TORQUE RIPPLE IN A DARRIEUS, VERTICAL AXIS WIND TURBINE

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ABSTRACT

Interaction between a steady wind and a rotating, Darrieus, vertical axis wind turbine produces time periodic aerodynamic loads which cause time dependent torque variations, referred to as torque ripple, to occur in the mechanical link between the turbine and the electrical generator. There is concern for the effect of torque ripple upon fatigue life of drive train components and upon power quality. An analytical solution characterizing the phenomenon of torque ripple has been obtained which is based upon a Fourier expansion of the time dependent features of the problem. Numerical results for torque ripple, some experimental data, determination of acceptable levels and methods of controlling it, are presented and discussed.

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INTRODUCTION

Recent years have seen a renewed and accelerating interest in the development of wind energy as an alternative to presently used, rapidly depleting energy resources. Wind energy conversion has enjoyed national attention because of its emergence as an alternative which is nearly, if not already, economically competitive in today's utility markets. An example of the technological developments which have contributed to the optimistic outlook generally held for wind energy systems is contained herein.

In a utility grid application, power gathered from the wind by a Darrieus, vertical axis wind turbine (VAWT), operating synchronously, is in the form of mechanical torque at a specified rotational speed. Interaction of the rotating blades with the incident wind causes a time periodicity in the net torque acting on the turbine, which is obtained by integrating torque producing aerodynamic loads over all blades present. Under the ideal conditions of a steady wind from a fixed direction, the applied torque may be viewed as a deterministic oscillation called torque ripple, (which may contain many harmonics) superimposed on a steady, mean torque, which is relatable to overall turbine performance. Depending upon turbine operating conditions (such as wind speed and turbine RPM) and upon drive train characteristics (such as component inertia properties and torsional rigidities, gear ratios and generator slip), the magnitude of the oscillations may be either amplified or attenuated at various locations along the drive train. In view of extended component fatigue life and high power quality requirements, attenuation of torque ripple to acceptable levels is highly desirable.
Recognition of the torque ripple problem and its consequences, and attempts to characterize it analytically and demonstrate control over it are not new \(^{(1,2)}\). Two of the assumptions upon which early analytical work on torque ripple in VAWT systems was based \(^{(1)}\) are as follows: 1. The wind is steady and from a fixed direction, and 2. The net torque applied to the turbine is a simple harmonic function of time. Models based on these assumptions captured torque ripple behavior trends as parameters were changed \(^{(1)}\) and permitted at least initial insights toward understanding the problem.

However, recent aerodynamic models \(^{(3)}\), from which come the magnitude and time dependence of the net aerodynamic torque applied to the turbine, demonstrate that the assumption of a simple harmonic form for the applied torque is not always justified. Asymmetries in the upwind and downwind aerodynamics \(^{(3)}\), and the temporal influence of stall at high wind speeds, (a previously known result \(^{(4)}\)), cause multiple harmonics to appear in the applied torque, even for a fixed wind. By using a Fourier expansion of the time dependent characteristics of the torque ripple problem, a general solution has been obtained which permits full representation of the consequences of upwind and downwind aerodynamic asymmetries and blade stall. This approach, along with numerical results, a limited amount of data correlation, and a discussion of how acceptable torque ripple levels are determined and achieved, is presented in the present paper. With appropriate modifications, this analysis may be used to study torque ripple in horizontal axis and other vertical axis wind energy systems.
THE TORQUE RIPPLE MODEL

A typical VAWT drive train consists of the turbine rotor (blades and rotating tower), a transmission and a generator, connected in series by various torque transmitting shafts and couplings. Additional components may be present depending upon the specific turbine design, purpose and installation. For example, the DOE/Sandia 17-meter research turbine (5) located in Albuquerque, NM, has a secondary gear ratio change capability in the form of interchangeable pulleys and a timing belt, located between the transmission (which has a fixed gear ratio) and the generator. This feature permits incremental changes in the turbine operating speed and allows field evaluation of aerodynamic, structural and system performance, in a synchronous mode, under a variety of operating conditions. "Operating conditions" refers collectively to combinations of incident wind velocity and turbine operating speed. A popular parameter characterizing operating conditions is tip speed ratio, $\lambda$, which is equal to maximum blade speed, $R_{\text{MAX}}\Omega$, divided by incident wind speed, $V$. When $\lambda > 3.5$ the simple harmonic representation of applied torque and drive train response is justified (1,3,4), as seen in Fig. 1. However, when $\lambda < 3.5$, blade stall effects and upwind and downwind aerodynamic asymmetries become strong (3,4), thus compelling a Fourier expansion of torque ripple time characteristics, see Fig. 2. Since peak turbine power and, therefore, peak mean torque occurs at a tip speed ratio in the range of 1.0 to 3.0, (5), it is essential that dynamic behavior of the turbine be well understood for low values of $\lambda$. 

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The torque ripple model consists of three essential elements. The first is a simplified, physical representation of the important characteristics of the entire drive train for which differential equations of motion can be written. Figure 3 shows the physical model chosen. The turbine rotor is represented by two rotational inertias, each with one-half of the total rotor inertia, and connected together by a torsional shaft representing the rotor tower, with a stiffness, $K_T$, chosen to yield the correct counter-rotating frequency for the rotor. Continuing downstream, the equivalent low speed shaft (stiffness = $K_L$) transmits torque to the transmission with its inertia, $J_T$, and fixed gear ratio, $n_1$. The interchangeable pulleys and timing belt have an incrementally adjustable, but operationally fixed gear ratio, $n_2$, and are connected upstream to the transmission with an equivalent intermediate speed shaft (stiffness = $K_I$) and downstream to an electric generator with an equivalent high speed shaft (stiffness = $K_H$). The electric generator (inertia = $J_G$) may be either synchronous or induction, with torque reactions proportional to rotational position or speed, respectively. The proportionality constant is $K_S$ for a synchronous generator, and $D_I$ for an induction generator. Although results in this work are limited to those for an induction generator only, $K_S$ is retained in the solution for generality. This physical representation of the drive train captures torsional vibration modes of interest.

The second element of the torque ripple model embodies a decomposition (6) of the functional dependence upon time of the applied aerodynamic torque as predicted by the vortex model (3) for low tip speed ratios and the stream tube models (4) for high tip speed ratios. (Changing from the vortex to the
stream tube models is done to conserve computer time and reduce computation cost). The applied torque can be distributed fractionally between the two rotor inertias in order to account for wind shear, if necessary, and have the form

\[ T_{1A} = T_{10} + \sum_{i=1}^{N} T_{1i} \cos \omega_i t + \sum_{i=1}^{N} \overrightarrow{T}_{1i} \sin \omega_i t \]

\[ T_{2A} = T_{20} + \sum_{i=1}^{N} T_{2i} \cos \omega_i t + \sum_{i=1}^{N} \overrightarrow{T}_{2i} \sin \omega_i t \]

\( T_{1A} \) and \( T_{2A} \) are applied to the top (upstream) half and bottom (downstream) half of the rotor, respectively.

The third model element consists of a solution to the equations of motion, taken in the form

\[ \theta_j = A_{j0} + \sum_{i=1}^{N} A_{ji} \cos \omega_i t + \sum_{i=1}^{N} \overrightarrow{A}_{ji} \sin \omega_i t + \Omega_j t \]

where \( \theta_1 \) and \( \theta_2 \) are the angular positions of the top and bottom rotor halves, respectively, \( \theta_3 \) is the angular position of the slow speed end of the transmission and \( \theta_4 \) is the angular position of the generator. \( \Omega_1 (= \Omega_2) \) is the mean operating speed of the turbine, \( \Omega_3 \) is the mean speed of the slow speed end of the transmission, and \( \Omega_4 \) is the mean operating speed of the generator (note that \( \Omega_3 = \Omega_4/n_1n_2 \)). Since torsional modes of the turbine system which are reacted by torque in the drive train are even multiples of the operating speed (7), \( \omega_i = 2i\Omega \).
Equations of motion for the physical representation of the torque ripple model depicted by Fig. 3 are

\[
\begin{align*}
\frac{1}{2} J_B \ddot{\theta}_1 + K_T (\theta_1 - \theta_2) &= T_{1A} \\
\frac{1}{2} J_B \ddot{\theta}_2 + K_L (\theta_2 - \theta_3) + K_T (\theta_2 - \theta_1) &= T_{2A} \\
J_C \ddot{\theta}_3 + K_3 (\theta_3 - \frac{\theta_4}{n_1n_2}) + K_L (\theta_3 - \theta_2) &= 0 \\
J_C \ddot{\theta}_4 + K_4 (\theta_4 - \frac{\theta_4}{n_1n_2}) + K_S (\theta_4 - \omega_s t) + D (\dot{\theta}_4 - \omega_s) &= 0
\end{align*}
\]

After a substitution of (1) and (2) into (3), and a substantial amount of algebra, the following results are obtained for determination of the unknown constants.

\[
\begin{align*}
A_{11} &= \frac{\lambda_{2i} \lambda_{3i} - \lambda_{1i} \lambda_{4i}}{\lambda_{1i}^2 + \lambda_{2i}^2} , \quad \bar{A}_{11} = \frac{\lambda_{1i} \lambda_{3i} + \lambda_{2i} \lambda_{4i}}{\lambda_{1i}^2 + \lambda_{2i}^2} \\
A_{21} &= \frac{\phi_{1i} A_{11} - T_{11}}{K_T} , \quad \bar{A}_{21} = \frac{\phi_{1i} \bar{A}_{11} - \bar{T}_{11}}{K_T} \\
A_{31} &= \frac{1}{K_T K_L} \left[ (\phi_{1i} \phi_{2i} - K_T^2) A_{11} - (K_T T_{2i} + \phi_{2i} T_{11}) \right] \\
\bar{A}_{31} &= \frac{1}{K_T K_L} \left[ (\phi_{1i} \phi_{2i} - K_T^2) \bar{A}_{11} - (K_T \bar{T}_{2i} + \phi_{2i} \bar{T}_{11}) \right]
\end{align*}
\]
\[ A \begin{array}{c} 41 = \frac{n_1 n_2}{K_T K_L K_3} \left\{ \left[ \phi_{3i} \left( \phi_{1i} \phi_{2i} + K_T^2 \right) - \phi_{1i} K_L^2 \right] A_{1i} \\
+ \left[ K_L^2 T_{11} - \phi_{3i} \left( K_T T_{2i} + \phi_{2i} T_{11} \right) \right] \right\} \\
\end{array} \]

\[ \bar{A}_{41} = \frac{n_1 n_2}{K_T K_L K_3} \left\{ \left[ \phi_{3i} \left( \phi_{1i} \phi_{2i} + K_T^2 \right) - \phi_{1i} K_L^2 \right] \bar{A}_{1i} \\
+ \left[ K_L^2 \bar{T}_{1i} - \phi_{3i} \left( K_T \bar{T}_{2i} + \phi_{2i} \bar{T}_{1i} \right) \right] \right\} \]

where

\[ \phi_{1i} = \left( K_T - \omega_1^2 J_B / 2 \right) , \quad \phi_{2i} = \left( K_T + K_L - \omega_1^2 J_B / 2 \right) \]

\[ \phi_{3i} = \left( K_L + K_3 - \omega_1^2 J_T \right) , \quad \phi_{4i} = \left( K_4 + K_S - \omega_1^2 J_G \right) \]

\[ \lambda_{1i} = \omega_1 D_1 \left[ \phi_{3i} \left( \phi_{1i} \phi_{2i} - K_T^2 \right) - \phi_{1i} K_L^2 \right] \]

\[ \lambda_{2i} = \left( \phi_{3i} \phi_{4i} - K_3 K_4 \right) \left( \phi_{1i} \phi_{2i} - K_T^2 \right) - \phi_{1i} \phi_{4i} K_L^2 \]

\[ \lambda_{3i} = \left( \phi_{1i} \phi_{2i} - K_3 K_4 \right) \left( K_T T_{2i} - \phi_{2i} T_{1i} \right) - \phi_{4i} K_L^2 T_{1i} \]

\[ - \omega_1 D_1 \left[ K_L^2 \bar{T}_{1i} - \phi_{3i} \left( K_T \bar{T}_{2i} + \phi_{2i} \bar{T}_{1i} \right) \right] \]
\[ \lambda_{41} = (\phi_{31}\phi_{41} - K_3 K_4) \left( K_L T_{T21} + \phi_{21} T_{L11} \right) - \phi_{41} K_L^2 T_{L11} \]

\[ + \omega_1 D_I \left[ K_L^2 T_{L11} - \phi_{31} \left( K_L T_{T21} + \phi_{21} T_{L11} \right) \right] \]

\[ K_3 = \frac{n_1^2 n_2^2 K_I K_H}{K_I + n_2^2 K_H} \]

\[ K_4 = K_3 / n_1^2 n_2^2 \]

which completes the solution derivation.

NUMERICAL RESULTS AND ALLOWABLE LEVELS

Numerical results presented here are based on drive train properties of the present DOE/Sandia 17-meter research turbine. They are:

\[ J_B = 2.92 \times 10^5 \text{ lb-sec}^2 \text{-in} \left( 3.30 \times 10^4 \text{ N-sec}^2 \text{-m} \right) \]

\[ J_T = 2.15 \times 10^3 \text{ lb-sec}^2 \text{-in} \left( 2.43 \times 10^2 \text{ N-sec}^2 \text{-m} \right) \]

\[ J_M = 27.1 \text{ lb-sec}^2 \text{-in} \left( 3.06 \text{ N-sec}^2 \text{-m} \right) \]

\[ D_I = 824.0 \text{ lb-in-sec/rad} \left( 93.1 \text{ N-m-sec/rad} \right) \]

\[ K_I = 1.46 \times 10^8 \text{ lb-in/rad} \left( 1.65 \times 10^5 \text{ N-m/rad} \right) \]

\[ K_L = 2.39 \times 10^6 \text{ lb-in/rad} \left( 2.69 \times 10^5 \text{ N-m/rad} \right) \]

\[ K_I = 1.25 \times 10^6 \text{ lb-in/rad} \left( 1.41 \times 10^5 \text{ N-m/rad} \right) \]

\[ K_H = 1.86 \times 10^4 \text{ lb-in/rad} \left( 2.10 \times 10^3 \text{ N-m/rad} \right) \]

\[ n_1 = 35.6 \]

\[ n_2 = \frac{1800}{n_1(\Omega)} \]

\[ T_R = 8.35 \times 10^3 \text{ ft-lb} \left( 1.13 \times 10^2 \text{ N-m} \right) \]
where 1800 is the rotational speed of the generator, \( \Omega \) is the rotational speed of the turbine, both in units of RPM, and \( T_R \) is the torque rating of the turbine. Before defining torque ripple explicitly, it is necessary to derive an expression for torque as a function of time for some specified drive train location. With the prior knowledge that, for the above set of properties, torque ripple in the drive train is essentially independent of location, it is only necessary to know the torque in the low speed end, \( T_L(t) \). It is given by \( T_L(t) = K_L(\Theta_3 - \Theta_2) \), and with the above solution

\[
T_L(t) = K_2 \left[ \sum_{i=1}^{N} (A_{3i} \cos \omega_1 t + \overline{A}_{3i} \sin \omega_1 t) - \sum_{i=1}^{N} (A_{2i} \cos \omega_1 t + \overline{A}_{2i} \sin \omega_1 t) - \frac{T_{10} + T_{20}}{K_L} \right]
\]

Torque ripple is defined in two ways. The first, labeled \( \tilde{T}_M \), is the ratio of the mean-to-peak value and the mean value of torque, and is a convenient form when considering fatigue characteristics of the drive train components. The second, labeled \( \tilde{T}_R \), is the ratio of the mean-to-peak value and the turbine's rated torque, and is relatable to power quality. Thus, from (4)

\[
\tilde{T}_M = \frac{T_{LMAX} - T_{LMIN}}{T_{LMAX} + T_{LMIN}}
\]
In order to facilitate numerical evaluation of torque ripple a computer code, named FATE, was written. Applied torque coefficients, found in (1), are used as input to the code and results for $\tilde{T}_M$ and $\tilde{T}_R$ are calculated for discrete values of $\lambda$. (The coefficients of (1) vary with $\lambda$). Figure 4 shows how torque ripple, using both definitions, varies with tip speed ratio for the DOE/Sandia research turbine operating at 50.6 rpm.

Because of the rapid changes in $\tilde{T}$ at low values of $\lambda$, calculated points are connected by straight lines. Three data points, based on the $\tilde{T}_M$ definition, are shown in the figure and agree closely with predicted values of $\tilde{T}_M$. These data are obtained by a torque sensor located in the low speed end of the drive train. More data are not presented because of the difficulty in obtaining experimental information not influenced by the random nature of the wind. Notice that $\tilde{T}_M$ increases with $\lambda$. This occurs because even though the oscillating portion of the torque is diminishing with $\lambda$, the mean value is diminishing faster, thus causing $\tilde{T}_M$ to increase. $\tilde{T}_R$ shows the change in only the oscillating portion of torque (since it is normalized by a constant--the turbine rated torque), where it is seen to decrease with increasing $\lambda$.

To determine what level of torque ripple might be allowable from a fatigue or life expectancy standpoint, assume that drive train components follow the Goodman law for fatigue strength (8). This law imposes a straight line relationship between fatigue strength for purely alternating

\[
\frac{\tilde{T}_R}{2 \tilde{T}_{LRATED}} = \frac{T_{\text{MAX}} - T_{\text{MIN}}}{T_{LRATED}}
\]
stress (the dependent variable) and mean stress (the independent variable).

Using this law and the above definition of torque ripple expressed as a % of mean torque, $T_M$, an expression for allowable $T_M$ in terms of expected fatigue strength, $\sigma_N$, mean stress, $\sigma_M$, and ultimate strength, $\sigma_U$, of drive train components can be derived.

$$\tilde{T}_M \leq \left( \frac{\sigma_N}{\sigma_U} \right) \left( \frac{\sigma_U}{\sigma_M} - 1 \right)$$  \hspace{1cm} (7)

Taking the fatigue limit for $\sigma_N$, a typical value of the ratio, $(\sigma_N/\sigma_U)$, for structural steels is 0.4. Using this value, (7) can be plotted versus the ratio $(\sigma_U/\sigma_M)$ as in Fig. 5. Since $(\sigma_U/\sigma_M)$ may be viewed as a safety factor for design of drive train components, whatever value is used can be located on the ordinate of Fig. 5, and as long as the $T_M$ calculated from (5), falls on or below the line in Fig. 5, infinite life can be expected. By taking the ratio, $\tilde{T}_R/\tilde{T}_M$, for specific values of $\lambda$ (for example from Fig. 4), it can be seen that as $\lambda$ increases, $\sigma_M$ decreases. Thus, increasing $\lambda$ corresponds to an increase in $(\sigma_U/\sigma_M)$ and, therefore, an increase in acceptable levels of $T_M$. For the DOE/Sandia research turbine, a design safety factor of 2.0 was used for drive train components. Since maximum torque occurs at $\lambda = 1.5$, $(\sigma_U/\sigma_M) = 2.0$ on the abscissa in Fig. 5 corresponds to $\lambda = 1.5$. Using Fig. 4, it can be seen that $(\sigma_U/\sigma_M) = 3.0$ corresponds to $\lambda = 3.0$, $(\sigma_U/\sigma_M) = 4$ corresponds to $\lambda = 4.0$, and $(\sigma_U/\sigma_M) = 6.5$ corresponds to $\lambda = 6$. This demonstrates that the allowable values of $\tilde{T}_M$ increase rapidly with $\lambda$. Examination of the values of $\tilde{T}_M$ in Fig. 4 indicates that the DOE/Sandia research turbine does not have a fatigue problem.
Power companies have determined that power quality may be determined by the amount of "light flicker" that people will tolerate for extended periods of time, (9). They have also determined that the "borderline of irritation" with 60 cycle power corresponds to a voltage variation of 0.5% of the line voltage. Since torque ripple in a generator is equivalent to current ripple in the line, acceptable torque ripple (expressed as a % of rated torque) can be related to voltage ripple. In the case of the DOE/Sandia research turbine, line impedance is approximately 4% of the load impedance. A maximum voltage ripple of 0.5%, therefore, corresponds to an allowable $\tilde{T}_R$ of 12.5%. Results in Fig. 4 indicate that the research turbine does not have a power quality problem.

CONTROL OF TORQUE RIPPLE

Among the properties which characterize the torque ripple problem, the most readily and easily modified are drive train torsional rigidities and perhaps, generator slip. Figure 6 shows numerical results for $\tilde{T}_M$ versus $\lambda$ for the research turbine and values which would have resulted from a doubling and a halving of the torsional rigidity of the low speed end of its drive train. While fatigue life does not appear to be reduced even with a doubling of the low speed stiffness, additional rigidity increases could cause problems. Since $\tilde{T}_R = \tilde{T}_M$ when $\lambda = 1.5$, doubling the stiffness of the low speed end could cause a noticeable reduction in power quality.

To see how a change in low speed torsional stiffness effects torque ripple, consider the results in Fig. 7, where $\tilde{T}_M$ is plotted, for three low speed rigidities, as a function of turbine operating speed, $\Omega$. Notice how the peak (which corresponds to the first critical drive train frequency)
moves to the left with a reduction in low speed stiffness and to the right for an increase in drive train stiffness. The effect that this has on torque ripple at a specified operating speed is obvious. (This figure does not depict what occurs during start up. It provides torque ripple values in the drive train at specified operating speeds.) The behavior of $T_R$ with $\Omega$ is similar to that shown for $T_M$ in Fig. 7. Other methods of controlling torque ripple exist. An increase in generator slip tends to lower torque ripple values at moderate $\Omega$, and increase them at higher $\Omega$ (above ~ 40 RPM). An increase in inertia properties tends to lower torque ripple at a given operating condition, but this may be costly. A reduction in gear ratio tends to lower apparent drive train rigidities and, thus, lower torque ripple. However, the most effective means of reducing torque ripple is through reduction of low speed rigidity. This can be shown as follows.

Let $K_1$ represent either the low speed (between the rotor and the transmission) drive train stiffness or the high speed (between the transmission and the generator) stiffness, and let $K_2$ represent the other. Assume that the high speed stiffness has been corrected to the low speed end by multiplying it by the square of the drive train speed ratio. Let the entire drive train stiffness be represented by $\bar{K}$. Then

$$\bar{K} = \frac{K_1 K_2}{K_1 + K_2} \quad (8)$$

The change in $\bar{K}$ can be expressed in terms of $K_1$ and $K_2$ and a change in either of these, say $\Delta K_1$. 

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Now, let $K_1$ represent the high speed stiffness and recognize that $K_1 \gg K_2$.

From (9)

$$\lim \frac{\Delta K}{K} \to 0 \text{ as } \frac{K_2}{K_1} \to 0$$

which implies that, for a given change in the high speed stiffness, the net effect is nearly zero. Now let $K_2$ represent the high speed stiffness and recognize that $K_2 \gg K_1$. From (9)

$$\lim \frac{\Delta K}{K} \to \frac{\Delta K_1}{K_1} \text{ as } \frac{K_1}{K_2} \to 0$$

This implies that a change in the low speed stiffness will result in approximately an equivalent change in the overall drive train stiffness.

Therefore, drive train stiffness changes are most effective when made at the low speed end. This result depends upon the high speed stiffness being much greater than the low speed stiffness, a condition which is nearly always true because of the effect that the speed ratio has on the high speed stiffness.
CONCLUSIONS AND RECOMMENDATIONS

Currently, the deterministic torque ripple problem is well understood. The source of torque ripple, its behavior with operating conditions, its response to property changes, and its allowable levels have been analytically predicted and experimentally verified. (Also, see (1)). Torque ripple in two-bladed VAWT systems can be maintained at acceptable levels.

As mentioned earlier, collection of data for correlation with the deterministic solution is difficult. This is due to the stochastic nature of the wind which tends to increase measured torque ripple in the turbine drive train above values predicted by the deterministic model. As turbines increase in size, their natural frequencies are reduced and their response times more nearly match the frequency content of the wind, thus aggravating the problem. Logically, the next step in torque ripple modeling should deal with the stochastic nature of the wind, in terms of both its magnitude and its direction. It is this author's feeling, however, that this additional characterization will have to begin with a modification of the aerodynamic codes which predict the torque applied to the turbine.

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