fission spectrum.

The activation rate of a material as previously presented is:

$$r = N_T \int_0^{\infty} \sigma(E) \phi(E) dE$$

$$r = \frac{Q}{\sum_s \xi} \int_{E_1}^{E_2} N_T \sigma(E) \frac{dE}{E} \quad (\text{where } E_1 \text{ to } E_2 \text{ represents the slowing down region})$$

The assumption is made that the scattering cross section remains constant energy. The upper limit of the resonance activation integral, E_2 can be considered equal to infinity since all the activation in the resonance detectors will take place at the lower energies of the $1/E$ spectrum. The constant in front of the activation integral is the neutron flux in any $\ln e$ energy interval (1 to 2.72 ev, 10 to 27.2 ev, etc.)

A detector placed in the core of a reactor is subject to activation from the whole neutron spectrum. To measure the ratio of the thermal response of a detector to its resonance response a cadmium ratio must be made. The cadmium ratio is defined as:

$$R_{cd} = \frac{\text{thermal response} + \text{resonance response}}{\text{resonance response}}$$

If the detectors sensitivity to the neutron flux is known, the response of the detector will be a measure of the neutron flux.

$$R_{cd} = \frac{\int_0^{\infty} \phi(E) \sigma(E) dE}{\int_{E_1}^{\infty} \phi(E) \sigma(E) dE} = \frac{\text{Bare detector activation}}{\text{Cd covered activation}}$$

$$R_{cd} = \frac{\int_0^{E_1} \phi(E) \sigma(E) dE + \int_{E_1}^{\infty} \phi(E) \sigma(E) dE}{\int_{E_1}^{\infty} \phi(E) \sigma(E) dE}$$

$$R_{cd} = \frac{\int_0^{E_M} n(E) v \sigma dE + \int_{E_M}^{E_1} \phi(E) \sigma dE}{\frac{0}{\Sigma_s \xi} + \int_{E_1}^{\infty} \frac{\sigma dE}{E}}$$

The first term in the numerator is the activation caused by thermal neutrons in the Maxwellian distribution. The second term is the activation caused by the $1/E$ spectrum from E_M , the energy at which the $1/E$ spectrum joins the Maxwellian distribution to some cadmium cut-off energy E_1 . Actually the true distribution of the flux near E_M will be quite difficult to describe since it is a transition region. The thermal cut-off energy (E_M) is normally defined by the condition that at E_M the slowing down density equals the absorption under the Maxwell distribution at lower energies.

$$\int_0^{E_M} \phi(E) \Sigma(E) dE = q = \phi(E_M) \xi \Sigma_s E_M$$

$$\Sigma_a \int_0^{E_M} \phi(E) dE = \xi \Sigma_s E_M \phi(E_M)$$

$$\phi(E) = vn(E) \quad n(E) = \text{Maxwell distribution}$$

$$\phi(E_M) = vn(E_M)$$

The flux per unit energy, $\phi(E)$, in the $1/E$ spectrum and the Maxwellian distribution are of course equal at E_M . By substituting the Maxwell neutron distribution for $\phi(E)$ and integrating it can be shown that:

$$\frac{\Sigma_a}{\Sigma_s \xi} = \left[\frac{E_M}{KT_N} \right]^2 e^{-E_M/KT_N}$$

Where, T_N is the neutron temperature. Using the constants for the Argonaut reactor similar to those found in reference 4, page 11, for a one slab loading:

$$\Sigma_a \approx .0722 \text{ cm}^{-1}$$

$$\xi \Sigma_s \approx 1.06 \text{ cm}^{-1}$$

$$kT_N = 0.0256 \text{ ev} \quad (T_N = 25^\circ\text{C})$$

E_M is found to be equal to 0.205 ev.

If the crude assumption is made that the neutron flux has a $1/E$ distribution from E_M to E_1 a calculation can be made to determine whether or not this region's activation is appreciable for a $1/v$ detector.

$$\begin{aligned} N_T \int_{E_M}^{E_1} \phi(E) \sigma(E) dE &= \frac{N_T Q}{\Sigma_s \xi} K \int_{E_M}^{E_1} \frac{dE}{E^{3/2}} \\ &= N_T \frac{Q}{\Sigma_s \xi} 2K \left[\frac{1}{\sqrt{E_M}} - \frac{1}{\sqrt{E_1}} \right] \end{aligned}$$

With $E_M = .205 \text{ ev}$

$$E_1 = .500 \text{ ev}$$

$$K = \sigma_{th} \sqrt{E_{th}} = \sigma_{th} \sqrt{.025}$$

$$\begin{aligned} \text{Activation rate from } E_M \text{ to } E_1 &= \frac{Q}{\Sigma_s \xi} N_T \sigma_{th} (0.25) \end{aligned}$$

The ratio of thermal activation to the calculated activation is:

$$\frac{v n_o \sigma_{th}}{\frac{Q}{\Sigma_s \xi} .25 \sigma_{th} \phi_r (.25 \sigma_{th})} = \frac{\phi_{th} \sigma_{th}}{\phi_r (.25 \sigma_{th})}$$

Since ϕ_r is much larger than ϕ_{th} (neutron flux per $\ln e$ interval), it can be seen that the resonance activation in this region is

The lower limit of the resonance activation integral will be an effective cut-off energy depending on the thickness of the cadmium cover and if the detector is placed in isotropic or beam neutron flux.

A cadmium thickness of 20 mils corresponds to an 0.5 ev effective cut-off energy in an isotropic flux. Some neutrons above 0.5 ev will be absorbed and others which are below the 0.5 ev will be transmitted through the cadmium. The neutron flux that would pass through the cadmium cover would have a shape similar to the one in the figure below:

Even if the shape of $\phi(E)$ is not known inside the foil cover it is still possible to determine (providing the cadmium cover eliminates all thermal activation) the effective cut-off energy. By using a thin $1/v$ cadmium covered detector, the activation rate below the effective cut-off energy can be made equal to the activation rate lost above the effective cut-off energy

$$\int_0^{E_1} \phi(E) \sigma(E) dE = \int_{E_1}^{\infty} \phi^*(E) \sigma(E) dE - \int_{E_1}^{\infty} \phi(E) \sigma(E) dE$$

$\phi(E)$ = flux/unit energy inside cd cover

$\phi^*(E)$ = flux/unit energy outside the cover which is a $1/E$ spectrum

$$\sigma(E) \approx 1/v = \frac{K}{\sqrt{E}}$$

$$\int_0^{\infty} \phi(E) \sigma(E) dE = \int_{E_1}^{\infty} \phi(E) \sigma(E) dE$$

$$\begin{aligned}
\text{Measured activation rate} &= \frac{QN_T}{\sum_s \xi} \int_{E_1}^{\infty} \sigma \frac{dE}{E} \\
&\quad (\text{in the core of a reactor}) \\
&= \frac{QN_T}{\sum_s \xi} K \int_{E_1}^{\infty} \frac{dE}{E^{3/2}} \\
&= \frac{QN_T}{\sum_s \xi} K \cdot 2 \left(\frac{1}{\sqrt{E_1}} \right)
\end{aligned}$$

If the neutral flux per $\ln e$ interval ($\frac{Q}{\sum_s \xi}$) and the thermal cross section of the detector are known, E_{eff} can be evaluated from a single irradiation in the core of the reactor. The value of the effective cadmium cut-off energy for a certain thickness of cadmium should hold for any detector whose $\sigma(E)$ exhibits a $1/v$ dependence in the cut-off energy region. If a detector has a resonance activation peak near this cut-off energy the error in a cadmium ratio measurement would become quite large. An approximate correction for the loss of resonance activation (extra gain in thermal activation) in the cadmium cover could be determined by irradiating the detector in different thicknesses of cadmium. A plot of activation versus cadmium cover thickness with a linear extrapolation back to zero cadmium thickness yield the approximate correction. The ratio of A_0/A_{20} is the correction factor for a 20 mil cadmium cover. This factor when multiplied by the resonance activation gives the approximate total resonance response. This quantity subtracted from the bare detector activation gives the true thermal response.

In considering the resonance response of the detector, the activation resonance integral can be divided in two separate parts, the $1/v$ component and the resonance component:

$$\int_{E_1}^{\infty} \sigma(E) \frac{dE}{E} = \int_{E_1}^{\infty} \sigma_{1/v} \frac{dE}{E} + \int_{E_1}^{\infty} \sigma_r \frac{dE}{E}$$

The $1/v$ component is easily related to the thermal cross section

$$\frac{\sigma_{th}}{\int_{E_1}^{\infty} \sigma_{1/v} \frac{dE}{E}} = \frac{K \frac{1}{(E_{th})^{1/2}}}{K \int_{0.5}^{\infty} \frac{dE}{E^{3/2}}} = \frac{1/\sqrt{.025}}{2 \left[\frac{1}{\sqrt{.5}} \right]} = \frac{1}{.448} = 2.23$$

$$\int_{E_1}^{\infty} \sigma_{1/v} \frac{dE}{E} = .45 \sigma_{th}$$

As previously calculated, the resonance activation lost for a 1/v detector covered with 20 mil cadmium is

$$\int_{E_M}^{E_1} \sigma(E) \frac{dE}{E} = .25 \sigma_{th}$$

Therefore, when considering a 1/v detector, which has no resonance activation peaks approximately one third of the resonance activation is lost in the cadmium cover. This loss need not be considered when making a cadmium ratio measurement since the thermal activation is so much greater than the resonance activation.

The resonance component of the activation integral is due to the resonance peaks of the detector. The type of resonance detector can be of two varieties: (1) With many resonance peaks which give an accumulative effect to the total resonance integral, (2) A single resonance peak attributing to the resonance integral. Only the latter type shall be considered here. The resonance integral for this type can be calculated from the Breit-Wigner formula. In most cases, the resonance integrals have been found by directed measurement using a material with a well known $\sigma(E)$ as a standard detector. The values of the resonance activation integrals will be taken from reference 3, where gold was taken as the standard. Its major resonance at 5 ev was measured with a crystal spectrometer and the small contribution from the next six resonance are taken from neutron time-of-flight spectrometer studies. Gold's cadmium ratio was compared with other resonance detectors, and from this comparison the relative ratio of the resonance integral can be determined.

$$\frac{[R_{cd}^{-1}]_{Au}}{[R_{cd}^{-1}]_x} = \frac{\phi_{th} \sigma_{thAu} / \phi_r \int_{Au} \sigma \frac{dE}{E}}{\phi_{th} \sigma_{thx} / \phi_r \int_x \sigma \frac{dE}{E}}$$

$$\int_x \sigma \frac{dE}{E} = \frac{[R_{cd}^{-1}]_{Au}}{[R_{cd}^{-1}]_x} \frac{\sigma_{th} x}{\sigma_{th} Au} \int_{Au} \sigma \frac{dE}{E}$$

The cadmium cut-off is chosen so that the $1/v$ component of the resonance integral is equal to $0.44 \sigma_{th}$ or $E_1 \cong 0.5$ ev in an isotropic flux.

The standard detector, gold, has a resonance activation integral of 1558 barns of which the resonance component attributes 1513 and the $1/v$ component 45 barns. The thermal cross section for gold was adopted at 98 barns for the measurements.

If it is desired to obtain the slowing down density at a certain energy, resonance detectors of the type just discussed could be used. The saturated activity of the detector (Au or In) would be expressed:

$$A_{\infty} = N_T \int_{E_1}^{\infty} \sigma(E) \phi(E) dE$$

$$= \frac{q N_T}{\xi \Sigma_s} \int_{E_1}^{\infty} \sigma(E) \frac{dE}{E}$$

Although the activation is noted as an integral effect, almost all the activation will take place in a small energy region around the detectors resonance peak. It must be remembered that there are many assumptions made in a measurement of this kind. (1) The medium in which the detector is placed has negligible absorption in the slowing down region. This will depend on the resonance escape probability of the system. In the Argonaut reactor, p has been found to be between 0.95 and 0.97, both by direct measurement and calculations. The loss of neutrons by absorption while slowing down is attributed mostly to capture in U^{238} . Since the Argonaut is a heterogeneous system the slowing down neutron distribution in the fuel elements shall be depleted at the absorption resonance peaks of U^{238} (its first resonance peak lies at 6.7 ev and contribution over half the total resonance absorption integral). As the neutrons move away from the fuel plates into the H_2O moderator the depleted portion of the neutron slowing down spectrum becomes less apparent since neutrons of higher energies are slowed down into this region. Hence, inside the moderator the resonance absorption effect should be smeared out with a net loss of neutrons below the energy at which the U^{238} absorption resonance peaks are

located. Since this loss is small in the Argonaut ($< 5\%$) it can be neglected.

(2) Because our system is of finite size, leakage occurs as the neutrons are being slowed down. Neglecting this leakage could lead to a large error. In a light water moderated reactor, the fast neutron leakage is the predominant leakage factor. Thermal leakage is small since the diffusion length (L) is very small. The so-called age (τ) in a light water system is small also, but it definitely makes up most of the migration length of the neutrons. The fast leakage, therefore, is expected to strongly effect the slowing down density in the assumed $1/E$ spectrum. Since the first flight of the fission neutrons makes up a large portion of the age (fission to thermal), most of the leakage occurs for neutrons of high energy. This is true because the mean free path of fast neutrons in H_2O is long compared to that for lower energy neutrons. This indicates that the assumption that the scattering cross section for the system is constant with energy is incorrect. The scattering cross section for H actually varies quite sharply at high energies increasing down to 10,000 ev. The cross section then levels off until the thermal region is reached; it then rises sharply due to the chemical binding effect of the water molecules.

Each assumption that has been made to describe the slowing down region does lead to an error which would be difficult to determine in most cases.

Returning again to the resonance type detectors, the activation equation should be stated:

$$A_{\infty} = N_T \int_{E_1}^{\infty} \sigma(E) \frac{q(E)}{\xi \Sigma_s(E)} \frac{dE}{E}$$

Where $q(E)$ and Σ_s are functions of energy. To determine $q(E)$ at a certain energy it would be necessary to have a detector which would only absorb neutrons over a small energy interval. This would allow the assumption that $q(E)$ and $\Sigma_s(E)$ are constant over this small energy region. The resonance detectors (Au and In) to a first approximation do possess this quality. The contributions other than the main resonance peak can normally be assumed small. If these assumptions hold, the measurement of ϕ_r will determine the flux at the resonance energy. The value of ϕ_r/E_r will be the flux per unit energy at E_r . If the flux spectrum does exhibit a $1/E$ dependence over a large portion of the resonance region, the differential flux can be stated as ϕ_r/E .

The cadmium difference method can be used to determine whether or not a $1/E$ neutron slowing down spectrum is present. This method requires the use of a number of resonance detectors which have different responses over the resonance region. The cadmium ratio of a resonance detector has been defined as, (14)

$$R_{cd-11} = \frac{\int_0^{E_M} \phi(E) \sigma dE + \int_{E_M}^{E_1} \phi(E) \sigma dE}{\frac{Q}{\sum_s \xi} \int_{E_1}^{\infty} \sigma \frac{dE}{E}}$$

The resonance activation can be divided into:

$$\begin{aligned} \phi_r \int_{E_1}^{\infty} \sigma \frac{dE}{E} &= \phi_r \left[\int_{E_1}^{\infty} \sigma \frac{1}{v} \frac{dE}{E} + \int_{E_1}^{\infty} \sigma_r \frac{dE}{E} \right] \\ &= \phi_r (1 + \alpha) \int_{E_1}^{\infty} \sigma \frac{1}{v} \frac{dE}{E} \quad \text{where } \alpha = \frac{\int \sigma_r \frac{dE}{E}}{\int \sigma \frac{1}{v} \frac{dE}{E}} \end{aligned}$$

for an 0.20 mil cadmium cover $E_1 = 0.5$ ev

$$\int_{0.5}^{\infty} \sigma \frac{1}{v} \frac{dE}{E} = 0.45 \sigma_{th}$$

and as previously calculated

$$\int_{E_M}^{E_1} \phi(E) \sigma dE \cong 0.25 \sigma_{th} \phi_r$$

hence,

$$R_{cd-11} = \frac{\phi_{th} \sigma_{th} + 0.25 \phi_r \sigma_{th}}{0.45 \sigma_{th} \phi_r (1 + \alpha)}$$

Assuming $\phi_{th} \gg \phi_r$

$$\frac{\phi_r}{\phi_{th}} = \frac{2.22}{(R_{cd}-1)(1+\alpha)} \quad (15)$$

Cadmium ratio measurements are made with a number of resonance detectors at a certain position in the reactor core. If α is known for each detector and the slowing down spectrum is of the $1/E$ type, ϕ_r/ϕ_{th} determined for each detector should be approximately the same.

If the thermal flux is known at this position, ϕ_r can be determined, and the differential flux over the range of energy determined by the detectors can be given as:

$$\phi(E) = \frac{\phi_r}{E}$$

Fast Region

The fast neutrons are produced in the fission process. These neutrons have not been moderated by collisions. The energy range of the fission neutrons varies from over 10 MEV down to the thermal region. The fission neutrons are divided into two categories, prompt and delayed neutrons. The prompt neutrons represent over 99 per cent of the total fission neutrons. The energy distribution of the delayed and prompt neutrons differs considerably. The delayed neutrons are produced at much lower energies than the average energy of the prompt neutrons. The effects of the delayed neutrons will not be considered, since measurement of the fast neutron flux will be made above 2.9 MEV. The fast neutron spectrum (prompt neutrons) for U^{235} fission is well represented by the

$$\phi(E) \sim e^{-E/0.965} \sinh \sqrt{2.29E} \quad (16)$$

The region of interest will be above 2.9 MEV, which is above the average energy of this fast neutron energy spectrum. The higher energy neutrons which are slightly moderated but still remain in the region of interest will not distort the flux spectrum. This is true because the number of neutrons per unit energy in the fission spectrum increases rapidly with decreasing energy above the average neutron energy.

The method which will be used to measure the integrated fast flux is called the "Trice" method. The reaction rate upon irradiation in an unknown flux $\phi(E)$ is:

$$r = \int_0^{\infty} N_T \sigma(E) \phi(E) dE$$

If an element is considered that exhibits a $\sigma(E)$ which remains equal to zero until some threshold energy is reached and then rises to some saturated value:

$$r = N_T \int_{E_T}^{\infty} \sigma(E) \phi(E) dE$$

The reaction that takes place is endothermic since the incident neutron must reach a certain energy before the reaction can take place. The threshold reactions that can be initiated by fast neutrons are (n,p) , (n,α) , (n,n') and $(n,2n)$. These reactions can be detected by activation methods only if the product nucleus is radioactive. If the Q value is found for a certain threshold reaction by mass consideration the threshold energy of the reaction will be:

$$E_T = Q \frac{(A + 1)}{A}$$

where A is the atomic weight of the target nucleus. The threshold energy is greater than the Q value since a fraction of the incident neutrons energy is imparted to translational energy of the compound nucleus. If the incident neutron possesses energy above E_T , the additional energy will appear as kinetic energy of the products. (Inversely proportional to their masses.) As the energy of the incident neutron is increased above E_T the cross section will increase to a saturated value for the (n,p) and (n,α) reactions. This is true because the compound nucleus possesses an electrostatic potential barrier which the alpha particle or proton must penetrate. The ability of the particle to penetrate the barrier will be dependent on the velocity, electronic charge of the particle and the charge of the final product nucleus.

The threshold detector which will be used to measure the fast neutron flux in the Argonaut will utilize the $S^{32}(n,p)P^{32}$ reaction. The cross section used for this reaction was idealized to a step function and assumed constant at some saturated value after an effective threshold energy is reached.

The effective threshold energy was determined from the following calculational technique:

$$r = N_T \int_0^{\infty} \sigma(E) \phi(E) dE = N_T \sigma_0 \int_{E_e}^{\infty} \phi(E) dE$$

σ_0 = constant saturated cross section for the reaction

Rearranging

$$\int_{E_e}^{\infty} \phi(E) dE = \frac{r}{N_T \sigma_0}$$

The activation rate will be determined by numerical integration. The reaction cross section variation with energy is known, and the neutron flux per unit energy $\phi(E)$ is assumed to have the previously stated U^{235} fission spectrum. The value of $\phi(E)$ versus energy is tabulated in reference 5, where:

$$\int_0^{14} \phi(E) dE = 1$$

For the $S^{32}(n,p)P^{32}$ reaction,

$$N_T \int_0^{\infty} \sigma(E) \phi(E) dE = r = N_T (0.0672)$$

$$\sigma_0 = 300 \text{ mb}$$

$$\frac{r}{N_T \sigma_0} = \frac{.0672}{.300} = \int_{E_e}^{\infty} \phi(E) dE = .224$$

from the $\phi(E)$ tabulation $E_e = 2.9 \text{ MEV}$

The figure below shows that this idealized method actually assumes that no neutrons below the effective energy contribute to the reaction and all the neutrons above E_e are assumed to have the same cross section σ_0 . The reaction rate is held constant with area A being made equivalent to area B.

If a U^{235} fission spectrum is definitely present and detector's cross section for the reaction has the proper shape, the accuracy of the measurement will depend on the determinations of the detector's activity. The fast flux above 2.9 MEV for the sulfur detector will be expressed:

$$\phi_f = \int_{2.9}^{\infty} \phi(E) dE = \frac{r}{N_T \sigma_o} \quad (17)$$

3. 1961 ASEE-AEC Reactor Laboratory Institute Writeup

NEUTRON SPECTRA IN REACTORS

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NEUTRON SPECTRA IN REACTORS

In the field of pile neutron research the neutron spectra is generally defined as the neutron energy spectrum. Naturally since the energy and the velocity of a neutron are directly related the term spectra also means the neutron velocity distribution.

Normally the neutron spectrum is reported as the number of neutrons per unit energy as a function of neutron energy. In mathematical terms this is represented by $N(E)$ versus E .

Traditionally the neutron spectrum is divided into three regions of neutron energy, these being the fast, resonance, and thermal regions. Each of these regions will now be discussed in more detail.

The fast neutron region actually deals with neutrons which have not been moderated, and does not in general mean a specified energy increment. This anomaly arises from the wide range of energies of neutrons emitted in the fission process. Early study of these neutron energies seemed to set an upper energy limit of 6 Mev for fission neutrons. However more recent and exact research indicates that fission neutrons have a broader energy spread and indeed range from zero to 15 Mev in energy. From an analysis of a model for neutron boil-off from highly excited fission fragments an empirical relationship between $N(E)$ and E has been developed which corresponds to the observed distribution within experimental error, namely

Figure 1. The fission spectrum as shown by (1) Bonner, et al, Phys Rev 87 1032-8 (1952).

Figure 2. Spectrum in $1/E$ region (From Horst & Sailor, Rev Sci Inst; Hughes p65)

$$N(E) = \sinh (2E)^{0.5} e^{-E}$$

where

$$\begin{aligned} N(E) &= \text{number of neutrons per unit energy} \\ E &= \text{neutron energy in Mev.} \end{aligned}$$

The most reliable fission spectrum is shown in Figure 1 (1).

As the neutrons from the fast or fission region collide with moderator atoms they are rapidly slow down into the $1/E$ energy distribution region. Another name for this region is the resonance neutron region. The actual energy spectrum in the resonance region requires that many $1/E$ distributions be summed extending from the thermal energy region up to the energy of each fission neutron. A convenient method of representing the distribution or spectrum of neutrons in the resonance region is by a relationship between neutron flux and neutron energy. Such a plot is shown in Figure 2 where the flux distribution does indeed have a variation as $1/E$ as predicted from theory.

In a relative way the number of neutrons having energies in the resonance region is many magnitudes greater than the number having fission energies. This is directly due to the long slowing down time and therefore longer time spent in the moderator medium. Alternatively, the relation between the resonance flux and thermal flux can be represented by the so called cadmium ratio, that is (2)

$$\frac{\text{thermal flux}}{\text{resonance flux}} = R_{Cd}^{-1}$$

where R_{Cd} is the cadmium ratio for a particular detector.

In the thermal region the neutron energy spectrum becomes the well known Maxwellian distribution which relates neutron density, n neutrons per cubic centimeter, or neutron flux, nv neutrons per square centimeter per second, with neutron velocity (i.e. energy)

$$dn = \frac{4n}{v_o^3 \sqrt{\pi}} v^2 e^{-\frac{v^2}{v_o^2}} dv$$

A plot of the Maxwellian distribution is shown in Figure 3.

In summary the energy distribution or spectra for neutrons from their origination as fission neutrons to thermal energies can be described by a physical picture of a neutron followed through its lifetime. The initial neutron density at high energies is very small due to the almost instantaneous

Figure 3. Maxwellian spectrum in thermal region.

Figure 4. Relative neutron energy spectrum in Argonaut reactor.

moderation of fission neutrons. As the moderation process continues the density of neutrons of lower and lower energies increases into the thermal regions until the thermal fission capture in fuel depletes the neutron density until the number of neutrons per unit energy for energies below thermal decreases very rapidly to zero. Thus in general $N(E)$ starts very low, increases to maximum at thermal energies and then slowly decreases with increase in neutron energy as shown in Figure 4.

R.L.I. EXPERIMENT NUMBER 9

REACTOR POWER CALIBRATION

INSTRUCTOR: K. C. RUZICH

ABSTRACT

The method used to calibrate the neutron flux monitoring instruments of the AGN-201 reactor to a given power level will be presented in this experiment. The experiment involves a measurement of the average thermal neutron flux at a certain power level. This measurement will be used to calculate the fission rate occurring in the reactor knowing the fuel mass and fission cross section and the core volume. Since each fission occurring in the reactor can be assumed to release an average amount of energy, the fission rate calculation can be easily converted to the power level of the reactor. The resulting power level will then correspond to the readings indicated by the flux monitoring instruments during the flux measurement. The accuracy and the corrections involved in the calibration will be brought out during the course of the experiment.

AGN-201 REACTOR POWER CALIBRATION

OUTLINE

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 - 2. Unpublished Report, "Flux Measurements and Power Calibration with AGN-201 Reactor" by Samuel Helf, et al.
 - 3. Argonaut Reactor Power Calibration, Memo by W. J. Sturm and L. L. Lawyer
 - 4. Boiling Water Power Reactors, 1961
 ASEE-AEC R.L.1.
 Student Results

I. INTRODUCTION

In high power reactors, it is possible to determine a given power level by calorimetric methods; core and coolant temperature measurements, coolant flow rates, etc. Low power reactors do not have this advantage since the heat generated during normal operation is small. Therefore, the approach used to measure a given power level of the AGN-201 reactor will not involve the product of nuclear fission, heat, but the initiator of the reaction, neutrons. Each fission which occurs in the reactor core releases an average energy of 201 mev. The rate at which energy is being released, the power level, is proportional to the fission rate. A measurement of the fission rate will therefore constitute a power level determination.

The method involved will employ the use of activation detectors to measure the neutron flux in the reactor core, hence the fission rate. Dysprosium-aluminum wire has been chosen to measure the thermal neutron spatial distribution. Dysprosium has a high thermal neutron activation cross section and a convenient half life of 140 minutes. Thermal flux distributions will be made through the glory hole on both sides of core center and along the fine control rod when it is fully inserted. An absolute thermal flux

measurement will be made at the core center with a standard gold foil. This foil will be counted on an end-window G-M counter whose efficiency for the standard gold foil has been determined from a previous standard pile irradiation. The results of the two activation detectors will yield the average thermal flux in the reactor core as described in the Theory section. A cadmium ratio measurement of AGN-201 fuel can also be made in order to determine the fraction of the total fission rate due to epithermal neutrons.

II. THEORY

Assuming an all thermal homogeneous reactor model, the following expression describes the total fission rate occurring when the reactor is in a steady state condition:

$$R_f \text{ (Fissions/Sec)} = \Sigma_f \int_{\text{Vol.}} \phi \, dv \quad (1)$$

where ϕ = the thermal neutron flux, the total neutron density times the most probable value of the thermal neutron velocity distribution at standard temperature.

Σ_f = macroscopic fission cross section of the fuel at the most probable velocity corrected if necessary for a non- $1/v$ behavior.

The neutron flux will have a certain spatial distribution in the core depending on the geometry of the system. If the average thermal neutron flux can be determined, equation (1) will simplify to

$$R_f = \Sigma_f \bar{\phi} (\text{Vol})_{\text{core}}$$

with $\bar{\phi} = \frac{\int_v \phi \, dv}{\int_v dv}$, the spatial average of the thermal flux.

The power level of the reactor corresponding to the average thermal flux is then equal to the total fission rate divided by the number of fissions per second required to produce one watt of power. For a reactor utilizing U^{235} as fuel,

$$\text{Power} = \frac{R_f}{3.1 \times 10^{10}} = \frac{\Sigma_f \bar{\phi} V}{3.1 \times 10^{10}} \text{ WATTS} \quad (2)$$

A good approximation of the average thermal flux can be found by measuring the thermal flux spatial distribution along the three lattice axes and determining

the absolute thermal flux at the core center. When it is possible, the core center should represent a common point for all three distribution measurements for normalization purposes. The average flux will be expressed as

$$\bar{\phi} = \phi_o K_x K_y K_z$$

where ϕ_o = absolute thermal flux at the core center and K_x , K_y , and K_z are the numerically averaged fluxes in the particular direction divided by the flux corresponding to the core center in the same direction. For the AGN-201 reactor which has a cylindrical core, $K_x = K_y = K_{\text{radial}}$, so that

$$\bar{\phi} = \phi_o K_r^2 K_z \quad (3)$$

Although equation (2) has been derived for a simplified model, its accuracy when used for the AGN-201 reactor power calibration will depend primarily on the average thermal flux measurement and to a lesser extent on a correction made for non-thermal fission. The energy distribution of the neutrons in the core actually covers a wide range, from fission energies down to the thermal region. Before the neutrons are slowed down into the thermal region, some will be captured in U^{235} and U^{238} and hence cause additional fissions. The fission occurring in U^{238} is caused only by neutrons of high energy. The threshold reaction's contributions to the total fission rate can be assumed small for the AGN-201 reactor since its moderator to uranium volume ratio is appreciable and its fuel is enriched with the U^{235} isotope. Very fast fission is normally accounted for in the four factor formula by the factor ϵ , the number of neutrons produced by all fissions divided by the number produced by thermal fission. In the AGN-201 non-thermal fission is predominately resonance fission since U^{235} has finite fission cross sections at all energies. The amount of epithermal fission can be determined by a simple cadmium ratio measurement of AGN-201 type fuel. The fission product activity of a bare and cadmium covered fuel sample can be counted on a proportional counter after two similar irradiations in the reactor core. Their ratio will yield the amount of non-thermal fission to the total fission after proper corrections for sample weight differences, irradiation times and power level differences have been made. The final power level expression then becomes

$$P = \frac{\Sigma_f \bar{\phi} V}{3.1 \times 10^{10}} \left[\frac{CR}{CR-1} \right]_{\text{Fuel}} \quad \text{WATTS} \quad (4)$$

where CR = fuel sample's cadmium ratio.

The average spatial thermal flux is actually the total neutron density times a velocity of 2200 m/sec. This is true because the absolute thermal flux is measured with a $1/v$ absorber using its cross section at this standard velocity. The macroscopic fission cross section of U^{235} is that of the same

velocity. Actually, since this cross section does not have a $1/v$ dependence, a correction can be made to make it an equivalent $1/v$ cross section, which will give the correct fission rate in equation (4). This correction is based on a Maxwellian neutron distribution being present and is a function of the neutron temperature. The temperature of the neutrons in the AGN-201 is approximately 450°K which corresponds to a correction factor of 0.98.

The absolute flux measurements at the core center with a standard gold foil requires the knowledge of fundamental activation analysis. The activity of the foil immediately after the irradiation can be expressed as

$$A_o (\text{dis/sec}) = N_T \sigma_{\text{act}} \phi k (1 - e^{-\lambda t}) \quad (5)$$

where N_T = total number of detector atoms

σ_{act} = thermal neutron activation cross section at 2200 meters/sec.

ϕ = neutron flux (nv₂₂₀₀)

k = thermal flux depression, self shielding factor

λ = detector's decay constant

t = irradiation time.

If the detector is counted on a system whose inverse efficiency, $E(\text{dis/sec/count/sec})$, is known at a time t_c after irradiation, the detector's activity becomes

$$EC = A_o e^{-\lambda t_c} \quad (6)$$

where C = counts/sec at t_c .

Solving (5) and (6) for the neutron flux

$$\phi = \frac{EC e^{\lambda t_c}}{N_T \sigma_{\text{act.}} k (1 - e^{-\lambda t})} \quad (7)$$

The resulting neutron flux is the proper flux to be used in equation (4) only if the activity is caused by thermal neutrons alone. Since the cross section of gold has a strong resonance activation peak, a correction must be made to separate the resonance activation from the thermal activation. This can be done by making a cadmium ratio measurement with the standard gold foil. If CR is the measured value, the thermal activity of the detector becomes

$$A_{o\text{th}} = A_o \left[\frac{CR-1}{CR} \right] \text{ Au Foil}$$

Hence, the flux in (7) must be multiplied by the factor in the brackets to yield the correct thermal flux.

III. APPARATUS

1. AGN-201 reactor
2. Dysprosium - aluminum wire for flux distribution measurements
3. End window G-M counter and wire scanning equipment
4. Standard gold foil and cadmium covers
5. Standard end window G-M counter

IV. PROCEDURE

1. Dysprosium wire and standard gold foil irradiation:

Two dysprosium-aluminum wires of the proper length are placed in the reactor core. One wire will be positioned in the glory hole for a radial flux traverse, the other in the fine control rod for a partial vertical traverse. The glory hole wire will be placed in the center of a 7/8 inch diameter polystyrene rod and positioned so as to traverse the core region completely. Care must be taken to mark the wire positions at the center and edges of the core. The second wire is attached to a piece of polystyrene which is placed in the fine control rod fuel capsule. The top of the polystyrene rod will serve as the reference for the wire's position in the core. A standard gold foil will be placed at the core center for an absolute flux measurement. The foil will be contained in a small aluminum container and shall be mounted in the polystyrene rod.

The reactor is then started up following the proper procedures. The fine control rod should be moved and left completely inserted in the core. A measurement of the reactor period, after the source was removed, should be made and recorded on the data sheet. Once full power has been reached, the reactor should be made critical by adjustment of the course control rod. The wires and foil are to be irradiated for approximately one hour at full power. The control rod settings, the readings on each of the three flux monitoring instruments and the irradiation times should be recorded on the data sheet.

After the reactor is scrammed, the wires and foil are removed from the core and taken to the south counting room. During their removal, survey instruments should be used to monitor the irradiation levels of the detectors. After a period of 20 minutes has elapsed from scram time, the wires will be

counted on the G-M wire scanner to obtain their respective relative flux traverses. The wire should be scanned in 1/2 or 1 inch segments. The decay of wires' activity during the counting period must be taken into consideration with the initial counting time taken as a reference. A plot of the corrected counting rates versus core position should be made for each wire. For convenience, the plot can be normalized to unity at the core center. The vertical plot will yield the flux distribution in that direction for the lower part of the core; it will be assumed that the upper part has the same distribution. The error involved in this assumption will not be large. For a more detailed description of the vertical flux distribution, see the unpublished report in Appendix 2. From these two plots, the average flux relative to the core center can be numerically determined yielding K_r and K_z in equation (3) of the theory section.

The standard gold foil will be counted on the standardized G-M end window counter. The efficiency of this counter has been measured for the gold foil from previous standard pile irradiations. The counter's background and dead time must be taken into consideration to obtain the correct counting rate. The times involved and the counting rates should be recorded on the data sheet.

2. Cadmium covered standard gold foil irradiations:

The second irradiation of the experiment will be started immediately after the first is completed. A standard gold foil, cadmium covered, is placed at the core center in the polystyrene rod. The procedure which will be followed for this irradiation will be identical to that of the first irradiation. Care must be taken to bring the reactor to the same power level on approximately the same period. After the hour irradiation, the reactor is scrammed and the foil is taken to the counting room and counted on the standardized G-M end window counter.

AGN-201 REACTOR POWER CALIBRATION

DATA SHEET

Run	Start Up Period	Critical FR	Settings CR	Time Full Power is Reached	Channel 1	Channel 2	Channel 3	Scram Time
1								
2								

Wire Segment	Start Time	Counting Rate	Corrected Counting Rate	Wire Segment	Start Time	Counting Rate	Corrected Counting Rate

Gold Foil Weight	Scram Time	Counting Time	CPM	Corrected CPM
Al				
Cd				

V. RESULTS

The results of the experiment will include the power calibration, a plot of the vertical and radial flux distribution, a determination of the average flux in the core, the absolute flux at the core center, and a cadmium ratio measurement with the standard gold foil. The calculations necessary to obtain these results will involve the equations presented in the Theory section of this experiment. The fuel cadmium ratio has been found to equal 19.7. The flux depression and self shielding of the standard gold foil can be determined from ANL-5800, page 487. This correction is quite small and can be neglected if desired.

VI. ANALYSIS AND CONCLUSIONS

A brief analysis of the experiment together with significant conclusions is left as an exercise for the participant.

VII. BIBLIOGRAPHY

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Experiment Written By: Ken Ruzich

VIII. APPENDICES1. DISCUSSION QUESTIONS

1. The measurement of the reactor power output will involve computation of the fission rate in the reactor core. What other method or methods might be used to measure the power output of this or another reactor?
2. For the purpose of this experiment, we will consider that a fission rate of 3.1×10^{10} fission/sec is equivalent to 1 watt. Analyze this in terms of Mev per fission.
3. Analyze the total Mev/fission derived in (2) above in terms of the amount of energy carried off by the various particles and photons, i.e., fission fragments, gamma rays, etc.
4. Assuming the fuel distribution in the reactor to be homogeneous and the distribution of neutron flux to be known, how may we arrive at the average neutron flux?
5. Why is it desirable to use ideally thin foils in measuring neutron flux?
6. Outline the steps involved in using a standard pile to calibrate a counting system which will be used for absolute neutron flux measurements with standard foils.
7. What is beta ray self-shielding in foils, and how may its effect be eliminated?
8. We propose to measure the cadmium ratio of gold and a fuel sample during this experiment. Discuss the need and give the direction of this correction. How does the absorption of epithermal neutrons by cadmium affect the cadmium ratio?
9. What factors affect the energy contribution due to fission in U²³⁸?
10. The final result of the experiment yields a given power level which corresponds to certain readings on the flux monitoring instruments. How may this calibration be used to determine the power level at other instrument readings?

2. FLUX MEASUREMENTS AND POWER CALIBRATION WITH AGN-201 REACTOR

by

Samuel Helf, Dirceu Coutinho and K. C. Ruzich

Introduction

Neutron physics experiments involving activation of foils are limited in the AGN-201 Reactor because of (a) the low neutron flux available and (b) the small excess reactivity of the core which restricts the size of sample to be irradiated. Nevertheless, by prudent choice of size and type of materials, such experiments can be successfully performed in this reactor. This report describes the determination of the thermal neutron flux distribution in various regions of the AGN-201 core and reactor by the use of dysprosium wire activation techniques. From this flux data and appropriate cadmium ratio corrections, a power output calibration for the reactor was made.

Summary of Results

Thermal Neutron flux distributions are given for traverses (a) horizontally through the glory hole on both sides of core center (b) vertically along the fine control rod when fully inserted and (c) vertically along the south side of the core tank in the reflector region. The peak thermal neutron flux in the center of the glory hole at 100 milliwatts operation is 4.9×10^6 n/cm²-sec. Corresponding instrument readings are given for 79 and 100 milliwatts of operation.

The cadmium ratio of dysprosium in the AGN-201 Reactor is 83 and that of AGN-201 fuel is 19.7.

Thermal Neutron Flux Measurements

The core of the AGN-201 Reactor is a right circular cylinder 10.15" in diameter and 9.34" high. For previous power calibrations of this reactor (Refs. 1, 2), the core was approximated as a sphere and only a radial flux distribution through the glory hole was measured using either indium or gold foils. In this determination, dysprosium-aluminum alloy wire was used and both radial and vertical flux traverses were made. The important pertinent characteristics of this detector are as follows:

Dysprosium-aluminum wire - 30 mil diameter; 10% Dy by weight; 1.34 mg Dy per cm; source - Michigan Chemical Corporation, Saint Louis, Michigan.

Dy¹⁶⁴ - Isotopic abundance = 28.2%
Isotopic σ_{act} (2200 m/sec.) = $2620 \pm 10\%$ barns

$$\text{Dy}^{165} = T_{1/2} = 140 \text{ min.}; B^- = 1.25 \text{ mev}; \gamma = 0.09\text{-}1 \text{ mev}$$

The radial flux distribution was obtained by irradiating a 36" piece of wire centered in the glory hole. Since no provision exists for placing a wire centered through the core in a vertical position, two vertical traverses were made by attaching one wire to the fine control rod and another along the outside of the south side of the core tank. The irradiation was performed for 60 minutes with the reactor at approximately 75% of maximum power. (All irradiations for this report were performed with 1" diameter polystyrene rod in the core region of the glory hole and with the fine control rod consisting of polystyrene instead of actual fuel.)

A wire counting assembly under an end-window GM counter was used to determine the radioactivity in the wires, which were scanned in 3/4" segments for each inch of length. As an additional check, the core section of the radial wire and the fine rod traverse wire were also cut into 3/4" and 1/2" segments respectively and counted with an end-window proportional flow counter. All counts were normalized to the latter counter which was previously calibrated for absolute flux measurement using several 3/4" pieces of the same wire irradiated to saturation in the Standard Pile (Building 203).

Figure 1 shows the relative radial flux distribution through the glory hole from the center of the core extending 18" south in the reactor. Figure 2 shows a more detailed plot of the flux distribution of the entire core region of the glory hole. A small dip in flux at the core center, as reported in Ref. 2, was not observed in this particular experiment. In Figure 3, the vertical flux distribution along the fine control rod is shown, and Figure 4 shows the vertical flux distribution along the outside of the core tank. The slight flattening of the flux towards the top of the core, in this last run, can be attributed to the presence of 1 cm of pure polyethylene moderator and 1 cm of void at that particular position.

Estimation of Peak Neutron Flux

The absolute flux, at the center of the core, was determined by counting several pieces of wire in the end-window proportional counter previously calibrated with similar wires irradiated to saturation in the Standard Pile. No epicadmium activation of the wire was observed in the Standard Pile. In the AGN-201 reactor, the cadmium ratio for dysprosium-aluminum wire, determined by irradiating bare and 20 mil cadmium-covered wires under similar conditions was found to be 83. The flux plots as shown in Figures 1 - 4 can therefore be taken as essentially thermal distributions.

The value for the peak absolute thermal flux at the core center, after correcting for saturation, decay and cadmium ratio was 3.86×10^6 n/cm²-sec. for the power calibration run.

Estimation of Average Neutron Flux

The average neutron flux in a reactor core can be expressed by the relation

$$\bar{\phi} = \phi_{\max} K_x K_y K_z$$

where K_x , K_y and K_z are the numerically averaged fluxes in the particular direction divided by the $\phi_{\max}$ in the same direction. For the AGN-201 reactor core $K_x = K_y$ so that

$$\bar{\phi} = \phi_{\max} K_x^2 K_y \text{ where } K_x \text{ is the radial and } K_y \text{ the vertical average distribution factors.}$$

From the radial plot in Figure 2, $K_x = 0.831$. The vertical flux distribution factor, K_y , was obtained from a combination of the two vertical flux traverses as shown in Figures 3 and 4. The flux distribution along the fine control rod is essentially the same as that of the bottom half of the traverse along the outside of the core tank. It was therefore assumed that the vertical distribution of the top half of the core was the same as that shown by the upper portion of the traverse along the outside of the core as well. By combining the data from these two plots, $K_y = 0.888$ and

$$\bar{\phi} = (3.86 \times 10^6) (0.831)^2 (0.888) = 2.37 \times 10^6 \text{ n/cm}^2\text{-sec.}$$

Cadmium Ratio of AGN-201 Fuel

To determine the fraction of the total fission rate due to epithermal neutrons, the cadmium ratio of a simulated sample of AGN-201 fuel was measured. This was made by blending a small quantity of lucite powder and 19.8% enriched UO_2 in the ratio of 3.30 to 1 by weight and pressing into a pellet; 495 mg of lucite powder and 150 mg of UO_2 resulted in a pellet 1" diameter by 1 mm thick. This was then cut into smaller pieces approximately 7 X 7 mm, weighing 40-50 mg for the cadmium ratio measurement.

Bare and cadmium-covered (20 mil) foils were irradiated for 1 hour in the glory hole at approximately 75% of full power at a point exactly 1/2 radius from core center. Separate runs were made for each foil using 1/2" dysprosium-aluminum wires placed 6-1/2" from core center as flux monitors. To avoid an exceedingly long positive period in reaching the desired power level, the area of the cadmium cover used could not exceed 1.6 cm^2 .

The irradiated samples were counted for gross fission product activity using an end-window gas flow proportional counter. Care was taken to subtract the original background activity of the foils and to count both bare and covered foils at exactly the same time after the end of the irradiation period. It was also necessary to wait at least 130 minutes after irradiation

before counting to allow for the U^{239} formed by the resonance activation of U^{238} to decay out. For the levels of activation involved in this experiment, the build-up of Np^{239} , the daughter product of U^{239} decay, is not sufficient to affect the results of the cadmium ratio determination (see Figure 5). After correcting for differences in power level, the cadmium ratio obtained for AGN-201 fuel was 19.7.

The above value was also determined using 93.2% enriched uranium foil. Bare and cadmium covered foils, 2 X 2 mm square, 5 mil thick and weighing 10 mg were irradiated for 15 minutes in exactly the same manner as above. A value of 19.9 was obtained for the cadmium ratio.

Power Calibration

The reactor power was calculated from the relation:

$$p = \frac{\bar{\phi} \sigma U^{235} N U^{235}}{3.1 \times 10^{10}} \left(\frac{CR}{CR-1} \right)$$

where $\bar{\phi}$ = average thermal neutron flux

σU^{235} = thermal fission cross-section for U^{235}

$N U^{235}$ = total number of U^{235} atoms in the core

3.1×10^{10} = fissions per watt

CR = cadmium ratio of U^{235} in fuel

$\left(\frac{CR}{CR-1} \right)$ = fraction of total fissions due to epithermal neutrons

Therefore

$$p = \frac{(2.37 \times 10^6) (582 \times 10^{-24}) \left(\frac{659.93 \times 6.023 \times 10^{23}}{235} \right)}{3.1 \times 10^{10}} \left(\frac{19.7}{18.7} \right)$$

$$= 0.079 \text{ watts}$$

The reactor conditions and instrument readings at 79 milliwatts were as follows:

Safety rods 1 and 2: In

Fine rod: 23.10 cm

Coarse rod: 21.96 cm

Channel 1: 198,000 cpm

Channel 2: 3.10×10^{-9} amp.

Channel 3: 1.80×10^{-9} amp.

Figure 1. Thermal Neutron Flux
Traverse through AGN-201
Reactor Tank Peak Flux at 100
milliwatts = 4.9×10^6 n/cm²-sec

Figure 2. Thermal Neutron Flux
Traverse through AGN-201
Reactor Core Peak Flux at 100
milliwatts = 4.9×10^6 n/cm²-sec

Figure 3. Thermal Neutron Flux Traverse along Fine Control Rod of AGN-201 Reactor

Figure 4. Thermal Neutron Vertical Flux Traverse along outside of AGN-201 Reactor Core Tank (through Reflector)

Figure 5.
Decay of U^{239}
and Buildup of
 N_p^{239} Starting
from Pure U^{239}

For 100 milliwatt operation: (extrapolated)

Channel 1: 251,000 cpm

Channel 2: 3.92×10^{-9} amp.

Channel 3: 2.28×10^{-9} amp.

Peak thermal neutron flux: $4.89 \times 10^6 \text{ n/cm}^2\text{-sec.}$

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L. L. Lawyer and W. J. Sturm.

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Oct., 1957.

3. ARGONAUT REACTOR POWER CALIBRATION

W. J. Strum and L. L. Lawyer

December, 1959

I. Abstract

A method for determining the power output of a small thermal reactor, such as the Argonaut, is described. The method depends upon the evaluation of the average neutron flux in the reactor. Combining this with the knowledge of the total amount of U^{235} in the core and its fission cross section, the power output can be calculated. A typical measurement is made and discussed; corrections are evaluated and the precision of the result is estimated. A correlation between instrument readings, flux levels and power output for the one-slab Argonaut is given.

II. Introduction and Theory

Power output of a reactor can be measured in any of several ways. The method chosen for the measurements to be discussed involved the computation of fission rate in the reactor core from knowledge of flux distribution, fission cross section and fuel content of the reactor core. On the assumption that the reactor core is homogeneous, one evaluates experimentally a spatially average flux of neutrons and calculates the fission rate from the relation

$$V = \overline{nv} \sigma_{f_{eff}} (U^{235}) N \quad (1)$$

In this relation $\overline{nv}$ is the experimentally averaged thermal neutron flux; $\sigma_{f_{eff}}(U^{235})$ is the fission cross section for U^{235} at .025 electron volts; N is the total number of U^{235} atoms in the reactor. Because it is conventional and related to certain practical aspects, we will convert the fission rate to a power unit. In order to do this we will take the energy per fission as 201 Mev and evaluate the total fission power in watts from

$$p = \frac{\overline{nv} \sigma_{f_{eff}} (U^{235}) N}{3.1 \times 10^{10}} \quad \text{watts,} \quad (2)$$

in which p is the power in watts and 3.1×10^{10} is the number of fissions per second required to produce one watt of power.

III. Experimental Flux Measurements

All of the experimental measurements were determinations of neutron flux and can be divided into the following classes.

(a) Spatial neutron flux distribution: In order to obtain the average flux required for the power evaluation, traverses were made of neutron flux in the one-slab reactor core of the Argonaut. Three distributions were measured; one across the radius of the slab in a central box, an anular distribution at the reactor core midline, and the third, vertically centrally in a central fuel box. Identical indium foils were used for the distribution measurements. With this method of measurements, all three distributions have a common central point to which the data can be normalized.

(b) Flux depression: The flux distribution measured as discussed above is taken to be qualitatively correct, but foil activations are lower than would be measured with ideally thin indium foils. Since we convert these foil activations to absolute fluxes, it is necessary to determine the amount of depression due to the presence of the finite indium foils during the irradiation. The quantitative estimate of the amount of flux depression was made by identically irradiating two gold foils at the center of the lattice core with and without the indium foils in place. The ratio of the two activations was taken as a measure of the flux depression and was assumed to be fractionally uniform throughout the reactor.

(c) Cadmium ratio measurements: In order to estimate the relative contributions of thermal and epithermal neutrons to the reactor fissions, a cadmium ratio measurement of a 3/4" diameter section of Argonaut fuel plate was made. A cadmium ratio measurement was also made of gold foils used for absolute flux determination in order to determine the fractional gold foil activation in the thermal and epithermal flux regions. 20 mil cadmium was used in these measurements.

(d) Absolute flux measurement: Absolute flux determination was made by irradiating a gold foil of known thickness and weight at the center of the reactor core at the point common to the three distributions referred to in Section III (a). The foil was counted absolutely in equipment designed for this purpose⁽¹⁾ in which the absolute number of disintegrations per unit time was derived from an observee counting rate. After ocrrection for saturation activation and decay, this number was converted to absolute neutron flux. The valve of nv thus obtained, corrected for its gold-cadmium ratio (Section IV-6), represents the flux of thermal neutrons at the center of the reactor. The cross section of gold was taken to be its value at .025 ev, 96 barns.⁽³⁾ This cross section value was decreased by 12 ⁽²⁾ on the assumption that the gold cross section in the region of the Maxwellian distribution is essentially 1/v.

IV. Results

A. Equation 2 was used to evaluate the reactor power output under the circumstances of the experiment. $\overline{nv}$, the average thermal neutron flux, was obtained from the distribution and absolute flux determinations of the previous sections. The indium foil distributions (Figures 1, 2, 3) were numerically averaged in each of the three lattice axes to yield the following results normalized to the flux level at the gold foil position.

Radial	.984
Vertical	.775
Azimuthal	.864

The average flux, therefore, is .66 of that indicated at the gold foil position.

The value of σ_f was taken from BNL-325 and N was obtained from the weight of U^{235} in the lattice.

The uncorrected value of the reactor power during the experiment (chamber readings given in Section V-B) is as follows from equation (2):

$$p = \frac{\overline{nv} \sigma_{f \text{ eff}} (U^{235}) N}{3.1 \times 10^{10}}$$

from the absolute flux measurement:

$$\overline{nv} = nv \times .66 = 1.47 \times 10^7 \times .66$$

from $\sigma_f (E) U^{235}$ (BNL-325):

$$\sigma_{f \text{ eff}} (U^{235}) = \sigma_{f, 0.025} (U^{235}) = 570 \times 10^{-24}$$

from the known weight of fuel in the lattice:

$$\begin{aligned} N &= \text{gms } U^{235} \times \text{atoms } U^{235} \text{ per gm} \\ &= \frac{1,898.59 \times .602 \times 10^{-24}}{235} \end{aligned}$$

hence:

$$p = \frac{.602 \times 10^{24} \times 570 \times 10^{-24} \times 1.47 \times 10^7 \times .66 \times 1898.59}{3.1 \times 10^{10} \times 235}$$

$$p = 0.87 \text{ watts}$$

B. Corrections and estimated precisions.

1. Disadvantage factor: The disadvantage factor, $\overline{nv}H_2O/\overline{nv}_{fuel}$ plate was taken to be 1.056. This value was obtained by P3 calculation assuming twenty grams of U^{235} per fuel plate (4). This correction, felt to be correct within 2%, is to be applied to N.

2. Foil thickness corrections: Since the indium foils for the flux scanning were identical in size and results were not interpreted absolutely, no correction for foil thickness was made. Similarly, since the standard gold foil was counted through approximately one-quarter inch of lead, no correction for beta ray self-shielding in the foil was pertinent. An estimation of self-shielding in the standard gold foil during irradiation was made and found to be approximately 1% in a thermal neutron flux. As such, it was neglected.

3. Flux depression: An evaluation of the depression of the reactor flux by the indium foils used for neutron distribution determination was also made (Section III-b). Because the standardizing gold foil was irradiated at the same time as the indium foils, this correction was necessary. To do this, identical gold foil irradiations, as determined by the clock and ion chambers, were made with and without the indium foils in place. The indium foils were found to lower the flux to .84 of the undepressed value. The uncertainty in this evaluation is taken to be about 4% and the correction must be applied to $\overline{nv}$.

4. Effective cross sections: As in the case of the absolute flux determination with gold, a correction for effective fission cross section was made to convert from the measured .025 ev value to the effective $1/v$ cross section in a Maxwellian distribution. For this purpose it is again assumed that the fission cross section in the thermal region was essentially $1/v$. As such a 12% correction will be made to the kT value, probably valid within 5%.

5. U^{238} fission contribution: The high energy induced fission in U^{238} is taken to be completely negligible. The effective cross section for a fission spectrum for U^{238} of .28 barns increased by a factor of four for the relative abundance of U^{238} - U^{235} , gives an effective cross section of approximately one barn to be compared with 580 for 235. Without taking into account the very much reduced fission spectrum, the U^{235} fission is already seen to be less than .2% and is negligible.

6. Cadmium rates: Measurement of cadmium ratios using 20 mil cadmium sheet were made of the standard gold foil and of an element of fuel plate in order to estimate the epithermal contribution to flux measurement and to U^{235} fission. The values were obtained as follows:

Gold foil (49.8 mg/cm ²)	$1.89 \pm .05$
Fuel plate element	$13.5 \pm .5$

Corrections for the epithermal gold activation have been applied to the standard gold activation in order to obtain $\bar{n}\bar{v}$, the thermal neutron flux. The epithermal fuel element activation as estimated from the cadmium ratio of the fuel plate element will be such as to increase the effective fission cross section for U^{235} . The effective epithermal component is evaluated within 2% in this way.

7. Absorption of epithermal neutrons by cadmium: Irradiations in cadmium boxes are assumed to remove all neutrons below .4 ev. However, due to the high energy tail on the cadmium resonance, there is also attenuation of neutrons above .4 ev. This component has been estimated for gold foils⁽⁵⁾ to be less than 2%. Since it is very small, this correction will not be applied.

V. Discussion, Comparison & Conclusions

A. Assumptions. The assumptions which have been made through this discussion are the following:

1. The spatial energy spectrum of neutrons throughout the reactor core is assumed to be constant.
2. Control rod perturbations are assumed to be constant. This implies that the power measurement is most accurate for k_{ex} of .52% and one can expect small variations for different one-slab loadings.
3. It has been assumed that the interaction between the indium foils did not depend on their center-to-center placement distances. This varied from one-half inch in the radial case to several inches in the vertical case.
4. It is assumed that the variables are separable in the relative flux distribution determination. This assumption was used in evaluating the average neutron flux.
5. The fission product decay spectrum and time parameters were assumed to be independent of the energy of the neutron causing the fission.
6. No hardening of the Maxwellian component of the reactor flux was assumed to be produced by the selective absorption of low energy neutrons in fission, control rod and other capture processes. Of course, such hardening exists, but since it affects both the standard gold activation and U^{235} fission the direction of changes are opposite and tend to cancel each other.

B. Conclusions. The finally corrected value for power level and the conditions of the experiment are given below.

<u>Chamber No.</u>	<u>Current Reading</u>	<u>Chassis Serial No.</u>
1	.18 X 10 ⁻⁸ amp	56267
2	.8 X 10 ⁻⁸ amp	Special test unit
3	.11 X 10 ⁻⁸ amp	
Log	.5.0 X 10 ⁻⁸	56266

All chambers were operated with +500 volts on the collector and -128 volts on the compensating electrode. Prior to the above run, these electronic instruments were aligned in accordance with Reference (6).

Under these conditions and applying all of the corrections of Section IV, the power level is calculated to be $p = 1.05$ watts. The corresponding thermal neutron flux at the position of the standard gold foil in the center of the lattice was

$$nv(th) = 1.47 \times 10^7 \pm 5\%.$$

A description of the chamber locations is given in Figure 4.

The overall precision of the determination is significantly limited by the nv determination by standard counters. The error here alone is ± 5 (7). Assuming the additional errors to be random, their net effort is about 7.2. The combined error is thus ± 8.7 . Applying the above corrections, we find:

$$p = (0.87) \frac{(1.08) U^{235} \text{ epithermal fission} \times (1.12) \sigma_f U^{235}}{(0.84)_{\text{flux dep.}} \times (1.06)_{\text{dis. fr.}} \times (1.12) \sigma_{aAu}},$$

$$p = 1.05 \text{ watts} \pm 8.7\%.$$

C. The value presently obtained for a single slab reactor having an excess k of 0.52% and located on the north side of the reactor annulus is lower by a factor of two than that traditionally used with the Argonaut. The origin of the traditional number is lost in obscurity, but probably antedates all Argonaut quantitative power assignments previous to the date of this memo.

In part, the variation mentioned above is due to a variation in the sensitivity of the log amplifier circuit over the span of time involved. However, there have been other indications as well that the traditional power level number was excessively high. Since the present power determination is associated with the output of four chambers as well as the activation induced in a central

gold foil, it is hoped that there need be no further deterioration of its significance in time.

The measurement can be simplified easily for use as a student experiment. In order to do this one would measure the neutron flux distributions and perform the central absolute gold foil measurement; to determine the largest correction involved, a measurement of flux depression made in the method discussed in this paper could be added as time permits.

References

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- (6) Electronic Instrument Alignment Procedures for the Argonaut Reactor, Argonaut File, ANL.
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2" Dia. 9-1/4" lg.
N. end of Chamber
16-7/8" from So. face of
Reflector

4. 1961 ASEE-AEC REACTOR LABORATORY INSTITUTE WRITEUP

June, 1961

BOILING WATER POWER REACTORS

by

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The execution of power producing boiling water reactors has progressed rather slowly due to the fear of control problems. Their stability and high level of safety having been investigated in the BORAX and SPERT experiments and proven with the EBWR, the boiling water reactors are coming into the power reactor field with a technology gradually and carefully advanced compared to other reactor types.

The advantages of boiling water reactors are many, not the least of which are the economics of eliminating the steam generator if a direct cycle system is used. This direct cycle also reduces the pumping power required by about 90%. Direct steam generation results in a higher steam temperature for a specified fuel temperature, and the lower pressures result in savings on pressure vessel construction. The safety of such systems is inherent in itself, in that the excess steam formation around fuel elements reduces reactivity before temperature rise becomes high enough to cause structural damage.

With advantages come disadvantages; the direct cycle introduces radioactivity into the turbine, condenser and pumps, creating a maintenance problem. A fuel element failure could prove very troublesome. Operation at low pressures limits the power density of the core by the boiling itself. Increased performance has been noted with increased pressure, thereby allowing smaller cores for the same power production or increased power for same core size. The developments of recent pressurized water reactors have been towards the introduction of localized boiling in the core to take advantage of the heat of vaporization of water. A merger of the two types seems possible for the future.

The inherent stability of a boiling water system at any operating power level appears attractive. This comes about due to the increase of bubble formation as an upward power perturbation is introduced. This change in moderator density effects several core parameters in various ways.

An increase in the thermal utilization is noted as bubbles increase either in size or number. This can be easily seen from the relationship

$$\frac{1}{f} = 1 + \frac{\Sigma_{a1} V_1}{\Sigma_{a0} V_0} d.$$

It is seen that with a change in moderator density, both Σ_{a1} and V_1 will decrease and $\frac{1}{f}$ will decrease with an increase in f . The disadvantage factor, d , will also decrease slightly but this can be ignored.

The resonance escape probability will decrease with bubble formation as from the approximation

$$p = \exp - \left[\frac{N_u}{\Sigma_s} \right] l_{\text{eff}}$$

where l_{eff} is proportional to $k \left(\frac{\Sigma_s}{N_u} \right)^{0.42}$. The value of the factors incorporated in k is assumed to remain constant if no appreciable temperature rise is noted. The resonance escape probability then reduces to

$$p = \exp - \left[k^1 \left(\frac{N_u}{\Sigma_s} \right)^{0.58} \right]$$

An increase in bubble formation is then reflected by an increase in N_u

$\frac{\text{Uranium Atom}}{\text{cm}^3 \text{ fuel + mod}}$ and decrease of Σ_s of moderator resulting an increase in the value of the bracket and a decrease of the resonance escape probability.

The decrease of moderator density is also reflected by an increase in L and τ as well as the buckling resulting in an increase in the leakage of neutrons out of the system at all energies. This further is reflected by a decrease of reactivity.

A very slight increase in the fast fission factor may be noted, but this may generally be ignored.

In heavy water boiling reactors, the thermal utilization is very high and any change in moderator density may be taken as being reflected only by p and the leakage. Both factors being negative, the reactor will become sub-critical until a critical configuration is again reached at the previous power level if coolant flow remains constant.

With light water, the situation may be reversed. The large increase in f may offset decreases from the other factors and inverse of reactivity may be noted. This obstacle can be easily overcome by proper design of

reactors so that at operating steam production, the reactor is not over-moderated, which necessitates the moderator to fuel ratio be kept below certain value. 2/1 for large power reactors; 2.5/1 for small reactors; 3.6/1 with enriched elements.

Changes in moderator density can be accomplished by either increasing the bubble formation rate, which, taking place near fuel, is mostly reflected by p . Expansion of existing bubbles as a result of decrease of pressure is strongly reflected in f and L . Those effects naturally overlap and are commonly incorporated in one void coefficient.

The presence of bubbles requires the use of fuel enriched slightly higher and therefore a higher fuel cost. The use of D_2O would allow the use of natural uranium as fuel, but the cost of D_2O would be prohibitive for economic operation.

The hope then seems to lie in very large reactors operating at higher pressures and higher power densities. This can be exemplified by the EBWR, which as initially designed would produce electricity at about 33 mills/kwh, an increase of power output would decrease this value tremendously since about 19 mills is in capital costs. Despite a reasonable thermal efficiency of about 24%, this high value can be attributed in part to the experimental nature of the reactor system.

Plans to raise the power of the EBWR to 100 mw thermal with a net cycle efficiency of 30% would result in an estimated cost of electricity of about 8.5 to 9 mills/kwh.

Plans for a larger boiling water station were reported by Zinn, et al. With a thermal reactor power of 242 mw and a conventional fuel fired super-heater, having an overall cycle efficiency of 37.1% would produce commercial electricity at a cost of 9 to 10 mills/kwh.

STUDENT RESULTS

Experiment Performed By E. Abele, J. Herbst, T. R. Rehm, C. J. Roberts, S. Suwannarat, July 6, 1961. Submitted by J. Herbst.

AGN-201
POWER CALIBRATION
J J HERBST 7-6-61

A. MAXIMUM FLUX DETERMINATION

$$\phi = \frac{ECe^{\lambda t_c}}{N_T \Gamma_{act} K (1 - e^{-\lambda t_c})}$$

$$E = 6.87 \pm 11.6\% \quad \frac{\text{DISINTEGRATIONS}}{\text{UNIT COUNT}}$$

$$C = 6127 \text{ cpm} = 102.1 \text{ cps} \quad \text{--- BARE FOIL}$$

$$= 3079 \text{ cpm} = 51.3 \text{ cps} \quad \text{--- COVERED FOIL}$$

$$K = \text{DEPRESSION CORRECTION} - \text{ASSUMED } \sim 1$$

$$t_c = \text{COUNT TIME} = 1 \text{ min.}$$

$$t_e = \text{EXPOSURE TIME} = 60 \text{ min.}$$

$$N_T$$

$$\text{BARE} = 99.5 \times 10^{-3} \text{ gm}$$

$$\text{COVERED} = 99.6 \times 10^{-3} \text{ gm}$$

$$\text{FOR Au} \quad 19.3 \frac{\text{gm}}{\text{cm}^3} \left(0.058 \times 10^{-24} \frac{\text{atom}}{\text{cm}^2} \right)^{-1} = 3.05 \frac{\text{atom}}{\text{gm}} \times 10^{21}$$

$$N_T = 3.05 \times 10^{21} (99.5 \times 10^{-3})$$

$$= 1.48 \times 10^{20} \text{ atoms.}$$

$$N_T = 3.05 \times 10^{21} (99.6 \times 10^{-3})$$

$$= 1.483 \times 10^{20} \text{ atoms.}$$

$$\lambda = 178 \times 10^{-4} \text{ min}^{-1}$$

$$e^{\lambda t_c} = e^{0.000178} \sim 1$$

$$1 - e^{-\lambda t_c} = 1 - e^{-0.00178(60)} = 1 - e^{-0.107} = 1 - 0.8938 = 0.1062$$

BARE FOIL

$$\phi = \frac{ECe^{\lambda t_c}}{N_T \Gamma_{act} K (1 - e^{-\lambda t_c})}$$

$$= \frac{6.87 (102.1) (\sim 1)}{(1.48 \times 10^{20}) (96 \times 10^{-24}) (\sim 1) (0.1062)}$$

$$\phi = 4.65 \times 10^6 \frac{\text{m}}{\text{cm}^2 \text{ sec}}$$

COVERED FOIL

$$\phi = \frac{6.87 (51.3) (\sim 1)}{(1.483 \times 10^{20}) (96 \times 10^{-24}) (\sim 1) (0.1062)}$$

$$\phi = 2.325 \times 10^6 \frac{\text{m}}{\text{cm}^2 \text{ sec}}$$

$$\text{CADMIUM RATIO} = \frac{4.65}{2.325} = 2$$

THERMAL FLUX

$$\phi_{th} = \phi \left(\frac{CR-1}{CR} \right)$$

$$= 4.65 \times 10^6 \left(\frac{1}{2} \right)$$

$$\phi_{th} = 2.325 \times 10^6 \frac{\text{m}}{\text{cm}^2 \text{ sec}}$$

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B. AVERAGE NEUTRON FLUX.

$$\bar{\phi} = \phi_{max} \left(\frac{\phi_{ave}}{\phi_{max}} \right)_R^2 \left(\frac{\phi_{ave}}{\phi_{max}} \right)_Z$$

$$\left(\frac{\phi_{ave}}{\phi_{max}} \right)_R = .785$$

$$\left(\frac{\phi_{ave}}{\phi_{max}} \right)_Z = .745$$

BY GRAPHICAL INTEGRATION OF
FLUX OVER CORE
D = 10.15"
H = 7.34"

$$\bar{\phi} = \phi_{max} (.785)^2 (.745)$$

$$\bar{\phi} = 0.460 \phi_{max}$$

$$\bar{\phi}_{eb} = 0.460 (2.325) \times 10^6 = 1.07 \times 10^6 \frac{ne}{cm^2 \cdot sec}$$

C. POWER CALIBRATION

$$P = \frac{\bar{\phi} V_c^{.235} N^{.235}}{3.1 \times 10^{10}} \left(\frac{CR}{CR-1} \right)$$

$$P = \frac{(1.07 \times 10^6) (6.82 \times 10^{24}) \left(\frac{652 \times 10,023 \times 10^{23}}{235} \right)^{.235}}{3.1 \times 10^{10}} \left(\frac{2}{1} \right)$$

$$P = 0.067 \text{ watts}$$

AT METER READING

CHANNEL	1	- 195,000 cpm
	2	- 2.2×10^{-9} amp
	3	- 1.25×10^{-9} amp

EXTENDED TO 0.100 WATTS

CHANNEL	1	- 291,000 cpm
	2	- 3.23×10^{-9} amps
	3	- 1.87×10^{-9} amps

WRITEUP VALUES

$(\phi_{eb})_{max}$	$3.47 \times 10^6 \frac{ne}{cm^2 \cdot sec}$	$4.89 \times 10^6 \frac{ne}{cm^2 \cdot sec}$
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9-5-61

RADIAL FLUX TRAVERSE OF AGN-201

Wire Pos	Wire Time	Counts	Dead Time* Corrected	Byd	Net Counts	$\epsilon^{2\sigma}$	Correct Counts	A/ A ₀
0	0	3488	7.43	24	3507	—	3507	.2739
1/2	2	5276	97	24	5349	1.010	5462	.4320
1	3.30	6377	143	24	6496	1.0125	7680	.5944
1 1/2	5	7413	193	24	7582	1.025	7772	.6071
2	7.30	8214	237	24	8437	1.038	8815.4	.6841
2 1/2	9.00	9111	290	24	9425	1.060	9885.4	.7701
3	10.30	9733	332	24	10010	1.058	10546	.8238
3 1/2	12	10458	385	24	10819	1.062	11490	.8975
4	15.30	10859	413	24	11248	1.060	12148	.9459
4 1/2	18.00	11047	427	24	11450	1.068	12458	.9731
5	18.30	11066	427	24	11469	1.0965	12576	.9823
5 1/2	20.00	11120	434	24	11530	1.105	12741	.9952
6	21.30	11095	431	24	11502	1.130	12802 *	1
6 1/2	23.00	10503	427	24	10906	1.127	12291	.9601
7	25.00	9818	397	24	10131	1.133	11478	.8966
7 1/2	26.30	9583	332	24	9881	1.141	11274	.8806
8	28.00	8971	283	24	9230	1.150	10615	.8292
8 1/2	29.30	8290	241	24	8507	1.158	9851	.7695
9	31.00	7332	188	24	7496	1.167	8748	.6833
9 1/2	32.30	6440	145	24	6561	1.173	7712	.6024
10	34.00	5332	96	24	5404	1.181	6398	.4998
10 1/2	35.30	4261	64	24	4301	1.193	5131	.4008
11	37.00	2770	24	24	2770	1.202	3330	.2601

* Resolving time of system - 210 nsec.

AXIAL FLUX TRAVERSE

Length of wire - 9 1/2

Wire Pos	Wire Time	Counts	Dead Time* Correct	Byd	Net Counts	$\epsilon^{2\sigma}$	Correct Counts	A/ A ₀
0	0	1977	8	24	1961	—	1961	.209
1	4	1966	19	24	1955	1.020	1994	.285
2	6	2254	17	24	2247	1.030	2314	.331
3	8	2793	26	24	2795	1.041	2910	.446
3 1/2	10	2923	39	24	2928	1.051	3077	.439
4	12	3368	40	24	3384	1.062	3594	.513
4 1/2	14	4198	62	24	4236	1.072	4541	.648
5	16	4781	80	24	4837	1.083	5238	.748
5 1/2	18	4924	85	24	4985	1.094	5454	.780
6	20	5453	99	24	5528	1.105	6108	.873
6 1/2	22	5654	112	24	5742	1.116	6408	.915
7	24	5852	120	24	5948	1.127	6703	.958
7 1/2	26	5848	114	24	5943	1.138	6763	.966
8	28	5958	124	24	6058	1.150	6967 *	.996
8 1/2	30	5672	113	24	5761	1.161	6689	.956
9	32	5767	117	24	5860	1.173	6874	.983
9 1/2	34	5676	113	24	5765	1.184	6826	.976
10	36	181	—	24	—	—	—	—

Axial Flux Ref.

Au foil	Weight	Irradiation time	CPM - Bgd
Bare	48.5	60 min	6127
Ed cov.	48.6	"	3079

R.L.I. EXPERIMENT NUMBER 10

ACTIVATION CROSS SECTION

INSTRUCTOR: W. F. GILBERT

ABSTRACT

A method of computing activation cross section by determination of the absolute disintegration rate of a foil irradiated in a known thermal flux is described. The method can also be used to determine an unknown thermal flux in which a foil has been irradiated provided that an activation cross section for the foil material is assumed. The method used could be applied to many materials which become activated by thermal neutrons and whose resulting beta activity has a sufficiently long half life so that counting of the sample is not a problem. The techniques involved in absolute beta counting are emphasized.

ACTIVATION CROSS SECTION

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VII. APPENDICES
 - 1. Discussion Questions
 - 2. Beta Counting Considerations
 - 3. Thermal Neutron Self-Shielding Effects in Thin Foils

I. INTRODUCTION

This experiment yields by measurement two fundamental reactor parameters - absorption cross section and thermal neutron flux - plus the important experience of doing absolute counting.

The absolute number of disintegrations per unit time in an irradiated foil is a function of flux, time, foil weight, characteristic decay constant, and activation cross section. If the first four are known and also the absolute disintegration rate, the activation cross section can be determined. And similarly if all are known except the thermal flux it can be determined.

II. THEORY

If a material which becomes activated by a thermal neutron flux is irradiated in such a flux the net rate of increase of radioactive nuclei is:

$$\frac{dN'}{dt} = \phi_{th} N \sigma_{act} - \lambda N' \quad (1)$$

where ϕ_{th} = the thermal neutron flux in neutrons/cm²-sec

N = number of nuclei in the sample subject to activation by thermal neutrons

σ_{act} = effective cross section of each nucleus for the activation reaction in cm²

N' = the number of radioactive nuclei present in the sample at any time t

λ = decay constant of the radioactive nuclei in sec^{-1}

t = time in seconds

The term $\phi_{\text{th}} N \sigma_{\text{act}}$ is the rate of formation of radioactive nuclei in the sample and $\lambda N'$ is the rate of decay of these same nuclei.

If the sample is irradiated to saturation the rate of decay equals the rate of formation and thus:

$$\frac{dN'_s}{dt} = 0 = \phi_{\text{th}} N \sigma_{\text{act}} - \lambda N'_s$$

or

$$N'_s = \phi_{\text{th}} N \sigma_{\text{act}} \quad (2)$$

where N'_s = the number of radioactive nuclei in the sample at saturation.

If the sample is then removed from the flux the initial activity is:

$$\frac{dN'_s}{dt} = -\lambda N'_s$$

or, from eq. (2)

$$\frac{dN'_s}{dt} = \phi_{\text{th}} N \sigma_{\text{act}} \quad \text{disintegrations/sec} \quad (3)$$

N is easily determined for a given sample and, if the saturated disintegration rate is determined, the activation cross section may be found if the thermal flux in which the sample was irradiated is known. Conversely, the thermal flux may be computed if an activation cross section is assumed.

Most neutron activated nuclei decay with beta particle emission and, since one beta particle is emitted for each disintegration, an absolute beta emission rate determined for the sample is equal to the disintegration rate.

If the sample used has a $1/v$ absorption cross section, as is true of most materials, the reaction rate is proportional to neutron density in the sample. The neutrons may be assumed to have any convenient single velocity but the cross section must correspond to the velocity chosen. ϕ_{th} is then:

$$\phi_{\text{th}} = N_o v_o \quad (4)$$

where N_0 = neutron density in neutrons/cm³

v_0 = arbitrary velocity of the neutrons usually chosen as the most probable velocity of a Maxwellian thermal neutron distribution at room temperature. (2.2×10^5 cm/sec)

N is computed as

$$N = \frac{N_0 W}{A} \quad (5)$$

where N_0 = Avogadro's number

A = atomic weight of sample

W = weight of sample in gm.

Equations (1), (2) and (3) are correct only if each atom of the sample is exposed to the same thermal flux. If the sample is in the usual form of a foil which has appreciable thickness the thermal flux is decreased inside the foil due to the absorption of thermal neutrons as they traverse the foil. This self-shielding effect will be compensated for in computing the equivalent activity the foil would have had if the thermal flux were not depleted by the self absorption of the foil.

When a foil is irradiated at a location where there is an appreciable epi-thermal flux present the activity observed may have to be corrected to determine the portion of the induced activity due only to the thermal flux. The activity of the foil due to thermal neutrons and corrected for the self shielding of the foil for thermal neutrons can be expressed as:

$$A_{th} = \frac{AR}{\alpha} \quad (6)$$

where A_{th} = equivalent thermal-neutron induced activity of the foil

A = observed total activity of the same foil at the end of irradiation

R = fraction of total foil activity due to thermal neutrons

α = thermal neutron self-shielding correction factor (relative activation per unit mass)

R is related to the usual cadmium ratio (CR) as follows:

$$A'_{th} = A - FA_{cd} \quad (7)$$

where A'_{th} = that part of the observed activity A due to thermal neutrons

A_{cd} = observed activity of the same foil when irradiated under the same conditions but cadmium covered

F = correction factor to compensate for resonance neutrons absorbed by the cadmium cover

rearranging: $A - A'_{th} = A_{cd}F$

dividing by A :

$$1 - \frac{A'_{th}}{A} = \frac{A_{cd}F}{A}$$

but $\frac{A}{A_{cd}} = (CR)$ by definition and $\frac{A'_{th}}{A} = R$

therefore:

$$1 - R = \frac{F}{(CR)}$$

or

$$R = \frac{(CR) - F}{(CR)} \quad (8)$$

The correction for the activity due to epi-thermal neutrons can be made by using either R in equation (6) or directly using equation (7).

The thermal neutron self-shielding factor of the foil, α , is computed using diffusion theory. This factor is based upon an isotropic flux and therefore implies that the foil was irradiated in a medium having good scattering properties for thermal neutrons. The calculation of the self-shielding factor is considered in Appendix 3.

III. APPARATUS

1. Reactor
2. Standard Pile
3. 2π and end window counters
4. Suitable foils and cadmium covers

IV. PROCEDURE

An aluminum covered indium foil is irradiated in the reactor and is counted in an end window counter to determine its initial activity. A thick aluminum backing for the foil is used while counting and the observed count rate is corrected to saturation.

A count rate vs absorber curve for the foil is found by using aluminum absorbers to about 2 gm/cm^2 . The gamma portion of the curve is extrapolated back to zero absorber to determine the gamma correction factor f_γ . The gamma contribution is subtracted from the curve and the resulting beta count rate vs absorber curve is plotted for absorber thicknesses to 50 mg/cm^2 . This curve is extrapolated to determine the counter window correction factor, f_w .

The foil is counted in a known geometry counter and in the end window counter using a thick aluminum backing in each case. Using suitable gamma and counter window corrections the geometry factor, f_g , for the end window counter is found. The foil is counted in the end window counters with and without aluminum backing to determine the backscattering correction factor, f_{bs} .

An aluminum covered indium foil is irradiated in the standard pile and counted with aluminum backing in the end window counter. The observed count rate is corrected to saturation.

Cadmium covered indium foils are irradiated in the reactor and the standard pile and counted with aluminum backing in the end window counter. The count rates are corrected to saturation and the cadmium ratio determined for the reactor and the standard pile. The saturated count rate of both aluminum covered foils is corrected for epi-thermal activity and the thermal flux in the reactor is computed using the known thermal flux in the standard pile.

The equivalent saturated thermal-neutron induced activity of the aluminum covered foil irradiated in the reactor is computed and the absolute thermal neutron induced disintegration rate of the saturated foil is determined. The activation cross section of indium is then computed.

V. RESULTS

Results obtained in the experiment are primarily the values obtained for the reactor thermal flux and the activation cross section of indium. The correction factors which must be found as part of the experiment are also of considerable interest and the data leading to them should be carefully presented.

VI. ANALYSIS AND CONCLUSIONS

The value obtained for the activation cross section of indium should be compared with the accepted values obtainable from various sources and possible sources of error in the experiment may be investigated.

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Write-up by: W. F. Gilbert

VIII. APPENDICES1. DISCUSSION QUESTIONS

The following propositions should be considered prior to your arrival at the Argonaut. If possible, the questions will be discussed before the beginning of experimental work.

1. When a foil has been irradiated and is then counted, what factors must be considered if the saturated count rate of the foil is to be found?
2. Explain the physical concepts involved in microscopic absorption, scattering, and activation cross sections.
3. What relationship necessarily exists between absorption and activation cross sections?
4. Are cross sections in any way related to the radius of the nucleus?
5. Do (n, γ) reactions in the foil contribute to the observed count rate?
6. A foil which emits both beta-particles and gamma-rays is counted in a counter which is sensitive to both. What shape would you expect the curve of observed count-rate vs absorber thickness to have?
7. How can we determine the count-rate due only to beta-particles from the count-rate vs absorber curve?
8. How can we determine the counter window correction factor for beta-particles from the count-rate vs absorber curve?
9. The beta-particle emission from a foil of appreciable thickness is not isotropic but rather is found to be predominant in the direction perpendicular to the plane of the foil. What is the reason for this observed effect?
10. What effect does the preferential forward scattering of beta-particles from a foil have on the apparent geometry factor of the counter when the foil is counted in an end window counter?
11. Would the gamma correction factor for indium be a function of shelf position when an end window counter is used?
12. What effect would we expect the foil thickness to have on the back-scattering correction factor?

13. Why must the saturated count-rate of a cadmium covered indium foil be corrected before it is subtracted from the bare foil saturated count-rate to determine the activity due only to thermal neutrons?
14. Why is the relative activity per unit mass of a foil less when irradiated in an isotropic flux as compared to a collimated flux?
15. The self-absorption of beta-particles by a foil is found to increase rapidly with foil thickness for thin foils and then to increase more slowly for thicker foils. What property of beta-particles produces this effect?
16. In this experiment the flux depression in the medium surrounding the foil has been neglected. What conditions in the experiment cause this effect to be very small? Under what condition might the effect have to be considered?

2. BETA COUNTING CONSIDERATIONS

Since each disintegration in the foil produces one beta particle, the corrected observed activity is related to the absolute disintegration rate by the many factors associated with the counting of the foil. This may be expressed as

$$A_{th} = \frac{dN}{dt} f_g f_w f_e f_\gamma f_{bs} f_s \quad (9)$$

where A_{th} = equivalent thermal-neutron induced activity of the foil

$\frac{dN}{dt}$ = absolute disintegration rate of the foil at the end of irradiation due to thermal neutrons

f_g = counter geometry correction factor

f_w = counter window absorption correction factor

f_e = counter efficiency for β particles

f_γ = gamma background correction factor

f_{bs} = correction factor for beta-particle back scattering from the foil support material

f_s = foil beta-particle self-absorption and self-scatter correction factor.

f_g , the counter geometry correction factor, cannot in general be computed if an end window counter tube is used. This is because beta emitting foils

exhibit a very strong preferential forward scattering effect which causes many more beta particles to enter the counter window than would be expected on the basis of the solid angle subtended by the window. The equivalent geometry factor can be obtained experimentally by counting a similar foil in the counter and also in another counter having known geometry, such as a 2π counter. An additional correction for different gamma backgrounds may have to be made in the two counting rates since the gamma emission of the foil does not exhibit the strong forward scattering effect of the beta particles. Different counter window correction factors may also have to be considered.

f_γ , the gamma background correction factor, can be found by counting the foil using absorbers of various thicknesses. After a certain critical thickness of absorber has been reached the beta particles are all removed and the remaining count rate is due to the gamma emission of the foil. The result will be a straight line if plotted on semi-log graph paper for absorbers above the critical thickness. This straight line can be extrapolated back to zero absorber to determine the gamma contribution in the original count rate.

f_w , the counter window correction factor, is found by subtracting the gamma contribution from the total count rate vs absorber curve and then plotting the first part of the resulting beta-activity vs absorber curve on semi-log paper. The result is a straight line which can be extrapolated back to give the correction for the counter tube window and the counter air window.

f_e , the counter efficiency for beta particles, can be considered as unity.

f_{bs} , the correction factor for beta particle back scattering from the foil support material, can be found by counting the foil on the supporting material and on a film support of negligible thickness.

f_s , the foil beta particle self absorption and self scatter correction factor, is the only correction factor which is a very strong function of the foil thickness. The absorption increases very rapidly with increasing foil thickness for thin foils due to the absorption of low energy beta particle and then increases more slowly for greater foil thicknesses. This correction factor can be found by using foils of various thicknesses and observing their relative unity as the foil thickness approaches zero gives the relative count rate to be expected at any foil thickness due to the foil self absorption and self scattering. The relative count rates are usually determined in 2π or 4π counting systems to eliminate extraneous effects such as preferential forward scattering, etc.

When $\frac{dN'}{dt}$, the disintegration rate of the foil at the end of irradiation has been found from equation (9) it can be converted to $\frac{dN'_s}{dt}$, the saturated disintegration rate, by the usually relationship:

$$\frac{dN'}{dt} = \frac{dN'_s}{dt} (1 - e^{-\lambda t_1})$$

where λ = decay constant

t_1 = irradiation time of the foil.

The thermal flux in which the foil was irradiated can be found by comparing the saturated activity of similar foils exposed in the unknown flux and in a known flux in a standard pile. In both cases, the cadmium ratio must be found and each activity corrected to determine the thermal component. If similar foils are used and they are counted under identical conditions all other correction factors pertaining to the counting of foils are identical for both and therefore do not need to be considered.

3. THERMAL NEUTRON SELF-SHIELDING EFFECTS IN THIN FOILS

Introduction

Materials which become active when exposed to neutrons are commonly used to detect and measure neutron flux. In order to obtain useful counting rates in moderate amounts of the irradiated materials the cross section for activation must be reasonably large and the total absorption cross section will be as large or larger.

It is usually assumed that the material to be activated is irradiated in a uniform neutron flux, either collimated or isotropic as the case may be. This condition can only be approached in practice by using a sample which captures a negligible fraction of the neutrons which enter the sample. Because of the relatively large absorption cross section the sample must be extremely thin to meet this condition. Such samples are difficult to prepare and very likely to be damaged in handling.

For convenience in handling, samples to be irradiated are commonly prepared as foils which are physically rather thin but which do not satisfy the requirement that they absorb a negligible number of neutrons. In such foils the neutron flux in the interior region of the foil is decreased by the absorption of neutrons in the outer regions of the foil. Since the activation produced at any point in the foil is proportional to the neutron flux at that point the total activation produced in such a foil is obviously less than that which would have been produced had the flux been constant throughout the foil.

It is the purpose of this report to determine correction factors by which the observed activity of a foil can be converted to the activity which would have been obtained in the absence of self-shielding in the foil. Such corrections are usually necessary only when the absolute disintegration rate in the foil is to be determined from the neutron flux in which the foil was irradiated and the activation cross section.

Correction for the self-shielding in a foil may or may not be necessary depending on the nature of the foil and the precision of the measurement to be made. For an indium foil five mils thick exposed in an isotropic flux the error introduced by neglecting the self-shielding is about 16 per cent. For the same foil in a collimated flux the error is only about 5 per cent.

I. Collimated Neutron Flux

Consider a collimated thermal neutron flux ϕ_0 to be incident upon a foil of thickness t perpendicular to the surface of the foil as shown in Figure 1.

For such a collimated flux the neutron current J_0 into the surface of the foil is equal to the flux ϕ_0 . If the foil has an absorption cross section which is not constant with respect to neutron energy (such as the $1/v$ cross section of many materials) neutrons of certain energies are absorbed preferentially. As a result the neutron flux observed anywhere in the foil or that emerging from the foil will have a different energy distribution than that entering the foil and the number of neutrons captured at any point is difficult to compute.

If the foil is sufficiently thin so that relatively few neutrons are captured in the foil, the shift in the energy distribution of the flux

Figure 1

throughout the foil is small and may be neglected for many calculations. The assumption of a sufficiently thin foil is made in deriving the following expressions.

The number of neutrons removed from the beam per cm^2 per second an incremental foil thickness dx at x is $\phi_x \Sigma dx$ and therefore:

$$d\phi_x = -\phi_x \Sigma dx \text{ or } \frac{d\phi_x}{\phi_x} = -\Sigma dx \quad (1)$$

integrating:

$$\ln \phi_x = -\Sigma x + \ln A \text{ or } \phi_x = A e^{-\Sigma x} \quad (2)$$

$$\text{when } x = 0, \phi_x = \phi_0, \text{ therefore } A = \phi_0 \text{ and } \phi_x = \phi_0 e^{-\Sigma x} \quad (3)$$

and at $x = t$:

$$\phi_t = \phi_0 e^{-\Sigma t} \quad (4)$$

The transmission, T_{coll} , of the foil for this collimated flux is defined as the ratio of the transmitted flux to the incident flux or:

$$T_{\text{coll}} = \frac{\phi_t}{\phi_o} = e^{-\Sigma t} \quad (5)$$

The number of reactions, R , which occur for each cm^2 of foil surface per second is equal to the reduction in flux in passing through the foil or:

$$R = \phi_o - \phi_t = \phi_o (1 - T_{\text{coll}}) \quad (6)$$

The number of reactions, R' , which would occur for each cm^2 of foil surface per second if the flux were ϕ_o throughout the foil is:

$$R' = \phi_o \Sigma t = \phi_o \frac{t}{\lambda} = \phi_o x' \quad (7)$$

Where λ is the mean free path of a neutron in the foil material and x is the foil thickness in mean free paths.

The ratio of actual reactions in the foil to those which would have occurred if the flux had remained constant at ϕ_o throughout the foil is known as the self-shielding factor of the foil, α . Thus:

$$\alpha_{\text{coll}} = \frac{R}{R'} = \frac{1 - T_{\text{coll}}}{x'} \quad (8)$$

Since any activity induced in the foil is proportional to the number of neutron reactions in the foil this factor, α , is also the ratio of the activity induced in the foil to that which would have been induced by a constant flux ϕ_o throughout the foil.

The average mean free path of thermal neutrons in the foil having a $1/v$ cross section is the mean free path of neutrons having the average velocity of the Maxwellian thermal velocity distribution. For such a distribution:

$$\bar{v} = 1.128 v_p \quad (9)$$

where v_p is the most probable neutron velocity

For a $1/v$ absorber

$$\bar{\sigma} = \frac{v_p}{1.128} \quad (10)$$

where σ_p is the microscopic absorption cross section at the velocity v_p .

T_{coll} and α_{coll} are plotted as a function of foil thickness in mean free paths in Figure 3 and Figure 4 at the end of this report.

II. Isotropic Neutron Flux

Consider a foil of thickness x mean free paths in an isotropic scattering medium and assume that the foil has a negligible effect on the flux in the medium. Also consider an imaginary hemisphere of radius r (much smaller than the foil) as shown in Figure 2.

Figure 2

Since the scattering is isotropic the number of neutrons dN' scattered into the hemisphere through any area dA is proportional to dA . Thus:

$$dN' = K dA = K 2\pi r^2 \cos \theta d\theta \quad (11)$$

Of this number, those which will pass through the incremental surface of the foil ds at O is proportioned to the solid angle subtended by ds at the point of entrance through the surface of the sphere. This angle is

$$d\Omega = \frac{ds \sin \theta}{r^2} \quad (12)$$

Since the neutrons scattered into the hemisphere have a uniform distribution over 2π steradians the probability of scattering through ds is

$$\frac{d\Omega}{2\pi} = \frac{ds \sin \theta}{2\pi r^2} \quad (13)$$

Thus the number of neutrons scattered inward through dA and passing through ds is

$$\begin{aligned} dN &= dN' \frac{d\Omega}{2\pi} = \frac{K ds 2\pi r^2 \cos \theta}{2\pi r^2} \sin \theta d\theta \\ &= K ds \cos \theta \sin \theta d\theta \end{aligned} \quad (14)$$

The total number of neutrons passing through ds is obtained by integrating over the surface of the hemisphere.

$$\begin{aligned} N &= \int dN = k ds \int_0^{\pi/2} \cos \theta \sin \theta d\theta \\ N &= \frac{K ds}{2} [\sin^2 \theta]_0^{\pi/2} = \frac{K ds}{2} \end{aligned} \quad (15)$$

The neutron current, J_o , through the surface is:

$$J_o = \frac{N}{ds} = \frac{K ds}{2 ds} = K/2 \quad (16)$$

but we know from diffusion theory (Glasstone & Edlund, Chap. V, p. 94) that this current in one direction through a plane surface in an isotropic scattering medium is $\phi/4$ in the absence of any flux gradient.

Thus:

$$\frac{K}{2} = \frac{\phi}{4} \text{ or } K = \frac{\phi}{2} \quad (17)$$

The restriction originally placed upon r is seen to be justified as the results are independent of r .

If we now consider these same neutrons which were scattered through the surface of the foil and find J_x , the number which pass through the foil and out the opposite side we can define the transmission factor T_{iso} for the foil in an isotropic flux:

$$T_{iso} = \frac{J_x}{J_o} \quad (18)$$

Of the number dN which passed through dS at an angle θ the number which emerge from the other side of the foil is

$$dN_x = dN e^{-x' \sec \theta} \quad (19)$$

Substituting for dN from equation (14) and for K from equation (17)

$$dN_x = \frac{\phi}{2} dS e^{-x' \sec \theta} \cos \theta \sin \theta d\theta \quad (20)$$

N_x can then be evaluated as

$$N_x = \int dN_x = \frac{\phi}{2} \int_0^{\pi/2} e^{-x' \sec \theta} \cos \theta \sin \theta d\theta \quad (21)$$

and

$$J_x = \frac{N_x}{ds} = \frac{\phi}{2} \int_0^{\pi} e^{-x' \sec \theta} \cos \theta \sin \theta d\theta \quad (22)$$

Integration by parts yields

$$J_x = \frac{\phi}{2} e^{-x'} - \frac{\phi x'}{4} \int_0^{\pi/2} e^{-x' \sec \theta} \sin \theta d\theta \quad (23)$$

a second integration by parts yields

$$J_x = \frac{\phi}{2} e^{-x'} - \frac{\phi x'}{2} e^{-x'} + \frac{\phi x'^2}{2} \int_0^{\pi} e^{-x' \sec \theta} \frac{\sin \theta}{\cos \theta} d\theta \quad (24)$$

Multiplying and dividing the remaining integral by $x' \cos \theta$ we have

$$J_x = \frac{\phi}{2} e^{-x'} (1-x') + \frac{\phi x'^2}{2} \int_0^{\pi} e^{-x' \sec \theta} \frac{x' \sin \theta \cos \theta}{x' \cos^2 \theta} d\theta \quad (25)$$

this can be reduced to

$$J_x = \frac{\phi}{2} e^{-x'} (1-x') + \frac{\phi x'^2}{2} \int_{x'}^{\infty} \frac{e^{-t}}{t} dt \quad (26)$$

where $t = x' \sec \theta$

and thus

$$J_x = \frac{\phi}{2} e^{-x'} (1-x') + \frac{\phi x'^2}{2} E_1(x') \quad (27)$$

where $E_1(x') =$ the exponential integral $\int_{x'}^{\infty} \frac{e^{-t}}{t}$

This can be rewritten

$$J_x = \frac{\phi}{2} [e^{-x'} (1-x') + x'^2 E_1(x')] \quad (28)$$

the transmission is then

$$T_{iso} = \frac{J_x}{J_o} = \frac{\phi/2 [e^{-x'} (1-x') + x'^2 E_1(x')]}{\phi/2} \quad (29)$$

or

$$T_{iso} = e^{-x'} (1-x') + x'^2 E_1(x') \quad (30)$$

For a foil exposed in an isotropic flux the number of reactions in the foil per cm^2 of surface and per second due to the flux on one side is

$$J_o - J_x = \frac{\phi}{4} - \frac{\phi}{4} T_{iso} = \frac{\phi}{4} (1 - T_{iso}) \quad (31)$$

considering the neutrons entering the foil from both sides the number of reactions, R , is twice the above or

$$R = \frac{\phi}{2} (1 - T_{iso}) \quad (32)$$

If the foil were exposed in such a way that no self-shielding were possible the number of reactions per cm^2 of surface per second would be

$$\phi \Sigma t = \frac{\phi t}{\lambda} = \phi x' \quad (33)$$

where t is the thickness of the foil in cm, and λ is the mean free path of neutrons in the foil in cm. x' is the thickness of the foil expressed in mean free paths. The ratio of the actual reactions in the foil to the number of reactions in the absence of self-shielding is α , the thermal neutron self-shielding factor.

Figure 3 - THIN FOIL TRANSMISSION FACTOR

Figure 4 - RELATIVE FOIL ACTIVATION

$$\alpha_{\text{iso}} = \frac{\frac{\phi}{2} (1-T_{\text{iso}})}{\phi x'} = \frac{1-T_{\text{iso}}}{2x'} \quad (34)$$

As in the case of the collimated flux the average mean free path for a $1/v$ absorber in a Maxwellian thermal neutron flux is the mean free path for neutrons having the average velocity of the distribution.

T_{iso} and α_{iso} are plotted as a function of foil thickness in mean free paths in Figure 3 and Figure 4 at the end of this report.

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NEUTRON DETECTION BY ACTIVATION METHODS

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Several different methods for measuring neutron flux and neutron energy spectra in and around reactors include boron acid fission ionization chambers, proton recoil spectroscopy, crystal spectras copy using thallium, activated lithium iodide, liquid scintillators, photographic plates and emulsions, calorimeters, and activation analyses. For irradiation tests in high flux reactors where space is severely limited and gamma intensity is high, activation analysis appears to offer the best means for the determination of the irradiation test flux environment. The principle of this method essentially involves activation of materials having different energy ranges or threshold energies for neutron capture. The neutron flux in various energy increments is then determined by absolute counting techniques. By this method, neutrons ranging from thermal through resonance to fast can be measured.

Since precise measurements of neutron spectra are difficult to make, several factors must be taken into consideration in the selection of specific foils used for spectral measurements. These include:

1. the type selected depends on the energy region to be measured
2. the use of thin foils to minimize any flux perturbation or self shielding
3. The half-life of the radioactive nuclei formed

4. freedom from interfering activities
5. physical and chemical properties of materials consistent with irradiation conditions.

The half-life of the induced activity is a determining factor of the length of time over which a flux measurement can be made. Although several foils are available for measuring thermal-neutron flux over periods of time for most irradiation studies, many of the fast neutron detectors have short half-lives. Thus irradiations are made before or during a portion of a radiation experiment, introducing a considerable error into the results of absorbed dose because of possible undetected variations in neutron flux. If interfering activities are induced during activation, it may be necessary to carry out a radiochemical separation and to use suitable counting equipment. In certain types of experiments, factors such as melting point, thermal stability, and chemical activity may influence the choice of a detector.

Any type of material for use to map flux profiles of a reactor, however, may be used such as wires, pellets, powders, or compounds.

The activation method also has certain limitations or disadvantages. These include a limited amount of cross section data available, especially for (np) and (n α) reactions. In the energy range of 1 to 25 Mev where the (np) and (n2n) reactions predominate, the cross sections are often very small in the neighborhood of the threshold energy. Phosphorous and sulfur are exceptions to this generalization.

Furthermore, this technique is restricted because:

1. information is not supplied continuously
2. information is supplied on the integrated flux over the period of exposure rather than on the instantaneous flux
3. time delay between irradiation and availability of data is long
4. complex mechanical equipment may be required
5. difficulty in the interpretation of measurements to synthesize the incident spectrum.

While foil technique may involve the above difficulties and may not give very high accuracy, it is the only method available at present which is capable of providing both neutron flux and energy spectra data in high neutron fluxes. In addition, this method provides excellent discrimination against gamma radiation of energy less than 6 Mev. It also provides information from which neutron exposure whether it be total energy absorbed, burnup of reactor fuels, or fraction of energy dissipated in producing atomic displacement.

Thermal-Neutron Detectors

Detectors activated by thermal neutrons are summarized in Table 1.

Table 1

Thermal-Neutron Detectors

Nuclear Reaction	Capture Cross Section, barns	Half-life
$\text{Au}^{197} (n \gamma) \text{Au}^{198}$	98	2.7 days
$\text{In}^{115} (n \gamma) \text{In}^{116}$	145	54.3 minutes
$\text{Al}^{27} (n \gamma) \text{Al}^{28}$	0.23	2.3 minutes
$\text{Co}^{59} (n \gamma) \text{Co}^{60}$	36	5.27 years

Many thermal-neutron detectors also exhibit some resonance-absorption peaks in the higher energy region as described below.

Thermal-neutron flux is sometimes defined ambiguously and has been used to describe two different energy ranges: (1) the total neutron flux in the thermal neutron region as determined by a $1/v$ absorber for which the cross section varies inversely with the neutron velocity such as cobalt 60, or (2) the neutron flux below the cadmium cut-off energy of about 0.4 ev.

For the first case the thermal-neutron flux is determined from the specific activity A_0 of a foil of N number of isotopic atoms per gram involving the following relationship

$$A_0 = N \sigma \phi [1 - e^{-\lambda t}]$$

where ϕ = neutron flux
 T = irradiation time
 λ = decay constant

The specific activity in practice is usually found as the interceptor in a semilog plot of observed count rate of the foil as a function of time after irradiation, including appropriate corrections for the counter efficiency. The true neutron flux ϕ based on a Maxwellian neutron energy distribution is 12 per cent less than the thermal neutron flux represented as nv_0 .

The second case involves measurement of foils with and without a cadmium cover. With this combination it is possible to measure both the neutron flux in the epithermal region above the cadmium cutoff and the flux below the cadmium cutoff. The cadmium ratio is the ratio of saturated

activity values of the bare foils to those of the covered foil. The flux measured with the cadmium cover represents neutrons in the thermal region above 0.4 ev plus neutrons in the resonance absorption region of the particular foil used.

Resonance - Neutron Detectors

The resonance detectors are used to measure the epithermal neutron spectra and usually exhibit one or more narrow energy bands in which the neutron capture cross section is much higher than either side of this energy region. Thermal-neutron detectors in many cases are used in this region in which cadmium is employed as a shield for measurements in the resonance region. Table 2 lists several possible resonance neutron detectors.

Table 2

Resonance Detectors

Nuclear Reaction	Resonance Energy, ev	Half-life
$\text{In}^{115} (n \gamma) \text{In}^{116}$	1.4	54.3 minutes
$\text{Au}^{197} (n \gamma) \text{Au}^{198}$	4.9	2.7 days
$\text{Co}^{59} (n \gamma) \text{Co}^{60}$	120	5.27 years
$\text{Ce}^{137} (n \gamma) \text{Ce}^{138}$	180	37.5 minutes
$\text{Mn}^{55} (n \gamma) \text{Mn}^{56}$	260	2.6 hours
$\text{Cu}^{63} (n \gamma) \text{Cu}^{64}$	562	12.8 hours
$\text{Na}^{23} (n \gamma) \text{Na}^{24}$	1710	15 hours
$\text{V}^{51} (n \gamma) \text{V}^{52}$	3000	3.74 minutes
$\text{Al}^{27} (n \gamma) \text{Al}^{28}$	9100	2.3 minutes

For measuring fluxes in the epithermal region, the method of Danncoff may be used. For this method a neutron spectral function ϕ_0/E is written in terms of the cadmium ratio R , true thermal flux ϕ_{th} , a number characteristic of the resonance α , a normalization constant for the $1/v$ part of cross section σ_0 , the thermal activation cross section σ_{th2} at 0.025 ev and the energy of the cadmium cutoff E_{Cd} . In units of $n \text{ cm}^2 \text{ sec}^{-1} \text{ Mev}^{-1}$, this function is

$$\frac{\phi_o}{E} = \frac{\phi_{th} (E_{cd})^{1/2} \sigma_{th}}{2 \sigma_o (1+\alpha)(R-1)E}$$

This expression is used to determine whether or not the epithermal flux is inversely proportional to energy E . If several values of ϕ_o obtained from several different materials, and having resonances located at different energy points so as to span the region of interest are not equal, the method cannot be used. If they are equal within experimental error, then the spectrum is very close to an E^{-1} type.

In practice the actual cadmium ratio observed is a function of the sensitivity of the detector to resonance and thermal neutrons and does not lead to the true flux ratio unless this sensitivity is known. Corrections have to be made for absorption and scattering of resonance neutrons by cadmium cover and flux depression by foils and cover. These corrections can be obtained by extrapolating a plot of foil activity vs. thickness of cover to zero thickness and dividing this activity by that at any given cover or foil thickness. Corrections for leakage neutrons through the cadmium cover also have to be made.

Indium foils have been used extensively to measure fluxes in the thermal and epithermal regions and may be used to measure fluxes within region of $1 \text{ n cm}^{-2} \text{ sec}^{-1}$.

Threshold Detectors

Atomic species which undergo nuclear reactions of neutron capture in higher energy regions and for which the capture cross section is small below a certain threshold energy and much larger above this energy may be used for fast neutron measurement.

Threshold detectors for fast neutrons fall into two general classes: radioactivants and fission detectors. For the radioactivant type, materials are usually selected which by fast neutron capture form a radioactive species different from that formed by thermal neutron capture. For the case of fission detectors thermal-neutron contributions are reduced by proper shielding.

To evaluate threshold measurements, two factors must be taken into consideration: (1) the threshold energy is not a sharply defined value, and (2) the capture cross section above the threshold energy is neither constant nor a simple function of neutron energy. The simplest method for determining the fast neutron flux involves replacing the actual cross section versus neutron energy curve with a step function method. This procedure is based on the selection of an effective threshold energy below which the

capture cross section is assumed to be zero and above which it is assumed to be some constant value which is equivalent to the actual cross-section variation for the particular neutron spectrum being measured. This calculational method leads to an uncertainty in the flux value of about ± 50 per cent.

In employing a step function on a threshold detector the following equation is used to determine the effective threshold energy and a representative cross section in a region of greatest stability

$$\Phi(E_f) = \int_{E_f}^{\infty} \phi_f(E) dE = \int_0^{\infty} \frac{\phi_f(E) \sigma(E) dE}{\bar{\sigma}}$$

where: $\Phi(E_f) = n \text{ cm}^{-2} \text{ sec}^{-1}$ flux above 2.9 Mev

E_f = effective threshold energy, Mev

$\phi_f(E)$ = fission spectrum $n \text{ cm}^{-2} \text{ sec}^{-1} \text{ Mev}^{-1}$

$\sigma(E)$ = cross section, $\text{cm}^2 \text{ Mev}^{-1} \text{ nucleons}^{-1}$

$\bar{\sigma}$ = representative value of cross section region where
(E) is flattest $\text{cm}^2 \text{ Mev}^{-1} \text{ nucleon}^{-1}$

To obtain more precise flux values, two other methods, the polygonal and polynomial can be used. In the first method the spectrum is divided into as many regments (minus 2) as there are foils or activated materials. The spectrum is then approximated by a polygon whose parameters are determined from the data and is represented by a combination of linear functions of energy. The polygonal method is the more difficult to use since the differential energy selections must be made before the proper matrix can be calculated. The polynomial method uses, for the energy spectrum expression, the product of a weighting function and a polynomial, the degree of which is one less than the number of different foils used in the measurements. A digital computer is conveniently used for the required calculations. Results indicate that the considerable improvement is made over the older method, the error being reduced to ± 10 per cent.

Another method, the orthonormal expansion method expands the neutron spectrum into a series of orthonormal functions that are composed of linear combinations of the cross sections used in the measurements. The orthonormal functions are constructed by means of the Schmidt-Hilbert process of orthonormalization and the coefficients of the expansion terms are determined from the activation data together with the cross sections. The method gives an optimum fit to the data inasmuch as an orthonormal

expansion is always equivalent to a least squares approximation. This method is capable of much higher resolution but the work required both experimentally and analytically is an order of magnitude tougher.

In practice errors due to absorption and scattering of neutrons by foils and foil holders and to the flux depression by single and multiple foil packs must be taken into account in calculating the neutron energy spectrum.

Table 3 lists a number of threshold detectors of the radioactivant type.

Table 3

Threshold Radioactivant Detectors

Nuclear Reaction	Threshold Energy, MEV	Convenient Form of Material	Cross Section barns	Product Activity	Half Life
$P^{31} (np) Si^{31}$	2.4	$(NH_4) H_2PO_4$	0.075	β	2.62 hr
$S^{32} (np) P^{32}$	3.0	Fused sulfur	0.30	β	14.5 da
$Al^{27} (np) Mg^{27}$	4.6	Al metal	0.039	γ	9.8 min
$Ni^{58} (np) Co^{58}$	5.0	Ni metal	1.23	γ	72 days
$Si^{28} (np) Al^{28}$	5.5	Silicate	0.080	γ	2.3 min
$Mg^{24} (np) Na^{24}$	6.3	Mg metal	0.048	γ	14.9 hr
$Al^{27} (n\alpha) Na^{24}$	8.1	Al metal	0.011	γ	14.9 hr
$I^{127} (n2n) I^{126}$	9.4	KI		$\beta - \gamma$	13.0 day
$Cu^{63} (n2n) Cu^{62}$	11.7	Cu metal		$\beta + \gamma$	10 min
$Mo^{92} (n2n) Mo^{91}$	12.7	Mo Metal		$\beta +$	15 min
$Cl^{35} (M\alpha) P^{32}$	1.8	K Cl		$\beta -$	14.3 day

The values obtained on cross sections and threshold energies are subject to errors. These presented above are those most often reported in the literature, even though in certain cases other values have been given. For accurate correlation of radiation damage studies, cross sections and threshold energies need to be standardized.

Threshold detectors of the fission type were used initially to measure tissue dose. The entire neutron spectrum was measured by a series of detectors consisting of gold and gold with cadmium cover for the thermal and epithermal flux, Pu^{239} shielded with B^{10} for total fast flux, Np^{237} for total flux above 0.75 Mev, U^{238} for total flux above 1.5 Mev, and S^{32} for total flux above 2.5 Mev with these series of detectors, the fast neutron spectrum can be constructed with reasonable accuracy for energy distributions, not too different from the fission spectrum. The absorbed dose for a material can be calculated from this spectral data. The advantage of using the above-mentioned elements is that their cross sections rise rapidly at the threshold energy and remain essentially constant for higher energies. Comparison of measurements made with proportional counters and threshold detectors in region of comparable application has shown agreement within 10 per cent.

In table 4 are presented fission detectors. Any combination of radio-activant and fission detectors may be used to obtain a detailed energy spectrum as described above.

Table 4
Threshold Fission Detectors

Isotope	Threshold Energy, Mev	Cross Section, barns
Pu^{239} (shielded with B^{10})	10 ⁻³	1.90
Np^{237}	0.75	1.50
U^{238}	1.45	0.54
Th^{232}	2.0	0.598
U^{235}	Thermal	580

Although threshold detectors used for neutron measurements are excellent discriminators against gamma radiation with energy up to several Mev, this type of detector may be used for gamma spectral measurements at higher energies since the (γn) threshold is the same as the (n, n) threshold. The measurement of pure gamma fields is simplified since the competing (n, γ) and (n, p) reactions are absent. The neutrons produced by the (γn) may be detected with boron proportional counters. Proper selection of materials will permit the determination of gamma spectral data from 1.68 Mev (beryllium) to 18.6 Mev (carbon). Photofission reactions may be used to measure high energy gamma radiation. The threshold being about 5 Mev in uranium and thorium.

Aerojet-General Nucleonics Company has reported the design and calibration of five standard foils for measuring neutron spectra in form energy increments. These include B^{10} shielded Pu^{239} , S^{32} , Au^{197} , and Co^{59} . Aerojet is also developing techniques for measuring neutrons with energies above 6.3 and 8.6 Mev by using the (np) and $(n\alpha)$ reactions (respectively) in Aluminum-27. Convair is also working with Aerojet in attempts to standardize neutron flux and spectra by activation analysis method.

From this discussion, it can be seen that the measurement of neutron flux by utilization of radioactivants has widespread application. However, the measurements are often hampered by the lack of reliable neutron cross-section data. In many cases the error in the reported cross sections will limit the accuracy of the results. As more reliable cross-section information becomes available, of course, the accuracy of the general method will be improved.

R.L.I. EXPERIMENT NUMBER 11

TOTAL NEUTRON CROSS SECTION

INSTRUCTOR: J. SPALEK

ABSTRACT

The theory and technique of the measurement of the total cross section by the transmission method is presented. The cross sections of various materials are measured. The measurement of the cross section of different thicknesses of a material illustrates the hardening of the neutron spectrum with increasing penetration into the material. Finally, the effect of the sensitivity of the type of detector used on the cross section obtained is illustrated.

TOTAL NEUTRON CROSS SECTION

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
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 - 2. Total Neutron Cross Section Theory

I. INTRODUCTION

The experiment is performed at the Argonaut reactor using a beam of neutrons coming out of a 1 inch² hole in the graphite stringer. The hole comes within a few inches of the reactor core. The beam is collimated and passed through a sample of the material whose cross section is to be measured. The emerging attenuated beam is detected by a B¹⁰F₃ detector, and count rates are taken on a Multi-Purpose Counter (MPC). Also the count rates of the initial (not attenuated) beam are taken, by removing the sample from the beam. From the ratio of these two count rates properly corrected for background, the cross section of the material can be calculated.

This experiment shows how total cross sections can be obtained by a simple direct measurement. Many early cross sections have been measured by the transmission method. Also the method is the basis of many time of flight and spectrometer measurements. The simplicity of the method, the accuracy of the average total cross section values obtained, and their usefulness in simplified reactor computations make the transmission method still important.

II. THEORY

The derivation of the equation involving the cross section to be measured is presented below. For a detailed discussion, see Appendix 2 and/or reference books.

A beam containing I_0 neutrons/cm² is attenuated in passing through a material of thickness x . The decrease ($-dI$) in neutrons/cm² due to passage

through thickness dx of material is proportional to the number of neutrons/cm² (I) incident on the thickness dx , and to the number of nuclei/cm³ (N) in the thin layer dx . The proportionality constant is the cross section (σ) to be measured. The above put into equation form is:

$$- \frac{dI}{dx} = IN\sigma$$

Integrating, and substituting the conditions $I = I_0$ when $x = 0$, the result is the equation

$$I = I_0 e^{-N\sigma x}$$

where the symbols were defined above.

The transmission (T) of a material is defined as the ratio of neutron flux at point x to the initial flux:

$$T = \frac{I}{I_0} = \frac{\text{relative flux at point } x}{\text{relative initial flux}} = \frac{\text{count rate with sample in beam}}{\text{count rate without sample in beam}},$$

so that $T = e^{-N\sigma x}$. Taking natural logarithm of this equation, the cross section is:

$$\sigma = - \frac{\ln T}{NX}.$$

The T and x are measured by this experiment, and N is calculated from the knowledge of the density of the material whose transmission was measured, its atomic weight A , and the Avogadro's number, N_0 , so that

$$N = \frac{\rho N_0}{A}.$$

For the interpretation of the cross section thus obtained (which is actually an average), and the correction for hardening of the beam, see again Appendix 2.

III. APPARATUS

Table with detector mount, collimator and sample holder. Samples of Au, In, Fe, A .

Beam catcher and shutter

Neutron detector (1 inch diameter, 14 inches long, 40 cm Hg pressure of B¹⁰F₃ gas, operating voltage 1100).

Preamplifier, amplifier, scalar (multi-purpose counter), H. V. supply,
Optional: oscilloscope.

IV. PROCEDURE

Close the beam shutter and position the apparatus in line with the expected neutron beam path. Place a beam catcher behind the $B^{10}F_3$ detector.

Open the beam shutter and align the apparatus so that neutrons will pass through the collimator and sample into the detector.

Discriminate against the γ -rays with the Pulse Height Selector on the Multiple Purpose Counter.

Take count rates without sample in beam, with sample in beam, and epi-cadmium background count rates with and without sample in beam. (Epi-Cd background is obtained with a thin (0.010" or 0.020") Cd sheet in the beam.

Do the above for as many of the different materials provided, and for several thicknesses of each (to show the hardening effect), as desired or as time permits.

Plot, calculate and interpret the data (see Results section and Appendix 2, page 249, for the various "correction" factors and interpretation of the cross section obtained).

V. RESULTS

When applicable, correct for hardening by means of the curve on the following page.

In order to compare the experimental values with the one in tables which are usually listed at the thermal velocity (σ_{v_0}), use the expression

$$\bar{\sigma}_{Texp} = 1.128 \sigma_{v_0}$$

which is derived in Appendix 2.

Sample: Au (or In) $N = \frac{\rho N_0}{A} = \dots$

Thickness	I	BG	IBG	$\bar{\sigma}_{Texp}$	$\bar{\sigma}_a$	$\bar{\sigma}_a$ corrected for hardening	$\sigma_{v_o} exp$
0	$= I_0$						
0.2032 cm							
0.1016 cm							
0.0508 cm							
0	$= I_0$						

The second I_0 count rate on the end of sample readings checks if the neutron flux did not change during the measurements.

Correction for hardening of the Maxwell distribution in the transmission measurement, for $\frac{1}{v}$ absorber and $\frac{1}{v}$ detector.

Sample: Al (or Fe) $N =$

Plot the following data on semilog paper:

<u>Thickness (cm)</u>	<u>I</u>	<u>BG</u>	<u>I-BG</u>
0	$= I_0$		
15.0			$\bar{\Sigma} T_{\text{exp}}$ from plot = ...
12.0			$\bar{\sigma} T_{\text{exp}} = \dots$
9.0			
6.0			
3.0			$\sigma(\text{tables}) = \dots$
0	$= I_0$		

Additional samples provided: Indium (3 samples)
Iron (5 samples)

VI. ANALYSIS AND CONCLUSIONS

To be done by the student. (Also see Appendix II)

VII. BIBLIOGRAPHY

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Experiment Written by: J. Spalek

VIII. APPENDICES1. DISCUSSION QUESTIONS

The following propositions should be considered prior to your arrival at the Argonaut. If possible, the questions will be discussed before the beginning of experimental work.

1. State the essentials of the transmission method. (What determines the cross section?)
2. Should the distance between the sample and the detector be small or large? Justify your answer.
3. What is "efficiency" and "sensitivity" of a detector? Distinguish between the two.
4. Do we need to know the efficiency of the detector used in the transmission experiment in order to calculate the cross section?
5. Do we need to know the energy sensitivity of the detector in order to interpret the results?
6. What is "hardening" of the beam? To what is it due?
7. Why is the beam collimated? How much (to what size) should the beam be collimated compared to sample diameter?
8. Does it make any difference which materials are used to support the sample and counter?
9. What does a detector see? (To what does a detector respond?)
10. Why do we subtract the count rate due to epithermal neutrons from the measured count rates? What kind of cross section would be obtained by subtracting the room background only (with epi-Cd neutrons blocked off)?
11. How would we eliminate the fast neutrons for a room background count?
12. Show how you would calculate the thickness of Cd that would eliminate the thermal neutrons.
13. Why do we not use a very thick Cd for the epicadmium background counts?
14. Is the average cross section and the cross section for the average velocity one and the same thing? If not, try to write down the relationship between them.

15. As can be seen, this experiment has a number of samples of different transmission values for each material. Is more than one sample of each material essential for determining its cross section?

2. TOTAL NEUTRON CROSS SECTION

THEORY

Introduction

Since the knowledge of the various cross sections is very important in the study of the materials used on a reactor, and practically all of neutron research deals with the measurement of cross sections, a basic detailed discussion of the subject is presented.

This discussion can be found in many books dealing with nuclear physics. (See references). The following section is an extract from the "many books" intended to save time for those who wish to review the subject.

NUCLEAR CROSS SECTION

To describe quantitatively the interactions of bombarding particles with atomic nuclei the concept of cross section was introduced. In this sense the term "cross section" is a measure of the probability of occurrence of a given process under certain conditions.

To illustrate the concept of nuclear cross section, it can be visualized as the cross-sectional (or target) area presented by a nucleus to an incident neutron. If we further visualize the nucleus as a sphere of radius r cm, and think of the neutrons as point projectiles, then the target area or cross section σ of each nucleus is: $\sigma = \pi r^2 \text{ cm}^2$.

This simple picture illustrates only one type of cross section, the geometrical, and considers only collisions of neutrons with target nuclei. But it serves to derive a formula for cross section that can be extended to any kind of interaction of neutrons with atomic nuclei.

Today, there are experimental values for many kinds of cross sections (like absorption, scattering, activation, etc.) for different materials, and several types of each (microscopic, macroscopic, atomic, nuclear, differential).

The following historical sketch may serve to illustrate the different cross sections.

1. In the early studies of x-rays it was observed that the intensity (I) of a parallel beam of x-rays was reduced by passing through a material. The decrease in intensity ($-dI$) was proportional to the thickness dx of the material, and to the intensity of the beam. Put into an equation:

$$- dI = \mu I dx \quad (1)$$

where μ is the factor of proportionality called the absorption coefficient, or the total absorption coefficient of the material.

This coefficient μ for x-rays is analogous to the macroscopic absorption cross section Σ_a for neutrons. The only difference is that μ represents the fraction of energy removed from the beam of x-rays per cm length, while Σ_a represents the fraction of neutrons absorbed per unit length.

The equation for the attenuation of a beam of neutrons is identical in form to the one for x-rays (eq. 1):

$$- \frac{dI}{I} = \Sigma dx \quad (2)$$

Written in this form, since the left side represents the fraction of neutrons absorbed in the path dx , Σdx is equal to this fraction, but is preferably referred to as the probability that neutrons will be absorbed in path dx .

To describe what was happening microscopically, the x-ray experimenters introduced the atomic absorption coefficient μ_a defined in terms of the total absorption coefficient μ as:

$$\mu_a = \frac{\mu A}{\rho N_0} \quad (3)$$

where: A is the atomic weight, ρ is the density of the element, and N_0 is Avogadro's number.

The dimensions of μ_a are those of an $\frac{\text{area}}{\text{atom}}$. As an example, the atomic absorption coefficient for copper is about $50 \times 10^{-21} \frac{\text{cm}^2}{\text{atom}}$. If we picture the atom as a sphere of radius 2×10^{-8} cm, then we can say that this atom has a geometrical cross section of about $12 \times 10^{-16} \text{ cm}^2$. From this example it is seen that the atomic absorption coefficient, called also the absorption cross section, is very much smaller than the geometrical cross section, in this case.

2. The concept of nuclear cross section can be extended to other types of nuclear processes. Since we are primarily interested in neutrons, we define the nuclear cross section σ for a specific reaction as the average number of individual processes occurring per incident neutron in the beam per nucleus:

$$\sigma = \frac{C}{N_A I} \text{ cm}^2 \text{ per nucleus} \quad (4)$$

where: C = number of individual nuclear processes
 N_A = number of atoms per cm^2 of material
 I = number of neutrons in the incident beam per cm^2

For the purpose of the above definition we assumed a uniform beam of I neutrons impinging perpendicularly on a target one atomic layer thick. By rearranging equation (4),

$$\sigma N_A = \frac{C}{I} \quad (5)$$

the right side represents the fraction of the incident neutrons which interact with the target nuclei. Therefore $N\sigma$ can be thought of as the fraction of the surface which is capable of undergoing the given reaction. Thus in 1 cm^2 of surface $N\sigma \text{ cm}^2$ is effective. Since 1 cm^2 contains N nuclei, the quantity $\sigma \text{ cm}^2$ is the effective area per single nucleus for the reaction. This interpretation of σ leads to the use of the term cross section, but it is generally not related to the geometrical cross section.

Because nuclear cross sections are usually in the range of 10^{-22} to 10^{-26} cm^2 per nucleus, a handy unit was introduced and called the barn - thus by definition:

$$1 \text{ barn} = 10^{-24} \text{ cm}^2 \text{ per nucleus}$$

For the purpose of general definition of cross section given by equation (4) there was assumed a uniform, that is, monoenergetic, beam of neutrons striking a target one atomic layer thick, or, which is the same, only the surface of the target material was considered.

Now to determine the cross section experimentally, we will have to measure the attenuation of neutrons through a material of finite thickness x.

3. Derivation of transmission equation.

A beam containing I_0 neutrons per cm^2 is striking the material with N nuclei per cm^3 from the left. (Figure 1).

The number of nuclei present in a thin layer dx of the material is Ndx nuclei per cm^2 . This is equivalent to the quantity N_A in the defining equation (4), therefore $Ndx \sigma$ is the fraction of the neutrons falling on this layer which react. $N \sigma dx$ is then set equal to $-\frac{dI}{I}$, where $-dI$ is the decrease in neutrons per cm^2 due to passage through thickness dx of the material.

Figure 1

Hence
$$-\frac{dI}{I} = N\sigma dx \quad (6)$$

Integrating

$$\ln I = N\sigma x + \ln I_0 \quad (6-1)$$

where I_0 is the intensity of the beam before entering the material (when $x = 0$).

Therefore:

$$I_x = I_0 e^{-N\sigma x} \quad (6-2)$$

where: I_x is the number of neutrons that get through the material of thickness x ,

I_0 is the number of neutrons falling on the material (initial flux)

N is the number of nuclei per cm^3 in the target material.

The last equation (6-2) is used in the transmission experiment.

The transmission (T) of a material is defined as the ratio of flux at point x to the initial flux:

$$T = \frac{I_x}{I_0} = e^{-N\sigma x} \quad (7)$$

In different notation, since the intensity of the beam I is the flux density nv (usually called flux), n = number neutrons in beam per cm^3 , v = velocity of neutrons in cm/sec , equation (7) can be written:

$$T = \frac{nv}{(nv)_0} = e^{-N\sigma x} \quad (7-1)$$

or taking natural logarithms of both sides, $\ln T = -N\sigma x$, so

$$\sigma = - \frac{\ln T}{Nx} \quad (7-2)$$

$$\begin{aligned} \text{where: } T &= \frac{I_x}{I_0} = \frac{nv}{(nv)_0} = \frac{\text{relative flux at point } x}{\text{relative initial flux}} \\ &= \frac{\text{count rate with sample in beam}}{\text{count rate without sample in beam}} \end{aligned}$$

N = number of nuclei per cm^3 in the target material

x = thickness of material

If the thickness x of the material is small, the value of transmission T may be close to unity. This would greatly reduce the accuracy of the cross section obtained. (Accuracy is discussed on pg. 261). So it would be preferable to use a sample thick enough to attenuate the beam more so that T would be much less than 1. But with increasing sample thickness the Maxwell velocity distribution (see pg. 255) is increasingly distorted due to greater (preferential) absorption of slow neutrons by $\frac{1}{v}$ capture. This change in the Maxwellian velocity distribution is called hardening of the beam.

Figure 2

4. The cross section given by equation (7-2) is called microscopic, to distinguish it from $N\sigma$, called the macroscopic cross section and symbolized Σ . Therefore $\Sigma = N\sigma \text{ cm}^{-1}$, where N is the number of nuclei per cm^3 . Σ is consequently the total cross section of the nuclei in 1 cm^3 of material. (its dimensions are cm^{-1}).

Given ρ , the density of the target material and its atomic weight A , then $\frac{\rho}{A}$ is the number of gram atoms per cm^3 . The number of atomic nuclei N per cm^3 is then:

$$N = \frac{\rho}{A} N_0 \quad \begin{array}{l} (N_0 = \text{Avogadro's number}) \\ N_0 = 6.02472 \times 10^{23} \end{array}$$

and

$$\Sigma = \frac{\rho N_0}{A} \sigma \quad (8)$$

is the resultant relationship between macroscopic and microscopic cross section.

The transmission method is the most direct procedure for the measurement of cross sections. It gives the total cross section with good accuracy, for it involves only the ratio of counting rates with and without sample in beam. The efficiency of the detector is not needed. But the sensitivity (with respect to neutron velocity) of a detector does enter into the interpretation of measured cross section, for it changes the effective flux distribution, as will be seen on pg. 257.

The total cross section (σ_T) determined experimentally is the sum of the absorption (σ_a) and scattering (σ_s) cross sections, for it makes no difference to the detector how the neutrons are removed from the beam in the attenuation process.

5. Furthermore, since the cross section depends on the neutron velocity or energy, the experiment gives σ_T of a material for the particular energy or energies of neutrons used. When the beam intensity is low it is not practical to use a velocity selector (crystal spectrometer or a chopper), to select a particular energy. So the whole energy spectrum of the neutrons coming out of a reactor is passed through the sample material, and the total cross section obtained (σ_T) should be called experimental ($\sigma_T \text{ exp}$), for then it is an average over the above spectrum.

The variation of $\sigma_T \text{ exp}$. is mostly due to its one component, the absorption cross section σ_a . The σ_a varies not only for different materials, but also usually varies greatly with the neutron energy. There are two types of variation of the absorption cross section σ_a .

In the first one σ_a varies according to the " $\frac{1}{v}$ law":

$$\sigma_a = \frac{(\sigma_a)_0 v_0}{v} \quad (9)$$

where: σ_a = absorption cross section at velocity v

$(\sigma_a)_0$ = absorption cross section at velocity $v_0 = 2200$ m/sec.

The second type is called "resonance" absorption. It is a sharp rise in absorption at one or more particular energies.

Finally, the two components of the total cross section can be still subdivided further. Thus there are two types of scattering, elastic and inelastic:

$$\sigma_s = \sigma_e + \sigma_i$$

And two types of absorption, capture and fission: $\sigma_a = \sigma_c + \sigma_f$

At capture of a neutron by a nucleus, the compound nucleus may be radioactive, and the measurement of this activity gives the activation cross section (see activation cross section experiment). And the fission cross section is the measure of the probability that a compound nucleus will fission into two almost equal parts.

6. Thermal Neutron Distributions

The fission neutrons from a fuel assembly surrounded by a moderator are slowed down by a number of elastic scattering collisions with the moderator atoms. The final average kinetic energy of the neutrons is approximately the same as that of the atoms (or molecules) of the moderator. Since the number of moderator atoms having speeds within a certain range is described by the Maxwell distribution function of molecular speeds (derived in any book on Kinetic Theory of Gases ref. No. 6), this same function may be used to describe the neutrons that are in thermal equilibrium with the moderator atoms (that is, thermal neutrons):

$$dn = \frac{4n}{\sqrt{\pi} v_0^3} v^2 e^{-\frac{v^2}{v_0^2}} dv \quad (10)$$

where: dn = number of neutrons per unit volume in the speed range dv at the speed v

n = total number of neutrons per unit volume

v_0 = most probable speed, that is, the speed at which $n(v)$ is a maximum;

graph of (10):

Figure 3

The ordinate on this graph (the ratio $\frac{dn}{dv}$, or the coefficient of dv in (10) is the Maxwell distribution of molecular (neutron) speeds, while the number of neutrons per unit volume with speeds between v and $v + dv$ is represented on the graph by the area of the narrow vertical strip of width dv .

The average kinetic energy of the thermal neutrons is

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} k T \quad (11)$$

where k is the Boltzmann constant ($= 1.380 \times 10^{-16}$ erg/degree K), m is the neutron mass, and T is the absolute temperature in $^{\circ}$ K. The speed corresponding to this energy is the root mean square speed

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \quad (11-1)$$

However the most probable speed v_0 in the Maxwellian distribution (10) corresponds to the energy kT of the Neutrons:

$$\frac{1}{2} m v_0^2 = kT \quad (12)$$

and therefore the value of the most probable speed (shown in Figure 3) is

$$v_0 = \sqrt{\frac{2kT}{m}} \quad (12-1)$$

Equation (10) and its graph in Figure 3 give the density of neutrons with speed v as a function of v . But the experimental measurements are done in terms of flux. Since flux is nv , the flux distribution can be obtained from eq. 10 by multiplying by v , giving:

$$d(nv) = \frac{4n}{\sqrt{\pi} v_o^3} v^3 e^{-\frac{v^2}{v_o^2}} dv \quad (13)$$

This equation describes the flux distribution within the reactor core. Its graph in Figure 4 shows that the most probable flux is at a speed greater than the most probable speed (v_o). To find the value of the speed at which the flux is maximum, differentiate eq. (13) and set the result equal to zero giving:

Figure 4

$$v = \sqrt{\frac{3}{2}} v_o = 1.22 v_o \quad (14)$$

The actual (measured) speed distribution of thermal neutrons in a reactor core was obtained by Ringo at Argonne, and it was seen to follow the Maxwellian fairly closely.

The flux distribution in a beam hole reaching through the reflector to the reactor core will then also be given by eq. 13. But the flux distribution as seen by a detector of a particular sensitivity placed in the beam will be given by equation (13), the flux distribution in the beam, multiplied by the sensitivity (with respect to neutron speed) of the detector.

If a detector has $\frac{1}{v}$ sensitivity (such as $B^{10}F_3$ detector), the measured distribution of count rate against neutron velocity will be:

$$d(nv) = \frac{4n}{\sqrt{\pi} v_o^3} \frac{v^3}{v} e^{-\frac{v^2}{v_o^2}} dv$$

which is again just the Maxwell speed distribution (10).

If in conjunction with a $\frac{1}{v}$ detector, a $\frac{1}{v}$ absorber of cross section $\frac{k}{v}$ is used in the transmission measurement, then the observed cross section for the whole beam will be the average cross section $\bar{\sigma}$ for the Maxwell distribution:

$$\bar{\sigma} = \frac{\int \frac{k}{v} n(v) dv}{\int n(v) dv} = \frac{\frac{4nk}{\sqrt{\pi} v_o^3} \int_0^{\infty} \frac{v^2}{v} e^{-\frac{v^2}{v_o^2}} dv}{\frac{4n}{\sqrt{\pi} v_o^3} \int_0^{\infty} v^2 e^{-\frac{v^2}{v_o^2}} dv} = \frac{2}{\sqrt{\pi}} \frac{k}{v_o} = 1.128 \sigma_{v_o}$$

$$f(n) = \int_0^{\infty} x^n e^{-ax^2} dx \quad (15)$$

n	f(n)	n	f(n)
0	$\frac{1}{2} \sqrt{\frac{\pi}{a}}$	1	$\frac{1}{2a}$
2	$\frac{1}{4} \sqrt{\frac{\pi}{a^3}}$	3	$\frac{1}{2a^2}$
4	$\frac{3}{8} \sqrt{\frac{\pi}{a^5}}$	5	$\frac{1}{a^3}$
6	$\frac{15}{16} \sqrt{\frac{\pi}{a^7}}$	7	$\frac{1}{a^4}$

$$\text{If } n \text{ is even, } \int_{-\infty}^{+\infty} x^n e^{-ax^2} dx = 2 f(n)$$

$$\text{If } n \text{ is odd, } \int_{-\infty}^{+\infty} x^n e^{-ax^2} dx = 0.$$

7. In order not to confuse the several average cross sections, a clear idea of the three particular speeds of the Maxwell speed distribution must be reached. Also, the validity of many formulas dealing with thermal neutrons depend on the correct interpretation of the speed v appearing in them as a particular speed in the Maxwellian distribution.

Two of the speeds, the root mean square v_{rms} , and the most probable speed v_o appeared before in terms of energy (eqs. 11, 12). For clarification, the above two speeds, and the arithmetic average speed $\bar{v}$, will now be derived.

Using the table of integrals on page 258 the average speed:

$$\bar{v} = \frac{\int v \, dn}{n} = \frac{\int_0^{\infty} v \frac{4n}{\sqrt{\pi} v_o^3} v^2 e^{-\frac{v^2}{v_o^2}} dv}{n} = \frac{2}{\sqrt{\pi}} v_o = 1.128 v_o \quad (16)$$

where dn is the number of neutrons per unit volume with speed v from the Maxwellian speed distribution (10).

The root mean square speed v_{rms} , as in the average kinetic energy of neutrons (eq. 11) is:

$$v_{rms} = \sqrt{\overline{v^2}} = \left[\frac{\int v^2 dn}{n} \right]^{\frac{1}{2}} = \left[\frac{4n}{n v_o^3 \sqrt{\pi}} \int_0^{\infty} v^4 e^{-\frac{v^2}{v_o^2}} dv \right]^{\frac{1}{2}} = \sqrt{\frac{3}{2}} v_o \quad (17)$$

Finally the most probable speed v_o is the maximum of the Maxwell distribution therefore setting the most derivative with respect to v of equation (10) equal to zero:

$$\frac{d}{dv} \left(\frac{dn}{dv} \right) = \frac{4n}{\sqrt{\pi} v_o^3} \left[2v e^{-\frac{v^2}{v_o^2}} - \frac{2v^3}{v_o^2} e^{-\frac{v^2}{v_o^2}} \right] \stackrel{\text{set}}{=} 0 \quad (18)$$

$$\therefore v_{max} = v_o$$

The relative magnitudes of these speeds are shown on the speed distribution $\left(\frac{dn}{dv} \right)$ graph in Fig. 5.

Figure 5

Now it is seen that the observed average cross section from eq. 15, pg. 258, is 1.128 times larger than the cross section at the most probable speed $\sigma v_0 = \frac{k}{v_0}$. Also, the observed average cross section $\bar{\sigma}$ is not the same as the cross section for the average velocity $\sigma_{\bar{v}}$, for:

(19)

A use of the average cross section as given by eq. 15 is to determine experimentally the neutron temperature. Obtaining $\bar{\sigma}$ experimentally determines v_0 from (15) and therefore the temperature T from eq. (12), provided $\frac{k}{v}$ is known.

8. Various correction factors; interpretation of the experimental cross section; accuracy.

The total experimental cross section for a sample material is obtained (from the two count rates) by equation:

$$\sigma_{T_{\text{exp}}} = \frac{\ln \frac{1}{T}}{N x} \quad (\text{see pg. 253})$$

If a very thin sample of $\frac{1}{v}$ absorber is used (transmission close to unity), and a detector of $\frac{1}{v}$ sensitivity, then the $\sigma_{T_{\text{exp}}}$ obtained is the average $\bar{\sigma}$ for the Maxwell spectrum, and if we wish to compare it with the listed value of σv_0 for the material, which is tabulated at the most probable speed v_0 , the relating factor is:

$$\bar{\sigma}_{T_{\text{exp}}} = 1.128 \quad v_0 \quad (\text{see pg. 258})$$

But in the above case of high transmission, the accuracy is very poor, as can be seen from the following:

Refer to eq. 6, pg. 252, $-dI = IN \sigma dx$

The number of neutrons removed from the beam ($-dI$) becomes

$$\Delta I = - IN \sigma \Delta x$$

and for a thin sample $\Delta x \approx x$, $\therefore \Delta I = IN \sigma x$.
to $I(1-T)$ therefore:

The number ΔI is also equal

$$\Delta I = I(1-T) = IN \sigma x$$

or:

$$1 - T = N \sigma x \quad (20)$$

So if the transmission of say 0.95 is measured with 1% accuracy,

$$1 - 0.95 = 0.05 = \sigma_1 N x$$

or:

$$1 - 0.9545 = 0.0455 = \sigma_2 N x$$

The difference between σ thus being .01 out of .05, or 20% accuracy in σ .

Therefore for better accuracy thicker samples with smaller transmission should be used.

Taking then a thick sample, of transmission 0.50 measured with 1% accuracy.

From:	$\ln T = N \sigma_T x$	
	$\ln 0.50 = - 0.693 = - NX\sigma$	$\left. \begin{array}{l} \text{difference in} \\ \sigma = 0.010 \text{ or} \\ 1.46\% \end{array} \right\}$
1% higher	$\ln 0.5050 = - 0.683 = - Nx\sigma$	

So from 0.50 transmission of 1% accuracy results a cross section of 1.46% accuracy. Thus a thicker sample is preferable for accuracy in σ .

9. Correction for hardening

With sample thickness increased, however, hardening (distortion) of the Maxwellian distribution occurs. So the $\sigma_{T_{\text{exp}}}$ is no longer only the average cross section $\bar{\sigma}$. Because the Maxwellian distribution changes shape as it penetrates the absorber, the $\sigma_{T_{\text{exp}}}$ is therefore an average over and average distribution.

The average cross section $\bar{\sigma}$ cannot be measured directly as a function of absorber thickness. Nevertheless, H. A. Bethe obtained a curve for this function by numerical integration for a $\frac{1}{v}$ material and a $\frac{1}{v}$ detector shown in the graph on pg. 246. This curve gives the ratio of the experimental absorption cross section (σ_{exp}) to the σ at v_0 , plotted against transmission. It can

Since this value of transmission gives also very good accuracy, samples of 50% transmission are therefore most commonly used in cross section measurements.

The above applies to the absorption cross section only and only for a $\frac{1}{v}$ material. Nevertheless if the $\frac{1}{v}$ material has an absorption cross section much larger than the scattering cross section, the curve on pg. 246 can be used for the hardening correction.

However, if a material is predominantly a scatterer, the total cross section may be used to obtain the average thermal scattering cross section by subtracting the average absorption $\bar{\sigma}_{\text{abs}} = 1.128 \times \sigma_{v_0}$, if known from other measurement (ex. pile oscillator), for there is almost no hardening in a predominant scatterer. But the average scattering cross section, which is fairly constant in the thermal region, has no simple relationship to cross section at particular velocity (σ_s), because σ_s , unlike σ_a , varies irregularly with velocity in the thermal region.

All the preceding theory considers only a special case of a general quantum mechanical treatment of neutron cross sections. (Ref. 2). With high neutron-flux sources (reactors), and the development of velocity selectors (choppers, spectrometers) which permit measurements at chosen velocities, the cross section values obtained are more accurate, but the methods much more complex. The above theory shows how the first cross section measurements were made. The presentation of this simple method is further justified by the fact that many cross sections have been measured as averages in the same way, and for some materials this simple method is still very useful.

- References: 1.) Glasstone and Edlund - Nuclear Reactor Theory
- 2.) Hughes - Pile Neutron Research
- 3.) Semat - Introduction to Atomic and Nuclear Physics
- 4.) Murray - Introduction to Nuclear Engineering
- 5.) Hoag - Nuclear Reactor Experiments
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3. 1961 ASEE-AEC REACTOR LABORATORY INSTITUTE WRITEUP

SURVEY OF NEUTRON VELOCITY SELECTORS

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Introduction

We have discussed the data for total cross section for thermal neutrons by the transmission method. The same technique may be applied to this measurement using the devices described below.

The most satisfactory procedure for this determination is based on the use of a stream of neutrons in which all the neutrons have the same velocity. This condition is approached by several instruments which may be called, collectively, neutron velocity selectors.

The type of instrument or device used will depend upon the range of energy of the neutrons under consideration. This survey will be based on that classification.

For this purpose the range of neutron energies may be divided into three principal regions:

1. High energy, about 10 Mev. to 10,000 ev., equivalent to fast neutrons.
2. Intermediate, 10,000 ev. to 1 ev., equivalent to intermediate neutrons.
3. Slow, less than 1 ev., equivalent to slow or thermal neutrons.

Monoenergetic High Energy Neutrons

Production by Nuclear Reactions

Very intense gamma ray emitters can be made in nuclear reactors. When such a source of gamma rays is placed in a container consisting largely of beryllium or deuterium in some suitable form or combination, neutrons are produced that are practically monoenergetic.⁽¹⁾

Deuterium ice or paraffin containing deuterium bombarded by deuterons from a Van de Graaff electrostatic generator produce good yields of approximately monoenergetic neutrons under properly controlled conditions. Other targets such as carbon may be used. They can provide beams of approximately monoenergetic neutrons in the range 5 kev. to 20 mev. with a precision of (5 to 10) %.

Monoenergetic Intermediate Neutrons

Production by Fast Choppers

Until quite recently, the production of monoenergetic neutron beams with energies lying within the range of 10,000 to 1,000 ev. presented a problem. The development of the Argonne Fast Chopper⁽²⁾ has provided an answer that is now satisfactory. The range of neutron energies covered by this instrument extends from 0.002 ev. to 20,000 ev. Table I below gives the precision to be expected.⁽³⁾

TABLE I

<u>Energy of Neutron Beam</u>	<u>Range</u>
20.0 ev.	± 0.03 ev
1,000.0	± 10.5
10,000.	± 24.0
50,000.	± 38.75

The Argonne fast chopper or better "sweeper", was designed as a general purpose instrument for measuring both total and partial neutron cross sections. The range of applicability was extended by the use of two interchangeable rotors, which will be presently described.

The device consists of three major components:

1. The mechanical chopper
2. The neutron detector
3. The multichannel time analyzer

This mechanical chopper is a rotor in which a set of 7 parallel slits is cut. It is rotated at a given speed in a stream of neutrons. Neutrons may pass through the chopper only when the slits are in line with the stream. Thus, pulses of neutrons are released at specified intervals. These pulses travel down a tube to the detector. Since the time of travel down the tube is given by the distance divided by the velocity, the members of the pulse of different speeds arrive at the detector at different times. The different energies are distinguished according to arrival by the multichannel time analyzer.

Rotor No. 1 weights about 190 lbs; No. 2 about 150 lbs. The rotor is hung vertically by a thin rod directly from a DC motor. The zero-time signal to the time analyzer is provided by the reflection of a light beam from a plane mirror mounted near the top of the rotor.....The rotor speed may be varied from about 100 to 15,000 rpm.

Each rotor consists of an outer shell of monel metal (70% nickel and 30% copper) around a core of uranium, forming a disk of 10 inch diameter. The first rotor was made of two parts bolted together. The second rotor was made by forming the slits in solid metal by the technique of electrical disintegration. The locations of the surfaces of these slits are accurate to about 0.001 inch. These slits are "cigar-shaped" with flat walls. All of the slits in both rotors are 2-1/16 in. high.

The sheet steel drift tube emerging from the chopper is filled with helium at atmospheric pressure. The tube passes through two detector stations at 30 to 60 meters from the chopper, and then into the final station at 120 meters.

The neutron bursts are received by boron-loaded liquid scintillation detectors.

The first station is used primarily for the measurement of partial cross sections, the other two for transmission measurements. Samples are usually in the form of powder. They are placed in an aluminum capsule 0.035 inches in width and of arbitrary length and height. Samples having areas from 50 mm to as low as 1 mm may be studied.

This apparatus is in use at present at the CP-5 Reactor.

Monoenergetic Slow Neutron Beams

Production by Slow Neutron Choppers

One form of chopper for slow neutrons was a cylinder built up of alternate layers of aluminum and cadmium. A pulsed neutron stream was produced when the cylinder was rotated about its long axis in a stream of thermal neutrons. In later models components of the pulse were distinguished by a time analyzer actuated by photocells.

This chopper is useful for neutron energies in the thermal region below about 0.4 ev.

Production by Mechanical Monochromators

A true mechanical monochromator is one in which neutrons of a narrow velocity band are produced by mechanical means, with no time of flight measurement involved. Several disks of cadmium containing radial slits are mounted on the same axis, which is rotated parallel to the neutron beam. The neutrons in a collimated beam pass through the slits if the neutron velocity bears the correct relationship to the speed of rotation. The resolution would be expected to be comparable to that of a chopper with size of openings and rotational speeds of the same order of magnitude. Of course, neutrons of only one velocity band are produced. Earlier designs were found deficient in resolution or in intensity. A modified rotating disk monochromator at Los Alamos has produced neutrons with wave lengths extending to 30 Å. The device seems adapted to studies in the sub-thermal region. Ringo at Argonne has developed a mechanical monochromator consisting of two slotted disks of cadmium each 2 feet in diameter, which rotate in opposite directions.

By Crystal Spectrometers

The deBroglie wave length of neutrons of velocity 2,200 meters per sec is calculated to be (1.8×10^{-8}) cm. This value is within the range of common x-ray wave lengths. Hence the array of plates of atoms in copper or lithium fluoride, for example, should provide a suitable diffraction grating for thermal neutron beams. Higher energy neutron beams, equivalent to shorter deBroglie waves may be diffracted by using the crystal at grazing incidence angles.

With a sufficiently intense source of the thermal neutrons from a reactor, a crystal and slit system may be arranged to select neutrons from the spectrum with good resolution from about 0.02 to about 10 ev.

It is interesting to note that an experiment is now in progress at the CP-5 reactor using this technique. A single crystal of copper serves as a diffraction grating for a beam of thermal neutrons from the reactor. The monochromatic neutron beams are used to study certain parameters of crystalline metallic uranium.

References

1. Irving Kaplan, Nuclear Physics, Addison Wesley 1955, p. 446
2. L. M. Bollinger, R. E. Cote and G. E. Thomas, Second U. N. International Conference on the Peaceful Uses of Atomic Energy, 14 (239) 1958
3. G. E. Thomas, private communication

STUDENT RESULTS

Experiment Performed By T. R. Rehm, K. Rothschild, H. A. Shadduck,
L. C. Wilbur, W. E. Wilson, Jr. June 23, 1961. Submitted by H. A. Shadduck

TABLE I
EXPERIMENTAL OBSERVATIONS.

SUBST.	THICK. CM.	COUNTS PER MIN.			TRANS. T	TOTAL BARNES EXP.	TOTAL BARNES LIT.
		WITH Cd	WITHOUT Cd	NET			
Au	0.0	6805	79669	72864	1.00		
	0.2032	8131	27858	24727	0.339	94.5	
	0.1016	4521	45697	41176	0.564	94.0	
	0.0508	5534	59717	54183	0.743	94.0	
	0.0	6862					
						Av. 94.2	107.3
						± 0.2	± 2.0
IN	0.0	6805	79669	72864	1.00		
	0.2032	2484	21814	19330	0.266	182.5	
	0.1016	3939	39735	35796	0.492	183.5	
	0.0508	4999	55559	50560	0.695	180.0	
	0.0	6797					
						Av. 182.0	192.2
						± 1.04	± 10.5
Al	0.0	3291	23861	20570	1.00		
	15.0	917	6613	5666	0.277	142	
	6.0	1993	14397	12404	0.604	139	
	3.0	2585	18112	15527	0.755	155	
	9.0	1452	11002	9550	0.464	141	
	12.0	1180	8342	7162	0.349	146	
						Av. 145	14
						± 0.03	± 0.1
Fe	0.0	6567	78273	71706	1.00		
	1.5875	1303	14183	12880	0.180	12.8	
	1.270	1871	19811	17940	0.250	12.9	
	0.9525	2624	28623	25999	0.363	12.5	
	0.6350	3583	40546	36968	0.515	12.3	
	0.3175	5003	56421	51418	0.717	12.4	
	0.0	6809					
						Av. 12.6	13.5
						± 0.09	± 1.06

* CORRECTED FROM CURVE BY BETHE.

TABLE II
SAMPLE CALCULATION

AU SAMPLE NO. 1.

$$T = \frac{I}{I_0} = \frac{\text{NET COUNTS WITH SAMPLE}}{\text{NET COUNTS WITHOUT SAMPLE}} = \frac{24727}{72864} = 0.339$$

$$N = \text{No. ATOMS AU IN 1 cc.} = \frac{(0.602 \times 10^{24})(19.3)}{197} = \frac{(N_A)(\text{DENSITY})}{\text{AT. WT.}} = (0.059 \times 10^{24})$$

$$\text{THICKNESS } x = 0.2032 \text{ cm.}$$

$$\sigma \text{ IN BARNS} = -\frac{\ln T}{x N}$$

$$= -\frac{(-1.08716)(10^{24})}{(0.2032)(0.059)(10^{24})}$$

$$\approx 90.3$$

CORRECTING FACTOR FROM CURVE = 0.955

$$\sigma_{\text{TOTAL}} = \frac{90.3}{0.955} = 94.5 \text{ BARNS.}$$

R.L.I. EXPERIMENT NUMBER 12

NEUTRON TEMPERATURE

INSTRUCTOR: J. SPALEK

ABSTRACT

The experiment is performed at the AGN-201 reactor. The temperature (kinetic energy) of the neutrons in the reactor core is measured by two methods. The basis of both methods is the variation of the absorption cross sections of materials with neutron velocity and therefore kinetic energy or temperature, depending on the viewpoint. The temperature of the neutrons determined by the variation in the measured cross section is emphasized. The experiment demonstrates that the temperature of neutrons in the core is higher than the temperature of the moderator atoms.

NEUTRON TEMPERATURE

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
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 - 2. Derivation of the Number of Neutrons of all Velocities Striking or Crossing a Surface Per Unit Area Per Unit Time.
 - 3. Hardening Correction Curve

I. INTRODUCTION

The temperature of neutrons in the core is of importance to reactor design and to the study of the mechanism of moderation, while the beam neutron temperature is of great value to the scientist using the neutron beam for research.

Temperatures of the beam neutrons have been measured at Argonne and Oak Ridge early after World War II. The experimental values showed a marked difference, even those obtained from the same reactor but with different method. Values of 255°K to 330°K for beam neutrons have been measured; the temperature of neutrons in the core varied from 90°C to 140°C above the moderator temperature. Taking the discrepancies of neutron temperature measurements into account, the neutrons, nevertheless, exhibited about 100°C higher temperature than the moderator atoms. A higher temperature was expected because of the presence of some fast neutrons which are in the process of slowing down to thermal velocities, and therefore not yet in equilibrium with the moderator atoms. Also, a preferential absorption of the slow neutrons by the uranium fuel may be taking place.

II. THEORY

The average kinetic energy of thermal neutrons is given by the relation

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \quad (1)$$

which is derived in any book on kinetic theory of gasses (see reference (4)). T will be referred to as neutron temperature.

To determine the neutron temperature would therefore involve the measurement of the average kinetic energy of the neutrons, or their "root mean square" velocity ($v_{\text{rms}} = \sqrt{v^2}$). These quantities, however, are practically not measureable. Nevertheless, the root mean square velocity is related to the most probable velocity v_0 of the Maxwell distribution of thermal neutrons, the latter velocity being a measureable quantity.

The relating factor is here derived:

$$v_{\text{rms}} = \sqrt{v^2} = \left[\frac{\int v^2 dn}{\int dn} \right]^{\frac{1}{2}} = \left[\frac{\frac{4n}{\sqrt{\pi} v_0^3} \int_0^\infty v^2 v^2 e^{-\frac{v^2}{v_0^2}} dv}{N} \right]^{\frac{1}{2}} = \left[\frac{3}{2} v_0^2 \right]^{\frac{1}{2}} = \underline{\underline{1.224 v_0}} \quad (2)$$

where $dn = \frac{4n}{\sqrt{\pi} v_0^3} v^2 e^{-\frac{v^2}{v_0^2}} dv$ is the Maxwellian distribution of speeds.

Therefore,

$$\overline{v^2} = \frac{3}{2} v_0^2 \quad (3)$$

and substituting (3) into (1) gives $1/2 m (3/2 v_0^2) = 3/2 kT$, which after simplification is no longer the average kinetic energy of the neutrons. The relation obtained,

$$1/2 m v_0^2 = kT \quad (4)$$

means that the most probable velocity of the Maxwellian distribution corresponds to a kinetic energy kT of the neutrons. Equation (4) is also used to define the "temperature" of neutrons.

The problem of determining T thus becomes one of measuring v_0 . This velocity could be obtained by measuring the relative number of neutrons as a function of velocity with a crystal spectrometer or chopper. The maximum of the distribution thus measured would give v_0 and therefore T from equation (4). The numerous difficulties associated with this direct method make it impractical for this experiment.

The two methods for measuring v_0 used in this experiment are based on the absorption cross section of materials which are dependent on the neutron velocity and therefore temperature.

The first method measures the cross section of a $1/v$ dependence such as of boron at the average velocity ($\bar{v}$), $\sigma_{\bar{v}}$, in the reactor core (glory hole). This cross section, ($\sigma_{\bar{v}}$) is related to the cross section at the most probable velocity σ_{v_0} (or σ_{kT}) by the same factor which relates the two velocities (v and v_0) in the Maxwell velocity distribution:

$$\bar{v} = \frac{\int v dn}{\int dn} = \frac{\frac{4n}{\sqrt{\pi} v_0^3} \int_0^{\infty} v v^2 e^{-\frac{v^2}{v_0^2}} dv}{n} = \frac{2}{\sqrt{\pi}} v_0 = 1.128 v_0 \quad (5)$$

Since the absorption cross section of a $1/v$ material is k/v ,

$$\sigma_{\bar{v}} = \frac{k}{\bar{v}} = \frac{k}{1.128 v_0} = \frac{1}{1.128} \sigma_{v_0} \quad (6)$$

From equation (6) the most probable velocity v_0 of the actual Maxwellian distribution in the reactor (at the time $\sigma_{\bar{v}}$ was measured) can be calculated, provided k is known. But k can be determined from a known cross section measured at a given velocity (by crystal spectrometer or chopper). Such a cross section is very well known for boron, and is

$$\sigma_{v_0} (\text{B}) = 755 \text{ barns at } v_0 = 2200 \text{ m/sec} ,$$

so that

$$k = 755 \text{ barns} \times 2200 \text{ m/sec, for boron} . \quad (7)$$

The v_0 of the present distribution in the reactor is thus

$$v_0 = \frac{k}{1.128 \sigma_{\bar{v}}} = \frac{755 \text{ barns} \times 2200 \text{ m/sec}}{1.128 (\sigma_{\bar{v}} \text{ measured, in barns})} . \quad (8)$$

Also it is known from (4) that the neutron temperature corresponding to 2200 m/sec neutron speed is 293°K , so that

$$1/2 m (2200 \text{ m/sec})^2 = k 293^\circ \text{K}, \quad \text{at } v_0 = 2200 \text{ m/sec}$$

and

$$1/2 m v_0^2 = kT, \quad \text{at } v_0 \text{ just measured.}$$

Hence

$$T = 293 \left(\frac{v_0}{2200 \text{ m/sec}} \right)^2 {}^\circ \text{K}$$

is the present neutron temperature in the reactor core. The method used to measure $\sigma_{\bar{v}}$ is described in the procedure from the point of view of cross sections. It is the same one as in the "Measurement of the Absorption Cross Section by the Danger Coefficient Method" experiment, where it is described in detail from the point of view of the neutron absorption rate and reactivity. (See reference 6).

The second method measures the average cross section of a $1/v$ material in the beam issuing out of a reactor (either a beam hole in graphite or a glory hole) by the transmission method. (Described briefly, (see reference No. 5 for details), the method consists of measuring the attenuation of the neutron beam at passing through a sample of thickness x ; the ratio of the intensity I_0 of the incident beam to the intensity I of the beam emerging from the sample determines the total cross section:

$$\frac{I}{I_0} = e^{-N\sigma_T x} \quad , \quad \text{or}$$

$$\sigma_T = \frac{\ell n \frac{I_0}{I}}{Nx} \quad (9)$$

Subtracting a known scattering cross section from σ_T gives the absorption cross section desired. Using a sample of 50% transmission, no correction for the hardening of the beam is necessary and the σ_T obtained is equal to σ_{v_0} . The basis for this second method is the same as for the first one, that is, comparing a measured velocity dependent cross section with one at a known velocity to obtain v_0 of the actual neutron velocity distribution in the reactor, and thus the temperature of the neutrons. But the difference in method gives a different type of cross section than the one measured by the first method, related by different factors to the cross section at the most probable velocity. A brief derivation of the relating factors necessary in this method is presented below. (For details, see again reference No. 5, Total Neutron Cross Section.)

The flux distribution of the neutrons in the beam is obtained by multiplying the Maxwellian velocity distribution of the neutrons in the core (dn) by v . Thus

$$d(nv) = \frac{4n}{\sqrt{\pi} v_0^3} v v^2 e^{-\frac{v^2}{v_0^2}} dv \quad (11)$$

However, using a $1/v$ detector ($B^{10}F_3$), the distribution as seen by this detector is again the velocity distribution as in the core (obtained by multiplying the flux distribution by $1/v$ sensitivity of the detector.) Now, putting a $1/v$ absorber that does not harden the neutron spectrum, i.e., of zero

thickness (of cross section k/v) in the beam, and passing the beam consisting of neutrons of all velocities through it, the observed cross section will be the average cross section $\bar{\sigma}$ for the Maxwell velocity distribution.

$$\bar{\sigma} = \frac{\int \frac{k}{v} n(v) dv}{\int n(v) dv} = \frac{\frac{4nk}{\sqrt{\pi} v_0^3} \int_0^\infty \frac{v^2}{v} e^{-\frac{v^2}{v_0^2}} dv}{\frac{4n}{\sqrt{\pi} v_0^3} \int_0^\infty v^2 e^{-\frac{v^2}{v_0^2}} dv} = \frac{2}{\sqrt{\pi}} \frac{k}{v_0} = 1.128 \sigma_{v_0} \quad (12)$$

$$\bar{\sigma} = \frac{2}{\sqrt{\pi}} \frac{k}{v_0 \exp} = 1.128 \sigma_{v_0} \quad (12.1)$$

Equation (12.1) shows that in this case the average cross section $\bar{\sigma}$ is not equal to the cross section at the average velocity $\sigma_{\bar{v}}$ (equation 6). Also, equation (12) was derived under the assumption that the $1/v$ absorber did not harden the neutron spectrum. A thin sample of high transmission (transmission $T = I/I_0$) would approximate the above condition, but then the accuracy of the cross section would be poor due to any small error in the count rates I_0 and I , which differ by a small amount. Therefore, a thicker sample has to be used to increase the accuracy. But the thick sample hardens the neutron beam spectrum that passes through it, and so equation (12) no longer holds. A correction for hardening now has to be applied to correctly relate the experimentally obtained cross section $\bar{\sigma}_{\text{exp}}$ with the cross section at the most probable velocity σ_{v_0} . The hardening correction was calculated for samples of different transmission by H. A. Bethe, (ref. 2, p. 237) and presented in the form of a curve, which is shown on page 312. It is a fortunate circumstance that for a sample of 50% transmission ($\ln T = 0.693$) which gives good accuracy, the hardening correction factor is unity $\left(\frac{\sigma_{\text{exp}}}{\sigma_{v_0}} = 1 \right)$, as can be seen from the graph.

The experimental cross section $\bar{\sigma}_{\text{exp}}$ is then equal to σ_{v_0} directly. It should be pointed out that the relating factor of equation (12.1) was calculated for the flux distribution in the beam modified by the $1/v$ detector so that the resultant distribution was again the same as in the core. However, the actual distribution of neutrons in the beam is the flux distribution of equation (11) whose $\bar{v}^2$ is higher than $\bar{v}^2$ of the velocity distribution of neutrons in the core. Correspondingly, the temperature of the neutrons in the beam is higher than of those in the core. To compute the temperature of the beam neutrons, their $\bar{v}^2$ must first be calculated. Taking a beam of neutrons issuing out of a core, where the neutrons have a Maxwell velocity distribution, only those neutrons that hit the surface of the core will get into

the beam. The number of neutrons with speed v striking a surface of a container is $1/4 v dn$ (see Appendix II). So the v^2 of the neutrons in the beam is

$$\overline{v^2} = \frac{\int v^2 \frac{1}{4} v dn}{\int \frac{1}{4} v dn} = \frac{\frac{4n}{4\sqrt{\pi} v_0^3} \int_0^\infty v^5 e^{-\frac{v^2}{v_0^2}} dv}{\frac{4n}{4\sqrt{\pi} v_0^3} \int_0^\infty v^3 e^{-\frac{v^2}{v_0^2}} dv} = 2v_0^2 \quad (13)$$

Summarizing: for the beam:

$$\overline{v_b^2} = 2v_0^2$$

for the core:

$$\overline{v_c^2} = 3/2 v_0^2$$

Taking the measured velocity v_0 , the temperature in the core is from

$$1/2 m \overline{v^2} = 3/2 kT,$$

$$1/2 m (3/2 v_0^2) = 3/2 k T_c$$

while the T in the beam:

$$1/2 m (2v_0^2) = 3/2 kT_b, \text{ so that}$$

$$T_{\text{beam}} = 4/3 T_{\text{core}} \quad (14)$$

If the detector used in the beam method is not of a $1/v$ sensitivity, but is black (like a long $B^{10}F_3$ detector), equation (12) no longer holds and an average cross section over the flux distribution must be computed.

III. APPARATUS

<u>First Method</u>	<u>Second Method</u>
AGN-201 Reactor	Reactor
Boron sample	Au sample
3 Cadmium wires	In sample
BF ₃ detector	BF ₃ detector
Preamplifier	Preamplifier
Amplifier	Amplifier
MPC counter	MPC counter
	Detector and sample mounts

IV. PROCEDURE

First Method: The cross section at the average velocity ($\sigma \bar{v}$) is obtained by the effect of an absorber on the reactivity of a reactor.

The absorption area (cross section) of a black absorber (like Cd) will not change with neutron velocity (or temperature). The absorption "area" (cross section) of boron (a $1/v$ absorber), however, will change with neutron temperature. Since an absorber will change the reactivity, we may calibrate the sensitivity (reactivity/area of absorber) of a reactor using a black absorber whose absorption area is constant.

In a given average flux $\bar{n}\bar{v}$, the number of neutrons hitting a unit surface per second is $1/4 n \bar{v}$ (see Appendix II for derivation). Therefore, the number of neutrons being absorbed per second by a Cd wire of length ℓ and diameter d ($d \ll \ell$) is equal to the surface of the wire times the number hitting this surface, or $\pi \ell d \times 1/4 n \bar{v}$. Said in other words, the absorption area of Cd wire for a given average flux $\bar{n}\bar{v}$ is

$$\frac{\pi \ell d}{4}$$

This is the "area of absorber" to be used in the sensitivity calibration.

Since the Cd wire depresses the flux, the sensitivity is measured for three Cd wires of different lengths, and a plot of reactivity/cm² against the absorption area of the wires is made. The extrapolated value (to "zero" absorption area) gives the sensitivity (ρ/cm^2) for no flux depression. Now, the effect of the boron atoms on reactivity is measured, giving

$$N = \frac{\text{number of boron atoms}}{\text{reactivity}}$$

The microscopic cross section of B at average velocity can then be calculated by dividing the total area per reactivity by the number of B atoms/reactivity:

$$\frac{\frac{\text{Area (cm}^2\text{) of absorber}}{\text{reactivity}}}{\frac{\text{number of B atoms}}{\text{reactivity}}} = \text{area of absorption of a single B atom} = \sigma \bar{v}(\text{B})$$

Using the relating factors from "theory" section, calculate T.

Second Method: Mount the sample and detector in line with the glory hole. Measure the transmission of a $1/v$ absorber (of close to 50% transmission to "eliminate" the hardening correction) by taking count rates with sample in and out of beam, and take the corresponding background counts.

Calculate $\bar{\sigma}_{\text{exp}}$ using equation (10); and this $\bar{\sigma}_{\text{exp}} = \sigma_{v_0}$ in this case. Then calculate the temperature.

Calculate also the temperature of the neutrons in the beam.

V. RESULTS

First Method

For Cd Wires:	Wire	Length (cm)	Diam. (cm)	Physical Area	Absorption Area	Sensitivity ($\mu\rho/\text{cm}^2$)
	1					
	2					
	3					

From the plot of sensitivity vs. absorption area, sensitivity = $\frac{1}{\frac{\mu\rho}{\text{cm}^2}}$

Calculate "inverse sensitivity" $A = \frac{\text{cm}^2}{\mu\rho}$

For boron samples (a and b):

Number of B atoms = _____ (sample a)

= _____ (sample b)

$N = \frac{\text{No. of B atoms}}{\text{reactivity corrected for epi-Cd absorption}}$, $N(a) = \frac{\text{atoms}}{\mu\rho}$

$N(b) = \frac{\text{atoms}}{\mu\rho}$

Epi-Cd correction $\approx 4\%$

$N(\text{average}) = \frac{\text{atoms}}{\mu\rho}$

Calculate:

$\sigma_{\frac{A}{N}}(B) = \frac{A}{N} = \text{_____ cm}^2 = \text{_____ barns}$

$v_0 = \frac{755 \text{ barns} \times 2200 \text{ m/sec}}{1.128 \sigma_{\frac{A}{N}}(B) \text{ barns}} = \text{_____ m/sec}$

$$T_{\text{core}} = 293 \frac{v_o}{2200}^2 \text{ } ^\circ\text{K} = \underline{\hspace{2cm}} \text{ } ^\circ\text{K}.$$

Second Method

$$I_o = \underline{\hspace{2cm}} \text{ counts/min, } I_o \text{ Background} = \underline{\hspace{2cm}} \text{ c/m}$$

Sample	Thickness x(cm)	N $\frac{\text{atoms}}{\text{cm}^3}$	I (c/m)	I - BG	$\ln \frac{I}{I_o}$	$\bar{\sigma}_{\text{exp}}$
Au						
In						

$$\sigma_{2200 \text{ m/sec}} (\text{Au}) = \underline{\hspace{2cm}}, k (\text{Au}) = 2200 \times \sigma_{2200} (\text{Au}) = \underline{\hspace{2cm}}$$

$$\bar{\sigma}_{\text{exp}} (\text{Au } 50\% \text{ transmission}) = (1) \sigma_{v_o \text{ exp}} = \frac{k}{v_o \text{ exp}}$$

$$v_o \text{ exp} = \frac{k}{\bar{\sigma}_{\text{exp}}} = \underline{\hspace{2cm}} \text{ m/sec}$$

$$T_{\text{core neutrons}} = 293 \left(\frac{v_o \text{ exp}}{2200} \right)^2 \text{ } ^\circ\text{K} = \underline{\hspace{2cm}} \text{ } ^\circ\text{K}$$

Calculate T using In cross section data.

Read on reactor thermometer: T(moderator) = $\underline{\hspace{2cm}}$ $^\circ\text{C}$ = $\underline{\hspace{2cm}}$ $^\circ\text{K}$

VI. ANALYSIS AND CONCLUSIONS

This section is to be done by the student. However, the following facts, besides the basic assumptions, should be considered before the analysis:

The neutron beam at the AGN-201 reactor was produced by inserting a polystyrene rod midway into the glory hole.

The B^{10}F_3 detector used may not have been of a $1/v$ sensitivity, depending on its length, pressure and orientation. If it was positioned with its axis parallel to the beam, its sensitivity is somewhere between a $1/v$ and that of a black detector, depending on its gas pressure and length.

There may have been impurities in the B sample; the boron was mounted in polystyrene.

Any other factors which may have effected the results should also be discussed.

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6. (AGN-201) Danger Coefficient Experiment

Experiment Written by: J. Spalek

VIII. APPENDICES1. Questions

1. Explain why the thermal neutron temperature is expected to be higher than the moderator temperature.
2. Explain what is meant by a black absorber.
3. Measuring the average cross section in the beam, $(\bar{\sigma})$, and using equation (12.1) to calculate v_0 , the most probable velocity, to which distribution (that is, the velocity distribution, or flux distribution) does this velocity belong?
4. If the detector used in the beam method of measuring were black instead of $1/v$, how would the temperature of the neutrons in the core be calculated?
5. Discuss how the variation of temperature would be detected using a $1/v$ and a black absorber, from the viewpoint of cross sections. Do the same from the viewpoint of neutron absorption rate and reactivity.
6. Discuss the possible deviations of the experimental setup from the theory (from basic assumptions like Maxwell distributions, isotropic flux, equilibrium conditions), and their causes, for the first and second method.
7. Why does the first method give the cross section at the average velocity $(\bar{\sigma})$ and not at some other velocity (say at the most probable velocity σv_0 , as in the second method)?
8. In what particular case only is the total cross section, as measured by the transmission method by passing the whole spectrum through the sample, equal to the cross section at the most probable velocity?

2. Derivation of the Number of Neutrons of All Velocities Striking
or Crossing A Surface Per Unit Area Per Unit Time

Assumptions:

1. All directions of neutron velocities are equally probable (isotropic flux).
2. Large number of neutrons in a container ("neutron gas"), exerting no forces on one another except during collision.

Assumption (1) can be put into analytic form by assigning to each neutron a vector representing the magnitude and direction of its velocity,

and transferring all the vectors to the origin of a sphere of arbitrary radius . The points of intersection of the velocity vectors (prolonged if necessary) with the surface of the sphere will be uniformly distributed due to assumption (). The number of these points (which represent the number of neutrons N) per unit area is therefore

$$\frac{N}{4\pi r^2} \quad (1)$$

The number of points in any element of area dA is then $dN = \frac{N}{4\pi r^2} dA$.

Using spherical coordinates, $dA = r^2 \sin \theta d\theta d\psi$, so that the number of points in area dA , which represents the number of neutrons having velocities in a direction between θ and $\theta + d\theta$, and $\psi + d\psi$, (as is evident from Figure 1), call it $dN_{\theta\psi}$, is

$$dN_{\theta\psi} = \frac{N}{4\pi r^2} r^2 \sin \theta d\theta d\psi = \frac{N}{4\pi} \sin \theta d\theta d\psi, \quad (2)$$

Figure 1

or the number of neutrons per unit volume with velocities in directions θ and $\theta + d\theta$, ψ and $\psi + d\psi$, (call it $dn_{\theta\psi}$), is

$$dn_{\theta\psi} = \frac{dn_{\theta\psi}}{V} = \frac{n}{4\pi} \sin \theta d\theta d\psi$$

But these neutrons have different velocities (not all the velocity vectors terminate on the sphere).

Let the number of neutrons per unit volume with speeds between v and $v + dv$ (in Figure 1 it is represented by the number of velocity vectors

terminating in a thin shell of radius $r = v$ and thickness dv) be represented by dn_v . Therefore, the number of neutrons per unit volume traveling with speeds between v and $v + dv$ in directions between θ and $\theta + d\theta$, ψ and $\psi + d\psi$, symbolized $dn_{\theta\psi v}$, is

$$dn_{\theta\psi v} = \frac{1}{4\pi} dn_v \sin \theta d\theta d\psi \quad (3)$$

How many of the neutrons with these particular velocities specified above will strike a surface/unit area/time? As many as there are in a volume element of length vdt and area dA , inclined in the directions θ and ψ . (See Figure 2.)

Figure 2

The volume of the element dV is $dV = dA \cdot vdt \cos \theta$, the number of neutrons in this element dV is

$$\begin{aligned} dn_{\theta\psi v} \cdot dV &= \frac{1}{4\pi} dn_v \sin \theta d\theta d\psi \cdot dA vdt \cos \theta \\ &= \frac{1}{4\pi} v dn_v \sin \theta \cos \theta d\theta d\psi dt dA \quad , \end{aligned} \quad (4)$$

or, the number of neutrons striking a surface per unit area per unit time, in the specified directions and with speed v is

$$\frac{dn_{\theta\psi v}}{dA dt} = \frac{1}{4\pi} v dn_v \sin \theta \cos \theta d\theta d\psi \quad (5)$$

Then the total number of neutrons striking a surface from all directions per unit area per unit time with speed v is

$$\frac{dn_v}{dA dt} = \frac{v dn_v}{4\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta d\psi = 1/4 v dn_v \quad (6)$$

The total number of neutrons of all velocities colliding with the surface $/dA/dt$

$$\frac{1}{4} \int v dn_v \quad (7)$$

But the average speed $\bar{v}$ of neutrons in any distribution dn_v is

$$\bar{v} = \frac{\int v dn_v}{n} \quad , \quad (8)$$

therefore, $\int v dn_v = n \bar{v}$, and the result of what was to be derived, that is, the total number of neutrons of all velocities striking a surface per unit area per unit time is

$$\frac{1}{4} (n \bar{v}) \quad (9)$$

The last equation was derived irrespective of what the velocity distribution dn_v of the neutrons is.

R. L. I. EXPERIMENT NUMBER 13

MEASUREMENT OF THE TRANSFER FUNCTION OF THE ARGONAUT

INSTRUCTOR: G. S. PAWLICKI

This document is not included here. It is available upon special request from Dr. G. S. Pawlicki, Argonne National Laboratory, Argonne, Illinois.

Laboratory Report on
MEASUREMENT OF THE TRANSFER FUNCTION OF THE
ARGONAUT REACTOR

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Experiment performed on July 27, 1961 during ASEE-AEC Reactor Laboratory Institute by T. R. Rehm, K. Rothschild, H. A. Shadduck, L. C. Wilbur, W. E. Wilson, Jr..

Introduction

The transfer function of the Argonaut reactor was measured by the pile oscillation technique utilizing the Argonaut pile oscillator designed by Dr. G. S. Pawlicki, et al. The theory behind this method of measuring a reactor's transfer function along with a description of the Argonaut pile oscillator and the experimental procedures used are given in the syllabus for this experiment. Consequently, this information will not be repeated and only the experimental results will be discussed.

Results

1. Worth of Oscillator

The pile oscillator was rotated to the 90 degree position and the reactor control rods adjusted to yield a constant power level of approximately 100 watts. The oscillator was then jogged to the 270 degree position and the stable reactor period measured. The measured period was 108 seconds, which yields a maximum worth, i.e., peak to peak of the reactivity sine wave, of the pile oscillator of 0.084\$ or 0.063 $\Delta k/k$.

2. Argonaut Transfer Function

The gain and phase characteristics of the Argonaut reactor were measured in the interval 0.08 to 30 C.P.S. using the Argonaut pile oscillator and the associated electronic equipment shown in Figure 1 of the syllabus. The measurements were taken starting at 30 C.P.S. and working downward in logarithmic frequency steps with the pile oscillator rotating in opposite directions for every other data point. The resultant gain data was normalized to zero db gain at 0.5 C.P.S. and used to construct the Bode gain plot shown in Figure 2. The phase data was used to construct the Bode phase

plot shown in Figure 3. The curve midway between the curve for forward and reverse oscillator rotations is the experimental phase shift curve for the Argonaut.

3. Mean Effective Neutron Lifetime

The solid curve on the Bode gain plot shown in Figure 2 is the theoretical transfer function of a U-235 fueled reactor with a neutron lifetime of 160 micro-seconds. The theoretical curve fits the experimental data quite well and thus it is inferred that the lifetime of neutrons in the Argonaut is 160 micro-seconds. This result does not compare very favorably with the lifetime of 200 micro-seconds measured by Dr. Pawlicki. However, it does compare very favorably with the results of other groups in the Reactor Laboratory Institute. No explanation for the comparably low values of neutron lifetime is readily apparent.

The theoretical phase shift curve for a neutron lifetime of 160 micro-seconds compares reasonably well with the experimental curve shown in Figure 3. The difference between the theoretical and experimental curves at low frequencies may be explained by the fact that measurements in this region are much less accurate than those at higher frequencies. This error is caused by difficulties in adjusting the oscilloscope pattern to indicate the null position at very low frequencies.

At the high frequency end of the measured transfer function, errors introduced by the electronics become important and could very easily be the cause of the difference between the theoretical and experimental phase shift in this region.

TRANSFER FUNCTION		EXPERIMENTAL DATA	
FREQUENCY (cps)	PHASE ANGLE IN DEGREES		GAIN in db
	ϕ_{f+s} δ ϕ_r	ϕ_{f+s} δ ϕ_r	(30cps=0) (0.5cps=0)
0.030		40 0 40	16.0 +3.5
0.065	10 0 10		14.5 +2.0
0.080		36 0 36	14.0 +1.5
0.10	4 0 4		13.5 +1.0
0.125	5 0 5		13.0 +0.5
0.16		22 0 22	12.5 +0
0.22	0 0 0		13.0 +0.5
0.25		22 0 22	12.5 +0
0.33	0 0 0		12.5 +0
0.40		22 0 22	12.5 +0
0.50	-2 0 -2		12.0 -0.5
0.63		22 0 22	12.5 -0
0.80	-1 0 -1		12.0 -0.5
1.00		25 0.5 24.5	12.0 -0.5
1.25	2 0.5 1.5		12.0 -0.5
1.6		27 0.5 26.5	11.5 -1.0
2.0	7 0.6 6.4		11.5 -1.0
2.5		36 0.7 35.3	11.0 -1.5
3.2	16 1 15		11.0 -1.5
4.0		45 1.4 43.6	10.5 -2.0
5.0	28 1.8 26.2		10.0 -2.5
6.3		58 2.3 55.7	9.5 -4.0
8.0	42 3 39		7.5 -5.0
10.0		72 3.5 68.5	6.0 -6.5
12.5	59 4.5 54.5		4.5 -8.0
16		87 5.7 81.3	2.5 -10.5
20	74 7.0 67		0 0 -12.5
30		112 10.6 101.4	
30	84 10.6 73.4		

ϕ_f = PHASE ANGLE WITH FORWARD OSCILLATOR ROTATION
 ϕ_r = PHASE ANGLE WITH REVERSE OSCILLATOR ROTATION
 δ = LOW PASS FILTER PHASE SHIFT CORRECTION

K₂E SEMI LOGARITHMIC 359 B1
REPRESENTATIVE
CIRCUIT

K₂E SEMI LOGARITHMIC 359 B1
REPRESENTATIVE
CIRCUIT

VIII. APPENDICES1. 1961 ASEE-AEC Reactor Laboratory Institute Writeup

June, 1961

APPLICATIONS OF TRANSFER FUNCTION MEASUREMENTS
TO REACTOR DESIGN AND OPERATION

by

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INTRODUCTION

The ultimate purpose for the construction and operation of a nuclear reactor is to provide a useful source of energy in the form of heat and/or nuclear radiations. In order to be useful, a nuclear reactor must be controllable under a set of known conditions and circumstances. One of the most important aspects of a reactor's controllability that must be known to insure safe operation of a reactor is the reactor's dynamic response to changes in operating conditions. Consequently, the analysis of the dynamic behavior of a reactor and reactor power plants is of the utmost importance in both the design and the operation of nuclear reactors and nuclear reactor power plants.

The most frequently employed method of analyzing the dynamic response of a system is to observe the system's response in terms of gain and phase angle between input and output for a sinusoidal varying input. This response information is then utilized to plot the observed transfer characteristic or transfer function of the system. The observed transfer function may then be analyzed to yield information about the stability of the system or may be compared with a theoretically determined transfer function. In the latter case, the comparison between an observed and a theoretical transfer function tests the accuracy with which the theoretical model chosen describes the actual system.

OPEN LOOP TRANSFER FUNCTION

The transfer function of a nuclear reactor may be defined as the ratio of the Laplace transforms of the complex variables that describe the

reactor's output flux variation to those describing the reactor's reactivity inputs for a sinusoidal varying input reactivity. That is,

$$R(s) = \frac{\mathcal{L}(\text{Output Flux Variation})}{\mathcal{L}(\text{Input Reactivity Variation})} = \frac{\phi(s)}{\Delta K(s)} \quad (1)$$

where $R(s)$ is the transfer function, $\phi(s)/\Delta K(s)$ is the Laplace transform of the function that describes all of the input reactivity variations. The term, transfer function, is derived from the fact that the transform of the output response of a system is equal to the transform of input times the transfer function of the system. Equation (2) and Figure 1 depict what

$$\phi(s) = \Delta K(s) \cdot R(s) \quad (2)$$

is called an open loop transfer function.

Figure 1
Block Diagram of an Open Loop Transfer Function

The open loop or zero power transfer function of a nuclear reactor is the transfer function of the reactor based only on delayed neutron effects and neglecting all feedback effects. This type of transfer function is easily measured by means of pile oscillation techniques and may be theoretically formulated quite simply from the space independent reactor kinetic equations (1). The practical utility of an experimental measurement of a reactor's open loop transfer function is that such a measurement yields information about the reactor's delayed neutron characteristics. Specifically, the ratios $\Sigma a_i / \lambda_i$ and $\Sigma a_i \beta_i / \ell^*$ may be evaluated (2), where a_i is the relative abundance of the i^{th} group of delayed neutrons, β is the fraction of the delayed neutrons of the i^{th} group per neutron emitted per fission, λ_i the decay constant of the i^{th} group of delayed neutrons, and ℓ^* is the means effective lifetime of neutrons in the reactor.

CLOSED LOOP TRANSFER FUNCTION

The closed loop or power transfer function of a nuclear reactor is the complete transfer function of a reactor including the delayed neutron effects as well as all feedback effects. A simple closed loop transfer function of a reactor with one feedback loop is depicted in Figure 2.

Figure 2
Block Diagram of One Feedback Loop Transfer Function

Mathematically, the transfer function of Figure 2 is given by

$$R(s) = \frac{\phi(s)}{\Delta K(s)} = \frac{K_1 G(s)}{1 + K_1 G(s) \cdot K_2 H(s)} \quad (3)$$

where K_1 is the frequency independent portion of the reactor power function $K_1 G(s)$ and K_2 is the frequency independent portion of power-to-reactivity feedback function $K_2 H(s)$.

The closed loop transfer function of a reactor describes the dynamic behavior of a reactor and its associated equipment and, consequently, is of more practical importance to reactor design and operation than a simple open loop transfer function. However, the closed loop transfer function of a large reactor and its associated equipment, power plant, is very complex and more difficult to theoretically formulate due to the presence of numerous feedback loops. The number and arrangement of these loops is dependent upon the particular type of reactor, the amount and arrangement of the reactor's associated equipment, and the analytical model chosen to represent the system. The adequacy with which the model represents the reactor system's power transfer function can only be effectively evaluated by actually measuring the closed loop transfer function and comparing the experimental and theoretical results (3).

REACTOR STABILITY FROM TRANSFER FUNCTION ANALYSIS

The procedure for obtaining and analyzing the theoretical transfer function of a nuclear reactor is well documented in the literature (1, 2, 3, 4 and 5) and may be generalized as follows:

1. Determine and write down the time-dependent differential equations that describe the reactor's kinetics, hydraulics, heat transfer, and reactivity feedback loops.

2. Linearize these equations by assuming small variations about a steady state condition.
3. Take the Laplace transforms of these linearized equations, converting the set of simultaneous differential equations to a set of algebraic equations in which the "unknowns" are the Laplace transforms of the time-dependent variations of the neutron flux, reactor temperature, etc.
4. Combine the algebraic equations and eliminate all transforms except those of the output flux variations, $\phi(s)$, and the input reactivity $\Delta K(s)$. Obtain the ratio $\phi(s) / \Delta K(s)$ which is the transfer function of the nuclear reactor described by the initial set of differential equations.
5. Determine the roots of the denominator of the transfer function, the p_i , and see if any has a positive real part. That is, the transfer function may be represented by

$$R(s) = \frac{\phi(s)}{\Delta K(s)} = \frac{Z(s)}{P(s)} = \frac{\pi_i(s - z_i)}{\pi_i(s - p_i)} \quad (4)$$

where $Z(s)$ and $P(s)$ are polynomials in s and where $P(s)$ is of higher order. Under these conditions $Z(s)$ and $P(s)$ may be factored into the product of terms containing $(s - z_i)$ and $(s - p_i)$ respectively. The z_i are the roots of $Z(s) = 0$ and are the zeros of the transfer function, and the p_i are the roots of $P(s) = 0$ and are the poles of the transfer function.

6. Determine the region of stability of the reactor. The reactor is stable in the region of operation in which the denominator of the transfer function does not have any zeros with positive real parts, ie, positive real p_i 's.
7. Plot the transfer function by one or more of the standard methods and investigate the boundaries of the regions of stability and instability.

TRANSFER FUNCTION PLOTS

Unfortunately, no completely satisfactory method is available for plotting on a single sheet of paper all of the information obtainable from a transfer function. Consequently, more than one of the many methods that have been devised to display particular characteristics of a transfer function must be employed to obtain a maximum amount of information. The three most useful types of plots are the Bode plot, the Nyquist plot, and the Root-locus plot.

Bode Plot

The Bode or log-amplitude plot is the least complicated and that which is most frequently encountered and is simply a plot of the amplitude and of the phase log of the transfer function versus frequency. The amplitude is plotted in units of decibels gain, 20 times the logarithm to the base 10 of the absolute value of the gain, of the transfer function against the frequency in radians per second. The phase lag between the input reactivity and output flux variation is plotted in units of degrees against the frequency in radians per second.

The Bode plot is most useful in displaying experimental results in a form from which an approximate transfer function can readily be derived. The Bode plot is also useful for comparing experimentally and theoretically determined transfer function data. The Bode plot may also be utilized to determine the stability of a transfer function in the vicinity of zero gain. That is, if the slope of the transfer function exceeds 40 db per decade as the transfer function crosses the zero db axis, the system is unstable.

Nyquist Plot

The Nyquist plot is the oldest of the transfer function graphical representations and employs the polar coordinate system. In the Nyquist plot the amplitude of the transfer function is denoted by the radius and the phase lag by the angle. This type of plot is constructed for a single set of reactor parameters by evaluating the transfer function for different values of driving frequency and plotting the resultant data on a sheet of polar graph paper. The data points are then connected by an appropriately shaped line and the plot examined for possible instabilities. If the plot encircles the point $-1, 0$, the system is unstable and will oscillate.

Root-Locus Plot

The root-locus plot, as the name implies, focuses attention on the roots of the denominator of the transfer function. In this method the roots of the denominator of the transfer function are evaluated for various values of one parameter and the points plotted on a graph in the complex plane. A line is then drawn connecting the various values of each root thereby tracing out the locus of each root in the complex plane. The threshold of stability occurs at the point where one of the loci of the roots crosses the imaginary axis, i.e. from all purely imaginary roots to at least one root with a positive real part.

APPLICATIONS OF TRANSFER FUNCTION ANALYSIS

In general, reactor transfer function analysis may be utilized for general reactor stability analysis, investigating particular types of reactor

instabilities, and studying the various feedback loops in a complete reactor system such as a nuclear power plant. More specifically, each of these three general types of applications may be subdivided into the following applications:

1. Applications to General Reactor Stability Analysis

- a. Reactor Design - Adjustment in the size and arrangement of the various components and parameters of a reactor and associated equipment that maximize stability.
- b. Control and Safety System Design - Determining the maximum rate of control rod withdrawal and the required response times of safety circuits for reasonably safe operation and for protection against plausible accidents.
- c. Determining Operating Procedures and Safety Limits - Evaluation of the limits of the reactor system's stability and the maximum rate of change of conditions for safe operation.

2. Applications to Specific Types of Instability Analysis

- a. Divergent - Studying the conditions and reactor parameter values that produce or could produce a prompt positive reactivity coefficient.
- b. Convergent - Studying the conditions and reactor parameter values that produce or could produce a prompt negative reactivity coefficient.
- c. Oscillatory in Time - Studying the conditions, reactor parameter values, and feedback mechanisms that produce oscillations in the reactor total power level.
- d. Oscillatory in Time and Space - Studying the conditions, reactor parameter values, and feedback mechanisms that produce oscillations in the specific power distribution in a reactor.

3. Applications to Specific Reactivity Feedback Mechanisms

- a. Temperature Coefficient of Reactivity - Studying the effect of various reactor parameters and reactor operating conditions on the various components of the reactor's temperature coefficient.
 1. Doppler broadening of fuel resonances
 2. Change in surface resonance absorption
 3. Change in thermal utilization due to density changes
 4. Change in neutron spectrum
 5. Change in nonleakage probability
- b. Void Coefficient of Reactivity - Studying the effect of various reactor parameters and reactor operating conditions on the various components of the reactor's void coefficient.
 1. Power Void effects
 2. Boundary Void effects
 3. Flashing effects
 4. Evolution of radiolytic gases

- c. Fission Product Poisoning - Studying the reactivity effects of the production of fission products that poison the reactor.
 - 1. Xenon
 - 2. Samarium
- d. Mechanical Motion of Core Components - Studying the reactivity effects by changes in heating rate, heat transfer rate, and coolant hydrodynamics that give rise to mechanical motion of various components of the core of a reactor.
- e. Moderator Flow Pattern - Studying the reactivity effects of changes in the reactor's moderator flow pattern.
- f. Automatic Control System - Studying the effects of an automatic reactor control system on the reactor's transfer function during automatic operation.

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R. L. I. EXPERIMENT NUMBER 14

ARGONAUT AUTOMATIC FLUX CONTROLLER

INSTRUCTOR: G. S. PAWLICKI

The descriptive text for this laboratory experiment is:

ARGONAUT AUTOMATIC FLUX CONTROLLER DESIGN REPORT,
A. Gerba, Jr., ANL-6110, 1960

This document is not included here. It is available through the Office of Technical Services, U. S. Department of Commerce, Washington 25, D.C.

R. L. I. EXPERIMENT NUMBER 15

MEASUREMENT OF THERMAL NEUTRON DIFFUSION LENGTH IN H_2O

INSTRUCTOR: L. L. LAWYER

ABSTRACT

This experiment is a measurement of thermal neutron diffusion length in H_2O . This measurement utilizes reactor leakage neutrons as a plane source, Indium foils as thermal neutron detectors, and cadmium absorption coupled with the Indium resonance as an epi-thermal neutron detector. The thermal neutron diffusion length is evaluated from the parameters of a cadmium covered Indium distribution.

MEASUREMENT OF THERMAL NEUTRON DIFFUSION LENGTH IN H₂O

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - 1. General
 - 2. Infinite plane source in infinite medium
 - 3. Physical significance of diffusion length and relation to relaxation length
 - 4. Effect of finite dimensions
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS OF RESULTS AND CONCLUSIONS
- VIII. APPENDICES
 - 1. Discussion Questions

I. INTRODUCTION

A neutron life cycle in a thermal reactor moderator or reflector may be divided on an energy basis into two main categories -- thermal and epithermal. The distance a neutron travels while its energy is in the thermal portion of this cycle is of prime importance. The thermal neutron diffusion length is proportional to this distance.

Due, among other things, to the widespread usage of H₂O as a moderator and reflector in current reactors, we have chosen it as the diffusing medium.

Experimental methods of measuring thermal neutron diffusion length can be separated into two main divisions. The first is the spatial variation of neutron flux from a steady source and the second is the time variation of neutron flux following a pulse of neutrons. The former method will be employed in this experiment.

II. THEORY

1. General:

The general diffusion equation states

$$D\nabla^2\phi - \Sigma_a\phi + S = \frac{\partial n}{\partial t}$$

where $-D\nabla^2\phi$ is the neutron leakage per unit volume per second, $\Sigma_a\phi$ is the neutron absorption per cm^3 per second, and S is the rate of neutron production.

We concern ourselves with the special case in which the system is in steady state and the time rate of change of neutron density is zero. In addition, we will restrict ourselves to the region outside the source ($S = 0$). Under these conditions, equation (1) becomes

$$\text{or} \quad D\nabla^2\phi - \Sigma_a\phi = 0$$

$$\nabla^2\phi - K^2\phi = 0 \quad (\text{wave equation})$$

where $K^2 = \frac{\Sigma_a}{D}$ and has units of reciprocal length squared.

2. Infinite plane source in infinite medium:

Assuming that the source is infinite in the x and y plane, then at any given value of z from the source, the flux will be independent of x and y so that ∇^2 will be $\frac{d^2}{dz^2}$ and the diffusion equation can be written

$$\frac{d^2\phi}{dz^2} - K^2\phi = 0 \quad (2)$$

with the boundary conditions

1. The flux must be finite except at $z = 0$.

2. The net current density at the source is $1/2$ the source strength in neutrons/ cm^2/sec . The general solution of (2) can then be written

$$\phi = A e^{-Kz} + C e^{Kz} \quad (3)$$

By imposing the above boundary conditions, C and A can be evaluated. This leads to $C = 0$ and $A = \frac{1}{2KD}$. Introducing these results, (3) becomes

$$\phi = \frac{e^{-Kz}}{2KD} \quad (4)$$

which describes the steady state flux distribution in the z direction from an infinite plane source emitting 1 neutron per cm^2 per second into an infinite medium. We now define the thermal neutron diffusion length L as

$$L \equiv \frac{1}{K} = \sqrt{\frac{D}{\Sigma_a}} \quad (5)$$

3. Physical significance of diffusion length and relation to relaxation length:

To give this quantity physical significance, we consider the mean square distance traveled by a neutron from the point at which it just becomes thermal to the point at which it is absorbed. This is just the second spatial moment of the flux distribution given by (4).

If ϕ is the flux in neutrons per cm^2 per second at a distance z from the plane source, then the rate at which neutrons will be absorbed in the medium will be equal to $\Sigma_a \phi$ per centimeter cubed per second, where Σ_a is the macroscopic absorption cross section of the medium. In an element of area A , thickness dz at a distance z from the plane source, the number of neutrons captured per second will be $\Sigma_a \phi A dz$. This is a measure of the probability that a neutron will be absorbed within the element. Therefore, the mean square distance that a neutron travels from its source to where it is absorbed is given by

$$\overline{z^2} = \frac{\int_0^\infty z^2 \Sigma_a \phi A dz}{\int_0^\infty \Sigma_a \phi A dz} \quad (6)$$

substituting the spatial relationship for the flux as given by (4) we have

$$\frac{\int_0^\infty z^2 \Sigma_a \frac{e^{-Kz}}{2KD} A dz}{\int_0^\infty \Sigma_a \frac{e^{-Kz}}{2KD} A dz} = \frac{\frac{2}{K^3}}{\frac{1}{K}} = \frac{2}{K^2} \quad (7)$$

Therefore

$$L^2 = \frac{1}{K^2} = \frac{1}{2} \overline{z^2} \quad (8)$$

Also since $\frac{1}{L} = K$ equation (4) may be written in the form

$$\phi = A e^{-\frac{z}{L}} \quad (9)$$

in which form we see that the diffusion length is equivalent to the "relaxation length," i.e. the distance in which the neutron flux is decreased by a factor of e.

4. Effect of finite dimensions:

If it were possible for us to experimentally duplicate an infinite plane source in an infinite medium, we could determine the thermal neutron diffusion length from a set of measurements of the spatial variation of the neutron flux. From equation (9), we see that $\frac{d \ln \phi}{dz} = -\frac{1}{L}$ and therefore a log plot of the spatial variation of the neutron flux from an infinite plane source into an infinite medium against z, the distance from the source, should yield a straight line of slope $-\frac{1}{L}$. However, practical measurements must be made with a finite source in a finite medium, and the analysis of the results is therefore not as simple as indicated by our consideration of equation (9).

In a manner very similar to that in which we derived equation (4), it is shown in Reference (3) that for thermal neutrons diffusing into a long rectangular parallelepiped of moderator, the flux distribution along any line in the z direction that is not too close to a boundary is given by

$$\phi = \text{const.} \times e^{-\gamma z} \quad (10)$$

where γ is related to the dimensions of the block by

$$\gamma^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \frac{1}{L^2} \quad (11)$$

where a and b are the dimensions, including twice the extrapolation distance, in the plane of the plane thermal neutron source. In order to determine the effect of finite dimensions on the measurement of L^2 without reproducing the rather lengthy derivation required to obtain equations (10) and (11), let us examine the comparatively simple case of a single finite dimension.

If the neutrons from an infinite plane source diffuse into a slab of infinite extent but of finite thickness, by the same arguments used in the finite dimension case we find the general solution given by equation (3) to be applicable. To evaluate the arbitrary constants A and C it is necessary to introduce an additional boundary condition that the flux shall vanish at the

extrapolated boundary. If a is the finite dimension of the slab including this extrapolation distance then

$$\begin{aligned}\phi_a &= A e^{-Ka} + C e^{Ka} = 0 \\ C &= -A e^{-2Ka}\end{aligned}\tag{12}$$

and equation (3) becomes

$$\phi = A \left[e^{-Kz} - e^{-K(2a-z)} \right]\tag{13}$$

Using the boundary condition on the neutron current density as before, we evaluate A to be

$$\frac{1}{2KD(1 + e^{-2Ka})}$$

hence

$$\phi = \frac{e^{-Kz} - e^{-K(2a-z)}}{2KD(1 + e^{-2Ka})}\tag{14}$$

If, in equation (14) we let a be very large, then we see that equation (14) becomes identical, as we would expect, with equation (4). Of more importance to us at this time, however, is determining an approximate value for a which is sufficiently large to make equation (14) essentially equal to equation (4). Let us select the dimension a such that $Ka = 3$, then equation (14) is

$$\phi = \frac{e^{-Kz} - e^{-6 + 3\frac{z}{a}}}{2KD(1 + e^{-6})}$$

Since we know $z \leq a$, let us choose for comparison a z of one diffusion length, i.e., $z = L = \frac{1}{K}$. Then equation (14) becomes

$$\phi = \frac{e^{-1} - e^{-5}}{2KD(1.0025)} = \frac{.368 - (0.0067)}{2KD(1.0025)}$$

and the correction terms, in parenthesis, are seen to be very small for $Ka = 3$. Thus, when the thickness of the medium, including the extrapolation distance, is about $\frac{3}{K}$, or more, the behavior of the flux at $\frac{1}{K}$ from the source, is essentially that of an infinite medium. It is generally true that if the thickness is at least three times the diffusion length, it may be treated as an infinite medium at distances greater than one diffusion length from the boundary.

We may explain the effects we have seen as follows: If the thickness is three or more times the diffusion length, most of the neutrons are scattered back before they reach the boundary and the leakage is very small.

Referring to equations (10) and (11), which are the working equations of this experiment, and employing intuition in lieu of mathematical rigor, we note that the only essential change as compared to the infinite source infinite medium case is the replacement of K by γ . Also we note that the expression for γ^2 equation (11) contains, in addition to K^2 , two terms which we will term the "leakage terms," which are identical to the geometrical buckling if we consider leakage in only two dimensions (since we measure along the third dimension).

III. APPARATUS

The apparatus for this experiment consists of neutron detectors, a diffusion box, and a large volume of H_2O with a source of thermal neutrons. The general method employed is that of the spatial variation of neutron flux from a steady source.

The neutron detectors are Indium foils, (see photograph). The diffusion box consists of a 60.9 cm x 60.9 cm x 60.9 cm Al frame covered with 20 mil Cadmium. This box has provision for 24 foils on its center line perpendicular to the front face and for 7 foils through the same centerline but parallel to the front face. The Cd cover on the front face is removable. There are provisions on the box for suspending it in a known location in the shield tank facility.

The H_2O volume required is provided by filling the shield tank facility of the Argonaut with H_2O and suspending the diffusion box in this medium. The Argonaut's shield tank facility is approximately 4' square on the face next to the reactor and 5' deep.

IV. PROCEDURE

Mount the Indium foils on the foil holder for the horizontal distribution. These foils will be used to determine the spatial variation of neutron flux inside the diffusion box. If deemed necessary, an additional set of seven foils may be distributed parallel to the face of the diffusion box. This distribution would be used to correct the physical dimensions of the diffusion box to its extrapolated boundaries. Two Al foils, one Cd covered and one bare, may be placed on the front face of the shielding tank facility as minitor and Cd ratio detectors.

The diffusion box should then be suspended in the shield tank facility with its front face parallel to the front face of the shield tank facility.

By operating the reactor at a suitable power level, the foils may be irradiated to activities proportional to the spatial distribution of neutron flux within the H_2O in the shielding facility.

Using the gas flow-end window counters provided, count the activities of the irradiated foils and compute their specific activity. The data sheet included will serve as a guide.

Name

Irradiation Period

Date

Reactor Power

 $(1 - e^{-\lambda\tau})$

Background

1 Foil Loc.	2 Act.	3 Act. -Bg.	4 Time	5 $e^{-\lambda t}$	6 Wt.	7 $1 - e^{-\lambda\tau}$	8 Saturated Activity

V. RESULTS

Using the data from IV above, make a plot of the spatial variation of the thermal neutron flux as a function of distance from the front face of the diffusion box. The slope of the plot on semi-log paper of the measured flux as a function of distance along the z direction yields the constant γ , from which L may be computed. If applicable, plot the transverse thermal neutron flux using the appropriate data. Also, it may prove useful to compute and list the absolute flux as determined by the Au foil irradiation along with the Au Cd ratio.

VI. ANALYSIS OF RESULTS AND CONCLUSIONS

This determination of the thermal diffusion length can be expected to agree with the published value ⁽²⁾ of 2.85 cm to approximately $\begin{matrix} +0.0\% \\ -8.0\% \end{matrix}$

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Experiment Written By: L. L. Lawyer

VIII. APPENDICES1. Discussion Questions

The following propositions should be considered prior to your arrival at the Argonaut. A discussion of them will be considered a part of this experiment.

1. What is the physical meaning of neutron diffusion length?
2. What facets of reactor design are primarily affected by the value of diffusion length?
3. What background corrections do you expect to make in this experiment and what is their need?
4. Explain the general physical meaning of L^2 & M^2 .
5. In what direction would our value of L^2 be affected by each of the following:
 1. Impurities whose only appreciable cross section is that for absorption.
 2. The foil distribution at an angle other than 90° to the plane of the plane thermal neutron source.
6. A Cd covered box 2' x 2' x 2' is available for measuring L^2 . In order to determine the In foil activation due to the fast neutrons present, it is suggested that we cover the foils with Cd. Discuss the merits of covering the entire box with Cd as compared to covering each individual foil with Cd.
7. From your knowledge of the thermal neutron diffusion length, calculate roughly the effect of the 2' dimension on L^2 . Will a box 2' x 2' be large enough?

2. 1961 ASEE-AEC Reactor Laboratory Institute WriteupMEASUREMENT OF DIFFUSION LENGTHS BY PULSED
NEUTRON TECHNIQUES

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The recent appearance on the market of numerous models of pulsed neutron sources may be partly due to their suitability for determining nuclear properties² such as absorption cross sections, diffusion coefficients, and diffusion lengths in moderators. Many of these sources employ the H^3 (d,n) He^4 reaction to obtain beams of neutrons from pulsed deuteron accelerators. Others⁶ have used pulsed accelerators to produce neutrons from a uranium slab by γ , n and γ , f reactions. Some systems have repetition rates which are widely variable, while the less expensive models generally operate in the range of a few cycles per second.

The experimental method used usually consists of irradiating one face or end of a convenient geometric shape such as a cylinder or parallelepiped which is shielded with a neutron absorber to prevent backscatter. The neutron density as a function of time is determined by neutron counters such as BF_3 tubes or LiI^4 crystals placed in the material or along a side adjacent to the face being irradiated. When placed in the side position, there is less disturbance of the flux due to the counter. Figure 1 shows the experimental method used by von Dardel and Sjostrand⁷ for water measurements in cylinders of 14.5 and 18 cm diameter filled to heights ranging from 3.5 to 15 cm.

Figure 1
Experimental Apparatus Used by von Dardel⁷

Small sizes of the moderator are advantageous in reducing the amplitude of harmonic modes but do give greater leakage of fast neutrons. This leakage of fast neutrons is called a neutron cooling effect for which temperature

corrections have been given.^{3,7} However, Campbell and Stelson⁴ found no evidence of curvature in their data due to diffusion cooling.

The decay of the thermal neutron intensity following injection of a pulse of fast neutrons in a finite moderator may be expressed in terms of simple diffusion theory by the relation

$$1/t = \lambda = v \Sigma_a + DB^2$$

where v is the thermal neutron velocity

Σ_a is the thermal neutron absorption cross section

B^2 is the geometric buckling calculated with extrapolated distances

D is the diffusion constant for thermal neutrons and is related to the transport mean free path by the expression $D = \lambda_t v/3$

From a plot of $1/t$ versus B^2 , Σ_a and D may be obtained as the intercept the slope, respectively. The diffusion length, L , may be calculated from the relation

$$L^2 = \frac{\lambda_t \lambda_a}{3} = \frac{D}{v \Sigma_a}$$

or obtained as $1/L^2$ at the extrapolated intercept on the buckling axis.

Multichannel time analyzers are used to obtain thermal neutron densities as a function of time. Counting rates plotted on logarithmic paper as a function of time should yield a straight line after harmonic modes and fast neutrons have decayed. The straight portion of these graphs is used to calculate the mean lifetime of thermal neutrons and the reciprocal of the mean lifetime is plotted against the buckling for different moderator sizes, giving a correlation similar to Figure 2.

Figure 2

Decay Constants Versus Buckling for H_2O Cylinders as Measured by Campbell and Stelson⁴ Using $Li^6I(Eu)$ Scintillation Counter

Since the fast neutrons escape soon after the pulse of source neutrons, only slow neutron densities are counted in the fundamental decay mode.

As mentioned earlier, some investigators have found the data drop below a linear relation at higher values of B^2 . This has been attributed to diffusion cooling resulting from excessive leakage of fast neutrons in small moderator sizes.

Beckhurts³ corrects for cooling effects by using the factor CB^2 in the relation

$$\lambda = \Sigma_a v + [D - CB^2] B^2$$

in which he assumes Maxwellian distribution. This gives an average energy of

$$\overline{E_a} = 3/2 kT + KT^2 \frac{d \ln D}{dT}$$

With the heat transferred from the moderator given as

$$-dE/dT = 3/2 K \gamma (T - T_0)$$

the equilibrium condition is

$$3/2 K \gamma (T_0 - T) = B^2 kT^2 (dD/dT)$$

As a first approximation

$$D = D(T_0) - CB^2$$

where

$$C = \frac{2 T_0^2 (dD/dT)^2}{3 \gamma}$$

Assuming λ_{tr} in graphite to be independent of temperature he obtains

$$(dD/dT) = D/2T_0 \text{ and } C = D^2/6\gamma$$

Antonov¹ also discusses the cooling effects and reports $C/D_0 \sim_s \lambda_t \lambda$ for H_2O , Be and graphite. However, as he points out the tangent to the $1/t-B^2$ correlation can be drawn in rather easily at low bucklings (large moderator sizes) if the correlating line does depart from linearity because of the fid diffusion cooling effect.

If counters are placed at two different positions in the moderator and used for simultaneous counts, the slowing down age, τ , can be obtained.

from $q(x, \tau) = \frac{S}{\sqrt{4\pi\tau}} e^{-x^2/4\tau}$

one can obtain the following equation given by Antonov¹

$$\ln n(x_1)/n(x_2) = x_2^2 - x_1^2/4\tau$$

When τ is plotted against the elapsed time, a straight line relationship is obtained which has an intercept of $\tau = \tau$.

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STUDENT RESULTS

Experiment Performed By E. Abele, G. H. Burnham, S. E. Liverhant, C. O. Reiser, L. C. Wilbur. July 7, 1961. Submitted by C. O. Reiser

R. L. I. EXPERIMENT NUMBER 16

NEUTRON AGE IN WATER

INSTRUCTOR: L. L. LAWYER

ABSTRACT

This experiment is designed to demonstrate the concepts involved in the slowing down process of neutrons from fission energy to the energy of the indium resonance (1.46 ev). The neutron "age", a parameter describing the behavior of these neutrons in this energy range is evaluated from a measurement of a spatial distribution of neutrons within a moderator. Thermalized reactor leakage neutrons impinging on a set of Argonaut fuel plates produce a plane source of fission energy neutrons. The moderating medium is water and the detecting devices are cadmium covered indium foils.

NEUTRON "AGE" IN WATEROUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - 1. DISCUSSION QUESTIONS

1. INTRODUCTION

The average crow-flight distance a fission neutron travels while slowing down has a direct bearing on the critical size of a reactor. If this average distance is large, then the reactor must be large in order to reduce the fast leakage factor. Therefore, the spatial distribution of neutrons resulting from the diffusion process determines the probability of loss of epi-thermal neutrons by leakage during the slowing down process.

A neutron's life cycle in a thermal reactor, moderator, or reflector can be considered to be divided on an energy scale into thermal and epi-thermal regions. The Argonaut experiment "Measurement of Thermal Neutron Diffusion Length in H_2O " concerns itself with the fate of the neutron in the thermal portion of such a cycle; this experiment completes the picture by considering the neutron's behavior during their epi-thermal career.

II. THEORY

The spatial distribution of neutrons of various energies resulting from diffusion during the slowing down process, as mentioned earlier, is important because the average (net vector) distance a fission neutron travels while slowing down has a direct bearing on the critical size of a reactor. An approximate approach to the investigation of this distribution is based on the "continuous slowing down model" or "Fermi age model".

Consider a neutron produced at fission energy E_0 in a medium containing only relatively heavy nuclei. It will travel with this energy for a certain time before it collides with a nucleus. After the collision, its energy will be decreased as the square of the mass ratio and will then travel with this constant lower energy until it again has a collision. Due to the lower

velocity after each collision, the time between collisions will, on the average, increase. This process will continue until the neutron is thermalized at energy E_{th} . This model is depicted in Figure 1.

Figure 1

It should be noted that in Figure 1 for a given material the vertical steps in energy are merely ξ , the average logarithmic energy decrement per collision, and are therefore independent of energy; whereas, the horizontal steps become increasingly large with decreasing energy.

If the medium consists of nuclei of moderate or high mass number, the spread of energies among the neutrons of the same generation is relatively small; and it is possible to represent the behavior of a large number of individual neutrons by means of an average. As a result, the series of steps may be replaced, without serious error, by a continuous curve; such as that shown in Figure 1. This is the basis of the continuous slowing down (Fermi age) model. Here it is assumed that, during slowing down, each neutron's velocity is a smooth and continuous function allowing the use of differential equations to describe a process that is actually discrete. It must be emphasized that this model, even as an approximation, is good only for moderators consisting of high mass number elements and definitely is not valid for light element moderators containing hydrogen, i.e., light water. The concepts and mathematical derivations involved in the Fermi age model, however, will prove themselves useful in the case of a light element moderator as will be seen later.

The spatial distribution of the slowing density, q , in a non-absorbing, non-multiplying medium is represented by the Fermi Age equation⁽¹⁾.

$$\nabla^2 q = \frac{\partial q}{\partial \tau} \quad (1)$$

where the energy dependent variable $\tau(E)$ is defined by:

$$\tau(E) \equiv \int_E^{E_0} \frac{D}{\xi \Sigma_s E} dE \quad (2)$$

In equation (2)

D = diffusion coefficient

ξ = average logarithmic energy decrement per collision

Σ_s = macroscopic cross section for scattering

E = neutron energy

$\tau(E)$ = Fermi age.

At this point it is interesting to note that, if $q(E)$ is a solution of equation (1) for the case of no absorption, then the corresponding slowing down density $q'(E)$ for the case with weak absorption is merely $q(E)$ times the resonance escape probability.

The solution⁽¹⁾, $q(E)$ (or $q'(E)$), of the Age equation for the slowing down density in the case of a plane source of neutrons in a moderating region of infinite extent is

$$q(x, \tau) = \frac{S}{\sqrt{4\pi\tau}} e^{-\frac{x^2}{4\tau}}$$

where q = slowing down density at a particular energy E

S = source strength

τ = the age of the neutrons at the energy E .

To give a more precise physical significance to this "age" we may calculate the second spatial moment of the slowing down density, (see section II.3 of "Measurement of Thermal Neutron Diffusion Length in H_2O "), denoted $\overline{x^2}(\tau)$.

$$\overline{x^2}(\tau) = \frac{\int_0^\infty x^2 e^{-\frac{x^2}{4\tau}} dx}{\int_0^\infty e^{-\frac{x^2}{4\tau}} dx} = 2\tau \quad (4)$$

The "age" τ is, therefore, one half the mean square (net vector) distance traveled by a neutron from the time of its emission from the source, i.e., age zero to the time at which its age is τ .

In considering the slowing down process applied to neutrons in hydrogen, a better approach is through a two group treatment⁽²⁾ in which the fast group is taken to include all neutrons of energies above a given energy E . Diffusion theory predicts that the flux, ϕ , of neutrons of energy E should vary with the distance, x , from the plane of an infinite plane source in an infinite medium according to the relation:

$$\phi_E = \text{constant } e^{-\frac{x}{L_f}} \quad (5)$$

where L_f = diffusion length of the fast group. Then in a manner analogous to that used in the case of thermal neutrons, it can be shown⁽³⁾ that for a finite source in a finite medium

$$\phi_E = \text{constant } e^{-\gamma x} \quad (6)$$

where γ is given by the relation

$$\gamma^2 = \frac{1}{L_f^2} + \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 \quad (7)$$

where a and b are the extrapolated dimensions of the moderator in the plane of the source.

If we now calculate the mean square (net vector) distance that the neutrons of the fast group have diffused in reaching the energy E , we get

$$\overline{x^2} = \frac{\int_0^\infty x^2 e^{-\gamma x} dx}{\int_0^\infty e^{-\gamma x} dx} = \frac{2}{\gamma^2}$$

or if we assume leakage to be negligible, i.e., if

$$\gamma^2 = \frac{1}{L_f^2}$$

then

$$\overline{x^2} = \frac{\int_0^{\infty} x^2 e^{-\frac{x}{L_f}} dx}{\int_0^{\infty} e^{-\frac{x}{L_f}} dx} = 2L_f^2$$

We can then, by accepting the definition of "age" for slowing down in hydrogen to be $\frac{x^2}{2}$, get the result that the "age" τ at the energy E is equal to the square of the diffusion length of the neutron group having energies between the source energy and the energy E .

Thus, the age of neutrons of specified energy in a given medium can be determined experimentally, provided a material is available which has a large activation cross section for neutrons principally of that energy. Indium provides us with such a material, due to its 1.46 ev energy resonance. The saturated activity A of a cadmium covered indium foil can be taken to be proportional to the flux ϕ of neutrons having energies in the narrow region of this resonance.

III. APPARATUS

The shielding tank facility, located on the south side of the reactor will be utilized for this experiment. A plane source of fission neutrons will be generated by a number of Argonaut fuel plates. The detecting devices are indium foils 1.9 cm in diameter ~ 80 mg/cm² enclosed in 0.0457 cm thick (18 mil) cadmium cans. These foils will be mounted on a holder provided. (See page 381). A pair of Au foils may be used for monitor foils and to determine the Au-Cd ratio if desired.

IV. PROCEDURE

Using the apparatus outlined above, a neutron slowing down distribution in water will be determined by activating a group of indium foils distributed perpendicular to the plane of the plane fission source. The saturated activity of these foils will then be determined by counting them using end window proportional flow counters.

Since some of the neutrons detected in the above procedure may have originated in the reactor core rather than in the plane source, the magnitude

FIGURE 1

of this activity should be investigated. To do this, we repeat the above procedure with the fuel plates removed from the tank.

The saturated activity of each foil due to neutrons originating in the plane source can then be calculated.

Name				Irradiation Period			
Date				Reactor Power			
$(1 - e^{-\lambda\tau})$				Background			
1 Foil Loc.	2 Act.	3 Act. -Bg.	4 Time	5 $e^{-\lambda t}$	6 Wt.	7 $1 - e^{-\lambda\tau}$ $\times 5 \times 6$	8 Saturated Activity

V. RESULTS

Plot the saturated activity of the background corrected indium foils versus the first power of their distance from the plane fission source and compute τ from the relations indicated in equations (6) and (7).

Next compute τ from the saturated activity data using the relationship in equation (4).

Finally, considering equation (3), plot the saturated activity of the background corrected foils as a function of the square of the distance of their position from the plane source of fission neutrons using semilog graph paper and compare the resulting curve with that predicted by Fermi Age theory for a non-hydrogenous medium.

VI. ANALYSIS AND CONCLUSIONS

In the analysis of our results we must note particularly that equation (3), like the continuous slowing down model is not applicable to a hydrogenous

medium such as H_2O and that equation (5) is for an infinite medium. With these assumptions in mind, we can then explain the deviations at the end of our plotted curves. Other possible sources of error to be considered are impurities in the water, inaccuracy in foil position, source thickness, and the transverse distribution of the fission neutrons.

VII. BIBLIOGRAPHY

1. S. Glasstone and M. C. Edlund, "The Elements of Nuclear Reactor Theory," (D. Van Nostrand Co., Inc., Princeton, New Jersey) p. 176.
2. H. Etherington, Ed., "Nuclear Engineering Handbook," (McGraw Hill Book Co., Inc., New York, 1958), p.1-35.
3. Ibid., p. 6-54.

Experiment Written by: L. L. Lawyer

VIII. APPENDICES

1. Discussion Questions

The following propositions should be considered prior to your arrival at the Argonaut. A discussion of them will be considered a part of this experiment.

1. What is the physical meaning of neutron age?
2. What facets of reactor design are primarily affected by the value of age?
3. What will be the energy spectrum over which we measure age if we use cadmium covered indium as the detecting device?
4. B^{10} ionization chambers are in place behind the water tank in our experiment. What effect, if any, will the presence of these chambers have on our results?
5. Express the slowing down density as a function of neutron age for the following geometries:
 - a. A point source in non-absorbing media.
 - b. A plane source in non-absorbing media.
6. Why is the term "age" not strictly proper for hydrogenous media?
7. What background corrections must be made in this experiment, and what is their need?
8. Can you suggest a better foil material than cadmium covered indium for this experiment?
9. How will you correct the data for leakage?
10. In view of your experience on the Argonaut, do you feel that it is necessary to remove some fuel from the core to compensate for the eight fuel plates used as our plane source of fission neutrons? Give supporting evidence.
11. Explain the general physical meaning of L^2 , M^2 and τ .
12. Qualitatively, what discrepancies might arise from assuming $q(r, \tau)$ proportional to the saturated activity of the indium foils we irradiate where $q(r, \tau)$ is the slowing down density?

13. From what sources, other than non-moderated neutrons, might we expect to get background neutrons during a background run with detecting foils in the water tank? What is the energy of these neutrons?
14. What factors must be considered in choosing the indium foil thickness and the cadmium cover thickness to be used for these experiments?
15. How will the results of this experiment be affected by flux leakage on the sides of the tank? Will this be caused by leakage into or out of the water tank?
16. How will the σ_a of hydrogen affect our value of τ ?
17. There is specified in the accompanying figure which shows experimental apparatus location, a dimension of four inches between the plane source of fission neutrons and the edge of the water tank. Do you expect this dimension to be of any importance? Would you increase this distance or decrease it?

2. 1961 ASEE-AEC Reactor Laboratory Institute Writeup

CONSIDERATIONS INVOLVED IN REACTOR MODERATOR SELECTION

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When a heavy nucleus fissions it splits into two fragments which liberates a large amount of energy. Also in the fission process several neutrons are liberated. If one of these neutrons will cause another fission there will be a self sustaining or chain reacting system. However, the neutrons liberated in the fission process come away with very high energies and the probability that they will cause a fission in the fuel increases for lower energies. Thus to utilize efficiently the fast neutrons in propagating the chain reaction, the neutrons must be slowed down to lower energies. The purpose of a moderator in a reactor is, then, to slow down the fast fission neutrons and thus increase the probability that they will cause further fissions.

When a neutron collides with a nucleus there are two processes which can occur, absorption and scattering. In the absorption process the neutron is captured by the nucleus and lost to the chain reaction. It should be noted that in such things as the fission process and $(n, 2n)$ reactions in which the neutron is absorbed, other neutrons are liberated, so in a sense,

instead of being lost, the neutron has been multiplied. However cross sections for (n, 2n) reactions are very small and the reason for a moderator is to increase the probability of fission, so when a fission occurs the purpose has been accomplished. Thus these will be considered as special cases and absorption will be understood to mean parasitic capture in which the neutron is lost to the system with no net return. In the case of scattering the neutron collides with a nucleus, there is an interchange of energy, and then the two particles separate, usually with energies different from those which they had before the collision. There are two possibilities, elastic scattering in which the total kinetic energy of the particles is conserved and inelastic scattering in which some kinetic energy is lost in exciting the nucleus to a higher energy state. This energy is usually liberated later in the form of gamma rays when the excited nucleus returns to the ground state. However the energy levels of nuclei are of the order of thousands of electron volts (kev) and therefore, a neutron must have this much energy to be inelastically scattered. To increase the fission probability a neutron should be slowed down to thermal energies which are of the order of a hundredth of an electron volt. Thus, because of its threshold characteristic, the inelastic scattering process is not the most important consideration in the selection of a moderator. However, in considerations concerning the determination of neutron spectra in a reactor, inelastic scattering should not be neglected because the process is significant in slowing down very fast neutrons. Inelastic scattering can be particularly important in heavier nuclei such as those in the fuel and the structural materials of the core.

A consideration in the selection of a reactor moderator is the amount of energy lost by a neutron in an elastic collision. It can easily be shown⁽¹⁾ that the neutron can lose the most energy in head on collisions with light nuclei and, specifically, in a collision with a particle of the same mass as the neutron, the neutron can lose all of its energy. One analogy is the familiar one of shooting marbles. It is possible to make the shooting marble stop completely when it strikes squarely a marble of equal size. However, small marbles shot at bigger marbles will rebound with only small loss of energy. The same is true when a neutron collides elastically with a heavier nucleus. Thus the lighter nuclei should provide the best moderators.

The quantity which is used to describe the energy loss per collision is the average logarithmic energy decrement per collision,

$$\xi = \overline{\ln E_1 / E_2}$$

where E_1 is the energy of the neutron before and E_2 is that after a collision.

(1) Glasstone and Edlund, Principles of Nuclear Reactor Engineering, D. Van Nostrand Co., Inc., 1955

It turns out that this quantity is dependent only upon the atomic number of the struck nucleus, i.e., independent of the neutron's initial energy.

A good moderator should also have a high scattering cross section, Σ_s . The macroscopic cross section is the product of the probability that the neutron will be scattered times the number density of scattering atoms. Thus the moderator should be reasonably dense. This immediately eliminates gases from consideration as good moderators. For economic reasons gases pressurized to the liquid state are usually not considered.

Table 1 lists some of the lighter elements and their moderator characteristics. The table lists the thermal neutron scattering and absorption cross sections, the average logarithmic energy decrement ξ , the slowing down power which is defined as the product ξ times the macroscopic cross section, and the moderating ratio which is the slowing down power divided by the absorption cross section. The slowing down power and moderating ratio are both defined such that a good moderator should have high values for these quantities, e.g., a good moderator should not be a good absorber and the higher the absorption cross section the lower the moderating ratio. Lithium and boron, both good absorbers, have fairly good figures of slowing down power but their moderating ratios are terribly low and thus are not good moderators. Also it should be noted that the moderating ratio is only dependent on nuclear constants and independent of macroscopic physical constants. Thus the gases helium and oxygen have high moderating ratios but, because of their low densities at atmospheric pressure, have very poor slowing down powers. By the same token hydrogen and deuterium gases would be poor moderators, however they both combine readily with oxygen to form liquids which make good moderators. It is important to note that some of the gases such as helium, carbon dioxide, and air, while not good moderators themselves are often used as reactor coolants and because of their good moderating ratios aid in neutron thermalization in the reactor.

The table is arbitrarily stopped at silicon. It can be seen that as the atomic weight increases the slowing down power decreases and the elements become less effective as moderators.

From the table it is seen that the best moderators are water, heavy water, beryllium and carbon. The highest moderating ratio is possessed by heavy water which is presently widely used as a moderator. The chief drawback to heavy water is the expense of separating it from naturally occurring water and then keeping it separate. The next best moderators are light water and carbon both of which are overly abundant and extensively used. Beryllium, beryllium oxide and beryllium carbide make good moderators, however their use has been limited because beryllium is very expensive and very toxic. Organic substances which are pure hydrogen, carbon and oxygen compounds also would make good moderators, however

Table 1

Element	Atomic Weight	Absorption Cross Section Barns	Scattering Cross Section Barns	Avg. Log Energy decrement ξ	Slowing Down Power -	Moderating Ratio
Hydrogen	1.0	0.33	20.80	1.000	1.38(H ₂ O)	70.0
Deuterium	2.0	0.00046	7.0	0.726	0.180(D ₂ O)	11000.0
Helium	4.0	0.0070	0.8	0.428	0.000015	49.0
Lithium	6.9	71.0	1.4	0.264	0.017	0.000052
Beryllium	9.0	0.010	7.0	0.208	0.156	160.0
Boron	10.8	755.	4.0	0.178	0.0985	0.000093
Carbon	12.0	0.0032	4.8	0.158	0.0595	170.0
Nitrogen	14.0	1.88	10.0	0.137	0.000068	0.726
Oxygen	16.0	0.00019	4.2	0.121	0.000025	2680.0
Sodium	23.0	0.505	4.0	0.085	0.0086	0.067
Magnesium	24.3	0.063	3.6	0.087	0.035	4.95
Aluminum	27.0	0.23	1.4	0.073	0.0061	0.49
Silicon	28.0	0.13	1.7	0.070	0.0061	0.92

these hydrocarbons are chemically unstable in high radiation fields so have found only limited use.

In summary, a reactor moderator must have the property of efficiently slowing down fast neutrons to thermal energies without destroying, or capturing, them. The elements which best accomplish this are hydrogen, deuterium, beryllium, carbon, and oxygen. The forms of these elements which are most widely used as moderators are water, heavy water, graphite, and to a lesser extent, beryllium, beryllium carbide, beryllium oxide and organic materials.

STUDENT RESULTS

Experiment Performed By R. W. Clack, J. J. Herbst, C. J. Roberts,
H. A. Shadduck, L. C. Wilbur, T. G. Williamson. July 13, 1961. Submitted by
T. G. Williamson

13 July
T. G. Williamson
C. J. Roberts

Foil #	wt.	Time (min)	Age Measurement			Data		1st bckgnd	2nd set
			cpm	net cpm	$e^{-\lambda t}$	A_0	$\frac{A_0}{wF}$		
3	.1267	30	1849	1833	.6804	2700	21300	19836	19819
4	.1374	28	1155	1139	.6131	1630	11870	10947	10676
5	.1366	26	654	638	.7162	890	6520	5780	5605
6	.1313	24	435	419	.7349	570	4250	3791	3692
7	.1362	22	246	230	.7540	305	2240	2000	1761
8	.1411	20	155	139	.7736	180	1273	963	1036
9	.1398	18	133	117	.7937	147	1056	925	640
10	.1387	16	77	664	.8143	81	589	448	455
11	.1270	14	? 310 ?	All covered by mistake maybe					_____
12	.1377	12	43	27	.8572	31	229	_____	_____

Background Run

3	.1360	21	164	152	.7637	199	1464
4	.1398	19	113	101	.7836	129	923
5	.1396	17	95	83	.8039	103	740
6	.1294	15	61	49	.8249	59	459
7	.1330	13	39	27	.8463	32	240
8	.1301	11	47	35	.8683	40	310
9	.1370	9	28	16	.8909	18	131
10	.1323	7	29	17	.9141	18	141

Distance From Source						
i	x	cm	x^2	$A(x)x^2$	$\Sigma A(x)x^2$	$\Sigma A(x)$
1	2.54	6.45	128000	128000	1988	6.46
2	5.08	25.8	279000	407	3639	13.29
3	7.62	58.0	330000	737	36331	20.3
4	10.16	103.2	386000	1123	40072	28.1
5	12.70	161.2	303800	1426	41952	34.0
6	15.24	232	232000	1658	42952	38.6
7	17.78	316	247000	1905	43734	43.5
8	20.32	412	188000	2091	4148	47.3

Extrapolate above using $A = A_0 e^{-\sigma x}$ with $\sigma = 0.193$

x-cm	0.193x	e ^{-0.193x}	A(x)	x ²	A(x)x ²	ΣA(x)x ²	ΣA(x)	27
20	3.88	.021	378	40x10 ²	15.1x10 ⁴	224x10 ⁴	445x10 ⁴	50.4
25	4.83	.0081	146	6.25	91	233	447	51.1
30	5.80	.0031	56	9.0	51	238	448	53.1
35	6.75	.0012	21.6	12.25	265	241	448	53.7
40	7.73	.00045	8.1	16.0	1.3	242	448	54.0
45	8.70	.00017	31	20.25	0.6	243	448	54.2

$$27 = 54.4$$

$$\underline{\underline{r = 27.2 \text{ cm}^2}}$$

Neutron Age in Water Summary

Plot A_5 vs x ($e^{-x/L}$, $\frac{1}{L} = x^2 \cdot \frac{1}{2L^3}$) $L = 27.8$
 Plot A_5 vs x^2 slope = $-\frac{1}{2L}$ $L = 26.2$
 $2L = \frac{1}{\text{slope}} = \frac{\sum A(x) x^2}{\sum A(x)}$ $L = 27.2$

$$A_{90} = \underline{\underline{27.1 \text{ cm}^2}}$$

This is the age of fission neutrons to indium resonance (1.4 eV). The age to thermal is
 $L_{th} = 29.1 \text{ cm}^2$

The best measured value today is 27.7 cm^2
 or a 5% difference.

R.L.I. EXPERIMENT NUMBER 17

RESONANCE ESCAPE PROBABILITY

INSTRUCTOR: D. A. DAAVETILA

ABSTRACT

Considering a uranium fueled thermal reactor, the neutrons born in a thermal fission event have energies from 10 mev to thermal with the majority being in the 1 to 3 mev range. In order to cause another thermal fission event these neutrons must slow down, and during this slowing down process some will leak and some will be captured. The resonance escape probability (p) is a measure of the neutron's probability of escaping non-fission capture while slowing down. Because of the relatively huge resonance peaks of U^{238} this capture will take place essentially completely in U^{238} yielding radioactive Np^{239} as a decay product, therefore the probability of escaping U^{238} capture (p^{28}) is equivalent to p .

In this experiment the resonance escape probability in the core is determined through a measurement of the U^{238} cadmium ratio using depleted uranium foils. A number of core parameters such as thermal utilization and non leakage probabilities are required in this determination and are to be calculated by the student with the aid of appendix 2. A technique for resolving the Np^{239} activity from the measured foil activity which is due in part to U^{235} and U^{238} fissions is given in Appendix 3.

RESONANCE ESCAPE PROBABILITY

OUTLINE

- I. INTRODUCTION
- II. THEORY
 - 1. Derivation of Equations
 - 2. True Cadmium Ratio Determination
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - 1. Discussion Questions
 - 2. Constants Evaluation
 - 3. Measurement of Np^{239} Activity in Irradiated Uranium Foils

I. INTRODUCTION

The infinite multiplication factor k_{∞} is a basic reactor constant, and it is the ratio of the average number of neutrons in one generation to that of the preceding generation in an infinite reactor core. For a finite reactor core an effective multiplication factor k_{eff} is given in terms of k_{∞} as follows

$$k_{\text{eff}} = k_{\infty} \times \text{total non leakage probability}$$

When k_{eff} is equal to one the reactor is critical.

On the average in a critical reactor one of the neutrons born in a fission event must cause another fission. From a consideration of this fission chain or cycle, the following important reactor (four factor) formula is readily developed.

$$k_{\infty} = \eta \epsilon p f$$

Where η is the average number of fast neutrons born per thermal neutron captured in the fuel, ϵ (fast fission factor) is the average number of neutrons slowing down past the U^{238} fast fission threshold of about 1.1 mev) per fast neutron from thermal fission, p (resonance escape probability) is the probability of these fast neutrons escaping non-fission capture while slowing down to thermal energy, and f (thermal utilization) is that fraction of thermal neutrons absorbed that are absorbed in the fuel which completes the cycle.

Given the core materials a reactor designer attempts to maximize k_{eff} as long as this aim is compatible with the proposed use and purpose of the reactor.

In uranium fueled thermal reactors the non fission capture of neutrons during the slowing down process occurs primarily in U^{238} . Its cross section versus neutron energy response has huge resonance peaks in the energy region between 200 and 6 ev. By comparison the capture in other core materials is negligible.

The numerical value of p which is always less than one tends to increase with fuel enrichment, fuel lumping, and moderator to fuel ratio. Enrichment increases p by reducing the amount of U^{238} . As uranium is a poor moderator, fuel lumping reduces the resonance flux in the inner volume of the lump through self shielding. Increasing the moderator to fuel ratio with proper core geometry increases the probability of a neutron slowing down through the resonance region while being in the moderator.

In this experimental determination of p , the cadmium ratio of U^{238} foils is measured, and from this ratio the resonance escape probability is calculated according to the theory and equations developed in the next section.

II. THEORY

1. Derivation of Equations

From the following sketch of the slowing down scheme of fast neutrons in the reactor core, the relationship of P to the U^{238} non fission cadmium ratio will be derived. For a more complete discussion, read entries 1 and 3 of the bibliography.

$$F = \frac{\sum U^{238}}{\sum U^{238} + \sum U^{235}}$$

Figure 1

From the relative activity of bare and cadmium covered U^{238} foils located within the fuel, the ratio of neutron captures in the resonance and thermal energy regions can be determined. The U^{238} foils are activated because under neutron bombardment U^{238} is converted to Pu^{239} according to this decay scheme.

A ratio "R" is defined as follows:

$$R = \frac{\text{Pu nuclei formed by resonance neutrons}}{\text{Pu nuclei formed by thermal neutrons}}$$

The ratio "R" is also expressed as follows in terms of the cadmium ratio of the U^{238} foils.

$$R = \frac{A_c}{A_b - A_c} = \frac{1}{CR - 1}$$

where A_c and A_b are the activities of the bare and covered foils and CR is the cadmium ratio.

$$R = \frac{(1 - p) U \mathcal{L}_0 \mathcal{L}_1}{U \mathcal{L}_0 \mathcal{L}_1 \mathcal{L}_2 \mathcal{L}_3 p f F}$$

Hence:

$$\frac{1 - p}{\mathcal{L}_2 \mathcal{L}_3 p f F} = \frac{1}{CR - 1}$$

From which

$$p = \frac{1}{1 + f F \frac{1}{CR - 1} \mathcal{L}_2 \mathcal{L}_3} \dots \dots \dots$$

Where

f = thermal utilization

$$F = \frac{\sum_a U^{238}}{\sum_a U^{235} + \sum_a U^{238}}$$

$\mathcal{L}_2$ = non leakage probability between resonance and thermal regions

$\mathcal{L}_3$ = thermal non leakage probability

In the above equation for p , the only quantity that is experimentally determined is the cadmium ratio. The others are calculated. Obviously the result for p will depend strongly on the accuracy of these calculated quantities.

In BNL-486, page 17, a more exact equation for "p" is given after similar derivation as presented for equation (1). In the Argonaut core the equations give nearly the same answer.

From BNL-486;

$$p = \frac{1 + \frac{(\epsilon - 1) \alpha_{28}}{\epsilon(\nu_{28} - 1 - \alpha_{28}) \mathcal{L}_1}}{1 + fF \mathcal{L}_2 \mathcal{L}_3 \left(\frac{1}{CR - 1} \right) \left[\frac{1 - \delta (CR - 1)}{1 + \delta} \right]}$$

The new quantities are $\mathcal{L}_1$ (fast non leakage probability), α_{28} (capture to fission ratio above fast fission threshold), ϵ (fast fission factor), and δ which is approximately the measured epi-cadmium to total fission rate ratio of U^{235} (BNL 486, page 32 and 33) and is defined by the equation:

$$\delta = \frac{1/v \text{ capture of } U^{238} \text{ foil Cd-covered}}{\text{sub-cadmium capture of } U^{238} \text{ foil}}$$

The term $1 - \frac{\delta (CR - 1)}{1 + \delta}$ therefore corrects for the effect of the $1/v$ capture in U^{238} in the epi cadmium range as $1/v$ capture is considered in this derivation to be thermal absorption. From fast fission work in the Argonaut core the value of δ has been experimentally determined to be 0.072. It is important to note that the $\mathcal{L}_2$ quantities of equations 2 and 1 are not identical because they differ in their upper energy cut offs which are about 200 and 6 ev respectively.

2. True Cadmium Ratio Determination

The U^{238} non fission capture cadmium ratio is measured by determining the relative Np^{239} activity in bare and cadmium covered uranium foils. Np^{239} in its decay to Pu^{239} results in a 103 Kev X-ray, and a gamma ray spectrometer is used to resolve the 103 Kev energy region out of the total activity. The measurement is complicated by the contribution to the activity in the 103 Kev range by U^{235} and U^{238} fission products as well as the natural uranium activity.

The latter correction is made by obtaining the 103 Kev activity of each foil before irradiation or of identical reference foils and subtracting this activity from the value after irradiation. The significance of the correction is reduced by long high power reactor runs. A satisfactory run is 100 watts for one half hour.

When highly depleted uranium foils are used for the cadmium ratio measurement, the fission product activity correction is insignificant, and it

will be neglected in this experiment. However, natural uranium foil is available to include in the bare and cadmium covered foil packages to enable the student to study the significance of the fission product correction following the procedure outlined in Appendix 3.

III. APPARATUS

1. Reactor
2. Two natural and two depleted uranium foils each about 1/4 square inch in area and 5 mil or less in thickness
3. Two - 2 mil thick - aluminum shield foils
4. Foil holders designed to keep foils positioned in the core. The design of these holders may be as simple as a lucite rod with the foils taped to it.
5. Counting system that includes a NaI scintillation counter, single channel analyzer, count rate meter, and a Brown recorder.

IV. PROCEDURE

Bare and cadmium covered depleted uranium (383 ppm U^{235}) foils will be irradiated in the core for one half hour at 100 watts. The foils are to be located in the water space between the fuel plates, and two inches above and below the core vertical center line. The particular fuel box and water space between plates will be determined on the day of the experiment. Perhaps more than one location can be investigated.

After the reactor scram, 1-1/2 hours will be allowed to pass before the foils are removed from the core and counted. The reason for waiting is to enable the U^{239} to decay practically completely to Np^{239} . During this waiting period, the quantities listed in Table 2 of the next section are to be evaluated by the student with the aid of Appendix 2.

When the waiting period is over, the foils will be removed from the core and separated into individual paper envelopes for ease in handling. The activity of the four uranium foils will be measured with a single channel automatic sweep type gamma ray spectrometer. A discussion of the counting apparatus will be given on the day of the experiment. A sketch of the apparatus with the important control variables is shown in the following sketch. A photograph of the apparatus is on page 188 in Experiment 8.

The foil activities are measured in count rate meter recorder chart divisions which run from 0 to 100. These divisions are related to percentages of the full scale setting of the count rate meter, however, the actual recorder chart zero must be determined as it will not be at the zero division. The peak height in the 103 Kev region is read as the Np^{239} activity after subtracting its activity before irradiation. It is generally considered that the area beneath the photo peak is more significant, so if you have time, calculate the cadmium ratio using the foil activities as determined from both methods. A typical chart recorder trace of uranium foil before and after irradiation is shown in the next figure.

When the counting is complete, the evaluation of the U^{238} cadmium ratio and the calculation of p proceed as outlined in the next section.

V. RESULTS

To keep your results in order, make use of the following tables.

TABLE 1

Foil Data

Foil	Weight	Activity Before Irradiation	Activity After Irradiation	Net Activity Due To Irradiation

NOTES:

1. Record peak height and full scale setting of the count rate meter.
2. The reference foil R is used to provide data to correct the activities before and after irradiation to the same standard in case the spectrometer drifts between the two sessions of foil counting.

TABLE 2

Calculation of p

$$p = \frac{1}{1 + fF\mathcal{L}_2\mathcal{L}_3 \left(\frac{1}{CR - 1} \right)}$$

where $f =$ _____ $\mathcal{L}_2 =$ _____ $F =$ _____ $\mathcal{L}_3 =$ _____and $fF\mathcal{L}_2\mathcal{L}_3 =$ _____

$$CR = \frac{\text{Bare Foil Activity}}{\text{Covered Foil Activity}} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

$$\left(\frac{1}{CR - 1} \right) = \frac{\quad}{\quad}$$

$$p = \frac{1}{1 + \left(\frac{\quad}{\quad} \right) \left(\frac{\quad}{\quad} \right)} = \frac{1}{1 + \quad}$$

$$p = \frac{\quad}{\quad}$$

VI. ANALYSIS AND CONCLUSIONS

This is left as an exercise for the student.

VII. BIBLIOGRAPHY

Resonance escape probability being a fundamental reactor parameter has been extensively investigated, and the following list is not one iota of the total literature. It is intended to provide a start for those interested in further reading peculiar to this technique of measuring "p", and also the source of the constants.

1. Volume 5 of the 1955 Geneva Conference Papers - Physics of Reactor Design; particular interest pg. 58, 211.
2. Proceedings of the Brookhaven Conference on Resonance Absorption of Neutrons in Nuclear Reactors; BNL-433; September, 1956.
3. Experimental Studies of Slightly Enriched Uranium, Water Moderated Lattices; H. Kouts and R. Sher; BNL-486; September, 1957.
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5. Two Dimensional Two-Group Calculation of the Argonaut One Slab Loading; D. P. Moon; ANL-6154; April, 1960.

Experiment written by: D. A. Daavettila

VIII. APPENDICES

1. Discussion Questions

1. Consider the cross section versus energy plot of U^{238} and discuss the following aspects of the plot, resonance region, $1/v$ absorption, non $1/v$ factor, and thermal cross section.
2. Compare the epi cadmium non fission absorption cross sections of U^{238} , U^{235} , Al, C, and H_2O at a few energies from 0.4 ev to 2 mev by referring to BNL-325. Generalize to rank them in order.
3. The atom or molecular ratios of the core constituents are $U^{235} = 1$, $U^{238} = 4$, Al = 116, $H_2O = 203$, C = 181, and O = $13(U_3O_8)$. On the basis of your answer to question one, what can you conclude about U^{238} absorption to total epi-cadmium absorption?
4. Define the quantity ν . How is it related to r_1 of $k = r_1 \epsilon$ pf. Is ν dependent on neutron energy?
5. Give a definition in words of ϵ , f , and F .
6. $\mathcal{L}_1, \mathcal{L}_2$ (equal $e^{-\tau B^2}$) and $\mathcal{L}_3$ (equal $\frac{1}{1 + L^2 B^2}$) are non leakage probabilities. How is B^2 related to core dimensions? Explain how and why a reflector affects the non leakage probability.
7. In general would you expect "p" to be larger in homogeneous or heterogeneous reactor and why?

2. Constant Evaluation

In this appendix the evaluation of f , F , $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ are outlined.

Tables 1 and 2, and the equation for L^2 and the values of B^2 , τ_1 , and τ_2 have been taken completely from the work of D. P. Moon in ANL 6154 (Bibliography entry 5). Although this work was based on a one-slab loading, the materials constants will hardly change for a two-slab loading because the fuel to moderator ratio and geometry are the same. The calculations are based on the assumption that the core is homogeneous.

TABLE 1

Atoms per cm³ in the core

$$\begin{aligned}
 N_{H_2O} &= 0.0193339 \times 10^{24} \text{ molecules/cm}^3 \\
 N_G &= 0.016944 \times 10^{24} \text{ atoms/cm}^3 \\
 N_{Al} &= 0.012375 \times 10^{24} - 0.0008020 \times 10^{24} W_{U25} \text{ atoms/cm}^3 \\
 N_{U25} &= 0.000048045 \times 10^{24} W_{U25} \text{ atoms/cm}^3 \\
 N_{U28} &= 0.00019215 \times 10^{24} W_{U25} \text{ atoms/cm}^3 \\
 N_{O16} &= 0.00064036 \times 10^{24} W_{U25} \text{ atoms/cm}^3 \\
 W_{U25} &= \text{kilograms of } U^{235} \text{ in reactor core}
 \end{aligned}$$

TABLE 2

Microscopic Thermal Cross Sections

Material	σ_{a2200}	σ_{f2200}	f	$\bar{\sigma}_{a_{49^\circ C}}^{37^\circ C}$	$\sigma_{f_{49^\circ C}}^{37^\circ C}$	σ_s
H ₂ O	0.664	0	1	0.562 0.573	0	*
C	— *	0	1	.00675	0	4.8
Al	0.230	0	1	0.195 0.199	0	1.4
O ¹⁶	<0.2x10 ⁻³	0	1	≈0	0	4.2
U ²³⁸	2.71	<0.5x10 ⁻³	1.02	2.34	≈0	8.3
U ²³⁵	694	582	.975	572 583	480 489	10

TABLE 3

Evaluation of fKilograms of U^{235} in core (W_{25}) _____

Material	$\frac{\text{Atoms}}{\text{Cm}^3}$	$\bar{\sigma}_a$	Σ_a	
H_2O (in molecules)				Σ_{H_2O}
Graphite				Σ_G
Aluminum				Σ_{Al}
U^{235}				Σ_5
U^{238}				Σ_8
Oxygen				Σ_0

The actual core volume cancels out of the calculation of f. Assume the following disadvantage factors due to ANL-6154 $\phi_H/\phi_f = 1.06$ and $\phi_g/\phi_f = 1.10$.

$$f = \frac{\Sigma_a 5 + \Sigma_a 8}{\Sigma_a 5 + \Sigma_a 8 + \Sigma_a G(1.10) + \Sigma_a H_2O(1.06) + \Sigma_a Al}$$

$$f = \underline{\hspace{10cm}} =$$

$$f = \underline{\hspace{10cm}}$$

Evaluation of F

$$F = \frac{\Sigma_a 8}{\Sigma_a 5 + \Sigma_a 8} =$$

$$F = \underline{\hspace{10cm}}$$

Evaluation of $\mathcal{L}_2$, and $\mathcal{L}_3$

$$\tau_2 \text{ from 6 Kev to 0.181 Kev} = 3.4 \text{ cm}^2$$

$$B^2 = 0.0090 \text{ cm}^2$$

$$\mathcal{L}_2 = e^{-\tau_2 B^2}$$

$$\mathcal{L}_2 = \underline{\underline{\underline{\hspace{10em}}}}$$

$$L^2 = \frac{1}{0.064973 + 0.11955 W_{U25}} =$$

$$\mathcal{L}_3 = \frac{1}{1 + L^2 B^2} = \frac{1}{1 +}$$

$$\mathcal{L}_3 = \underline{\underline{\underline{\hspace{10em}}}}$$

3. Measurement of Np Activity in Irradiated Uranium Foils

Internal memo by

W. Grant and D. Daavettila

INTRODUCTION

When uranium foils are irradiated a number of neutron induced reactions occur. In the case of measuring resonance escape probability by the cadmium ratio method only the Np 239 activity is significant, yet other activities primarily due to U^{235} fission will be present having initial or compton scattered rays within the 104 kev range. The greater the U^{235} content within the foil the larger will be the fission product contribution to the Np peak. A two foil irradiation method for resolving the Np activity from the measured activity in the 104 kev range is presented.

THEORY

Two foils irradiation method.

Let foils 1 and 2 be of different U^{235} enrichment but irradiated under the same flux conditions. Let N be the number of atoms of a particular isotope in a foil. Let suffixes 9, 5 and 8 denote Np^{239} , U^{235} and U^{238} . Let "a" denote the specific activity measured by the counting mechanism in the Np resonance peak, i.e., 104 kev.

Hence ${}_1N_9$ will denote the number of Np^{239} atoms in foil 1, and ${}_1N_9a_9$ will denote the total activity measured by the counter due to decay of Np^{239} in foil 1. Similarly let a_5 and a_8 denote the specific activities measured by the counter due to fission per atom of U^{235} and U^{238} in the Np resonance peak.

Let "A" be the measured activity.

Hence, the measured activity in foils 1 and 2 are:

$$\begin{aligned} A_1 &= {}_1N_9a_9 + {}_1N_5a_5 + {}_1N_8a_8 \quad) \\ & \quad) \dots \dots \dots (1) \\ A_2 &= {}_2N_9a_9 + {}_2N_5a_5 + {}_2N_8a_8 \quad) \end{aligned}$$

Let the ratio of fast fission per atom U^{238} to thermal fission per atom U^{235} be δ for the particular spectrum involved.

Hence in equation (1) we may substitute the following:

$$a_8 = \delta a_5 \dots \dots \dots (2)$$

assuming the fission products to have the same specific activity in the Np resonance region.

Equation (1) then becomes:

$$\begin{aligned} A_1 &= {}_1N_9a_9 + {}_1N_5a_5 + {}_1N_8\delta a_5 \quad) \\ & \quad) \dots \dots \dots (3) \\ A_2 &= {}_2N_9a_9 + {}_2N_5a_5 + {}_2N_8\delta a_5 \quad) \end{aligned}$$

Furthermore, because the flux intensity and spectrum is the same for the two foils concerned, the same amount of Np will be produced per U^{238} atom.

We may thus say:

$$\frac{{}_1N_9}{{}_2N_9} = \frac{{}_1N_8}{{}_2N_8} \quad \text{or} \quad {}_1N_9 = {}_2N_9 \frac{{}_1N_8}{{}_2N_8} \dots \dots \dots (4)$$

Substituting this in equation (3) yields:

$$\begin{aligned} A_1 + {}_2N_9 \left(\frac{{}_1N_8}{{}_2N_8} \right) a_9 + {}_1N_5 a_5 + {}_1N_8 \delta a_5 & \quad) \\ A_2 = {}_2N_9 a_9 + {}_2N_5 a_5 + {}_2N_8 \delta a_5 & \quad) \end{aligned} \dots \dots \dots (5)$$

or

$$\begin{aligned} A_1 &= {}_2N_9 \left(\frac{{}_1N_8}{{}_2N_8} \right) a_9 + a_5 ({}_1N_5 + {}_1N_8 \delta) \quad) \\ A_2 &= {}_2N_9 a_9 + a_5 ({}_2N_5 + {}_2N_8 \delta) \quad) \end{aligned} \dots \dots \dots (5a)$$

$$\text{Let } C_1 = {}_1N_5 \left[1 + \frac{{}_1N_8}{{}_1N_5} \delta \right], \text{ and } C_2 = {}_2N_5 \left[1 + \frac{{}_2N_8}{{}_2N_5} \delta \right] \dots \dots \dots (6)$$

The value of δ can be readily obtained from the fast fission factor measurement. Aluminum catcher foils in contact with uranium foils of two different enrichments will have different gross fission product activations, so that from a plot of gross activity versus enrichment the following factor "F" is determined, from the equation:

$$F_1 = \frac{{}_1N_8 a_8}{{}_1N_5 a_5} \dots \dots \dots (7)$$

(For greatest accuracy ${}_1N_8 a_8$ and ${}_1N_5 a_5$ should really be measured in the energy spectrum of interest, 104 kev range in this case.)

Substituting $a_8 = \delta a_5$ in equation (7) and solving for

$$\delta = \frac{{}_1N_5 F}{{}_1N_8}$$

Substituting this value of δ into equation (6) for C_1 and C_2 and simplifying, the following equations are obtained:

$$C_1 = {}_1N_5 [1 + F_1] \quad C_2 = {}_2N_5 \left[1 + \frac{{}_2N_8}{{}_1N_8} \times \frac{{}_1N_5}{{}_2N_5} F_1 \right]$$

Now rewriting the equations expressing the foil activities:

$$A_1 = {}_2N_9 \left(\frac{{}_1N_8}{{}_2N_8} \right) a_9 + a_5 C_1$$

$$A_2 = {}_2N_9 a_9 + a_5 C_2$$

or

$$C_2 A_1 = {}_2N_9 a_9 \left(\frac{{}_1N_8}{{}_2N_8} \right) C_2 + a_5 C_1 C_2$$

$$C_1 A_2 = {}_2N_9 a_9 C_1 + a_5 C_1 C_2$$

$$C_2 A_1 = C_1 A_2 = {}_2N_9 a_9 \left[\left(\frac{{}_1N_8}{{}_2N_8} \right) C_2 - C_1 \right]$$

Hence the Np activity in foil No. 2 is:

$${}_2Np = {}_2N_9 a_9 = \frac{C_2 A_1 - C_1 A_2}{\frac{{}_1N_8}{{}_2N_8} C_2 - C_1} \dots \dots \dots (7)$$

or

$${}_2Np = {}_2N_9 a_9 = \frac{\frac{C_2}{C_1} A_1 - A_2}{\frac{{}_1N_8}{{}_2N_8} \frac{C_2}{C_1} - 1} \dots \dots \dots (7a)$$

where the value of ${}_2Np$ is the true Np^{239} activity in foil number 2.

4. 1961 ASEE-AEC Reactor Laboratory Institute Writeup

EVALUATION OF RESONANCE INTEGRAL

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A. Empirical Approach:

The resonance escape probability in a moderator-fuel system can be represented to a very good approximation by

$$p(E) = \exp \left[- \int_E^{E_0} \frac{\Sigma_a}{\Sigma_a + \Sigma_s} \frac{dE'}{\xi \cdot E'} \right] \quad (1)$$

if the absorber resonances are widely spaced, as is the case in uranium for the most part.

By slightly modifying the terms in the integrand, expression (1) can be rewritten in the form

$$p(E) = \exp \left[- \frac{N_0}{\xi \cdot \Sigma_s} \int_E^{E_0} \frac{\sigma_{a0}}{1 + \sigma_{a0} (N_0/\Sigma_s)} \frac{dE'}{E'} \right] \quad (2)$$

where E_0 is the energy of the source neutrons, σ_{a0} and σ_s , for all practical purposes, are the absorption cross section of the absorber and the macroscopic scattering cross section of the moderator respectively. If there are more than one type of moderator atoms present ξ represents the average logarithmic energy decrement for the moderator mixture suitably weighted by the appropriate scattering cross sections.

The integral in (2) is the effective resonance integral and its value is seen to depend on Σ_s/N_0 . The two limiting cases of a) infinite dilution (i.e. $\Sigma_s/N_0 \rightarrow \infty$) and b) complete absence of moderator (i.e. $\Sigma_s = \Sigma_{s0}$) lead in the former case to the resonance integral of U-238 and in the latter to the effective resonance integral for pure uranium, showing that the resonance absorption is determined by the resonance integral only in strongly diluted absorber-moderator combinations. Otherwise, the resonance absorption depends on the effective resonance integral. Expression (2) also shows that the effective resonance integral remains finite if σ_{a0} is infinite over a finite energy interval although the resonance integral itself will be infinite in that case. The numerical values of the integrals for the two cases a) and b) have been measured and the most recent figures given are 282 barns for a) and 8.6 barns for b). For other moderator/absorber proportions the value of the effective resonance integral will lie between these two extreme values.

If one writes the neutron absorption rate $\phi \Sigma_a$ in the form

$$\phi \cdot \Sigma_a = \frac{N_0}{\xi \cdot \Sigma_s} \int_{E_1}^{E_2} \sigma_a(E) \cdot \frac{dE}{E} \quad (3)$$

one gets an expression which involves the resonance integral and which is correct if it can be assumed that the dE/E spectrum has not been distorted by the presence of the absorber. This assumption will generally be valid if the absorber is highly diluted in the moderating medium. If the presence of the absorber distorts the $1/E$ -spectrum to a marked extent the integral over the distorted spectrum becomes the effective resonance integral, which will depend in a rather involved way on the amount of scattering material present, amount and physical shape of absorber present and last but most importantly on the nature of the absorption resonances.

The simplest case arises for a homogeneous mixture where the absorber is uniformly dispersed in some moderating material, so that the actual shape of the absorbing material need not be taken into account. An empirical relation which has been found satisfactory for a homogeneous absorber-moderator mixture and which is valid for $\Sigma_s/N_0 \leq 1000$ barns is the following:

$$\int_{E_{\text{thermal}}}^{E_0} \frac{\sigma_{a0}}{1 + \sigma_{a0} \frac{N_0}{\Sigma_0}} \frac{dE'}{E'} = 3.9 \left(\frac{\Sigma_s}{N_0} \right) \quad (4)$$

which shows that the effective resonance integral for a given absorber is independent of the mass of the scattering nucleus and determined entirely by its scattering properties; more specifically, by the Σ_s/N_0 ratio alone.

The observed 0.415-power variation agrees well with the theoretically predicted value of 0.5 which arises from an analysis of the slowing down process when the flux modification by resonance absorption is taken into account in the calculation.

For the heterogeneous absorber-moderator arrangement the state of affairs is more complicated because the nuclei in the thin surface layer of the absorber are exposed to a much higher resonance flux than those inside the absorber and will, as a consequence, have a much higher resonance absorption. Although the resonance neutrons will be removed from the dE/E spectrum by absorption in the surface layer they are constantly being replenished by elastic scattering of neutrons by the moderator atoms. The equilibrium flux thus established will be an only slightly perturbed dE/E flux spectrum.

However, in the interior of the absorber the equilibrium neutron spectrum will be considerably depleted of its resonance energy component, because no sufficiently large influx of resonance energy neutrons can take place to compensate for their disappearance in the absorber. Any moderation of neutrons inside the absorber would have to be provided by the absorber (uranium) which is clearly insufficient. One can then expect all resonance energy neutrons that pass from the moderator to the absorber and diffuse into

it to be captured by a thin surface layer of absorber so that this component will be completely absent from the neutron flux that penetrates into the interior. Whatever resonance flux that is present in the interior will have come from the fast neutrons which have diffused into the lump and whose energy has then been degraded to resonance energies by collision with absorber nuclei in the interior.

This contrasting behavior of the neutron flux at the absorber surface and in its interior and the different mechanisms responsible for the equilibrium resonance flux in the two regions suggests a division of the resonance absorption integral into two contributing parts; one proportional to the surface area of the absorber and the other proportional to its volume.

In particular, one can write the effective resonance absorption as

$$(\sigma_{a0})_{\text{eff}} \equiv \frac{\sigma_{a0}}{1 + \sigma_{a0} \frac{N_0}{\Sigma_s}} = a(E) + b(E) \frac{S}{M} \quad (5)$$

and seeks to obtain some appropriate expressions or values for $a(E)$ and $b(E)$ that will apply to the particular absorber under consideration.

The volume portion of (5), i.e. $a(E)$, is ascribed to the effective absorption cross section within the absorber, or fuel lump, so that

$$a(E) = \sigma_{a0} \frac{\Sigma_{s0}}{\Sigma_{s0} + \Sigma_{a0}} = \frac{\sigma_{a0} \cdot \sigma_{s0}}{\sigma_{s0} + \sigma_{a0}} \quad (6)$$

This reduced volume absorption reflects the fact that the neutron flux $\phi_{1,0}(E)$ incident on the absorber lump has been depleted and reduced to the fraction $(\Sigma_{s0}/\Sigma_{s0} + \Sigma_{a0})$ of its value in the adjacent moderator region. The fraction of resonance flux entering the surface region will therefore be

$$\phi_{1,0}(E) = \left(1 - \frac{\Sigma_{s0}}{\Sigma_{s0} + \Sigma_{a0}}\right) \phi(E) = \frac{\Sigma_{a0}}{\Sigma_{a0} + \Sigma_{s0}} \phi(E) \quad (7)$$

The number of neutrons striking unit area of the absorber lump per sec is $\approx \frac{1}{4} \phi(E)$. Of these, the fraction $(\Sigma_{a0}/\Sigma_{a0} + \Sigma_{s0})$ will be absorbed on first collision, so that the total number of neutron absorptions per unit area of surface per sec is

$$\frac{1}{4} \frac{\Sigma_{a0}}{\Sigma_{a0} + \Sigma_{s0}} \times \frac{\Sigma_{a0}}{\Sigma_{a0} + \Sigma_{s0}} \quad (8)$$

In terms of absorptions per atom per cm^2/g per sec this becomes

$$\frac{1}{4} \left(\frac{\Sigma_{a0}}{\Sigma_{a0} + \Sigma_{s0}} \right)^2 \times \frac{d}{N_0} \quad (d_0 = \text{density of absorber}) \quad (9)$$

and for the number of absorptions over the total surface this has to be multiplied by the surface area S .

The fraction of absorptions taking place in the surface as compared to the overall absorptions in the lump becomes then

$$\left(\frac{\Sigma_a}{\Sigma_s + \Sigma_a} \right)^2 \times \frac{d}{4N_0} \times \frac{S}{M} \quad (M = \text{mass of fuel lump}) \quad (10)$$

With the expressions (6) and (10) the effective resonance integral can then be written as

$$\int_E^{E_0} (\sigma_a)_{\text{eff}} \frac{dE'}{E} = \int_E^{E_0} \frac{\sigma_{a0}}{\sigma_{a0} + \sigma_{s0}} \frac{dE'}{E'} + \frac{S}{M} \int_E^{E_0} \frac{d}{4N_0} \left(\frac{\sigma_{a0}}{\sigma_{a0} + \sigma_{s0}} \right)^2 \frac{dE'}{E'} \quad (11)$$

The quantities expressed as integrals have been determined by experiment and found to be 8.6 barns and 24.7 barns gm/cm^2 , respectively. Combining these values one arrives at the standard formula for uranium metal

$$\int_E^{E_0} (\sigma_a)_{\text{eff}} \cdot \frac{dE'}{E'} = \left(8.6 + 24.5 \frac{S}{M} \right) \text{ barns} \quad (12)$$

which has been used extensively. Similar formulas with somewhat different numerical constants have been obtained for the uranium oxides and for thorium and its oxides.

More recently, a somewhat different approach to the problem has led to slightly different standard forms in which the S/M ratio appears to the power $\frac{1}{2}$ in conjunction with different values of the constants. The remarkable fact is that on purely experimental grounds there is very little to choose between the two formulations, since within the practical ranges of fuel element dimensions used in nuclear reactors both lead to almost identical results.

A word should be said about the range of the resonance region and the limits of integration to be used for the effective resonance integral evaluation.

On the basis of early measurements of the partial widths of the resonance levels the logarithmic energy range was taken as 5.6 for absorption by uranium metal and 7.3 for absorption by uranium oxide. More recent work has reduced this range to 2.6 for the metal and about 3.7 for the oxide. (Apparently, the presence of oxide makes the absorption band wider.) The only criterion one can use is to pick a resonance range interval which gives the most accurate value of the resonance escape probability. The value recommended by Weinberg and Wigner is 3 for both metal and oxide.

B. Theoretical Approach

It should be possible, on the basis of the known absorber resonances, to calculate the effective absorption cross section and various attempts along these theoretical lines have been made. The problem boils down to a calculation of the energy spectrum and of the spatial dependence of the collision density in the presence of strong absorbers. If the resonance escape probability is written as

$$p = \exp \left[- \int_E^{E_0} \frac{F(E) \cdot \Sigma_a(E)}{\Sigma_t(E)} \cdot dE \right] \quad (13)$$

where $F(E)$ is the collision density, the effective resonance integral to be evaluated is then

$$I_{\text{eff}} = \frac{\Sigma_s}{N_0} \int_E^{E_0} \frac{F(E) \cdot \Sigma_a(E)}{\Sigma_t(E)} \cdot dE \quad (14)$$

In the Narrow Resonance approximation (NR) which applies to those absorber resonances which are narrow as compared to the average energy loss of a neutron in collision with an absorber atom, the asymptotic form of the col-

lision density $F(E) \simeq \frac{1}{\xi \cdot E}$ can be used in the effective resonance integral

(14), and will be quite satisfactory. If, however, the absorber resonances are not narrow in comparison with the average energy loss of neutrons in collisions with absorber nuclei, although narrow as compared to the average energy loss in collisions with moderator nuclei, the Narrow Resonance, Infinite Absorber (NRIA) approximation should be used. The collision density to be introduced in (14) in this case is:

$$F(E) \simeq \frac{\Sigma_t(E)}{\Sigma_t(E) - \Sigma_{s0}(E)} \cdot \frac{1}{\xi \cdot E}$$

In the case of U-238 the two approximations can be used to evaluate the contribution to the effective absorption integral up to about 500 ev. In the case of Th-232 the limit of resolved lines is about 20% lower.

For the lowest resonances in U-238 (6.7; 21; 36.9; 66.3; 103.5; 192 ev) and in Th-232 (22.0; 27.7; 70.1; 174 ev) the NR1A expression should be used whereas for most other lines the NR approximation would be preferable.

In addition, there are still unresolved resonance which cannot be handled with these approximations and which account for about 30% of the resonance absorption, which is far from negligible.

In order to decide between the NR or NR1A approximation for a given resonance line and also to make a choice that will minimize the error introduced in the simplified collision density one uses as a criterion the practical width of a resonance line. This is the energy interval within which the cross section is at least twice greater than the background cross section between adjacent resonance lines.

It is also necessary, when evaluating I_{eff} from the contributions of the individual resonance lines, to take account of the Doppler broadening of the resonance lines. Although the area under the resonance line is unaffected by the Doppler broadening, so that the absorption integral itself (i.e. the integral for infinite absorber dilution) is temperature independent, I_{eff} is affected by the broadening of the absorption lines and is therefore temperature dependent, as can be verified by inspection of equation (2).

Finally, it should be pointed out that the contribution to the effective integral of the individual resonance lines is more nearly equal than their contribution to the actual resonance integral in consequence of the fact that strong resonances go hand in hand with strong flux depressions. Thus, the collision density which involves the product of absorption cross section and depleted resonance flux remains fairly constant over a rather wide energy range, and so does the average absorption over a wide range.

R.L.I. EXPERIMENT NUMBER 18

FAST FISSION FACTOR

INSTRUCTOR: D. A. DAAVETILA

ABSTRACT

When uranium and aluminum foils are tightly pressed together and irradiated, some of the fission products will become imbedded in the aluminum foils which are called catcher foils. As these fission products are radioactive, the measured activity of a catcher foil will be related to the number of fissions that occurred in the contact uranium foil. From a measurement of the activity ratio of catcher foils that were in contact with uranium of two different enrichments, one can determine the ratio of U^{238} fissions to U^{235} fissions and then readily calculate the fast fission factor " ϵ ". This catcher foil method of measuring ϵ is presented, and an extension of the experiment to determine the enrichment of a depleted uranium foil and the episcadmium U^{235} fission ratio in the core is included.

FAST FISSION FACTOR

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSION
- VII. BIBLIOGRAPHY
- VIII. APPENDICES
 - 1. Discussion Questions
 - 2. Epi Cadmium Fission Ratio of U^{235} and Depleted Uranium Foil Enrichment Analysis

I. INTRODUCTION

The fissioning of U^{238} atoms by fast neutrons contributes to the neutron multiplication of uranium fueled thermal reactors, although the significance of the contribution strongly depends on both fuel enrichment and reactor design. The measure of the contribution is given by the fast fission factor ϵ and according to the following important fundamental reactor formula.

$$k_{\infty} = \eta \epsilon p f \tag{1}$$

which gives the ratio of neutrons absorbed in one generation to that absorbed in the preceding generation for an infinite reactor lattice. In the equation k_{∞} is the infinite multiplication factor, η is the number of fast neutrons born per thermal neutron captured in the fuel, ϵ is the number of neutrons slowing down past the U^{238} fast fission threshold per fast neutron from thermal fission, p is the probability of a fast neutron escaping capture while slowing down, and f is the fraction of thermal neutrons absorbed that are absorbed in the fuel. Calculations of critical reactor size often start with calculations of the four factors. For a finite reactor lattice the actual multiplication factor k_{eff} is less than k_{∞} because of neutron leakage, $k_{\text{eff}} = k_{\infty}$ multiplied by the total non leakage probability, when k_{eff} equals one the reactor is critical.

The name fast fission factor arises from the fact that where as U^{235} will fission for neutrons with energies from thermal on up, U^{238} has a fission threshold at about 0.65 mev and will only fission appreciably for neutrons with energies exceeding 1 mev. At higher neutron energies the fission cross sections (σ_f) of both U^{235} and U^{238} are quite constant, U^{235} levels off at about 1.8 barns above 1 mev while σ_f for U^{238} continues to increase until it levels off at 0.55 barns beyond 2 mev. Between 6 and 7 mev the σ_f for both U^{235} and U^{238} roughly doubles and then again levels off.

Fast fission is a bonus because some of the neutron causing fast fission in the U^{238} would have been lost through leakage and capture while slowing down. The value of ϵ for natural uranium reactors is nominally 1.03 and for certain reactor designs it may be as much as 1.10, but for highly enriched reactors like the Argonaut it is essentially one.

The fast fission effect tends to be larger in a heterogeneous reactor than in a homogeneous reactor, because in the homogeneous reactor the fast neutrons immediately have a good chance of suffering a scattering collision with moderator atoms after which the neutrons will likely have less than the fast fission threshold energy. Within the fuel lumps of a heterogeneous reactor elastic scattering collisions with uranium nuclei generally leave the neutron above the threshold energy but inelastic scatterings do not. Nevertheless the gross result is that neutrons within the lump exist for relatively more scattering collisions above the threshold.

A representative series of cross sections for fast neutrons in uranium metal (from Bibliography entry #3, page 278) are the following in barns, $\sigma_f = 0.29$, $\sigma_i = 2.47$, $\sigma_c = 1.5$, and $\sigma_{sc} = 0.04$. For comparison the total microscopic cross section of carbon is 2 barns.

II. THEORY

For more complete discussions of fast fission measurements, one is referred to BNL-486 and P-600 of the 1955 Geneva papers.

The equation for the experimental determination of ϵ is simple in form and obtained from the following analysis starting with one fast neutron from a thermal fission birth. Let n = probability of fast fission, r = probability of non-fission capture and leakage, and hence $(1-n-r)$ is the probability of neither of preceding events occurring neglecting the probability product of nr .

The fast fission factor according to the definition and from the above sketch is:

$$\begin{aligned}\epsilon &= (1-n-r) + n\nu_{28}(1-n-r) + n^2\nu_{28}^2(1-n-r) + \text{-----} \\ \epsilon &= (1-n-r) [1 + n\nu_{28} + n^2\nu_{28}^2 + \text{-----}] \\ \epsilon &= \frac{1-n-r}{1-n\nu_{28}} \text{-----} \quad (1)\end{aligned}$$

The U^{238} fissions per neutron from thermal fission is given by the sum:

$$n + n^2\nu_{28} + n^3\nu_{28}^2 + n^4\nu_{28}^3 + \text{-----}$$

which can be rewritten as

$$\frac{n}{1-n\nu_{28}}$$

The fast neutrons resulting from a thermal fission is the value of ν_{25} , and inversely $1/\nu_{25}$ is the U^{235} fissions per neutron from thermal fission. A factor F is defined by the following equation:

$$F = \frac{U^{238} \text{ fissions}}{U^{235} \text{ fissions}} = \frac{\frac{n}{1-n\nu_{28}}}{\frac{1}{\nu_{25}}} = \frac{n\nu_{25}}{1-n\nu_{28}} \quad (2)$$

If fast leakage is neglected,

$$r = \alpha_{28}n \quad (3)$$

where α_{28} is capture to fission ratio averaged over the fission neutron spectrum and U^{238} fission cross section above the threshold.

Subtracting 1 from each side of equation (1) the following equation is obtained:

$$\epsilon - 1 = \frac{1-n-r}{1-n\nu_{28}} - 1 = \frac{1-n-r-1+n\nu_{28}}{1-n\nu_{28}} = \frac{n\nu_{28}-n-r}{1-n\nu_{28}} \quad (4)$$

Multiplying equation (4) by $n\nu_{25}/n\nu_{25}$ and by making use of equations (2) and (3) the final equation for ϵ is obtained.

$$\epsilon = 1 + \frac{F}{\nu_{25}} (\nu_{28} - 1 - \alpha_{28}) \quad (5)$$

The values of ν_{25} , ν_{28} , and α_{28} are taken to be 2.50, 2.65 and 0.107 respectively. The aim of the experiment is to determine F , and then ϵ is readily calculated from equation 5.

The basis of the measurement of F is that when a uranium atom fissions the fission fragments fly a significant though extremely short distance. A foil package containing uranium foils of two enrichments and numerous aluminum foils is employed in this measurement, see Figure 3. The aluminum foils are tightly pressed in contact with the uranium and will capture some of the fission fragments. These aluminum foils are called catcher foils. The nominal fission fragment has an atomic mass of 100, atomic number of 45, net electric charge of 20, and 100 mev of energy. Their range in air is about one inch, but it is less than 0.0005 inches in aluminum.

As U^{235} has a much larger fission cross section than U^{238} the catcher foil activity will be very sensitive to uranium enrichment providing the uranium foils have equal surface areas.

Other aluminum foils are included in the foil package to serve as fission fragment shields and to provide the background activity correction. The background foil activity is the difference between the measured catcher foil activity and the true fission product activity. The background activity is due to activation of the impurities in the aluminum as well as (n,p) , $(n, 2n)$ and (n, α) reactions in the aluminum itself.

Within the foil package there are catcher foils on each side of the uranium foils. Let the average true fission product activity of catcher foils in contact with the natural uranium be given by A_N , and that of the depleted by A_D . Then a plot of foil activity in counts per minute versus foil enrichment in ppm U^{235} is constructed, as shown in the following sketch.

Extrapolation of the curve to zero ppm U^{235} yields the activity that is due to U^{238} fissions (f_8). Subtracting $0.993 f_8$ from A_N yields that fraction of the activity of A_N that is due to U^{235} fissions (f_5). The value of 0.993 is the factor by which the local U^{238} atom density of natural uranium differs from pure U^{238} . It follows that the factor F sought is given by.

$$F = \frac{f_8}{f_5} \quad (6)$$

Equation (6) is based on the assumption that the activity ratio of U^{238} and U^{235} fission products is equal to the fission rate ratio. This assumption is in error by a few per cent because the decay schemes of U^{238} and U^{235} fission products are not identical. The determination of the exact magnitude of the error is not simple, and for a thorough discussion of the technique read pages 8 and 9 of reference 1.

Using the value of F from equation (6) in equation (5) one obtains the value of the fast fission factor of natural uranium in the Argonaut neutron spectrum. The U^{238}/U^{235} atom ratio of Argonaut fuel is 4 to 1 and in natural uranium it is 140 to 1. Assuming that $\epsilon - 1$ varies as the U^{238}/U^{235} atom ratio, all other considerations being equal, it follows that the " ϵ " of the core equals 4/140 times the measured value for natural uranium.

III. APPARATUS

A photograph of the foils and foil holders is shown on the next page.

1. Reactor
2. Ten - one inch diameter - one mill thick - aluminum foils
3. One - one inch diameter - five mill thick - depleted uranium foil (400 ppm U^{235})
4. One one inch diameter - five mill thick - natural uranium foil
5. Foil holder designed to keep foils tightly pressed together.
6. Foil counter - gas flow proportional counter or Geiger Muller counter
7. Note, if Appendix one experiment is also done, items 2, 3, 4, and 5 are to be doubled.

IV. PROCEDURE

A foil package like that shown in the sketch will be made up and irradiated for 1/2 hour at 100 watts in the reactor core. The foils are 1 inch diameter circles with the following thicknesses; depleted uranium 0.003, natural uranium 0.005, and aluminum 0.001. The uranium foils have to be cleaned with nitric acid to remove the oxide coating.

FAST FISSION FACTOR APPARATUS

NOTE—NOT ALL REQUIRED FOILS ARE SHOWN

Figure 3

The foil covers have been drilled and tapped so that the foils can be tightly pressed together. In the sketch the shield foils are 1, 4, 7 and 10, the background foils are 8 and 9, the depleted catcher foils are 5 and 6, and the natural uranium catcher foils are 2 and 3.

After allowing at least an hour for the decay of short life activity, the foils will be removed from the package. The wrinkled foils should be made flat, and the foils arranged in order 2, 3, 5, 6, 8 and 9 on a sheet of paper. The foils are to be counted for one minute each on the second shelf of the gas flow proportional counters. After each foil has been counted once, the cycle is to be repeated counting each foil on the opposite side and then continue counting for at least six cycles in order to obtain data to correct for fission product decay and to increase statistical accuracy.

As only the relative foil activities are significant, the fission product decay can be corrected for in either of the following two ways. The first is to plot foil activity versus time for each foil and at some single arbitrary time ordinate pick off the relative foil activities. Another technique would be to sum the activity of each foil over the number of cycles and then consider the sums or the sums divided by the number of cycles as the relative foil activities.

From the relative foil activities, the values of A_N and A_D are determined as follows:

$$A_D = \frac{A_5 + A_6}{2} - \frac{A_9 + A_8}{2}$$

$$A_N = \frac{A_2 + A_3}{2} - \frac{A_9 + A_8}{2}$$

With the value of A_N and A_D make the plot of activity versus foil enrichment as shown in Figure 2, and then determine f_8 , f_5 and F for the natural uranium foil. Finish up by calculating ϵ for both natural uranium and the core.

V. RESULTS

To keep your calculations straight make use of the following tables.

DATA AND RESULTS

1

Reactor Scram Time _____

Foil Weights DU _____ gms NU _____ gms

Raw Foil Activity
(cpm)

Zero time for counting _____

Foil Number	Cycles									
	1	2	3	4	5	6	7	8	9	10
2										
3										
5										
6										
8										
9										

Relative Activities
(cpm)

2 _____ 3 _____ 5 _____ 6 _____

9 _____ 8 _____ $\frac{2+3}{2}$ _____ $\frac{5+6}{2}$ _____

9+8 _____ A_N _____ A_D _____

Activity vs Enrichment Sketch

f₈ _____ f₅ _____ F _____
 ε core _____ ε nat uranium _____

VI. ANALYSIS AND CONCLUSIONS

This section is left as the main exercise for the student. Include with your discussion the estimated precision of the results, and discuss the correctness of the extrapolation of the natural uranium ε value to the core ε value that was presented in the Theory section.

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Experiment Written by: D. A. Daavettila

VIII. APPENDICES

1. Discussion Questions

The following questions will be discussed as part of the general experiment discussion. The student is urged to contribute more questions and general comments to the discussion.

1. What is the important practical use of the non-fission capture in U^{238} ?
2. The non-fission capture of U^{238} is the significant contribution to one of the four factors. Which one and what is the nature of the contribution?
3. If the Argonaut fuel was natural uranium would you expect it to go critical?
4. Why is it important in this experiment that the DU and NU foils have equal surface areas?
5. Why are the aluminum foils sensitive to which side of the foil is being counted? Why does this sensitivity decrease with increasing geometric distance between foil and detector?
6. It was noticed one day that the change in measured foil activity versus distance from detector was different for an unirradiated uranium foil compared to a catcher foil. Why was this?
7. What is general shape of the fission yield curves for U^{235} and U^{238} ?
8. In general how does $\sigma_f U^{238}$ compare to $\sigma_f U^{235}$? Does fast fission occur in U^{235} ? What is spontaneous fission?
9. In general, how does the " ϵ " of a homogeneous core compare to the " ϵ " of a heterogeneous core? Explain.
10. Explain why ϵ approaches one, as the fuel enrichment increases.
11. If the experiment is extended to include Appendix Two, one will notice the epi-cadmium fission ratio of U^{235} is about 5%. How is this epi-cadmium fission included in core calculations?
12. The U^{235} - U^{238} fission product activity ratio is not equal to the fission rate ratio. What is the reason for this difference?

2. Enrichment Analysis And Epi Cadmium U^{235} Fission Ratio

If another foil package identical to the one previously described except for a cadmium cover is included in the experiment, the enrichment of the depleted uranium foil and the epi-cadmium fission ratio of U^{235} can be determined. The relative foil activity and analysis is exactly as in the fast fission experiment. In this discussion, let subscripts B and C refer to the bare and covered foil packages.

First consider the enrichment analysis. Because the U^{238} fast fission threshold is far above the cadmium cutoff, the U^{238} fission rate is independent of foil package if both are in identical flux.* That is f_{8C} is equal to f_{8B} . With this constraint and from the activity versus enrichment plots for both foil packages (the depleted enrichment is plotted as X), the following equation is readily derived.

$$x = \frac{A_{DB} - A_{DC}}{A_{NB} - A_{NC}} (7120) \text{ ppm } U^{235} \quad (1)$$

Where 7120 is the ppm U^{235} in natural uranium and X is the unknown enrichment.

The epi-cadmium fission ratio of U^{235} (ρ) is simply:

$$\rho = \frac{f_{5C}}{f_{5B}} \quad (2)$$

* To take into account whether or not the foil packages are in identical fast fluxes, sulfur foils will be placed in the immediate vicinity of each foil package. Sulfur becomes P^{32} through a (n, p) reaction with a threshold of about 1.3 mev. Choosing one package or the other as the reference, the f_8 values are adjusted according to the ratio of the sulfur activities.

R. L. I. EXPERIMENT NUMBER 19

DETERMINATION OF β/ℓ FOR THE ARGONAUT REACTOR BY
PILE NOISE ANALYSIS

INSTRUCTORS: C. E. COHN AND S. G. KAUFMANN

ABSTRACT

This experiment is a measurement of β/ℓ for the Argonaut reactor. The measurements comprise the harmonic analysis of the noise component of a signal derived from a high efficiency ion chamber exposed to the reactor flux. The spectrum of the noise can be used to yield β/ℓ through a simple application of the autocorrelation theorem.

DETERMINATION OF β/ℓ FOR THE ARGONAUT REACTOR BY
PILE NOISE ANALYSIS

OUTLINE

- I. INTRODUCTION
- II. THEORY
- III. APPARATUS
- IV. PROCEDURE
- V. RESULTS
- VI. ANALYSIS AND CONCLUSIONS
- VII. BIBLIOGRAPHY
- VII. APPENDICES
 - 1. Discussion Questions
 - 2. Answers to Discussion Questions
 - 3. Determination of Reactor Kinetic Parameters
by Pile Noise Analysis by C. E. Cohn

I. INTRODUCTION

The quantities β and ℓ are of prime importance in reactor control because of their effect on the dynamic properties of a reactor. The measurements of either β or ℓ present considerable difficulty; however, the ratio β/ℓ may be determined through a relatively simple measurement. This quantity has been measured by three general methods: (1) Studying the frequency dependence of the reactor transfer function ⁽¹⁾, (2) By studying a reactor's time response to a pulse of neutrons ⁽²⁾⁽³⁾, and (3) Studying the time correlation between individual fission events ⁽³⁾⁽⁴⁾.

The random processes and resulting power fluctuations, herein termed "pile noise," which are associated with the propagation of neutron chains in multiplying media were first discussed by Feynman, De Hoffmann and Serber at Los Alamos and by Courant and Wallace at Chalk River. ⁽⁵⁾⁽⁶⁾ Since that time, a number of experimenters have investigated Pile Noise. Recently, simple and successful measurements have been made by Cohn ⁽⁷⁾; our experiment will follow his work.

II. THEORY

a) In a critical reactor, the neutron population is exactly sustained by the contributions of fission neutrons, which appear promptly upon fission, plus the delayed neutrons which are emitted by the fission fragments and appear seconds or minutes after the fission event. This condition of neutron balance is called "delayed critical", denoting the fact that prompt and delayed neutrons are needed to balance neutron losses from absorption and leakage

and thus to sustain the chain reaction. A "prompt critical" condition is one in which the neutron economy is balanced by the contribution of the prompt neutrons alone, without the need of the delayed neutrons.

Figure 1 shows a typical fluctuation of the neutron population after an initial neutron has caused a fission event and has thus, instantaneously increased the neutron population by, say, three neutrons. Note that the time scale is very short so that delayed neutrons from this fission event have not had time to make their appearance. Since the reactor is delayed critical so that the delayed neutron contribution is necessary to sustain the neutron population, it will show a behavior which is characteristic of a subcritical pile, until the delayed neutrons appear. Note also that the origin of the figure is shifted in such a way that the average neutron population is omitted. The figure represents one, typical, time development of the fluctuation of the neutron population, triggered by an initial fission event and dying out for need of delayed neutrons.

A single neutron chain cannot be described mathematically: however the average properties of a very large number of such chains can be expressed in terms of the statistical concepts of k_{eff} , ℓ , and n

where k_{eff} = the ratio of the number of neutrons present in any generation to the number present in the preceding generation, including delayed neutrons.

ℓ = neutron lifetime

n = neutron density

Figure 1

The description of the statistical properties of neutron chains such as shown in Figure 1 is particularly simple, since it need not take into account the delayed neutron contribution, and may thus be written⁽⁸⁾ as

$$\dot{n} = \frac{k_{ex}}{\ell} n \quad (1)$$

In a delayed critical pile, the prompt neutrons fall short of sustaining the neutron population by β , the fractional contribution from the delayed neutrons. Thus, in this case:

$$k_{ex} = -\beta, \text{ and } n(t) = n_0 e^{-\frac{\beta}{\ell} t} \quad (2)$$

It is thus apparent that the average of many neutron chains of the type shown in Figure 1 would define the exponential decay shown as the smooth, dashed curve on the diagram.

b) We may express the difference between the signal output $V(t)$ from a detector exposed to pile flux at some time t and the average output $\bar{V}$ over a time interval T as a Fourier expansion.

$$\phi(t) \equiv V(t) - \bar{V} = \sum_{n=1}^{\infty} a_n \cos \omega_n t + b_n \sin \omega_n t \quad 0 \leq t \leq T \quad (3)$$

The coefficients a_n and b_n are accessible to measurement through the harmonic analysis, using tuned circuits of known frequency and response, of the time dependent component, $\phi(t)$, of the signal derived from the ion chamber inside the reactor.

We now find it convenient to define the auto-correlation function of $\phi(t)$ as

$$\psi(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \phi(t) \phi(t+\tau) dt \quad (4)$$

depends on the product of the value of ϕ at time t by the value of ϕ at a time τ seconds earlier. If there is absolutely no correlation between the value of the function at t and at $t + \tau$, (e.g. in the case of random noise) then $\psi(\tau)$ is zero, unless τ is zero; i.e., $\psi(\tau) \sim \delta(\tau)$.

If $\phi(t)$ is a sinusoidal function, $\psi(\tau)$ represents a measure of the power dissipation in an AC circuit where τ is the phase shift between current and

voltage. Thus, if τ is zero, the power dissipation is maximized as in a purely resistive circuit where current and voltage are in phase. On the other hand, if τ corresponds to a 90° phase shift, the power dissipation is minimized as in a purely inductive or capacitive circuit and $\psi(\tau)$ vanishes.

It is shown in Reference (9) that

$$(a_n^2 + b_n^2) \sim \int_0^\infty \psi(\tau) \cos 2\pi f_n(\tau) d\tau \quad (5)$$

Where a_n and b_n are amplitudes of the frequencies f_n found in the harmonic analysis of a function ϕ and where $\psi(\tau)$ is the autocorrelation of $\phi(t)$. In the specific situation which we discuss here, the ϕ is the pile noise, containing a random component and also a coherent component made up of fission chains of the type shown in Figure 1 depending on the time through

$\phi(t) \sim e^{-\beta t} \frac{\phi_{\text{coherent}}}{\ell}$. Upon substituting this ϕ into the relations (4) and (5) we obtain a frequency dependence of the power spectrum of the form

$$(a_n^2 + b_n^2) \sim A + \frac{B}{(2\pi f)^2 + (\beta/\ell)^2} \quad (6)$$

It is this relation which we propose to employ to find the exponent β/ℓ once the $a_n^2 + b_n^2$ have been measured for many values of n .

III. APPARATUS

The instrumentation consists of five units, a detector, a preamplifier, an intermediate amplifier, an analyzer, and a recorder. Auxiliary equipment includes a well-charged storage battery, an oscilloscope and a reactor. Some of the following optional equipment will be necessary; scissors, an analytical balance, and/or a planimeter.

The detector is an ion chamber which is heavily loaded with boron enriched in the isotope 10 in order to maximize its capture cross section for thermal neutrons. This detector feeds the preamplifier and receives its HV from an external shielded source (batt.).

A photograph of the equipment is shown in Photo #1. A schematic diagram of the preamplifier is shown in Figure 1. The preamplifier uses a 12J7GT operated as a cathode follower with reduced plate and filament voltages to reduce grid currents. The DC component of the voltage produced across the grid resistor by the ion chamber current can be bucked out by a negative bias controlled by the "preamp bias" control on the front panel of

the intermediate amplifier, to maintain the 12J7GT at its correct operating point. This should be about 90-100 M amps.

The intermediate amplifier has on its front panel, in addition to the bias control, a meter for reading the preamp cathode current. This amplifier consists of a 5693 and a 6J5GT, which amplify the signals from the preamp to the level required by the analyzer. The signal on the cathode of the 6J5 is brought out to a connector on the front panel labeled "scope A" to allow for examination of the signal with an oscilloscope.

The analyzer has, in order to allow for a wide range of pile fluxes, an attenuator with six steps which change the gain by factors of five. The attenuator feeds a 6SJ7 voltage amplifier, which in turn feeds a bandpass filter. This is a simple parallel resonant circuit consisting of a high Q audio choke across which either of 18 condensers may be connected to give a range of 18 resonant frequencies. A resistor is inserted in series to prevent loading of the tuned circuit by the preceding stage, and is made approximately 10 times the resonant impedance of the tuned circuit for each setting.

The output of the filter feeds a linear feedback type vacuum tube volt meter consisting of the 6FS5 and one half of the 6SL7. The meter on the front panel indicates the output of this circuit. A shunt in the meter circuit provides a signal for a ten millivolt recorder. This signal is brought out to an AN connector on the back of the chassis. The "TIME CONSTANT" switch on the front panel places a large condenser across the meter circuit to damp the fluctuations in the output. The other section of the 6SL7 is a cathode follower which feeds the "SCOPE 'B'" terminal on the front panel to allow observation of the filtered noise components with an oscilloscope.

The setup should be connected to a good Earth ground at one and only one point to prevent hum pickup. Grounding at more than one point should be avoided, because it can set up ground loops which can introduce a considerable amount of hum. When the setup is complete, it should be turned on and an oscilloscope connected to the SCOPE "A" terminal to check for hum pickup. This can be done by synchronizing the oscilloscope sweep with the line frequency and looking for any signals which remain stationary. The total sweep time for this test should be around 50 or 100 milliseconds. If everything is alright, the predominant signal that should appear is the noise from the input resistor.

IV. PROCEDURE

The experiment is performed in two stages. The first stage is the calibration of the equipment with reactor white noise, while the second stage is the reactor pile noise run. The operating procedure for the two stages is identical, the only difference being in the exposure of the chamber. The

Pile Noise Analyzer

Fig. 3
* RESISTORS 1%

particular placement of the chamber for each stage will be prescribed by the instructor.

Set the FREQUENCY SELECTOR switch to the first position. Set the GAIN switch to the highest position that will allow the volt meter to remain on scale at all times with the TIME CONSTANT in. Note that this circuit also requires some waiting time to come to equilibrium after each adjustment, in this case about one half a minute. When the proper settings are obtained, the recorder may be started and a trace taken. The trace should be taken for about three minutes, this being sufficient to average out most of the random fluctuations in noise component amplitude. Following this, all of the other FREQUENCY SELECTOR settings may be covered in a similar manner. When the calibration is complete, the reactor pile noise run may be done. When you are through with the equipment, be sure to turn off all units to avoid premature battery exhaustion. The on-off switch for the intermediate amplifier and preamplifier is on the intermediate amplifier front panel, while that for the analyzer is on the back of the analyzer chassis.

With the runs completed the recorder traces for each frequency setting must be averaged. This may be done by the use of a planimeter, the cut and weight system, or as noted in the accompanying paper "Determination of Reactor Kinetic Parameters by Pile Noise Analysis," by C. E. Cohn. Dividing the averaged reading for the reactor run by that for the calibration run at each FREQUENCY SELECTOR setting (allowing for differences in GAIN setting for the two runs), and squaring the ratio, will yield the mean square noise amplitude in arbitrary units for each of these frequencies. Using the frequencies corresponding to each FREQUENCY SELECTOR setting of the analyzer, the reactor noise spectrum can be directly plotted.

V. RESULTS

Determination of β/ℓ :

As noted earlier and in the accompanying paper by Dr. Cohn, the square modulus of the transfer function is proportional to $\frac{1}{\alpha^2 + \omega^2}$ where

$\alpha = \beta/\ell$ and therefore our observed noise spectrum (normalized means square amplitude vs. ω) can be represented by the expression $A + \frac{B}{\alpha^2 + \omega^2}$

where A is the white noise component and $\frac{B}{\alpha^2 + \omega^2}$ represents the pile noise component.

VI. ANALYSIS OF RESULTS AND CONCLUSIONS

The data may be fitted and α determined as outlined by Dr. Cohn in Appendix 2. However, in the interest of time, an approximate value for α can be obtained as indicated by the following analysis.

The expression $A + \frac{B}{\alpha^2 + \omega^2}$ at high frequency should asymptotically approach the value A and therefore the white noise component can be determined. As we approach zero frequency it will approach $A + \frac{B}{\alpha^2}$ and by subtracting the previously determined A we can determine $\frac{B}{\alpha^2}$ in terms of normalized mean square amplitude. Finally when $\alpha = \omega$ our expression takes the form $A + \frac{B}{2\alpha^2}$ for which the pile noise component $B/2\alpha^2$ is one half its value at zero frequency. By reading this value of ω from our plot and equating it to α we can thus determine β/ℓ .

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VIII. APPENDICES1. DISCUSSION QUESTIONS

The following propositions should be considered prior to your arrival at the Argonaut. These will be discussed in a group discussion just prior to or during the experiment. It should be stressed that these propositions are meant as a guide to informative discussion and, although they are tailored to a particular procedure, it is intended that the student be allowed maximum flexibility in his choice of experimental procedure.

1. Compare the two reactor conditions "delayed critical" and "prompt critical."
2. Defend the statement "On a microsecond time scale, a delayed critical reactor is subcritical."
3. Justify that the expression $dn/dt = nk_{ex}/\ell$ describes correctly the change of neutron density n with time t on the microsecond time scale. k_{ex} is the excess prompt reactivity and ℓ is the neutron lifetime.
4. Show from all of this that the neutron population of a delayed critical reactor decays on the microsecond time scale according to $\exp(-\beta t/\ell)$.
5. Use this result to make a picture for yourself of the noise component on top of the reactor flux. Compare this type of noise with the noise which you would expect to find superimposed on the current passing through a vacuum tube.
6. An important function in the description of noise is the autocorrelation function

$$\psi(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \phi(t) \phi(t + \tau) dt$$

If $\phi(t)$ is a sinusoidal function, can you picture for yourself how the autocorrelation function of $\phi(t)$ will vary with τ ?

7. If we interpret $\phi(t)$ as the reactor flux, time dependent through the noise component, do we have an experimental way to approach and to measure the quantities a_n^2 and b_n^2 ?
8. Do we have the form of the autocorrelation function $\psi(\tau)$ for reactor noise? (See 6).

9. Summarize: How may we measure the coefficients a_n and b_n and derive from them the relation β/ℓ ?
10. Inspect the circuit diagram and try to draw a block diagram. Show the importance of each block.
11. Since the circuits comprise amplifying stages and resonating circuits, it is to be expected that the overall circuit response will be different for different frequencies. Can you suggest a method of determining this frequency dependence?

2. ANSWERS TO DISCUSSION QUESTIONS

1. "Delayed critical" is the condition where the neutron economy in a reactor is balanced by the contributions of the fission neutrons which appear promptly upon fission, and also by the delayed neutrons which are generated by fission products and appear many seconds or minutes after the fission event.

"Prompt critical" is the condition where the neutron economy in a reactor is balanced by the contribution of the prompt neutrons alone, without the need of delayed neutrons. The prompt fission neutrons are those neutrons which appear immediately after the fission event.

If the signal on the strip chart recorders is constant, then the neutron economy as a whole of the reactor is balanced. The balance is brought about through the contribution of both the delayed and the prompt neutrons, so that this condition is the condition of delayed criticality.

2. The neutron economy in a "delayed critical" is balanced through the combined benefits of the delayed and the prompt neutrons. On a microsecond time scale we do not see the delayed neutrons because they appear long after a fission event. On the microsecond time scale we only see the prompt neutrons, and we therefore see a subcritical reactor.
3. If dn/dt is the rate of change of neutron density on the microsecond time scale, we need not take into account the complicated relations accounting for the effect of delayed neutrons. The rate of change of the neutron population is proportional to the number of times that a fission occurs per second per initial neutron. The number of times that such a fission occurs is to be expected to be inversely proportional to the lifetime of such a neutron, or to $1/\ell$. The rate of change of the neutron density is also proportional to the neutron population increase from one generation to the next one. This is given by k_{ex} , or the excess k . Since all of these factors refer to one initial neutron that causes a fission, the total number of neutrons, n , (Bonilla, p. 214, 6-109, 6-110).

4. If a reactor is delayed critical, then k_{ex} (delayed) is zero. However, on the prompt time scale the k_{ex} of a delayed critical reactor is less than zero; that is, it is $-\beta$ where β is the fraction of the neutrons that appear as a delayed neutron and are therefore not counted into the k_{ex} on the prompt time scale. Thus, if we describe what happens on the microsecond time scale in a delayed critical reactor, we substitute $-\beta$ for k_{ex} . Once we have done this, we can integrate the differential equation $n = \frac{n}{\ell} k_{\text{ex}}$.
5. We shall imagine a reactor that is delayed critical and whose power is steady. We shall also imagine that a stray neutron, either from an outside source or a delayed neutron from earlier fissions enters this reactor, and thereby increases the neutron population of that reactor by exactly one. This neutron may cause fission or may be absorbed, but on the average its effects will decay down to zero, in time, according to $e^{-\frac{\beta t}{\ell}}$. The diagram 1 shows the neutron population of a reactor on the microsecond time scale, (or without the benefit of the delayed neutrons) as it might look after an original neutron has caused a fission event. In the complete random noise, which we might find in a vacuum tube, one noise excursion does not know of the existence of any of the others; thus, noise pulses are completely incoherent.
6. The autocorrelation function, ψ , really shows you the product of a given function, ϕ , by the same function as it looked τ seconds earlier. If there is absolutely no correlation between the appearance of the function at t and that of the function at $t + \tau$ (random noise), then we will expect that this autocorrelation function is zero, except if τ is zero. If ϕ is the sinusoidal function, then the autocorrelation function takes on the shape of the power dissipation in an AC circuit where τ is the phase difference between current and voltage. We know that if τ is zero, the power dissipation is maximized, because current and voltage are in phase, on the other extreme, if τ corresponds to a quarter cycle, the product of ϕ at t and ϕ at $t + \text{a quarter cycle}$ is minimized. This corresponds to the power dissipated either in an inductor or a capacitor where there is continuous displacement of a quarter cycle between voltage and current (ANS Journal, Vol. 3, 4, April, 1958, p. 389, rel. 4 to 6, 9, 10).
7. The quantities a and b represent the amplitudes of the components of a given frequency in the reactor noise. If we can analyze the reactor noise for its individual frequency components, we can measure a and b . In our experiment, we do it through a frequency analyzer or a harmonic analyzer.

8. We should be able to find the autocorrelation function of the reactor noise by forming $\psi(\tau)$ according to the recipe given in equation (1) and equation (4) of Section II. We only have to substitute in equation (4) for $\phi(t)$, namely the time dependence of the flux, the exponential $e^{-\frac{\beta t}{\ell}}$. We will then find that the autocorrelation function, ψ , is simply $e^{-\frac{\beta \tau}{\ell}}$ times some constant.
9. Here we should go back to Question 7. We should find that we have an experimental access to a and b through our harmonic analyzer circuit which gives us the amplitudes of each frequency component. We should know that $\psi(\tau)$ is the exponential $e^{-\frac{\beta \tau}{\ell}}$, and we should find that the only unknown, once a and b have been measured, is the ratio β/ℓ .
10. We would expect that our circuit consists first of some stages which amplify the output of our counter, then a harmonic analyzing stage. The harmonic analyzer stages are expected to comprise simple inductance-capacitor circuits of different combined values which will resonate at different frequencies, and select out of the entire noise spectrum only their own characteristic frequency. Behind the filter circuits we have some amplifying stages which amplify the output of the harmonic analyzer and feed a signal proportional to the amplitude of the output into the meter displayed on the chassis front.
11. If the circuit is going to respond to a different degree to different frequencies, (perhaps because the RC coupling between amplifying stages is a frequency bias, or because the resonating circuits have different Q values or different band passes), the best way to correct this is to feed into the input of our circuit a white noise which contains all frequencies to the same extent. The output is then a measure of the frequency bias of our entire circuit. We have to use the output as the correction factor to the final results.
12. A low efficiency counter in a high flux is a counter which will see many of the fast fission chains overlapping, and will therefore begin to see essentially white noise. A high efficiency counter in a low flux should be a counter looking into a flux whose fission chains are on the average well separated, so that it is almost a certainty that two pulses entering the counter within milli- or microseconds belong to the same fission chain. In the second situation we expect to see characteristic pile noise; whereas in the first situation we will see white noise.

3. DETERMINATION OF REACTOR KINETIC PARAMETERS BY PILE NOISE ANALYSIS

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Determination of Reactor Kinetic Parameters by Pile Noise Analysis*

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An experimental investigation was made of the statistical fluctuations in neutron intensity which occur in a nuclear reactor. An ion chamber was exposed to reactor flux, and the fluctuations in its output current were analyzed in a tunable bandpass filter to get the frequency spectrum of these fluctuations, which has the shape of the square modulus of the transfer function. Results are presented of some measurements made on various low-power experimental reactors at Argonne National Laboratory. For reactors with prompt neutron lifetime between 15 and 70 μsec , the quantity l/β was determined within 5 per cent or better from a least squares fit to the transfer function thus measured.

INTRODUCTION

Since a nuclear reactor is a statistical system, it will show fluctuations in neutron intensity. These fluctuations, or "pile noise," are not commonly considered of interest in themselves, but only as interference to other experiments. However, since the nature of the pile noise depends strongly on important reactor parameters, its study can enable the determination of quantities less easily accessible by other means. In particular, Moore (1) points out that the noise spectrum of such a system, that is, the mean square noise amplitude per unit band width, is proportional to the square modulus of the transfer function or to the Fourier cosine transform of the autocorrelation function. Thus, observation of the noise spectrum of a reactor could yield information about the shape of its transfer function. To test this technique, pile noise analyses were done on various low-power experimental reactors at Argonne National Laboratory. Since these reactors operate at such a low level that power effects on reactivity do not appear, the shape of the low-frequency portions of their transfer functions would depend only on fairly well-known delayed neutron parameters, and thus would be of little interest. However, the high-frequency rolloff portion of the transfer function is strongly dependent on the quotient of the effective delayed neutron fraction over the prompt neutron

lifetime. Since this quantity was of interest to the experimental programs of some of the reactors involved, it was decided to measure it.

APPARATUS AND METHODS

The measurements were made by exposing an ionization chamber to the reactor flux. The fluctuations in the current signal from this chamber were analyzed in a tunable bandpass filter to obtain the frequency spectrum. These fluctuations are made up of two portions. The first is the pile noise, and the second is the white noise resulting mainly from the statistics of neutron collection in the chamber. The amplitude of the signal resulting from the pile noise is proportional both to the square root of the power level of the reactor, and to the fraction of reactor neutrons absorbed in the chamber. The amplitude of the white noise, however, is proportional to the square root of the total number of neutrons per second entering the chamber. Thus, it is proportional to the square root of the reactor power and to the square root of the fraction of neutrons absorbed. For a good measurement it is necessary to have a high ratio of pile noise to white noise, which thus requires that the chamber absorb a sufficient fraction of the neutrons in the reactor. To achieve this, the chamber must be heavily loaded with boron and be placed in a high flux region in the reactor. The chamber used in these experiments was a parallel-circular-plate type and was loaded with about 4 gm of boron 10, in the form of both boron

* Work performed under the auspices of the U. S. Atomic Energy Commission.

coating on the plates and BF_3 gas filling. In the reactors tested, this chamber removed a few tenths of a per cent of reactivity.

A schematic of the electronics used in these measurements is shown in Fig. 1. The output of the chamber was taken to a cathode follower preamplifier using a 12J7GT tube operated with reduced filament and plate voltages to minimize grid current. A battery and potentiometer were provided to insert a bucking voltage to the grid circuit to oppose the voltage drop across the grid resistor produced by the dc chamber current and thus to hold the tube at the correct operating point. The output of the preamplifier fed the intermediate amplifier, which in turn fed the analyzer. The 6SJ7 voltage amplifier stage in the analyzer drove the bandpass filter, consisting of the components L , C , and R shown inside

type linear vacuum tube voltmeter. In theory, since it is the mean square amplitude that is of interest, a square law voltmeter should be used. However, the reading of a square law voltmeter and the square of the reading of a linear voltmeter, when averaged, will always be proportional on a Gaussian signal. The linear voltmeter thus gives the spectrum correctly, and is much more convenient than the square law circuit. The indicating meter had a large condenser which could be switched across it to provide a time constant of about 1 sec, and a shunt to provide a signal for a 10 mv Brown strip chart recorder. To eliminate hum, all supply voltages for the equipment were obtained from dry batteries, except for the filaments of all tubes but the preamplifier, which were powered by a storage battery. Other precautions

FIG. 1. Schematic diagram of pile noise analyzer.

the dotted line. Different combinations of L and C could be switched in to obtain a wide range of resonant frequencies—from 9.4 to 438 cycles per second. The low frequency limit was determined by the characteristics of available components, while the high frequency limit was selected to be around that point where the pile noise in the reactors tested became small compared with the white noise due to the rolloff of the transfer function. The inductances L were high Q audio chokes of 3 and 20 henries, while the condensers C ranged from 0.05 to 12 μf . For each combination, R was chosen to be about ten times the resonant impedance of the tuned circuit in order to prevent loading by the preceding stage. The Q of the tuned circuit varied from 10 to 50 for different settings.

The amplitude of the frequency components selected by the filter was measured by a feedback-

to avoid hum included the grounding of the setup at one point only to avoid ground loops.

Noise spectra obtained with this equipment had to be corrected for its frequency response, which was dependent upon many factors, including the resonance curve of the filter, the frequency response of the amplifier, and the collection properties of the chamber. To allow for all these factors, the system was calibrated with white noise in the form of a 40-curie Co^{60} source. With the chamber exposed to the source, readings were taken at all the filter settings to be used in subsequent runs. Then, by dividing readings obtained from a reactor by readings obtained from the Co^{60} source at corresponding frequencies, the reactor noise spectrum was directly obtained. Care was taken to establish that the chamber voltage ($67\frac{1}{2}$ volts) was sufficient to give saturation at the chamber currents used, since it was

found that lack of saturation in the chamber could give rise to odd spectrum shapes.

Since the amplitude of each frequency component of the noise was itself fluctuating, it was necessary to average it over a period of time to obtain an accurate reading. (According to Van der Ziel (2), the fractional standard deviation of a noise measurement, n , is given by the equation

$$\delta n/n = \sqrt{K/BT} \quad (1)$$

where B is the filter bandwidth, T the averaging time, and K a constant dependent on the dynamic characteristics of the averaging mechanism.) Thus, during each run, voltmeter output was recorded for several minutes at each frequency setting. The recorder trace was read at about $1\frac{1}{2}$ sec intervals and the points were averaged to give the reading.

The shape of the noise spectrum observed with a given setup should be independent of reactor power level, as pointed out previously. In practice, though, there are upper and lower limits to the power that may be used. It must be sufficiently high so that the pile noise is large compared with the background noise of the apparatus, but must not be so large that the chamber behaves improperly or the signal overloads the amplifiers. Also, in order to get an accurate value of the lifetime, the power level should be high enough so that the reactor is truly critical and not merely multiplying some extraneous or inherent source.

In the frequency range used, delayed neutron effects on the transfer function are negligible. This allows the zero power transfer function $ZP(\omega)$ to be represented by the equation

$$ZP(\omega) = (1 - \beta)/(\beta + j\omega l)$$

where β is the effective delayed neutron fraction, l is the prompt neutron lifetime, and ω is the angular frequency. The square modulus of the transfer function then becomes proportional to the function

$$1/(\alpha^2 + \omega^2) \quad \text{where } \alpha = \beta/l.$$

The observed noise spectrum can then be represented by the expression $A + B/(\alpha^2 + \omega^2)$, where the first term is the white noise component and the second term is the pile noise component. This is the function which it was desired to fit to the experimental data.

The fitting was done by minimizing the least squares sum

$$S = \sum_i \left(n_i^2 - A - \frac{B}{\alpha^2 + \omega_i^2} \right)^2$$

where n_i^2 is the normalized mean amplitude squared observed at the frequency ω_i . The weight for each

point was taken as unity, since the presence of other errors such as reactor and instrument drifts tended to mask the statistical errors of Eq. (1) and to make the standard errors of all points about the same. Differentiating S with respect to A , B , and α and setting the derivatives equal to zero gave three equations in the three unknowns. Straightforward elimination of A and B yielded an equation for α of the form

$$F(\alpha, n_i^2, \omega_i) = 0$$

This was solved for α by the Newton-Raphson method. Then, working backwards, A and B were determined, allowing S to be calculated. The standard error on α was obtained using the method of Deming (3), in which the square of the standard error is proportional to S . The calculations were done on an IBM-704 computer.

EXPERIMENTAL RESULTS

Figure 2 shows the noise spectrum obtained in a run on the internal exponential ZPR-V (4). This was done with a fast core loading of 5 to 1 U^{238} to U^{235} enrichment ratio and a 1-in. natural uranium blanket. The chamber was placed in an instrument tube in the thermal section. For this case, α was $180.4 \pm 8.5 \text{ sec}^{-1}$ with a corresponding lifetime value of $38.8 \pm 1.8 \text{ } \mu\text{sec}$. This agrees within experimental error with several other measurements done on this facility, including a $1/v$ poison measurement which yielded $39.0 \pm 3.0 \text{ } \mu\text{sec}$; a measurement using the counting statistics method of deHoffmann which yielded 39.2 to 42.8 μsec ; and a measurement using Rossi alpha techniques which yielded $36.8 \pm 4.0 \text{ } \mu\text{sec}$. These values are all based on an effective de-

FIG. 2. Pile noise spectrum from ZPR-V.

FIG. 3. Pile noise spectrum from ZPR-IV'.

FIG. 5. Pile noise spectrum from Argonaut.

FIG. 4. Pile noise spectrum from the coupled critical experiment done in ZPR-III.

FIG. 6. Pile noise spectrum from ZPR-VII.

layed neutron fraction of 0.007. Figure 3 shows the spectrum obtained from the thermal source reactor ZPR-IV' (5). For this run, the chamber was placed in an instrument tube in an empty fuel element position in the middle of the core. Here, the value of α obtained was $100.6 \pm 2.2 \text{ sec}^{-1}$. Figure 4 shows the noise spectrum obtained from the coupled fast thermal critical experiment (6) in the Argonne Fast Critical Facility, ZPR-III. The value of α obtained was $446 \pm 21 \text{ sec}^{-1}$. With an assumed effective delayed neutron fraction of 0.0074, the lifetime was $16.5 \pm 0.8 \mu\text{sec}$. This agrees with a value of $16.5 \pm 1.7 \mu\text{sec}$ obtained from Rossi alpha measurements.

Figure 5 shows the spectrum obtained from the Argonaut low-power research reactor (7), which

yields an α of $45.3 \pm 6.3 \text{ sec}^{-1}$. Assuming an effective delayed neutron fraction of 0.007, a lifetime of $154 \pm 21 \mu\text{sec}$ was obtained. It can be seen that here α is not as well determined as it was in the previous cases. The reason for this is that the part of the transfer function observed in this run is largely on the 6 decibel per octave tail. Since this portion of the transfer function is identical for all transfer functions with rolloffs below a certain frequency, the ability to discriminate between such functions is limited. This is seen even more strongly in Fig. 6, which is the noise spectrum obtained for the heavy water critical ZPR-VII (8), which gave an α of $20.4 \pm 10.2 \text{ sec}^{-1}$ and a lifetime of $252\text{--}755 \mu\text{sec}$, assuming an effective delayed neutron fraction of 0.0077. Better results might have been obtained in

both these cases if the range of the analyzer had extended to lower frequencies.

CONCLUSIONS

Pile noise analysis is a convenient method for obtaining the shape of reactor transfer functions, and for determining reactor prompt neutron lifetimes. The method is especially useful at frequencies which are inconveniently high for other methods, and requires much simpler equipment than many existing techniques. Other possible applications of this technique include the observation of the shape of the transfer function of power reactors in the study of instabilities, and the determination of prompt neutron lifetime in fast reactors. The latter application would require a detector with high sensitivity to fast neutrons, and the former a chamber that would operate well at the high fluxes involved.

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R.L.I. EXPERIMENT NUMBER 20

STANDARD PILE EXPERIMENT

INSTRUCTOR: HARVEY CASSON

PURPOSE:

- (1) To familiarize the student with the use of a standard pile;
- (2) To standardize a foil plus counter system and to use the results in
 - (a) the determination of unknown thermal neutron fluxes and
 - (b) the calibration of other thermal neutron detectors.

INTRODUCTION:

In reactor and other neutron work, where one is interested in measuring the intensities of thermal neutrons with reasonable accuracy over an extended period of time, it is found that one has to contend with detection equipment whose operating characteristics change with time. It becomes desirable to provide a long-lived, stable supply of thermal neutrons of accurately known intensities, which can be used as a reference standard in the calibration or re-calibration of the detection equipment. The standard graphite pile, or standard pile as it is commonly called, meets this need.

The standard pile in Building D-203 is the reference standard for thermal neutron work at Argonne, and consists of an assembly of long graphite bars forming a solid structure, 7 feet x 7 feet x 10 feet high, with a radium-beryllium source accurately positioned in the interior. (See Figure 1.) The source (Source #38) contains 0.504 gm of radium in the form of radium bromide mixed with 3 gms. of beryllium. The outer faces of the pile are covered with a layer of cadmium of sufficient thickness to absorb any thermal neutrons which impinge on it. Foils are activated in known thermal neutron flux positions directly below the source by inserting them at the numbered positions. The source is located at position #20. (For a discussion of the Argonne Standard Pile, see chapter 3 of Pile Neutron Research by D. J. Hughes (Addison-Wesley, 1953).)

The fast neutrons emitted by the source are slowed down by collisions with the carbon nuclei of the graphite. After a sufficient number of collisions, the neutrons arrive at thermal velocities, achieving an equilibrium velocity distribution determined by the temperature of the graphite. (A neutron having the most probable velocity has an energy of about 0.025 electron-volts.) These thermalized (or thermal) neutrons diffuse through the graphite until they are absorbed either in the interior of the pile or by the cadmium at the surface. At every point in the pile, there is a mixture of thermalized and non-thermalized neutrons, with the relative composition of the mixture depending on the distance from the source.

When one activates a foil in the pile, it is the activity induced by the thermal neutrons which is of interest. One must in general consider how much of the total activity was induced by the fast ("epithermal") neutrons and

STANDARD PILE

Figure 2

Table I: STANDARD PILE THERMAL NEUTRON FLUX

Slot	Standard Pile in D-203	
	Source	38
18	6,611 n/cm ² -sec.	
16	4,497	
14	2,444	
12	1,343	
10	698	

use means to eliminate their effect, if necessary. By activating a second foil, surrounded by a suitable thickness of cadmium, the strong absorption by the cadmium for the neutrons in the thermal region allows the foil to "see" only the faster neutrons. The activity induced in the cadmium-covered foil may then be subtracted from the activity in a bare foil. This is known as the "cadmium-difference method" for determining the effect of thermal neutrons.

DESCRIPTION OF EXPERIMENT:

Part I (Standardization): Two foils of indium, one without, and one with cadmium covering are irradiated in a known flux position in the pile, are counted in a counter system, and the cadmium-difference method employed to determine the calibration of the (foil plus counter system) vs. thermal flux.

Part II (Unknown flux): A BF₃ proportional counter tube, with indium foil attached, is inserted into the pile in the same position as for Part I, for a similar irradiation and foil counting period as in Part I. This operation is repeated with another foil attached to the counter tube, but this time with cadmium surrounding the combination. This part, in conjunction with Part I, determines the effective thermal neutron flux as seen by the counter.

Part III (Calibration of BF₃ counter): The BF₃ counter is calibrated in the thermal neutron flux of Part II.

The method of foil-counting employed in this experiment is called the "running count" method. It involves taking repeated readings of the counter without resetting it.

This method has the following advantages over the usual method of counting for fixed time intervals and resetting the counter after each count: (1) no data is lost because of the time spent in recording the count and re-setting the counter (2) the counts in each interval can be used to check whether the foil activity is decaying with the proper half-life (3) the accumulated count over the total time that the half-life is proper is used to determine the activity of the foil, and provides a convenient way to estimate the statistical accuracy of the results.

PROCEDURE: (See modified procedure on page (6))

- (1) Check the counting system. Run a "plateau curve," using a standard source, and find a suitable operating point for the Geiger tube.
- (2) Determine the "background" of the counter (i.e. counting rate without source.)

Part I (a)-Bare foil:

- (3) Determine the counting rate with the unactivated foil in position.
- (4) Insert the foil into position 18 of the standard pile and record the time. See step (7).
- (5) After about a one hour irradiation, remove the foil from the pile, record the time and place in the counter.
- (6) Start counting at a definite time after the end of the irradiation (say, 2 minutes.) (Be sure that the counter time and scaler are set to zero, and that you have recorded the clock time on your Part I a data sheet.)

Part I (b)-Cadmium-covered foil:

- (7) Before proceeding with step (5) place the unactivated part I b foil in the counter and count as in step (3).
- (8) Cover the part I b foil with cadmium and proceed with irradiation and counting as in step (4), (5) and (6).

Part II (a) Bare foil attached to BF_3 counter tube:

- (9) Between steps (4) and (5) of part I b determine the counting rate of the part II a foil.
- (10) Cover the part II a foil with paper and tape it to the side of the BF_3 counter tube. Irradiate and count as in step (4), (5) and (6).

Part II (b)-Foil attached to BF_3 counter. Combination covered with cadmium.

- (11) Between steps (4) and (5) of part II a determine the counting rate of the part II b foil.

- (12) Cover the part II b foil with paper, tape to the side of the BF_3 counter tube and cover the combination with cadmium. Irradiate and count as in steps (4), (5) and (6).

Part III.

- (13) During the irradiation of step (10), turn on the high voltage on the BF_3 counter and run a plateau curve. Repeat later with cadmium covered tube.

Calculations

If the data has been recorded in accordance with the enclosed sample data sheet, the notation will be in accord with equation (10). A subscript 0, 1, 2, 3, etc. is assigned to each row and corresponds to the successive times t_0, t_1 , etc. that the running count is recorded. (The time intervals need not necessarily be equal). Row 0 corresponds to time $t = t_0$, background = 0, etc. and should be filled in like the other rows.

For each of the four foils,

- (A) By subtraction, obtain the gross counts accumulated in each interval. Subtract the appropriate background and obtain the net count accumulated in each interval t_i to t_{i+1} and record in row i .
- (B) Compute $\lambda, \lambda t_i$ and then the exponentials $e^{-\lambda t_i}$, using a table of exponentials. Exponential factors for indium have been tabulated and may be used if available. See, for example, Hoag, J. B., Nuclear Reactor Experiments (D. Van Nostrand, 1958), p. 434.
- (C) Compute the successive differences of the exponentials $e^{-\lambda t_i} - e^{-\lambda t_{i+1}}$ and record in row i (for each i)
- (D) Using equation (10), compute $\frac{C_0}{\lambda}$ for each interval t_i to t_{i+1}
- (E) Observe the various $\frac{C_0}{\lambda}$ computed in step (D). They should be equal within statistics, unless impurities of half-lives much shorter, or longer than the indium 54.0 minute half-life are present.
- (F) Taking the largest interval where $\frac{C_0}{\lambda}$ is approximately constant, such as putting $i = 0$ and $j = 12$ in equation (10), compute the average value for $\frac{C_0}{\lambda}$. (Be sure to correct for the background.

(G) Using the $\frac{C_0}{\lambda}$ for step (F), compute C_0 and then C_∞ using equ. (11).

Having obtained, for the two foils of part I, C_∞ and $(C_\infty)_{cd}$, and for the two foils of part II, C'_∞ and $(C'_\infty)_{cd}$, compute

- (1) $(C_\infty)_{\text{thermal}}$ and $(C'_\infty)_{\text{thermal}}$ using equation (20)
- (2) the foil calibration factor K from part I, using equation (23)
- (3) the effective thermal neutron flux for part II, using the K of part I
- (4) Calculate percentage reduction in flux in part II as compared with part I
- (5) the net counting rate for the proportional counter in part II for thermal neutrons (Do not use the cadmium correction factor F used for the foils)
- (6) the thermal calibration factor for the proportional counter (counts per minute per unit flux) using the flux obtained in part II
- (7) Compute the cadmium ratio $\frac{C_\infty}{(C_\infty)_{cd}}$ for each pair of foils and the BF_3 proportional counter.
- (8) Estimate the flux depression produced by the bare indium foil in part I a.
- (9) Calculate the indium foil calibration factors to be used if the standardized foils plus counting system used in this experiment were to be used to determine the thermal neutron flux in the following moderating media: (1) H_2O (b) paraffin (c) D_2O (d) graphite. (See for example, Price, W. J., Nuclear Radiation Detection, McGraw-Hill, pp. 285-290 (1958).

Modified procedure and calculations in the event that two counter systems are available for simultaneous standardization and use in the experiment:

An indium foil is activated in the pile and steps (1) and (2) are followed for both systems. The foil is then counted alternately in the two systems, using 5 minute counting intervals, until approximately four counts have been obtained for each system. Compute the average $\frac{C_0}{\lambda}$ for each system, and determine their ratio. This ratio gives the relative efficiency of the two systems. (Evaluate the statistics for this result.)

The remainder of the experiment is carried out as indicated in the single counter system procedure, except that half of the foils may be counted using one system and half using the other. One of the systems is designated as the "standard system" and all foil counts are reduced to counts on this system by use of the relative efficiency factor, when necessary.

DISCUSSION AND DERIVATION OF FORMULAS

When a foil is inserted in the pile, activated for a time T , and then allowed to decay, the number of radioactive atoms, $N(t)$, at time, t , follows a growth and then a decay curve as indicated in Figure 2. In the derivation of foil activity which follows (and in the notation of the experimental data sheets), the end of the irradiation corresponds to time $t=0$ so that the beginning of the irradiation occurred at time $t = - T$.

Figure 2

The notation employed in the derivation is summarized below.

σ = activation cross section, in barns

M = mass of foil, in grams

A = atomic weight of foil

λ = decay constant of foil activity $= \frac{\ln}{T_{1/2}} = \frac{0.69315}{T_{1/2}}$

$T_{1/2}$ = half-life of foil activity

n = number of neutrons per cm^3

v = speed of neutrons, cm/sec.

ϵ = fraction of disintegrations in foil which are counted by the counter system

During the irradiation $\frac{dN}{dt} = -\lambda N + P$ (1)

where the production rate $P = nv(\sigma \times 10^{-24}) \frac{M}{A} 6.02 \times 10^{23}$ (2)

Hence $\frac{dN}{dt} = -\lambda N + \frac{nv \sigma M}{A} \times .602$ (3)

For an infinite irradiation time ($T = \infty$) $P = \lambda N$ and $N_{\infty} = \frac{nv \sigma M}{A} (.602)$.

At the end of the irradiation ($t = 0$), $N(0) = N (1 - e^{-\lambda T}) = \frac{nv \sigma M}{A} (.602) [1 - e^{-\lambda T}]$ (4)

After the irradiation ($t > 0$), $\frac{dN}{dt} = -\lambda N$
so that $N(t) = N(0) e^{-\lambda T}$ (5)

Let the counting rate of the foil at end of the irradiation ($t = 0$) = $C_0 = \lambda N(0) \varepsilon$ (6)

where ε is the ratio of the number of counts records by the counter system to the number of disintegrations in the foil.

Had the irradiation time been infinite ($T = \infty$), the counting rate for such a foil at the end of the irradiation ($t = 0$) would have been $C_{\infty} = \lambda N \varepsilon$ (7)

C_{∞} is called the saturated activity.

So that, using equation (6) $C_0 = \lambda N_0 \varepsilon = \lambda N_{\infty} \varepsilon (1 - e^{-\lambda T}) = C_{\infty} (1 - e^{-\lambda T})$ (8)

If the counter registers counts between time $t = t_i$ and $t = t_j$, then the counts accumulated in this interval equal $C_{ij} = \varepsilon [N(t_i) - N(t_j)]$, where the factor $N(t_i) - N(t_j)$ represents the number of decays that take place and ε , the fraction of such decays that are counted by the counter.

Using equation (5) $C_{ij} = \varepsilon [N(t_i) - N(t_j)] = \varepsilon N_0 (e^{-\lambda t_i} - e^{-\lambda t_j})$ (9)

Using equation (6), we can express C_{ij} in terms of the counting rate C_0 at time $t = 0$: $C_{ij} = \frac{C_0}{\lambda} (e^{-\lambda t_i} - e^{-\lambda t_j})$ (10)

Using the experimentally determined C_{ij} values, we first determine C_0 from equation (10).

Using this value of C_0 and equation (8), we determine

$$C_{\infty} = \frac{C_0}{1 - e^{-\lambda T}} \quad (11)$$

From equations (7) and (4) we also have that

$$C_{\infty} = \frac{nv\sigma M}{A} (.602) \times \varepsilon \quad (12)$$

From equation (12) it is seen that the determination of C_{∞} enables us to calculate the efficiency factor ε , provided we know the cross section and for a neutron of velocity v .

From equation (12) it is seen that the determination of C_{∞} enables us to calculate the efficiency factor ε , provided we know the cross section σ for neutrons of velocity v , the flux (nv) and the mass of the foil.

The above derivation applies strictly to the case where the neutrons all have the same velocity v . For a continuous distribution of neutron velocities, as is the case for thermal neutrons, the term $nv\sigma$ must be replaced by an integral over all the velocities in the distribution. In this case, the true total flux is given by

$$\int_0^{\infty} n(v)v \, dv = n \bar{v} \quad (13)$$

$$\text{where} \quad \int_0^{\infty} n(v) \, dv = n = \text{total neutron density} \quad (14)$$

And $\bar{v}$ is the average velocity.

By comparison with equation (12), it is seen that

$$C_{\infty} = \left[\int_0^{\infty} n(v) v \sigma(v) \, dv \right] \frac{.602 \, \varepsilon \, M}{A} \quad (15)$$

Because of the way in which equation (15) depends on the cross section $\sigma(v)$, C_{∞} is not, in the general case, proportional to the neutron flux.

However, for most foil materials, including indium, the activation cross section $\sigma(v)$ is proportional to $1/v$ in the thermal region and we may write

$$\sigma(v) = \frac{k}{v} \quad (16)$$

Putting (16) into (15), we obtain with the use of (14),

$$C_{\infty} = \left[\int_0^{\infty} n(v) v \frac{k}{v} \, dv \right] \frac{.602 \, \varepsilon \, M}{A} = nk \left(\frac{.602 \, \varepsilon \, M}{A} \right) \quad (17)$$

From (17), it is seen that C_∞ is proportional to the total neutron density, n .

$$\text{Equation (17) can be re-written in the form } C_\infty = \frac{nv_0 k}{v_0} \left(\frac{.602 \varepsilon M}{A} \right) \quad (18)$$

where v_0 is any constant.

Using (16) equation (18) can be written in the form of a "flux" multiplied by a "cross section"

$$C_\infty = [(nv_0) \sigma(v_0)] \frac{.602 \varepsilon M}{A} \quad (19)$$

Thermal Flux Convention

Instead of putting $nv_0 = \bar{n}v$ (= the true thermal neutron flux), it has become customary to put $v_0 = 2200$ meters/sec and to call nv_0 the thermal "flux." The cross section $\sigma(v_0)$ is then evaluated at the energy 0.0252 electron volts, which corresponds to $v_0 = 2200$ meters/second. This is the convention employed in the "flux" values determined for the standard pile. It has the advantage in activation work with thermal neutrons using "1/v" cross section materials that only one, quantity, namely the conventional thermal "flux" need be specified in order to compute the activity. On the other hand, if the true flux $\bar{n}v$ were used in equation (19), then two quantities, namely $\bar{n}v$ and $\bar{v}$ would both have to be specified.

Net Thermal Activity

In employing the cadmium difference method, the fast neutrons are partly absorbed by the cadmium, so that the activity obtained with the cadmium-covered foil must be multiplied by a correction factor, F_{cd} , before subtracting:

$$(C_\infty)_{\text{thermal}} = (C_\infty)_{\text{bare foil}} - F_{cd}(C_\infty)_{cd} \quad (20)$$

The correction factor F_{cd} varies with both the indium foil and the cadmium thicknesses. In part I of the experiment, the cadmium thickness is $\approx .017$ " so that $F_{cd} \approx 1.05$; in part II, $F_{cd} \approx 1.12$, (c.f. Price, W. J., Nuclear Radiation Detection (McGraw-Hill, 1958) p. 290.

$$\text{The cadmium ratio, } R_{cd}, \text{ is defined as } R_{cd} = \frac{(C_\infty)_{\text{bare}}}{(C_\infty)_{cd}} \quad (21)$$

The cadmium ratio is useful in providing a measure of how well thermalized the neutrons are. Using equation (21), equation (20) can be written in the form

$$(C_{\infty})_{\text{thermal}} = (C_{\infty})_{\text{bare foil}} \left[1 - \frac{F_{\text{cd}}}{R_{\text{cd}}} \right] \quad (22)$$

The ratio of the thermal neutron "flux" to the saturated net thermal activity of the foil is called the calibration factor, K , of the foil plus counter system.

$$K = \frac{nv}{(C_{\infty})_{\text{thermal}}} \quad (23)$$

From (19) it is seen that K depends on the mass of the foil, and the efficiency factor, ε . If the "standard" indium foils vary somewhat in mass, it is necessary to express the calibration factor K with respect to some standard mass. The calibration factor K as given here can be applied to determine the flux in any graphite moderating medium without further correction. If the "flux" in another moderating medium is to be determined, then K must be corrected for the relative flux depression produced by the foil in the graphite (of the standard pile) and the other moderator. (c.f., Price, Nuclear Radiation Detection or Hughes, Pile Neutron Research.)

Dead-time Correction

When an ionizing event occurs in the G-M tube, and produces a pulse which is ultimately counted in the scaler, there is a certain time interval (which may vary from event to event) during which the occurrence of a second ionizing event would not be counted. This counting loss is due partly to the properties of the G-M tube and partly to the finite time resolution of the electronic circuitry. The counting errors due to these effects are called "dead-time" or "resolving time" losses and must be considered in any counting experiment. The true counting rate, C_{true} , can be obtained from the observed counting rate, C_{obs} , by use of the formula

$$C_{\text{true}} = \frac{C_{\text{obs}}}{1 - C_{\text{obs}} \tau} \quad (24)$$

where τ is the average total "dead time" (including both G-M tube and electronic effect) and is generally determined experimentally.

In the following the decay of a high activity foil (either irradiated in a high neutron flux, or counted in a high efficiency counting system), the counting intervals should each occupy a small fraction of the half-life and a separate "dead-time" correction must be made for each interval.

In the present experiment, fortunately, the counting rates are sufficiently small that the counting losses should be negligible. However, this problem should be borne in mind with regard to any other activations that are undertaken.

STUDENT RESULTS

Experiment Performed By: E. Abele, G. Burnham, S. Liverhant,
C. Reiser, J. Wilson. June 21, 1961. Submitted by E. Abele

ACTIVATED FOIL # 2571 MASS: 25716N REMOVED AT 11:05									
COUNTER # 1									
TIME	COUNTS	BKGRD	NETT COUNTS	C_1	$e^{-\lambda t_1}$	$C_1 e^{-\lambda t_1 + 5}$	$\frac{-\lambda t_1}{e} - \frac{-\lambda t_2}{e}$	$\frac{C_2}{\lambda}$	
11:07-11:12	13 749	111	13 638 ± 117	7	.97 465	.91406	.06 059	225 086 ± 1931	
11:21-11:26	11 349	111	11 238 ± 107	21	.81 434	.76 371	.05 063	221 963 ± 2113	
11:35-11:40	9 607	111	9 496 ± 96	35	.68 037	.63 810	.04 229	224 544 ± 2317	
				6		$\frac{1}{6^2}$		223 864 ± 1200	
				(2 251 ± .019) × 10 ⁵		2770		(2.239 ± .012) × 10 ⁵	54%
				(3.220 ± .021) × 10 ⁵		2268			
				(2.245 ± .023) × 10 ⁵		1890			
				$\frac{15513}{6928}$		$= (2.239 \pm .012)$			
							0.54%		
COUNTER # 2									
11:14-11:19	10 892	128 ± 5	10 764 ± 104	14	.89 089	.83551	.05 538	194 366 ± 1877	
11:28-11:33	9 187	128	9 059 ± 96	28	.74 436	.69 808	.04 628	195 747 ± 2074	
11:42-11:47	7 228	128	7 100 ± 88	42	.62 192	.58326	.03 866	199 168 ± 2276	
				6		$\frac{1}{6^2}$		196 427 ± 1200	
				(1.944 ± .019) × 10 ⁵		2770		(1.964 ± .012) × 10 ⁵	0.61%
				(1.957 ± .021) × 10 ⁵		2268			
				(1.992 ± .023) × 10 ⁵		1890			
				$\frac{13588}{6928}$		$= 1.964 \pm .012$			
							0.61%		
$K = \frac{\text{COUNT \# 1}}{\text{COUNT \# 2}} = \frac{2.239 \pm .012}{1.964 \pm .012} = 1.140 \pm .009$									

Part 1A Indium foil wo cadmium
Position in pile # 18

Indium foil # 2546
Mass of foil - .2546 gms.
Dia of foil = .75"

6-21-61

Counter #/

BACKGROUND RUNS:

(a) With Foil (Before Irradiation)

(b) Without Foil

112 cts. per 5 Min.

142 cts. per 5 Min.

<u>112</u>	cts. per	<u>5</u>	Min.
114	cts. per	5	Min.

127 cts. per 5 Min.

108 5

$$\frac{114}{5}$$

Irradiation conditions:

$$\frac{114}{12.8 + 7 \text{ c/su}}$$

Clock time of foil put into pile = 11:10

Irradiation time, T = 35 min.

Clock time of foil out = 11:45

Irradiation time,

111496

Clock time start counting = 11:47

t - 2 min.

subscript	count	running time [min]	count = RC	Δt_{tot}	Δt_{i+1}	Bkgd calc. for Δt_{RC}	Δt_{RC}	t_i [min]	λ_i	$e^{-\lambda_i}$	$(e^{-\lambda_i} - e^{-\lambda_{i+1}})$	$\frac{C_0}{\lambda}$
0	0	0	0					2		97.465	0.534	
1	5	6.004	6.004	111		5.893		7		91.406	0.6059	97.260
2	10	11.624	5.620			5.509		12		85.124	0.5682	96.955
	15	16.800	5.176			5.065		17		80.395	0.5329	95.045
4	20	21.674	4.864			4.753		22		75.397	0.4998	95.098
5	25	26.216	4.542			4.431		27		70.710	0.4687	94.538
6	30	30.508	4.292			4.181		32		66.315	0.4395	95.131
7	35	34.576	4.068			3.957		37		62.192	0.4123	95.974
8	40	38.433	3.857			3.746		42		58.326	0.3866	96.896
9	45	42.034	3.601			3.490		47		54.700	0.3626	96.249
10	50	45.477	3.443		Y	3.332		52		51.300	0.3400	98.000
11												
12	$t_i = 2 \rightarrow 52$					$\frac{C_0}{\lambda} = \frac{45.477 - 1110}{37.362 - 3.332} = \frac{44.367 \pm 279}{.46765} = 96.403 \pm 475$						
						$C_0 = 1215 \pm 16 \%$						
						$C_{00} = \frac{C_0}{1 - e^{-\lambda \Delta t}} = \frac{1215 \pm 6}{.86189} = 1398 \pm 17 \%$						

 $T = 35^\circ \text{mm}$

6-21-61

Part / B Indium foil w/ cadmium
Position in pile # 18Indium foil # 2543
Mass of foil = 2543 gms.
Dia of foil = .75"Part 2A Indium foil w/o cadmium
Position in pile # 18Indium foil # 2503
Mass of foil = 2503 gms.
Dia of foil = .75"

COUNTER #2

BACKGROUND RUNS:

(a) With Foil Before Irradiation

134 cts. per 5 Min.
142 cts. per 5 Min.
(2)

Irradiation conditions:

Clock time of foil put into pile = 12:00
Clock time of foil out = 12:45Clock time start counting = 12:49

(b) Without Foil

115 cts. per 5 Min.
129 cts. per 5 Min.
AVE: 122 ± 8 %/MINIrradiation time, T = 45 min.t₀ = 4 min.

abscrip t	counter time min	running count = RC	ΔRC calc. for t _i to t _{i+1}	Bkgrd ΔRC calc. for t _i to t _{i+1}	t _i (min)	λ t _i	e ^{-λ t_i}	e ^{-λ t_{i+1}}	C ₀ λ
0	0	0	0		4	.94996			
1	5	1150	1150	132	9	.89089	.05906	17.237	
2	10	2286	1136	1004	14	.83551	.05538	18.129	
3	15	3322	1036	904	19	.78357	.05194	17.405	
4	20	4353	1031	899	24	.73486	.04871	18.456	
5	25	5312	959	827	29	.68918	.04568	18.104	
6	30	6206	894	762	34	.64634	.04284	17.787	
7	35	7040	834	702	39	.60616	.04018	17.471	
8	40	7819	779	647	44	.56848	.03768	17.171	
9	45	8533	714	582	49	.53314	.03534	16.468	
10	50	9202	669	537	54	.50000	.03314	16.204	
11	55	9875	673	541	59	.46891	.03109	17.401	
12	60	10509	634	502	64	.43976	.02915	17.221	
i = 2 → 62									
C ₀ = $\frac{10509 - 1584}{.94996 - .43976} = \frac{8925 \pm 95}{.51020} = 17493 \pm 186$									
C ₀ = <u>2247 ± 2.4 %/sec</u>									

$$T = 45 \text{ min } C_{00} = \frac{C_0}{1 - e^{-\lambda T}} = \frac{2247 \pm 2.4}{.43876} = 512 \pm 6 \frac{\text{c}}{\text{sec}}$$

$$\text{Count \#1 } C_{00} = \text{Count \#2 } C_{00} \times K_R = (512 \pm 6) / (1.14 \pm 0.009) = 584 \pm 7 \frac{\text{c}}{\text{sec}}$$

COUNTER #2

BACKGROUND RUNS:

(a) With Foil Before Irradiation

134 cts. per 5 Min.
142 cts. per 5 Min.
(2)

Irradiation conditions:

Clock time of foil put into pile = 13:06
Clock time of foil out = 13:56Clock time start counting = 13:58

(b) Without Foil

115 cts. per 5 Min.
129 cts. per 5 Min.Irradiation time, T = 50 mint₀ = 2 min.

abscrip t	counter time min	running count = RC	ΔRC calc. for t _i to t _{i+1}	Bkgrd ΔRC calc. for t _i to t _{i+1}	t _i (min)	λ t _i	e ^{-λ t_i}	e ^{-λ t_{i+1}}	C ₀ λ
0	0	0	0		2	.97465	.02534		
1	5	5816	5816	132	7	.91406	.06592	92.817	
2	10	11098	5282	5150	12	.85724	.05682	90.637	
3	15	16074	4976	4844	17	.80395	.05329	90.899	
4	20	20666	4592	4460	22	.75397	.04998	89.236	
5	25	25022	4356	4224	27	.70710	.04687	90.122	
6	30	29104	4082	3950	32	.66315	.04395	89.875	
7	35	33010	3906	3774	37	.62192	.04123	91.535	
8	40	36684	3674	3542	42	.58326	.03866	91.619	
9	45	40053	3369	3237	47	.54700	.03626	89.272	
10	50	43274	3221	3089	52	.51300	.03400	90.853	
11	55	46300	3026	2894	57	.48111	.03189	90.149	
12									
i = 2 → 57									
C ₀ = $\frac{46300 - 1584}{.97465 - .43976} = \frac{44716 \pm 212}{.53489} = 90890 \pm 430$									
C ₀ = <u>1129 ± 6 %/min</u>									

$$C_{00} = \frac{1129 \pm 6}{1 - e^{-\lambda T}} = \frac{1129 \pm 6}{.47865} = 2384 \pm 13 \frac{\text{c}}{\text{min}}$$

$$\text{Count \#1 } C_{00} = \text{Count \#2 } C_{00} \times K_R = (2384 \pm 13) / (1.140 \pm 0.009) = 2118 \pm 15$$

Indium foil $\frac{2502}{\text{Mass of foil}} \frac{.2502}{.75''}$ gms

BACKGROUND RUNS

2. 124 5 Min
114 5 Min
119 ± 6 % MIN

b. Without Foil
115 5 Min
129 5 Min
122 ± 8 % MIN

Clo¹ " fo l 2 7 1 f 14 02
Cl¹ K " fo l 14 52

Irradiation time, T = 50 min

Clo¹ " H M g 14 54
t₀ 2 min

absorpt i	averaging con p	ΔRC total	Bkgrd calc for ΔRC	ΔRC total	t_1 min	λt_1	$e^{-\lambda t_1}$	λ	C_0 λ
0	0	0			2		.97465	.02534	
1	5	1062	1062	119	943	7	.91406	.06592	15.564
2	10	2099	1037		918	12	.85724	.05682	16.156
	15	3118	1019		900	17	.80395	.05329	16.889
4	20	4042	924		805	22	.75397	.04998	16.106
5	25	4893	851		732	27	.70710	.04687	15.618
6	30	5739	846		727	32	.66315	.04395	16.342
7	35	6572	833		714	37	.62192	.04123	17.314
8	40	7290	718		599	42	.58326	.03866	15.494
9	45	8011	721		602	47	.54700	.03626	16.602
10	50	8652	641	Y	522	52	.51300	.03400	16.353

$$12 \quad 1 = 2 \rightarrow 52 \quad \frac{\bar{C}_0}{\bar{N}} = \frac{8652 - 1190}{97465 - 51399} = \frac{7462 \pm 87}{46066} = 16125 \pm 188$$

$$C_p = 207 \pm 3 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

$$C_{\infty} = \frac{207 \pm 3}{1 - e^{-2T}} = \frac{207 \pm 3}{0.7365} = 281 \pm 4 \text{ } \mu\text{m}$$

$T = 50 \text{ MIN}$

PROPORTIONAL COUNTERS

RESULTS

PART	(C _∞) _{SYST I}	(C _∞) _{SYST II}	CONVERTED TO COUNTS/INST - Y)	CONVERTED TO HITS AND JUST - X)	(ADJ) UN ADJ R _{CD}
Ia	3358 ± 17	—	3358 ± 17	3358 ± 17	575 ± .07
Ib	—	512 ± 6	—	584 ± 7	584 ± 7
Ia	—	2384 ± 13	2718 ± 15	2764 ± 15	443 ± 6
Ib	436 ± 6	—	436 ± 6	—	625 ± .075

x) CONVERSION FACTOR FROM JUST #2 TO JUST #1 MULTIPLY BY 1.40 ± .009 (SEE INTERCALIBRATION SHEET)

y) CONVERSION FACTOR FROM PART #2 TO PART #1 ON HITS MULTIPLY BY 1.017

PART	(C _∞) _{bare}	(C _∞) _{cond}	RATIO F _{cd}	F _{cd} (C _∞) _{cd}	(C _∞) _{NET THERMAL}	K Y)	(mV) ₁₀₀₀ at 0C	FLUX CORRECTION IN %
I	3358 ± 17	584 ± 7	1.05	613 ± 7	2745 ± 18	—	66.11	—
II	2764 ± 15	443 ± 6	1.05	465 ± 7	2299 ± 17	2.41	55.41 ± .41	16.2

y) THE FOL CALIBRATION FACTOR FROM PART I - $K = \frac{mV_{1000}}{(C_{\infty})_{CD}} = \frac{66.11}{2745} = 2.41$ (NIM FOR FOL HAS #18 OF GRID PLE)

PART III - B F₃ COUNTER

DATA cts/min	R_d covered cts/min	F_{cd}	$F_{cd}/(C_{\infty})_{cd}$	NET THERMAL c/min	R_{cd}
1.21×10^6	$.035 \times 10^6$	112	$.039 \times 10^6$	1.179×10^6	34.8

THE THERMAL CALIBRATION FACTOR FOR THE PROPORTIONAL COUNTER

$\left(\frac{\text{CPM}}{\phi_{th}}\right)$ USING FLUX OBTAINED IN PART II

$$F = \frac{(C_{m})_{th}}{(\phi)_{th}} = \frac{2297 \pm 17}{5541 \pm 91} = .416 \pm .004^+$$

THERMAL DEPRESSION FACTOR PRODUCED BY THE BARE INDIUM POIL IN PART I a

$$t = \frac{m}{A} = \frac{.2546 \text{ gm}}{\pi (1.905)^2} = .0888 \frac{\text{gm}}{\text{cm}^2}$$

a) GRAPHITE

$$\sum_a \alpha = \frac{t N_a \Sigma_a}{W} = \frac{.0888 (.602 \times 10^{24}) / (175 \times 10^{-24})}{115} = .081$$

$$\alpha = .13 \quad (\text{FROM TABLE PRICE - N.R.D. pg 288})$$

$$F_{th} = 1 + \frac{.34 \alpha R}{\lambda_{th}} \quad \text{for } R < \lambda_{th}$$

$$= 1 + \frac{.34 (.13) (.95)}{2.7} = 1.016 \quad 1.6\% \text{ in GRAPHITE.}$$

b) D₂O: (R = .953, $\lambda_{th} = 2.4$, $R < \lambda_{th}$)

$$F_{th} = 1 + \frac{.34 (.13) (.953)}{2.4} = 1.018$$

c) WATER: ($\rho_{th} = .953 \text{ cm}$, $\lambda_{th} = .425 \text{ cm}$, $L = 2.74 \text{ cm}$, $R > \lambda_{th}$)

$$F_{th} = 1 + \frac{\alpha}{2} \left[\frac{3RL}{2\lambda_{th}(R+L)} - 1 \right] = 1.097$$

d) PARAFFIN: (R = .953; $\lambda_{th} = .395$, $L = 2.42$, $R > \lambda_{th}$)

$$F_{th} = 1 + .065 \left[\frac{3(.953)(2.42)}{2(.395)(.953+2.42)} - 1 \right] = 1.103$$

MODERATOR	THERM. CAL. FACT.
GRAPHITE	1.016
D ₂ O	1.018
WATER	1.097
PARAFFIN	1.103

ABSOLUTE NEUTRON SOURCE STRENGTH MEASUREMENT.

(Hughes-File Neutron Research, Addison-Wesley, Ch.3, pg.72)

With the absolute neutron source strength one understands the actual amount of neutrons emitted by the source. Such a source is called a standard source, and any other source strength can be determined by comparison.

Standard sources can be based on the following reactions:

1. Ra-alpha-Be source: ${}_4\text{Be}^9(\alpha, n){}_6\text{C}^{12}$

Disadvantage: emission rate changes by shifting the powder.

2. Ra- γ -Be source: Ra is surrounded by Be, not in contact. α from Ra do not reach Be.

The reaction is ${}_4\text{Be}^9(\gamma, n){}_4\text{Be}^8$

Characteristics: a) More constant than the first source for there is no intimate contact between Ra and Be.

b) Has about 10 times smaller neutron rate than the Ra- α -Be source

c) Has lower average neutron energy.

3. Photoneutron sources:

High specific activity γ emitters are used to construct γ -Be or γ -D₂O sources. Some emitters (Na-D₂O, Na-Be) have single energy γ above photodisintegration threshold, therefore produced neutrons are more monoenergetic than Ra- γ -Be neutrons.

Argonne Ra- α -Be source calibration was done by F.O.P. Seidl & S.P. Harris Oct. 1944. Calibration was done by determining the amount of He produced when the neutrons were absorbed in a boron solution ${}_5\text{B}^{10}(n, \alpha){}_3\text{Li}^7$.

The amount of He produced by the neutrons from the source itself is too small to detect. A relative method was used. The neutron flux from the source was compared with the much higher flux in the pile by the means of activation of a manganous solution. The amount of He produced by the high flux in a known amount of Boron was measured. The He produced by Standard source now can be determined resp. absolute neutron source strength.

Argonne source #38 is composed of .504 gm Ra and 3 gm Be. Q = $(5.5 \pm .4) 10^6$ n/sec in Oct. 1944.

Q increases by 2.54 percent per year due to the growth of α emitting Po^{210} .

R. L. I. EXPERIMENT NUMBER 21

ISOTOPE PRODUCTION

INSTRUCTOR: A. J. CASSELL

PROCEDURE:

Select suitable isotopes for activation - sodium nitrite and manganese acetate will be used in this experiment.

With the scissors, cut a small piece of aluminum foil (about 5 cm x 6 cm) and weigh it to the nearest 0.0001 gm on the analytical balance. Next, place approximately 0.2 gm of NaNO_2 on the foil, exact weight to be determined to an accuracy of 0.0001 gm. Repeat with $\text{Mn}(\text{C}_2\text{H}_3\text{O}_2)_2 \cdot 4\text{H}_2\text{O}$ using a sample weight of approximately 0.14 gm.

Carefully fold the salt in the foil to form a tight seal, taking precaution not to lose any of the salt. Carefully place the packet into an aluminum container and secure the cap. (See instructor for this procedure).

(Containers are now ready to be placed in the CP-5 Reactor for activation. However, in this case, previously prepared samples of NaNO_2 and $\text{Mn}(\text{C}_2\text{H}_3\text{O}_2)_2 \cdot 4\text{H}_2\text{O}$ which have been activated will be ready for completion of the experiment).

On receiving the activated samples, monitor the aluminum can in order to determine whether excess dose rate is present. (Excess dose-rate, in this case, would be 100 mR/hr or higher). Carefully remove the aluminum packet from its container and empty the salt into a 30 ml beaker. Wash the foil several times with distilled water using the squeeze bottle, collecting the washings in the beaker, until the foil shows no more activity. Pour the solution from the beaker into a 50 ml volumetric flask (do not spill) and rinse the beaker with distilled water. Pour the rinsings into the volumetric flask and repeat until the beaker shows no more activity. Now carefully fill the remainder of the volumetric flask with distilled water and replace the stopper. Repeat the above procedure with the other sample. From your data determine the concentration of the solutions expressed in mg/ml.

Remove an aliquot portion, 10 λ volume, of the NaNO_2 solution with the pipet and place on a stainless steel planchet under the infra-red lamp. Allow the sample to dry; then heat the planchet over a Bunsen burner until the planchet becomes red hot and allow it to air cool. Repeat with the $\text{Mn}(\text{C}_2\text{H}_3\text{O}_2)_2 \cdot 4\text{H}_2\text{O}$ solution using a 25 λ sample.

Determine an initial activity of each isotope using the proportional flow counter. Determine the half-life of Na^{24} and Mn^{56} by taking counts of each every 10 minutes and 5 minutes respectively. Plot a graph on semi-log paper with Counting Rate (cpm) as ordinate vs. Time (min) as abscissa. Because of the difference in half-lives of the two isotopes, use a separate sheet of paper for each, making the time scale of appropriate length in order to show a good slope.

DATA:

Weight of foil and salt _____ mg

Weight of foil and salt _____ mg

Weight of foil _____ mg

Weight of foil _____ mg

Net weight of salt _____ mg

Net weight of salt _____ mg

Conc. of solution _____ mg/ml

Conc. of solution _____ mg/ml

Initial activity (Time O) _____ mc

Initial activity (Time O) _____ mc

Sample No. _____

Irradiation Time _____

Background _____ cpm.

Time (min)	Gross Counts	Gross - Bkg.	Net Counts

R. L. I. EXPERIMENT NUMBER 22

ANALOGUE COMPUTER I

INSTRUCTORS: L. T. BRYANT, L. C. JUST, N. F. MOREHOUSE, JR.

The descriptive text for this laboratory experiment is:

INTRODUCTION TO ELECTRONIC ANALOGUE COMPUTING,
L. T. Bryant, L. C. Just, and G. S. Pawlicki, ANL-6187, 1960

This document is not included here. It is available through the Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C.

Laboratory Report on
CALCULATION OF XENON BUILD-UP USING AN ANALOGUE COMPUTER

The Analogue Computer - Its Use in Nuclear System Analysis

Report Prepared by

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It seems inevitable that one of the basic courses that every engineer and scientist will find it necessary to take and understand is that of system analysis. Some colleges are already presenting such a course, at least from the viewpoint of practical applications, on the sophomore level. The justification for the presentation of this course before the student has achieved a sufficient theoretical background, particularly in mathematics, is the ever-growing recourse to the use of the computer, both analog and digital, for the solution of engineering problems.

Such a course in System Analysis would roughly include:

- a. various methods of unit or block diagram representations of complete systems.
- b. mathematical abstraction of these physical diagrams into easily-manipulated forms such as LaPlace transforms of integro-differential relationships. Various mathematical tools can then be used leading to partial or complete solutions.
- c. the representation and solution of the original system through the construction of an analogous physical system by means of the mathematical transforms. This latter operation is the function of the electronic computer. When the solution is a time-varying function the device is known specifically as an analog computer.

It is unfortunate that, at least for the next few years, we will not be able to balance the weight of the slide-rule on our belts by a solid-state, non-linear, distributed-parameter analyzer of equivalent size and weight. For the present we must be satisfied with the cumbersome electronic analog computer whose basic parts are high-gain tubes or transistors, resistors, capacitors and potentiometers. The tube is essentially limited to its use as a linear, uni-lateral voltage amplifier. The resistor, by Ohm's Law, defies

the property of impedance called resistance as $R = e/i$. The capacitor, by definition, leads to the property of capacitance, $C = q/e = \int idt/e$, which can be written in transform form as $C = 1/s \ i/e$. Hence the impedance of a capacitor becomes $Z = e/i = i/s \ i/c$. The potentiometer, if loaded down by an equivalent infinite impedance, acts as a voltage divider or a constant-coefficient multiplier by a value less than one.

If we now represent a voltage amplifier consisting of an odd number of tube stages together with a multiple input and a feedback path, we can diagram it in the following manner:

By Kirchhoff's Law:

$$i_1 + i_2 + \dots + i_n + i_f = 0$$

since the current into the amplifier is zero.

Using Ohm's Law this equation can be written as:

$$\frac{e_1 - e_g}{Z_1} + \frac{e_2 - e_g}{Z_2} + \dots + \frac{e_n - e_g}{Z_n} + \frac{e_o - e_g}{Z_f} = 0$$

Since the gain of the amplifier (e_o/e_g) is in the order of 10^8 and the ratio (e_o/e_n) is usually less than 10^2 , the e_g terms in the above expression can be equated to zero. Then the expression can be reduced to the following form:

$$e_o = -\frac{Z_f}{Z_1} e_1 - \frac{Z_f}{Z_2} e_2 - \dots$$

Hence, according to the value of the impedance ratio ($A = Z_f/Z_m$), this device can act as one or more of the following:

- a. an inverter or sign changer
- b. a summer

- c. a multiplier ($A = R_f/R_n$)
- d. an integrator or a differentiator depending on whether the impedance ratio is

$$\frac{Z_f}{Z_n} = \frac{1/s C}{R_n} = \frac{1}{s} \frac{1}{RC}$$

its inverse.

Ordinarily in an integro-differential problem we can express the basic relationships in terms of a set of simultaneous differential equations. Any terms involving integration can be removed by raising the degree of the respective differential equation. As a consequence the programming of the problem on the computer will avoid the use of differentiators which are troublesome due to their unstable voltage characteristics and noise levels.

As an example of the use of the use of the analog computer in the solution of a problem in Nuclear Engineering, there is an experiment concerning iodine and xenon build-up in a reactor which is described in the report ANL-6187 on pages 51 to 58. Starting with the kinetic equations of the original system, the complete programming of the problem on the Pace Analog Computer is described. Note that, in order to program the problem, the set of given kinetic equations must be time-dependent just as is the analog computer itself. The number of original equations must, at least, equal the number of time-varying dependent variables. However, on the computer there is but one dependent variable; namely, voltage. So each of the original variables must be converted to a corresponding voltage on the computer program by a unit-converting factor. This factor also serves partially as an amplitude-converting factor in order to stay within the linear range of the amplifiers. This necessitates that the maximum time-varying output of any amplifier must never exceed the absolute value of one hundred volts. Likewise the analog computer and the recording devices have an optimum operating speed for maximum accuracy and efficiency. If this does not correspond to the actual or real time of the original system a time-converting factor must be used. This latter factor also influences the amplitude factor.

Besides the problem of conversion factors a question arises as to the feasibility of using the kinetic equations as derived from the original system or mathematically manipulating them into a more concise form before proceeding with the programming. This question is usually resolved on the basis of the following considerations:

- a. the analog equipment on hand
- b. the information desired
- c. the mathematical ability to derive the new relationships or transfer functions.

Usually, as was done in this experiment, the program placed on the computer matches the original system as closely as possible. In this manner a maximum amount of information is obtained.

With these considerations in mind the problem was programmed as described in the report, ANL-6187, and the desired graphical results were obtained.

STUDENT RESULTS

Experiment Performed By R. W. Clack, G. H. Burnham, C. O. Reiser, B. J. Stewart, J. A. Wilson, July 6, 1961, Submitted By B. J. Stewart.

Iodine and Xenon Concentration in a Reactor

The programming of the kinetic equations necessary to express the iodine and xenon concentration as a function of time is outlined in the report ANL-6187, pages 51 to 58. The graphical results shown in this paper are obtained by the flux levels defined by the following time relationship:

$$\phi = \phi(0) \quad [t \leq 0]$$

$$\phi = 0 \quad [0 < t < t_1]$$

$$\phi = \phi(0) \quad [t_1 < t]$$

The two curves if continued beyond the time limit of the graph paper would maintain an exponential rise towards the steady-state level corresponding to a flux, $\phi(0)$. The break-away section of the xenon curve represents a separate run where the flux was restored to the zero-time level at about the time, $t = 6.5$ hours. It is to be noted that all curves are normalized by the zero-time value of the iodine concentration.

FODINE
 AND
 XENON
 Poisoning
 from
 Trace Ampley Composite

R. L. I. EXPERIMENT NUMBER 22

ANALOGUE COMPUTER II

INSTRUCTORS: L. T. BRYANT, L. C. JUST, N. F. MOREHOUSE, JR.

The descriptive text for this laboratory experiment is:

ENGINEERING APPLICATIONS OF ANALOG COMPUTERS
L. T. Bryant, M. J. Janicke, L. C. Just and Alan L. Winiecke, ANL-6319, 1961

This document is not included here. It is available through the Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C.

Laboratory Report on
CALCULATION OF REACTOR KINETIC BEHAVIOR USING AN
ANALOGUE COMPUTER

Report Prepared by

George H. Burnham

Department of Physics
 Norwich University
 Northfield, Vermont

Experiment performed on July 10, 1961 during ASEE-AEC Reactor Laboratory Institute by: George H. Burnham, Robert W. Clack, Castle O. Reiser, Joseph Stewart, John A. Wilson.

Experiment C - Reactor Kinetics (pp. 59-66, ANL 6187) was performed by the group. Graphical solutions of the reactor kinetic equations were obtained for step and ramp insertions of various positive and negative values of k_{ex} and for a sinusoidally varying k_{ex} of two frequencies and two amplitudes. Experiment III - Reactor Kinetics Over Many Decades with Thermal Feedback (pp. 18-26, ANL 6319) as modified by L. Bryant (to extend the decade range beyond that available in the circuit diagram of Figure 8) was also performed with considerable assistance from L. Bryant. Graphical results ($\ln n(t)/n(d)$ as a function of time and K_{eff} as a function of time) for two different ramp insertions of $K(t)$ were obtained.

In Experiment C, on which most of the time was spent, considerable attention was given to the development of the Analog Diagram (Figure 56). The basic equations for the time dependent behavior of a bare zero power reactor (see, for example, pages 152-154, Murray, Nuclear Reactor Physics, but note that Murray's k_e is not identical with k_{ex} used in ANL 6187) are first altered by normalizing n (to $n_1 = n(t)/n(o)$) and each C_i (to $C_{i1} = C_i(t)/C_i(o)$). Consideration of these equations leads to a basic analog circuit, subject to modification in the interest of minimizing the number of components used. A brief introduction to this procedure follows:

Basic equations:

$$A. \quad \frac{dn_1}{dt} = \frac{\beta}{\ell} \left[(1 - \beta) k_{lex} n_1 - n_1 + \sum_{i=1}^6 \frac{\beta_i C_i}{\beta} \right]$$

$$B. \quad \frac{dC_{i1}}{dt} = \lambda_i \beta_i k_{lex} n_1 + \lambda_i n_1 - \lambda_i C_{i1}$$

Solution of Equation A requires an integrator, the input to which (dn_1/dt) is the sum of three terms:

$$\beta/\ell [(1 - \beta) k_{lex} n_1] , \quad -n_1 \beta/\ell \quad \text{and} \quad \sum \beta_i C_{i1}/\ell$$

Equation B actually consists of six equations, each requiring an integrator whose input (dc_i/dt) is the sum of three terms:

$$\lambda_i \beta_i k_{lex} n_1 , \quad \lambda_i n_1 \quad \text{and} \quad -\lambda_i C_{i1}$$

The output of the integrators are the values of $-C_i$.

Schematically:

It is also necessary to supply a k_{lex} term which can be varied with time in a desired arbitrary manner (step, ramp, sinusoidal, etc.). With the required multiplications, all of the terms appearing at the inputs of the integrators can now be produced. It will be noted that, except for the two terms involving the product $k_{lex} n_1$, all multiplications are by constants. These latter can be accomplished by potentiometer voltage dividers alone if the constant factor is less than 1 but still large enough to be set on the potentiometers with the required accuracy. If the factors do not fall within this range, the multiplications can be carried out in two or more steps involving the use of operational amplifiers, with which multiplications by factors greater than one can be carried out, combined with potentiometers.

In the special case where k_{lex} is a step function consisting of an abrupt change from one constant value to another, the multiplication of k_{lex}

and n_1 can also be carried out using a tapped potentiometer arrangement of the following sort.

$$k_{1ex} = \frac{R_1}{R_1 + R_2 + R_3} (\pm n_1)$$

If, as in our case, it is desired to vary k_{1ex} in other ways, the multiplication of $k_{1ex}(t)$ by $n_1(t)$ is accomplished by a servo-multiplier in which the position of the slider on a potentiometer is controlled by an electrical input voltage to a servo system.

The desired input to the servo may be generated by the use of switches and/or operational amplifiers (see Experiment B, p. 57, ANL 6187).

Having established a general scheme by which all of the necessary terms for internal consumption as well as the desired variable (n_1) can be derived. There remains the more detailed problem of establishing scale factors for converting the equation variables into voltages which will, for the time interval of interest, remain within the range which can be handled by the operational amplifiers. An upper limit to these voltages is set by the overload point of the amplifiers, a lower limit is set by amplifier "noise". (In practice these limits are about ± 100 volts and ± 1 volt). At the same time detailed consideration must be given to meeting the requirements for

multiplication for potentiometers previously mentioned. Experience and a knowledge of the general nature of the expected solution are necessary if cut-and-try experimentation is to be minimized.

Initial conditions, translated into actual voltages must be introduced at the proper points. In this experiment, the initial condition for the equation variables are that

$$n_1(0) = \frac{n(0)}{n(0)} = 1$$

and

$$C_{i1} = \frac{C_i(0)}{C_i(0)} = 1 \quad i = 1, 2, \dots, 6$$

The recorder contains no drive motor for x displacement of pen (or paper) but is servo driven. An electrical input ($v = ct$) to this system must, therefore, be provided. This consists of an integrator whose input is a constant voltage (e) and whose output (v) is given by

$$v = \int_0^t e dt = et$$

(See Experiment B, Figure 4, p. 56, ANL 6187)

The second experiment (No. III in ANL 6319) is an extension of the first in two ways. (1) the variable corresponding to n ($n = \ln n_1$) goes through a very much wider range of values and (2) a temperature coefficient type of feedback is introduced so that the reactor eventually stabilizes itself after a change in K .

The extension of the flux-related variable from one decade to 6 or more is made possible by the substitution of a logarithmic function for n and by the use of sequentially operating set of differential relays. To the arbitrarily time-dependent k_{ex} of the first experiment is added to a second term $k(n,t)$ which, in its n -dependence, corresponds to a temperature coefficient since temperature is also a function of n .

The normalized delayed neutron precursor concentrations (C_1) are replaced by an exponential variable

$$\psi_i = e^{-n} C_{i1} - 1$$

The change of variables introduces a set of complicating $\psi_i \frac{dn}{dt}$ terms necessitating the use of additional multipliers. The generation of the n -dependent part of k provides an introduction to the use of the diode function generator and the differential relays.

 K_{eff}

Time