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DYNAMICS EXPERIMENTS ON THE AMF MILLSTONE POINT
FLASH EVAPORATOR*

MASTER

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ABSTRACT

Dynamics experiments were performed on the three-stage AMF flash evaporator at Millstone Point, Conn., in June 1967 to verify theoretical calculations of large MSF plant dynamic behavior. Frequency response tests were run at operating temperatures between 100 and 140°F, tray-brine flowrates between 0.4 and 0.7 x 10⁶ lb/hr per foot of width, and with both plain and baffled interstage orifice designs. Pseudorandom binary sequences were used as input perturbation signals for the recirculating brine and brine heater steam flow control valves.

The measured results indicate that the fixed-coefficient, single-phase flow orifice equation is inadequate for describing transient interstage orifice flowrates. Improved models were derived for the tray brine vaporization rate and flow dynamics. The scope of the experiments was reduced due to equipment difficulties and to excessively large inherent temperature fluctuations under certain operating conditions.

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1. INTRODUCTION

Dynamics tests of the three-stage AMF flash evaporator at Millstone Point, Conn., in June 1967 were a first step in an experimental verification of theoretical models for prediction of multistage flash (MSF) plant dynamics. Theoretical studies of the stability characteristics of large MSF plants show that the predicted behavior is very sensitive to variations in certain assumed models and parameters.^{1,2} The most important relationships appear to be those governing the brine flowrate through the interstage orifices, particularly the effect of tray brine level on inlet brine flowrate. If the full effect of the liquid head is considered (i.e., no baffling effect), calculations show that the behavior of a typical large MSF plant would be inherently very oscillatory or even unstable. The greater the baffling effect, i.e., the less dependent the inlet brine flowrate is on the tray brine level, the more stable and well-behaved the predicted dynamic behavior becomes. This aspect of the importance of careful interstage brine-flow orifice design is also cited by Silver,³ who qualitatively discusses the stability problem.

Other factors relating stage temperature and vapor pressure responses to input perturbations are important in predicting large MSF plant operating characteristics. Examples of such factors are the model that represents the transport of tray brine through the stage and the equation that is used to calculate the transient vaporization rate.

Hence the primary objective of the Millstone Point tests was to obtain quantitative information about the dynamics of a single stage operating at conditions typical of advanced MSF plant designs. The AMF

¹S. J. Ball, Nuclear Desalination Dual-Purpose Plant Control Studies Interim Report, ORNL-TM-1618, Part 1, (Oct. 1966).

²T. W. Kerlin et al., Nuclear Desalination Plant Dynamics: Modeling and Analysis of a Multi-Stage Flash Evaporator, Univ. of Tenn. NEUT-2806-1 (June 1967).

³R. S. Silver, "Fresh Water From the Sea," Proc. Inst. Mech. Engrs. 179 (5), Part 1, 135-54 (1965).

evaporator permitted investigation at temperature conditions from 100 to 150°F and at brine flowrates up to 10^6 lb/hr per foot of tray width. The AMF plant was particularly desirable for these tests since it has only three stages (the middle one, stage 2, being the "test stage"), and separate tray-brine and coolant loops. Since there were only a small number of stages and the loops were separate, the interaction between stages caused by a test input perturbation was minimal, and it was easier to infer the effect of a single input disturbance (such as tray-brine-inlet temperature) on the behavior of a stage.

The first tests were run with a plain orifice at the entrance to the test stage. These were followed by tests at higher temperature and with a 12-in. high vertical baffle plate installed 2 ft downstream from the entrance.

During most of the tests, a standard set of parameters was monitored for the purpose of analyzing the interstage flowrate behavior and obtaining transfer functions for the response of the single test stage to perturbations of inlet brine temperature and flowrate. These are hereafter referred to as "standard" tests. In other tests, called "probe" tests, the conditions at the inlet were perturbed to obtain the frequency response of tray-brine and vapor temperatures at different locations in stage 2. The probe used had six thermocouples and was mounted vertically. We had planned to install the probe in four different axial positions, but we didn't have time to do this.

2. DESCRIPTION OF TESTS

2.1 Preliminary Tests and Observations

In preliminary tests, the operation of the data logging system, the special flow, level, and temperature transducers, and the pneumatically operated control valves was checked out. During these tests, and later on during the dynamics tests, several major unforeseen problems forced a reduction in the scope of the tests and limited the usefulness and accuracy of the data:

1. The evaporator tended to be very erratic at times, often for no

apparent reason. A possible explanation is as follows. The tests were run just after an unseasonably cold spring had given way to very mild weather, and this sudden and marked change had upset the bacteria and seaweed life in Long Island Sound. On several occasions, this apparently caused excessive foaming and carryover of the flashing brine through the demister screens, forcing a plant shutdown. Once during stormy weather, seaweed plugged the seawater intake line and caused the plant to be shut down for the rest of the day. Hence a striking and perhaps significant feature of the plant was its marked response to external weather and seawater conditions. In the initial control studies we treated the MSF plant as a completely deterministic system, with primary concern being given to controlling temperatures, pressures, and flows at optimum conditions for best operating economy. It may be, however, that a more profitable activity would be to find ways of neutralizing the effects of external conditions on the MSF plant so that the plant could be kept on line, since a few shutdowns could easily cancel the profits from several months of fine tuning.

2. In a previous visit to the AMF plant, we had noted inherent brine temperature fluctuations of roughly 0.2°F peak-to-peak. In planning the tests we assumed that background fluctuations of this magnitude would be present in the signals from the evaporator. As it turned out, the magnitude of the inherent fluctuations varied considerably with operating conditions, and they were, in fact, up to ten times as large as our original observations indicated. Little useful information was obtained from the particularly noisy data.
3. The dynamic response of the 6-in. double-seated, air-operated brine recirculation valve (with positioner) was very sluggish (~ 20 sec response time vs a design response time of 2 sec), and from the very first day of operation, the valve stem tended to stick in certain positions. This sticking was later found to be due to rusting of the type 400 stainless steel valve guide and type 316 valve seats and stem. Monel trim would be better suited for seawater service.

4. All five strain-gauge type differential pressure transducers failed upon being exposed to the brine and vapor. Examination revealed that the strain-gauge wiring was exposed in each low-pressure chamber, and hence only a clean dry gas should have been used on that side. Four of the five transducers were replaced with sealed, force-balance type d/p cells.
5. After the sealed d/p cells were installed, several other operational problems were encountered, including discrepancies with corresponding steady-state manometer readings and anomalous sudden drops in indicated liquid level. Further research has been done on these problems, which are discussed in more detail in Sect. 6.

The performance of the temperature measuring systems and the digitizer, on the other hand, was very satisfactory. Initially, electrical "noise" produced large spikes in the recordings, particularly in the AMF unshielded iron-constantan thermocouple measurements. Virtually all noise was eliminated by

1. replacing the AMF thermocouples with sheathed, ungrounded Chromel-Alumel thermocouples having shielded leads;
2. removing the major source of the noise (the 110-v switches controlling two solenoid valves) from the instrumentation room area;
3. putting an additional load on the digitizer regulated-power supply (these devices typically perform better when loaded to near capacity).

After these changes we did not detect any electrical noise component on the temperature signals (i.e., noise was less than $1 \mu\text{v}$, or $\sim 0.04^\circ\text{F}$).

The Beckman digitizer, modified by ORNL, logged over 2.5 million data points with less than a dozen errors.

2.2 Random Fluctuation and Steady-State Measurements

Observations of the large random fluctuations in temperature accompanying high tray-brine levels prompted us to digitize the signals for 5- to 10-min periods while the AMF personnel took steady-state readings with their standard instrumentation. Digitized data was subsequently processed to determine statistical properties such as power spectral density and variance (see Sect. 5).

The variables for which steady-state data was recorded during these tests are listed in Table 1. Table 1 also lists the variables which were recorded by the digitizer for both the standard PRBS tests and the temperature-probe PRBS tests.

A summary of all the steady-state run conditions is given in Table 2.

2.3 Pseudorandom Binary Sequence Tests

Input perturbations were introduced by varying the flow of steam to the brine heater or the flow recirculating brine, or both, as shown in Fig. 1. Pneumatically operated valves were installed for the tests, and the valve positioning signals during the tests were generated by an on-line hybrid computer. Input signals known as pseudorandom binary sequences (PRBS) were used because of their desirable harmonic properties and the relative ease with which they could be produced. The 10-channel digitizer was used to record the data on magnetic tape for later analyses, including frequency-response calculations of single-stage transfer functions and determinations of interstage brine-flowrate equations.

The actual tests were preceded by extensive practice tests on an analog computer simulation of the AMF plant. Effects of nonlinearities and extraneous noise on the data-collecting and analysis methods and computer codes were checked in detail for the assumed plant model. Analysis of these practice tests indicated that there was a good chance of obtaining useful information from the actual tests. Analysis of simulated-flow equations corroborated the (known) model equations with reasonable accuracy, and analysis of the transfer functions showed good agreement with the model. The only significant discrepancy was that there were excessive phase lags in the response of stage temperatures to perturbations of inlet brine flowrate, as calculated from the practice tests with the analog computer.

A pseudorandom binary sequence is a periodic signal. Its frequency spectrum is very desirable for frequency response testing, in that typically up to several hundred harmonics of the fundamental frequency have strong spectral content. Consequently for a single PRBS test, the

Table 1. Variables Recorded for AMF Tests

	Steady- State	On Digitizer	
		Standard PRBS	Probe PRBS
Temperatures:			
Brine Heater:			
Brine inlet	X		
Brine outlet	X	X	
Condenser Coolant:			
No. 3 inlet	X		
No. 2 inlet	X		
No. 1 inlet	X		
No. 1 outlet	X		
Stage No. 1: Brine outlet	X	X	X
Stage No. 2:			
Brine outlet	X	X	
Vapor	X	X	
Probe (1 to 6)		(No. 3 only)	X
Differential Temperature: Vapor ΔT , Stg. 1-2			
		X	
Stage No. 3: Brine outlet	X		
Pressures:			
Absolute vapor pressure, Stg. 2	X		
Differential vapor pressure, Stg. 1-2	X	X	
Differential vapor pressure, Stg. 2-3	X		
Levels:			
Brine level, Stg. 1	X	X	X
Brine level, Stg. 2	X	X	X
Brine level, Stg. 3	X		
Flowrates:			
Brine recirculation	X	X	X
Stg. 2 distillate	X		
Miscellaneous:			
Orifice heights	X		
Brine concentration	X		
Distillate concentration	X		

Table 2. Summary of AMF Steady-State Run Conditions

Run	Temperatures, (°F)							Stg. 1 Brine Out	Stg. 2 Brine Out	Stg. 3 Brine Out	Abs. Press. (mm Hg) Stg. 2	Vapor ΔP (in. H ₂ O) Stg. 2		Tray Brine Level (in.)			Flowrates (lb/hr)		Outlet Orifice Heights (in.)	
	Brine Htr.		Condenser Coolant				Stg. 2 1-2					Stg. 2 2-3	Stg. 1	Stg. 2	Stg. 3	Brine Recirc.	Stg. 2 Distillate	Stg. 1	Stg. 2	
	In	Out	3 In	2 In	1 In	1 Out														
1	141.5	149.8	99.5	102.2	103.7	104.4	148.0	145.2	142.9	-	149	7	-	23.5	24.5	15	706,200	2450	8.75	15.5
6 ^c	140.5	150.0	98.4	101.8	107.2	109.7	147.7	143.5	143.2	-	162.3	8	11	19	14	20	696,600	870	6.25	9.5
16 ^a	102.2	109.5	65.0	66.8	69.2	70.1	108.4	104.4	102.4	-	53	3.3	2.3	23	21	15	477,000	1592	6.5	7.5
25	105.3	113.1	68.1	69.8	72.4	73.7	111.3	107.5	106.0	-	57.5	4	2.1	23.5	21.5	16	466,100	1750	6.5	7.5
26	112.1	120.9	71.3	73.0	75.6	77.5	118.3	114.3	113.0	-	70.2	5.5	3.5	14.5	16	14	458,500	1720	6.5	7.5
27	105.1	109.0	61.8	63.9	65.4	66.0	108.7	107.0	105.3	104.9	58	2	2.5	21.2	19	21	702,200	1515	9.25	13
34	135.8	147.0	99.1	102.0	106.6	108.6	144.2	139.2	139.1	136.7	142.5	9	10	15.8	14	14	517,000	2280	4.75	6
40 ^b	132.0	143.8	93.2	96.5	101.3	103.4	141.2	136.1	135.9	133.4	131.3	9	9	24.3	12.1	10.5	518,800	2418	7.5	5.5
45 ^b	133.4	145.3	93.6	96.7	103.4	103.9	142.8	137.3	137.8	134.4	139	10	11.3	24.8	21.2	19.5	506,700	2575	7.5	5.5
50 ^{a,b}	134.8	146.8	96.1	98.9	103.9	106.0	143.8	138.7	138.7	135.8	139.5	8.5	11	26	21	18.5	524,500	2450	7.5	5.5
55 ^{a,b}	131.2	142.7	80.5	84.3	90.2	93.0	140.2	135.2	135.9	132.0	129.5	3	12	28	23.5	21	574,500	3320	11.5	7.25
60 ^b	134.3	145.2	81.2	85.1	91.1	93.9	142.8	138.0	138.6	134.7	138	3	13	28.5	22.5	19.5	692,200	3345	11.5	7.25
65 ^b	138.0	149.2	85.4	89.5	95.7	98.6	146.8	141.3	141.6	138.4	149.2	-	-	23.8	13.8	5.5	685,700	3390	12	5.5

^aTemperature probe run.^bBaffle installed.^cConditions for Run 15 same as for Run 6.

the system is excited at many different known frequencies, and the frequency response can be measured at these frequencies. Thus, the PRBS has advantages over both the sine wave or square wave inputs, which give retrievable information at only one frequency per test, and the step or pulse inputs, which have a continuous rather than discrete spectra and hence poorer signal-to-noise ratios.

The PRBS is generated by a shift register with a special feedback configuration,⁴ and gives an alternating on-off output signal in the form of a contact closure. A PRBS is characterized by the number of bits in the sequence and the bit duration. If the number of bits in the sequence is Z and the basic bit duration Δt , then the fundamental radian frequency ω_0 and the spacing of the harmonics $\Delta\omega$ are given by $\omega_0 = \Delta\omega = 2\pi/Z\Delta t$.

The PRBS signal controlled the position of the pneumatically operated recirculation and steam-supply valves by means of three-way solenoid valves, as shown in Fig. 2.

Fig. 2. PRBS Control of Process Valves.

⁴T. W. Kerlin, The Pseudo-Random Binary Signal for Frequency Response Testing, ORNL-TM-1662 (Sept. 1966).

A typical test at given evaporator operating conditions consisted of four separate runs of about 15 min each. In the first run, the brine recirculation rate was perturbed by changing the position of the recirculation valve. In the second run, only the steam supply to the steam heater in the brine recirculation line was varied. In the third run, the two valves were positioned in phase, i.e., the recirculation and steam flows were increased or decreased simultaneously, and in the fourth run, the two valves were positioned 180° out of phase. The rationale behind this procedure is explained in Sect. 3.

In all tests, the basic bit time Δt was 5 sec and the sequence length Z was 63 bits, giving a total period of 315 sec. The fundamental radian frequency and harmonic spacing were

$$\omega_0 = \Delta\omega = \frac{2\pi}{(63)(5 \text{ sec})} = 0.019947 \text{ radian/sec}.$$

The amplitude of the harmonics in a PRBS typically decreases quite slowly with increasing frequency. An approximate expression for the harmonic number k_h of the harmonic whose amplitude is half the amplitude of the fundamental is⁴

$$k_h \approx 0.44 Z.$$

For these tests, the idealized input-signal harmonic amplitude was half that of the fundamental at $\omega = 0.55$ radian/sec. The effective input signals, however, were attenuated significantly at the higher frequencies, because the valve positioner responded slowly to positioning signals. The effective input from the supply-steam flow perturbations, i.e., the test stage inlet-brine temperature, was also attenuated by thermal and mixing lags in the brine heater, inlet piping and plenum, and in stage 1.

Fig. 3 illustrates the idealized 63-bit PRBS input signal, a typical recirculation flow input signal showing the long time response and sticking valve problem, and a trace of test-stage inlet-brine-temperature perturbations due only to changes in the recirculating brine flow.

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Fig. 3. Ideal PRBS Signal Compared with Actual Recirculation Flowrate (W_{BR}) and Test Stage Inlet Temperature (T_{i2}) for AMF Run 35.

3. RESULTS OF TRANSFER FUNCTION ANALYSES

3.1 Analytical Method for Determining Single-Stage Transfer Functions

The single-stage transfer function measurements were made to improve on the mathematical models presently used at ORNL in MSF dynamic studies. To estimate the accuracy of MSF plant mathematical models from tests on pilot units, it is almost necessary to work with a single stage. This is because there is considerable interaction between sets of coupled stages, so it would be very difficult to determine the causes of the response to an input perturbation in a many-stage unit. For example, a change in inlet tray-brine temperature at the upper stage of an evaporator train causes transients in the brine flowrates and levels, vapor pressures, and cooling water temperatures in each stage, and these changes in turn affect the neighboring stages in a distressingly complex fashion. Such tests, then, would give yes-or-no verification of the overall behavior of a given plant model, but wouldn't necessarily provide information which could be used to improve the model.

In most of the tests, measurements were made of the frequency response of stage 2 tray-brine temperature and vapor temperature to variations in inlet tray-brine temperature and flowrate. These variations were produced by changing the flowrate of the recirculating brine and the flowrate of the steam to the steam heater. In each case these variations affected both the inlet tray-brine temperature and flowrate to stage 2. Thus any given stage transfer function cannot be determined from a single test.

The method used to analyze the data is illustrated in Fig. 4, which shows stage 2 with inputs $\hat{W}_i$ and $\hat{T}_i$ and an output $\hat{T}_o$. We define the single-stage transfer functions G as

$$\hat{T}_o \equiv G_1 \hat{T}_i + G_2 \hat{W}_i . \quad (1)$$

Fig. 4. Illustration of Transfer Function Estimation with Multiple Inputs.

Frequency response measurements can be made directly of the relationships $\hat{T}_o/\hat{T}_i$ and $\hat{W}_i/\hat{T}_i$ for any given test. Thus for two different tests, G_1 and G_2 can be solved from two simultaneous equations as follows:

Rewriting (1) to accommodate the measurements:

$$G_1 + \left(\frac{\hat{W}_i}{\hat{T}_i} \right)_1 G_2 = \left(\frac{\hat{T}_o}{\hat{T}_i} \right)_1, \quad (2)$$

$$G_1 + \left(\frac{\hat{W}_i}{\hat{T}_i} \right)_2 G_2 = \left(\frac{\hat{T}_o}{\hat{T}_i} \right)_2,$$

where the subscripts 1 and 2 indicate tests 1 and 2.

Solving for G_1 , we get

$$G_1 = \frac{\left(\frac{\hat{T}_o}{\hat{T}_i} \right)_1 - \frac{\left(\frac{\hat{T}_o}{\hat{T}_i} \right)_2 \left(\frac{\hat{W}_i}{\hat{T}_i} \right)_1}{\left(\frac{\hat{W}_i}{\hat{T}_i} \right)_2}}{1 - \frac{\left(\frac{\hat{W}_i}{\hat{T}_i} \right)_1}{\left(\frac{\hat{W}_i}{\hat{T}_i} \right)_2}} \quad (3)$$

An examination of the denominator of Eq. (3) shows the advantage of making the two tests sufficiently different. Because of the inherent random fluctuations in the signals and the relative ease of performing the tests at any given set of operating conditions, we made four frequency-response tests at each condition, and then computed the least-squares fit of the desired transfer functions, as follows: Let

$$H_k \equiv \begin{pmatrix} \hat{W}_i \\ \frac{1}{\lambda} \\ T_i \end{pmatrix}_k,$$

and

$$J_k \equiv \begin{pmatrix} \hat{T}_o \\ \frac{1}{\lambda} \\ T_i \end{pmatrix}_k.$$

Expressing Eq. (2) in matrix form for $\underline{n}$ tests:

$$Ax = b \quad (4)$$

where

$$A \equiv \begin{pmatrix} 1 & H_1 \\ 1 & H_2 \\ 1 & H_3 \\ \vdots & \\ \vdots & \\ 1 & H_n \end{pmatrix},$$

$$x \equiv \begin{pmatrix} G_1 \\ G_2 \end{pmatrix},$$

$$b \equiv \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ \vdots \\ \vdots \\ J_n \end{pmatrix}.$$

The least-squares fit value of x is found by premultiplying both sides of Eq. (4) by the transpose of A :

$$A^T A x = A^T b, \quad (5)$$

where

$$A^T = \begin{pmatrix} 1 & 1 & 1 \\ H_1 & H_2 & H_3 & \dots & H_n \end{pmatrix}.$$

Since $A^T A$ is a 2×2 matrix and $A^T b$ is a 1×2 column vector, Eq. (5) can be solved for x directly.

The four different pseudorandom binary input perturbations used are

1. changing the recirculating brine flowrate W_R only,
2. changing the steam-heater steam flowrate W_S only,
3. changing both W_R and W_S in phase,
4. changing both W_R and W_S 180° out of phase.

This technique was used to analyze both the standard PRBS and temperature probe PRBS test data.

The analysis requires a measurement of the tray-brine flowrate into the test stage (W_{TBi2}). Although the flowrate cannot be measured directly, it can be calculated using the mass balance equation for stage 1:

$$\frac{dM_{TB1}}{dt} = W_{TBi1} - W_{TBF1} - W_{TBi2}, \quad (6)$$

where

W_{TBi1} = measured flow rate into stage 1, lb/sec,

W_{TBF1} = flashing flowrate (vaporization rate) in stage 1, lb/sec,

M_{TB1} = mass of tray brine in stage 1, lb (obtained from level measurement).

From simulated tests we determined that $\hat{W}_{TBF1}$ was always less than 0.5% of $\hat{W}_{TBi2}$ even when W_{TBi1} was not perturbed; therefore, W_{TBF1} was assumed to be negligible in this calculation. Hence,

$$\hat{W}_{TBi2} \approx \hat{W}_{TBi1} - \frac{d\hat{M}_{TB1}}{dt}. \quad (7)$$

The method used to convert the time-response data into frequency response form is known as the "direct method," and by narrow band-pass filtering, cross-power spectral density functions are obtained, as described by Chang.⁵ The computer code for this purpose is a modified version of one used in dynamic analyses of the Molten-Salt Reactor Experiment.⁶

3.2 Description of AMF Digital Simulation Program

An analog computer simulation of the AMF plant was tested and analyzed extensively to check the data collecting methods, analysis techniques, and computer codes before the dynamics tests of three-stage flash evaporator at Millstone Point were begun. To analyze the dynamics test data more extensively by comparing those data with data from tests of a simulated plant, a digital simulator program was written. This digital simulation was more versatile than the analog simulation, in that digitizing was not required and many more nonlinear effects could be added to the digital model.

The digital simulator code was written as a set of subroutines to MATEXP, a general-purpose program for solving differential equations.⁷ The simulation model (Fig. 5) consists of 23 first-order differential equations and two transport lags. Nonlinear effects (such as variable flow and heat transfer), interstage flowrates proportional to ΔP , and steam table relationships are included. PRBS input perturbations to the model were generated by a special subroutine that simulates the shift register used in the AMF experiments. Data from the simulator was stored on magnetic tape and processed in the same manner as the experimental data.

⁵S. S. L. Chang, Synthesis of Optimum Control Systems, Chap. 3, McGraw-Hill, New York, 1961.

⁶T. W. Kerlin and S. J. Ball, Experimental Dynamic Analysis of the Molten-Salt Reactor Experiment, ORNL-TM-1647 (Oct. 1966).

⁷S. J. Ball and R. K. Adams, "MATEXP", A General Purpose Digital Computer Program for Solving Ordinary Differential Equations by the Matrix Exponential Method, ORNL-TM-1933 (Aug. 1967).

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① = EQUATION 1

T = TEMPERATURE (HEAT BALANCE) EQUATION

M = MASS BALANCE EQUATION

Fig. 5 AMF-Block Diagram for Matexp Equations.

The equations solved by the simulator, which are based on the model used in the large MSF plant dynamics study,⁸ are given in the Appendix, Sect. 8.1.

3.3 Results of Five Selected Standard Transfer Function Tests

Eight complete sets of PRBS tests were made during which the standard set of variables was digitized. Five of these sets were with a plain orifice at the test-stage inlet and three were with a 12 in. vertical baffle plate 2 ft downstream from the inlet orifice. As noted in Sect. 2, in each of the runs with the plain orifice and with a high tray-brine level in the test stage, there were excessively large random temperature fluctuations. These tended to bury the signal fluctuations due to the PRBS inputs. At the time of the tests it was hoped that by running each PRBS for three cycles and using the least-squares fit method for four separate tests, the analysis would recover enough of the signal from the noise background. It was apparent from the analysis, however, that longer time records or larger signal-to-noise ratios, or both, would be required to get good results at these conditions. Only one set of results from the plain orifice, high brine level will be presented as an example of the poor quality obtained. A summary of the standard test run conditions is shown in Table 3. More detailed information on the nominal operating conditions during any set of PRBS runs can be obtained from Table 2 from the corresponding steady-state run conditions.

The experimental and theoretical frequency response results for runs 35-38, with high temperature, low flowrate, low brine level and a plain orifice, are shown in Figs. 6 through 8. Figure 6, which indicates the response of the tray-brine outlet temperature from stage 2, $\bar{T}_{TB2}$, to changes in the inlet brine temperature, T_{i2} , shows that

⁸S. J. Ball, Nuclear Desalination Dual-Purpose Plant Control Studies Interim Report Appendix, ORNL-TM-1618, Part 2, (Jan. 1967).

Table 3. Standard PRBS Transfer Function Runs

Run Nos.	Corresponding Steady-State Run No.	Was PRBS Analysis Included	Was Baffle Installed	Temperature	Flow	Stage 2 Brine Level
2-5	1	No	No	High	High	High
7-10	6	Yes	No	High	High	Low
21-24	25	Yes	No	Low	Low	High
28-31	27	No	No	Low	High	Middle
35-38	34	Yes	No	High	Low	Low
41-44	40	Yes	Yes	High	Low	Low
46-49	45	Yes	Yes	High	Low	High
61-64	60	No	Yes	High	High	High

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Fig. 6. Frequency Response of $\bar{T}_{TB2}/T_{i2}$ for Runs 35-8.

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Fig. 7. Frequency Response of T_{v2}/T_{i2} for Runs 35-8.

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Fig. 8. Frequency Response of T_{v2}/W_{i2} for Runs 35-8.

the experimental magnitude ratio is slightly below the theoretical response at low frequencies (0.04 to 0.14 radian/sec) and above at higher frequencies (0.14 to 0.5 radian/sec). This result could be explained by assuming that part of the inlet brine flowed along the bottom of the tray ("short circuiting" the tray) rather than undergoing perfect mixing, as is assumed in the theoretical model. The smaller values of experimental phase lag above 0.1 radian/sec also tend to corroborate this explanation. For example, assume there is a small step increase in T_{i2} and some of the inlet flow short circuits the tray without mixing with the rest of the cooler brine. This would result in a larger fraction of the total temperature change appearing sooner at the outlet (and hence the higher gain and smaller phase lag at high frequencies) than it would with the perfect-mixing model. This short-circuiting effect has been seen clearly in flow model studies at the University of Tennessee and at the 3-stage BLH evaporator at Wrightsville Beach, N.C., for inlet orifices with no baffles.⁹

The response of the vapor temperature in stage 2, T_{V2} , to perturbations in T_{i2} (Fig. 7) demonstrates another important characteristic of the test-stage response that was apparent from the transient response curves and from the frequency response analysis as well. This is the rapid response of the vapor temperature, relative to the tray-brine temperature, to tray-brine inlet temperature changes. In the original ORNL model,¹ it was assumed that the vaporization rate was dependent on the difference between the mean temperatures of the tray brine and vapor:

$$W_{TBF} = K (\bar{T}_{TB} - T_V) . \quad (8)$$

Since vapor temperature changes in turn depended on the difference between vaporization and condensation flow rates, the response of the vapor temperature would be expected to lag behind the response of the mean

⁹T. W. Kerlin et al., Nuclear Desalination Plant Dynamics: Modeling and Analysis of a Multi-Stage Flash Evaporator, Progress Report No. 2, Univ. of Tenn. NEUT-2806-2 (July 1968).

tray-brine temperature to inlet brine temperature changes. The tests showed uniformly, however, that this was not the case. In Fig. 7, the values of phase lag are very small at the higher frequencies relative to the theoretical results. To adjust the model so that it would better fit the data, the evaporator digital simulator program was modified to calculate the flashing flowrate from the following equation:

$$W_{TBF} = K W_{i2}^{0.5} (T_{i2} - T_{V2}). \quad (9)$$

The half-power dependence on inlet brine flowrate W_{i2} was suggested by Dickson and Silver,¹⁰ and the dependence on T_{i2} instead of $\bar{T}_{TB2}$ was apparent from the data.

The results of a frequency response analysis of a simulated set of PRBS tests incorporating Eq.(9) are also shown in Fig. 7. While the phase lag agreement is excellent, the magnitude ratio of the revised model does not show the proper attenuation at high frequencies. It is evident that a compromise between the two theoretical models is in order. This would appear to be reasonable, in that some of the flashing would naturally occur downstream from the inlet.

Figure 8 shows theoretical and experimental results for T_{V2} response to changes in inlet brine mass flowrate W_{i2} . In earlier practice tests with an analog simulation of the plant, analysis of the data gave reasonably accurate indications of the known model except for excessive phase lags in the response of the stage temperatures to W_{i2} . Excessive phase lags were also evident in the experimental results, as seen in Fig. 8. In all cases, the excessive lag appeared to correspond to the fluid transport lag from the brine heater to the test stage. This is an anomaly that does not make any sense physically, since any additional energy input into the stage from inlet flowrate changes alone would tend to propagate throughout the stage at approximately the same rate as any other inlet disturbance. Several tests on known digital simulator models were run, and it was concluded that the errors were due to insufficient

¹⁰A. N. Dickson and R. S. Silver, "The Flow of Flashing Water in Conduits", Desalination, 2, 175-195 (1967).

response of the stage temperatures to flowrate changes as compared to the other input changes inherent in the tests. For example, in run 35, where only the recirculating brine flowrate was varied, T_{i2} fluctuated $\sim 2^\circ\text{F}$, causing nearly as much variation in $\bar{T}_{TB2}$ and T_{V2} . These T_{i2} fluctuations were due mainly to the variations of the brine-heater outlet temperature that occurred when the brine flowrate changed. Since typical flowrate variations were about 20 lb/sec, and the stage temperature low-frequency response (theoretical or experimental) was typically about $0.02^\circ\text{F lb}^{-1}\text{sec}^{-1}$, the signal strengths for measurements of W_{i2} transfer functions were only about 20% as much as those due to T_{i2} perturbations, and the quality was degraded proportionately. Hence the systematic errors in the measured phase responses are attributed to those brine-heater outlet temperature changes that were due to flowrate changes. These T_{i2} perturbations are strongly correlated to the flowrate changes themselves, but are delayed by the brine transport lag time. Hence any small measurement errors could lead to a situation where the T_{i2} signal would influence the W_{i2} response analysis. Because of these errors, it appears that there is very little physical significance in the experimental results of W_{i2} response; therefore, no more of them will be shown.

In future tests, we plan to try to make W_{i2} vary with as little change in T_{i2} as possible, and to do more extensive error analyses of the practice test simulation.

Figures 9 and 10 show the frequency response of $\bar{T}_{TB2}$ and T_{V2} to T_{i2} perturbations for runs 7-10, which were run at conditions very similar to those of runs 35-38, except for higher brine flowrate in runs 7-10. The same features noted for runs 35-38 (Figs. 6 and 7) are also apparent here, in particular the very small phase lag of T_{V2}/T_{i2} .

Figures 11 and 12 show the results for a high-brine-level test with no baffle (runs 21-24), where the effects of the poor signal-to-noise ratios are apparent. The higher-brine level and lower flowrate resulted in a longer tray-brine bulk holdup time (~ 10 sec vs 5 or 6 sec in the previously described cases), and this can be seen, with a little imagination, in the magnitude ratio plot in Fig. 11, where

Fig. 9. Frequency Response of $\bar{T}_{TB2}/T_{i2}$ for Runs 7-10.

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Fig. 10. Frequency Response of T_{v2}/T_{i2} for Runs 7-10.

Fig. 11. Frequency Response of $\bar{T}_{TB2}/T_{i2}$ for Runs 21-4.

ORNL-DWG 68-5401

Fig. 12. Frequency Response of T_{v2}/T_{i2} for Runs 21-4.

there is greater attenuation at lower frequencies than in Figs. 6 and 9. While some of the peaks and valleys in the curves look potentially interesting, it would be risky to try to attach any physical significance to them.

For runs 41-44, the evaporator conditions were approximately the same as for runs 35-38, except for the addition of the baffle plate. As would be expected, the flow short-circuiting effect postulated for the plain orifice tests was reduced significantly, as seen in Fig. 13. The phase lag for T_{V2}/T_{i2} (Fig. 14) is small, which would again substantiate a conclusion that flashing is more dependent on brine inlet temperature than bulk brine temperature. However, the magnitude ratio curve for T_{V2}/T_{i2} clearly shows first-order lag attenuation characteristics. Rarely would we ever be upset about such excellent agreement of theoretical and experimental results; however, first-order-lag attenuation is not usually accompanied by such small phase lags in physical systems, and we have not been able to devise a satisfactory explanation.

Frequency response results for baffled-orifice runs 46-49, with high temperature, high brine level, and low flowrate, are shown in Figs. 15 and 16. The large random temperature fluctuations that occurred in high-brine-level runs 21-24 made without a baffle were not present in these runs, and hence relatively good results were obtained. The large attenuation in the magnitude ratio curve of Fig. 15 is very similar to that of Fig. 11 for runs 21-24, and can again be attributed to the larger holdup time of the tray brine. Figure 16 shows the same perplexing behavior for T_{V2}/T_{i2} as was noted for runs 41-44.

In conclusion, the transfer function results were informative and interesting, although they clearly have not answered all of the important questions about MSF evaporator temperature dynamics. With these tests as a background, however, we are in a much better position to intelligently plan and execute future tests, which we hope will provide more conclusive results.

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Fig. 13. Frequency Response of $\bar{T}_{TB2}/T_{12}$ for Runs 41-4.

Fig. 14. Frequency Response of T_{v2}/T_{i2} for Runs 41-4.

ORNL-DWG 68-5402

Fig. 15. Frequency Response of $\bar{T}_{TB2}/T_{i2}$ for Runs 46-9.

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Fig. 16. Frequency Response of T_{v2}/T_{i2} for Runs 46-9.

3.4 Results of One Temperature-Probe Transfer Function Test

The original intent of the temperature-probe transfer function tests was to measure temperature response data throughout the test stage by placing the probe in four axial positions, and thus provide information about the accuracy of the lumped-parameter models of the tray brine and vapor temperature dynamics. Due to time limitations, however, we left the probe in one axial position (4 ft 2 in. downstream from the inlet orifice) while runs were made at several different evaporator operating conditions. Consequently, the results of the probe tests did not provide conclusive evidence about the validity of the single-node models. On one hand, results typically showed a gradual transition in the dynamic characteristics from the low point (3 in. from the tray bottom) to the high point (36 in. from the bottom), where single-node representations would have given space-independent response within either the brine or the vapor. This gradual change in response is shown in Figs. 17 to 21 for AMF probe runs 11 to 14, where the evaporator was operating at the same conditions as in runs 7 to 10: high brine-inlet temperature (148°F) and high flowrate (697,000 lb/hr), and low tray-brine level (14 in).

On the other hand, the frequency response of T_{P1} (3 in. from the bottom) to inlet brine temperature changes (T_{i2}) shows amplitude attenuation and phase lags very similar to those shown by the outlet temperature response (runs 7-10, Fig. 9), which tends to substantiate the single-node model for the brine temperature. For example, the measured phase lags for both response functions at 0.2 radian/sec are 35 ± 5 deg. The calculated total tray-brine bulk holdup time for this case is 4.3 sec, and the holdup time between the probe and the outlet is 2.7 sec. If one assumes either a plug flow or a first-order mixing lag model for bulk brine transport between the probe and the outlet, the difference in phase lags at that frequency would be about 30 deg. Hence it appears that, while an axially space-independent lumped-parameter model would be fairly accurate, there may be sufficient vertical temperature dependence to require a multinode

Fig. 17. Frequency Response of Probe 1 Temperature (3 in. from Bottom) as a Function of T_{i2} for Runs 11-14.

ORNL-DWG 68-5405

Fig. 18. Frequency Response of Probe 2 Temperature (9 in. from Bottom) as a Function of T_{i2} for Runs 11-14.

Fig. 19. Frequency Response of Probe 3 Temperature (15 in. from Bottom) as a Function of T_{i2} for Runs 11-14.

ORNL-DWG 68-5407

Fig. 20. Frequency Response of Probe 4 Temperature (21 in. from Bottom) as a Function of T_{i2} for Runs 11-14.

Fig. 21. Frequency Response of Probe 6 Temperature (36 in. from Bottom) as a Function of T_{i2} for Runs 11-14.

or distributed parameter approximation.

Further tests of space-dependent temperature responses would clearly be of interest.

4. RESULTS OF ORIFICE FLOW EQUATION ANALYSES

4.1 Analytical Methods for Determining Orifice Flow Equation Model and Parameters

A wide variety of assumptions and approximations is typically used to predict and correlate the steady-state flow through MSF interstage orifices. It is generally acknowledged that a "complete" theoretical calculation of such flows is not practical, particularly when flashing occurs in a region close enough to the orifice to affect the flowrate. Hence plant designers and operators rely on orifice equations in forms that are derived from simpler flow situations but which have experimentally derived coefficients, or "fudge factors". This approach has proved adequate to the point that a given plant design can usually be built and operated at desired conditions after appropriate adjustment of the orifice heights.

The problem of determining how stage parameters govern flow through the interstage orifices for the purpose of predicting large MSF plant dynamic behavior, however, is more demanding. Although a 20 or 30% error band in a log-log plot of flow versus some effective differential pressure is considered sufficiently accurate for plant design purposes, it may just be that the scatter is due to a parameter not considered in the equation but which would, however, have a significant effect on the dynamics. Dynamics studies of the very large MSF evaporators as designed for nuclear desalting plants show that the overall plant stability characteristics are extremely sensitive to the flow equations, particularly the effect of downstream tray brine level. Specifically, the more dependent the flow is on downstream level, the less stable the plant.¹¹ Most commonly used empirical flow equations attach equal

¹¹S. J. Ball, Nuclear Desalination Dual-Purpose Plant Control Studies Interim Report, ORNL-TM-1618, Part 1 (Oct. 1966), p 42.

importance to the downstream static head and the vapor pressure head, in which case the predicted inherent large plant behavior is quite unsatisfactory.

Intuitively it would appear that the effect of downstream level is not so simply related to flow, even for plain orifices and single-phase flow, and particularly for baffled orifices and with some dependence on two-phase flow.

Hence the purpose of the orifice flow analysis was to obtain accurate relationships for describing flow perturbations as a function of stage parameters, particularly downstream level, for both plain and baffled orifice configurations.

To clarify the assumptions implicit in the various approximate flow equations, a brief development of the conservation equations for the single-phase submerged jet in an open channel will be presented.

Figure 22 shows a schematic representation of flow through a submerged orifice.

Fig. 22. Schematic of Single-Phase Flow Through a Submerged Orifice.

The following derivation is an extension of the case developed by Henry¹², modified to include the effect of a vapor pressure difference. It is assumed that there is: no energy loss between sections 1 and 2; uniform velocity distribution at sections 1 and 3; and, at section 2, uniform velocity in the jet and zero velocity above the vena contracta. The flow is computed from a simultaneous solution of the continuity equations at all three sections, the conservation of energy equation for flow between sections 1 and 2, and the conservation of momentum equation for sections 2 to 3.

Continuity (at sections 1, 2, and 3):

$$q = V_1 L_1 = V_2 C_c B = V_3 t \quad (10)$$

Energy (Borda-Carnot formula)(between sections 1 and 2):

$$V_2^2 - V_1^2 = 2g (h - t') \quad (11)$$

Momentum (between sections 2 and 3):

$$\frac{1}{2} g t'^2 + C_c B V_2^2 = \frac{1}{2} g t^2 + t V_3^2 \quad (12)$$

where

q = volume flowrate, ft³/sec per ft of width,

V = velocity, ft/sec,

L_1 = upstream liquid depth, ft,

C_c = contraction coefficient,

B = orifice height, ft,

t = liquid depth of section 3, ft,

g = acceleration due to gravity, ft/sec²,

h = total upstream head $\Delta P_V + L_1$, ft of water,

t' = liquid depth at section 2, ft,

ΔP_V = differential vapor pressure across orifice, ft of water.

It is convenient to solve for t'/B , resulting in the quadratic solution

$$\frac{t'}{B} = \frac{K^2 \pm \sqrt{K^4 + 4A [A(t/B)^2 - (h/B)K^2]}}{2A} \quad (13)$$

¹²H. R. Henry, "Discussion of Paper by M. L. Albertson, Y. B. Dai, R. A. Jensen, and H. Rouse, 'Diffusion of Submerged Jets,'" Trans. Am. Soc. Civil Eng. 115, 639-697 (1950).

where

$$K \equiv \left[1 - \left(\frac{C_c B}{L_1} \right)^2 \right]^{-1/2},$$

$$A \equiv \frac{1}{4C_c \left(1 - \frac{C_c B}{t} \right)}.$$

Solving for the flowrate from Eqs. (10) and (11):

$$q = C_c B K \sqrt{2g(h-t')}. \quad (14)$$

Although Henry's experimental results agreed well with his predictions, he suggested that the small discrepancies were due to air entrainment and to accelerative forces within the "roller", or the reverse flow near the surface of the jet. For the case of flashing flow, additional steam bubble entrainment and accelerative forces due to the flashing would be expected to increase the error in the predictions. In particular, for cases where flashing occurs near the orifice region, this treatment breaks down, in that the quantity within the square-root sign in Eq. (13) becomes negative. This implies, based on effective (measured) ΔP , that the flow should be greater than the theoretical free-discharge (no submergence) value. The additional energy loss not accounted for by this treatment is apparently due to flashing forces impeding the axial flow. Experimental correlations will be shown in Sect. 4.2.

The most commonly used simplification of Henry's treatment involves the assumption that the submergence depth t' equals the downstream depth t , and elimination of the momentum equation [Eq. (12)]. This results in

$$q = C_c B \sqrt{\frac{2g(h-t)}{1 - \left(\frac{C_c B}{L_1} \right)^2}}, \quad (15)$$

where the quantity $[1 - (C_c B/L_1)^2]$ (or its square root) is commonly known as the velocity-of-approach factor. Typically, an additional energy loss coefficient C_v is included based on empirical data, and the net

discharge coefficient C_d is taken as $C_v C_c$. Theoretically then, C_d should never exceed C_c (≈ 0.6), otherwise an energy gain, rather than loss, would be implied. Actually, however, measured values of C_d often exceed 0.6, particularly for plain orifices. Two possible reasons for this would be the error in the assumption $t' = t$ (which would overestimate the downstream static head), and the possible augmentation of the axial kinetic energy by flashing. Of the two, the former appears to be more likely in view of the fact that correlations with Henry's equation indicate that flashing serves as an impedance to the flow.

An empirical equation in common usage is that derived by AMF¹³ for plain orifices:

$$\frac{w}{A} = 400,000 \left(\frac{\Delta P_T}{1.25} \right)^{0.455}, \quad (16)$$

where

w = interstage flowrate, lb/hr,

A = interstage orifice area, ft^2 ,

A_1 = cross-section area of brine leaving upstream stage, ft^2 ,

$\Delta P_T = (\Delta P_V + \Delta h) / (1 - A^2/A_1^2)$, in. of water,

ΔP_V = interstage vapor pressure difference, in. of water,

Δh = interstage static liquid level difference, in. of water.

Equation (16) gave relatively good predictions for steady-state flow for the plain orifice runs; however, both Eqs. (15) and (16) suffer from the simplifying assumption that $t' = t$, which could not be uniformly accurate, particularly for low-brine-level cases approaching the free-discharge situation. In view of the importance of the downstream level effect in the large MSF plant stability calculations, some further refinement of these equations would be in order.

Our analysis of the transient flow data was based on yet another approach to the problem, which was to equate the difference between total upstream and downstream energy to a friction loss term proportional to some power of mass flowrate:

¹³Saline Water Conversion Report for 1966, U. S. Dept. of the Interior, p 179-180.

$$\left(P_{V1} + P_{H1} + \frac{V_1^2}{2g}\right) - \left(P_{V2} + K_2 P_{H2} + \frac{V_2^2}{2g}\right) = K \left(\frac{W}{A_o}\right)^{1/n} \quad (17)$$

Rearranging Eq. (17):

$$\frac{W}{A_o} = C (\Delta P_{EFF})^n, \quad (18)$$

where in units convenient for the dynamic tests

W = brine mass flowrate, lb/sec,

A_o = orifice area, ft²,

C = orifice flow factor, $\frac{\text{lb/sec}}{(\text{ft}^2)(\text{psi})^n}$,

$$\Delta P_{EFF} = \Delta P_V + \Delta P_H + \Delta P_m,$$

and where

ΔP_V = vapor pressure difference between stages, psi,

ΔP_H = liquid head pressure difference $\equiv$

$$\frac{M_{TB1}}{A_{T1}} - K_2 \frac{M_{TB2}}{A_{T2}}, \text{ psi},$$

$$\Delta P_m = \text{velocity head term} = \rho_L (V_1^2 - V_2^2)/2g,$$

M_{TB1}, M_{TB2} = mass of brine in trays of upstream and downstream stages, lb,

$A_{T1,2}$ = base areas of stage 1 and 2 trays, in.²,

K_2 = importance factor for downstream liquid level,

ρ_L = brine density, lb/in.³,

$V_{1,2}$ = upstream and downstream tray brine bulk velocities in./sec,

g = acceleration due to gravity, in./sec².

We attempted to determine the downstream level importance factor, K_2 , in the AMF evaporator for a plain rectangular orifice for various combinations of temperature, flow, and brine-level ranges by using frequency-response measurements. The use of frequency-response measurements implied the use of linearized equations, which may be of

questionable value for determining a factor in a nonlinear orifice equation. In defense of the assumption, however, we point out that the MSF plant stability studies for which this data is to be used, at least at first, are based on linearized models. The validity of MSF plant model linearization is still being researched, but it is clear from the results of simulations using the assumed single-phase orifice equation that linear stability estimates are adequate.

When Eq. (18) is linearized,

$$\hat{W} = A_o C_n (\Delta P_{EFF-0})^{n-1} \hat{\Delta P}_{EFF}, \quad (19)$$

where the circumflex (^) indicates a perturbation quantity, and the zero subscript indicates a mean, steady-state value. Since C can be evaluated from steady-state conditions, Eq. (19) can be written as

$$\hat{W} = \frac{n W_o}{\Delta P_{EFF-0}} \hat{\Delta P}_{EFF} \equiv K_1 \hat{\Delta P}_{EFF}. \quad (20)$$

Comparisons of the nonlinear and linearized Eqs. (18) and (20) for an exponent n value of 0.455 (per AMF) indicates that the error due to linearization is less than 2% of W for up to $\pm 30\%$ changes in ΔP_{EFF} , as shown in Fig. 23.

Fig. 23. Linear Approximation of Orifice Equation for Exponent $n = 0.455$.

The original objective of the flow equation analysis was to estimate the values of the effective orifice coefficient K_1 and the downstream level-effect coefficient K_2 . Since we had assumed that the flow relationships were instantaneous, K_1 and K_2 would not be frequency dependent, and an estimate could be made for each frequency analyzed. It appears reasonable to try to determine many estimates and, hence, good statistics on K_1 and K_2 for each set of evaporator operating conditions.

It was also noted that while the velocity head term ΔP_m could contribute significantly to the steady-state flow conditions, the perturbations $\hat{\Delta P}_m$ were negligible. Hence $\hat{\Delta P}_m$ was not needed for evaluation of K_1 and K_2 ; thus

$$\hat{W}_{TBi2} \approx \hat{W}_{TBi1} - \frac{dM_{TB1}}{dt} \approx K_1 \left[\hat{\Delta P}_V + \frac{\hat{M}_{TB1}}{A_T} - K_2 \frac{\hat{M}_{TB2}}{A_T} \right]. \quad (21)$$

From the simulated frequency-response tests, we obtained the Fourier transform values of each perturbation variable in Eq. (21), and thus the two unknowns K_1 and K_2 are derived from the single equation. This is because each variable is represented by a complex number (i.e. a real plus an imaginary part).

Since K_1 is a real number, we solved for the value of K_2 which makes the phase of $\hat{W}_{TBi1} - \frac{dM_{TB1}}{dt}$ equal to the phase of $\hat{\Delta P}_V + \frac{\hat{M}_{TB1}}{A_T} - K_2 \frac{\hat{M}_{TB2}}{A_T}$. Arbitrarily specifying the phase of $\hat{M}_{TB1}$ as the reference phase (0°), and noting that $-\frac{dM_{TB1}}{dt} = -j\omega \hat{M}_{TB1}$, we write

$$F\{\hat{W}_{TBi2}\} = \text{Re}\{\hat{W}_{TBi1}\} + j[\text{Im}\{\hat{W}_{TBi1}\} - \omega \hat{M}_{TB1}], \quad (22)$$

where

$F\{\hat{W}\} \equiv$ Fourier transform of $\hat{W}$,

$\text{Re}\{\hat{W}\} \equiv$ real part of $\hat{W}$,

$\text{Im}\{\hat{W}\} \equiv$ imaginary part of $\hat{W}$,

$\omega =$ frequency, radians/sec,

$j = \sqrt{-1}$.

Equating the tangents of $F\{\hat{W}_{TBi2}\}$ and $F\{\hat{\Delta P}_{EFF}\}$, we get

$$\tan X = \frac{\text{Im}\{\hat{W}_{TBi1}\} - \omega \hat{M}_{TB1}}{\text{Re}\{\hat{W}_{TBi1}\}} = \frac{\text{Im}\{\hat{P}_V\} - \frac{K_2}{A_T} \text{Im}\{\hat{M}_{TB2}\}}{\frac{\hat{M}_{TB1}}{A_T} + \text{Re}\{\hat{P}_V\} - \frac{K_2}{A_T} \text{Re}\{\hat{M}_{TB2}\}} \quad (23)$$

Equation (23) is solved for K_2 , and K_1 is found from the ratio

$$K_1 = \frac{F\{\hat{W}_{TBi2}\}}{F\{\hat{\Delta P}_{EFF}\}} \quad (24)$$

The ratios of $F\{\hat{M}_{TB2}\}$, $F\{\hat{P}_V\}$, and $F\{\hat{W}_{TBi1}\}$ to $F\{\hat{M}_{TB1}\}$ are calculated by a digital filter method⁶ which computes cross-power spectral densities (CPSD's) and transfer functions by the "direct" method.

Extensive practice analyses of test data taken from an analog simulation of the AMF plant indicated that the "correct" values of K_1 and K_2 could be predicted from the frequency-response results to within about 5%. Although the model that we had assumed was later shown to be deficient, the general method of deriving coefficients in the flow equation appeared to be sound and was applied to an analysis of a more detailed flow equation model.

It may be noted that while this initial analysis was based on the equation assuming kinetic energy losses proportional to a power of W , Eq. (17), it could easily be applied to AMF's equation, Eq. (16), by including another (calculable) coefficient in the upstream static head term in Eq. (21). This would be less than unity, due to the effect of the velocity-of-approach factor.

Although in many cases we obtained fairly good correlations for measured versus theoretical static flowrates, there developed some strong evidence that the coefficients of the flow equation were not constant. The first indication of a deficiency of the fixed-coefficient orifice equation was the complete lack of correlation between theory and experiment, or of any reasonable answers when the data was analyzed by the techniques just described. But perhaps the most striking

indication of the inadequacy of the simple model was seen in several series of time-response plots on the 8-channel oscillograph records of the tests. They showed that the net ΔP change (vapor plus liquid head) was clearly not directly related to the change in flowrate into stage 2, $\hat{W}_{\text{TBi2}}$. In one case, the net ΔP dropped about 0.29 psi in 10 sec, and yet W_{TBi2} remained fairly constant. Such evidence led to the conclusion that the flow must have a significant temperature dependence, and perhaps even a dependence on the rate of change of temperatures and brine levels. For this to be the case, we have postulated that voids would have had to be present due to flashing in the effective orifice region.

The theoretical studies of Leslie¹⁴ and subsequent experiments by Hawes and Leslie¹⁵ on void formation in MSF plant orifices are most valuable contributions to the understanding of the mechanisms of flashing in low-quality systems. Their experiments indicate the strong dependence of flashing on the presence of small entrained bubbles acting as nucleation sites. The striking dependence of bubble growth rates on saturation temperature (e.g., an order of magnitude faster at 100°F than at 200°F) was also shown. These factors indicate some of the difficulties unfortunately present in accurately predicting pressure-drop behavior of flow through an orifice.

Since no specific relationships were available for predicting transient flowrate in the presence of voids, we used a rather brute force approach by considering all of the gross effects that might be important, finding the coefficients for this more general model that gave the best agreement with the experimental results, and then checking to see if the results looked reasonable (i.e. possible). As a further check, we incorporated the revised model, with a sample set of experimentally derived coefficients, into the digital time-response simulation of the AMF plant so that both time-domain and frequency-domain comparisons could be made.

The assumption of the presence of a significant void fraction in the orifice region is admittedly questionable, but an argument can be made that such void fractions are at least possible. The total void

¹⁴D. C. Leslie, The Development of Flashing Flow from Existing Nucleation Sites, A.E.E. Winfrith Report R-505 (Oct. 1966).

¹⁵R. L. Hawes and D. C. Leslie, "A Study of the Mechanism of Flashing Flow by Experiment and Theoretical Analysis," Desalination 2, 329-36 (1967).

fraction in the orifice region is the sum of the subcooled or entrained voids plus the voids due to the very low, but quite likely significant quality of the inlet brine. Typically the maximum possible quality for $\sim 3^\circ\text{F}$ flashdown, $C_p \approx 1.0 \text{ Btu lb}^{-1}\text{F}^{-1}$, and $h_{fg} \approx 1000 \text{ Btu/}^\circ\text{F}$ would be only $(3 \times 1)/1000$, or 0.3%. At the low temperatures in an MSF plant, however, extremely small fractions of this quality could result in quite significant void fractions, as shown in Fig. 24 which was derived directly from properties of saturated steam and water.

To include voids in the flow equation analysis, it was assumed that the quality (and hence the void fraction) is affected by an "effective" temperature difference between stages, i.e. $T_{TB1} - T_{V2}$, and the liquid-head pressures in stages 1 and 2, i.e. P_{H1} and P_{H2} , where

T_{TB1} = stage 1 tray brine temperature, $^\circ\text{F}$ (measured 3 in. upstream of the interstage orifice and 3 in. from bottom of tray),

T_{V2} = stage 2 vapor temperature, $^\circ\text{F}$,

$P_{H1,2}$ = liquid head in stages 1 and 2, psi.

A steady-state increase in the void fraction in the orifice region would reduce the effective flow area and, hence, increase the pressure drop for a given mass flow rate of inlet brine. A transient increase in the void fraction would then result in an increase of mass flow out of the region due to the displacement of the brine, the flow change being proportional to the time rate-of-change of void fraction. This would also result in an increased pressure drop.

Since we are dealing with a linearized model (of uncertain form), we can revise Eq. (21) directly to include the additional effects of voids on the pressure drop:

$$\hat{W}_{TBi2} = C \left\{ \hat{\Delta P}_V + K_1 \hat{P}_{H1} - K_2 (\hat{T}_{TB1} - \hat{T}_{V2}) - K_3 \hat{P}_{H2} - j\omega [K_4 (\hat{T}_{TB1} - \hat{T}_{V2}) - K_5 \hat{P}_{H1} - K_6 \hat{P}_{H2}] \right\}, \quad (25)$$

where the coefficients K_{1-6} might include several effects. For example, K_1 and K_3 might include a static pressure effect plus a static void-suppression effect. The effects of liquid-level changes on saturation

Fig. 24. Saturated Water: Void Fraction vs Quality.

temperatures are also quite large in the range tested, varying from about 1°F/in. of water at 100°F to about 0.5°F/in. of water at 140°F.

Although the two unknowns in the original flow Eq. (21) could be evaluated directly, the six unknowns in Eq. (25) required a different treatment. The computer code RANOPT¹⁶ was used which implements the pattern search method of Hooke and Jeeves¹⁷ with additional options such as random starts and specification of constraints. RANOPT was set up to minimize the differences in phase angles between the measured transforms of $\hat{W}_{TB12}$ and the effective ΔP (the right-hand side of Eq. (25) by varying K_{1-6} . Typically, between 10 and 25 sets of transformed PRBS test data at selected frequencies (for various input perturbations) would be used in a RANOPT case. Data was selected on the basis of the coherence function¹⁸, as computed by the CPSD code. Although methods for finding explicit values for the uncertainties in the transfer function measurements are apparently not yet developed for periodic signal analysis, the coherence function gives a rough indication of accuracy. Typically, transformed data at frequencies above 0.15 radian/sec and coherences less than 0.5 (on a 0 to 1.0 scale) would be rejected.

4.2 Results of Static and Dynamic Flowrate Correlations

Table 4 summarizes the comparison between the static flowrate measurements and the values predicted by the various equations, as derived in Section 4.1. Also shown are the orifice coefficients, C_d in Eq. (15) and C in Eq. (18), which ideally should be constant for each set (i.e., no baffle, or baffle) of runs. As noted in the results for Henry's equation, in two of the no-baffle runs and in all but one of the baffle runs the quantity under the square root sign in Eq. (13) is negative, implying that the flowrate should be greater than the theoretical free-discharge value. Both of these no-baffle runs, 6 and

¹⁶RANOPT, an unpublished Code by S. J. Ball.

¹⁷R. Hooke and T. A. Jeeves "Direct Search Solution of Numerical and Statistical Problems", J. Assoc. Computing Mach. 8, 212-29 (1961).

¹⁸L. D. Enochson, Frequency Response Functions and Coherence Functions for Multiple-Input Linear Systems, NASA-CR-32 (April 1964).

34, were runs in which the test stage brine level was low and the flashdown relatively high, so it is likely that flashing occurred close to the orifice and affected the net flowrate. To arrive at reasonable solutions to Henry's equations for the negative square-root cases, the value of the contraction coefficient was reduced until the square-root argument went to zero. This occasionally resulted in some clearly unrealistic values of C_c and typically inconsistent values of computed flow. By arbitrarily setting the square-root term to zero, however, good consistent results were obtained. There is no apparent reason for this, however. The scatter in the resulting correlations is within $\pm 4\%$ for all tests.

The scatter for the AMF equation correlations was also small, $\pm 4\%$, for the no-baffle runs; this equation did not apply as well to the baffle runs where the scatter was about $\pm 20\%$.

The orifice coefficients for the conservation of kinetic energy Eq. (15) and the kinetic energy loss Eq. (18) typically showed more scatter than the other two did. The assumption that $t' = t$ in Eq. (15) was probably responsible for its poor showing. In Eq. (18) it is implied that the factor relating kinetic energy loss to the square of mass flowrate is constant, but actually it would depend somewhat on flashdown and downstream level. A simple such correlation, however, was not readily apparent from the data. More experimental and theoretical work on static flowrate correlations is in progress.

The analysis of the dynamic flow correlations will be limited to four of the five tests discussed in the transfer function analysis section (Sect. 3.3), the exception being the test at low temperature (runs 21-24). The data recorded for the brine level in stage 2 in this test contained about 18 sizable spurious transients, and these were sufficient to ruin the analysis. The form of each transient was typically a sudden decrease of about 2 in. in the recorded level (within about a second), followed by a gradual recovery of the recorded level to near the initial mean value (in about 5 min.). The probable causes of these transients have been determined from subsequent tests and are discussed in detail in Sect. 6.

Table 4. Comparison of Measured Brine Recirculation Flow W_R with Calculated Flows. Test Stage Inlet Orifice.

Run No.	Measured W_R (lb/sec)	Henry Eq. (14) W_H (lb/sec)	W_H/W_R	K.E. Eq. (15) C_d	AMF Eq. (16) W_{AMF} (lb/sec)	W_{AMF}/W_R	K.E. Loss Eq. (18) C (lb/sec)/(ft ² psi ^{0.5})
1	196	199.2	1.016	.752	176.8	.902	580.2
6	194	209.2*	1.078	.712	176.8	.911	561.7
16	132	133.0	1.007	.729	120.5	.913	562.2
25	129	141.0	1.093	.670	127.3	.987	515.9
26	127	129.2	1.017	.789	113.0	.890	612.6
27	195	198.6	1.018	.832	163.5	.838	664.3
34	144	154.0*	1.069	.766	122.4	.850	591.7
Baffle Runs:							
40	144	284.4*	1.975	.346	263.3	1.828	270.3
45	141	284.3*	2.016	.423	214.7	1.522	325.5
50	145	295.2*	2.036	.402	230.2	1.588	308.8
55	160	343.4	2.145	.415	261.5	1.634	325.5
60	192	389.4*	2.028	.456	283.2	1.475	358.2

*In these cases the quantity under the square root sign of Eq. (13) was negative, and the square-root term was arbitrarily set equal to zero.

Regarding the four remaining "good" tests, we emphasize that the quantitative results should at best be considered as tentative. Our measurements and test methods were set up to determine two unknown coefficients in an assumed single-phase-flow orifice equation. However, the data has shown that to rationally describe transient orifice flow behavior it must be assumed that voids are present in the orifice region. This requires the finding of six unknown coefficients. Other detractors from the credibility of the results are the uncertainty in the form of the revised model, the possible nonlinear effects due to changes in temperature and nucleation site conditions, and the ambiguities in the absolute d/p cell level measurements (as compared with the manometer measurements). Thus in order to obtain high quality results from low quality orifice tests, much more extensive and careful measurements would need to be made. Measurements at the orifice, such as local levels, temperature profiles, and void fractions, would be required.

Table 5 shows the experimentally determined flow coefficient values for the model described by Eq. (25). The average phase error values $|\overline{\epsilon}|$ indicate the differences between the measured phase of the inlet flow $\hat{W}_{TBi2}$ and the phase of the resultant $\hat{\Delta P}_{EFF}$ vector. While fairly good agreement is indicated by the phase-angle-matching criterion, it is the sensitivity of this criterion function to changes in parameters, $\frac{\partial |\overline{\epsilon}|}{\partial K_i}$, that indicates how accurately the parameters K_i

can be inferred. These sensitivity values are also shown in Table 5. For example, the sensitivities for the upstream-level static coefficients K_1 for runs 35-38 and runs 41-44 were 0.03 and 4.3, respectively. This means that the K_1 value for runs 41-44 is likely to be much more accurate than that for runs 35-38.

Another important point in interpreting Table 5 is that the inferred expression is linearized; i.e., the total steady-state flowrate cannot be calculated from steady-state values of the variables in the $\hat{\Delta P}_{EFF}$ expression, only the flow perturbations can be calculated from small changes in ΔP_{EFF} .

Table 5. Measured Coefficients for Dynamic Orifice Flow Model*

	Runs 7-10	Runs 35-38	Runs 41-44	Runs 46-49
	High Temperature	High Temperature	High Temperature	High Temperature
	High Flowrate	Low Flowrate	Low Flowrate	Low Flowrate
	Low Level	Low Level	Low Level	High Level
	No Baffle	No Baffle	Baffle In	Baffle In
Coefficients				
C	25	26	64	45
K ₁	4.0	2.6	2.3	2.0
K ₂	0.0	0.2	0.0	0.0
K ₃	1.9	2.7	-0.6	9.0
K ₄	6.9	0.0	2.3	1.3
K ₅	82	70	42	50
K ₆	0.0	0.0	0.0	0.0
Avg. Phase Error, ε , deg.	6	4	5	3
Sensitivities				
∂ ε /∂K ₁	0.25	0.03	4.3	1.6
∂ ε /∂K ₂	3.9	3.1	24	4.1
∂ ε /∂K ₃	0.17	0.16	0.31	0.3
∂ ε /∂K ₄	0.01	0.21	0.05	1.1
∂ ε /∂K ₅	0.02	0.01	0.26	0.09
∂ ε /∂K ₆	0.03	0.01	0.04	0.02

*Coefficients correspond to those in Eq. (25):

$$\hat{W}_{12} = C \left[\hat{\Delta P}_{V1-2} + K_1 \hat{P}_{H1} - K_2 \hat{\Delta T} - K_3 \hat{P}_{H2} - j\omega (K_4 \hat{\Delta T} - K_5 \hat{P}_{H1} - K_6 \hat{P}_{H2}) \right]$$

Significantly, there is an indication that the rate of change of stage 1 level has a very strong effect on $\hat{\Delta P}_{EFF}$, i.e., K_5 is large. This means that an increase in level at the rate of 1 psi/sec would, by suppressing voids in the "effective" orifice region (i.e. replacing the voids with liquid), cause the mass flow W_{i2} to be increased CK_5 lb/sec. Table 5 shows that the product CK_5 varies from 1800 lb/psi in runs 35-38 to 2700 lb/psi in runs 41-44. These large coefficients can be shown to be "possible", though perhaps not credible, by the following argument:

1. From the graph of the void fraction VF vs quality Q (Fig. 24) for $T = 141.5^\circ\text{F}$, $Q = 0.00001$, and $VF = 0.1$, the slope of the curve $\Delta VF/\Delta Q$ is 2500.
2. From steam tables, at 141.5°F the slope of $\Delta Q/\Delta P_{sat} = 0.0133 \text{ psi}^{-1}$.
3. The effect of a small change in saturation pressure on the void fraction $\Delta VF/\Delta P_{sat}$ is then $(2500)(0.0133) = 33.3 \text{ psi}^{-1}$.
4. Thus, if we arbitrarily assume that the effective orifice region volume is 1.3 ft^3 with the water density $= 61.4 \text{ lb/ft}^3$, the weight of liquid in the volume (when full) would be $(1.3)(61.4) = 80 \text{ lb}$. Therefore, the change in mass in the orifice with saturation pressure would be $(33.3 \text{ psi}^{-1})(80 \text{ lb}) = 2670 \text{ lb/psi}$, which is about equal to the value of CK_5 for runs 41-44.

The large possible effect of saturation temperature on void fraction, corresponding to the product CK_4 , can also be rationalized by a similar (more conservative) argument.

The flow equation model, with the coefficient values derived from runs 35-38, and the coefficient-seeking method were checked for internal consistency by using the digital simulation of the AMF plant.

The original intent of the orifice flow equation study was to derive relationships that could be used in calculations of the dynamic behavior of large MSF plants, where stage temperatures range from 100 to 250°F . The present AMF tests succeeded only in providing some questionable results at 140°F , and indicated the existence of a much more complex phenomenon than we had anticipated. Since the large-plant dynamics studies to date have shown that the orifice flow relationships are extremely crucial in determining the stability and operability of the plant, we are now planning more careful and extensive tests.

5. STATISTICAL ANALYSIS OF AMF TEMPERATURE FLUCTUATIONS

In a preliminary visit to the AMF evaporator, we had observed random fluctuations in the brine and vapor temperature signals of about $\pm 0.1^{\circ}\text{F}$, and had planned frequency response tests so that the signals would be able to overcome this much noise. On the first run of the dynamic tests, however, with the evaporator conditions at steady-state, we observed random temperature fluctuations as much as an order of magnitude greater, or about $\pm 1.0^{\circ}\text{F}$. Consequently, before each frequency-response test, we digitized the temperature, level, and flow signals (at 5 samples/sec) for periods of 5 to 10 min while the AMF operators were taking steady-state data. It turned out that the magnitude of the inherent temperature fluctuations varied from about $\pm 1.0^{\circ}\text{F}$ in tests with high brine levels to less than $\pm 0.1^{\circ}\text{F}$ with low brine levels.

- There are two ways in which statistical studies of these data can be useful to the analysis and understanding of MSF plant behavior:
1. In MSF plants equipped with automatic data loggers for monitoring stage temperatures and temperature profiles, there would be a problem in obtaining accurate mean or steady-state data by taking just a few samples at long intervals such as a minute or 100 sec. Since overall stage ΔT 's are in the order of 3 to 5°F , random variations of 2°F in a given signal mean that some type of filtering and smoothing would be called for. In Sect. 5.1, an analysis using AMF data is presented, along with recommendations and techniques for obtaining accurate steady-state temperatures from a logger.
 2. Although the mechanisms that cause the temperature fluctuations are not fully understood, certain factors that affect the characteristics of the noise can be deduced from studies of frequency spectra. Observations which may be pertinent to the understanding of the tray brine dynamics are given in Sect. 5.2.

5.1 Problems in Obtaining Accurate Steady-State Data Using Data Loggers

From observing the magnitude of the random fluctuations present in the AMF brine and vapor temperature signals, we believed that it would be difficult to consistently obtain accurate mean values of MSF stage temperatures by use of a data logger. In order to substantiate this, a statistical analysis was performed on the AMF stage 2 outlet brine temperature signal for various plant operating conditions. The analysis revealed that the confidence level (which we will arbitrarily define as the probability that the average value of a given number of samples will not differ from the actual mean value by more than 0.1°F) was occasionally quite poor, varying between 32.5% and 99.5% for the runs studied, using six samples of raw data. If the temperature signals were filtered and smoothed as recommended below, the confidence level would be between 98.8% and 100% for the same runs. While this accuracy problem would be present in any MSF plant with an automatic data logger, we have used as an example the particular case of the BLH three-stage unit at Wrightsville Beach and BLH proposed plan of operation.¹⁹

A 700-sec data sample showing the AMF evaporator inlet brine temperature (T_{TBI}), the outlet brine temperature of stage 1 (T_{TB1}), and the outlet brine temperature of stage 2 (T_{TB2}) taken from AMF run 1 is shown in Fig. 25. The fluctuations in the data are quite large, and it would be very unlikely that an accurate mean value for the brine temperature in stage 2 could be obtained from a small number of samples. In contrast to Fig. 25, Fig. 26 shows a 340-sec record of the same three temperatures for run 6, where the only significant difference in the operating conditions was lower brine levels.

To ensure accurate mean-value measurements, we low-pass filtered the temperature signal with a 50-sec time constant and obtained accurate results even in the worst case (i.e. run 1). For resistance thermometers with an effective resistance of 440 ohms, such as those used on the BLH evaporator, a 50-sec time constant can be obtained by placing a 115,000-mf low-voltage capacitor across the output of

¹⁹Design and Construction of the High-Flow Flash Devices Test Plant, Final Report to the Office of Saline Water, Oct. 1967, Baldwin-Lima-Hamilton Corp., Eddystone, Pa.

Fig. 25. Stage 1 Inlet, Stage 2 Inlet, and Stage 2 Outlet
Brine Temperatures for AMF Run 1.

Fig. 26. Stage 1 Inlet, Stage 2 Inlet, and Stage 2 Outlet
Brine Temperatures for AMF Run 6.

the bridge circuit. These capacitors are commercially available for about \$5.00 each. For thermocouples, the 50-sec time constant might be obtained by placing the thermocouple in a specially designed ungrounded sheath with a low-conductivity material surrounding the thermocouple.

In analyzing the AMF data for the steady-state runs, we noted that in some cases there was slow drift of the mean value of the temperature, as seen in Fig. 25. If several samples of the record were taken, e.g. at 1-min or 100-sec intervals, even with a 50-sec low-pass filter the mean temperature data could vary significantly over a 5- or 10-min period. To circumvent this problem, the method used by the AMF personnel²⁰ could readily be adapted to the data logger. Periodically during the sampling of the stage temperatures, they take a reading of a reference temperature which (by experience) they found accurately represents the slow temperature drift of the plant. Intermediate readings of stage temperature are corrected by subtracting the drift term. In the BLH operation, this would mean that the temperature drift would be approximated by a straight line during 100-sec intervals (the time required for a complete set of samples to be logged). Based on typical drift rates observed in the AMF tests, this approximation was found to be sufficiently accurate. Removal of the drift errors from the data is referred to as "smoothing".

The statistical analyses performed on the AMF data were based on the assumption that the signals were ergodic. However, this was not a good assumption when the mean value of the temperature probability distribution changed significantly with time. Since we had no recording of an accurate "drift" temperature, we smoothed the AMF data by subtracting a calculated regression line from the measured data.

The regression line is determined by fitting the data with a straight line, using a minimum mean-square error criterion. The equation for the straight, or regression, line is

²⁰Personal communication from Fred W. Gilbert, AMF Corp., Millstone Point, Conn.

$$T_{RL}(t) = C + Bt, \quad (26)$$

where

T_{RL} = value of regression line, °F,

t = time in sec,

C = intercept of regression line at t equal to zero, °F,

B = slope of regression line, or the drift in data, °F/sec.

Since the data are obtained by sampling at constant time intervals, the mean square error is expressed as a summation, as indicated in Eq. (27):

$$MSE = \sum_{n=0}^N [T(n\Delta t) - C - Bn\Delta t]^2, \quad (27)$$

where

MSE = mean square error, (°F)²,

$T(n\Delta t)$ = value of temperature at time $n\Delta t$, °F,

Δt = sample interval (0.2 sec),

n = an integer equal to the sample number,

N = total number of samples in the record.

The values for C and B are determined by taking the derivative of the mean square error with respect to C and B and setting them equal to zero:

$$\frac{\partial MSE}{\partial C} = \sum_{n=0}^N (-2) [T(n\Delta t) - C - Bn\Delta t] = 0, \quad (28)$$

and

$$\frac{\partial MSE}{\partial B} = \sum_{n=0}^N (-2n\Delta t) [T(n\Delta t) - C - Bn\Delta t] = 0. \quad (29)$$

Equations (28) and (29) are evaluated to find the values of C and B , giving

$$C = \frac{\left[\sum_{n=0}^N T(n\Delta t) \right] \left[\sum_{n=0}^N (n\Delta t)^2 \right] - \left[\sum_{n=0}^N n\Delta t T(n\Delta t) \right] \left[\sum_{n=0}^N n\Delta t \right]}{[N + 1] \left[\sum_{n=0}^N (n\Delta t)^2 \right] - \left[\sum_{n=0}^N n\Delta t \right]^2} \quad (30)$$

and

$$B = \frac{[N + 1] \left[\sum_{n=0}^N n\Delta t T(n\Delta t) \right] - \left[\sum_{n=0}^N n\Delta t \right] \left[\sum_{n=0}^N T(n\Delta t) \right]}{[N + 1] \left[\sum_{n=0}^N (n\Delta t)^2 \right] - \left[\sum_{n=0}^N n\Delta t \right]^2} \quad (31)$$

The probability distribution of the smoothed data is assumed to be Gaussian, and hence is completely determined by two parameters, the mean value and the variance. The mean value is estimated from the average value for all the temperature samples taken during a given run:

$$\bar{T} = \frac{1}{N + 1} \sum_{n=0}^N T(n\Delta t), \quad (32)$$

where

$\bar{T}$ = mean value of the brine temperature, °F.

The variance σ^2 is estimated from the average squared deviation of the samples from the mean value:

$$\sigma^2 = \frac{1}{N + 1} \sum_{n=0}^N [T(n\Delta t) - \bar{T}]^2. \quad (33)$$

The confidence level is determined by the error function, since the probability distribution is Gaussian as:

$$\begin{aligned}
 P (|T_{\text{est}} - \bar{T}| < a) &= \frac{1}{2\pi} \int_{-\sqrt{M} \frac{a}{\sigma}}^{\sqrt{M} \frac{a}{\sigma}} e^{-\frac{x^2}{2}} dx , \\
 &= \Phi_M \left(\sqrt{M} \frac{a}{\sigma} \right) , \quad (34)
 \end{aligned}$$

where

M = number of samples used in estimating the mean value,

a = tolerance placed on estimate, 0.1°F ,

σ = square root of the variance,

$\Phi_M \left(\sqrt{M} \frac{a}{\sigma} \right)$ = error function, or confidence level, for M samples,

T_{est} = estimated mean value using M samples.

The analysis was performed by use of Eqs. (26) - (34) and as indicated by Fig. 27. These calculations were performed for the case of the total record and for 100-sec samples from the data record for some of the AMF steady-state runs. The results shown in Table 6 were obtained by calculating the average confidence level for all the 100-sec samples in the record. The results clearly indicate that to obtain accurate steady-state data with a high confidence level using a data-logger, filtering the temperature signal with a low-pass network prior to sampling would be required. It is also apparent that only two samples of the filtered data would be required to obtain accurate results; hence, the amount of data collected and plant operating time could be reduced accordingly.

5.2 Interpretation of Power Spectral Density Characteristics

The power spectral density (PSD) describes the statistical behavior of fluctuating data in the frequency domain. The PSD indicates the distribution of the mean-square value (M.S.) of the fluctuations with respect to frequency. For example, the mean-square value of fluctuations in the frequency range from ω_1 to ω_2 is equal to the integral of the PSD with respect to frequency from ω_1 to ω_2 , as shown by the following equations:

FIG. 27. Block Diagram of the Statistical Analysis.

TABLE 6. Confidence Levels for Mean Value of Stage 2 Brine Outlet Temperature^a

AMF Run No.	Mean Value (°F)	Stage 2 Brine Level (in.)	Brine Flowrate (lb/hr)	Confidence Levels		
				Raw Data	Filtered Data	
				$\bar{\Phi}_6(X)$ (%)	$\bar{\Phi}_6(FX)$ (%)	$\bar{\Phi}_2(FX)$ (%)
1	142.9	24	700,000	32.53	99.99	98.59
6	144.5	14	700,000	99.53	99.91	97.33
25	106.4	21	460,000	66.92	100	99.90
26	113.4	16	460,000	86.32	99.86	97.58
27	109.9	19	700,000	77.87	100	99.56
34	141.0	14	520,000	99.07	100	100

^aSix samples, each, of raw and filtered data and two samples of filtered data were used.

$$\Delta\omega = \omega_1 - \omega_2 , \quad (35)$$

$$\text{M.S.}(\Delta\omega) = \int_{\omega_1}^{\omega_2} \text{PSD } d\omega. \quad (36)$$

By using the PSD along with one's knowledge about the types of parameters affecting the dynamic response of the system (i.e. brine flow rate, brine temperature, brine level, etc.), some conclusions can often be obtained about how input signals are transmitted through the system. Such conclusions are based on the assumption that the system response is linear over the range of the fluctuations. In attempting to interpret the PSD of the test stage outlet brine temperature (measured at a point 2-in. upstream from the exit orifice and 3-in. from the tray bottom), it is helpful to have a general idea of the possible mechanisms that can produce fluctuations in brine temperature.

A thermocouple, when measuring temperature in a moving liquid, senses only the temperature of the liquid in the small adjacent region. Temperature gradients and changing flow patterns cause the thermocouple output to fluctuate, because liquid at different temperatures is being moved past the thermocouple. Determining the cause of fluctuations is further complicated by the presence of entrained vapor having a slightly higher temperature, and the unknown effect of the vapor on the brine temperature and on the flow patterns in the region around the thermocouple. Relatively large steady-state temperature gradients in the brine were noted in an AMF report,²¹ and changing flow patterns were readily apparent. From the above discussion, it is apparent that the mechanisms affecting the dynamic response of local temperatures in the system are not well known. However, by examining the PSD

²¹F. W. Gilbert, Summary Report on Non-Equilibrium Allowances and Interstage Flows at Low and High Temperatures, AMF Corp., Millstone Point, Conn. (Oct. 1966).

for various steady-state operating conditions and noting trends in the PSD characteristics with changes in brine level, flowrate, and temperature, we are in a better position to speculate about how the parameters affect the dynamic response of local temperatures. The method used to calculate the PSD's is described in the Appendix, Sect. 8.2.

PSD plots were obtained for several AMF steady-state runs (Fig. 28-36). The parameters which were varied in these runs were tray-brine inlet temperature, flowrate, and the level in stage 2. The PSD plots in Fig. 34-36 are for runs in which the 12-in. high baffle was placed 2 ft downstream from the stage 2 inlet orifice. The trends or changes in PSD were determined by comparing the PSD's from two similar runs in which only one of the three input parameters was significantly different, or where the only difference was the presence of the baffle.

The effect of changing brine level in stage 2 is shown in Figs. 28-31. First, Fig. 30 and 31, show the effect of dropping the level (5.5 in.) while a low inlet temperature and flowrate are maintained. In this comparison, each PSD has three peaks: for the low level case the peak frequencies are 0.06, 0.15, and 0.33 radian/sec; and for the high level case the frequencies are 0.13, 0.42, and 0.8 radian/sec. The mean-square value of the fluctuations over most of the frequency range, as well as the peaks, is greater by about a factor of 4 for the high level case. Figures 28 and 29 show the effect of changing the level while a high inlet temperature and flow rate are maintained. For the low level case there are three peaks, at frequencies of 0.05, 0.15, and .25 radian/sec; for the high level case there are only two peaks, at 0.08 and 0.8 radian/sec. The mean-square value of the fluctuations at each peak is again greater for the high level case, by a factor of 8 or 10. The magnitude of the changes in PSD's for different brine levels indicates that the fluctuations are very sensitive to level. This general, very striking, dependence of brine level on PSD can be seen in Fig. 37, which summarizes Figs. 28-33. The six runs were made without the flow baffle but at a variety of temperatures and flowrates; yet the gross dependence of the high-frequency

Figs. 28-30. PSD's of Test Stage Tray Brine Outlet Temperature Fluctuations.

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FIG. 31.
RUN 26

FIG. 32.
RUN 27

FIG. 33.
RUN 34

Figs. 31-33. PSD's of Test Stage Tray Brine Outlet Temperature Fluctuations.

FIG. 34.
RUN 40

FIG. 35.
RUN 45

FIG. 36.
RUN 60

Figs. 34-36. PSD's of Test Stage Tray Brine Outlet Temperature Fluctuations.

Fig. 37. PSD of Test Stage Tray Brine Outlet Temperature Fluctuations as a Function of Brine Level. No Baffle.

part of the curve on the brine level would almost permit the PSD to serve as a level indication. Clearly, then, increasing the brine level causes both the amplitude and the frequency of the outlet temperature fluctuations to increase. There are probably two factors that contribute to this behavior. First, when no baffle is present, there is a tendency for the hot inlet brine to short circuit the stage along the bottom of the tray (as noted in the transfer function description, Sect. 3.3). With deeper brine, this hotter jet would be farther from the surface and, hence, less likely to flash. Therefore, a greater temperature difference between surface brine and brine at the bottom could be sustained, and the extremes in brine temperature leaving the stage would be widened. Thus, the temperature gradient would be dependent on the saturation conditions as a function of depth:

$$T_{\text{gradient}} \sim \left. \frac{\partial T}{\partial h} \right|_{\text{sat}}, \quad (37)$$

where

$$\left. \frac{\partial T}{\partial h} \right|_{\text{sat}} = \text{variation of saturation temperature with liquid height, } ^\circ\text{F/in. } (\sim 1.0 \text{ at } 100^\circ\text{F}, \sim 0.5 \text{ at } 140^\circ\text{F}).$$

A second factor affecting the level-dependent fluctuations is that the height of surface waves increases at higher brine levels, which would cause a corresponding increase in the fluctuation of the vertical component of flow near the exit. This would tend to direct the flow streamlines from brine at different elevations up and down across the thermocouple. While the brine level is clearly the dominant factor, it also appears that effects of changing the brine temperature and flowrates can be seen from a closer inspection of the results.

The effect of inlet brine temperature on PSD can be inferred by examining Figs. 31 and 33, which show the PSD for low and high inlet brine temperatures, respectively, with both flowrate and level at low values. In each PSD curve three peaks are noted. In Fig. 31, for a low temperature run, the peaks are at 0.06, 0.15, and 0.33 radian/sec; and in Fig. 33, for the high temperature run, the peaks are at 0.06, 0.13, and 0.27 radian/sec. The mean-square value of fluctuations at

each peak is less for the high temperature case. One interpretation of this result is that the inlet temperature apparently affects the amplitude of the fluctuation; that is, when the inlet temperature was increased, the amplitude of the fluctuations decreased. It should be noted, however, that there was a difference of 2 in. in brine level for these two runs, which tends to reduce one's confidence in the validity of the interpretation. Possible reasons for this trend in PSD behavior is that the term $\partial T / \partial h$ shown in Eq. (37) is larger at lower temperatures, which indicates a tendency for higher gradients to occur at lower temperatures.

The effect of brine flow rate on PSD is seen by comparing Figs. 30 and 32; however, a 2.5-in. difference in brine level in the two runs might mask part of the flowrate effect. In the brine level comparison, higher levels apparently caused larger fluctuations with higher frequencies. The mean-square values of the fluctuations in Fig. 30 are higher than those in Fig. 32, which is opposite to the observation in the level comparison. One interpretation of this observation is that increasing the brine flow rate increases the frequency of fluctuations because there is a corresponding increase in the rate of change in flow patterns around the thermocouple.

The effect of the baffle on PSD is seen by comparing Figs. 28 and 36 and Figs. 34 and 35. The brine inlet temperature, flowrate, and level are approximately the same value in Figs. 28 and 36. Figure 36 shows the PSD for a run in which a 12-in. vertical baffle was installed 2 ft downstream from the inlet orifice, and Fig. 28 shows the PSD for a run made without a baffle. The mean-square value of the fluctuations for the run with the baffle is smaller by a factor of approximately 100 than for the run without the baffle. The PSD's shown in Figs. 34 and 35 are both with the baffle installed and with a difference of 9 in. in level. Comparing the PSD's, it is seen that there is little difference between them. This indicates that the baffle essentially eliminates the effect of level on the fluctuations, possibly because the hotter brine which tends to flow along the bottom of the stage is disrupted by the baffle and reduces the vertical temperature gradients downstream from the baffle.

Quantitatively, the effects on stage efficiency of large temperature fluctuations at the outlet are not immediately apparent. However, intuitively, it is clear that, if significant fractions of the brine leaves the stage a degree or so hotter than the stage's equilibrium saturation temperature, some efficiency is lost. The PSD is perhaps a rather unique indication of the disadvantage of deep brine.

6. DISCUSSION OF DIFFERENTIAL-PRESSURE MEASUREMENT PROBLEMS

Three types of information were transmitted to the data logger by means of differential-pressure (d/p) cells: recirculation brine flowrate, two tray brine levels, and interstage vapor differential pressure. Only the flow signal gave consistent, unambiguous results which agreed with manometer readings from AMF's standard instrumentation. Considerable discrepancies were occasionally noted when the brine level and vapor d/p cell outputs at steady state were compared with readings from water manometers connected in parallel with the d/p cells. Repeated attempts at calibrating the d/p cells and then reconnecting them back into the operating evaporator failed to reveal the sources of the problem. During the runs, it did appear that the d/p cells and manometers agreed on the changes in ΔP , indicating that the errors were bias rather than gain errors. Constant bias errors would not affect the frequency response analyses, which consider only perturbations from steady state.

Some spurious transients were also observed in the level signals, particularly during the low-temperature runs ($\sim 100^\circ\text{F}$). Typically there were large, sudden decreases in indicated level, followed by a gradual return over a period of 10 to 20 min to near the original level indication.

Subsequent tests on a two-stage deaerator at ORNL were run using a mockup of the AMF d/p cell setup for level measurement. Anomalous transient behavior similar to that encountered in the AMF tests was observed during programmed level transients at high vacuum conditions and near-saturation temperatures. Sudden decreases in the indicated level were conclusively attributed to a sudden flashing of liquid from the bottom (high pressure) leg of the d/p cell connection

tube, explained as follows: When a bubble first forms at the top of the tube there is less pressure on the liquid in the tube. Hence there is a greater tendency for this liquid to flash, so the emptying of the tube is rapid. Natural refilling of the 1/4-in.-diam tube with liquid is quite slow (~ minutes) due to strong surface tension effects. During this refilling period, however, the d/p transmitter continues to respond to level variations. In the AMF tests, the runs at low temperature indicated 10 to 20 such transients, compared with 0 or 1 transient in runs at higher temperatures. Data with such transients were not used in the analyses presented in Sect. 3-5.

The low-pressure taps on the level d/p cells and both taps on the interstage vapor d/p cell were connected to nominally dry or evacuated legs. Although the evidence is by no means conclusive, it appears that the steady-state bias errors were due to vapor condensing in the 1/4-in.-diam tubing and "hanging up" there, rather than draining out, again due to significant surface tension forces. If a column of water were formed in this way, it would cause an error in the indicated pressure. The possibility of this effect is easily verified.

Since the problems with both the transient and bias errors were aggravated by the strong surface tension effects in small-bore tubing, larger, 1/2-in. copper tubing was also tested in the level mockup. Under certain conditions, the liquid in the tube could be made to flash. Although the tube refilled and the level signal returned to normal almost instantaneously, such spurious signals would not be desirable. We believe that a flush-diaphragm d/p cell connected directly to the evaporator would be required for satisfactory performance.

7. CONCLUSIONS AND FUTURE PLANS

Results from analyses of single-stage transfer functions and orifice flow equations, while they were generally quite informative and interesting, did not provide all of the conclusive, quantitative data required for accurate predictions of large MSF plant dynamics. Improved models for the vaporization rate and tray brine flow dynamics were determined, and strong indications of the importance of void formation in the interstage orifice flow characteristics were noted. Many measurement difficulties encountered in this first test of evaporator dynamics were resolved, and problems with the acquisition of data, such as stage temperature-to-flowrate transfer functions, were recognized. The characteristics of the inherent temperature fluctuations that need to be coped with are also better understood.

Based on this experience, we are planning to run similar tests on the BLH-OSW 3-stage flash evaporator at Wrightsville Beach, N. C. More extensive orifice-region measurements, including local temperature, pressure, and void fraction, will be made. Other transfer function information relating temperatures, vapor pressures, and flows will also be obtained. We expect that the data obtained from both the AMF and BLH evaporator tests will be sufficient to make significant improvements in the accuracy of predicting large MSF plant transient behavior.

8. APPENDIX

8.1 AMF MATEXP Equations

8.1.1 Tray Brine Mass Balance:

$$\frac{dM_{TB}}{dt} = W_{TBi} - W_{TBo} - W_{TBF} (+ W_{FEED} - W_{BLEED})^* .$$

*This term is included for M_{TB3} only: $W_{BLEED} = 0.01 (M_{TB3} - M_{TBo})$.

8.1.2 Tray Brine Interstage Flowrate:

$$\text{Linear: } W_{TBo} = W_{BIAS} + K1 \Delta P ,$$

$$K1 = \frac{W_0}{2\Delta P_0} ,$$

$$W_{BIAS} = W_0 - K1 \Delta P_0 .$$

$$\text{Nonlinear: } W_{TBo} = XW1 (\Delta P)^{0.5} .$$

8.1.3 Effective Pressure Drop for Interstage Brine Flowrate:

$$\Delta P_{1-2} = P_{V1} - P_{V2} + \frac{M_{TB1}}{AFC_1} - K2_1 \frac{M_{TB2}}{AFC_2} ,$$

$$\Delta P_{2-3} = P_{V2} - P_{V3} + \frac{M_{TB2}}{AFC_2} - K2_2 \frac{M_{TB3}}{AFC_3} - XM3B .$$

8.1.4 Flashing Brine Flowrate:

$$W_{TBF} = 0.02 \text{ GF } (\bar{T}_{TB} - T_V) ,$$

where 0.02 is a scale factor to correspond to analog gains.

8.1.5 Cell Vapor Mass Balance:

$$\frac{dM_{CV}}{dt} = W_{TBF} - W_{COND} ,$$

$$W_{COND} = \frac{h_{oo} A_o}{h_{fg}} (T_V - \bar{T}_{TUBE}) .$$

8.1.6 Tray Brine Heat Balance:

$$\frac{d\bar{T}_{TB}}{dt} = \frac{W_{TBi}}{M_{TB}} (T_{TBi} - \bar{T}_{TB}) - \frac{W_{TBF} h_{fg}}{M_{TB} C_{PTB}} .$$

8.1.7 Tube Flow Heat Balance (Condensers and Steam Heater):

$$\frac{d\bar{T}_T}{dt} = \frac{2W}{M} (T_i - \bar{T}_T) + \frac{h_i A_i}{(MC_P)_T} (\bar{T}_{TUBE} - \bar{T}_T) .$$

$$\frac{dT_{To}}{dt} = \frac{2W}{M} (\bar{T}_T - T_{To}) + \frac{h_i A_i}{(MC_P)_T} (\bar{T}_{TUBE} - \bar{T}_T) .$$

$$\frac{d\bar{T}_{TUBE}}{dt} = \frac{h_i A_i}{(MC_P)_{TUBE}} (\bar{T}_T - \bar{T}_{TUBE}) + \frac{h_o A_o}{(MC_P)_{TUBE}} (T_S - \bar{T}_{TUBE}) .$$

8.1.8 Holdup Equations (Inlet Plenum and Cooling-Loop Holdup):

$$\frac{d\bar{T}}{dt} = \frac{W}{M} (T_i - \bar{T}) .$$

8.1.9 Transport Lags:

$$T_o(t) = T_i(t - \tau),$$

$$\tau = \text{Holdup Time, sec.} = \frac{\rho_T V_{\text{HOLDUP}}}{W_{BR}} .$$

8.1.1.10 Nomenclature:

A	surface area, ft^2
AFC	base area of flow channel, ft^2
C_p	specific heat, $\text{Btu/lb-}^\circ\text{F}$
GF	flashing gain factor, $[(\text{lb/sec})/^\circ\text{F}] \times 50$
h	heat transfer coefficient, $\text{Btu sec}^{-1} \text{ft}^{-2} \text{ }^\circ\text{F}^{-1}$
h_{fg}	heat of vaporization, Btu/lb
K	constant
M	mass, lb
P	pressure, psi
T	temperature, $^\circ\text{F}$
t	time, sec
τ	holdup time, sec
V	volume, ft^3
W	mass flowrate, lb/sec
XM3B	bias term accounting for curvature of stage 3 sides, psi
XW1	flow coefficient, $(\text{lb/sec})/(\text{psi})^{0.5}$
ρ	density, lb/ft^3

Subscripts:

COND	condensate
CV	cell vapor
F	flashing
i	inside, inlet
o	outside, outlet
S	steam
T	water in tubes
TB	tray brine
V	vapor
0	initial or steady-state value
1	stage 1
2	stage 2
3	stage 3

8.2 Power Spectral Density Calculations

The power spectral density (PSD) of the fluctuating temperature signal is calculated by using the digitized noise signal as the input, or forcing function, to differential equations representing a narrow bandpass filter, and the resulting output of the filter is squared and averaged. This is known as the direct method,²² as opposed to the indirect method which involves calculation of the autocorrelation function and subsequent Fourier transformation. The matrix exponential technique⁷ is used to solve for the transient response of the filter, which has the characteristics of a quadratic lag and a transfer function

$$H(j\omega) = \frac{j\omega}{\omega_o^2 + 2 \xi \omega_o j\omega - \omega^2} . \quad (38)$$

The center, or resonant, frequency is ω_o (radians/sec), and the band width increases with increasing damping factor ξ .

The PSD of the signal $F(j\omega)$ in (units)²/(radian/sec) is related to the mean-square filter output $\overline{q^2}$ by²³

$$\overline{q^2} = F(j\omega) \int_0^{\infty} |H(j\omega)|^2 d\omega , \quad (39)$$

if it is assumed that the PSD is constant within the bandwidth of the filter. For this filter,

$$\int_0^{\infty} |H(j\omega)|^2 d\omega = \frac{\pi}{4\xi\omega_o} . \quad (40)$$

Equation (39) gives the PSD in an engineering rather than in a mathematical sense.²⁴ Some prefer to use $(-\infty)$ as the lower limit of integration, which implies the presence of a passband in the negative

²²R. B. Blackman and J. W. Tukey, The Measurement of Power Spectra, Dover, New York, 1958, pp. 25-28.

²³J. H. Laning and R. H. Battin, Random Processes in Automatic Control, McGraw-Hill, New York, 1956, pp. 194-201.

²⁴S. S. L. Chang, Synthesis of Optimum Control Systems, McGraw-Hill, New York, 1961, pp. 46-47.

frequency region. This approach would result in doubling the value of the integral of Eq. (40) and halving the calculated PSD.

Calculations of the percentage standard deviations (% SD) of the PSD estimates are also made using Eq. (41):

$$\% \text{ SD} \equiv \frac{100\sigma}{\overline{X^2}} = \frac{100}{\sqrt{BT}} , \quad (41)$$

where

σ = standard deviation of mean-square value,

$\overline{X^2}$ = mean-square value,

B = equivalent noise bandwidth, radians/sec,

T = integration time, sec.

The equivalent noise bandwidth²⁵ for $H(j\omega)$ is

$$B \text{ (radians/sec)} = \pi \xi \omega_0 . \quad (42)$$

Compensation is also provided for the low-pass filtering that the true temperature fluctuation undergoes before being digitized. This filtering, or smoothing, is done both by the response-time lag of the sheathed thermocouples and the low-pass filter network installed in the digitizer input system to prevent aliasing. The effective transfer function $G(j\omega)$ of the thermocouple plus the filter was measured and found to be well approximated by

$$G(j\omega) \approx \frac{1}{(j\omega T_1 + 1)(j\omega T_2 + 1)} . \quad (43)$$

For the 1/16 in. sheathed thermocouples (used on the probe), T_1 and T_2 were as follows: $T_1 = 0.6$ and $T_2 = 0.2$; for the 1/8 in. thermocouples, $T_1 = 1.0$ and $T_2 = 0.2$. The corrected PSD $F_c(j\omega)$ is related to the measured PSD $F(j\omega)$ by

$$F_c(j\omega) = \frac{F(j\omega)}{|G(j\omega)|^2} . \quad (44)$$

²⁵T. J. Karras, Equivalent Noise Bandwidth from Transfer Functions, NASA, TN-D-2842 (Nov. 1965).

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