FRONT-FORM CALCULATION OF $7d^{*}np$ REACTIONS
AT HIGH ENERGIES

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Abstract

A front-form calculation of $7d \to np$ reaction has been performed and compared with the data at 90°.

To investigate nuclear dynamics in the high energy and/or high momentum-transfer regions, it is necessary to develop a relativistic formulation of the problem. The most well developed relativistic formulation is the lagrangian local quantum field theory. Although significant progress has been made in recent years, rigorous theoretical methods for predicting bound states of a system of strongly interacting particles, such as the deuteron, from a given lagrangian are still being developed. The traditional way to circumvent this difficulty is to assume that the nuclear dynamics can be effectively defined within the nonrelativistic particle quantum mechanics and the interaction potentials and operators for observables are "derived" from taking equal-time limits of a set of field theory amplitudes. This phenomenological approach has been very successful in investigating low energy nuclear physics. In this talk I shall describe a similar procedure, developed in a collaboration with Coester and Kondratyuk, to investigate the photo-disintegration of the deuteron at high energies.

Our approach is to assume that the nuclear dynamics can be defined within the relativistic particle quantum mechanics. This relatively unfamiliar theoretical framework has recently been explicitly presented in an excellent review by "Eister and Polyzou [1]. To introduce this short talk, I only want to point out that this approach allows us to use the "existing" meson-exchange two-nucleon models, such as the Paris potential and coupled-channels $NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN\otimes NN$ models, to carry out a fully relativistic calculation of $7d^{*}np$.
reaction. Our formulation follows closely that developed by Chung, Coester, Keister and Polyzou [2] in their study of e-d elastic form factors.

The form of the relativistic quantum mechanics is not unique since the energy-momentum relation $P^2 = P#P# = M^2$ is of a bi-linear form. Our formulation is a front-form defined by a light-like vector $n# = \{1,Îô\}$ with $|Îô| = 1$. Any four vector is then defined with respect to this choice as

$$A# : \{A_-, A_T\} \quad \text{with} \quad A : \{A_-, A_T\} \quad \text{and} \quad A_T \cdot Îô = 0 .$$

The scalar product of two four vectors is defined as

$$A \cdot B = \frac{1}{2} \left[ A^+ B^- + B^+ A^- - 2A_T \cdot B_T \right] .$$

It is common to choose $Îô \parallel \frac{1}{2}$ and hence in terms of the usual canonical components $A# = (A_0, A_1, A_2, A_3)$ we have $A# = A_0 \pm A_3$ and $A_T = A_1 Îô + A_2 îô$. We then have for a particle with mass m

$$p#p# = p^- p^+ - p_T^2 = m^2 .$$

The front-form Hamiltonian is defined as

$$H = p^- = \frac{m^2 + p_T^2}{p^+} \quad (1)$$

To describe a two-particle system, we first write in the absence of interactions

$$p# = p_1 + p_2 = (p^+, p_T) \quad (2)$$

$$H_0 = \frac{M_0^2 + p_T^2}{p^+} \quad (3)$$

where the mass operator $M_0$ can be expressed in terms of the intrinsic momentum defined by a front-form Lorentz boost defined by

$$L_F(p) \{p^0, p^1, p^2, p^3\} = \{M_0, 0, 0, 0\}$$

Explicitly we have
\[ M_0^2 = 4 \left( m^2 + \vec{k}^2 \right) \]  

where

\[ \vec{k}^2 = k_n^2 + k_T^2 \quad , \quad k_n = \vec{k} \cdot \hat{n} \ . \]

The components of front-form vector \( \vec{k} \{ k^+, k_T \} \) are defined by

\[ k^+ = \{ L_f(p)p_1 \}^+ = M_0 \xi \]

\[ \vec{k}_T = \{ L_f(p)p_1 \}_T = \vec{p}_1 - \vec{p}_T \xi \]

\[ k_n = M_0 \left( \xi - 1/2 \right) \]

where \( \xi = p^+ / P^+ \) is the momentum fraction of the first nucleon. The spin of the system is defined as

\[ J = i \, \vec{\sigma} \times \vec{k} + R_M \left( \xi, \vec{k}_T, m \right) \vec{s}_1 + R_M \left( 1 - \xi, \vec{k}_T, m \right) \vec{s}_2 \]

where

\[ R_M(\xi, \vec{k}_T, m) = \frac{m + \xi M_0 - i \, \vec{\sigma} \cdot (\vec{n} \times \vec{k}_T)}{[(m + \xi M_0)^2 + \vec{k}_T^2]^{1/2}} \]

is the Melosh transformation.

The dynamics are introduced by modifying the mass operator

\[ M_0^2 + M^2 = M_0^2 + 4m V_{12} \]

As discussed in details in Refs. [1] and [2], the relativistic invariance can be achieved if we require that \( V_{12} \) is a function of only \( \vec{k}, \vec{V}, \vec{s}_1, \vec{s}_2 \) and satisfies \( [V_{12}, J] = 0 \). Then the eigenfunctions of the four momentum

\[ p^\mu : \{ H, p \} \quad \text{with} \quad H = \frac{M^2 + p_T}{p^+} , \quad p^\mu = \{ p^+, p_T \} \]

can be written as
\[ H | \psi_{pd, \mu_d} \rangle = \frac{M_d^2 + p_d^2}{p_d^+} | \psi_{pd, \mu_d} \rangle \] (9.a)

\[ \hat{p} | \psi_{pd, \mu_d} \rangle = \hat{p} | \psi_{pd, \mu_d} \rangle \] (9.b)

The main point is that the wave function, Eq. (9), can be directly constructed from the existing NN models. This is explicitly given in Eqs. (2.27)-(2.31) of Ref. [2] for the deuteron ground state. Extensions of these equations to calculate np scattering state is straightforward. This practical simplicity is due to the fact that the form of \( M^2 \chi = E \chi \) can be cast into the form of the usual nonrelativistic Schrödinger equation in momentum space, and the front-form boost transformation is only kinematic.

The amplitude of the \( 7d+np \) reaction is

\[ T_{fi} = \epsilon_{\mu} \langle \gamma | J^\mu | i \rangle \]

where \( \epsilon_{\mu} \) is the photon polarization vector and

\[ \langle \gamma | J^\mu | i \rangle = \langle \chi_{pd, \mu} | J^\mu | \psi_{pd, \mu_d} \rangle = \sum_{\mu', \mu_2, \mu_2} \int d\vec{p}_1 d\vec{p}_2 d\vec{p}_1' d\vec{p}_2' \chi_{\mu}^{(-)} (\vec{p}_1, \mu_1 \vec{p}_2, \mu_2) \chi_{\mu}^{(-)} (\vec{p}_1', \mu_1' \vec{p}_2', \mu_2') \]

\[ \langle \vec{p}_1 \mu_1 \vec{p}_2 \mu_2 | J^\mu | \vec{p}_1' \mu_1' \vec{p}_2' \mu_2' \rangle \phi_{pd, \mu_d} (\vec{p}_1, \mu_1, \vec{p}_2, \mu_2) \] (10)

To proceed, we need to define the front-form matrix elements of the current operator \( J^\mu \). We choose the light-front vector \( n^\mu \) such that \( q^+ = 0 \) and hence \( \hat{q}_T = 0 \) because for a real photon \( q^+ q^- - \hat{q}_T^2 = 0 \). In the \( 7d \) cm frame, this choice of front-form dynamics requires setting \( \hat{p}_d = -\hat{q} \parallel \hat{n} = \hat{z} \). The current conservation then leads to the following condition

\[ q_\mu \langle \gamma | J^\mu | i \rangle = q^+ \langle \gamma | J^- | i \rangle + q^- \langle \gamma | J^+ | i \rangle - q_T \cdot \langle \gamma | J_T | i \rangle = 0 \]

Hence any acceptable model of the current operator must satisfy the following condition

\[ \langle \gamma | J^+ | i \rangle = 0 \] (11)
The total front-form momentum vector is conserved in any reaction. Since 
\( q^+ = 0 \) and \( \hat{q}_T = 0 \), we then have for the \( \gamma d + np \) reaction
\[
\hat{q} = (q^+, \hat{q}_T) = 0 \quad \text{and} \quad \hat{q} + \hat{p}_d \equiv \hat{p}_d = \hat{p} .
\] (12)

This means that the total front-form three vectors \( \hat{p}_d (p^+, \hat{p}_T) \) of the final np system is equal to \( \hat{p}_d \) of the initial deuteron state. Thus the impulse current matrix element takes the following "diagonal" form
\[
\langle \hat{p}_1 \mu_1 \hat{p}_2 \mu_2 | J^{\mu} | \hat{p}_1 \mu_1 \hat{p}_2 \mu_2 \rangle
\]
\[
= \delta(p_1^+ - p_1^-) \delta_{\mu_1 \mu_2} \delta(p_2^+ - p_2^-) \langle \hat{p}_2 \mu_2 | J^{(\mu)} | \hat{p}_2 \mu_2 \rangle + (1 \leftrightarrow 2) .
\] (13)

The one-body current matrix element can be directly related to the known electromagnetic properties of a single nucleon
\[
\langle \hat{p} \mu^- | J^{\mu} | \hat{p} \mu^- \rangle = \bar{u}_{\mu} \gamma_{\mu}^\dagger u_{\mu}
\] (14)

where \( u_{\mu} \) is the front-form Dirac spinor. It can be shown for the \( J^+ \) component that
\[
\langle \hat{p} \mu^- | J^+ | \hat{p} \mu^- \rangle = 2 \delta_{\mu \mu} .
\] (15)

Fig. 1 The predicted differential cross section of \( \gamma d + np \) is compared with the data [4]. The dotted curve is obtained when the final state interaction is neglected.
Substituting Eqs. (13)-(16) into (10) and noting the orthogonality of the scattering and bound state wave functions, we then have

\[ \langle f | \mu^+ | i \rangle \propto 2 \langle \chi^{(-)}_\mu \left| \phi_{p_d, \mu_d} \right\rangle = 0 \]  

(16)

The current conservation condition Eq. (11) is therefore satisfied.

The calculation was performed by using the Paris potential. The calculated differential cross sections at 90° are shown in Fig. 1. The dotted curve is obtained when the final state np interaction is neglected. Clearly, at GeV energies the main contribution is due to the np final state interaction. The predicted energy-dependence of the present front-form calculation is not too different from that of our earlier coupled-channel calculation [3]. It appears that the neglect of relativistic effects is not the main reason for the failure of the meson-exchange calculations to describe the data. Note that at 2 GeV incident photon energy the final np scattering energy is nearly 4 GeV in the laboratory system. The use of the Paris potential for the final state is obviously not correct. In order to explore whether the \( \gamma + \text{np} \rightarrow \text{pp} \) reaction can be described in terms of hadronic degrees of freedom, we need to develop a NN model which can account for the NN data up to about 5 GeV. It is also necessary to develop an approach to account for two-body currents in the front-form formulation.

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References


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