MASTER

SLAC-PUB-2584
August 1980
(M)

CONF-800724--26

SUPPRESSION OF SUPERHEAVY MAGNETIC MONOPOLES
IN GRAND UNIFIED THEORIES*

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ABSTRACT

The superheavy magnetic monopoles predicted by grand unified theories would not be produced in significant numbers if electromagnetic gauge invariance is spontaneously broken when the temperature T is greater than $T_c \gtrsim 1$ TeV.

Grand unified theories predict the existence of superheavy magnetic monopoles.¹⁻⁷ These monopoles are of the type discovered by 't Hooft and Polyakov.⁶ They exist if a semi-simple group is broken down to a subgroup which contains U_1 factor. The monopole mass M_m is of order M_X/α , where $\alpha = g^2/4\pi$, g is a gauge coupling and M_X is a typical mass of a gauge boson associated with a broken generator. For example, in Georgi-Glashow model $M_X \approx 10^{14}$ GeV and $M_m \approx 10^{16}$ GeV.

The problem of monopole production and their subsequent annihilation, in the context of a second order or weakly first order phase transition, was analyzed by Zeldovich and Khlopov¹ and by Preskill.² In Preskill's analysis, it was found that relic monopoles would exceed present bounds by roughly 14 orders of magnitude. Since it seems difficult to modify the estimated annihilation rate, one must find a way which suppresses the production of these monopoles.

One interesting solution to this problem, suggested by Preskill,² Einhorn et al.,³ and Guth and Tye⁴ is that the phase transition at which the U_1 factor occurs is strongly first order. The problem of monopole production in a strongly first order phase transition was treated in detail by Guth and Tye.

In this talk I will describe an alternative scenario for the suppression of monopoles, developed in collaboration with P. Langacker,⁷ in which the universe undergoes two or more phase transitions (which can be second order)

$$G \xrightarrow{T_1} H_1 \xrightarrow{T_2} H_2 \dots \xrightarrow{T_n} H_n \xrightarrow{T_c} SU_3^C \times U_1^{EM}, \quad (1)$$

where U_1^{EM} is not a subgroup of H_n . The critical temperature at which U_1^{EM} appears is $T_c \gtrsim 1$ TeV. For example, in SU_5 model,

$$SU_5 \xrightarrow{T_1 \lesssim M_X} SU_3^C \xrightarrow{T_c \gtrsim 1 \text{ TeV}} SU_3^C \times U_1^{EM}. \quad (2)$$

Since $T \ll M_m \approx 10^{16}$ GeV no monopoles will be produced.

We consider a model which at $T=0$ is the standard SU_5 model with symmetry breaking

$$SU_5 \rightarrow SU_3^C \times SU_2 \times U_1 \rightarrow SU_3^C \times U_1^{EM} \quad (3)$$

* Work supported by the Department of Energy, contract DE-AC03-76SF00515.
(Invited talk presented at the XXth International Conference on High Energy Physics, University of Wisconsin, Madison, July 17-23, 1980.)

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by an adjoint Higgs representation and three five Higgs representations. (It turns out that this is the minimum number of five Higgs representations required for our purpose.) The Higgs potential at $T=0$ is

$$V = V_\phi + V_{\phi\phi} + V_\psi \quad (4a)$$

$$V_\phi = -\frac{1}{2}m^2 \text{Tr} \phi^2 + \frac{1}{4}a(\text{Tr} \phi^2)^2 + \frac{1}{2}b \text{Tr} \phi^4 \quad (4b)$$

$$V_{\phi\phi} = \sum_{i=1}^3 \left[\alpha_i \phi_i^\dagger \phi_i \text{Tr} \phi^2 + \beta_i \phi_{ia}^\dagger \phi_{ab}^2 \phi_{ib} \right] \quad (4c)$$

$$V_\psi = \sum_{i=1}^3 \left[-\mu_i^2 \phi_i^\dagger \phi_i + \lambda_i (\phi_i^\dagger \phi_i)^2 \right] + \sum_{i<j} \left[\sigma_{ij} (\phi_i^\dagger \phi_i) (\phi_j^\dagger \phi_j) + \rho_{ij} (\phi_i^\dagger \phi_j) (\phi_j^\dagger \phi_i) + n_{ij} (\phi_i^\dagger \phi_j)^2 + n_{ij}^* (\phi_j^\dagger \phi_i)^2 \right] \quad (4d)$$

where ϕ is an adjoint Higgs representation and ϕ_i are five Higgs representations. We have imposed discrete symmetries $\phi \rightarrow -\phi$ and $\phi_i \rightarrow -\phi_i$ for simplicity.

For $0 \leq T \ll M_X$ we need only to consider the $SU_2 \times U_1$ part of the model. (We assume that SU_3 is never broken.) Therefore let us first consider $SU_2 \times U_1$ part of the model with Higgs potential V_ϕ , Eq. (4d), in which ϕ_i are SU_2 doublets.

At $T=0$ we choose the parameters in the potential such that the vacuum expectation values (VEV) of the Higgs fields are $\langle \phi_1(0) \rangle = (0 \ v_1)^T/\sqrt{2}$ and $\langle \phi_2(0) \rangle = \langle \phi_3(0) \rangle = 0$. $SU_2 \times U_1$ symmetry is broken down to U_1^{EM} . We also require the parameters satisfy the sufficient conditions for V_ϕ to be bounded below.⁷ We also take $\rho_{ij} > 2|\eta_{ij}|$ so that when two fields ϕ_i and ϕ_j develop VEV they want to be orthogonal, i.e., $\phi_i = (0 \ v_i)^T/\sqrt{2}$ and $\phi_j = (v_j \ 0)^T/\sqrt{2}$. We want U_1^{EM} to be unbroken at $T=0$ but broken for $T > T_c$.

At high temperatures, we have to calculate the finite temperature effective potential to study the symmetry behavior of the system.⁸⁻¹⁰ For sufficiently high T , the ensemble averages $\langle \phi_i(T) \rangle$ can be obtained by minimizing the effective potential^{9,10}

$$V_\phi(T) = V_\phi(0) + \sum_{i=1}^3 \frac{1}{2} T^2 F_i \phi_i^\dagger \phi_i \quad (5)$$

The functions F_i are given by⁷

$$F_i = (3g^2 + g'^2)/8 + \lambda_i + \sum_{j \neq i} \left[-\frac{\sigma_{ij}}{3} + \frac{\rho_{ij}}{6} \right] + \text{Yukawa terms} \quad (6)$$

For small fermion masses Yukawa terms are negligible. The effective mass terms at high temperature will be $M_i^2(T) = \mu_i^2 - \frac{1}{2} F_i T^2$. Therefore, if $F_i > 0$ then for $T^2 \geq 2\mu_i^2/F_i$ $SU_2 \times U_1$ will be restored. However, if $F_i < 0$ the symmetry will stay broken^{11,12} or may be further broken down to a lower symmetry⁹ at high temperature. We choose parameters so that $F_{1,2} < 0$. This turns out to require $F_3 > 0$ so that for sufficiently high T , we may have a phase transition to a phase where $SU_2 \times U_1$ is completely broken.

We have found a range of parameters such that at high temperature $\langle \phi_1(T) \rangle = (0 \ v_1(T))^T / \sqrt{2}$, $\langle \phi_2(T) \rangle = (v_2(T) \ 0)^T / \sqrt{2}$, $\langle \phi_3(T) \rangle = 0$ is (at least) a local minimum of $V_\phi(T)$.⁷ These parameters satisfy

$$\begin{aligned} \lambda_1 \approx \lambda_2 \approx \lambda \gg g^4, \quad |\rho_{ij}| \\ -\sigma_{13} \approx -\sigma_{23} \approx \sigma > 3\lambda + \sigma_{12} + 3X \\ |\mu_2^2| > \mu_1^2, \quad 2 > \sigma_{12}/\lambda > -1, \quad \lambda_3 > \sigma^2/\lambda \end{aligned} \quad (7)$$

where $X = (3g^2 + g'^2)/8 \approx 0.16$. The condition $\lambda \ll g^4$ allows us to neglect radiative corrections to V_ϕ . For a typical set of numbers, choose $\lambda \approx -\sigma_{12} \approx g^2 \approx 0.4$, $\sigma \geq 1.3$, $\lambda_3 \geq 4.1$. We see that there is a range of parameters which satisfy the above conditions, but a rather large value for λ_3 is required. The second order phase transition occurs at T_c such that $v_2(T_c) = 0$. T_c is given by

$$T_c = A\mu_1/\sqrt{\lambda_1} = (246 \text{ GeV})A \quad (8)$$

where A is a function of the parameters in the potential and is typically of order unity, but can be made much larger or smaller by adjusting parameters. We will assume $T_c \geq 1 \text{ TeV}$. We have therefore demonstrated the existence of a phase transition in which $SU_2 \times U_1$ is broken to U_1^{EM} at $T=0$ and $SU_2 \times U_1$ is completely broken for $T > T_c$.

Now I would like to describe how to embed our scheme to SU_5 . We study the complete SU_5 potential, Eqs. (4a)-(4d). The conditions that V is bounded below and have symmetry breaking Eq. (3) at $T=0$ are

$$\begin{aligned} b > 0, \quad 15a + 7b > 0, \quad \beta_i < 0, \quad 5\alpha_i + \mu_i > 0 \\ \lambda_i > 0, \quad \sqrt{\lambda_i \lambda_j} + \sigma_{ij} > 0 \end{aligned} \quad (9)$$

For $T > M_X$ we have to consider the heavy particle contributions to $V(T)$. The effective potential at high temperature, $T > M_X$, are given by

$$V(T) = V_\phi(0) + V_\psi(0) + V_{\phi\psi}(0) + \frac{1}{2} GT^2 \text{Tr} \phi^2 + \sum_{i=1}^3 \frac{1}{2} F_i' T^2 \phi_i^\dagger \phi_i \quad (10)$$

where

$$\begin{aligned} G &= \frac{1}{60} \left[13a + 94b + 75g^2 + \sum_{i=1}^3 (50\alpha_i + 10\beta_i) \right] \\ F_i' &= 2\lambda_i + \sum_{j \neq i} \left[\frac{5}{6} \sigma_{ij} + \frac{1}{6} \rho_{ij} \right] + \frac{6}{5} g^2 + \text{Yukawa terms} \end{aligned}$$

G is always positive for the parameters satisfying Eq. (9). Therefore, at sufficiently high T , the effective mass of ϕ , $-\frac{1}{2}m^2 + \frac{1}{2}GT^2 > 0$ so that $\text{VEV} \langle \phi(T) \rangle$ will vanish. The parameters λ_i , σ_{ij} and ρ_{ij} have been already chosen as Eq. (7) for the phase transition at $T_c \geq 1 \text{ TeV}$. For those λ_i , σ_{ij} and ρ_{ij} and any α_i and β_i in Eq. (9) F_3 will be always positive. F_1 and F_2 may be positive or negative depending upon the values of α_i and β_i . It is very likely that SU_5 symmetry have been restored in the very early universe. In this case α_i and β_i should be chosen such that F_1 and F_2 are also positive.

It is a difficult problem to study the effective potential near $T \lesssim M_X$. There may be intermediate phases between SU_5 and SU_3^C phases (for example, $SU_3^C \times SU_2 \times U_1$):

$$SU_5 \xrightarrow{T_1} \text{intermediate phases} \xrightarrow{T_n} SU_3^C \xrightarrow{T_c} SU_3^C \times U_1^{EM}. \quad (11)$$

There should be essentially no magnetic monopoles in our model. Any monopoles produced during intermediate phases at $T \lesssim M_X$ will become unstable once the SU_3^C phase is entered. They would presumably either decay or be confined in pairs which could subsequently annihilate. Stable monopoles of mass $M_m \approx 10^{16}$ GeV could, in principle, exist for $T < T_c$, but the number $r \approx \exp(-M_m/T_c)$ expected from thermal fluctuations when $T \approx T_c$ is extremely small.

For $T_n > T > T_c$ the U_1^{EM} is spontaneously broken. During this period the photon has a mass and electric charge is violated. Charge violating reactions are in equilibrium for $T \gtrsim T_c$.⁷ If there is a net charge density in the present universe left over from fluctuations from equilibrium as $T > T_c$ it is far smaller⁷ than the observational limit^{13,14} from galaxies and cosmology.

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