LA-8469-MS

Informal Report

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Fuel Temperature Determination for ICF Microspheres

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LA-8469-MS
Informal Report

UC-21
Issued: August 1980

Fuel Temperature Determination for ICF Microspheres

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FUEL TEMPERATURE DETERMINATION FOR ICF MICROSPHERES

by

D. B. Henderson and D. V. Giovanielli

ABSTRACT

The common heuristic expression for estimating thermonuclear burn in ICF microspheres far from bootstrap heating is often used for inversion to obtain peak temperatures from experimental data. It contains an ad hoc or fitted parameter and lacks some parameters of obvious influence in actual systems. We present an alternative expression, which may be usefully inverted and does not suffer these defects.

A common heuristic formula for the yield of an imploded ICF microsphere is

$$Y = \left(\frac{M}{M_A} \right)^2 \left(\frac{4}{3} \pi R_o^3 \right)^{-1} \left(\frac{1}{4} \langle \sigma v \rangle \right) \tau, \quad (1)$$

$$\tau \approx \tau_c = f R_o / C_s, \quad = f R_o (Y P_o / \rho_o)^{-1/2}, \quad (2)$$

where Y is the number of reactions (equal to the number of neutrons for DT and twice that number for DD). For DD the term $(\frac{1}{4} \langle \sigma v \rangle)$ is simply $\langle \sigma v \rangle$. M is the fuel mass, M_A is the average ion mass, τ_c is the characteristic burn time, and R_o is the compressed fuel radius. Here and throughout we use the subscript "o" to indicate conditions at maximum compression. Both the reactivity $\langle \sigma v \rangle$ and sound speed C_s are functions of the peak temperature, T_o ; P_o and ρ_o are the peak pressure and density of the homogeneous fuel; Y is taken to be 5/3. The factor f is used to account for the dynamics of the implosion. It can be estimated¹ as 0.25; more usually, it is obtained by fits to computer simulations. Clearly, this formula should be applied only for small fractional

yield, $Y \ll (M/M_A)$, which is appropriate for all experiments to date. While the above formulae are instructive, their deficiencies are soon apparent when inverted to obtain the burn temperature from data (number of neutrons and fuel density) and fixed parameters. Not only is the factor f fixed by an ad hoc procedure, but some obviously important parameters are absent. Actual ICF experiments are not conducted with simple spheres of DT, but involve layers of other materials.

We consider a model as shown in Fig. 1. Here a pusher shell of mass M_s is accelerated by some outer ablator material. This shell then compresses the fuel mass M to a minimum radius $R = R_0$. We take the fuel to be a homogeneous perfect gas in which the pressure rises with adiabatic compression to stop the shell and then to accelerate it outward. We integrate the yield throughout this process. The principal limiting assumption is that the heat deposited from the burn is insignificant; for experiments to date this is clearly satisfied. In this model no ad hoc parameters such as f are required.

Fig. 1. Target model used for the calculation. Ablated material is assumed to accelerate the pusher inward, causing it to adiabatically compress the fuel. No dynamics internal to the pusher shell are included and the fuel is taken to be homogeneous.

Taking the fuel temperature as T , we may simply express the force F accelerating the shell as

$$F = 4\pi R^2 \cdot P ,$$

$$P = (2 \cdot \rho / M_A) kT ,$$

$$\rho = M / \frac{4}{3} \pi R^3 ,$$

$$T = T_0 (\rho / \rho_0)^{\gamma-1} = T_0 (\rho / \rho_0)^{2/3} ,$$

where the 2 in the expression for pressure P accounts for a fully ionized fuel. Substituting, we obtain

$$F = F_0 (R_0 / R)^3 ,$$

$$F_0 = 6(M/M_A)(kT_0/R_0) .$$

Neglecting any internal dynamics within the pusher shell, we have

$$\frac{d^2}{dt^2} (R/R_0) = (F_0/R_0 M_S) (R_0/R)^3 .$$

Defining the origin of time $t = 0$ at turnaround, $dR/dt = 0$, and integrating we obtain

$$R = R_0 (1 + F_0 t^2 / R_0 M_S)^{1/2} . \quad (3)$$

Integrating the yield rate over the implosion and the explosion (bounce), we have

$$Y = 2 \left(\frac{M}{M_A} \right)^2 \int_0^\infty \left(\frac{4}{3} \pi R^3 \right)^{-1} \left(\frac{1}{4} \langle \sigma v \rangle \right) dt .$$

For the reactivity $\langle \sigma v \rangle$ we use a Gamow law fit:²

$$\langle \sigma v \rangle = A_1 T^{-2/3} \exp(-A_2 T^{-1/3}) , \quad (4)$$

where for DT, $A_1 = 3.68 \times 10^{-12} \text{ cm}^3 \text{ sec}^{-1} (\text{keV})^{2/3}$, $A_2 = 19.94 (\text{keV})^{1/3}$
 and for DD, $A_1 = 9.32 \times 10^{-14} \text{ cm}^3 \text{ sec}^{-1} (\text{keV})^{2/3}$, $A_2 = 18.76 (\text{keV})^{1/3}$.
 For applications of interest between 0.1 and 10 keV, $A_2 T^{-1/3}$ is always large,
 between 8.7 and 41.

Because of the steep dependence of $\langle \sigma v \rangle$ on temperature, we may substitute a
 power law fit within the integration:

$$\langle \sigma v \rangle = \langle \sigma v \rangle_0 (T/T_0)^\ell,$$

$$\langle \sigma v \rangle = \langle \sigma v \rangle_0 (R_0/R)^{2\ell},$$

where from Eq. (4)

$$\ell(T_0) = 1/3(A_2 T_0^{-1/3} - 2).$$

Integrating, we obtain

$$\begin{aligned} Y &= \frac{3}{8\pi} \left(\frac{M}{M_A} \right)^2 \langle \sigma v \rangle_0 R_0^{-3} \int_0^\infty (R/R_0)^{-3-2\ell} dt \\ &= \frac{3}{8\pi} \left(\frac{M}{M_A} \right)^2 \langle \sigma v \rangle_0 R_0^{-3} \int_0^\infty \left[1 + \frac{F_0}{R_0 M_s} t^2 \right]^{-\frac{3}{2}-\ell} dt. \end{aligned}$$

By substituting $X = (F_0/R_0 M_s) t^2$, the integral becomes one of the standard
 definitions of the Beta function B. Substituting for F_0 and ℓ , we find

$$Y = \left(\frac{M}{M_A} \right)^{3/2} \frac{M_s^{1/2}}{R_0^2} \tilde{Y}(T_0) = \left(\frac{4\pi}{3} \right)^{2/3} \frac{M^{5/6} M_s^{1/2} \rho_0^{2/3}}{M_A^{3/2}} \tilde{Y}(T_0), \quad (5)$$

where $\tilde{Y}(T_0)$ is a function only of the peak temperature T_0 :

$$\tilde{Y} = 609. \begin{bmatrix} x1 \text{ for DT} \\ x4 \text{ for DD} \end{bmatrix} A_1 T_0^{-7/6} \exp(-A_2 T_0^{-1/3}) B \left[\frac{1}{2}, \frac{1}{3}(1 + A_2 T_0^{-1/3}) \right] \quad (6)$$

The auxiliary functions $\ell(T_0)$ and $B[1/2, 1 + \ell(T_0)]$ are plotted for DT in Fig. 2. Over our range of interest we may evaluate B by differentiating and expanding for the large argument. We obtain

$$\frac{d \ln B}{d \ln T_0} = \frac{1}{6} \left\{ 1 - \frac{1}{4}(A_2^{-1} T_0^{1/3}) - \frac{55}{8}(A_2^{-1} T_0^{1/3})^2 + \dots \right\} = 0.162$$

for $A_2^{-1} T_0^{1/3} = 0.05$.

Fig. 2. The functions ℓ (the exponent in the power law fit of $\langle \sigma v \rangle$ to fuel temperature) and B (the Beta function) versus fuel temperature.

Thus a good fit is

$$B = A_3 T_0^{0.162} , \quad (7)$$

where $A_3 = 0.682$ for DT and 0.703 for DD. Making this substitution into Eq. (6), we have

$$\tilde{Y} = \begin{cases} 1.528 \times 10^{-9} T_0^{-1.00} \exp(-19.94 T_0^{-1/3}) & \text{for DT} \\ 3.99 \times 10^{-11} T_0^{-1.00} \exp(-18.76 T_0^{-1/3}) & \text{for DD} \end{cases} \quad (8a)$$

$$(8b)$$

It is interesting to compare Eqs. (1) and (6) and to solve for the effective confinement time τ , we obtain

$$\begin{aligned} \tau &= \frac{1}{\sqrt{6}} \sqrt{\frac{M_S}{M}} B \frac{R_G}{\sqrt{kT_0/M_A}} \\ &= \sqrt{\frac{5}{9}} \sqrt{\frac{M_S}{M}} B \frac{\tau_C}{f} , \end{aligned}$$

where τ_C and f are defined in Eq. (2) and B may be evaluated from Eq. (7). Figures 3 and 4 are plots of $\tilde{Y}$ for DT from Eq. (8a), which cannot be distinguished from Eq. (6). Figure 4 is especially useful for inversion of data. In addition to the function $\tilde{Y}(T_0)$ we have indicated with the dashed line a rather good fit $\tilde{Y}(T_0) = 4.29 \times 10^{-18} T_0^{-7.13}$, which has a simple algebraic inversion. For use in scaling experimental data by using the graph, we have indicated conveniently chosen typical values of $\tilde{Y}$. Other values of parameters may be scaled from these.

The Gamow fit for reactivity $\langle \sigma v \rangle$ has been supplemented by better fits for the cross section,^{3,4} which are, however, not in as general use. Over the range $0.3 < T_0 < 1$ keV we have fit the data of Hale³ for DT to Eq. (4), obtaining $A'_1 = 3.72 \times 10^{-12} \text{ cm}^3 \text{ sec}^{-1} (\text{keV})^{2/3}$ and $A'_2 = 20.16 (\text{keV})^{1/3}$. Using these values A'_3 becomes 0.678 for DT and Eq. (8a) becomes

$$\tilde{Y} = 1.536 \times 10^{-9} T_0^{-1.00} \exp(-20.16 T_0^{-1/3}) \text{ for DT} . \quad (8a')$$

The simple power law fit becomes $\tilde{Y} = 3.46 \times 10^{-18} T_0^{7.22}$. These are shown in Fig. 5. Over this range the temperatures implied for given data are higher by three to four percent. These values are probably more accurate; however, the differences are masked by typical experimental uncertainties.

Fig. 3. The normalized neutron yield, $\tilde{Y}$, versus peak fuel temperature, T_0 .

Fig. 4. The normalized neutron yield, $\tilde{Y}$, versus peak fuel temperature for the range of fuel temperatures of current interest in ICF experiments (solid curve). An easily inverted power law fit of $\tilde{Y}$ to T_0 is also shown (dashed curve), along with the typical values of $\tilde{Y}$ for two arbitrary choices of experimental parameters.

Fig. 5. The normalized neutron yield, $\tilde{Y}$, versus peak fuel temperature, T_0 , for the range of fuel temperatures of current interest in ICF experiments. These curves are little different from those of Fig. 4, but use more modern data in the evaluation of the reactivity $\langle \sigma v \rangle$.

REFERENCES

1. Fraley, G. S., Linnebur, E. J., Mason, R. J., and Morse, R. L., "Thermonuclear Burn Characteristics of Compressed Deuterium-Tritium Microspheres," *Phys. Fluids*. 17, 474 (1974).
2. Stoval, E. J., unpublished. Cited in Glasstone, S., and Lovberg, R. H., Controlled Thermonuclear Reactions, (Princeton, Van Nostrand, 1960), p. 20
3. Hale, G. M., Los Alamos Scientific Laboratory, private communication (1975).
4. Peres, A., "Fusion Cross Sections and Thermonuclear Reaction Rates," *J. Appl. Phys.* 50, 5569 (1979).