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THROUGH A TENUOUS MAGNETIZED PLASMA

By

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ABSTRACT

When an ion beam is propagated through a plasma, the question of charge neutralization is critical to its propagation. We consider such a problem where the plasma is magnetized with magnetic field perpendicular to the beam. The plasma-number density and beam-number density are assumed comparable. We reduce the problem to a two-dimensional model, which we solve. The solution suggests that it should be possible to attain charge neutralization if the beam density is properly varied along itself.

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1. INTRODUCTION

The question has been raised whether it is possible to send a low density, energetic ion beam through the earth's ionosphere. An essential unresolved problem is to determine the degree of charge neutralization of the beam by the ionospheric plasma electrons since the earth's magnetic field prevents charge neutralization in any other way. In this paper we consider the general problem of propagation of an ion beam across a magnetized plasma under conditions in which the beam density and plasma density are comparable. We show that by properly tapering the beam density along its length it is usually possible to charge neutralize it.

In Sec. II we set up a simplified model for treating the problem. In Sec. III we give the numerical solution of the model. In Sec. IV we derive an analytic solution for the case where beam density is small compared to plasma density. In Sec. V we consider the applications of our results.

II. THE MODEL FOR CHARGE NEUTRALIZATION

Consider an energetic charged ion beam with a density n_b and finite circular cross section of radius R propagating at a velocity v through a cold plasma of density n_0 and across a strong magnetic field B [Fig. 1(a)]. In the absence of the plasma the electrostatic field of the beam will tend to expand it transversely. This outward force is counteracted to some extent by the inward magnetic force produced by the beam current. However, it is well known (see e.g. Ref. 1) that in the absence of any neutralizing electrons this inward magnetic force is smaller than the outward electric force, and the beam must expand radially.

We consider beams whose current is so small that the beam-generated magnetic field is very small compared to the ambient field. That is, we

assume

$$\frac{2\pi R n e v}{c} < B \quad , \quad (1)$$

Further, we also assume that any polarization electric fields present in the beam are too small to carry the electrons across B at the beam speed. This amounts to

$$\frac{c E_{pol}}{B} < \frac{4\pi n_B e R c}{B} < v \quad , \quad (2)$$

where the left inequality gives an upper limit on E_{pol} that corresponds to total charge separation in the beam. Finally, we assume the electron gyroradius at velocity v is small compared to propagation distances of interest. For the specific example of Sec. V these conditions are well satisfied. If they are satisfied, the only way to charge neutralize the beam is by motion of the plasma electrons along the ambient field.

In the presence of the cold plasma through which the beam is considered to be passing, neutralization can be produced by motion of the plasma electrons. The very electric field that tends to expand the beam also causes the plasma electrons to move along the direction of the external magnetic field into the beam region, thereby, producing partial neutralization. Because the electrons are much more mobile than the ions, they will respond to a much greater extent to the electric field. The resulting neutralization will reduce the electric field to such an extent that a dynamic balance is reached between the net excess of ion density above the electron density and the electric field necessary to produce the required electron density in the beam. (In the first approximation the plasma ions are assumed to be too heavy

to respond to the electric field and thus remain static.)

In order to determine this balance, the goal of this paper, it is important to consider times during which the beam first propagates into the plasma. This is equivalent to asking about the variation of ion density at the head of the beam. If the ion density rises abruptly, in a time short compared to the electron-plasma period, the electrons will see a constant potential and oscillate through the beam with no loss of energy. Little charge neutralization will result. On the other hand, if the beam density at a fixed point rises slowly in time compared to the electron-plasma period, then a balance can be reached adiabatically, and no large scale oscillation of the system results. (The electrons are gradually drawn into the beam and are adiabatically trapped in the self-consistent potential well, the well produced by the ion beam as modified by the electron density.) This happens comparatively easily if the initial plasma density n_0 is larger than the beam density n_b . On the other hand, if n_b is larger than n_0 , neutralization is more difficult and the ion beam density must rise even more slowly to achieve it. A characteristic length for the beam head variation is v/ω_{pe} , where v is the beam propagation speed and $\omega_{pe} \equiv (4\pi n_0 e^2/m)^{1/2}$. [See Fig. 1(b)]. Since in all cases of interest this length is much larger than the beam radius, the beam density can locally be taken as approximately constant along its axis. This fact allows us to model the problem as two-dimensional.

To formalize the problem mathematically, we wish to determine the degree of neutralization of the ion beam as a function of the ratio of n_b to n_0 and as a function of the ratio of initial rate of rise of the beam density γ (at a fixed point) to ω_{pe} . We imagine an infinite cylinder of time-dependent ion density varying in time and parabolic in radius, see Figs. 2(a) thru 2(c)

$$n(r,t) = n_B f(t) g(r),$$

$$f(t) = 1 + \tanh \gamma t$$

$$g(r) = \left(1 - \frac{r^2}{R^2}\right) \quad (3)$$

(Note that the peak ion density at $r = 0$ is $2n_B$, but the average ion density in the beam is n_B .) This cylinder is embedded in a uniform cold plasma of density n_0 . The plasma electrons are constrained to move in the x direction by a magnetic field in that direction, $B = B_0 \hat{x}$. The z axis is taken along the cylinder. The ambient plasma ions are taken fixed in space. We take all quantities independent of z for our model. Typical electron trajectories are shown in Fig. 2(d).

The equation of motion of each electron is

$$m \ddot{x} = -e \frac{\partial \phi}{\partial x}(x, y) \quad (4)$$

If $n_B < n_0$ and the background electrons are cold, Eq. (4) may be linearized to solve for the displacement $\xi \equiv x - x_0$ where x_0 is the initial position of the electron. The perturbed electron density is

$$n_{e1} = -\frac{\partial \xi}{\partial x_0} n_0 \quad (5)$$

This perturbed density is combined with Poisson's equation in Sec. IV to obtain an analytical solution.

For $n_B > n_0$ we employ a particle-in-cell (PIC) code² to follow the solution of Eq. (4) for a large number of electrons and to calculate their

density. These electrons are started with random initial velocities and positions consistent with initial density n_0 and with a temperature T_0 .

The electrostatic potential ϕ is determined by solving Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 4\pi e \left[n_e(x, y, t) - n_0 - \begin{cases} n_B f(t) \\ 0 \end{cases} (1 - r^2/R^2) \right] \begin{cases} x^2 + y^2 < R^2 \\ x^2 + y^2 > R^2 \end{cases} \quad (6)$$

The numerical method is described in Sec. III.

We are primarily interested in the reduction of the electrostatic force on the beam ions. A figure of merit for this is the ratio of the mean charge density to the mean ion-beam density

$$\epsilon = \frac{\int [gfn_B + n_0 - n_e(x, y)] dx dy}{\int gfn_B dx dy} \quad (7)$$

where the integrals are taken inside the circle $x^2 + y^2 = R^2$ since this represents the degree of charge neutralization. At large times, ϵ becomes approximately a constant, ϵ_0 (see Fig. 3). The two significant parameters of the problem are n_B/n_0 and γ/ω_p . Results showing curves of constant ϵ_0 are given in Fig. 4.

For the beam to hold together we require $\epsilon_0 < (v/c)^2$. If this condition is satisfied, the repulsive electrostatic force is reduced to a value less than the attractive magnetic force. Note that because the electrons are constrained to move only in the x direction there is no reduction in the magnetic force. (There is charge neutralization, but no current neutralization.)

From Fig. 4 we see that smaller values of γ/ω_{pe} indeed favor charge neutralization while large values of the ratio n_B/n_0 oppose it. It appears

that for sufficiently small values of γ any degree of charge neutralization can be attained.

The analytical linearized solution of Eqs. (3) - (6) is given in the limit of small n_B/n_O , for $f(t) = e^{\gamma t}$ and n_B independent of r . The results of this linearized calculation support the numerical results of Sec. III, when account is taken of the different time and space behavior.

In Sec. V some of the physical results of our solutions are discussed as well as the validity of our model for an actual problem, the propagation of a very energetic ion beam through the earth's ionosphere.

III. THE NUMERICAL METHOD

Before describing the numerical code to solve Eqs. (3) - (6), it is convenient to write them in dimensionless form by choosing as units of time and space ω_{pe}^{-1} and R . Let $\Phi \equiv e\phi / (m\omega_p^2 R^2)$, $N = n_e/n_O$, $N_B = n_B/n_O$, and $\Gamma = \gamma/\omega_p$. Then we have

$$n \approx N_B f(t) g(r) ,$$

$$f(t) = [1 + \tanh(\Gamma t)] ,$$

$$g = 1 - r^2 , \tag{3a}$$

$$\frac{\partial^2 \Phi}{\partial t^2} = + \frac{\partial \Phi}{\partial x} , \tag{4a}$$

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = N - 1 - \left\{ \begin{array}{ll} N_B f(t) g(r) & r < 1 \\ 0 & r > 1 \end{array} \right\} \tag{5a}$$

and

$$\epsilon = \frac{1}{\pi N_B f(t)} \int_{x^2+y^2 < 1} [N_B f(t)g(r) + 1 - N(x,y)] dx dy \quad (6a)$$

and N is to be obtained from the PIC code.² Clearly the two parameters of the problem are N_B and Γ .

We expect electrons with initial positions $x < N_B$, $y < 1$ to be drawn into the beam while those with $x > N_B$ or $y > N_B$ to be relatively unaffected. Thus, we solve these equations on a grid that exceeds these bounds. The grid spacing is chosen such that there are 30 grid cells in the y direction and of order 10-15 cells in the x direction inside the ion beam. We match to a vacuum solution outside of this grid (see Fig. 4a). We then advance the position of each of the electrons at each time step Δt , determining N at each step. We do this by alternately advancing the velocities by Δt on the half time intervals $\Delta t/2$, $3/2 \Delta t$ etc., and then advancing the positions on steps Δt , $2\Delta t$ etc.

$$\begin{aligned} (\Delta v)_{i+1/2} &= \partial \Phi / \partial x (x_i + v_{i-1/2} \Delta t) \Delta t + v_{i-1/2} \\ (\Delta x)_{i+1} &= \Delta x_i + v_{i+1/2} \Delta t \end{aligned} \quad (8)$$

At each time step the electron density is calculated at each grid point by counting the total number of electrons that are within one-half x -grid space of it.

Once the electron density N is determined at a new time the total charge density is found by adding the plasma-ion density and the beam-ion density at that time. This gives the right-hand side of Eq. (5a) at all grid points x, y . The Poisson equation is solved by Fourier analyzing in y , and by employing Gaussian elimination to solve the appropriate difference equations in the x direction for these harmonics. The harmonics are then resummed to

find the values of Φ on the grid points. In order to accomplish this, we need boundary conditions on the edges of our numerical grid. For the y direction we assume symmetry about $y = 0$, so $\partial\Phi/\partial y$ vanishes at $y = 0$. We also assume periodicity in y, so $\partial\Phi/\partial y$ also vanishes at $y = y_{\max}$. This is, of course, not exact but if the extent of the grid in the y direction is sufficiently large compared to the beam radius, it should be an adequate approximation. In the x direction we assume $\partial\Phi/\partial x = 0$ at $x = 0$, again by symmetry.

To determine the boundary condition on at $x = x_{\max}$, we must somehow take into account that the net charge that the ion beam carries away from the accelerator gets back to the accelerator. This probably happens through electron diffusion through the ionosphere at a large distance from the beam. As a consequence, net electron charge flows along B into the region of beam. To take this into account in a rough way, we adjust the total charge in the integration region by adjusting the uniform background ion density at each step before solving Poisson's equation. In addition, we assume that the region $x > x_{\max}$ is a vacuum and apply the vacuum boundary condition

$$\frac{\partial\phi_K}{\partial x} = -k\phi_K \quad (x = x_{\max}) \quad , \quad (9)$$

where ϕ_K is the Fourier coefficient in y.

The boundary condition on the electrons are: Reflection at $x = 0$ by symmetry, at $x = x_{\max}$ outgoing particles are allowed to escape freely, and particles are injected into $x < x_{\max}$ at a rate and velocity distribution consistent with the temperature T_0 .

We start the time integration with t sufficiently negative so that $\epsilon \ll 1/N_B$, say $\Gamma t = -3$. The number of electrons followed is of order 30 per grid spacing. The initial random velocities of the particles were taken as

$[(\Delta v)^2]^{1/2} \approx 0.1 \omega_{pe} R$. These random velocities were small compared to the final random velocities in the beam. We continue the integration until $f \approx 1$, say $\gamma t = +5$. We find that ϵ approaches a constant in time. The variation of $\epsilon(t)$ with t is shown in Fig. 3 for the case $\gamma = 0.2$, $N_B = 4$. It is seen that the neutralization sets in almost immediately, and it is quite constant at a value 0.53 while n_B varies up to its maximum value of 4. The electron neutralization tracks the beam density quite well. This limit ϵ_0 is evaluated for a number of cases and is listed in Table I.

The set of numerical values for ϵ_0 are given in Table I. From these values the curves of constant ϵ_0 in Fig. 4 are obtained by linear interpolation.

In Fig. 5 is a phase space plot of the positions and velocities of the electrons on the line $y = 0$ for the above case ($\Gamma = 0.2$, $N_B = 4$). Inside the beam the electrons have a much higher density which balances the ion charge. Also, their larger random velocities balance the reduced electrostatic potential gradient by their pressure.

IV. THE LINEARIZED SOLUTION FOR $n_B \ll n_0$

If $N_B \equiv n_B/n_0$ is small enough, we may assume that the electrons do not move very far. This suggests a linearized solution. The solution will provide us with a check on the numerical result. To make this linearized solution tractable we assume the beam density increases exponentially in time and is uniform in radius. Equation (3a) is changed to

$$f(t) = e^{\Gamma t}, \quad g = 1. \quad (10)$$

Set $\Phi = \Phi_1(x, y)e^{\Gamma t}$ and replace x by x_0 in Φ_1 . Then the linearized

solution of Eq. (4a) is

$$x = x_0 + \frac{1}{\Gamma^2} \frac{\partial \Phi_1}{\partial x} e^{\Gamma t} \quad (11)$$

Substituting Eq. (11) into Eq. (5) and linearizing we have

$$N = \frac{1}{1 + 1/\Gamma^2 (\partial^2 \Phi_1 / \partial x_0^2) e^{\Gamma t}} = 1 - \frac{1}{\Gamma^2} \frac{\partial^2 \Phi_1}{\partial x^2} e^{\Gamma t} \quad (12)$$

Substitution of Eq. (12) into Eq. (6) yields

$$\frac{\partial^2 \Phi_1}{\partial x^2} + \frac{\partial^2 \Phi_1}{\partial y^2} = -\frac{1}{\Gamma^2} \frac{\partial^2 \Phi_1}{\partial x^2} - \left\{ \begin{array}{l} N_B \quad x^2 + y^2 < 1 \\ 0 \quad x^2 + y^2 > 1 \end{array} \right\} \quad (13)$$

Equation (13) can be reduced to Poisson's equation for the electrostatic potential about a uniformly charged elliptical cylinder by the transformation

$$x = \alpha x_1 = x_1/a \quad (14)$$

where $\alpha^2 \equiv 1 + 1/\Gamma^2 \equiv 1/a^2$, with the result

$$\frac{\partial^2 \Phi_1}{\partial x_1^2} + \frac{\partial^2 \Phi_1}{\partial y^2} = \left\{ \begin{array}{l} -N_B \quad x_1^2/a^2 + y^2 < 1 \\ 0 \quad x_1^2/a^2 + y^2 > 1 \end{array} \right\} \quad (15)$$

The solution of this problem is well known. It can be shown (see Appendix) that the solution for Φ , which vanishes at $x = 0$, $y = 0$, is

$$\Phi_1(x, y) = -\frac{N_B a}{4} \int_0^{\xi} \frac{ds}{[(a^2+s)(1+s)]^{1/2}} - \frac{N_B a}{4} \int_{\xi}^{\infty} \left(\frac{a^2 x^2}{a^2+s} + \frac{y^2}{1+s} \right) \frac{ds}{[(a^2+s)(1+s)]^{1/2}}, \quad (16)$$

where ξ depends on x and y and is defined to be 0 when $x^2 + y^2 < 1$, and to be the solution of

$$\frac{a^2 x^2}{a^2 + \xi} + \frac{y^2}{1 + \xi} = 1 \quad (17)$$

when $x^2 + y^2 > 1$. Carrying out the integrals in Eq. (16) yields

$$\begin{aligned} \Phi_1 = & -\frac{N_B x^2}{2(1-\alpha^2)} \left(\frac{1}{\alpha} - \frac{(1+\xi)^{1/2}}{(1+\alpha^2 \xi)^{1/2}} \right) \\ & - \frac{N_B}{2} \frac{y^2}{(\alpha^2 - 1)} \left(\alpha - \frac{(1+\alpha^2 \xi)^{1/2}}{(1+\xi)^{1/2}} \right) \\ & - \frac{N_B}{2\alpha} \ln \left(\frac{(1+\alpha^2 \xi)^{1/2} + \alpha(1+\xi)^{1/2}}{(1+\alpha)} \right). \end{aligned} \quad (18)$$

The x derivative of Eq. (13) is

$$\frac{\partial \Phi_1}{\partial x} = \frac{x N_B}{\alpha^2 - 1} \left(\frac{1}{\alpha} - \frac{(1+\xi)^{1/2}}{(1+\alpha^2 \xi)^{1/2}} \right), \quad (19)$$

so inside the beam region (where $\xi = 0$) the x component of the force on an electron is

$$\frac{\partial \Phi}{\partial x} = \frac{\partial \Phi_1}{\partial x} e^{\Gamma t} = - \frac{N_B x}{\alpha(1+\alpha)} e^{\Gamma t} \quad (x^2 + y^2 < 1) \quad . \quad (20)$$

This force is to be compared with the force exerted by the ion beam alone ($\alpha = 1$):

$$\frac{\partial \Phi_B}{\partial x} = - \frac{N_B x e^{\Gamma t}}{2} \quad (x^2 + y^2 < 1) \quad . \quad (21)$$

The ratio of Eq. (20) to Eq. (21) gives the reduction in the x component of the electrostatic force due to charge neutralization by the plasma:

$$f_x = \frac{2}{\alpha(1+\alpha)} = \frac{2}{(1+\omega_{pe}^2/\gamma^2)^{1/2} [1+(1+\omega_{pe}^2/\gamma^2)^{1/2}]} \quad . \quad (22)$$

For the y component of the force we have correspondingly

$$\frac{\partial \Phi_1}{\partial y} = \frac{\gamma N_B}{1-\alpha^2} \left[\alpha - \frac{(1+\alpha^2 \xi)^{1/2}}{(1+\xi)^{1/2}} \right] \quad (23)$$

and inside the beam region the y component of the force on an electron

$$\frac{\partial \Phi}{\partial y} = \frac{\partial \Phi_1}{\partial y} e^{\Gamma t} = - \frac{N_B y}{1+\alpha} e^{\Gamma t} \quad . \quad (24)$$

Comparing with the y component of the force exerted by bare ion beam,

$$\frac{\partial \Phi_B}{\partial y} = - \frac{N_B y e^{\Gamma t}}{2} \quad , \quad (25)$$

we see that the y force is reduced by

$$f_y = \frac{2}{1+\alpha} = \frac{2}{1+(1+\omega_{pe}^2/\gamma^2)^{1/2}} \quad . \quad (26)$$

Combining Eqs. (7a), (12), and (20) we find

$$\epsilon = \frac{1}{\alpha} = \frac{\Gamma}{(\Gamma^2 + 1)^{1/2}} = \frac{\gamma}{(\gamma^2 + \omega_{pe}^2)^{1/2}} \quad (27)$$

The results of Eq. (27) do not compare very well with those of Fig. (4) and Table I. For example, for $\gamma/\omega_p = 0.5$, $n_B/n_0 = 0$, Eq. (27) gives $\epsilon_0 = 0.45$ which is larger than the result $\epsilon_0 = 0.33$ of Fig. 4 at $n_B/n_0 = 1$, although it is expected to be smaller. The origin of this discrepancy lies in the different time behavior of $\epsilon(t)$ for the analytical and numerical results. A more appropriate numerical value for $\epsilon(t)$ would be its value $\epsilon(0)$ at $t = 0$ when $f(t)$ is still increasing approximately exponentially. This value is actually $\epsilon(0) = 0.5$, in agreement with Eq. (27). (The values given in Fig. 4 are saturated values.) In fact, the results of Eq. (27) are actually in good agreement with the numerical values $\epsilon(0)$, the unsaturated results, for all γ/ω_p that were calculated. As an additional check, the case $\gamma/\omega_p = 0.33$, $n_B/n_0 = 0.3$ was solved numerically with exponential time dependence $f = 3xp(\gamma t)$, and the resulting ϵ_0 agreed with Eq. (27) to two significant figures.

V. APPLICATION

To illustrate the results of this paper, we consider the propagation of an intense energetic beam of ions through the ionosphere at various heights and different times. To be specific let us choose the following parameters. We take the earth's magnetic field to be 0.5 gauss and vertical. We take the cross section of the ion beam to be of radius 10^2 cm and its central density to be $10^5/\text{cm}^3$. We choose the ions to be 200 Mev protons so that $v/c \approx 0.66$.

The geometry of the beam is a horizontal cylinder of finite length (10 - 100 km). The electron density of the ionosphere depends on height and time and ranges from 10^4 to $10^6/\text{cm}^3$, although it may sometimes be as small as $10^3/\text{cm}^3$. It is easily seen that the conditions Eq. (1) and (2) are satisfied so that the beam must be neutralized by the ionospheric electrons.

Let us suppose that charge neutralization of the beam does not occur. Then the outward electrostatic force on the beam will exceed the pinch forces and the beam expands. If the density is uniform in radius, the beam expands uniformly. The radial expansion is governed by

$$\frac{d^2 R}{dt^2} = [1 - (v/c)^2] \frac{2\pi n_B e^2 R}{m_B} = [1 - (v/c)^2] \frac{\omega_{pi}^2}{2} \frac{R^2}{R}, \quad (28)$$

where $\omega_{pi}^2 \equiv 4\pi n_{Bo} e^2 / m_B$, and use is made of the fact that $n_B R^2 = n_{Bo} R_o^2$ is a constant. Solving this equation in time one finds that the beam will double its radius in a time

$$t_{ex} = \frac{2.145}{\omega_{pi} [1 - (v/c)^2]^{1/2}} = 5.49 \mu\text{sec} \quad (29)$$

since $\omega_{pi} = 4 \times 10^5 \text{ s}^{-1}$ for protons. Thus, charge neutralization must occur in this time.

If the average electron density of the neutralized electrons is $(v/c)^2 n_B \approx 0.44 n_B$, the electrostatic outward force is reduced to be equal to the inner directed magnetic force, and a cold plasma beam will not change its radius. On the other hand, there will be some beam divergence which produces a pressure exerting an outward force. If the beam divergence is initially $\Delta\theta$, then the perpendicular pressure of the beam is

$$p_{\perp} = n_B m_B v_{B\perp}^2 \approx n_B m_B v^2 (\Delta\theta)^2 \quad (30)$$

The electrostatic outward force of the bare beam at the boundary is

$$F_E = 2\pi n_B^2 e^2 R \quad , \quad (31)$$

so comparing we have

$$\frac{p_{\perp}}{RF_E} \approx \frac{n_B m_B v^2}{2\pi n_B^2 e^2 R^2} (\Delta\theta)^2 = \frac{2v^2}{\omega_{pi}^2 R^2} (\Delta\theta)^2 \approx 3 \times 10^5 (\Delta\theta)^2 \quad (32)$$

Thus, if the excess electron density in the beam is $\epsilon_0 n_B$ a beam of radial divergence $\Delta\theta$ will be confined from expansion by magnetic forces if

$$\epsilon_0 < \left[\left(\frac{v}{c} \right)^2 + \frac{p_{\perp}}{RF_E} \right] \approx [0.44 + 3 \times 10^5 (\Delta\theta)^2] \quad (33)$$

For example, if $\Delta\theta \approx 0.5 \times 10^{-3}$, ϵ_0 needs to be less than 0.37.

Making use of these results, we may now employ Fig. 4 to estimate the extent to which the beam will be charge neutralized and its expansion will be controlled. Consider an ionospheric electron density of $2 \times 10^5/\text{cm}^3$ ($N_B = 2$). Then, from Fig. 4 or Table I, $\gamma/\omega_{pe} \approx 0.33$ gives $\epsilon_0 \approx 0.37$ or $n_e = 1.86 n_B$. At this density $\omega_{pe} = 2.4 \times 10^7 \text{s}^{-1}$ so $\gamma \approx 8 \times 10^6 \text{s}^{-1}$ and charge neutralization occurs in a time $\approx 1/\gamma = 0.1 \mu\text{sec}$, very short compared to the expansion time of Eq. (29).

In fact, from Fig. 5 we see that charge neutralization occurs long before the ion beam reaches its maximum density. Thus, in this case, charge neutralization effectively prevents any expansion of the ion beam.

The same results apply to other cases represented by Table I and

Fig. 4. At higher values of n_B/n_0 smaller values of γ/ω_{pe} ($\lesssim 0.01$) are probably necessary, but even for these values charge neutralization probably occurs fast enough to prevent beam expansion. In any event, if the ions have larger energies (larger values of v^2/c^2 and ϵ_0) they can be propagated without expansion.

The physical behavior of the electrons is interesting from other points of view. They are constrained to move along the ambient field in their self-consistent field. If one wishes to treat instabilities such as the Helmholtz instability, it is necessary to take this behavior into account and probably necessary to treat their behavior by a PIC code such as described in this paper.

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APPENDIX A

We wish to derive expression (18) for the potential of a uniformly charged cylinder, i.e., the solution of Eq. (17).

We start from the gravitational potential of an ellipsoid of uniform density

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} < 1 \quad . \quad (A1)$$

This is given e.g. by Eq. (96.11) in Landau and Lifshitz Classical Theory of Fields:³

$$\Phi = -\pi \mu abc k \int_{\xi}^{\infty} \left(1 - \frac{x^2}{a^2+s} - \frac{y^2}{b^2+s} - \frac{z^2}{c^2+s} \right) \frac{ds}{R_s} \quad , \quad (A2)$$

$$R_s = [(a^2+s)(b^2+s)(c^2+s)]^{1/2} \quad (A3)$$

where ξ depends on x , y and z and is given as the solution of

$$\frac{x^2}{a^2+\xi} + \frac{y^2}{b^2+\xi} + \frac{z^2}{c^2+\xi} = 1 \quad (A4)$$

outside the ellipsoid and 0 inside the ellipsoid. The density of the ellipsoid is μ , and k is the gravitational constant. To employ this solution to solve Eq. (17), we must choose $4\pi\mu k = -N_B$ and let c pass to ∞ . However, Φ as given by Eq. (A2) approaches 0 at ∞ , while the potential of an elliptical cylinder is logarithmically singular at ∞ . Thus, before passing to the limit $c \rightarrow \infty$ we subtract a constant to make Φ in Eq. (A2) 0 at the origin. Thus,

$$\Phi = \frac{N_B}{4} abc \left[\int_{\xi}^{\infty} \left(1 - \frac{x^2}{a^2+s} - \frac{y^2}{b^2+s} - \frac{z^2}{c^2+s} \right) \frac{ds}{R_s} - \int_0^{\infty} \frac{ds}{R_s} \right] \quad (A5)$$

Noting that $c/R_s \rightarrow [(a^2+s)(b^2+s)]^{-1/2}$ and $z^2/c^2 \rightarrow 0$ as $c \rightarrow \infty$ we see that Φ approaches Eq. (18) as $c \rightarrow \infty$.

Although Eq. (A2) is hard to derive, it is fairly easy to show it satisfies Poisson's equation. Inside the ellipsoid $\xi = 0$ so that

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 2\pi\mu abc k \int_0^{\infty} \left(\frac{1}{a^2+s} + \frac{1}{b^2+s} + \frac{1}{c^2+s} \right) \frac{ds}{R_s} \quad (A6)$$

a constant whose value is readily found to be $4\pi\mu k$. Outside the ellipsoid we have

$$\frac{\partial \Phi}{\partial x} = -\pi\mu abc k \int_{\xi}^{\infty} \frac{-2x ds}{\xi (a^2+\xi) R_s} \quad (A7)$$

the derivative with respect to the limit ξ vanishing in virtue of Eq. (A4).

Next

$$\nabla^2 \Phi = -\pi abc \mu k - 4 \int_{\xi}^{\infty} \left\{ \frac{1}{R^2} \frac{\partial R}{\partial s} + \left(\frac{2x}{a^2+\xi} \frac{\partial \xi}{\partial x} + \frac{2y}{b^2+\xi} \frac{\partial \xi}{\partial y} + \frac{2z}{c^2+\xi} \frac{\partial \xi}{\partial z} \right) \frac{1}{R_{\xi}} \right\} ds. \quad (A8)$$

The integral of the first term is easy to do and gives $4/R_{\xi}$ (where R_{ξ} is R_s at $s = \xi$). Carrying out implicit derivatives of Eq. (A4) easily gives $-4/R_{\xi}$ for the second term, so $\nabla^2 \Phi = 0$. Since Φ is clearly continuous at the surface of the ellipsoid, Eq. (A2) clearly gives the correct potential for a uniform density ellipsoid.

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TABLE I. Numerical Values of ϵ_0 .

n_B/n_0	$\gamma/\omega_p = 0.1$	0.2	0.33	0.5	1.0
1	0.25	0.27	0.29	0.33	0.35
2	0.30	0.32	0.39	0.45	0.59
4	0.40	0.48	0.55	0.60	
8	0.53	0.72	0.69	0.73	
16	0.70	0.76	0.79	0.84	

FIGURE CAPTIONS

FIG. 1. An energetic beam of ions propagating through a magnetized plasma. The beam is essentially undeflected by the magnetic field. The plasma electrons move freely along the B field under the influence of the electric field E to neutralize the ion beam charge. The plasma ions are essentially immobile. (b) The z dependence of the beam ions, plasma and electron densities in the beam at any instant. The scale of time variation at any point is γ . The scale of variation is v/γ . The electron density n_e essentially balances the ion densities, effectively neutralizing the charge density.

FIG. 2. The two-dimensional model of the beam-plasma problem. The ion beam is an infinite cylinder in the z direction whose density is time varying. All quantities are independent of z . (b) The radial profile of the beam density. (c) The time variation of the beam density. (d) Typical electron orbits for initial x positions $(x_0)_1$ and $(x_0)_2$, $y = 0$. Note the damped oscillations in the potential well formed by the ion beam.

FIG. 3. The variation of $\epsilon(t)$ (Eq. 7), the charge neutralization reduction factor, with t for $\gamma/\omega_{pe} = 0.2$, $n_B/n_0 = 4$. Initially $\epsilon \approx 1$ since the plasma is unneutralized. It quickly approaches a constant value $\epsilon_0 = 0.48$ during the early phases of the beam density rise. The oscillations are due to the abrupt turning on of the beam at $\gamma t = -6$. Since the beam density is low, they represent

very small oscillations in electron density. The initial value of $\epsilon \approx 1.15$ also is produced by the effect of the initial conditions.

FIG. 4. Curves of constant ϵ_0 in the γ/ω_{pe} , n_B/n_0 plane. The maximum averaged density is $\bar{n}_B$. The neutralization is more effective at small n_B/n_0 and small γ/ω_{pe} . The requirement for the ion beam to hold together is essentially $\epsilon_0 < (v/c)^2$. [See Eq. (33).]

FIG. 5. x , v_x phase space display of electrons at $y = 0$. For $x > 1$, the root-mean-square velocity is of the order of its initial value of 0.1. For $x < 1$, the electrons' density is larger and their pressure, represented by the increased velocities and densities, balances the reduced electric field of the beam. The streaks of electron density may represent weak two-stream instabilities. The unit of velocity is $\omega_{pe}R$ and the unit of distance is R .

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Fig. 1

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FIG. 2

Fig. 3

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Fig. 4

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Fig. 5

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