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Studies of End Loss from a Theta Pinch Using
a Twyman-Green Interferometer

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by

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ABSTRACT

The transient rate of flow of plasma from the ends of a short, high density, low temperature linear theta pinch was studied experimentally and analytically. Diagnostic devices were used to study the discharge and plasma properties; these included a Rogowski coil, a magnetic field probe and a Twyman-Green double pass interferometer which was illuminated by a 7 mW He-Ne laser. The interferometer was used to monitor the passage of fringes with respect to time at two radial positions simultaneously by the use of a photodetector consisting of two fast silicon photodiodes with supporting circuitry. One complete fringe represented a change in number density of $2.942 \times 10^{16} \text{ cm}^{-3}$.

Compatible interpretation of the above-mentioned diagnostics allowed evaluation of the loss rate of plasma. The major uncertainty in obtaining $1/e$ particle end loss times from the Twyman-Green interferometric data was associated with lack of data on the density profile. Two different techniques were developed to provide estimates of the profile. These led to loss times estimates of 1.82 μsec and 2.85 μsec for a 50 millitorr fill pressure of H_2 , and 2.39 μsec and 2.81 μsec for a 65 millitorr fill pressure. These loss times were compared with: results obtained from magnetic field probe data on plasma column oscillation which indicated the loss rate; loss times analytically determined by scaling experimental results from a larger theta pinch; and loss time predicted by a zero dimensional computer code. It was determined that, for the 65 millitorr fill pressure, agreement between loss time predicted by these four techniques was good; however, both experimental uncertainties and loss time uncertainties were greater for the 50 millitorr fill pressure.

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NOMENCLATURE

A	= cross sectional area
A_p	= plasma cross sectional area
A_s	= surface area of plasma column or current sheath parallel to the discharge chamber axis
b	= inside radius of discharge chamber
B	= magnetic field strength
B_{ext}	= external axial magnetic field strength
B'_{ext}	= external axial magnetic field strength after a change in plasma radius occurs due to radial plasma oscillations
B_{int}	= axial magnetic field strength internal to plasma column
B'_{int}	= axial magnetic field strength internal to plasma column after a change in plasma radius occurs due to radial plasma oscillations
B_{oint}	= equilibrium internal axial magnetic field strength
B_{max}	= maximum axial magnetic field strength
B_z	= magnetic field strength in z-direction
$\dot{B}_z = \frac{dB_z}{dt}$	= time rate of change in the magnetic field strength in z direction
C	= capacitance
c	= speed of light
F	= force
I	= current
J	= current density
L	= inductance
L_c	= inductance of magnetic field probe coil
l	= length
l_c	= length of theta pinch compression coil

ℓ_p	= length of plasma column
ℓ_p^{I-C}	= length of plasma column in Scylla I-C
ℓ_s	= length of current sheath
$\ell_T = 2\ell_p$	= total length of path traversed by laser beam through the plasma
m	= mass
m_e	= mass of an electron
m_i	= mass of an ion
m_i^{I-C}	= mass of an ion in Scylla I-C
m_s	= mass entrained by current sheath
M_L	= mass per unit length
M_T	= total mass contained in plasma column
n	= number density
n^*	= number of turns in either the Rogowski coil or the magnetic field probe coil
$n_a(t)$	= on axis number density at a time t
n_e	= electron number density
n_o	= initial fill density
$\bar{n}_p$	= initial average number density
$n_b(0)$	= initial number density at a radial position $r = b$
$n_r(t)$	= number density radial profile at a radial position r at a time t in μsec
$n_r'(2)$	= number density radial profile 2 μsec after implosion if only particle loss were to have occurred
$n_{\text{rout}}(0)$	= estimate on initial number density profile outside $r_p(0)$
$n_{rp}(0)$	= initial number density at a radial position $r = r_p(0)$
Δn_a	= calculated change in number density during the first two microseconds of plasma column lifetime, at a radial position $r = 0$

Δn_r	= calculated change in number density during the first two microseconds of plasma column lifetime, at a radial position r
$N(t)$	= particle inventory in plasma column at a time t
$N_{out}(0)$	= number of particles initially inside compression coil not swept up by imploding current sheath
N_{TOT}	= total number of particles in system at a time t
P_o	= initial fill pressure
r	= radius
r_c	= theta pinch compression coil radius
$r_{1/e}(t)$	= the radius where the number density is down by a factor $1/e$ of the on axis number density (n_a) for a Gaussian profile at a time t
r_o	= initial equilibrium plasma radius
$r_o(t)$	= equilibrium plasma radius calculated by computer simulation at a time t
$r_p(t)$	= plasma column radius at a time t , including radial oscillations
r_s	= radial position of current sheath
Δr	= initial perturbation of plasma radius by radial plasma oscillations
Δr_p	= change in plasma radius due to radial plasma oscillations
Δr_{pmax}	= maximum change in plasma radius due to radial plasma oscillations
R	= resistance
R_T	= terminating resistance
S	= effective Rogowski coil length
t	= time
t_p	= pinch time
T	= temperature
T_e	= electron temperature

T_{ep}	= average initial electron temperature
T_i	= maximum ion temperature
T_{ip}	= average initial ion temperature
T_i^{I-C}	= maximum ion temperature for Scylla I-C
T_p or kT_p	= average initial plasma column temperature ($= T_{ep} + T_{ip}$)
u_s	= normalized current sheath radial position
v	= velocity
v_s	= current sheath velocity
$\Delta v_s / \Delta t$	= current sheath acceleration
V	= voltage
V_{B_z}	= integrated magnetic field probe voltage output
$\dot{V}_{B_z}$	= unintegrated magnetic field probe voltage output
V_{ROG}	= integrated Rogowski coil voltage output
$X_T(t)$	= time rate of fringe movement at a radial position r
ΔX_T	= fringe movement during the first two usec of plasma column life, excluding movement due to radial plasma oscillation, at a radial position r
α	= damping coefficient
α_{os}	= damping coefficient for the radial plasma oscillation
β	= magnetic field efficiency factor
n	= fractional mass pick up factor
A	= ratio of the Debye shielding distance to the impact parameter
λ	= wavelength
λ_o	= wavelength of He:Ne laser light source for interferometer
λ_{ii}	= ion-ion mean free path
μ	= refractive index
μ_p or μ_{plasma}	= refractive index of a plasma

μ_{vacuum}	= refractive index of a vacuum
μ_0	= free space permeability constant
ω	= frequency
ω_l	= laser frequency
ω_p	= plasma frequency
ϕ	= magnetic flux
$\sigma_{\perp}$	= plasma conductivity perpendicular to the axial magnetic field
τ	= period of either the oscillating driving current or magnetic field produced in the theta pinch
τ_d	= diffusion time of magnetic field through pyrex tube shield
τ_p	= the 1/e particle end loss time
τ_p^{I-C}	= the 1/e particle end loss time for Scylla I-C
τ_{probe}	= response time of magnetic field probe coil
τ_{os}	= period of radial plasma oscillations
τ_s	= unitless time parameter associated with normalized current sheath radial positions, u_s
τ_o	= unitless time for current sheath to reach discharge chamber axis

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Chapter I

INTRODUCTION

1.1 Statement and Discussion of Problem

In recent years there has been a major technical effort directed towards the development of nuclear fusion power to supplement the world's available energy sources. Several devices are currently being investigated as candidate configurations for future nuclear fusion reactor power plants. These devices can be divided into two categories: those which use magnetic confinement and those which use inertial confinement. In the magnetic confinement category, the major devices are the tokamak, the magnetic mirror, the bumpy torus, the z-pinch and the torodial and linear theta pinches (1,2). In the inertial confinement category, laser ignited pellet fusion is being most actively pursued (1). Each one of these devices has inherent physics and engineering problems which have impeded reactor development (2).

This thesis will deal exclusively with an experimental study of a linear theta pinch device. While it is possible to generate thermonuclear plasma in such devices, one of the more serious problems confronting the linear theta pinch concept is the rapid streaming of plasma particles from the open ends of the plasma column. As a consequence, particle confinement time (τ_p), which is the time for the total number of particles in the plasma column to drop by a factor of $1/e$ from the initial value, is found to be several orders of magnitude too small to achieve controlled fusion in present-day machines (3). The most practical solution to this problem is to decrease the flow rate from the ends such that the confinement time is increased to an acceptable period. In order to

accomplish this in a systematic manner, a reasonable first step is to understand the flow processes in detail. In general, only gross plasma properties have been measured in experiments conducted to date (3,4). Detailed spatial measurements of plasma properties from which a basic physics understanding of the end loss phenomena may be determined, remain to be conducted. The main purpose of theta pinch research at The Pennsylvania State University is to investigate the particle loss phenomenon by developing and using unique diagnostics for making local measurements to determine plasma properties in the vicinity of the plasma column ends (5,6). In addition, there is a need to determine the change in the total number of particles contained within the plasma column as a function of time for reference purposes. This measurement has normally been made using interferometry, which averages properties over the column to yield an average line density (7). This line density can then be used to evaluate the total number of particles remaining in the plasma column, which in turn can be used to determine the $1/e$ particle loss time.

In the work being reported here, an interferometric diagnostic was designed, fabricated and used to obtain an estimate of the $1/e$ particle loss time in a very short (10 cm), highly transient ($\tau \approx 8 \mu\text{sec}$) theta pinch. This experimentally determined loss time was then compared to loss times obtained from $L/(T_i/n_i)^{1/2}$ scaling of larger present-day machines, from the use of a zero dimensional computer code being developed by Stover (6) and also from other experimental methods.

1.2 Experimental Approach

In this thesis, plasma particle end loss and plasma column behavior were studied using the results from several plasma diagnostics in a linear

theta pinch device. From these diagnostics, the following information was obtained: current sheath trajectory; percent mass pick up during collapse; column length; the frequency and damping rate of radial plasma oscillations; and the $1/e$ particle loss time. Plasma column properties such as electron and ion temperature, plasma column radius and average number densities were either calculated using a simple snowplow implosion model (4) or determined using the zero dimensional computer code mentioned earlier.

The $1/e$ particle loss time was determined experimentally using two independent methods. In the first method a Twyman-Green type interferometric diagnostic was developed as a major part of this experimental study. In the second method, an estimate of the $1/e$ particle loss time was determined by the use of a magnetic field probe positioned inside the plasma column. This probe was used to monitor radial plasma oscillation frequency, which can be used to evaluate the total number of particles in the column with respect to time (8) (also see Appendix A).

The Twyman-Green interferometer used was a double pass, two beam device. One beam served as a reference beam and the other served as the test beam which traversed the plasma column. Since the plasma electron density was continually varying during the column lifetime, the plasma refractive index also was changing (7). This caused variation in the optical path length of the test beam as compared with the reference beam. Therefore, when the reference beam was recombined with the test beam, there was a phase difference between the two. The phase difference varied in both space and time causing a spatially and temporally varying fringe pattern. This fringe pattern was monitored by placing a number of fast silicon photodiode detectors at various positions in the path of the

recombined beam. By appropriate interpretation of the fringe movement at these positions, the $1/e$ particle loss time was evaluated.

There are two important advantages that the interferometric set-up used here has over other more commonly used interferometers (7). The first advantage is the minimal cost of the laser, detection system, and optics resulting from the use of a He-Ne CW laser light source ($\lambda = 6328\text{\AA}$). Secondly, the most important advantage is that by expanding the laser beam to approximately the radial size of the plasma column both temporal and spatial resolution can be obtained simultaneously. This cannot be done using any of the commonly used interferometers without great sophistication (9).

1.3 Alternative Interferometric Techniques

Three major types of interferometric systems have been used previously in evaluating particle loss from plasmas. These three systems include: 1) two-beam interferometry; 2) holography; and 3) coupled cavity interferometry. A brief description of these devices and their use is given below.

There are two configurations of two beam interferometers which have previously been employed. The most commonly used configuration is the Mach-Zehnder interferometer; the other type is the Twyman-Green interferometer which will be discussed first.

Twyman-Green interferometry using a He:Ne CW laser light source was first proposed by Buser and Kainz (10). They found that the system frequency response was very high when using a photomultiplier detection system, making the system ideal for investigating rapidly changing indices of refraction. They also showed that either the 0.6328μ or 3.39μ

lines emitted by He:Ne lasers could be used in line density measurements. Deuchars et al. (11) were the first to apply the Buser and Kainz technique for investigating the time rate of change in density in a linear theta pinch device. Deuchars showed that the system could easily follow rapidly changing center line density and that the frequency response was governed by the detection system and not the interferometer itself. Molen (12) was the first to report the capability of Twyman-Green interferometry to measure both temporal and spatial changes in line density simultaneously. His investigations, however, were not conducted on a fast theta pinch discharge but rather on a long lived (70 usec), low temperature and high density plasma created by applying a current through two small ring electrodes placed 35 cm apart on a discharge chamber.

Mach-Zehnder interferometry (7, 13, 14) is the most widely used two-beam interferometer for making plasma measurements. One beam, the test beam, makes a single pass through the plasma while a reference beam of approximately the same length passes around the plasma. The two beams are then recombined and a photograph of the resulting fringe pattern can be taken. Since the test beam only makes a single pass through the plasma, the sensitivity is exactly half that of a double pass device such as a Twyman-Green interferometer. Of course, this assumes that the same light source is being used on both devices. However, it has been shown for the Mach-Zehnder interferometer that by tilting one of the mirrors in the reference leg, the sensitivity can be increased somewhat (7). Work has been done with the Mach-Zehnder interferometer using a pulsed CO₂ laser (13) for dense plasma ($n_e \approx 10^{16}-10^{17} \text{ cm}^{-3}$) and a HCN laser (12) for low density plasma ($n_e \approx 10^{14}-10^{15} \text{ cm}^{-3}$).

The Mach-Zehnder setup, in comparison with the Twyman-Green setup used in this study, has two major disadvantages. First, since pulsed lasers and photographic methods are used, the cost of the Mach-Zehnder setup is an order of magnitude greater than the Twyman-Green setup. Secondly, only spatial resolution can normally be obtained in one discharge and temporal resolution is achieved by taking data with many successive discharges. A schematic of the Mach-Zehnder setup can be found in reference 7.

Holographic interferometry (3, 7, 15-20) consists of making two holograms of the region which contains the plasma on the same photographic plate. One hologram is made when there is no plasma in the region in question and the second hologram is made when the plasma is present. When the photographic plate is illuminated, the images of the two holograms interfere such that only the differences between the two appear. In other words, one sees the fringe pattern caused by the phase difference of the laser light at any spatial position with the plasma present and when it is not present. The holographic method has been used extensively at Los Alamos Scientific Laboratory (3, 18-20) for plasma measurements in linear theta pinch devices. One of the advantages of holography is that the optics do not have to be very flat since the effects of the optical surface will be the same for both holograms and will cancel (7). The light source most commonly used was a pulsed ruby laser (15, 19, 20). Again, the disadvantages as compared to the Twyman-Green setup are overall cost and the fact that only spatial resolution can easily be obtained from one discharge. A schematic of a standard holographic interferometric setup can be found in reference 7.

The coupled cavity interferometer (7, 18, 20-26) works on the principle that the laser output is modulated if one allows some of the emitted laser light to reflect off an external mirror back into the laser. Light reflecting back into the laser cavity in phase with the laser light will tend to increase the laser output and light out of phase will tend to decrease the output. The laser beam passes through the plasma twice, once on the way to the external mirror and once on the return to the laser, so that its sensitivity is the same as a Twyman-Green setup. However it has been shown that if a spherical mirror is used instead of a flat mirror the sensitivity can be increased up to four times that flat mirror value (22). Also since a CW model laser is used, system costs are minimal. A disadvantage of this system over a Twyman-Green system is that only temporal resolution can be achieved in one discharge. Spatial resolution is usually obtained by repositioning the interferometer between each successive discharge.

Gas laser interferometers of this type have been made using the three He:Ne wavelengths 0.6328μ (7, 18, 20-22), 1.15μ (7, 23), and 3.39μ (7, 18, 20-22) using the 10.6μ wavelengths from a CO_2 laser (7, 26) and using the 337μ wavelength from a HCN laser (24). One limitation of this method is that the laser output has an inherent frequency cutoff which is approximately 1-3 mHz using 0.6328μ He-Ne wavelength (22) and 7 mHz using 3.39μ He-Ne wavelength (25). A schematic of the general coupled cavity setup can be found in reference 7.

Chapter II

APPARATUS AND TEST CONDITIONS

2.1 Introduction

The investigations of plasma behavior and the loss phenomenon in the linear theta pinch device were based on detailed experimental measurements. The discharge tube was cylindrical with an aluminum electrical discharge coil fitted around it. A time-varying axial magnetic field was produced in the discharge tube by an azimuthal current driven through the electrical discharge coil by a capacitor bank charged to 11 kV. An azimuthal diamagnetic current with direction opposite to the coil current was induced at the internal periphery of the discharge tube and separated the preionized plasma from the external magnetic field. This diamagnetic current interacted with the magnetic field to generate a radial $\vec{J} \times \vec{B}$ force which accelerated the plasma into the center of the discharge tube, resulting in the formation of a hot, dense plasma column on axis. The linear theta pinch was thus a logical configuration choice for development of a Twyman-Green diagnostic, since the compression coil was open ended, allowing unimpeded passage of the laser beam through the entire plasma column length with its radial distribution of plasma and related variation in the index of refraction.

2.2 Discharge Apparatus and Auxiliary Equipment

The discharge apparatus consisted of four fundamental components: the discharge chamber, the theta pinch electrical discharge coil, an energy storage capacitor bank and an open-air gas-triggered switch which connected the compression coil to the capacitor bank.

The pinch coil, fabricated from 0.08 cm thick aluminum, was 10.5 cm long and 11.5 cm in diameter. The compression coil was fitted around the middle of three standard pyrex tubes connected together to make up the discharge chamber. Each pyrex tube was 30.25 cm long and had an inside diameter of 10.16 cm. A 90.75 cm tube length was used so that any optical elements which has to be placed in the system would be positioned far enough away from the discharge so their surfaces would not be damaged or optically altered by the streaming plasma. Special end flanges were constructed to hold the optics and diagnostics in place. Between the second and third pyrex tubes a special connector was fabricated so that an Edward High Vacuum, Limited, Speedvac ED250 vacuum pump could be attached to the discharge chamber. With this vacuum pump, the chamber was evacuated to a base pressure of approximately 2.5 millitorr between each discharge. The pressure was monitored by a Bendix Pirani gauge calibrated for hydrogen gas; it was accurate for pressures between 0 and 2000 millitorr. A schematic of the discharge chamber and electrical discharge coil geometry is presented in Figure 2-1.

The discharge current was produced by two Aerovox Corporation capacitors (Type PX60D17) connected in parallel and charged to 11.0 kV using a Sorensen model 9030-S D.C. power supply. The combined capacitance and inductance of these capacitors were 9185 μf and 20 nH, respectively. The two capacitors were connected to the gas-triggered switch and the electrical discharge coil by a parallel plate transmission line made of 0.08 cm thick and 7 cm wide copper plate. The total inductance of the transmission line was determined to be 16.5 nH. The system was switched on by use of an open air gap, around which helium was bled to allow breakdown of the gap. The inductance of the gap was found to be

Figure 2-1: Discharge Chamber Geometry

approximately 42.7 nH. The electrical equivalent of the electrical discharge coil and supporting circuitry is presented in Figure 2-2.

The current waveform was determined to be a damped sinusoid, which is characteristic for a lumped LRC network. The equation which represents this waveform is:

$$I(t) = V_0 \omega C e^{-\alpha t} \sin \omega t$$

where α is the damping coefficient, experimentally determined to be $4.01 \times 10^4 \text{ sec}^{-1}$. All other parameters are defined and their values can be found in Table 2-1 (Theta Pinch Characteristics). An example of the current waveform with a vacuum in the discharge chamber is presented in Figure 2-3; this signal represents the integrated output of a Rogowski coil diagnostic.

2.3 Discharge Conditions

Hydrogen gas at several fill pressures was used throughout the investigation; hydrogen has the low atomic weight of interest in fusion studies, and it was readily preionized by the relatively low energy (595 Joule) storage system. The first half cycle of the discharge current was used to preionize the gas while the second half cycle generated a current sheath which imploded to form the plasma column on axis. There was no indication of trapped magnetic fields from the first half-cycle of the discharge. While discharge conditions with fill pressures from 30 to 200 millitorr were examined, fill pressures of 50 and 65 millitorr of hydrogen gas were investigated in detail. These particular fill pressures were chosen because higher fill pressures yielded loss times large compared to the half cycle time of the discharge current making interpretation of the interferometric data difficult, and

Figure 2-2: Electrical Equivalent of Electrical Discharge Coil and Supporting Circuitry

TABLE 2-1
THETA PINCH CHARACTERISTICS

Voltage (kV)	11.0
Capacitance (μ f)	9.85
Total source inductance (nH)	79.2
Coil length (cm)	10.5
Coil diameter (cm)	11.5
Coil inductance (nH)	81.3
Inside radius of discharge chamber (cm) . .	10.16
Maximum current (amps)	
1st half cycle	7.85×10^4
2nd half cycle	6.69×10^4
Maximum magnetic field (kG)	
1st half cycle	6.36
2nd half cycle	5.42
Maximum E_g (at tube I.D.) (V/cm)	
1st half cycle	137.3
2nd half cycle	107.9
Half cycle time (usec)	4.0

Calibration factor = 1.013×10^5 amps/volt

Figure 2-3: Integrated Rogowski Coil Output

and lower fill pressures produced plasmas with densities too low to be evaluated by the Twyman-Green interferometer.

Chapter III

DIAGNOSTIC DEVICES

3.1 Introduction

A number of diagnostic devices were used to investigate the properties of the theta pinch plasma column. These diagnostics can be placed into two categories: those which are placed outside the discharge tube, i.e. a simple current loop (Rogowski coil) and the Twyman-Green interferometer; and those which are inserted into the discharge tube, i.e. the magnetic field probe. The magnetic field probe was protected against the severe plasma column thermal environment to prevent deterioration of probe performance during successive discharges.

The operation and calibration of the Rogowski coil and magnetic field probe will be discussed briefly below. Details of the Twyman-Green interferometer will be discussed in Chapter IV.

3.2 Rogowski Coil

The total discharge current was obtained from a simple current loop (Rogowski coil) of the design discussed in the literature (27, 28). A sketch of the Rogowski coil and associated circuitry is shown in Figure 3-1. Basically a Rogowski coil is a multiturn solenoid bent into a torodial shape. The Rogowski coil acts as a magnetic induction pick up loop. The time varying current being measured passes through the loop cross section and produces a time varying magnetic flux which threads the coils and induces in them an emf force. Therefore a measure of the current can be obtained by integrating the coil's output, $d\phi/dt$, yielding

Figure 3-1: Rogowski Coil and Supporting Circuitry

$$V_{\text{ROG}} = \frac{1}{RC} \int_0^t \frac{d\phi}{dt} dt = \frac{\phi(t)}{RC} \quad (3.1)$$

where

$$\phi(t) = \frac{\mu_0 A}{S} n^* I(t)$$

therefore,

$$V_{\text{ROG}} = \frac{\mu_0 A n^*}{SRC} I(t) \quad (3.2)$$

where ϕ is the induced magnetic flux, A is the cross sectional area of a single turn, n^* is the total number of turns, S is the effective length of the bent solenoid and RC is the time constant of the integrator.

The most important feature of a Rogowski coil is that it will monitor the current passing through the coil regardless of coil configuration with respect to the current being measured. Thus, great flexibility can be used in the positioning of the coil. In this experiment, the coil was wrapped around the high voltage side of the parallel plate transmission line connecting the capacitor bank to the theta pinch electrical discharge coil through a switch.

The Rogowski coil output was integrated by using a RC integrator. The R and C were selected such that the resulting RC time constant was at least an order of magnitude greater than the time period of interest and signal to noise ratio was adequate. The former was necessary in order to avoid signal distortion due to RC decay. Since only the first two half cycles of the current were of interest (8 μsec), an RC time constant of 97 μsec was chosen. The output of the coil was terminated with a 50 Ω matching resistor at the input side of the integrator.

The coil itself was fabricated using 27 turns of #28 heavy Formvar wire wrapped in insulating tape to avoid coil to coil shorting. The effective length of the solenoid was 0.197 meters with each turn having

a cross sectional area of 5.59×10^{-6} meters². The coil was calibrated experimentally on the theta pinch device with the capacitor bank charged to 11 kV. Since the current variation can be represented by $I(t) = V_0 \omega C e^{-\alpha t} \sin \omega t$ for a simple LRC circuit, the calibration factor was calculated to be 1.013×10^5 amps/volt. This calibration factor was found to agree, within 0.60 percent, with that found using the theoretical Equation 3-2. This agreement was considered satisfactory.

3.3 Magnetic Field Probe

The magnetic field probe was used extensively in this experimental study. From magnetic field probe data, current sheath trajectory, plasma column length, percent mass pick up and radial plasma oscillations were identified. Total plasma column particle inventory was estimated as a function of time from the radial plasma oscillation results. This allowed a $1/e$ particle loss time to be calculated.

Probe theory and construction has been reported in the literature (27, 28), but a brief description of the probe will be presented here. Basically, the probe consists of a small diameter multiturn coil made of light gage heavy Formvar coated wire. The probe operates according to Faraday's law of magnetic induction. The coil was oriented such that the coil axis was parallel to the measured magnetic field. The emf generated in the coil by the enclosed time varying magnetic field is given by:

$$V_{B_z} = n^* A \frac{dB_z}{dt} \quad (3.3)$$

where n^* is the total number of turns and A is the cross sectional area of a single turn. The single turn cross sectional area A , had to be kept small enough so that the gradient of the magnetic field was small over

the coil area. This would give good spatial resolution which was needed to accurately identify current sheath position during the implosion phase of the discharge. A signal proportional to the magnetic field strength, B_z , was obtained by use of a simple RC integration. Lovberg (27) showed that the signal obtained can be related to the magnetic field by

$$\begin{aligned} V_{B_z} &= \frac{1}{RC} \int V_{B_z} dt \\ &\approx \frac{n^* A}{RC} B_z \end{aligned} \quad (3-4)$$

The coil was fabricated by wrapping 13 turns of #32 heavy Formvar wire around a 0.08 inch diameter shaft. The leads of the coil were tightly twisted together to minimize spurious inductive pickup. The coil was protected from the plasma column thermal environment by placing it in a 4 mm I.D., 6 mm O.D., 62.2 cm long pyrex glass tube sealed at one end to retain the vacuum in the discharge chamber. The leads were connected to a coaxial cable terminated with a 50Ω load. The coil output was integrated by a RC network with a time constant of 110 μsec. Again, this was at least an order of magnitude greater than the time period of the phenomenon investigated. Typical records of B_z and $\dot{B}_z$ are presented in Figure 3-2.

The coil also had to be designed such that its frequency response was high compared to the phenomenon being studied. It has been shown (27) that the frequency response of a magnetic field probe of this type can be represented by the response time approximated by:

$$\tau_{\text{probe}} \approx \frac{L_c}{R_T}$$

where L_c is the coil inductance and R_T is the terminating resistance. It

Figure 3-2: Typical Magnetic Field Probe Response

was determined that for the particular probe used, $\tau_{\text{probe}} \approx 4.36 \times 10^{-9}$ sec.; this was considered well within the required constraints. However, since the probe was placed inside the protective pyrex tube, magnetic flux must also diffuse through the pyrex into the coil. This diffusion time must also be rapid enough to insure adequate probe response. The diffusion time is given by (27)

$$\tau_d \approx \frac{1}{4} \mu_0 \sigma_{\perp} r^2$$

where r is the mean radius of the pyrex tube (2.5 mm) and $\sigma_{\perp}$ is the plasma conductivity perpendicular to the magnetic field. The Spitzer-Härm (29) resistivity perpendicular to the magnetic field was used here to calculate $\sigma_{\perp}$ which was found to be approximately 7.28×10^3 mhos/m at a 50 millitorr hydrogen fill pressure. In this case, $\tau_d \approx 1.43 \times 10^{-8}$ seconds which lies within the required constraints.

The magnetic field probe was calibrated using the pinch coil with the capacitor bank charged to 11 kV. The magnetic field strength at the geometrical center of the coil was calculated using the Biot-Savart law for large single turn solenoids, which states (30):

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \vec{r}}{r^3}$$

- . By integration, the magnetic field was found to be given by the relationship

$$B_z(t) = \frac{\mu_0 I(t)}{\ell_c} \cos\theta \quad (3-5)$$

where $I(t) = I_0 e^{-\alpha t} \sin \omega t$, ℓ_c is the length of the coil and $\theta = \tan^{-1} \frac{r_c}{\ell_c/2}$.

The probe calibration was determined to be 24.16 kG/volt which was found to agree within 1.5 percent of the theoretical value obtained from Equation 3-4. This agreement was considered satisfactory for this experimental study.

Chapter IV

TWYMAN-GREEN INTERFEROMETRY

4.1 Introduction

One objective of this experimental study was to develop a Twyman-Green interferometer which could be used to investigate the rate of particle loss from a linear theta pinch device. A brief description of both the experimental setup used and the principle on which the Twyman-Green interferometer operates will be presented below. As was stated above, the Twyman-Green interferometer is a double pass, two beam interferometer, which can be used to monitor changes in refractive index of a plasma. With this data, electron densities can be obtained as a function of time, from which the time varying total particle inventory in the plasma column can also be determined. The presentation and interpretation of data will be given in Chapter V.

4.2 Experimental Layout

The Twyman-Green interferometer setup (Figure 4-1) consisted of five major components: a laser light source; an expander and collimating lens apparatus; an isolation system; a set of optical components used to direct both interferometer beams; and two fast silicon photodiode detection systems. Each set of components will be discussed briefly below.

The light source was a He:Ne CW mode model 3224H-PC laser fabricated by the Hughes Aircraft Corporation. The useable power output was measured at 6.9 milliwatts (mW) on a Spectra Physics model 401B power meter. Tests showed no visible drift in power output over a three-hour testing period. This characteristic was necessary so that the light intensity reaching the

Figure 4-1: Twyman-Green Interferometer

detection system would remain constant during successive discharges. The laser beam was linearly polarized and had a divergence of approximately 1 mrad. The beam diameter, given by twice the radius where the intensity drops from the maximum value by e^{-2} , was 0.81 mm.

Connected directly to the laser head was a Spectra Physics collimator and expander, which consisted of a model 332 spatial filter and a model 331 collimating lens separated by a 15 micron aperture. This apparatus was used to expand and collimate the laser beam to a diameter of 19 mm, a dimension approximately equal to the radius of the plasma column being investigated. The spatial filter consisted of a converging lens with a focal length of 12.8 mm, which focused the laser beam to a spot size of approximately 13 microns at the 15 micron aperture. The aperture was used to cut off the radial edge of the beam's Gaussian profile, thereby making the emerging beam more uniform in intensity. After passing through the aperture, the beam was expanded to a diameter of 19 mm and then passed through the collimating lens for focusing to minimize beam divergence. The usable power output of the beam emerging from the collimator-expander was determined to have been approximately 4.8 mW.

A linearly polarized laser source was used so that an inexpensive optical system could be fabricated to isolate the laser from light reflected back by the external mirror cavities created by the two legs of the interferometer. This isolation was considered necessary since if light from an external cavity was reflected back into the laser cavity, modulation of the power output could occur. In other words, if light was allowed to re-enter the laser, the interferometer could exhibit behavior similar to the coupled cavity type interferometer discussed in Chapter I.

The isolation system consisted of an inexpensive Telesar Polaroid camera filter and a mica quarter wave plate. The system operated in the following fashion. The polaroid filter was aligned such that the linearly polarized laser light emanating from the laser would pass through the filter and onto the quarter wave plate. The quarter wave plate was adjusted to convert linearly polarized light to right circularly polarized light. Upon reflection from the external mirrors the light returned as left circularly polarized light and by passing it through the quarter wave plate again the light was converted back into linearly polarized light. The reflected light electric field vector was rotated 90° with respect to the electric field vector of the light leaving the laser cavity. Consequently, this reflected light could not pass through the polaroid filter and back into the laser. By use of this isolation system the intensity of laser light reflected back into the laser cavity was determined to be sufficiently small that modulation of the laser output could not be detected. The power of the beam after passage through the isolation system was found to have been 2.9 mW.

The optical components used were a beamsplitter, two optical windows, three totally reflecting mirrors and a line filter, all of which were designed for use with a He:Ne $6328 \text{ \AA}$ light source. The optical characteristic for each piece of optical equipment can be found in Table 4-1. A brief description of their function in the system will be presented here.

A beamsplitter was used to split the beam emerging from the isolation system into two beams; i.e., the reference beam and the test beam. It was experimentally determined that 52.4 percent of the light reaching the beamsplitter was reflected into the reference leg of the interferometer

TABLE 4-1
CHARACTERISTICS OF OPTICAL COMPONENTS

	Diameter (Inches)	Coatings	Material	Flatness	% Reflection	% Transmission	% Loss
Beamsplitter	2	dielectric	Fused Silica	$\lambda/10$ with 30 min. wedge	52.4	45.7	1.9
Mirrors	1	dielectric	Fused Silica	$\lambda/10$ with 30 min. wedge	99.44	---	0.56
Optical Windows	1	none	Fused Silica	$\lambda/10$	---	94.6	5.4
*Line Filter	2	dielectric	Fused Silica	---	---	60	40

*Line filter had a band width of 45 Å centered around 6328 Å

while 45.7 percent was transmitted into the test leg. The reference leg was 150 cm long and had a totally reflecting mirror at its end which reflected the light back onto itself and back to the beamsplitter. The test leg was also approximately 150 cm in length with a totally reflecting mirror at its end. Optical windows had to be placed at both ends of the discharge chamber in the test leg to allow unimpeded passages of the laser beam through the plasma column. The test beam was aligned such that it was parallel to the discharge chamber axis and was set off axis so that an entire radial side of the plasma column cross-section perpendicular to the discharge chamber axis was illuminated. This was done to enable a detailed radial survey of the plasma column to be performed. Again the test beam was reflected back onto itself and back onto the beamsplitter where the test beam then recombined with the reference beam. This recombined beam was then reflected off a third totally reflecting mirror after which it passed through a line filter and then into the detection system. The line filter was used to eliminate most of the incoherent plasma radiation reflected by the mirrors towards the detectors. Ideally the line filter should have been designed to allow only $6328 \text{ \AA}$ light to pass through. In practice the line filter allowed a $45 \text{ \AA}$ wide band of light centered around $6328 \text{ \AA}$ to pass through. This was determined to be satisfactory, as the ratio of laser light intensity to the intensity of discharge plasma radiation reaching the detectors was found to be very high. Most of the above mentioned optics were fabricated by the Valtec Corporation and had an optical flatness of $\lambda_0/10$, where λ_0 is the wavelength of the light source used ($6328 \text{ \AA}$).

The detection system consisted of circuitry supporting two EGG SGD-040-A fast silicon photodiodes each of which had a 0.815 mm^2 photosurface.

The output of each diode was amplified by an RCA CA-3015-A operational amplifier (Figure 4-2). The load resistance seen by the diodes was designed to be 1.47 k Ω , which gave each diode a band width of approximately 40 MHz. Amplifier gain was approximately 10 dB with the 3 dB cutoff occurring at approximately 18.3 MHz (Figure 4-3). The entire circuit was fabricated inside a 0.13 cm thick aluminum box wrapped with two layers of 3.81×10^{-3} cm thick aluminum foil to minimize electromagnetic noise pick up by the circuit components. The active surfaces of each diode extended out of the box with the centers spaced 8.5 mm apart.

The detector outputs were monitored by a Tektronix type 556 dual beam oscilloscope in which types 1A5 and 1L preamplifiers were used. The oscilloscope was used to record the passage of fringes with respect to time. Total spatial resolution was achieved by repositioning the dual detector between discharges, at different plasma column radii. Data were recorded at three sets of radial positions; $r = 0$ mm and 8.5 mm, 2 mm and 10.5 mm, and 4 mm and 12.5 mm with both positions in each set recorded simultaneously. A typical example of the detector's output is presented in Figure 4-4.

4.3 Theory of Operation

As previously mentioned, when the two beams of light within the interferometer recombine, they interfere with each other in such a way that a fringe pattern is formed which varies temporally and, with position of the detector, spatially. The reason for this interference, is that when the total optical path length of the test leg changes by a factor of λ_0 ($\lambda_0 = 6328 \text{ \AA}$), at any special position, one fringe shift occurs at that position (6). However, since the test beam passes through the plasma

Figure 4-2: Photodiode Detector Circuit Design

Figure 4-3: Amplifier Gain Vs. Frequency

$$P_0 = 65 \text{ mT of H}_2$$

$$r = 0 \text{ mm}$$

Figure 4-4: Typical Example of Photodetector Output

column twice, one fringe shift occurs when the optical path length of the plasma at that radial position changes by a factor of $\lambda_0/2$ (6). The total number of fringes to be seen at any radial position is

$$\chi_r(t) = \frac{\text{number of fringes passing a position in a given amount of time}}{\text{difference in optical path length between a vacuum and the plasma. wavelength of the light source.}} = \frac{\text{difference in optical path length between a vacuum and the plasma.}}{\text{wavelength of the light source.}} \quad (\lambda_0 = 6328\text{\AA})$$

The optical path length is equal to $\mu \ell_t$ where μ is the refractive index of the medium and ℓ_t is the total distance traveled by the laser beam in passing through the medium. For a vacuum, $\mu = 1$, but for a plasma, it has been shown (6) that $\mu_p \approx 1 - \frac{1}{2}(\omega_p^2/\omega_L^2)$ where;

$$\omega_p = \text{plasma frequency} = [(4\pi e^2/m_e)n_e]^{1/2}, \text{ (rad/sec),}$$

$$e = \text{electronic charge, (esu),}$$

$$m_e = \text{mass of electron, (gm),}$$

$$n_e = \text{electron density, (cm}^{-3}\text{)}$$

$$\omega_L = \text{laser frequency} = 2\pi c/\lambda_0, \text{ (rad/sec),}$$

$$c = \text{speed of light, (cm/sec),}$$

$$\lambda_0 = \text{wavelength of laser light source being used} = 632.8 \times 10^{-7} \text{ cm.}$$

Making the appropriate substitutions, the refractive index for a plasma medium is:

$$\mu_p = 1 - 4.477 \times 10^{-14} n_e \lambda_0^2.$$

The optical path length can be written as $\mu \ell_t = \int_0^{2\ell_p} \mu_p d\ell$, where ℓ_p is the length of the plasma column. The upper limit of the integration is $2\ell_p$ instead of ℓ_p because the system was a double pass type. The number of fringes expected can be written as:

$$\chi_r(t) = \frac{\int_0^{2\ell_p} \mu_{\text{vacuum}} d\ell - \int_0^{2\ell_p} \mu_{\text{plasma}} d\ell}{\lambda_0} \quad (4-1)$$

or

$$\chi_r(t) = \frac{\int_0^{2\ell} p(l) dl - \int_0^{2\ell} p[1 - 4.477 \times 10^{-14} n_e \lambda_0^2] dl}{\lambda_0}$$

therefore,

$$\chi_r(t) = 4.477 \times 10^{-14} \lambda_0 \int_0^{2\ell} p n_e dl. \quad (4-2)$$

By rearranging terms, the total change in the line density corresponding to a complete fringe was calculated to be,

$$\int_0^{2\ell} p n_e dl = \frac{1.117 \times 10^{13}}{\lambda_0} \text{ electrons cm}^{-2} \quad (4-3)$$

Since $\lambda_0 = 632.8 \times 10^{-7} \text{ cm}$, one fringe represents a change in average line density of $1.765 \times 10^{17} \text{ electrons cm}^{-2}$. The net change in average number density represented by one fringe can be obtained by dividing the change in average line density by the length of the plasma column, therefore, per fringe,

$$\frac{\Delta n_e}{\ell_p} = \frac{1.765 \times 10^{17}}{\ell_p} \text{ electrons cm}^{-3}. \quad (4-4)$$

It is mentioned, at this point, that in order for the interferometer to be governed by Equations 4-1 through 4-4, it must be properly aligned. The proper alignment for the Twyman-Green interferometer is obtained when the reference beam and the test beam are exactly reflected back upon themselves by the mirrors at the ends of each leg. If this is done correctly the two beams should interfere with each other, when there is no plasma present, in a way that is sometimes referred to as the infinite fringe mode. In the infinite fringe mode, the two beams will interfere with each other such that there is either total constructive or destructive interference across the entire cross section of the recombined beam. Theory predicts an infinite distance between adjacent fringes. In

reality this is very difficult to achieve. Therefore a reasonable approximation to this mode was obtained by aligning the system such that only one horizontal fringe appeared across the face of the beam before the plasma was formed in the discharge chamber. It was found that careful realignment of the system had to be carried out between each discharge to insure accurate data. The one fringe was aligned horizontally so that the detectors, which were also aligned horizontally, would both sense the same intensity light prior to the discharge.

4.4 Discussion of Expected Fringe Behavior

In the literature (31, 32) it has been shown that when a Twyman-Green interferometer is used to investigate the uniformity in the parallelism between two sides of an optical piece of glass, fringes form along regions of equal optical path length. In other words, fringes form along regions where the refractive index (n) times the thickness (d) of the piece of optics in question is equal to a constant. In a plasma this is analogous to regions where the average line densities are equal. Therefore, since a plasma is usually radially symmetric in line density, the fringe pattern expected upon illuminating the entire plasma column cross section perpendicular to the discharge chamber axis will be circular in nature. This type of fringe pattern is very similar to that found when using most other types of interferometric methods where spatial resolution is desired; i.e., Mach-Zehnder interferometry and holographic interferometry. Photographs of the fringe pattern obtained from both of these two types of interferometers have been reported (3, 7, 33). These photographs show that inside the dense regions of the plasma column circular fringes form along contours of equal density. The distance

between each fringe for the Twyman-Green interferometer used here should correspond to the distance between regions where the average line density differs by 1.765×10^{17} electrons cm^{-2} . Buser and Kainz (10) reported that they also expected a similar fringe pattern to be formed when investigating a plasma using a Twyman-Green type interferometer.

Fringe movement will be caused by either decreasing or increasing line density in the region in question. Radial movement of fringes with time should occur in a systematic manner. If the electron density is presumed to increase within a central region of the plasma column, already existing fringes should move radially outwards from the center of the plasma column, and new fringes will originate at the center where the density is a maximum. Of course, when the density decreases the opposite should occur. A relative reference condition in this experiment was the effectively zero electron number density at large radii.

In the linear theta pinch device investigated in this study, three processes were found to cause fringe motion during the lifetime of the plasma column. First, since dynamic sheath collapse occurred in about 1 μsec , the plasma column was formed before peak magnetic field was reached ($\sim 2 \mu\text{sec}$), and the column was then compressed until the magnetic field reached its peak value. This can be expected to cause a slight increase in line density near the center of the plasma column. After peak magnetic field, the plasma column predictably expanded against the decreasing magnetic field ($\sim 2 \mu\text{sec} - 4 \mu\text{sec}$) and a related decrease in line density would occur. Second, simultaneous with these processes, particle loss through the column ends occurred causing a decrease in the total plasma column particle inventory which also resulted in fringe movement. Lastly, the process resulting from radial oscillations

("bounce") of the plasma column also had a significant effect on fringe movement. These oscillations periodically compress and expand the plasma column causing a periodic change in the average plasma column line density.

From loss times found from scaling larger present day machines and line densities estimated from the snowplow model, the loss process was expected to be responsible for about one complete fringe movement or less at any radial position, during the first three microseconds of plasma column lifetime. However, radial plasma oscillations could account for more than one fringe movement during plasma column lifetime depending on fill pressure and magnetic field duration. Since this particular theta pinch was a weak field device ($B_{\max} \approx 5.42$ kG) the effect of compression and expansion of the plasma column against the confining magnetic field was expected to be minimized.

Chapter V

DATA ANALYSIS AND RESULTS

5.1 Introduction

In this chapter, data obtained from the axial magnetic field probe surveys and the Twyman-Green interferometer will be analyzed to predict plasma behavior during implosion, and plasma column formation and development. Along with these experimental methods, a computer simulation will be presented and used to predict aspects of plasma behavior which the diagnostic data could not be used to predict. Section 5.2 will deal with the implosion phase of the plasma formation, and Section 5.3 will deal exclusively with post implosion plasma column behavior.

5.2 Dynamic Current Sheath Collapse Phase of the Discharge

The implosion phase of the discharge was investigated using several diagnostic devices described previously. These devices were the axial magnetic field probe and the Twyman-Green interferometer. A radial survey of the time rate of change in the axial magnetic field, $\dot{B}_z$, was performed so that current sheath position with respect to time could be identified. The Twyman-Green interferometer was used to establish the time of mass arrival at the center of the discharge chamber. From these studies percent mass pick up by the imploding current sheath, and pinch time, t_p , the time for the current sheath to implode into the center of the discharge chamber were obtained for fill pressures of 50 and 65 millitorr of hydrogen gas.

The nonintegrated output of the magnetic field probe, proportional to $\dot{B}_z$, was monitored at five radial positions: $r = 3.81$ cm, 3.175 cm,

2.54 cm, 1.90 cm, and 1.59 cm. Since the magnetic field probe output indicated regions of intense current flow, the signal proportional to $\dot{B}_z$ was used to identify the time of the current sheath arrival at these various radial positions. Typical examples of $\dot{B}_z$ data are presented in Figures 5-1 and 5-2. These figures show that at the beginning of the second half cycle ($t = 4\mu\text{sec}$) a current sheath was formed at the periphery of the discharge chamber. The B_z field produced by the diamagnetic current sheath cancelled the external B_z field in the plasma causing the $\dot{B}_z$ component to go to zero. Then, when the current sheath imploded inwards due to the radial $\vec{J} \times \vec{B}$ force and passed over the magnetic field probe, the probe was again able to sense the external magnetic field causing a rapid change in $\dot{B}_z$. Thus the arrival time of the current sheath at any radial position coincided with a large spike in the $\dot{B}_z$ data at that position. A plot of the current sheath position with respect to time was obtained from the records of the radial survey of the axial magnetic field probe (Figure 5-3). Current sheath arrival times were not obtained for r less than 1.59 cm because severe oscillations in $\dot{B}_z$ caused by radial plasma oscillations masked the spike caused by the arrival of the current sheath.

Also in Figure 5-3, a plot of the arrival times of mass between the radial positions $r = 12.5$ mm and 0 mm is shown. These points were obtained from the Twyman-Green interferometric data. The interferometer was aligned such that data could be taken only for radii smaller than 12.5 mm. Since the interferometer was sensitive to changes in line density, it was used to sense the arrival of mass near the center of the discharge chamber. The arrival of the mass was indicated by a very quick deflection in the output of the detector due to rapid fringe motion

$r = 38.10 \text{ mm } \dot{B}_z$

↑
arrival of current
sheath

 $r = 31.75 \text{ mm } \dot{B}_z$

↑
start of second
half cycle

 $r = 25.40 \text{ mm } \dot{B}_z$

 $r = 19.00 \text{ mm } \dot{B}_z$

(Note: data taken at discharge compression coil midplane:
 $z = 5.25 \text{ cm}$)

Figure S-1: Radial Survey of Axial Magnetic Field Probe for a 50 Millitorr Fill Pressure of H_2

(Note: data taken at discharge compression coil midplane:
 $z = 5.25 \text{ cm}$)

Figure 5-2: Radial Survey of Axial Magnetic Field Probe for a 65 Millitorr Fill Pressure of H_2

Figure 5-3: Current Sheath Implosion Phase of Discharge

associated with the quick change in line density. Typical examples of Twyman-Green data are presented in Figure 5-4.

By use of the simple force balance equation $F = \frac{d}{dt}(mv)$ and the plot of the current sheath trajectory in Figure 5-3, estimates on the percent mass pick up by the imploding current sheath were obtained for fill pressures of 50 and 65 millitorr. Since,

$$F = m_s(t) \frac{dv_s(t)}{dt} + v_s(t) \frac{dm_s(t)}{dt},$$

where $m_s(t)$ is the mass swept-up by the current sheath, then

$$F/A_s = \frac{m_s(t)}{A_s} \frac{dv_s(t)}{dt} + \frac{v_s(t)}{A_s} \frac{dm_s(t)}{dt},$$

where A_s is the surface area of the current sheath parallel to the discharge chamber axis. The quantity F/A_s equals the external magnetic field pressure $B_{\text{ext}}^2(t)/8\pi$, so that

$$\begin{aligned} \frac{B_{\text{ext}}^2(t)}{8\pi} &= \frac{m_s(t)}{A_s} \frac{dv_s(t)}{dt} + \frac{v_s(t)}{A_s} \frac{dm_s(t)}{dt}, \\ &\approx \frac{m_s(t)}{A_s} \frac{\Delta v_s(t)}{\Delta t} + \frac{v_s(t)}{A_s} \frac{\Delta m_s(t)}{\Delta t}, \end{aligned} \quad (5-1)$$

where $\Delta m_s(t)/\Delta t = \eta m_i n_0 2\pi r_s(t) \ell_s v_s(t)$

m_i = mass of hydrogen ion, (gm),

n_0 = initial fill density, (cm^{-3}),

$r_s(t)$ = radial position of the current sheath, (cm),

ℓ_s = axial length of the current sheath, (cm),

η = fraction of the total mass entrained by the imploding current sheath,

$A_s = 2\pi r_s(t) \ell_s$,

$m_s(t) = \eta(\pi b^2 - \pi r_s^2(t)) \ell_s m_i n_0$, (gm),

b = inside diameter of the discharge chamber, (cm).

Figure 5-4: Typical Twyman-Green Interferometric Data

Substituting the expressions for $\Delta m_s(t)/\Delta t$, $m_s(t)$, and A_s into Equation 5-1 yields,

$$\frac{B_{\text{ext}}^2(t)}{8\pi} = n m_i n_o [v_s^2(t) + \frac{b^2 - r_s^2(t)}{2r_s(t)} \frac{\Delta v_s(t)}{\Delta t}] \quad (5-2)$$

Equation 5-2 was used to calculate average values of n between any two consecutive data points, i.e.

$$\bar{n} = \left(\frac{\bar{B}_{\text{ext}}^2}{8\pi} \right) \times \frac{1}{m_i n_o \left[\bar{v}_s^2 + \frac{b^2 - \bar{r}_s^2}{2\bar{r}_s} \frac{\Delta \bar{v}_s}{\Delta t} \right]} \quad (5-3)$$

From Figure 5-3 average values of v_s , $\Delta v_s/\Delta t$, and r_s were obtained, while the average values of B_{ext} were known. This enabled a percent mass pick up factor to be calculated from $r_s = 5.08$ cm to $r_s = 1.59$ cm. It was then assumed that 100 percent of the mass was picked up from $r_s = 1.59$ cm to $r = 0$ cm since these points lay well within the final plasma column radius of approximately 2.1 cm. It was determined by this procedure that approximately 81.3 percent of the total mass was entrained by the imploding current sheath at 50 millitorr fill pressure and 76.9 percent was entrained with 65 millitorr fill pressure.

The Twyman-Green interferometer was used to sense the change in line density caused by the current sheath "snowplowing" the mass towards the center of the discharge chamber. From this data it was determined that the mass reached the chamber axis in 0.948 μsec for 50 millitorr fill pressure and 1.010 μsec for 65 millitorr fill pressure. These times were the times when the mass first started to reach the discharge chamber axis. They were determined from the data by the times of initial fringe movement as indicated in Figure 5-4. However, this method of interpretation of data gives a lower limit on the pinch time. An upper

limit was obtained by measuring the time it took for peak density to occur at the center of the discharge axis during implosion. This time was indicated in the data by a half fringe shift deflection in the detectors output from the output level before implosion. This time is also indicated in Figure 5-4. The upper limit on the pinch time was determined to have been 1.043 μsec for 50 millitorr fill pressure and 1.107 μsec for 65 millitorr fill pressure. It was determined that the upper limit estimate on pinch time agreed more closely with arrival time of the current sheath found by extrapolating the plot of the $\dot{B}_2$ data to the $r = 0$ axis.

These pinch times were compared with pinch times predicted from a simplified snowplow implosion model given by York and McKenna (4), and characteristic pinch times calculated using snowplow implosion equations developed by Artsimovich (34). The York-McKenna snowplow theory predicts the pinch time to be governed by the following relationship (4),

$$\frac{B_{\text{max}}^2 \sin^2(\omega t_p)}{4} / 8\pi n_i n_o = b^2 / t_p^2 \quad (5-4)$$

where B_{max} = maximum external magnetic field strength, (gauss),

ω = frequency of the magnetic field, (rads/sec), and

t_p = pinch time, (μsec).

From the Equation 5-4 it was calculated that the pinch time was 0.929 μsec for 50 millitorr fill pressure and 0.984 μsec for 65 millitorr fill pressure.

Artsimovich (34) showed that $u_s(t)$, the normalized current sheath position equalling $r_s(t)/b$, satisfies;

$$r_s(t)/b \approx 1 - \frac{\tau_s^2}{\sqrt{12}} + \dots \quad (5-5)$$

where τ_s is the unitless time parameter equalling $t/[(n4\pi b^2 n_o m_i)^{1/4}/(\overline{dB_z}/dt)^{1/2}]$, and $\overline{dB_z}/dt$ = average time rate of change of the axial magnetic field during the implosion. Therefore the pinch time is found to be;

$$\tau_p = [(n4\pi b^2 n_o m_i)^{1/4}/(\overline{dB_z}/dt)^{1/2}]\tau_o \quad (5-6)$$

where τ_o is the value of τ_s found from Equation 5-5 when $\tau_s(t)$ is set equal to zero. It was determined that $\tau_o = 1.861 \mu\text{sec}$. From Equation 5-6 it was calculated that for 50 millitorr fill pressure $\tau_p = 1.006 \mu\text{sec}$, and for 65 millitorr fill pressure $\tau_p = 1.059 \mu\text{sec}$.

As can be seen, fairly good agreement was obtained between experimental and theoretically determined pinch times. Differences between snowplow predicted times and experimentally found times are thought to stem from the assumptions made in the snowplow models used here. The assumptions made in the York-McKenna theory were that acceleration effects were negligible during the implosion of the current sheath, and values of the velocity and magnetic field used were values averaged over the entire time of the implosion. Differences between Artsimovich's estimate on pinch times, which includes the effect of sheath acceleration, and experimental times are thought to have been caused by the use of the average $\overline{dB_z}/dt$ term, which was obtained from the assumption that B_z was linear during the implosion phase of the discharge. As can be seen, the lower limit on experimentally found pinch times agrees more closely with times calculated using the York-McKenna snowplow model while the upper limit agrees closely with times predicted by Artsimovich's snowplow implosion model.

5.3 Post-Implosion Plasma Column Behavior

5.3.1 Introduction

In this section, post-implosion plasma column behavior will be analyzed in detail. This detailed analysis will be performed using both theoretical and experimental results. Initial values of plasma column temperature, average density and radius will be calculated using a snowplow implosion model. Both plasma column length and time varying particle inventory, $N(t)$, will be estimated from axial magnetic field probe data. Plasma radius as a function of time, $r_p(t)$, will be estimated using both the magnetic field probe data and a zero dimensional computer code. Also, five different estimates--three based on experimental results --are presented for the $1/e$ particle end loss time. The first analytical method uses the zero dimensional computer code, while another estimate is based on $L/(T_i/n_i)^{1/2}$ end loss scaling from Scylla I-C (3, 4, 19), a larger collision dominated theta pinch experiment conducted at the Los Alamos Scientific Laboratory. Experimental loss times will be determined from magnetic field probe data obtained by probing internal to the plasma column, and, using two different analysis methods, from the Twyman-Green interferometric data. Finally, a brief comparison and discussion of the five loss time estimates will be presented.

5.3.2 Post-implosion initial conditions

Since the only diagnostics used to investigate plasma behavior after column formation were the axial magnetic field probe and the Twyman-Green interferometer, no information on initial plasma radius and temperature could be obtained. However, these quantities were needed as initial conditions for the computer simulations. A snowplow implosion model was

used to obtain the necessary parameters. It was shown in the previous section that the snowplow implosion model compared satisfactorily in estimate of pinch time, t_p , with experimentally found times. Also, the snowplow model is generally thought to be valid when the ion-ion mean free path, λ_{ii} , is much less than the current sheath thickness (4). As will be shown shortly, an upper limit on λ_{ii} is 0.072 cm. Experiments conducted on low temperature (~ 50 eV), highly collisional plasmas have shown that current sheath thickness is on the order of 1 cm (35). Therefore, since $\lambda_{ii} \ll 1$ cm it was decided that the snowplow model would best describe the plasma being investigated here.

The snowplow model used was developed by York and McKenna (4). In this model it was assumed that the collapse of the current sheath could be modeled by use of an average implosion velocity. The plasma then expanded due to thermal pressure to form the plasma column. From this model the post-implosion plasma temperature, kT_p , equalling the electron plus the ion temperature, can be approximated by;

$$kT_p = \frac{B_{\max}^2 \sin^2(\omega t_p) / 32\pi}{n n_0 (\frac{3}{2} + \frac{1}{\beta})}, \quad (5-7)$$

where $B_{\max}$ = maximum external magnetic field strength, (gauss),

n_0 = initial fill density, (cm^{-3}),

β = the magnetic field efficiency factor defined by

$$(1 - \beta) = \frac{B_{\text{int}}^2}{B_{\text{ext}}^2},$$

and B_{int} is the magnetic field strength inside the plasma column. The equilibrium radius (r_0) predicted by this model can be determined from;

$$r_o^2 = \frac{n}{\beta} n_o \frac{b^2 kT_p}{B_{\max}^2 \sin^2(\omega t_p) / 8\pi} \quad (5-8)$$

where kT_p is in ergs and r_o is in centimeters. Finally, the average number density is given by;

$$\frac{n}{n_p} = \frac{n_o b^2}{r_o^2} \quad (5-9)$$

Values for these parameters were calculated for fill pressures of 50 and 65 millitorr using the percent mass pick up factors calculated in the previous section (Section 5.2) and an estimated average plasma β of 0.3. This estimate of β was based on magnetic field probe data. The magnetic field probe data at $r = 25.4$ mm indicated that no plasma column oscillations were present; therefore, the probe had to have been situated outside the final plasma column radius. However, data taken at $r = 19.0$ mm showed signs of these plasma oscillations, indicating the probe was interior to the plasma column. These data are presented in Figures 5-1 and 5-2. By use of Equation 5-8, only β 's in the range of 0.25 to 0.4 yield plasma radii between the above specified limits. Accordingly, a β value of 0.3 was chosen. This value of β leads to estimates of t_p , τ_p , and $N(t)$ which are consistent with several experimental and computational methods. The results from calculations performed using Equation 5-7 through 5-9 are presented in Table 5-1.

From the estimates of plasma temperature presented in Table 5-1, an upper limit on the ion mean free path, λ_{ii} , was calculated using an equation given by Chen (36),

$$\lambda_{ii} = \frac{4.5 \times 10^{13} T_i^2}{n_o \ln \Lambda}$$

TABLE S-1
 SNOWFLOW IMPLOSION MODEL PREDICTIONS OF
 INITIAL PLASMA COLUMN PARAMETERS

Fill pressure (millitorr)	50	65
Plasma temperature (eV) ($T_e + T_i$)	6.46	5.76
Equilibrium plasma radius r_0 (cm)	2.11	2.11
Average number density $\bar{n}_p$ (cm ⁻³)	1.52×10^{16}	1.87×10^{16}

where T_i is the temperature of the ions during implosion (eV), and Λ , the ratio of the Debye shielding distance to the impact parameter, is

$$\Lambda = 3/2 \left(\frac{(kT_p)^3}{\pi n_0} \right)^{1/2} \frac{1}{e}$$

where e is the electronic charge, (esu). Since an estimate of ion temperature during implosion could not be made, it was assumed that this temperature was the plasma column temperature, kT_p , following implosion. For a 50 millitorr fill pressure, the ion-ion mean free path was calculated to be 0.072 cm.

Plasma column length, also essential for analyzing plasma column behavior, was estimated. In most larger theta pinch machines, the plasma column length is approximately equal to the length of the discharge compression coil. However, the machine considered here had a short compression coil length (10.5 cm), and an axial survey of the axial magnetic field indicated an identifiable plasma column existed only near the center of the coil. This was attributed to fringing effects of the field lines at the ends of the compression coil.

Examples of an axial survey of the axial magnetic field are presented in Figures 5-5 and 5-6. It was decided that the ends of the plasma could be defined at those locations where the magnitude of the internal axial magnetic field oscillations decreased by 67 percent from the value obtained at the compression coil mid-plane ($z = 5.25$ cm) (see Figure 5-5). Two checks were also made to verify that the criteria mentioned above were appropriate. The first of these is based on B_z behavior. At the midplane, $z = 5.25$ cm, Figure 5-5 showed that during implosion most of the magnetic field was excluded from the plasma. The evidence was that B_z was zero during implosion until the current sheath passed over the

$$P_0 = 50 \text{ mT of } H_2$$

$$r = 13 \text{ mm}$$

(Note: $z = 5.25$ cm denotes midplane of discharge compression coil)

Figure 5-5: Axial Survey of Axial Magnetic Field Probe Data (used as indication of plasma column length for a 50 millitorr fill pressure)

(Note: $z = 5.25 \text{ cm}$ denotes midplane of discharge compression coil)

Figure 5-6: Axial Survey of Axial Magnetic Field Probe Data (used as indication of plasma column length for a 65 millitorr fill pressure)

axial magnetic field probe. This indicated that the imploding current sheath was highly diamagnetic and was carrying a large amount of mass. However, B_z data at $z = 2.25$ cm revealed that a very weak current sheath formed with little magnetic field being excluded, indicating a low mass pick up. The second check was to estimate the mass pick up at these positions. The mass pick up was found to be less than 50 percent of the total mass available. This drop in mass pick up was found to occur around $z = 2.25$ cm. For these reasons it was felt that the criteria used was valid, and from this it was determined that the plasma column length was approximately 6 cm for fill pressure of 50 and 65 millitorr.

5.3.3 Predictions from the computer model

The next step taken in analyzing the post-implosion plasma column was to predict its behavior by use of a zero dimensional computer code developed by Stover (6). A brief description of this code and its output will be presented. Predicted plasma column behavior will then be compared with experimentally determined plasma column behavior.

The code simulates post-implosion plasma column behavior by numerical solution of four basic equations. These four equations for T_e , T_i , n and A_p , the plasma area, were obtained from equations for conservation of electron energy, conservation of ion energy, conservation of total number of particles in the system, and radial pressure balance. The model includes the effects of particle end loss, energy loss due to both ion and electron thermal conduction along magnetic field lines, electron-ion thermal equilibration, a diffuse radial number density profile, diffusion of external magnetic field into the plasma column, and resistive heating of the plasma column electrons. The model does

not include plasma column radial inertial effects associated with radial oscillations, or implosion dynamics.

The above-described code was used to predict plasma column behavior for fill pressures of 50 and 65 millitorr. Since the code does not model implosion dynamics, estimates on initial plasma column parameters r_0 , kT_p , and $\bar{n}_p$ immediately following implosion were determined as discussed earlier in this section. The initial electron and ion temperature were assumed to be equal, for the ion electron energy equilibration time was short with respect to both the implosion time and the magnetic field quarter cycle time.

The computer program estimated the time-varying pressure equilibrium radius, $r_0(t)$, including effects caused by diffusion of the magnetic field. Also determined were estimates on the time-varying particle inventory, $N(t)$, peak electron and ion temperatures, and 1/e particle end loss time, τ_p . It was found that for a 50 millitorr fill pressure, the particle inventory varied as;

$$N(t) = \exp(41.690 - 0.4430t) , \quad (5-10)$$

where t is in μsec . The predicted 1/e particle end loss time τ_p , was 2.26 μsec . At a 65 millitorr fill pressure $N(t)$ varied as;

$$N(t) = \exp(41.986 - 0.4057t) , \quad (5-11)$$

where the predicted 1/e particle end loss time was 2.47 μsec .

5.3.4 End loss time from analysis of magnetic field probe data

Axial magnetic field probe data taken at the discharge compression coil mid plane indicated that the plasma column experienced significant radial oscillations. These oscillations closely approximated a damped sinusoidal oscillation. Typical examples of such data are presented in

Figure 5-7. The oscillatory nature of both B_z and $\dot{B}_z$ following column formation indicated that the internal magnetic field lines were alternately compressed and expanded. These radial oscillations were caused by the overshoot of the imploding current sheath past the pressure equilibrium radius ($r_0 = 2.11$ cm) due to plasma inertial effects. This set the plasma column into oscillations around the equilibrium radius. It has been shown (37, 38) that the time dependent oscillation frequency can be related to the instantaneous total particle inventory. It was necessary, however, to rederive the governing relationship to fit the special constraint of the theta pinch device investigated here. A first order approximation for the relationship between $N(t)$ and the frequency of oscillation $\omega(t)$ was found to be satisfactory. The derivation is presented in detail in Appendix A and only the results will be presented below. The governing relationship is shown to be (A-14);

$$N(t) = \frac{B_{\text{ext}}^2(t)}{\omega^2(t)} \frac{l_p}{m_i} \frac{E}{4},$$

where $B_{\text{ext}}(t)$ = time-varying external magnetic field strength; (gauss),

$\omega(t)$ = time-varying frequency of oscillation, (rads/sec),

l_p = plasma column length, (~ 6 cm),

m_i = mass of a proton, (1.67×10^{-24} gms),

E = a constant

$$= \frac{(4 - \frac{2}{3}\beta)b^4 r_0^{16/3} - (4 - \frac{4}{3}\beta)b^2 r_0^{22/3} - \frac{2}{3}\beta r_0^{28/3}}{r_0^{28/3} + b^4 r_0^{16/3} - 2b^2 r_0^{22/3}}$$

r_0 = equilibrium radius of plasma column, (~ 2.11 cm).

Setting $b = 5.08$ cm and $\beta = 0.3$, and substituting the appropriate parameter values into Equation A-14, $N(t)$ becomes,

Figure 5-7: Axial Magnetic Field Probe Data Indicating Radial Plasma Oscillations

$$N(t) = 4.16 \times 10^{24} \frac{B_{\text{ext}}^2(t)}{\omega^2(t)} \text{ particles.} \quad (5-12)$$

$N(t)$, was calculated between each set of adjacent peaks (indicated by arrows in Figure 5-7, equalling one half cycle of oscillation) in the oscillatory part of the internal magnetic field. This was done by assuming first that the half cycle time represented one half of the average period of one oscillation $\bar{\tau}_{\text{os}}$. Then it was assumed that these average periods were the value of the period occurring at a time half way between the ends of the half cycle (half way between adjacent arrows indicated in Figure 5-7). Knowing the average period of oscillation enabled the average frequency for the half cycle of oscillation to be calculated; $\bar{\omega} = 2\pi/\bar{\tau}_{\text{os}}$. Using $\bar{\omega}$ and the known average external magnetic field, $N(t)$ was calculated. In Table 5-2, the average frequencies, times, external magnetic field strength and $N(t)$ are tabulated for both 50 and 65 millitorr fill pressures. A plot of these data points is presented in Figure 5-8.

A least squares fit was performed on the data points in Table 5-2 to obtain $N(t)$ for each fill pressure. At a 50 millitorr fill pressure, $N(t)$ was determined to vary as;

$$N(t) = \exp(41.793 - .4903t) , \quad (5-13)$$

where t is in μsec . For 65 millitorr fill pressure $N(t)$ was found to be described by;

$$N(t) = \exp(41.916 - .4088t) . \quad (5-14)$$

From Equations 5-13 and 5-14, an estimate of $1/e$ particle loss time was obtained. It was determined that $\tau_p \approx 2.04 \mu\text{sec}$ for a 50 millitorr fill pressure and $\tau_p \approx 2.45 \mu\text{sec}$ for a 65 millitorr fill pressure. The results of Equations 5-13 and 5-14 are also shown in Figure 5-8.

TABLE 5-2

TABULATION OF $N(t)$ CALCULATED FROM RADIAL PLASMA
OSCILLATION FREQUENCY

P_o (mT)	$\bar{t}$ (μ sec after implosion)	$\omega(\bar{t})$ (rads/sec)	$B_{ext}(\bar{t})$ (gauss)	$N(\bar{t})$ (particles)	$N(\bar{t})$ from computer program (particles)
50	0.353	8.795×10^6	4.698×10^3	1.187×10^{18}	1.077×10^{18}
50	0.679	1.065×10^7	5.226×10^3	1.002×10^{18}	0.919×10^{18}
50	0.962	1.166×10^7	5.403×10^3	0.894×10^{18}	0.803×10^{18}
65	0.305	8.203×10^6	4.711×10^3	1.373×10^{18}	1.368×10^{18}
65	0.660	9.613×10^6	5.264×10^3	1.248×10^{18}	1.162×10^{18}
65	0.976	1.032×10^7	5.419×10^3	1.148×10^{18}	1.010×10^{18}
65	1.267	1.130×10^7	5.241×10^3	0.895×10^{18}	0.888×10^{18}

Figure 5-8: Experimentally Determined Total Particle Inventory with Respect to Time

Comparing Equations 5-13 and 5-14 with the equations for $N(t)$ found from the computer simulations, i.e., Equations 5-10 and 5-11, it can be seen that there is good agreement in both $N(t)$ and r_p . Any differences were attributed to statistical error in the data, to the limitations in the validity of Equation A-14, and to the limitations on validity of the simulation. As a result of this good agreement between these two methods of determining $N(t)$, it is felt that the computer code is a reasonably accurate method of determining post-implosion plasma column behavior, except for oscillations.

The plasma column radius can be written as $r_o(t) + \Delta r_p(t)$, where $\Delta r_p(t)$ is the oscillatory part of the plasma radius caused by the radial plasma oscillations. $\Delta r_p(t)$ can be obtained by use of the frequency of plasma column oscillations, and the damping rate of these oscillations. Since the data for B_z presented in Figure 5-7 indicate that these oscillations closely approximate a damped sinusoid, $\Delta r_p(t)$ was assumed to vary as:

$$\Delta r_p(t) = \Delta r_{pmax} [\sin \omega(t)t] e^{-\alpha_{os}(t)t}, \quad (5-15)$$

where Δr_{pmax} is a constant, (cm), the damping rate $\alpha_{os}(t)$ is found experimentally, (sec^{-1}), and t is in seconds after implosion. An estimate on Δr_{pmax} was obtained by assuming that both compression and expansion of the plasma column were adiabatic and varied as (4);

$$r_p(t) = r_o(t) \left(\frac{B_{oint}}{B'_{int}} \right)^{0.6}$$

where $r_p(t)$ is the actual plasma radius, $r_o(t)$ is the pressure equilibrium plasma radius estimated by the computer simulation, B_{oint} is the equilibrium internal magnetic field strength, (gauss), and B'_{int} is the actual internal magnetic field strength, (gauss). Therefore, Δr_p at the

time of the first compression ($t \approx 0.2 \mu\text{sec}$ after implosion) equals $r_o(t) - r_p(t)$, with both evaluated at $t \approx 0.2 \mu\text{sec}$. From this, $\Delta r_{p\text{max}}$ can be calculated. $\Delta r_{p\text{max}}$ was determined to be approximately 0.250 cm for a 50 millitorr fill pressure and 0.349 from a 65 millitorr fill pressure. Since $r_o(t)$ was determined from the computer simulation, an estimate on the actual plasma radius was obtained. Plots of $\Delta r_p(t)$, $r_o(t)$ and $r_p(t)$ are presented in Figure 5-9.

5.3.5 End loss time from Twyman-Green interferometry

A fundamental goal of this research was the use of Twyman-Green interferometry to predict post-implosion plasma column behavior. The Twyman-Green interferometer was used to monitor changes in line density during the plasma column lifetime. An estimate of the 1/e particle end loss time was obtained by analyzing the interferometric data. However, interpretation of the data was difficult because changes in line density resulted from three effects: (1) particle end loss; (2) radial plasma oscillations; and (3) changes in plasma column radius due to the time-varying external magnetic field and the diffusion of magnetic field into the plasma column. The method of data reduction used to obtain τ_p will be generally described first, then the data will be presented and described, and lastly the detailed analysis of the data will be presented.

As a number of processes cause fringe motion during the lifetime of the plasma column a method was developed so that these data could be reduced to provide information relating to the loss phenomenon. From this reduced information an estimate of the 1/e particle end loss time can be obtained. The orders of magnitude and general behavior of the fringe shift components will now be discussed. After implosion, as

Figure 5-9: Estimated Plasma Column Radius with Respect to Time

mentioned above, the fringe motion is initially dominated by radial plasma column oscillations which tend to mask the fringe motion due to particle end loss. By waiting until these oscillations damp out (≈ 1.7 μsec after implosion), this source of fringe motion could be eliminated. Therefore, it was decided to consider data relating to loss over a time interval starting at the end of implosion and ending after the radial plasma oscillations had damped out. This is a significant fraction of the total plasma column lifetime, which ends at the start of the second half cycle of the driving current (≈ 3 μsec after implosion). Furthermore, data analysis is most natural at a time when the external magnetic field strengths were equal. Since pinch occurred approximately 1 μsec after the start of the second half cycle, and the magnetic field strength was symmetric about its peak (≈ 2 μsec after the start of the second half cycle), it was decided that the time period for which end loss would be evaluated would be the first 2 μsec of plasma column lifetime, i.e., from approximately 1 μsec to 3 μsec after the beginning of the second half cycle.

The first step in data reduction was a determination of the fringe motion excluding motion due to the radial plasma oscillations. It is useful to consider what would happen if there was no particle end loss and no diffusion of magnetic field into the plasma column. Since the radial plasma oscillations had damped out by the end of the 2 μsec time period, and the external magnetic field strengths were approximately equal at the start and end of the time period, the plasma column at $t = 2$ μsec after implosion would have been in the same state as it was at implosion ($t = 0$ μsec after implosion). The associated fringe pattern would also have been the same, and the photodetector would have indicated

this by having the identical voltage output at both times. Therefore, in the actual event, any difference between the voltage output of the photodetector at these two times can be associated with fringe movement due to particle end loss and any changes in the plasma column radius due to the combined effects of the compression of the plasma column by the external magnetic field and by magnetic field diffusion. The radial changes occur in order to maintain pressure equilibrium and to allow the expansion of the plasma column due to radial diffusion. Figure 5-9 indicates little difference between plasma radii at these two times, suggesting the two magnetic field effects tended to cancel each other. The fringe shift observed over this time period can be expected to be one fringe or less; any more change would indicate an unrealistically low $1/e$ particle end loss time, as discussed in Chapter IV.

The direct reduction of fringe data results in a calculation of number density change at a given radius which proceeds in the following manner: since the fringe shift between the two times of interest can be determined from the data at various radial positions, ΔX_r , the corresponding change in line density at a given radial position, Δn_r , can be determined using Equation 4-4 by substituting 6 cm for L_p , and multiplying this number by ΔX_r . The problem, however, is to relate the change in number density at a given radius, and the column radius, with end loss time. To accomplish this a method was devised for relating Δn_r to the change in the total particle inventory in the plasma column, $N(t)$. It was presumed, as is usually the case, that $N(t)$ varies as

$$N(t) = N(0)e^{-t/\tau_p}, \quad (5-16)$$

where $N(0)$ is the initial particle inventory and τ_p is the $1/e$ particle end loss time (3, 16). Also, since

$$N(t) = 2\pi \int_0^{r_p(t)} \int_0^L n_r(t) r dr d\ell, \quad (5-17)$$

where $n_r(t)$ is the radial number density profile at a time t , it is evident that the knowledge of the number density profile is necessary to determine τ_p . Since the number density profiles for $t = 0$ μsec and $t = 2$ μsec after implosion could not be determined experimentally, a functional profile which fits available data was used. It was assumed that the number density profile was Gaussian in shape, so that

$$n_r(t) = n_a(t) e^{-(r/r_{1/e}(t))^2}, \quad (5-18)$$

where $n_a(t)$ is the number density on-axis (i.e., $r = 0$ mm) at a time t , and $r_{1/e}(t)$ is the value of the radius, at a time t , where the number density has decreased by a factor of $1/e$ from the on-axis value. This assumption of a Gaussian distribution was based on the results of other experiments (15, 19). To obtain τ_p , one then needs $n_a(0)$, $r_{1/e}(0)$, $n_a(2)$ and $r_{1/e}(2)$.

In order to specify $n_a(0)$ and $r_{1/e}(0)$, two independent methods were considered. In the first of the two methods, it was assumed that the mass which was previously calculated to be swept-up by the imploding current sheath resides inside the initially predicted snowplow equilibrium plasma radius, $r_p(0)$, and that the plasma mass which was not swept-up is located between $r = r_p(0)$ and the discharge chamber inside wall, $r = b$. Although it was assumed that the number density distribution of the particles inside $r = r_p(0)$ is Gaussian in shape, it was not assumed that the particles located outside of $r = r_p(0)$ had to have a Gaussian distribution. Recent experiments (39) have indicated that some of the mass outside the column

can form a halo around the swept-up mass inside the plasma column. This implies that the mass distribution outside the plasma column does not drop off as rapidly as a Gaussian distribution would predict at radii near $r = r_p(0)$. In order to allow for this, a $1/r^2$ distribution was assumed for the unswept mass instead of a Gaussian.

The swept particle inventory at time $t = 0$ μsec , $N(0)$, equals ηN_{TOT} , and the unswept particle inventory, $N_{\text{out}}(0)$, equals $(1 - \eta)N_{\text{TOT}}$, where N_{TOT} is the total number of particles initially located in the discharge tube along the 6 cm plasma column length, and η is the percent mass pick-up factor. By use of Equation 5-17 and the assumption that $N(0)$ equals the initial particle inventory inside $r = r_p(0)$ it can be shown that;

$$\begin{aligned} N(0) &= \eta N_{\text{TOT}} = 2\pi k_p \int_0^{r_p(0)} n_a(0) e^{-(r/r_{1/e}(0))^2} r dr, \\ &= \pi k_p n_a(0) r_{1/e}^2(0) [1 - e^{-(r_p(0)/r_{1/e}(0))^2}] \quad (5-19) \end{aligned}$$

The $1/r^2$ distribution shape for particles located outside of $r = r_p(0)$ yields a distribution function of the form;

$$n_{\text{rout}}(0) = f \left(\frac{1}{r^2} - \frac{1}{b^2} \right) \quad (5-20)$$

where $n_{\text{rout}}(0)$ is the initial number density profile for particles outside of $r = r_p(0)$, and f is a constant to be determined. It was assumed that $n_{\text{rout}}(0)$ equals zero at $r = b$. From Equation 5-17 it can be shown that;

$$\int_{r_p(0)}^b f \left(\frac{1}{r^2} - \frac{1}{b^2} \right) r dr = \frac{N_{\text{out}}(s)}{2\pi k_p} = \frac{N_{\text{TOT}}(1 - \eta)}{2\pi k_p} \quad (5-21)$$

Performing the required integration and solving for f it is found that,

$$n_{\text{rout}}(0) = \frac{N_{\text{TOT}}(1 - \eta)}{2\pi k_p \left[\ln\left(\frac{b}{r_p(0)}\right) - \frac{b^2 - r_p^2(0)}{2b^2} \right]} \left[\frac{1}{r^2} - \frac{1}{b^2} \right], \quad (5-22)$$

and therefore the number density profile is known for r greater than or equal to $r_p(0)$. Using this distribution, the number density $n_{r_p}(0)$ can be determined. Then, from Equation 5-18;

$$n_a(0) = n_{r_p}(0) e^{+(r_p(0)/r_{1/e}(0))^2}. \quad (5-23)$$

Next, by solving Equation 5-19 for $n_a(0)$ and equating this result to the result in Equation 5-23, an explicit expression for $r_{1/e}(0)$ is obtained;

$$\left(\frac{\pi k_p n_{r_p}(0)}{\eta N_{\text{TOT}}} \right) r_{1/e}^2(0) = \frac{e^{-(r_p(0)/r_{1/e}(0))^2}}{[1 - e^{-(r_p(0)/r_{1/e}(0))^2}]} \quad (5-24)$$

Solving for $r_{1/e}(0)$ from Equation 5-24 and substituting the result into Equation 5-23 to solve for $n_a(0)$, the initial plasma column number density profile can be specified.

The second method used to specify $n_r(0)$ involves assuming an initial value of the number density at $r = b$, $n_b(0)$. Then it is assumed that the swept-up initial particle inventory, $N(0)$, forms a Gaussian distribution from $r = 0$ to $r = b$ instead of only to $r = r_p(0)$ as in the first method. Since $r_{1/e}(0)$ is expected to be less than $r_p(0)$, still approximately 90 percent of these particles will reside inside of $r_p(0)$. By this approach, the total number of particles inside $r = r_p(0)$ does not change appreciably from that in the first method, but the problem of having to specify a functional form for the non swept-up mass is now avoided since it will not

come into any of the calculations. The unswept mass is assumed to be accounted for by being added to the Gaussian distribution in the region for $r > r_p$.

By substituting b for r in Equation 5-18 and using the estimated value of the number density at $r = b$, $n_b(0)$, it can be seen that;

$$n_a(0) = n_b(0)/e^{-(b/r_{1/e}(0))^2} \quad (5-25)$$

Also from Equation 5-17 and the assumption that the total number of particles under the Gaussian number density profile from $r = 0$ to $r = b$ equals $N(0)$ it was found that;

$$n_a(0) = \frac{N(0)}{\pi l_p^2 r_{1/e}^2(0) [1 - e^{-(b/r_{1/e}(0))^2}]} \quad (5-26)$$

Equating the right hand side of Equations 5-25 and 5-26 an implicit expression for $r_{1/e}(0)$ is obtained;

$$\frac{e^{-(b/r_{1/e}(0))^2}}{[1 - e^{-(b/r_{1/e}(0))^2}]} = \frac{n_b(0)}{n(0)} \pi l_p^2 r_{1/e}^2(0) \quad (5-27)$$

Once $r_{1/e}(0)$ is determined, $n_a(0)$ can be calculated from either Equation 5-25 or 5-26. Therefore the initial number density profile is specified. However, since $r_{1/e}(0)$ relies directly on the specified value of $n_b(0)$, a method had to be developed for selecting $n_b(0)$ accurately. This was accomplished as follows: First, a value of $n_b(0)$ is specified; from this, as mentioned above, the initial number density profile can be determined. Next, assuming that the ratio of $r_{1/e}$ to the predicted plasma radius r_p remains constant with time, which is the same as assuming that $n_{rp}(t)/n_a(t) = \text{constant}$, $r_{1/e}(t)$ can be determined from;

$$r_{1/e}(t) = \frac{r_{1/e}(0)}{r_p(0)} r_p(t) \quad (5-28)$$

where $r_{1/e}(0)$ has been determined and $r_p(t)$ is presumed known. As can be seen from Equation 5-26, with zero replaced by t , if $r_{1/e}(t)$ is known, and if $N(t)$ can be estimated, then $n_a(t)$ can be calculated. The computer simulation is used to give an estimate on $N(t)$ (Equation 5-10 and 5-11). The use of the computer simulation was considered valid since it agreed very closely with the experimentally determined $N(t)$ found by the radial plasma oscillation study, so that discrepancies in this assumption is considered minimal. From this estimate of $N(t)$, $n_a(t)$ is calculated. The knowledge of $r_{1/e}(t)$ and $n_a(t)$ allowed the Gaussian profiles with respect to time to be reconstructed. Using these time varying number density profiles and knowing that a change in number density of $2.94 \times 10^{16} \text{ cm}^{-3}$ represents the movement of one fringe, the expected fringe motion with respect to time at a number of radial positions where data is obtained can be reconstructed. This reconstructed fringe behavior is then compared with the fringe motion actually indicated by the data. If there is good agreement between the two, then the specified $n_b(0)$ is considered correct, and the corresponding $r_{1/e}(0)$ is used in the calculations. If the agreement is not acceptable, a new value of $n_b(0)$ is assumed and the process is repeated. The decision as to when agreement is satisfactory is a matter of judgement.

After the initial number density profile is determined, it is a straightforward matter to obtain an estimate of the density profile variables at the end of the time period of interest ($t = 2 \text{ } \mu\text{sec}$ after implosion). By assuming again that the ratio of $r_{1/e}$ and r_p is a constant in time, and since $r_p(2)$ is known, then $r_{1/e}(2)$ can be calculated. Also,

$n_a(2)$ can be determined from $n_a(0)$ and the interferometric data obtained for the radial position $r = 0$. From the data, Δn_a , the change in number density at $r = 0$ for the 2 μsec time period of interest, can be determined by the procedure described at the beginning of the section. Therefore, $n_a(2)$ is known since

$$n_a(2) = n_a(0) - \Delta n_a \quad (5-29)$$

and, with $r_{1/e}(2)$, the number density profile at $t = 2 \mu\text{sec}$ after implosion can be determined.

Since τ_p , the 1/e particle end loss time, can be calculated from the relationship

$$N(2) = N(0)e^{-2/\tau_p}, \quad (5-30)$$

where $N(2)$ is the total plasma column particle inventory at $t = 2 \mu\text{sec}$ after implosion and τ_p is in microseconds, and since $N(2)$ can be related to the radial number density profile at $t = 2 \mu\text{sec}$ by use of Equation 5-17, it can be shown that;

$$\begin{aligned} \tau_p &\approx \frac{-2}{\ln \left[\frac{2\pi \int_0^{r^*} r n_a(2) e^{-(r/r_{1/e}(2))^2} r dr dL}{N(0)} \right]} \\ &= \frac{-2}{\ln \left[\frac{n_a(2) \pi r_{1/e}^2(2) (1 - e^{-(r^*/r_{1/e}(2))^2})}{N(0)} \right]} \end{aligned} \quad (5-31)$$

where r^* equals $r_p(2)$ if $n_a(2)$ and $r_{1/e}(2)$ are calculated by the first method and r^* equals b if these parameters are calculated by the second method. Since $N(0)$, $n_a(2)$ and $r_{1/e}(2)$ have been determined, τ_p can be calculated.

Data was obtained for each fill pressure at three pairs of positions: $r = 0$ and 8.5 mm, $r = 2$ and 10.5 mm, and $r = 4$ and 12.5 mm. Data obtained at two of these three pairs for fill pressures of 50 and 65 millitorr are

presented in Figures 5-10 and 5-11. These records are the time-varying changes of the photodetector voltage output. It was determined that the maximum voltage output was obtained when a dark fringe passed by the detector and the minimum output was obtained when a bright fringe passed by the detector. Therefore, a peak-to-peak vertical deflection represented the passage of half a fringe (i.e., either dark or light or light to dark). Also, it can be seen that the maximum voltage deflection obtained at radial positions near the 4 mm position are larger than at the other radii. This is because the intensity of the laser light source was Gaussian in shape and therefore the maximum intensity was largest near the center of the beam ($r = 55$ mm).

As can be seen from the data, no fringe movement occurred during the first half cycle of the current waveform. This was attributed to use of the first half cycle for plasma preionization without collapse. Also, the lack of fringe movement during the collapse time of the second half cycle suggests an imploding sheath, indicating that the implosion was acting as predicted by the snowplow model. The data indicates mass arrival at the center of the discharge chamber approximately 1 μ sec after the start of the second half cycle. This is associated with the quick initial deflection in the detector output. The end of this deflection is considered the end of the implosion phase of the discharge and the beginning of the plasma column lifetime.

The analysis of the data obtained from the Twyman-Green interferometer follows the procedure previously discussed. The first step is the determination of the fringe shift, excluding fringe motion due to radial plasma oscillation, during the first two microseconds of plasma column lifetime. In Table 5-3 the experimentally determined

Figure 5-10: Typical Interferometric Data at Various Radial Positions for a 50 Millitorr Fill Pressure of H_2

Figure 5-11: Typical Interferometric Data at Various Radial Positions for a 65 Millitorr Fill Pressure of H_2

TABLE 5-3

EXPERIMENTALLY DETERMINED FRINGE MOTION AND
CORRESPONDING CHANGE IN NUMBER DENSITY

P_0 (millitorr)	r (mm)	ΔX_r fringe movement $t = 0 \rightarrow t = 2 \text{ } \mu\text{sec}$	Δn_r change in number density (10^{-16} cm^{-3})
50	0	0.78 ± 0.12	2.29 ± 0.36
	2.0	0.80 ± 0.09	2.35 ± 0.27
	4.0	0.79 ± 0.15	2.32 ± 0.45
	8.5	0.45 ± 0.15	1.32 ± 0.45
	10.5	0.42 ± 0.15	1.24 ± 0.44
	12.5	0.25 ± 0.08	0.74 ± 0.23
65	0	0.94 ± 0.06	2.77 ± 0.17
	2.0	0.91 ± 0.10	2.68 ± 0.29
	4.0	0.87 ± 0.10	2.56 ± 0.29
	8.5	0.70 ± 0.13	2.06 ± 0.38
	10.5	0.62 ± 0.10	1.82 ± 0.30
	12.5	0.42 ± 0.07	1.24 ± 0.20

fringe shift, ΔX_r , and the corresponding change in number density, Δn_r , are presented at all radial positions where data was obtained for fill pressures of 50 and 65 millitorr. These numbers were obtained by taking the statistical average for a number of data records (ranging from 5 to 7 depending on the radial position) obtained for each radial position. Along with the statistical average the statistical error is also indicated.

The next step was to obtain an estimate for the initial number density profile. As mentioned earlier, two methods were used. It was calculated that for a 50 millitorr fill pressure the total number of particles initially contained in the discharge chamber along the 6 cm plasma column length was 1.57×10^{18} particles and for a 65 millitorr fill pressure N_{TOT} was determined to be 2.04×10^{18} particles. By the first method it was determined for a 50 millitorr fill pressure that $N_{OUT}(0) = 2.94 \times 10^{17}$ particles, from which $n_{rout}(0)$ was determined to be;

$$n_{rout}(0) = \left(\frac{1.67 \times 10^{16}}{r^2} - 6.49 \times 10^{14} \right) \text{ cm}^{-3}.$$

From this expression, $n_{rp}(0)$ was found to be $3.12 \times 10^{15} \text{ cm}^{-3}$. Substituting this value for $n_{rp}(0)$ into Equation 5-24, $r_{1/e}(0)$ was calculated to be 1.30 cm. Then $n_a(0)$ was determined from Equation 5-23 to be $4.31 \times 10^{16} \text{ cm}^{-3}$. Therefore, the first method yielded an estimate for the initial number density profile for the 50 millitorr case of;

$$n_r(0) = 4.31 \times 10^{16} e^{-(r/1.30)^2} \text{ cm}^{-3}, \quad (5-32)$$

where r is in centimeters. For the 65 millitorr case $N_{out}(0)$ was calculated to be 4.72×10^{17} particles, and $n_{rout}(0)$ was found to vary as;

$$n_{rout}(0) = \left(\frac{2.69 \times 10^{16}}{r^2} - 1.04 \times 10^{15} \right) \text{ cm}^{-3},$$

from which $n_{rp}(0)$ was determined to be $5.00 \times 10^{15} \text{ cm}^{-3}$. This value yielded a $r_{1/e}(0)$ of 1.41 cm, a $n_a(0)$ of 4.69×10^{16} and an initial number density profile of

$$n_r(0) = 4.69 \times 10^{16} e^{-(r/1.41)^2} \text{ cm}^{-3} \quad (5-33)$$

For the second method, time-varying Gaussian number density profiles were reconstructed for assumed $n_b(0)$ values of 1×10^{12} , 5×10^{11} , 1×10^{11} and $5 \times 10^{11} \text{ cm}^{-3}$. It was determined that a $n_b(0)$ estimate of 1×10^{11} yielded results that best reproduced the actual fringe data for both the 50 and 65 millitorr cases. It must be pointed out that no effort was made to check if more precisely determined values of $n_b(0)$ would give better results; the accuracy of the reproduction of fringe behavior was felt to be sufficient. These values of $n_b(0)$ led to an initial number density profile of the form;

$$n_r(0) = 3.33 \times 10^{16} e^{-(r/1.43)^2} \text{ cm}^{-3} \quad (5-34)$$

for a 50 millitorr fill pressure, and,

$$n_r(0) = 4.26 \times 10^{16} e^{-(r/1.41)^2} \text{ cm}^{-3} \quad (5-35)$$

for a 65 millitorr fill pressure. The plot of the time-varying Gaussian number density profiles obtained from these initial profiles are presented in Figures 5-12 and 5-13. Along with these profiles, densities which present a multiple of a half fringe difference in density from a zero density position are also labeled for reference points.

The next step was to determine the number density profile at $t = 2$ μsec after implosion. Again the methods by which $n_a(2)$ and $r_{1/e}(2)$ are to be determined have been previously stated. Since $n_a(2) = n_a(0) - \Delta n_a$ and $n_a(0)$ and Δn_a are known (i.e., $n_a(0)$ can be determined from Equations

Figure 5-12: Predicted Time-Varying Density Profiles and Fringe Position for a 50 Millitorr Fill Pressure of H_2

Figure 5-13: Predicted Time-Varying Density Profiles and Fringe Position for a 65 Millitorr Fill Pressure of H_2

5-32 and 5-33 if the first method of finding $n_r(0)$ is to be used in calculations, or from Equation 5-34 and 5-35 if the second method is to be used, and Δn_a is tabulated in Table 5-3), $n_a(2)$ was determined using the parameters found using both methods for calculations of $n_a(0)$. Using the first method $n_a(2)$ was determined to be $2.02 \times 10^{16} \text{ cm}^{-3}$ for the 50 millitorr case and $1.92 \times 10^{16} \text{ cm}^{-3}$ for a 65 millitorr case. Using parameters found by the second method, $n_a(2)$ was determined to be $1.04 \times 10^{16} \text{ cm}^{-3}$ and $1.49 \times 10^{16} \text{ cm}^{-3}$ for fill pressures of 50 and 65 millitorr, respectively. Since it has been assumed that the ratio of $r_{1/e}$ to r_p remains constant in time, $r_{1/e}(2)$ was determined from Equation 5-28, where $r_{1/e}(0)$, $r_p(0)$ and $r_p(2)$ are known. It was found that $r_{1/e}(2)$ equalled 1.34 cm and 1.55 cm for fill pressures of 50 and 65 millitorr, respectively, using the first method of calculations. Using the second method of calculations, $r_{1/e}(0)$ was determined to be 1.47 for a 50 millitorr fill pressure and 1.55 cm for a 65 millitorr fill pressure. Using the values of $n_a(2)$ and $r_{1/e}(2)$ calculated from the first method, the final number density profiles at $t = 2 \text{ } \mu\text{sec}$ after implosion were determined to be;

$$n_r(2) = 2.02 \times 10^{16} e^{-(r/1.34)^2} \text{ cm}^{-3} \quad (5-36)$$

for a 50 millitorr fill pressure, and

$$n_r(2) = 1.92 \times 10^{16} e^{-(r/1.55)^2} \text{ cm}^{-3} \quad (5-37)$$

for a 65 millitorr fill pressure. Using values found by use of the second method, the final number density profiles were;

$$n_r(2) = 1.04 \times 10^{16} e^{-(r/1.47)^2} \text{ cm}^{-3} \quad (5-38)$$

for the 50 millitorr case, and

$$n_r(2) = 1.49 \times 10^{16} e^{-(r/1.55)^2} \text{ cm}^{-3}, \quad (5-39)$$

for the 65 millitorr case.

Finally by use of Equation 5-31 and the results obtained by the first method τ_p was calculated to be 2.85 ± 0.61 μsec for a 50 millitorr fill pressure and 2.81 ± 0.26 μsec for a 65 millitorr fill pressure. Using results calculated by the second method, τ_p was determined to be 1.82 ± 0.67 μsec for the 50 millitorr case and 2.37 ± 0.35 μsec for the 65 millitorr case. The scatter indicated above in the calculations of τ_p resulted from the statistical error in the raw data itself. The much larger scatter in the 50 millitorr case was related to the fact that the short length of the plasma column being investigated caused the interferometer sensitivity to changes in number density to be low. Specifically, (Figures 5-12 and 5-13) with the small number of indicated fringes at a 50 millitorr fill pressure, normal experimental perturbations produced an exaggerated scatter in the data. It was found that a 50 millitorr case was the minimum fill pressure for which the interferometer could be used on this particular theta pinch device.

A brief comparison between the two methods used in the above calculations is in order here. Both methods have advantages and disadvantages. The first method has the advantage that it does not rely on the computer program to give estimates on many parameters used in calculations, and, except for $r_p(2)$ and the estimate on the shape of the distribution function for the non swept-up particles, all parameters used in calculations were found experimentally. Also, another advantage is that the total number of particles swept-up by the imploding current sheath entirely resided inside the predicted plasma column radius, which was considered more correct than allowing these particles to spread out

past this radius as was done in the second method. The disadvantages of the first method were that the distribution function used for the non swept-up particles had to be estimated, and no information pertaining to fringe motion with respect to time, which could be compared to the actual data, was obtained. With regard to the choice of the radial density distribution for $r > r_p$, it should be noted that different choices will lead to different values for τ_p . The choice of an r^{-2} function was felt to be intuitively reasonable, although it did lead to higher values for τ_p than found by the other methods.

The two main advantages which the second method has over the first method are, first, that the unswept mass distribution function does not have to be estimated since it is not used in calculations; and, second, the fringe motion with respect to time is determined and compared with the actual fringe motion obtained from the data. The disadvantages for this second method are that the $N(t)$ predicted by the computer program had to be used in calculations of the time dependent number density profiles and that some of the swept-up particles are allowed to reside outside of the predicted plasma radius. Also, $n_b(0)$ may differ slightly from 10^{11} , and could be one of the reasons for the differences between the two methods used to calculate τ_p .

5.3.6 End loss time from scaling

The final estimate for the l/e particle end loss time, τ_p , was achieved by scaling the loss time from the one-meter-long collision-dominated Scylla I-C theta pinch device (3, 4, 19). It has been suggested in the literature (39) that theta pinch machines scale as $l(T_i/m_i)^{1/2}$, where l is the plasma column length, T_i is the maximum ion temperature,

and m_i is the mass of the fill gas ion. Experiments on Scylla I-C (3) have determined that the $1/e$ particle end loss time was approximately 14.4 μsec , and that T_i was 50 eV. Since deuterium was used as the fill gas, m_i was 2.0141 amu. If $\ell/(T_i/m_i)^{1/2}$ scaling is used, the scaling relationship is;

$$\tau_p = \tau_p^{I-C} \frac{\ell_p^{I-C}}{\ell_p} \left(\frac{T_i^{I-C}/m_i^{I-C}}{T_i/m_i} \right)^{1/2} \quad (5-40)$$

where the superscript I-C denotes Scylla I-C parameters, and those without subscripts denote parameters of the theta pinch being investigated here. The maximum value of T_i for this experiment was estimated by use of the computer simulation to be 3.52 eV for a 50 millitorr fill pressure, and 3.10 eV for a 65 millitorr fill pressure. The appropriate plasma column length was 6 cm for this experiment. Since hydrogen was used in this work, $m_i = 1.00782$ amu. From Equation 5-40 it was determined that τ_p is approximately 2.30 μsec for a 50 millitorr fill pressure and 2.45 μsec for a 65 millitorr fill pressure.

In Table 5-4 the $1/e$ particle end loss times obtained from the various methods used to calculate τ_p are tabulated. As can be seen, all estimates agree rather well, except for the two methods of calculating the particle end loss time for a 50 millitorr fill pressure from the Twyman-Green data. However, the two values appear to bracket the loss times calculated by all other methods. This difference can be attributed to low sensitivity of the interferometer at this fill pressure and the effects of the estimates of the initial number density profiles used in calculations. Even with the slight discrepancies between the Twyman-Green calculated τ_p for a 50 millitorr fill pressure it was felt that the good general agreement implied that both the radial plasma oscillation techniques and

TABLE 5-4
COMPARISON OF 1/e PARTICLE END LOSS TIMES

Method of calculation	τ_p (usec)	
	Fill pressure <u>50</u>	(millitorr) <u>65</u>
1. Computer simulation	2.26	2.47
2. Radial plasma oscillations	2.04	2.45
3. Twyman-Green		
a. Use of first method for calculation of $n_r(0)$	2.85	2.81
b. Use of second method for calculation of $n_r(0)$	1.82	2.37
4. $\lambda/(T_i/n_i)^{1/2}$ scaling for Scylla I-C	2.31	2.45

the Twyman-Green interferometry are viable methods of evaluating the $1/e$ particle end loss times for the theta pinch used here. However, for future work, a measurement of $n_a(0)$ is considered essential for the interpretation of Twyman-Green interferometry. The more completely one can measure the density profile, the greater will be the confidence in the calculated loss times.

Chapter VI

DISCUSSION

6.1 Introduction

This work focused on the development of two experimental diagnostic techniques which could be used to obtain information on the particle end loss process inherent in all linear theta pinch devices. The two diagnostics used to determine τ_p were: (1) a magnetic field probe; (2) a Twyman-Green interferometer. The data obtained from these two diagnostics were discussed in the previous chapter. The most important information obtained from these diagnostics was the 1/e particle end loss time, τ_p . It should be recalled that both diagnostics yielded similar results, indicating that they could accurately monitor the end loss phenomenon. However, there are limitations on their applicability to the machine investigated here. These limitations will be discussed below, along with a discussion on how these limitations will be affected when applied to more conventional theta pinch designs.

6.2 Magnetic Field Probe Technique

The axial magnetic field probe was used to determine the total plasma column particle inventory as a function of time from recorded plasma column radial oscillations data. From these results τ_p was inferred; this technique was shown to be consistent with other methods of estimating τ_p , with the limitations on the technique being dependent on the ion temperature and the accuracy of the first order relationship used to calculate $N(t)$. The ion temperature plays an important role in the length of time these oscillations will persist, since the damping rate, as

postulated by Grossman (38), is proportional to the ion-ion viscosity which increases with increasing ion temperature. Therefore, the hotter the plasma, the shorter lived are these oscillations. Since these oscillations damp out, only the initial time dependence of N can be obtained. In order that the total time dependent behavior of N be estimated, it had to be assumed that this initial behavior could be extrapolated to longer times. The longer these oscillations exist, the more accurate is this assumption. This implies that the applicability of this analysis is limited to machines where the ion temperatures are low enough so that the oscillations last a significant percentage of the $1/e$ particle end loss time.

6.3 Twyman-Green Interferometric Technique

The Twyman-Green interferometer, used to monitor time-varying changes in line density at various radial positions, was also shown to be a valuable diagnostic for estimating $1/e$ particle end loss time. The most severe limitation on this method of analysis was that the initial density profile had to be estimated, as it could not be determined experimentally. Since the calculation of τ_p relied heavily on the estimated initial number density profile, this was a potential source of error; this could have been greatly reduced if the initial on-axis number density and the number density at any other radial position could have been determined experimentally. From these two known number densities, a Gaussian profile could have been constructed easily. One experimental technique which can be used to identify the plasma radial distribution is the Thompson scattering diagnostic technique (40).

Another limitation of this method was the accuracy in estimating the change in number density resulting from particle end loss. This was difficult because of the time-varying influence of radial plasma oscillations and the time-varying magnetic field on the plasma column number density. Since these effects accounted for a significant fraction of the total recorded fringe movement, the loss time was determined only after 2 μ sec after column formation. The ability to calculate the change in number density due solely to particle loss as a continuous function of time would greatly enhance the usefulness and accuracy of this diagnostic technique. In order to achieve this, the effects of the radial plasma oscillations and the time-varying magnetic field must be minimized. Increasing the temperature of the plasma would increase the damping rate of the radial plasma oscillations thereby decreasing the time scale of their influence and increasing the time for which particle loss calculations can be accurately made. This hotter plasma column would also exhibit a decrease in the rate at which the magnetic field diffused into the plasma thereby reducing the plasma column expansion rate due to the increase in internal magnetic field. The effects of the time-varying magnetic field on the plasma radius could be reduced also by incorporating a simple crowbarring mechanism which would tend to maintain a constant external magnetic field strength following peak magnetic field. If these three effects were minimized, loss calculations could be obtained after the radial plasma oscillation have damped out and terminating at the end of plasma column lifetime.

The present experiment was rather short for use of Twyman-Green interferometry. Since, as can be seen from Equation 4-4, the sensitivity of the interferometer increase linearly with increasing plasma column

length, by increasing the length of the device, one would be able to obtain more fringe movement for a given change in number density than in a shorter machine. The more fringes there are to be counted, the more accurate the count. This added length would also increase the $1/e$ particle end loss time, thereby increasing the time period over which end loss calculations can be made.

It is evident that the Twyman-Green interferometric technique is best suited for use on theta pinch devices where the column length is long and the plasma temperature is high. Of course it can be used on any machine but the accuracy of the results would be maximized on a long, hot machine.

Appendix A

DERIVATION OF RELATIONSHIP BETWEEN TOTAL PARTICLE INVENTORY AND RADIAL PLASMA OSCILLATION FREQUENCY

In Chapter V an estimate of the $1/e$ particle loss time (τ_p) was obtained from the frequency of radial plasma column oscillations. In this section the derivation of the relationship used will be presented. Also there will be brief discussions on the applicability of assumptions made in the derivation.

The oscillations were caused by the current sheath imploding inwards at such a large velocity that it implodes past the pressure equilibrium position due to momentum effects. After the current sheath ceases to move inwards the internal plasma column pressure, equaling $nkT + (B_{int}^2/8\pi)$, is larger than the external magnetic fields pressure, $B_{ext}^2/8\pi$, used to confine the plasma column. This pressure difference causes the plasma column to expand against the confining magnetic field and again, due to momentum effects, the plasma overshoots the equilibrium position. This type of phenomenon sets the plasma column into radial oscillations where the frequency varies with time and the oscillations slowly damp out ($\approx 2-3 \mu\text{sec}$). It has been suggested that the damping is caused by ion viscosity inside the plasma column (38).

In this derivation it will be assumed that the frequency of oscillation is independent of the damping rate, because the damping does not modify the real part of the frequency. This derivation will not be an exact solution to the problem but will be a first order approximation. It will be assumed that initially there is an internal magnetic field present in the plasma column ($\beta \neq 1$) and the flux in the column does not

change during the lifetime of the oscillations. It will also be assumed that the plasma column starts at an equilibrium radius r_0 and $t = t_p$. Therefore, at $t = t_p$ the equilibrium pressure balance equation can be written as:

$$\frac{B_{\text{ext}}^2}{8\pi} = \bar{n}_p k T_p + \frac{B_{\text{int}}^2}{8\pi} \quad (\text{A-1})$$

where

B_{ext} = external magnetic field strength,

B_{int} = magnetic field strength internal to the plasma column,

$\bar{n}_p$ = average equilibrium plasma density,

T_p = total average equilibrium temperature = $\bar{T}_{ip} + \bar{T}_{ep}$,

T_{ip} = average ion temperature,

T_{ep} = average electron temperature.

Equation B-1 can also be written as:

$$\beta \frac{B_{\text{ext}}^2}{8\pi} = \bar{n}_p k T_p \quad (\text{A-2})$$

where β is defined by the relationship:

$$\frac{B_{\text{int}}^2}{B_{\text{ext}}^2} = (1 - \beta).$$

Next, a small perturbation, Δr , in the equilibrium radius, r_0 , will occur due to the overshoot of the current sheath. After this perturbation the pressure balance equation becomes:

$$\frac{B_{\text{ext}}'^2}{8\pi} - \bar{n}_p' k T_p' - \frac{B_{\text{int}}'^2}{8\pi} = - \frac{M_T}{A_S} \frac{d^2 r_p}{dt^2}, \quad (\text{A-3})$$

where the ' terms are the new values after the perturbation; M_T is the total mass entrained in the plasma column; A_S is the total surface area

of the plasma column parallel to the discharge chamber axis, equalling $2\pi r_p \ell_p$; and r_p is the new plasma column radius equalling $r_0 - \Delta r$. Equation A-3 is the equation which is to be solved for Δr as a function of time.

Through flux conservation both B'_{ext} and B'_{int} can be related to B_{ext} and β in the following manner:

$$B'_{ext} = B_{ext} \frac{(b^2 - r_0^2)}{[b^2 - (r_0 - \Delta r)^2]}, \quad (A-4)$$

$$B'_{int} = B_{ext} \frac{r_0^2}{(r_0 - \Delta r)^2} (1 - \beta)^{1/2}. \quad (A-5)$$

where b is the inside radius of the discharge chamber. Assuming that the compression and expansion of the plasma column is adiabatic in nature it has been shown (4) by the "snowplow" model approximation that $\bar{n}_p$ and T_p vary with changing radius as:

$$\bar{n}_p' = \bar{n}_p \left(\frac{r_0}{r_0 - \Delta r} \right)^2, \text{ and} \quad (A-6)$$

$$T_p' = T_p \left(\frac{r_0}{r_0 - \Delta r} \right)^{4/3}. \quad (A-7)$$

Plugging relationships A-4 through A-7 into Equation A-3 the pressure balance equation becomes,

$$\begin{aligned} \frac{B_{ext}^2}{8\pi} \left[\frac{(b^2 - r_0^2)^2}{[b^2 - (r_0 - \Delta r)^2]^2} - \frac{r_0^4}{(r_0 - \Delta r)^4} (1 - \beta) \right] \\ - \bar{n}_p k T_p \left(\frac{r_0}{r_0 - \Delta r} \right)^{10/3} = - \frac{M_T}{2\pi (r_0 - \Delta r) \ell_p} \frac{d^2 r_p}{dt^2} \end{aligned} \quad (A-8)$$

Solving $\bar{n}_p kT_p$ in terms of B_{ext} and β from Equation A-2, defining M_L , the mass per unit length, as M_T/ℓ_p , and realizing that $(d^2 r_p)/(dt^2) = -(d^2 \Delta r)/(dt^2)$, Equation A-8 becomes;

$$\frac{B_{\text{ext}}^2}{8\pi} \left[\frac{(b^2 - r_0^2)^2}{[b^2 - (r_0 - r)^2]^2} - \frac{r_0^4}{(r_0 - r)^4} (1 - \beta) - \beta \left(\frac{r_0}{r_0 - r} \right)^{10/3} \right] =$$

$$= \frac{M_L}{2\pi(r_0 - \Delta r)} \frac{d^2 \Delta r}{dt^2} \quad (\text{A-9})$$

Taking Δr as small and performing a Taylor series expansion, one obtains, to the first order in Δr ;

$$\frac{d^2 \Delta r}{dt^2} + \frac{B_{\text{ext}}^2}{4M_L} \left[\frac{(4 - \frac{2}{3}\beta)b^4 r_0^{16/3} - (4 - \frac{4}{3}\beta)b^2 r_0^{22/3} - \frac{2}{3}\beta r_0^{28/3}}{r_0^{28/3} + b^4 r_0^{16/3} - 2b^2 r_0^{22/3}} \right] \Delta r = 0 \quad (\text{A-10})$$

By defining E as equal to the terms inside the brackets, Equation A-10 takes the form,

$$\frac{d^2 \Delta r}{dt^2} + \frac{B_{\text{ext}}^2}{4M_L} E \Delta r = 0$$

Defining $\omega^2 = B_{\text{ext}}^2 E/4M_L$, this equation becomes,

$$\frac{d^2 \Delta r(t)}{dt^2} + \omega^2(t) \Delta r(t) = 0$$

Assuming $\omega(t)$ is slowly varying in time, a WKBJ solution of the above differential equation is:

$$\Delta r(t) = \frac{A}{\omega^{1/2}} \sin \left[\int_0^t \omega(t') dt' + \phi \right] \quad (\text{A-11})$$

where A is a constant and ϕ is the phase angle. Assuming $\omega(t)$ varies slowly over the period of the oscillations, the integral $\int_0^t \omega(t') dt'$, can be approximated by $\omega(t) + \frac{\partial \omega(t)}{\partial t} \frac{t^2}{2}$. Experimentally it was determined

that use of the first term alone is good to within five percent over the time intervals in question. Therefore

$$\Delta r(t) = \frac{A}{\omega^{1/2}} \sin[\omega(t)t + \phi] \quad (\text{A-12})$$

Rearranging terms and solving for M_L ,

$$M_L(t) = \frac{B_{\text{ext}}^2(t)}{4\omega^2(t)} E \quad (\text{A-13})$$

However, since the total number $N(t)$ of particles contained inside the plasma column is $N(t) = [M_L(t)\ell_p]/m_i$, where ℓ_p is the length of the plasma column and m_i is the mass of a proton, then,

$$N(t) = \frac{B_{\text{ext}}^2(t)}{4\omega^2(t)} \frac{\ell_p}{m_i} E \quad (\text{A-14})$$

Since $B_{\text{ext}}(t)$, ℓ_p , E , and $\omega(t)$ can be either calculated or experimentally determined, $N(t)$ can be calculated and an estimate of the 1/e particle end loss time can be obtained.

Because this is a first order approximation some error in Equation A-14 exists due to the neglect of higher order terms. An estimate of this error can be obtained by assuming $\beta = 1$, solving for r_0 , using the snowplow model, and then solving for E in Equation A-14. This result is then compared with the exact solution for a $\beta = 1$ case. The exact solution from a similar derivation has been determined by Green and Niblett (37). They found that $N(t)$ varies as:

$$N(t) = \frac{B_{\text{ext}}^2(t)}{\omega^2(t)} \frac{\ell_p}{m_i} .$$

For $\beta = 1$, Equation A-14 predicts that $N(t)$ varies as:

$$N(t) = (.945) \frac{B_{\text{ext}}^2(t)}{\omega^2(t)} \frac{\ell_p}{m_i} .$$

This suggest that the approximate error due to neglecting higher order terms is about 5.5 percent.

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