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STABILITY OF FLOW FROM A NUCLEAR CAVITY

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Urbana, IL 61801

Final Report

of Research performed under

Subcontract No. 1281903

for

THE REGENTS OF THE UNIVERSITY OF CALIFORNIA

July 1977

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FOREWORD

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Dr. F. A. Morrison, Jr., Professor of Mechanical Engineering, acted as principal investigator. Mr. James M. Laude made major contributions to the program. In addition, G. Curry, L. Sharpe, and M. B. Stewart participated in the investigation.

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CHAPTER 1

INTRODUCTION

The stability of multiphase, multicomponent flow in porous media is considered. The flow issues from the cavity formed by an underground nuclear explosion. For a short period of time following the test, cavity gases, steam and air at high temperature and pressure, are driven into the surrounding porous medium and the stemming material. The extent of the fluid penetration must be analyzed in order to assure adequate containment of the flow and prevent venting.

Past analyses of cavity fluid penetration into the surrounding medium have addressed low [1] and high [2, 3] Reynolds number gas flow, convective dispersive transport [4], crack propagation [5], and multiphase, multicomponent flow [6, 7]. In each case, the assumption that permits calculation of the position of the interface is that the displacement occurs along a broad, smooth, stable front. Stable displacement is not, however, a conservative assumption. The petroleum industry, for example, has long been aware of unstable displacement processes. In such cases, the displacing fluid forms fingers penetrating into the displaced fluid and bypassing the displaced fluid. The displacing fluid then travels much further than would otherwise be the case. The ability to predict the presence of such fingers and to determine their growth rate would be quite valuable to containment.

Consider, as an example, the vertical displacement of one liquid by another [8]. A stable displacement would result in a horizontal undisturbed interface between the fluids. A small wavelike disturbance to the interface may grow or decay depending upon several factors. For a lower liquid (1) moving with a uniform velocity V and displacing an upper liquid (2), Saffman and Taylor determined the growth rate, σ , of such a disturbance.

$$\sigma = \alpha \frac{\left(\frac{\mu_1}{k} + \frac{\mu_2}{k}\right)V + (\rho_2 - \rho_1)g}{\frac{\mu_1}{k} + \frac{\mu_2}{k}} \quad (1)$$

α is the wave number, ρ and μ are density and viscosity, g is the acceleration of gravity, and k is the permeability of the medium.

Saffman and Taylor concluded that the mobility of the fluid, the ratio of effective permeability to fluid viscosity, influenced frontal stability. For a horizontal displacement, the disturbance grows when a fluid is displaced by one with lower mobility. For vertical displacement, gravity tends to stabilize the motion when the lower fluid is more dense, but destabilizes when the lower fluid is lighter.

While this example illustrates the phenomenon, the calculations are not sufficient to describe the flow from a nuclear cavity. In that case, condensation, heat transfer, and multi-component, multiphase flow may also have major effects.

Condensation, an effect not considered by Saffman and Taylor but present in the flow of interest here, may be anticipated to have a stabilizing effect. Miller [9] examined fluid displacements in porous media accompanied by a change of phase, steam condensing at a moving front while displacing water in a completely saturated bed. The phase change resulted in the flow being more stable than equation (1) would predict. Our method of analysis is similar to that employed by Miller and our notation largely follows his.

A description of cavity behavior has been given by Kennedy and Higgins [10]. The resulting flow through a stemming column has also been described [6, 7]. Steam and air enter the column from the cavity. This porous medium is initially at ambient temperature and may have some initial moisture content. Steam from the cavity condenses along a moving front causing an abrupt increase in temperature and in water saturation of the medium. Any noncondensable cavity gas passes through the condensation front. We seek to determine marginal stability conditions and growth rates for condensation front instabilities.

CHAPTER 2

THE UNPERTURBED CONDENSATION FRONT

Flow in the neighborhood of an unperturbed condensation front is shown in Fig. 1. Region 1 of the figure shows the steam and noncondensable cavity gas approaching the interface, while liquid and the solid matrix recede. Region 2, still at ambient temperature, shows the ground water initially present in the medium as well as the noncondensable gas. The condensation front moves with velocity V . The apparent velocity, volume flow rate per unit area normal to the flow, of the steam-air mixture in region 1 is U_1 . The apparent velocity of the air in region 2 is U_2 . Because of the comparatively small mobility, liquid motion may be neglected with little error while greatly simplifying the analysis.

Conservation relations are conveniently written in the frame of the interface for the unperturbed flow. Figure 2 shows the flow with a moving coordinate system. The apparent velocity of each component of the flow is shown with respect to the interface.

For a control volume about the interface shown in Fig. 2, a continuity equation for the mass flux of water, liquid and vapor, yields

$$\rho_v (U_1 - S_1 \epsilon V) = \rho_l \epsilon V (S_2 - S_1) \quad (2)$$

The continuity equation for air is, similarly,

$$\rho_{a1} (U_1 - S_1 \epsilon V) = \rho_{a2} (U_2 - S_2 \epsilon V) \quad (3)$$

ρ_v , ρ_l , and ρ_a are the mass densities of the vapor, liquid, and air. Subscripts 1 and 2 refer to region 1 or 2. ϵ is the porosity of the medium. S , the saturation of the gas-vapor mixture, is the void volume fraction occupied by the gaseous phase.

For the control volume about the interface, an energy equation for the unperturbed flow across the interface can be written. When the effective conductivity of the medium is not negligible, the control surface in region 2 is placed far from the interface and T_2 below is replaced by $T_{2\infty}$. With the zero conductivity assumption, the control surface for region 2 may be arbitrarily close to the interface. The resulting energy equation is

$$\begin{aligned} & \rho_{a2} (U_2 - S_2 \epsilon V) h_{a2} - \rho_{a1} (U_1 - S_1 \epsilon V) h_{a1} \\ & - \rho_v (U_1 - S_1 \epsilon V) h_v + \rho_l (1 - S_1) \epsilon V h_{l1} - \rho_l (1 - S_2) \epsilon V h_{l2} \\ & + \rho_s (1 - \epsilon) V C_s (T_1 - T_2) = 0 \end{aligned} \quad (4)$$

h is enthalpy per unit mass. ρ_s and C_s are the density and the specific heat of the solid, respectively. T is the temperature. The gas, liquid, and solid temperatures are assumed equal and local thermal equilibrium prevails.

Relations for the apparent velocities and the saturation of the gas-vapor mixture in region 1 can now be derived in terms of

the known interface velocity and the known saturation of the air in region 2. Solving equations (2), (3), and (4) for U_1 , U_2 , and S_1 yields,

$$U_1 = \epsilon V \left\{ S_2 + \frac{(\rho_l - \rho_v)}{\rho_l} \frac{[\rho_l(h_{l1} - h_{l2})(1 - S_2) + \rho_s \frac{(1 - \epsilon)}{\epsilon} C_s (T_1 - T_2)]}{[\rho_v h_{fg} + \rho_{a1}(h_{a1} - h_{a2})]} \right\} \quad (5)$$

$$U_2 = \epsilon V \left\{ S_2 + \frac{\rho_{a1}}{\rho_{a2}} \frac{[\rho_l(h_{l1} - h_{l2})(1 - S_2) + \rho_s \frac{(1 - \epsilon)}{\epsilon} C_s (T_1 - T_2)]}{[\rho_v h_{fg} + \rho_{a1}(h_{a1} - h_{a2})]} \right\} \quad (6)$$

and

$$S_1 = S_2 - \frac{\rho_v}{\rho_l} \frac{[\rho_l(h_{l1} - h_{l2})(1 - S_2) + \rho_s \frac{(1 - \epsilon)}{\epsilon} C_s (T_1 - T_2)]}{[\rho_v h_{fg} + \rho_{a1}(h_{a1} - h_{a2})]} \quad (7)$$

h_{fg} is the latent heat of vaporization in region 1.

CHAPTER 3

MARGINAL STABILITY ANALYSIS

Marginal stability will now be examined. Marginal stability implies that the perturbation to the front neither grows nor decays. The case where the perturbation is allowed to grow or decay will be examined later. Equations describing perturbations of velocity, saturation, and pressure will be written. The normal mode method [11] will be used to analyze the stability of the flow undergoing these small perturbations. Mass and energy conservation equations and an expression for pressure equality at the interface will be written for the perturbed flow.

The displacement front is originally in the plane $Z = 0$, as shown in Fig. 2. It is now assumed that the front undergoes a small wavy perturbation to its position

$$\bar{Z} = \delta f(x, y), \quad (8)$$

where the function f satisfies the wave equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = -\alpha^2 f \quad (9)$$

δ is the unknown amplitude of the displacement front perturbation. The wave number, α , is 2π divided by the mean wavelength, λ , of the perturbation. All perturbations are assumed to be products of $f(x, y)$ and the distance $\bar{Z}$ from the front.

The driving forces for the flow are the pressure gradient in the medium and gravity. For small Reynolds numbers, as are of interest here, Darcy's Law applies. Darcy's Law states that the apparent velocity of a component of the flow will be

$$U_i = \frac{-k_i}{\mu_i} \frac{\partial (P_i - \rho_i gZ)}{\partial Z} = \nabla \phi_i \quad (10)$$

ϕ is defined as the velocity potential.

Continuity equations for the perturbed flow are

$$\nabla^2 \phi_1 = 0 \quad \nabla^2 \phi_2 = 0 \quad (11)$$

The solutions of equations (11) that vanish with increasing distance from the front are, respectively,

$$\phi_1 = a_1 e^{\alpha Z} f(x, y) \quad \phi_2 = a_2 e^{-\alpha Z} f(x, y) \quad (12)$$

where a_1 and a_2 are undetermined constants. The perturbation of saturation leaving the interface is

$$s = s_1 f(x, y) \quad (13)$$

where s_1 is also an undetermined constant.

A mass balance for the perturbed flow of both water and air yields,

$$\rho_v [U_1 - (S_1 + s)\epsilon V + v_{1Z}] = \rho_g \epsilon V (S_2 - S_1 - s) \quad (14)$$

and

$$\rho_{a1} [U_1 - (S_1 + s)\epsilon V + v_{1Z}] = \rho_{a2} (U_2 - S_2 \epsilon V + v_{2Z}) \quad (15)$$

Subtracting the unperturbed flow equations, (2) and (3), from equations (14) and (15), and substituting the expressions given in (10), (12), and (13) for the perturbed velocities and saturation yields,

$$\rho_v \alpha a_1 + (\rho_l - \rho_v) \epsilon V s_1 = 0 \quad (16)$$

and

$$\rho_{a1} \alpha a_1 - \rho_{a1} \epsilon V s_1 + \rho_{a2} \alpha a_2 = 0 \quad (17)$$

Energy conservation for the perturbed flow shows that

$$\begin{aligned} & \rho_{a2} (U_2 - S_2 \epsilon V + v_{2Z}) h_{a2} - \\ & \rho_{a1} [U_1 - (S_1 + s) \epsilon V + v_{1Z}] h_{a1} - \\ & \rho_v [U_1 - (S_1 + s) \epsilon V + v_{1Z}] h_v + \\ & \rho_l \epsilon V (1 - S_1 - s) h_{l1} - \rho_l \epsilon V (1 - S_2) h_{l2} + \\ & \rho_s (1 - \epsilon) V C_s (T_1 - T_2) = 0 \end{aligned} \quad (18)$$

Subtracting the unperturbed energy expression, equation (4), and again substituting expressions (10), (12), and (13) for the velocity and saturation perturbations yields,

$$\begin{aligned} & (\rho_{a1} h_{a1} + \rho_v h_v) \alpha a_1 + \rho_{a2} h_{a2} \alpha a_2 \\ & + (\rho_l h_{l1} - \rho_v h_v - \rho_{a1} h_{a1}) \epsilon V s_1 = 0 \end{aligned} \quad (19)$$

The final constraint on the perturbed flow is pressure continuity along the wavy front.

$$(P_1 + p_1)|_{\bar{z}} = (P_2 + p_2)|_{\bar{z}} \quad (20)$$

and

$$P_{10} + \bar{z} \frac{dP_1}{d\bar{z}}|_0 + p_{10} = P_{20} + \bar{z} \frac{dP_2}{d\bar{z}}|_0 + p_{20} \quad (21)$$

where P_i is the initial pressure distribution and p_i is the pressure perturbation. From equation (10),

$$\frac{dp_i}{d\bar{z}} = \frac{-\mu_i U_i}{k_i} + \rho_i g \quad (22)$$

$$p_{i0} = \frac{-\mu_i \phi_i}{k_i} \quad (23)$$

Substitution of these expressions into equation (21) yields,

$$\frac{-\mu_1}{k_1} a_1 + \frac{\mu_2}{k_2} a_2 + \left[\frac{-\mu_1 U_1}{k_1} + \frac{\mu_2 U_2}{k_2} - (\rho_1 - \rho_2)g \right] \delta = 0 \quad (24)$$

where k_1 and k_2 are effective permeabilities, ρ_1 is the density of the mixture in region 1, and ρ_2 is the density of air in region 2. The effective permeability in a region is the absolute permeability, k , of the medium reduced by the relative permeability. Although not strictly functions of saturation only, relative permeability depends most strongly on saturation and is ordinarily taken to depend on saturation only.

Observe that equations (16), (17), (19), and (24) comprise four linear, homogeneous equations in the four unknown constants a_1 , a_2 , s_1 , and δ . The determinant of coefficients for these equations must be identically zero for a non-trivial solution to

exist. This produces the marginal stability condition

$$\frac{-\mu_1 U_1}{k_1} + \frac{\mu_2 U_2}{k_2} - (\rho_1 - \rho_2)g = 0 \quad (25)$$

In the limit of no condensation, the apparent velocities are equal and equal to the interface velocity. The stability condition, in this limit, reduces to the condition of Saffman and Taylor [8]. Their marginal stability condition is recovered by setting the numerator of equation (1) equal to zero. The fluid mobility again influences frontal instability. Here, the fluid flow will be stable when the product of the mobility and velocity of the displacing fluid is less than that of the corresponding product for the displaced fluid. Gravity also acts to stabilize the flow because the lower fluid, the steam-air mixture of region 1, is more dense than the air of region 2.

The stability condition, equation (25), also agrees with a result of Miller [9] for steam completely displacing water in an entirely saturated bed. In the limit of zero thermal conductivity, Miller's stability condition reduces to equation (25). A numerical evaluation of equation (25) appears later in the analysis.

CHAPTER 4'

GROWTH OF INSTABILITIES

It is of interest to analyze the growth rate of any potential instabilities, comparing them with the rate of frontal displacement. It is now assumed that the position of the perturbed interface is a function of time

$$\bar{z} = \delta f(x, y)e^{\sigma t} \quad (26)$$

where t is time and σ is the growth rate of the perturbation.

For the continuity equations (11), solutions that vanish far from the front are

$$\begin{aligned} \phi_1 &= a_1 e^{\alpha \bar{z}} f(x, y) e^{\sigma t} \\ \phi_2 &= a_2 e^{-\alpha \bar{z}} f(x, y) e^{\sigma t} \end{aligned} \quad (27)$$

The saturation perturbation at the interface is also a function of time

$$s = s_1 f(x, y) e^{\sigma t} \quad (28)$$

Mass conservation equations for water and air are

$$\begin{aligned} \rho_v [U_1 - (S_1 + s)\epsilon(V + \frac{d\bar{z}}{dt}) + v_{1z}] = \\ \rho_l \epsilon (S_2 - S_1 - s)(V + \frac{d\bar{z}}{dt}) \end{aligned} \quad (29)$$

and

$$\rho_{a1} [U_1 - (S_1 + s)\epsilon(V + \frac{d\bar{z}}{dt}) + v_{1z}] =$$

$$\rho_{a2} [U_2 - S_2 \epsilon (V + \frac{d\bar{z}}{dt}) + v_{2z}] \quad (30)$$

Subtracting the unperturbed mass equations, (2) and (3), and substituting relations (10), (12), and (13) for the perturbations of velocity and saturation yields,

$$\rho_v \alpha a_1 + (\rho_\ell - \rho_v) \epsilon V s_1 -$$

$$[\rho_v \epsilon S_1 + \rho_\ell \epsilon (S_2 - S_1)] \sigma \delta = 0 \quad (31)$$

and

$$\rho_{a1} \alpha a_1 + \rho_{a2} \alpha a_2 - \rho_{a1} \epsilon V s_1 +$$

$$(\rho_{a2} S_2 - \rho_{a1} S_1) \epsilon \sigma \delta = 0 \quad (32)$$

Similarly, an energy equation, neglecting conductivity and containing derivatives of $\bar{z}$ with respect to time, can be written for the perturbed flow. Subtracting the unperturbed energy equation, (4), and again substituting the necessary equations for the perturbations yields,

$$(\rho_{a1} h_{a1} + \rho_v h_v) \alpha a_1 + \rho_{a2} h_{a2} \alpha a_2$$

$$+ (\rho_\ell h_{\ell 1} + \rho_{a1} h_{a1} - \rho_v h_v) \epsilon V s_1 +$$

$$\{ \epsilon [\rho_{a2} h_{a2} S_2 - \rho_{a1} h_{a1} S_1 - \rho_v h_v S_1$$

$$+ \rho_\ell h_{\ell 2} (1 - S_2) - \rho_\ell h_{\ell 1} (1 - S_1)]$$

$$- (1-\epsilon) \rho_s C_s (T_1 - T_2) \sigma \delta = 0 \quad (33)$$

Pressure equality still exists at the interface. Equation (24) is still applicable. The growth does not alter this relation.

It should be noted that equations (24), (31), (32), and (33) are four linear, homogeneous equations in the four undetermined constants a_1 , a_2 , s_1 , and δ . For a non-trivial solution to exist, the determinant of the coefficient matrix must again be zero. This stipulation leads to the following relation for the growth rate of the instability

$$\sigma = \frac{\rho_l \rho_{a2} \alpha}{\epsilon} [\rho_v h_{fg} + \rho_{a1} (h_{a1} - h_{a2})] \left[-\frac{\mu_1 U_1}{k_1} + \frac{\mu_2 U_2}{k_2} - (\rho_1 - \rho_2) g \right]$$

$$\{ \rho_{a2} \rho_{s2} \left(\frac{\mu_1}{k_1} + \frac{\mu_2}{k_2} \right) [\rho_v h_{fg} + \rho_{a1} (h_{a1} - h_{a2})] \}$$

$$+ [\rho_{a2} (\rho_l - \rho_v) \frac{\mu_1}{k_1} + \rho_{a1} \rho_l \frac{\mu_2}{k_2}]$$

$$\left[\frac{(1-\epsilon)}{\epsilon} \rho_s C_s (T_1 - T_2) + \rho_l (1 - S_2) (h_{l1} - h_{l2}) \right] \} \quad (34)$$

Equation (34) can be simplified by substitution of equations (5) and (6) for U_1 and U_2 . The resulting expression for σ is

$$\sigma = \frac{\alpha V \left[\frac{-\mu_1 U_1}{k_1} + \frac{\mu_2 U_2}{k_2} - (\rho_1 - \rho_2)g \right]}{\left(\frac{\mu_1 U_1}{k_1} + \frac{\mu_2 U_2}{k_2} \right)} \quad (35)$$

This expression for σ is similar to the expression for the growth rate of instability found by Saffman and Taylor [8]. Again, in the limit of no condensation, the apparent velocities are equal to the interface velocity and equation (35) reduces to the expression given by Saffman and Taylor, equation (1). Observe that our expression for the marginal stability condition, equation (25), is recovered when σ is set equal to zero. When σ is negative, any disturbance to the interface will damp out. The flow will remain stable. Favorable fluid mobilities, $k_1/\mu_1 < k_2/\mu_2$, and a more dense fluid displacing a less dense fluid favor the decay of disturbances. A sample calculation appears later in the analysis.

Armento and Miller [12] have examined the stability of moving combustion fronts in porous media and the resulting growth rates of frontal instabilities. In their analysis, in the limit of no heat generation by combustion, their growth rate expression reduces to an expression similar to equation (35). In the absence of a gravity field, equation (35) and the expression of Armento and Miller reduce to the same expression.

CHAPTER 5

MARGINAL STABILITY

AND THE EFFECT OF THERMAL CONDUCTIVITY

The previous marginal stability analysis assumed that a sharp temperature drop occurred across the condensation front. Now, the possible effect of the conduction of heat through the porous medium will be considered. By increasing the heat transfer from hot protrusions, heat conduction will have a stabilizing influence.

A portion of the results of the previous marginal stability analysis will be applied to the present analysis. The equations written for the unperturbed mass conservation of water and air, equations (2) and (3), are equally applicable to this case. Equations (16) and (17) for the perturbed mass balances also remain unchanged. Pressure equality along the wavy front is still expressed by equation (24).

Energy transport in region 2 must, however, be reexamined. A one dimensional first law analysis for an infinitesimal slice in region 2 will yield

$$K_e \frac{d^2 T}{dz^2} = [\rho_{a2} (U_2 - S_2 \epsilon V) C_a - \rho_s (1 - \epsilon) V C_s - \rho_l \epsilon (1 - S_2) V C_l] \frac{dT}{dz} \quad (36)$$

K_e is the effective thermal conductivity of the porous medium. C_a is the specific heat at constant pressure of air. Defining the constant

$$\gamma \equiv -[\rho_{a2} (U_2 - S_2 \varepsilon V) C_a - \rho_s (1 - \varepsilon) V C_s - \rho_g \varepsilon (1 - S_2) V C_g] / K_e \quad (37)$$

we solve equation (36). With the substitution of γ and application of the boundary conditions $T = T_1$ at $Z = 0$ and $T \rightarrow T_{2\infty}$ as $Z \rightarrow \infty$, we find

$$T = T_{2\infty} + (T_1 - T_{2\infty}) e^{-\gamma Z} \quad (38)$$

A three dimensional energy equation for the perturbed flow will now be written. The temperature in region 2 may be expressed in terms of the unperturbed temperature distribution and a perturbation to the temperature.

$$T = T(Z) + \theta(Z) \cdot f(x, y) \quad (39)$$

where $\theta(Z) \cdot f(x, y)$ is the temperature perturbation. For a three dimensional differential element in region 2, the perturbed energy equation, invoking continuity for the perturbed flow and neglecting terms of higher order in perturbed quantities, reduces to

$$K_e \left[\frac{d^2 T}{dZ^2} + f \left(\frac{d^2}{dZ^2} - \alpha^2 \right) \theta \right] = - \rho_g (1 - \varepsilon) V C_s \left[\frac{dT}{dZ} + f \frac{d\theta}{dZ} \right]$$

$$\begin{aligned}
& + \rho_{a2} (U_2 - S_2 \epsilon V) C_a \left[\frac{dT}{dZ} + f \frac{d\theta}{dZ} \right] \\
& - \rho_l \epsilon V (1 - S_2) C_l \left[\frac{dT}{dZ} + f \frac{d\theta}{dZ} \right] \\
& + \rho_{a2} C_a v_{2Z} \frac{dT}{dZ}
\end{aligned} \tag{40}$$

Subtracting the steady state, unperturbed energy equation, equation (36), from equation (40) and substituting relations (10) and (12) for the velocity perturbation yields

$$\begin{aligned}
\left(\frac{d^2}{dZ^2} - \alpha^2 \right) \theta &= -\gamma \frac{d\theta}{dZ} + \\
& \frac{\rho_{a2} C_a \alpha a_2 (T_1 - T_{2\infty}) \gamma}{K_e} e^{-(\alpha+\gamma)Z}
\end{aligned} \tag{41}$$

The solution of equation (41) that vanishes far from the front is

$$\theta = a_3 e^{-\gamma_2 Z} + \frac{(T_1 - T_{2\infty}) \rho_{a2} C_a a_2}{K_e} e^{-(\alpha+\gamma)Z} \tag{42}$$

where

$$\gamma_2 = \gamma/2 + (\alpha^2 + \gamma^2/4)^{1/2} \tag{43}$$

and a_3 is an undetermined constant.

Temperature equality must exist along the perturbed front.

Neglecting higher order terms, the temperature is

$$T_1 = T_{20} + \bar{Z} \left. \frac{dT}{dZ} \right|_0 + \theta|_0 \cdot f(x, y) \quad (44)$$

Substitution of equations (7), (38), and (42) for $\bar{Z}$, T , and θ , respectively, yields

$$\frac{(T_1 - T_{2\infty}) \rho_a c_a}{K_e} a_2 + a_3 + -\gamma(T_1 - T_{2\infty}) \hat{c} = 0 \quad (45)$$

The final equation describing the flow is an equation for the energy flux across the interface. Unlike the zero conductivity case, it should be noted that $T_{20} = T_1$. Therefore, the flux of the solid matrix and the air do not contribute to an energy balance. An energy balance for the unperturbed water flux yields

$$K_e \left. \frac{dT}{dZ} \right|_0 = \rho_v (U_1 - S_1 \epsilon V) h_v + \rho_l \epsilon V (S_1 - S_2) h_{l1} \quad (46)$$

Substitution of equation (2), the water mass balance for the unperturbed flow, into equation (46) yields

$$K_e \left. \frac{dT}{dZ} \right|_0 = \rho_v (U_1 - S_1 \epsilon V) h_{fg} \quad (47)$$

For the perturbed energy flux across the interface, again, the solid flux does not contribute to the energy balance. The perturbed energy flux, neglecting perturbation terms of higher order, is

$$K_e \left[\left. \frac{dT}{dZ} \right|_0 + \bar{Z} \left. \frac{d^2 T}{dZ^2} \right|_0 + f \left. \frac{d\theta}{dZ} \right|_0 \right] =$$

$$\begin{aligned}
& \rho_{a2}(U_2 - S_2 \epsilon V + v_{22})h_{a2} - \rho_{a1}[U_1 - (S_1 + s)\epsilon V + v_{12}]h_{a1} \\
& - \rho_v[U_1 - (S_1 + s)\epsilon V + v_{12}]h_v \\
& + \rho_\ell(1 - S_1 - s)\epsilon V h_{\ell 10} - \rho_\ell(1 - S_2)\epsilon V h_{\ell 20} \quad (48)
\end{aligned}$$

Subtracting the steady state solution, equation (46), and substituting expressions for the perturbations of temperature, saturation, and velocity into equation (48) yields

$$\begin{aligned}
& \alpha(\rho_v h_v + \rho_{a1} h_{a1})a_1 + [\alpha \rho_{a2} h_{a20} - (\alpha + \gamma)(T_1 - T_{2\infty})\rho_{a2} C_a]a_2 \\
& - K_e \gamma_2 a_3 + (\rho_\ell h_{\ell 1} - \rho_v h_v - \rho_{a1} h_{a1})\epsilon V s_1 \\
& + K_e \gamma^2 (T_1 - T_{2\infty})\delta = 0 \quad (49)
\end{aligned}$$

It should be noted that $h_{a20} = h_{a1}$.

Equations (16), (17), (24), (45), and (49) represent five linear, homogeneous equations in the five undetermined constants a_1 , a_2 , a_3 , s_1 , and δ . The requirement for the existence of a non-trivial solution is that the determinant of the coefficients for these equations be zero. This yields the marginal stability condition

$$\frac{-\mu_1 U_1}{k_1} + \frac{\mu_2 U_2}{k_2} - (\rho_1 - \rho_2)g$$

$$+ \frac{K_e \gamma (\gamma - \gamma_2) (T_1 - T_{2\infty})}{\rho_{a2} \rho_l} \frac{[\rho_{a2} (\rho_l - \rho_v) \frac{\mu_1}{k_1} + \rho_{a1} \rho_l \frac{\mu_2}{k_2}]}{[\rho_v h_{fg} \alpha + \rho_{a1} C_a (T_1 - T_{2\infty}) (\alpha + \gamma - \gamma_2)]}$$

(50)

Observe that this expression for marginal stability recovers the previous marginal stability condition, equation (25), where conductivity was neglected. The expression, equation (50), modifies equation (25) by the addition of a single term accounting for the effect of heat transport. Equation (50) is a result similar to the result of Miller [9], where Miller examined steam condensing and displacing water. It will be shown in the sample calculations to follow that the conductivity effect may normally be neglected. The additional margin of stability is not necessary in these flows.

CHAPTER 6

SAMPLE CALCULATION

A computer code was written to evaluate the two marginal stability conditions and the growth rate of the instability.

The physics of the flow of interest, flow from the cavity formed by an underground nuclear explosion, have been described by Morrison [6]. To calculate such a flow, a numerical technique [7] was devised. A portion of the results of the calculations performed by the computer code [7] become the input parameters of the program.

Input parameters to the program are the temperature in region 1, the time after detonation, the distance of the condensation front from the cavity, the total pressure in region 1, and the initial water saturation in region 2. Medium properties, porosity and permeability, and the wave number, α , are also input parameters.

The program uses equations (5), (6), and (7) to calculate the apparent velocities, U_1 and U_2 , and the saturation in region 1, S_1 . Material properties are calculated by subroutines and function subprograms written by Morrison [6]. These results are used to evaluate the two marginal stability conditions, equations (25) and (50), and the growth rate of the instability, equation (35). A listing of the program and its sample output appears in the Appendix A1.

A typical flow to be evaluated assumes a cavity gas composition of 90% steam and 10% air. The ambient temperature of the medium is 300 K. The medium is initially dry. The porosity of the medium is the porosity of Overton sand with a porosity of 0.37 and a permeability of $1.96 \times 10^{-11} \text{ m}^2$. The thermal conductivity and specific heat of the sand backfill were assumed to be $4 \text{ W/m}\cdot\text{K}$ [13] and $900 \text{ J/kg}\cdot\text{K}$, respectively. The calculations for the flow described in trial one (see Appendix A2) show that the flow is stable for the zero conductivity case. Any instability that may arise will be damped out. As the expression for σ , equation (35), shows, the rate of growth of the instability is inversely proportional to the wavelength of the perturbation. As the wavelength decreases, the rate of decay of the perturbation increases. For a wavelength of 6.28 m, $\alpha = 1.0$, the rate of decay is $-7.16 \times 10^{-3} \text{ s}^{-1}$. With the effect of thermal conductivity included, it is shown that the flow is again stable.

Examination of the temperature decay constant, γ , equation (37), shows that the temperature in region 2 reaches ambient conditions in $4 \times 10^{-4} \text{ m}$. Thermal conductivity will have a significant effect only on perturbations with wavelengths of this order or smaller. It is, therefore, concluded that the assumption of a sharp temperature drop across the interface is valid and the effect of thermal conductivity may safely be neglected. The simpler zero conductivity analysis is appropriate and adequate for containment analysis.

CHAPTER 7.

CONCLUSION

The stability of flow from a nuclear cavity has been examined. The flow of interest consists of air and steam at high pressure and temperature moving through a porous medium. The steam condenses at a moving front.

The stability of the flow was analyzed using the normal mode method and a program was written to perform the necessary calculations.

The derived marginal stability conditions, equations (25) and (50), are in agreement with earlier work, in the proper limits. The expression for the growth rate of the instability, equation (55), also extends previous work, reducing to the earlier expressions in the appropriate limits.

From the results of the computer calculations, we find that the flow of interest is stable and any instabilities that occur will decay slowly.

Stability was influenced by gravity, fluid mobilities, and condensation. It was found that the stabilizing effect of conductivity is only important at negligibly small wavelengths.

NOMENCLATURE

- a = undetermined constant, defined in equations (12, 42)
 C = specific heat, $J/kg \cdot K$
 f = dimensionless perturbation of interface position
 g = gravitational acceleration, m/s^2
 h = enthalpy, J/kg
 k = permeability, m^2
 K_e = effective thermal conductivity of porous medium, $W/m \cdot K$
 P = pressure, N/m^2
 p = perturbation of pressure, N/m^2
 S = saturation
 s = perturbation of saturation
 T = temperature, K
 t = time, s
 U = apparent velocity, m/s
 V = velocity of interface, m/s
 v = perturbation of velocity, m/s
 x = spatial coordinate in the plane of the unperturbed interface
 y = spatial coordinate in the plane of the unperturbed interface
 z = spatial coordinate normal to the unperturbed interface
 $\bar{z}$ = perturbation to interface location

Greek Letters

- α = wave number, m^{-1}
 γ = temperature decay constant, m^{-1}
 γ_2 = perturbed temperature decay constant, m^{-1}

δ = amplitude of front displacement, m

ϵ = porosity

θ = perturbation in temperature, K

λ = wavelength, m

μ = viscosity, $\text{N}\cdot\text{s}/\text{m}^2$

ρ = density, kg/m^3

σ = growth rate of instability, s^{-1}

ϕ = velocity potential, m^2/s

Subscripts

0 = evaluated at interface

1 = region 1

2 = region 2

a = air

l = liquid

s = solid

v = vapor

z = z-component

fg = latent heat of vaporization

∞ = evaluated far from the interface in region 2

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Figure 1. Model of flow near the moving interface

Figure 2. Model of flow near the interface, frame at rest with respect to the interface

APPENDIX A1
PROGRAM LISTINGS

PROGRAM GANDALF

THIS PROGRAM CALCULATES VELOCITIES, SATURATION,
MARGINAL STABILITY CONDITIONS, AND GROWTH RATES
OF INSTABILITY FOR MULTIPHASE MULTICOMPONENT
FLOW IN POROUS MEDIA. STEAM, WATER,
AND AIR PROPERTIES ARE ALSO CALCULATED.

BY JAMES M. LAUDE
UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN 1977

DOUBLE PRECISION ALPHA, GAMMA, GAMMA2, DSQRT
REAL MUSTAR, MUAIR1, MUAIR2, MUWAP, KPERM, KPERM1, KPERM2,
C KCOND, NRGVAP, MOLWTG, MOLWTW, NRGLIO, LAMBDA, MUMIX

X IS DISTANCE IN METERS,
TIME IS IN SECONDS,
TEMP1 IS THE TEMPERATURE OF THE MIXTURE IN REGION ONE IN KELVIN,
ALPHA IS A WAVENUMBER EQUAL TO $2\pi/\text{WAVELENGTH}$ IN $1/\text{M}$,
SWO IS WATER SATURATION AND EQUAL TO 1. - S2,
PPSI IS THE TOTAL PRESSURE IN REGION ONE IN LBF/IN**2,
PORSTY IS THE POROSITY OF THE MEDIUM,
KPERM IS THE PERMIABILITY OF THE MEDIUM IN M**2.

NAMelist/BILBO/ X,TIME,TEMP1,ALPHA,SWO,PPSI,PORSTY,KPERM
KCOUNT=1
20 READ(5,BILBO)
IF(X.EQ.0.) GO TO 300
S2= 1.- SWO

THE FOLLOWING ARE CONSTANTS FOR A TYPICAL POROUS MEDIUM.

TEMP2 IS THE INITIAL BED TEMPERATURE IN KELVIN,

TEMP2=300.

C PPSI2 IS THE INITIAL MEDIUM PRESSURE IN LBF/IN**2.

PPSI2= 14.696

C RHO SOL IS THE DENSITY OF THE SOLID IN KG/M**3.

RHO SOL= 2400.

C MOLECULAR WEIGHT OF AIR,

MOLWTG= 28.969

C MOLECULAR WEIGHT OF WATER,

MOLWTW = 18.016

C KCOND IS EFFECTIVE CONDUCTIVITY OF THE MEDIUM IN W/M*K,

KCOND= 4.

C THE FOLLOWING ARE SPECIFIC HEATS FOR AIR, LIQUID WATER, AND
C THE SOLID IN J/KG*K.

CPAIR= 1004.

CPLIO= 4184.

CPSOL= 900.

C PPSIWV IS THE PARTIAL PRESSURE OF WATER VAPOR IN REGION ONE
C IN LBF/IN**2.

CALL VAPORP(TEMP1,PSAT,X)

PPSIWV= PSAT

C PPSIG IS THE PARTIAL PRESSURE OF THE AIR IN REGION ONE OR TWO.

PPSIG1= PPSI - PPSIWV

PPSIG2= 14.696

C CONVERT PRESSURES TO SI UNITS OF MEGA PASCALS.

SIPPSI= PPSI* 6.8948 E-3
SIPPMV= PPSIWV* 6.8948 E-3

CALCULATE PROPERTIES USING F.A. MORRISON'S PROGRAM "PORE".

RHAIR IS THE DENSITY OF AIR IN REGION ONE OR TWO IN KG/M**3

RHAIR1= PPSIG1/(41.6317 * TEMP1) * 1000.

RHAIR2= PPSIG2/(41.6317 * TEMP2) * 1000.

RHOLIQ IS THE DENSITY OF THE LIQUID WATER IN KG/M**3.

CALL WATERD(TEMP1,RHOLIQ,K)

RHOVAP IS THE DENSITY OF THE WATER VAPOR IN KG/M**3.

RHOVAP= SUPERD(TEMP1, PPSIWV)

CALCULATE ENTHALPIES IN J/KG.

NRGVAP= SUPERE(TEMP1)

HVAP= SUPERH(TEMP1, RHOVAP, NRGVAP) * 4184.

CALL WATERE(TEMP1, NRGLIQ,K)

HLIQ=(NRGLIQ + .00164679 * PPSI/RHOLIQ)* 4184.

HFG= HVAP - HLIQ

RHOLIQ= RHOLIQ * 1000.

RHOVAP= RHOVAP * 1000.

MUAIR IS THE VISCOSITY OF AIR IN REGION ONE OR TWO IN N*S/M**2.

MUAIR1= VISAIR(TEMP1) * 1.E-7

MUAIR2= VISAIR(TEMP2) * 1.E-7

MUVAP IS THE VISCOSITY OF STEAM IN N*S/M**2.

MUVAP= VISSTE(TEMP1) * 1.E-7

XGAS= RHAIR1/(RHAIR1 + MOLWTG*RHOVAP/MOLWTW)

MUMIX IS THE VISCOSITY OF THE AIR--STEAM MIXTURE IN REGION ONE.

MUMIX= VISMIX(MUAIR1,MUVAP, XGAS)

RHOMIX IS THE DENSITY OF THE MIXTURE IN REGION ONE IN KG/M**3.

RHOMIX= RHAIR1 + RHOVAP

RHO2 IS THE DENSITY OF THE FLUID IN REGION TWO.

RHO2= RHAIR2

CALCULATE VELOCITIES, SATURATION, AND GROWTH RATES

V IS THE VELOCITY OF THE INTERFACE.

V= X/(2. * TIME)

TERM = (RHOLIQ*CPLIQ*(1. - S2)*(TEMP1 - TEMP2) + RHOSOL*

(1. - PORSTY)*CPSOL*(TEMP1 - TEMP2)/PORSTY) / (RHOVAP*HFG +

RHAIR1 *CPAIR*(TEMP1 - TEMP2))

U1 IS THE APPARENT VELOCITY OF THE MIXTURE IN REGION ONE IN M/S.

U1= PORSTY*V* (S2 + (RHOLIQ - RHOVAP)*TERM/RHOLIQ)

U2 IS THE APPARENT VELOCITY OF THE FLOW IN REGION TWO IN M/S.

U2= PORSTY*V*(S2 + RHAIR1*TERM/RHAIR2)

S1 IS THE SATURATION OF THE MIXTURE IN REGION ONE.

S1= S2 - RHOVAP*TERM/RHOLIQ

CALCULATE RELATIVE PERMEABILITIES IN M**2

SW1= 1. - S1

CALL PERM(SW1, RELIQ, RELMIX)

KPERM1= RELMIX * KPERM

CALL PERM(SW2, RELIQ, RELMIX)

KPERM2= RELMIX * KPERM

ZERO CONDUCTIVITY CALCULATION

RESID1 IS THE RESIDUAL FROM THE MARGINAL STABILITY CONDITION.

GRAVITY IS THE SI CONSTANT OF ACCELERATION IN M/S**2.

GRAVITY= 9.8

```

      T1 = MUMIX*U1/KPERM1
      T2 = MUAIR2*U2/KPERM2
      T3 = (RHOMIX - RH02) * GRAVITY
      RESID1 = T1 + T2 + T3
C   SIGMA IS THE GROWTH RATE OF THE INSTABILITY IN 1./S.
      SIGMA = ALPHA*V*(RESID1)/(MUMIX*U1/KPERM1 + MUAIR2*U2/KPERM2)

C   CONDUCTIVITY NOT EQUAL TO ZERO CALCULATIONS.

C   GAMMA IS THE DECAY CONSTANT FOR THE TEMPERATURE PROFILE IN 1./M.
      GAMMA = RHOLIO*PORSTY*(S2 - S1)*V*HFG/((TEMP1 - TEMP2)*KCOND)
C   GAMMA2 IS THE DECAY CONSTANT FOR THE PERTURBATION IN TEMPERATURE.
      GAMMA2 = GAMMA/2. + DSQRT(.ALPHA**2 + GAMMA**2/4.)

C   RESID2 IS THE RESIDUAL FROM THE MARGINAL STABILITY CONDITION.
      RESID2 = RESID1 + KCOND*GAMMA*( GAMMA - GAMMA2 )*(TEMP1 -
C   TEMP2)*( RHAIR2*( RHOLIO - RHOVAP)*MUMIX/KPERM1 + RHAIR1
C   *RHOLIO*MUAIR2/KPERM2 ) / ( RHAIR2*RHOLIO*( RHOVAP*HFG*ALPHA
C   + RHAIR1*CPAIR*( TEMP1 - TEMP2 )*(ALPHA + GAMMA - GAMMA2)))
      LAMBDA = 2. * 3.1415927/ ALPHA

C
C
C   PRINT 98
      FORMAT(' ')
      PRINT 99
      FORMAT(' ')
      PRINT 100, KCOUNT
      FORMAT(' ',10X,'TRIAL NUMBER ',I3)
      PRINT 101
      FORMAT(' ',10X,'-----')
      PRINT 110, X, TIME
      FORMAT(' ',X = 'F7.1,' METER' ,5X,'TIME = ',F8.1,
C   ' SECONDS')
      PRINT 113, ALPHA, LAMBDA
      FORMAT(' ',ALPHA = 'D10.3,' 1./METER', 3X,'LAMBDA = ',
C   E10.3,' METER')
      PRINT 117, TEMP1, PORSTY
      FORMAT(' ',TEMPERATURE IN REGION ONE = 'F6.1, ' KELVIN',
C   3X,'POROSITY = ',F5.3)
      PRINT 118, SWD, SIPPSI
      FORMAT(' ',WATER SATURATION = 'F6.3,3X,'TOTAL PRESSURE = '
C   F5.2,' MEGA PASCAL')
      PRINT 120, SIPPHV
      FORMAT(' ',PARTIAL PRESSURE OF WATER VAPOR = 'F5.2,
C   ' MEGA PASCAL')
      PRINT 121
      FORMAT(' ')
      PRINT 122, V
      FORMAT(' ',VELOCITY OF THE INTERFACE = 'E13.6,
C   ' METER/SECOND')
      PRINT 124, U1
      FORMAT(' ',VELOCITY OF FLOW IN REGION ONE = 'E13.6,
C   ' METER/SECOND')
      PRINT 126, U2
      FORMAT(' ',VELOCITY OF FLOW IN REGION TWO = 'E13.6,
C   ' METER/SECOND')
      PRINT 128, S1
      FORMAT(' ',SATURATION OF MIXTURE IN REGION ONE = 'E13.6)
      PRINT 130
      FORMAT(' ')
      PRINT 131
      FORMAT(' ')

```

```

132 PRINT 132
132 FORMAT('0','FOR THE CONDUCTIVITY EQUAL TO ZERO CASE: ')
133 PRINT 133
133 FORMAT('0',' 1) MARGINAL STABILITY CONDITION: ')
134 PRINT 134
134 C FORMAT('0','RESIDUAL = U1*MUMIX/KPERM1+U2*MUAIR2/KPERM2 = ',
134 C '(RHOMIX-RHO2)*GRAVITY')
135 PRINT 135, T1, T2, T3, RESID1
135 FORMAT('0','RESIDUAL = ',E12,5,'+',E12,5,'+',E12,5,'*',E12,5)
136 PRINT 136
136 FORMAT('0',' 2) GROWTH RATE OF INSTABILITY: ')
137 PRINT 137
137 C FORMAT('0','SIGMA = ALPHA*V*( RESIDUAL )/(MUMIX*U1/KPERM1 + ',
137 C 'U2*MUAIR2/KPERM2)')
137 PRINT 137, SIGMA
137 C FORMAT('0','SIGMA = ',E13,6, ' 1./SECOND')
138 PRINT 138
138 FORMAT('0')
139 PRINT 139
139 FORMAT('0','FOR THE CONDUCTIVITY NOT EQUAL ZERO CASE: ')
140 PRINT 140
140 FORMAT('0',' 1) MARGINAL STABILITY CONDITION: ')
141 PRINT 141, RESID2
141 FORMAT('0','RESIDUAL = ', E13,6 )
142 PRINT 142
142 FORMAT('0','GAMMA--TEMPERATURE DECAY CONSTANT: ')
143 PRINT 143, GAMMA
143 FORMAT('0','GAMMA = ',D15,8, ' 1./METER' )
144 PRINT 144
144 FORMAT('0','GAMMA2 TEMPERATURE PERTURBATION DECAY CONSTANT: ')
145 PRINT 145, GAMMA2
145 FORMAT('0','GAMMA2 = ',D15,8, ' 1./METER' )
146 PRINT 146
146 FORMAT('0')
147 PRINT 147
147 FORMAT('0',' MATERIAL PROPERTIES FOR THIS TRIAL: ')
148 PRINT 148, RHAIR1, RHAIR2, RHOVAP
148 C FORMAT('0','RHAIR1 = ',E12,5,2X, 'RHAIR2 = ', E12,5,2X,
148 C 'RHOVAP = ', E12,5 )
148 PRINT 148, RHOLIQ, RHOMIX
148 C FORMAT('0','RHOLIQ = ',E12,5,4X, 'RHOMIX = ',E12,5,6X,
148 C 'KG/M**3')
149 PRINT 149, MUVP, MUAIR1, MUAIR2
149 C FORMAT('0','MUVP = ', E12,5,2X, 'MUAIR1 = ',
149 C 'E12,5,2X, 'MUAIR2 = ',E12,5)
149 PRINT 149, HUMIX
149 C FORMAT('0','HUMIX = ',E12,5,6X, 'N**3/M**2')
150 PRINT 150, HVAP, HLIQ, HFG
150 C FORMAT('0','HVAP = ',E12,5,2X, 'HLIQ = ',E12,5,2X, 'HFG = ',
150 C 'E12,5, ' J/KG')
151 PRINT 151, KPERM, KPERM1, KPERM2
151 C FORMAT('0','KPERM = ',E12,5,2X, 'KPERM1 = ',E12,5,2X,
151 C 'KPERM2 = ',E12,5, ' M**2')
151 C KCOUNT = KCOUNT + 1
151 GO TO 20
300 STOP
END

```

VARIOUS SUBROUTINES AND FUNCTION SUBPROGRAMS FROM "PORE" .

FUNCTION SUPERD (TK,PPSI)

C CALCULATES THE DENSITY OF SUPERHEATED STEAM, GIVEN THE

TEMPERATURE AND PRESSURE, USING CLAUSIUS EQUATION OF STATE
 $P(V - B) = R \cdot T$, B IS CHOSEN SO THAT PROPERTIES AGREE WITH
 TABULATED PROPERTIES OF DRY SATURATED STEAM EXCEPT NEAR THE
 CRITICAL POINT, ALSO SEE SUPERH AND SUPERP, TK IS TEMPERATURE
 IN DEGREES KELVIN, PPSI IS PRESSURE IN LBF/IN**2, SUPERD IS
 DENSITY IN G/CC,

B = -8.8
 R = 82.9353
 SUPERD = $1.7 / (B + R \cdot TK / PPSI)$
 RETURN
 END

FUNCTION SUPERE(TK)

CALCULATES THE INTERNAL ENERGY OF WATER VAPOR AT LOW PRESSURE
 GIVEN THE TEMPERATURE, USING A QUADRATIC FIT TO DATA IN STEAM
 TABLES BY KEENAN, KEYES, HILL, AND MOORE, TK IS TEMPERATURE
 IN DEGREES KELVIN, U IS INTERNAL ENERGY IN BTU/LBM, SUPERE IS
 INTERNAL ENERGY IN CAL/G,

TR = $1.8 \cdot TK$
 $U = 868.664 + .2907 \cdot TR + .00004 \cdot TR \cdot TR$
 SUPERE = $U / 1.8$
 RETURN
 END

FUNCTION SUPERH(TK, RHOCGS, ENERGY)

CALCULATES THE ENTHALPY OF SUPERHEATED STEAM, GIVEN THE ENERGY,
 TEMPERATURE, AND DENSITY USING CLAUSIUS EQUATION OF STATE
 $P(V - B) = R \cdot T$, B IS CHOSEN SO THE P, V, T AGREE WITH TABULATED
 PROPERTIES OF DRY SATURATED STEAM EXCEPT NEAR THE CRITICAL POINT,
 ALSO SEE SUPERD AND SUPERP, GAS CONSTANT FROM PAGE 134 OF KEENAN,
 KEYES, HILL, AND MOORE, R EQUAL TO .46151 J/G·K * 1. CAL/4.184 J
 AVOIDS 0/0 CALCULATION WHICH MAY RESULT FROM PRESSURE/DENSITY
 CALCULATION, TK IS TEMPERATURE IN DEGREES KELVIN, RHOCGS IS
 DENSITY IN G/CC, ENERGY HAS UNITS OF CAL/G, SUPERH IS ENTHALPY
 IN CAL/G,

R = 1103035
 B = -8.8
 SUPERH = $ENERGY + R \cdot TK / (1. - B \cdot RHOCGS)$
 RETURN
 END

FUNCTION VISAIR(TK)

CALCULATES THE VISCOSITY OF AIR AT ATMOSPHERIC PRESSURE, GIVEN
 THE TEMPERATURE, USING THE RELATION GIVEN ON PAGE 10 NBS
 CIRCULAR 564, TK IS THE TEMPERATURE IN DEGREES KELVIN, VISAIR
 IS VISCOSITY IN MICRPOISE,

A = 145.8
 B = 110.4
 VISAIR = $.1 \cdot A \cdot TK \cdot 1.5 / (TK + B)$
 RETURN
 END

FUNCTION VISSTE(TK)

CALCULATES THE VISCOSITY OF STEAM AT ATMOSPHERIC PRESSURE GIVEN THE TEMPERATURE, USING THE RELATION GIVEN ON PAGE 10 OF NBS CIRCULAR 564. TK IS TEMPERATURE IN DEGREES KELVIN. VISSTE IS VISCOSITY IN MICHOPOISE. THE CONSTANT D IS MISPRINTED IN NBS CIRCULAR 564.

```

      IF(TK.GT,800) GO TO 100
      A=0.361
      B=10.2
      VISSTE=A*TK*B
      RETURN
100  C=39.37
      D=3315
      E=.001158
      VISSTE=C*TK**1.5/(D+TK+E*TK*TK)
      RETURN
      END

```

FUNCTION VISMIX(MU1,MU2,X1)

CALCULATES VISCOSITY OF MIXTURE OF AIR AND STEAM USING EQ. 11 OF C. R. WILKE, J. CHEM PHYS. 18, 517-519, (1950). MU1 AND MU2 ARE VISCOSITIES OF AIR AND STEAM, RESPECTIVELY, AT MIXTURE TEMPERATURE. X1 IS THE MOLE FRACTION OF AIR. VISMIX IS MIXTURE VISCOSITY IN SAME UNITS AS MU1 AND MU2.

```

      DIMENSION MU(2), M(2), X(2)
      REAL M,MU,MU1,MU2
      MU(1)=MU1
      MU(2)=MU2
      X(1)=X1
      X(2)=1.-X1
      M(1)=28.969
      M(2)=18.016
      VISMIX=0.
      DO 200 I=1,2
      DENOM=0.
      DO 100 J=1,2
      DENOM=DENOM+X(J)*(1.+((MU(I)/MU(J))**.5)+
100  (M(J)/M(I))**.25)**2/(8.+(1.+M(I)/M(J))**.5)
200  CONTINUE
      VISMIX=VISMIX+X(I)*MU(I)/DENOM
      RETURN
      END

```

SUBROUTINE PERM(SW, PERMW, PERMG)

WHEN WATER SATURATION EXCEEDS IRREDUCIBLE WATER SATURATION, CALCULATES RELATIVE PERMEABILITIES OF WATER, PERMW, AND GAS, PERMG, USING RELATIONS GIVEN IN WYLLIE AND GARDNER, WORLD OIL 146, (1958). THESE RELATIONS ARE BASED ON DRAINAGE CYCLE TESTS. OTHERWISE, RELATIVE PERMEABILITIES OF GAS AND WATER ARE 1. AND 0, RESPECTIVELY. SW IS WATER SATURATION, SWI IS IRREDUCIBLE WATER SATURATION.

```

      SWI=.1
      IF(SW.GT,SWI) GO TO 100
      PERMW=0.
      PERMG=1.0

```

```

102 RETURN
   SWSTAR=(SW-SW1)/(1. + SW1)
   PERMW= SWSTAR**3
   PERMG=(1. + SWSTAR)**3
   RETURN
END

```

SUBROUTINE WATERD(TK,DFCGS,K)

```

C CALCULATES THE DENSITY OF SATURATED WATER, GIVEN THE TEMPERATURE,
C USING THE EQUATION 18 ON PAGE 21 OF STEAM TABLES BY KEENAN AND
C KEYES. RETURN (K=1) AND NO CALCULATION IF TEMPERATURE EXCEEDS
C CRITICAL TEMPERATURE. TK IS TEMPERATURE IN DEGREES KELVIN,
C DFCGS IS THE DENSITY IN G/CC.

```

```

TC= TK = 273.15
TCRIT= 374.136
IF (TCRIT,GT,TC) GO TO 100
  K=1
  RETURN
100 CONTINUE
  K=0
  VCRIT=3.1975
  A=-0.3151548
  B=-0.001203374
  C=7.48908/10,**13
  D=0.1342489
  E=-0.003946263
  X= TCRIT - TC
  ROOT3= X**(.1/3.)
  VFCGS = (VCRIT + A*ROOT3 + B*X + C*X*X*X*X) / (1. +
C D*ROOT3 + E*X)
  OFCGS= 1./ VFCGS
  RETURN
END

```

SUBROUTINE WATERE(TK,NRG,K)

```

C CALCULATES THE INTERNAL ENERGY OF LIQUID WATER, GIVEN THE
C TEMPERATURE, USING A QUADRATIC FIT TO DATA IN STEAM TABLES BY
C KEENAN, KEYES, HILL, AND MOORE. TK IS TEMPERATURE IN DEGREES
C KELVIN. U IS INTERNAL ENERGY IN BTU/LBM. NRG IS INTERNAL
C ENERGY IN CAL/G. RETURN (K=1) AND NO CALCULATION IF TEMPERATURE
C EXCEEDS CRITICAL TEMPERATURE.

```

```

REAL NRG
TCRIT=647.286
IF(TCRIT,GT,TK) GO TO 100
K=1
RETURN
CONTINUE
K=2
TR=1.8*TK
U=-448.1 + .8432*TR + .00014*TR*TR
NRG= U/1.8
RETURN
END

```

SUBROUTINE VAPORP(TK,VP,K)

C

C CALCULATES VAPOR PRESSURE, GIVEN THE TEMPERATURE, USING THE
 C RELATION ON PAGE 141 OF STEAM TABLES BY KEENAN, KEYES, HILL,
 C AND MOORE. RETURN (K=1) AND NO VAPOR PRESSURE CALCULATION IF
 C TEMPERATURE EXCEEDS CRITICAL TEMPERATURE. TK IS TEMPERATURE
 C IN DEGREES KELVIN. VP IS VAPOR PRESSURE IN LBF/IN**2.

```

      DIMENSION F(8)
      TC= TK - 273.15
      TCRIT= 374.136
      IF(TCRIT.GT.TC) GO TO 100
      K=1
      RETURN
100   CONTINUE
      K=0
      PCRT= 220.80
      TAU=1000./TK
      F(1)= -741.9242
      F(2)= -29.72100
      F(3)= -1.55286
      F(4)= -0.8685635
      F(5)= 0.1094898
      F(6)= 0.439993
      F(7)= 0.2520658
      F(8)= 0.05218604
      DUMMY= 0.
      DO 200 I=1,8
      J= I - 1
200   DUMMY= DUMMY + F(I)*( 0.65 - .01*TC)**J
      DUMMY= DUMMY*TAU*.00001*( TCRIT - TC)
      P = PCRT*EXP (DUMMY)
      VP= 14.5038*P
      RETURN
      END
  
```

APPENDIX A2
SAMPLE OUTPUT

TRIAL NUMBER 1

X = 9.6 METER TIME = 60.0 SECONDS
 ALPHA = 0.1000 01 1./METER LAMBDA = 0.628E 01 METER
 TEMPERATURE IN REGION ONE = 573.7 KELVIN POROSITY = 0.370
 WATER SATURATION = 0.000 TOTAL PRESSURE = 9.40 MEGA PASCAL
 PARTIAL PRESSURE OF WATER VAPOR = 8.65 MEGA PASCAL

VELOCITY OF THE INTERFACE = 0.801175E-01 METER/SECOND
 VELOCITY OF FLOW IN REGION ONE = 0.390141E 00 METER/SECOND
 VELOCITY OF FLOW IN REGION TWO = 0.153282E 01 METER/SECOND
 SATURATION OF MIXTURE IN REGION ONE = 0.162536E 00

FOR THE CONDUCTIVITY EQUAL TO ZERO CASE:

1) MARGINAL STABILITY CONDITION:

RESIDUAL = $-U_1 \cdot \mu_{MIX} / K_{PERM1} + U_2 \cdot \mu_{AIR2} / K_{PERM2} - (\rho_{MIX} - \rho_{H2O}) \cdot g$
 RESIDUAL = -0.68633E 08 + 0.14437E 07 + -0.48261E 03 = -0.67189E 08

2) GROWTH RATE OF INSTABILITY:

SIGMA = $\text{ALPHA} \cdot V \cdot (\text{RESIDUAL}) / (\mu_{MIX} \cdot U_1 / K_{PERM1} + U_2 \cdot \mu_{AIR2} / K_{PERM2})$
 SIGMA = -0.768169E-01 1./SECOND

FOR THE CONDUCTIVITY NOT EQUAL ZERO CASE:

1) MARGINAL STABILITY CONDITION:

RESIDUAL = $-0.671918E\ 03$

GAMMA--TEMPERATURE DECAY CONSTANT:

GAMMA = $0.26811941D\ 05\ 1./METER$

GAMMA2 TEMPERATURE PERTURBATION DECAY CONSTANT:

GAMMA2 = $0.26811941D\ 05\ 1./METER$

MATERIAL PROPERTIES FOR THIS TRIAL:

RHAIR1 = $0.45903E\ 01$ RHAIR2 = $0.11767E\ 01$ RHOVAP = $0.45832E\ 02$

RHOLIQ = $0.71138E\ 03$ RHOMIX = $0.50423E\ 02$ KG/M**3

MUVAP = $0.19691E-04$ MUAIR1 = $0.29286E-04$ MUAIR2 = $0.18460E-04$

MUMIX = $0.20309E-04$ N*S/M**2

HVAP = $0.30048E\ 07$ HLIQ = $0.13426E\ 07$ HFG = $0.16621E\ 07$ J/KG

KPERM = $0.19600E-10$ KPERM1 = $0.11545E-12$ KPERM2 = $0.19600E-10$ M**2

NOTICE

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