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PPL Report #116

THE ELECTROMAGNETIC PINCH EFFECT
FOR SPACE PROPULSION

by

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Plasma Propulsion Laboratory
Republic Aviation Corporation

August, 1959

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ABSTRACT

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The phenomenon of the electromagnetic pinch effect is used to accelerate ionized gases for space propulsion. Electrical energy, initially stored in capacitors, is discharged across two nozzle shaped electrodes wherein the radial pinch is converted to axial motion of the effected gases instead of confinement at the axis.

The gas dynamics of a pinch using the hydrodynamical model of a magnetic piston driving a shock wave, is combined with the electrodynamics of the circuit to calculate the ^{discharge} behavior, ~~the discharge.~~

Experiments on three different electrode designs are discussed and ~~the~~ results ~~obtained are~~ compared with ~~the~~ calculated values.

The results of ~~the study~~ are applied to one particular space propulsion system consisting of a nuclear energy source, a space radiator, a turbine-generator, capacitor, and a pinch tube. The specific mission analyzed is a one-way unmanned flight to a Mars orbit, starting from an Earth orbit.

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ANALYSIS OF THE ELECTROMAGNETIC PINCH

The Pinch Effect for Propulsion: A great deal of experimental work has been done in recent years on linear or straight discharges of large currents in low pressure gases (Figure 1). Capacitors are discharged across a cylindrical column of gas between two electrodes. Upon initiation of the discharge, the rapidly increasing current results in the well-known skin effect; the current in the gas is confined to a thin cylindrical shell at the outer radius of the gas column. The associated magnetic field which is confined entirely between this shell and the return current path, reacts with the axial current in the gas to constrict the gaseous column to a thin tube at the axis of the electrodes. This type of linear or straight discharge apparatus has been called a "Columbus" tube in this country.

The first published experiments of linear discharges of appreciable power were reported by the Russians in 1956.^{1,2} Electrodes 8 to 16 inches in diameter spaced 25 to 40 inches apart were used with initial voltage differentials of 20 to 50 KV. Peak currents of 1 million amperes were reached in periods of time ranging from 5 to 17 microsecnds. Initial gas pressures of .005 millimeters of mercury to one atmosphere have been reported. Recent reports in this country^{3,4} of experiments on the Columbus II and Columbus S-4 devices quote electrodes of 5" diameter, up to 24 inches apart. Capabilities of operating with 100 KV and peak currents of 2.5 million amperes have been reported but the test data indicates lower values.

Several models of the magneto-gas-dynamical behavior of the pinch effect have been proposed. Rosenbluth⁵ mentions the "snow-plow", the "free particle" and the usual hydrodynamic model of a magnetic piston driving a shock wave. Because the initial pressure of the gas considered in this paper is relatively high, and because more detailed information is desired, we limit the study in this paper to the hydrodynamic or shock model.

Since the magnetic field is confined entirely outside an almost infinitely conducting plasma in the skin, the charged particles within the plasma will be deflected from the magnetic field-plasma interface, and will not be able to penetrate it. Under such circumstances the skin will act like a solid piston, compressing the plasma into the center. If the plasma is not completely ionized and contains a large percentage of neutral atoms, it is quite possible that a number of these may escape without being ionized at the skin. In this case the piston would be porous to some degree. The skin will herein be likened to a non-porous "magnetic piston". The gasdynamical behavior will be described by a rapidly moving and contracting piston, driving a strong shock wave ahead of it into the center.

Three stages of a pinch are shown in Figure 2 illustrating how the magnetic piston and the shock wave move progressively inward. A complicated flow mechanism would occur as the inward moving shock wave would reflect from the axis of the electrodes and travel radially outward until it met the magnetic-piston moving radially inward. As long as the magnetic-piston continued to drive inward, cylindrical shock waves would travel in and out between the axis and the

piston, raising the pressure, temperature, and entropy of the gas with each pass. When the magnetic-piston pressure equaled the internal gas pressure, the motion would stop and the gas temperature would be a maximum value. All of the directed kinetic energy would be converted to thermal energy with considerable increase in entropy. Unlike the linear pinch, however, where the objective is to convert the electrical energy into thermal energy, the present objective is to design suitably shaped electrodes to permit the axial discharge of the pinch-gases with the maximum conversion of electrical to mechanical energy. In the linear case just described, if a hole were cut in one electrode, the gas would ultimately be discharged through the nozzle. This would be an extremely inefficient process.

An improved version of electrode design is shown in Figure 3. The sketches on the right indicate how the shock wave should travel when it is permitted to expand out of the nozzle. This design would practically eliminate shock reflections at the axis of the electrodes and would permit the transformation of radial kinetic energy to axial kinetic energy with the minimum increase in entropy. For the same amount of electric energy expended, the kinetic energy of the exhaust jet obtained in this manner would be much greater than for the plane cylindrical tube. Obviously very careful analysis is required for the proper design of electrodes and nozzle. Such configurations will influence the electrical characteristics, (such as initiation and profile of the discharge and its stability), the shock profile and also cause the possible formation of subsidiary shocks in the flow.

Three types of electrodes have been used for experimentation and they are illustrated in figure 4. The cylindrically symmetrical electrodes (figure 4a) are similar to the linear discharge

electrodes which have been the subject of much experimentation as indicated previously. The "one dimensional nozzle-electrodes" (figure 4b) are designed with a constant cross-sectional flow area throughout the annular path. The width of the annular path is considered small enough compared to the average radius of curvature of the path so that one dimensional flow analysis can be applied. The nozzle-electrodes of figure 4c were designed to permit the maximum length of radial travel before turning the driven gases in an axial direction. The results obtained with each of these electrodes will be considered after the general analysis applied to all three designs is described.

Electrical Circuit Analysis: Due to the contracting plasma between the electrodes a variable inductance is introduced into the electrical circuit. The energy expended per unit time by the condenser goes into an increase of the magnetic field energy, dissipation as heat, and work on the plasma. Neglecting the resistive and capacitive effects of the electrodes and the plasma resistance, the energy form of the circuit equation (see reference 6) can be written as

$$-\frac{d}{dt}\left(\frac{q^2}{2C}\right) = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) + Ri^2 + V_e i$$

or with $i = -\frac{dq}{dt}$ we get

$$\frac{q}{C} + L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + L_p \frac{d^2 q}{dt^2} + \frac{dq}{dt} \frac{dL_p}{dt} = 0$$

Preliminary calculations have shown that for the present experimental results the shock and "magnetic piston" velocities can be obtained with sufficient accuracy for the studies herein by neglecting the inductance and rate of change of inductance of the electrodes, and the resistance of the external circuit. In estimating the efficiency of the system (defined as the kinetic energy of gas leaving pinch tube divided by the electrical energy extracted from the condenser in the same time), the external resistance, plasma inductance and external inductance are all extremely important. The electrical circuit equation can then be written independently of the gas dynamical behavior and reduces to-- $L \frac{dq}{dt} + \frac{q}{C} = 0$

--the solution of which is

$$q = q_0 \cos \omega t$$

where q_0 is the initial capacitor charge

and $\omega = \sqrt{\frac{1}{LC}}$;

differentiating and with $i = -\frac{dq}{dt}$ we get

$$i = + q_0 \omega \sin \omega t \quad (1)$$

To understand why the plasma inductance effects can be neglected it is interesting to consider the value, η , where η is the amount of electrical energy from the condenser converted into thermal and kinetic energy of the gas. The value of η can be shown to depend upon the value of the damping factor $\zeta = R/2\omega L$, and the ratio of the maximum electrode inductance to the external inductance, $\alpha \approx \mu l / 2\pi L$ (Reference 7). The larger the value of α and the smaller the value of ζ , the more effectively can condenser energy be converted into mechanical energy (Figure 5).

It is apparent that for optimum efficiency we require both small R/L and small L . Coupling between electrodynamic and gas dynamic equations would then become important. For the present experiments we are operating inefficiently and in the range indicated in Figure 5. Therefore it has been possible to neglect the relatively small plasma inductance.

Gas Dynamical Analysis: It is found that at low initial densities there is a sharp delineation of the skin (current carrying plasma), whereas at higher densities the front suggests diffusion rather than the sharp discontinuity. Reference 8 indicates that the measured skin thickness varies from 1 to 2.5 cms at initial gas pressures from 0.1 to 10 mm Hg.

In the present analysis the assumption will be made that there is no escape of gas through the piston and the full volume of gas is compressed by piston.

Theoretically when the piston moves at a speed greater than five times the initial speed of sound, the gas behind is unable to follow it. A vacuum is created behind the piston. In actual fact we have evaporation of electrode material and quantities of adsorbed gas are stripped continually from the surface of the metals present. This in conjunction with diffusion through the piston due to the finite conductivity in skin, makes it possible to have a number of pinches

FIGURE 5. The efficiency of conversion of electrical energy into magnetic work per pinch (linear case).

as is evidenced by experiment, even after the first pinch when solid piston theory would indicate that all the gas should have been expelled.

The following additional assumptions will be made:

(1) The Rankine-Hugoniot equations are applicable and are simplified by assuming strong shock conditions, even at the commencement of the pinch.

(2) Ideal gas equations are applicable even though the temperatures reach values for which real gas effects become important. Fortunately these effects on pressure and velocity are negligible.

(3) A continuum gas dynamical analysis is considered throughout.

(4) The entire process is considered quasistatic so that at any time the conditions of the gas between the "magnetic piston" and the shock wave are uniform. (How valid such an approximation may be will only be apparent from actual experimental data and a more exact analysis.)

The state equations for a stationary strong shock in an ideal gas are -

$$P_2/P_1 = \frac{2\gamma}{\gamma+1} M_1^2 \quad (2)$$

$$T_2/T_1 = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} M_1^2 \quad (3)$$

$$\rho_2/\rho_1 = \frac{\gamma+1}{\gamma-1} \quad (4)$$

The value of γ for calculations of shock and piston velocities and for pressure, temperature and density ratios is the value ahead of the shock. Comparison of the results of this simplification with the results obtained by using one value of γ ahead of the shock and another value behind the shock depends, of course, on the actual value chosen for the high temperature gas. In the temperature range considered γ for air (used in the experiments) is of the order of 1.4 to 1.6 and a constant γ can be taken with very little error.

For moving shocks, with the gas ahead of the shock at rest (as in a pinch), the static pressure, temperature and density ratios remain the same but the value of M_1 is the ratio of the speed of the moving shock wave relative to the stationary gas divided by the speed of sound in the stationary gas.

For the cylindrically symmetrical electrodes (figure 4a), the mass of gas compressed between the shock and the magnetic piston must be equal to the mass originally enclosed in the region between the shock and the initial starting radius. Continuity yields

$$r_p^2 = R^2 \frac{\gamma-1}{\gamma+1} \left\{ 1 + \frac{2}{\gamma-1} \left(\frac{r_s}{R} \right)^2 \right\} \quad (5)$$

From

$$M_1^2 = \frac{U_1^2}{c_1^2} = \frac{P_1}{\gamma P_1} \left(\frac{dr_s}{dt} \right)^2$$

and

$$P_2 = \frac{2\gamma}{\gamma+1} M_1^2 P_1$$

we find

$$P_2 = \frac{2}{\gamma+1} \rho_1 \left(\frac{dr_s}{dt} \right)^2 \quad (6)$$

However, from electromagnetic considerations, if the current is uniform through a shell, the pressure on the inside of the shell (with zero pressure outside) is

$$P_2 = \frac{\mu i^2}{8\pi^2 r_p^2} = \frac{\mu i^2}{8\pi^2 R^2 \left(\frac{\gamma-1}{\gamma+1} \right) \left\{ 1 + \frac{2}{\gamma-1} \left(\frac{r_s}{R} \right)^2 \right\}} \quad (7)$$

If the pressure behind the piston is not zero, then accounting for a rarefaction zone behind the piston gives

$$P_2 = \frac{\mu i^2}{8\pi^2 r_p^2} + p_0 \left\{ 1 - \frac{\gamma-1}{2} \frac{u}{a_0} \right\}^{2\gamma/\gamma-1}$$

Hence $P_2 = p_0$ initially

$$P_2 = \frac{\mu i^2}{8\pi^2 r_p^2} + p_0 \left\{ 1 - \frac{\gamma-1}{2} \frac{u}{a_0} \right\}^{2\gamma/\gamma-1} \quad \left[\frac{u}{a_0} < \frac{2}{\gamma-1} \right]$$

$$P_2 = \frac{\mu i^2}{8\pi^2 r_p^2} \quad \left[\frac{u}{a_0} > \frac{2}{\gamma-1} \right]$$

based on continuum analysis extended down into vacuum conditions. Inasmuch as $u/a_0 > 2/\gamma-1$ for most of the piston travel, the latter value of P_2 is sufficient. Equating the two expressions for P_2 , [(6) and (7)], gives

$$\frac{2\rho_1}{\gamma+1} \dot{r}_s^2 = \frac{\mu i^2}{8\pi^2 R^2 \frac{\gamma-1}{\gamma+1} \left\{ 1 + \frac{2}{\gamma-1} \left(\frac{r_s}{R} \right)^2 \right\}}$$

On making the following change of variables -

$$g = \sqrt{\frac{2}{\gamma-1}} \frac{r_s}{R} ; \quad G = \frac{\gamma+1}{\gamma-1} \frac{1}{R^2} \sqrt{\frac{\mu}{8\pi^2 \rho_1}}$$

we get

$$\dot{g} = \frac{iG}{\sqrt{1+g^2}}$$

The electrodynamic equation (1) together with the above gas dynamical solution gives -

$$\frac{dg}{dt} = \frac{-qG\omega \sin \omega t}{\sqrt{1+g^2}}$$

which is easily integrated
by separation of variables.

For the case of the "one-dimensional" nozzle electrode of figure 4b, the only variation from the analysis above is in the form of the continuity equation. For moving shocks, with the gas in front of the shock remaining at rest, it can be shown by continuity arguments that

$$u_p = \frac{2}{\gamma+1} u_s$$

With the geometry of figure 4b -

$$\frac{d\theta_s}{dt} = \frac{\gamma+1}{2} \frac{d\theta_p}{dt}$$

From

$$\begin{aligned} M_1^2 &= \frac{u_1^2}{c_1^2} = \frac{\rho_1}{\gamma p_1} r_c^2 \left(\frac{d\theta_s}{dt} \right)^2 \\ &= \frac{\rho_1}{\gamma p_1} r_c^2 \left(\frac{\gamma+1}{2} \right)^2 \left(\frac{d\theta_p}{dt} \right)^2 \end{aligned}$$

$$\text{and } P_2 = \frac{2\gamma}{\gamma+1} M_1^2 P_1$$

we find

$$P_2 = \frac{\gamma+1}{2} \rho_1 r_c^2 \left(\frac{d\theta_p}{dt} \right)^2$$

From the same electromagnetic considerations used before, assuming that the average pressure at the working side of the "magnetic piston" is given by

$$P_2 = \frac{\mu i^2}{8\pi^2 r_p^2} = \frac{\mu i^2}{8\pi^2 (R - r_c \sin \theta_p)^2}$$

Equating the two expressions for P_2 gives -

$$\frac{\gamma+1}{2} \rho_1 r_c^2 \dot{\theta}_p^2 = \frac{\mu i^2}{8\pi^2 (R - r_c \sin \theta_p)^2} ;$$

letting

$$G_1 = \sqrt{\frac{2}{\gamma+1}} \frac{1}{r_c^2} \sqrt{\frac{\mu}{8\pi^2 \rho_1}}$$

we get

$$\dot{\theta}_p = \frac{iG_1}{\left(\frac{R}{r_c} - \sin \theta_p \right)}$$

The electrodynamic equation (1) together with the above gas dynamical solution gives -

$$\frac{d\theta_p}{dt} = \left\{ q_0 G_1 \omega \sin \omega t / \left(\frac{R}{r_c} - \sin \theta_p \right) \right\}$$

which is easily integrated by separation of variables.

EXPERIMENTAL RESULTS

Experimental Equipment: An experimental apparatus has been constructed to verify the above theoretical treatment (see figure 6). The capacitor storage bank is variable, but a typical bank as shown consists of 20 capacitors, each 30 microfarads, 0.1 microhenry, and rated at 8 kv dc. These capacitors can be shorted repeatedly without damage. The capacitors are connected to a collecting disc by means of coaxial cables which in turn are connected to the lower terminal of a vacuum switch. The switch can be seen through a cut out in the smallest diameter cylinder located above the table top. The pinch chamber is located directly above the vacuum switch and its axis of symmetry is vertical.

A mechanical vacuum pump can evacuate the system to a pressure of the order of .01 mm Hg. A streak camera mounted from above can be seen. This camera is used to take high speed continuous photographs of the illumination generated by the pinch. The coaxial cables, collecting disc, and vacuum switch are shown in a close-up view, figure 7. The copper fingers projecting from the upper terminal of the switch connect to the lower terminal of the pinch chamber.

Cylindrically Symmetric Pinch: A view from above, looking into the pinch chamber with the upper electrode removed to one side, is shown in figure 8. The lower and upper electrodes have center portions removed and replaced by an insulating section which prevents the current from being driven inward to the axis. On the upper electrode the slit through which streak photographs are taken is clearly visible. Various instrumentation are visible such as pressure

transducers, current density measuring coils and resistance thermocouples.

The parameters for a particular experiment which we shall examine are given in Table I together with the current wave form.

The stream photograph resulting from these initial conditions is shown in figure 9, and the current wave form is superimposed for reference. The horizontal coordinate is time, the scale is shown below and is the same for the current. The vertical coordinate is a view looking into the slit in the upper electrode. At any given instant of time the view seen through the slit in the electrode is reproduced in the streak photograph as a thin vertical strip. For instance, at 5 microseconds a view into the slit would produce a picture as indicated by the vertical strip so marked.

The most interesting results occur in the first 10 microseconds. Within this period the gas enclosed between the electrodes is pinched to the axis. The thickness of the illuminated region, measured vertically on the photograph gives some indication of the distance between the shock wave and the magnetic piston referred to earlier. A comparison of theory and experiment for the first pinch towards the axis is shown in figure 10. The theoretical position of the shock wave and the piston may be seen and compared with the observed region of high excitation. The close proximity for strong shocks of the shock wave and magnetic piston lends credence to the snow-plow model of reference 5 wherein this region is considered as an infinitely thin front. For a high enthalpy gas we have density ratios of the order 20, so piston and shock are even closer than present analysis indicates.

To calculate the time for the shock to reach the axis the following equation is integrated from time 0 to τ with the limits of g being $\sqrt{\frac{2}{\delta-1}}$ and 0.

$$-\int_{\sqrt{\frac{2}{\delta-1}}}^g \sqrt{1+g^2} dg = \int_0^t q_0 G \omega \sin \omega t dt$$

Solving for $t=\tau$, the time when the shock reaches the axis or $g=0$, gives $\tau = 7.9 \mu\text{sec}$. Experiment would indicate that the value of τ is $7.5 \mu\text{sec}$. Obviously the agreement is very good.

From equation (5) the piston position when $g=r_s=0$ is 1.63 inches from the axis of symmetry.

Assuming that the shock reflects from the axis as if from a solid wall, a reflected shock wave moving at approximately half the speed of the incoming wave will travel radially outwards, to leave a high pressure, high temperature region behind it (figure 11). The magnetic piston will continue to move inwards until it collides with the shock. The piston will be unable to sustain the high pressure so the compressed gas will expand, driving the piston outwards and sending a rarefaction front in towards the axis. Shortly afterwards a second, though much weaker, pinch takes place, and a series of wave interactions follows.

"One Dimensional" Nozzle Electrodes: In order to make a rough estimate of the available thrust from a "one dimensional" nozzle-electrode a very light disc was placed over the exit, and the impulse given to it by the pinched gases was estimated from the vertical movement of the disc.

The impulse of the disc was calculated for the time interval "bd" in figure 11. The pressure during this time corresponds to the stagnation pressure behind the reflected shock. This time interval is of the order of one microsecond. Such an estimate yields values of the impulse, $I^* \approx k m_g u_p$ where k is a function of the speed of the incident shock, m_g = mass of gas in the pinch tube, and u_p is the gas velocity behind the incident shock. The value of I^* calculated for an ideal gas (with an effective γ to account for real gas effects), is essentially the same as that calculated directly for a real gas. However, in the ideal gas case the pressures will be smaller and the time duration larger than for the real gas case. A typical experiment with

$$m_g = 0.345 \times 10^{-6} \text{ kg,}$$

$$m_{\text{disc}} = 0.036 \text{ kg,}$$

gave the initial disc velocity of 0.58 m/sec, and consequently an impulse of 0.021 newton sec.

The calculated exhaust velocity for a single pinch following the analysis indicated earlier is 19,600 m/sec, and the calculated impulse of 0.0115 newton sec. This would indicate, therefore, that about 50% of the total impulse obtained in the experiment derives from the theoretical stagnation pressure after the first shock reflection.

The calculations could not realistically be extended for a much longer period, since further pinches take place, the skin is much thicker, and in addition there may be a certain amount of evaporated metal and outgassing leading to an increase in mass of accelerated gas. Radiation effects from the hot gas may also become important.

Radially Dominant Nozzle Electrodes: A series of experiments were run with curved electrodes that were designed to maintain a radial pinch as much as possible and the curvature is introduced only near the axis of the electrodes (see figure 12). The axis of the electrodes was installed in the vertical position and a thrust plate was placed over the opening. A portion of the electrodes was covered with an insulating paint as indicated by the cross-hatched regions. The insulation was applied to induce the current-carrying plasma (magnetic piston) to turn through 90 degrees rather than remain in an axial direction. The expected position and nature of the magnetic piston is indicated for three different times during the discharge. The experimental conditions and results of a typical test are given in Table II.

To estimate the effect of the discharge on the thrust plate, the assumption is made that the velocity of the magnetic piston is increasing until it reaches the position marked, $t = t_1$. From this position until the exit of the nozzle, the current and the mean radius do not change appreciably. The short slug of gas between the magnetic piston and the shock wave is not considered to undergo any change in velocity beyond the position, $t = t_1$.

Despite the changing cross-sectional area in the early stages of the pinch, constant cross-section area is assumed here. It can be shown that such an assumption does not appreciably alter the velocity of the gas slug over the distance considered. Using the methods of analysis previously described, the conditions are solved at the time, $t = t_1$, when the piston reaches the insulated region of the electrodes. The calculated piston velocity at this point is

48,000 meters per second and the peak current is 290,000 amperes. The time, t_1 , is 9.7 μ sec. These results are in good agreement with the measured values as indicated in Table II. The stagnation pressure at the thrust plate after the outgoing shock has reflected, is 37×10^5 newtons/met² and the total time during which this stagnation pressure exists on the thrust plate is .82 μ sec. The velocity of the disc due to this impulsive force should be 1.8 meters per second. The measured disc velocity is 2.5 meters per second, indicating, as described earlier, that although the stagnation impulse is a good part of the total impulse acting on the disc, it is not the entire impulse. Obviously the thrust plate is not a good means of measuring thrust because the events, subsequent to the shock striking the thrust plate, are fictitious from the point of view of thrust.

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PROPOSED "PINCH ENGINE" FOR MARS RECONNAISSANCE

Discussion of Assumptions: The discharge of a capacitor suddenly placed in series with a closed electrical circuit containing resistance and inductance yields many waveforms of current and voltage. In the case of the electromagnetic pinch, part of the electrical circuitry is a conducting gas of varying resistance and inductance. The initial voltage across the capacitors must be large enough to cause electrical breakdown between the electrodes. The subsequent resistance and inductance of the plasma depends on its gas dynamic history. This complicated interaction is not yet completely analyzed. It has become obvious, however, from unpublished studies, that it is not efficient to attempt conversion of electrically stored energy into kinetic energy of the plasma in one pinch. It will probably take a minimum of six pinches or three complete waveforms of voltage and current. Either the stored energy in the capacitor can be made to operate six pinch tubes in rapid succession with the use of a novel switching device, or a single pinch tube can be made to resonate aerodynamically with a connecting plenum chamber so that six pinches can occur successively during the period of a single capacitor discharge. At present the latter method is favored and the calculations below are based on this mode of operation.

The current is assumed to be linearly increasing with time and at time, τ , the shock wave has moved through a 90° arc. No gas has, as yet, been discharged from the nozzle but efflux begins at time τ . The time of discharge of the compressed slug of ionized

gas, v_1 , has been determined by assuming a constant efflux velocity of the gas between the shock wave and the magnetic-piston. This efflux velocity is equal to the piston velocity at time, τ . During time, τ , the current is relatively constant and then rapidly falls to zero to complete one-half of one complete waveform.

Of the energy stored in the capacitors at full charge, only 50% will do work on the gas in six pinches. The balance is assumed to go into resistive losses in the circuitry. In all pinch experiments run to date, a much smaller amount of energy does work on the gas. However, these experiments have been inefficiently designed from this point of view and it is expected that in six pinches per discharge, the value quoted above can be obtained.

Analysis of thermal energy and kinetic energy behind strong shocks in an ideal gas indicate that 40% of the total energy is kinetic and the balance thermal.

From both considerations above, it is assumed that 20% of the energy in the capacitor goes into the useful form of kinetic energy. Hence, the kinetic energy in the slug of gas issuing from a single pinch must be multiplied by 30 to determine the capacitor energy (5 times for efficiency and 6 times for six pinches).

Pinch Engine Performance Data

Pinch cylinder dia.	16 cm
Radius of mean line, R	5.12 cm
Propellant	Oxygen
Propellant temperature, initial	460°R
Initial pressure in pinch tube	10 mm Hg
Pinch tube, exhaust area.	32.1 cm ²
Pinch tube, volume	258-cm ³
Pinch time, τ	4.4 μ sec.
Discharge time, τ_d	.29 μ sec.
Current, Rate of Growth, i	4x10 ⁸ amperes/sec.
Propellant discharge velocity	46.9 Km/sec.
Thrust during τ , (with pressure thrust)	22.7x10 ⁴ lbs. force
Impulse, per pinch	.066 lbs. per sec.
Specific impulse, sec. (with pressure thrust)	5730
Kinetic energy per pinch	5700 joules
Capacitor storage energy	171,000 joules
Capacitor voltage	1,000 volts
Capacitor charge time	1/4 sec.
Generator rating	684 Kw

Pinch Engine Weight Analysis

	<u>This Report</u> ¹¹	<u>Stuhlinger</u> ⁹	<u>Bussard</u> ¹⁰
Generator weight	5.0 lbs/Kw	4.0 lbs/Kw	1.0 lb/Kw
Turbine weight			.3
Compressor weight	1.0	1.8	
Radiator (condenser) & actuation	4.0	4.0	.2
Reactor	.8	1.5	.025
Shielding (for instruments)	1.5	3.0	-
Thrust chambers	.3	4.5	.025
Capacitors	(see below)	-	-
Working fluid for turbine	.2	1.0	-
Reactor Circuit (piping etc.)	.3	-	.5
Structure	2.0		
Electric circuitry	.3		
Fuel system & valves	.5	19.8	
	16.0		
Capacitor weight	.03 #/joule		
Guidance & information (payload)	6000. #		

It can be shown that the weight of the condensers can be reduced considerably by decreasing the initial gas pressure and pulsing much more often. Under such conditions switching and charging problems may become prohibitive, and much more serious than the weight penalty.

Mars Reconnaissance Vehicle Performance

The vehicle is considered to take off from a 1000 Km Earth orbit and to culminate in a 500 Km Mars orbit. The characteristic velocity for such a mission is given in Reference 12 as 10.03 Km/sec. Considering the propellant velocity to be 46.9 Km/sec the mass ratio (roughly corrected for gravitational forces based on Reference 9 is 1.61).

Engine & Structural weight (excluding capacitors)	11,000 lbs.
Capacitor weight	<u>5,100</u>
Total Engine & Structural weight	16,100 lbs.
Payload weight (guidance & information)*	6,000
Fuel weight	<u>13,400</u>
Total Take-off weight . . .	35,500 lbs.
Change in vehicle velocity in first day	273 ft./sec.
Travel time	approximately 1 year

* The vehicle will be equipped to photograph Mars and relay this photograph to Earth.

TABLE I

Pressure	= 0.09 m.m. Hg.
Capacitance, C	= 570 μ fd.
Inductance, L	= 0.10 uhy.
Resistance, R	= 0.003 ohms
Capacitor Voltage, E_c	= 2000 volts
Frequency	= 21 Kcs.

TABLE II

Parameters

Capacitance	= 570 μ fds
Voltage	= 4000 volts
Stored Energy	= 4600 joules
Inductance	= 0.10 μ hy
Circuit Resistance	= 0.003 ohms
Gas Pressure	= 0.85 mm Hg, Air Used
Disc Mass	= 0.043 kg
Mass of Gas	= 1.05×10^{-6} kg

Observations

Time to Maximum Current	= 11.0 μ sec.
Maximum Current	= 300,000 amperes
Disc Initial Velocity	= 2.5 meters/second
Measured Disc Momentum	= 0.108 newton-seconds

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FIGURE 1. LINEAR DISCHARGE TUBE

FIGURE 2. LINEAR DISCHARGE TUBE WITH NOZZLE

FIGURE 3 CURVED NOZZLE-ELECTRODES

(a)

Cylindrically Symmetric
Electrodes

(b)

One Dimensional Nozzle
Electrodes

(c)

Radially Dominant
Nozzle Electrodes

Figure 4 Three types of pinch experiments

Fig. 5. The efficiency of conversion of electrical energy into magnetic work per pinch. (linear case)

Fig. 6. Experimental Equipment

Fig. 7. Interior Close-up of Apparatus Above Table

Fig. 8. Pinch Chamber with Upper Electrode Removed

Fig. 9. Streak Photograph - Cylindrical Pinch

Figure 10. Comparison of Theoretical and Observed Pinch

Figure 11 Space-time plot illustrating the wave interactions present

Figure 12 Radially dominant electrode system with end disc

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