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STUDY OF THE SENSITIVITY OF CALCULATIONS FOR
FAST REACTORS FUELED WITH Pu^{239} - U^{238} AND U^{233} -Th
TO UNCERTAINTIES IN NUCLEAR DATA

by

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by

A. Gandini

I. INTRODUCTION

The precision obtained in the theoretical prediction of the fast reactors is still far from being satisfactory. The difficulties in making these predictions, besides being of mathematical and computational nature, are closely related to the uncertainties of our knowledge of the cross-section data. In this work an analytical study is made to evaluate the effect of uncertainties of the nuclear data upon the calculation of some integral quantities for a few significant fast breeder systems. The integral quantities chosen are the reactivity, the initial breeding ratio, and the initial internal breeding ratio.

Estimates of the uncertainties with which cross-section data are presently known are used to determine the separate reactivity uncertainties due to this present state of our experimental data. This information should indicate which data, and at what energies, are most lacking and how much of an improvement of the integral quantities could be obtained by reduction of some of these specific inaccuracies. In the analytical study the multigroup diffusion theory and the perturbation approximation were used. An evaluation was made of the importance of propagation of the errors of the nuclear parameters into the integral quantities considered by means of the usual error theory. The 16-group cross-section set of Yiftah, Okrent, and Moldauer⁽¹⁾ has been used for all the calculations.

II. FAST REACTORS STUDIED

A number of different medium and large power breeder reactor systems have been considered, as shown in Table 1. The fuels chosen were $\text{Pu}^{239}\text{-U}^{238}$, both as metal and oxide, and $\text{U}^{233}\text{-Th}$ as oxide. The sizes of the cores ranged from 300 to 2500 liter. All of the systems were taken as consisting of three regions: core, inner blanket, and inert outer reflector. The geometry chosen was spherical. In the core the fissile-fertile ratio varied, but the volume fraction of the fuel was kept constant, as were those of the other materials used. The composition used for all cores is as follows:⁽²⁾

Volume Fraction - Fuel	0.25
Volume Fraction - Iron	0.25
Volume Fraction - Sodium	0.5

The compositions of the blanket and outer reflector were kept constant.
For the inner blanket:

Volume Fraction of Fertile 0.6
Volume Fraction of Iron 0.2
Volume Fraction of Sodium 0.2

For the outer reflector:

Volume Fraction of Iron 0.6
Volume Fraction of Sodium 0.4

Table 1

FAST BREEDER REACTORS STUDIED

System	Core Volume (liters)	Fuel	Fuel Density, g/cc	Critical Mass (kg)	Volume Fraction of Fissile Material	Volume Fraction of Fertile Material	Breeding Ratio	Internal Breeding Ratio
1	300	Pu ²³⁹ -U ²³⁸ (Metal)	19	223	0.0391	0.2109	~1.85	~0.50
2	800	Pu ²³⁹ -U ²³⁸ (Metal)	19	431	0.0283	0.2217	1.82	0.73
3	1500	Pu ²³⁹ -U ²³⁸ (Metal)	19	686	0.0241	0.22593	1.79	0.91
4	2500	Pu ²³⁹ -U ²³⁸ (Metal)	19	1025	0.0216	0.2284	1.76	1.04
5	300	Pu ²³⁹ O ₂ -U ²³⁸ O ₂	8.4	208	0.0366	0.0609	1.661	0.16
6	1500	Pu ²³⁹ O ₂ -U ²³⁸ O ₂	8.4	565	0.0198	0.0777	1.46	0.43
7	1500	U ²³³ O ₂ -ThO ₂	8.4	544	0.0195	0.128	1.21	0.39
8	2500	U ²³³ O ₂ -ThO ₂	8.4	755	0.0164	0.133	1.18	0.48

Estimated

The fertile element of the blanket was taken to be the same as that of the core and in all the cases considered was of metallic composition. The clean case of no buildup of fissionable material was chosen. The blanket thickness was kept at 45 cm for the plutonium-uranium systems and at 54 cm for the uranium-thorium system. The outer reflector was 30 cm in all cases.

III. SENSITIVITY OF REACTIVITY TO VARIATIONS OF NUCLEAR PARAMETERS

The sensitivity of the reactivity k to the nuclear parameter may be obtained by increasing the value by 10% within each group and material and evaluating the resulting total effect. For performing the calculations, the real and adjoint fluxes obtained from a diffusion-theory calculation were used to feed a perturbation code. By this means, a set of coefficients were obtained which accounted, within each region and fine group, for the partial effects due to fission, absorption from the group, transport, and transfer from a group into a lower group. They are independent of the materials and are represented, respectively, in the following way:

Fission coefficients	$\frac{\partial k}{\partial (\nu \Sigma_f)}$
Absorption coefficients (total removal)	$\frac{\partial k}{\partial \Sigma_a}$
Transport coefficients	$\frac{\partial k}{\partial \Sigma_{tr}}$
Transfer coefficients [from group i to group (i+j)]	$\frac{\partial k}{\partial \Sigma_{i \rightarrow i+j}}$

where the Σ 's are macroscopic cross sections.

The following parameters were considered for the different materials and groups:

Fission cross section	Σ_f	
Capture cross section	Σ_c	
Transport elastic- scattering cross section	$\Sigma_{n,n}$	
Inelastic-scattering cross section	$\Sigma_{n,n}^{i \rightarrow i+j}$	(Max j = 6)
Number of neutrons per fission	ν	

These quantities will be denoted generally as $\Sigma(m, z, i)$ to indicate dependence on the material m, parameter z, and fine group i. To have coefficients consistent with these nuclear parameters in the sense of representing the whole effect due to a perturbation of one of them, the partial coefficients described above have been gathered in the following way:

1. Capture Coefficient $\frac{dk}{d\Sigma_c} = \frac{\partial k}{\partial \Sigma_a} + \frac{\partial k}{\partial \Sigma_{tr}}$
2. Fission Coefficient $\frac{dk}{d\Sigma_f}(m) = \frac{\partial k}{\partial \Sigma_a} + \frac{\partial k}{\partial \Sigma_{tr}} + \frac{\partial k}{\partial (\nu \Sigma_f)} \nu(m)$
3. Transport Elastic-scattering Coefficient $\frac{dk}{d\Sigma_{n,n}}(m) = \frac{\partial k}{\partial \Sigma_{tr}} + \frac{\partial k}{\partial \Sigma_{i \rightarrow i+1}} a_i \frac{\xi(m)}{1 - \mu_0(m)}$

4...9. Inelastic-scattering
Coefficient ($j=1 \dots 6$)

$$\frac{dk}{d\Sigma_{n,n'}^{i \rightarrow i+j}} = \frac{\partial k}{\partial \Sigma_{tr}} + \frac{\partial k}{\partial \Sigma_{i \rightarrow i+j}}$$

10. Inelastic-scattering
Coefficient ($j=0$)

$$\frac{dk}{d\Sigma_{n,n'}^{i \rightarrow i}} = \frac{\partial k}{\partial \Sigma_{tr}}$$

11. Nu Coefficient

$$\frac{dk}{d\nu}(m) = \frac{\partial k}{\partial (\nu \Sigma_f)} \Sigma_f(m)$$

The parameters with the index m are dependent on the material and the expression for $dk/d\Sigma_{n,n}$ is obtained by assuming that within a certain group the elastic removal is related to the transport elastic scattering by the expression

$$\Sigma_{er} = \frac{a_i \xi \Sigma_{n,n}}{1 - \mu_0},$$

where a_i is the ratio

$$a_i = \frac{\phi(E_{i,L}) E_{i,L}}{\int_{E_{i,L}}^{E_{i-1,L}} \phi(E) dE},$$

and $E_{i,L}$ is the low-energy limit of the i -th group.

To perform all these calculations a small program has been made in Fortran language. It incorporates all the nuclear data of the materials used, i.e., Pu^{239} , U^{233} , U^{238} , Th, Fe, Na, and O. The input consists of the following quantities:

- (a) the volume fractions of the materials in the core and inner blanket;
- (b) the partial reactivity coefficients $\partial k / \partial \Sigma$ obtained by the perturbation code for each region and for each of the 16 groups i used in this study;
- (c) the fractional change of the nuclear parameters in the three coarse groups obtained by collapsing the 16-group set. The coarse group structure is shown in Table 2. The fractional changes are indicated by $h(m, z, \ell)$ to denote dependence on the material, parameter, and coarse group, respectively. The

split-up of the coarse groups has been chosen on reactor-physics considerations. The first group defines the range of the fast fissions by U^{238} . The lower limit of the second coarse group has been chosen to include most of the inelastic-scattering processes of all the materials.

Table 2

FINE- AND COARSE-GROUP STRUCTURE

Coarse Groups	1			2							3					
Fine Groups	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E_L (kev)	3668	2225	1350	825	500	300	180	110	67	40.7	25	15	9.1	5.5	2.1	0.5

In the program the total reactivity coefficients $dk/d\Sigma$ described above are calculated for each region and for each fine group and afterward they are weighted with the volume fractions of each material in every region. The values obtained will be called generally $\kappa(m,z,i)$ to describe their general dependence on the material m , nuclear parameter z , and fine group i . Then the program, first, multiplies $\kappa(m,z,i)$ by the deviations of the nuclear parameters $\Sigma(m,z,i)h(m,z,\ell)$ and then performs a sum over i in each coarse group. The resulting values correspond to the reactivity deviations due to given fractional changes of the nuclear parameters within the coarse groups.

Problems have been run for most of the systems indicated in Table 1. The results are shown in Tables 3 through 9. The reactivity changes are shown for the core and the inner (fertile) blanket. No effect was considered in the outer reflector.

For the 800-liter plutonium-uranium metal system, the most "sensitive" parameters were found to be ν and Σ_f of Pu^{239} . Then, in order of decreasing importance, were Σ_c of U^{238} , Σ_f of U^{238} , $\Sigma_{n,n}$ and $\Sigma_{n,n'}$ of U^{238} , $\Sigma_{n,n}$ and $\Sigma_{n,n'}$ of iron, Σ_c of Pu^{239} , and $\Sigma_{n,n}$ and $\Sigma_{n,n'}$ of sodium. It is interesting to note how the sensitivity is affected by the size of the core and the kind of fuel, metal or oxide, used. With increasing size the importance of the transport effect decreases, the spectrum softens, and the capture cross sections become more important. The reduced transport effect in the bigger systems makes the inelastic cross section increase its negative effect on the reactivity. In the Pu oxide systems the reactivity appears to be much more sensitive to the transport effects of the cross sections than in the corresponding metal ones. This is in large part because of the presence of oxygen at the expense of U^{238} , whose contribution to the total transport cross section is much larger.

Table 3

SENSITIVITY TO INDICATED CHANGE, % $\Delta k/k$, FOR 300-LITER
Pu²³⁹-U²³⁸ METAL-FUELED REACTOR

Parameter Having 10% Increase	Coarse Group	Pu ²³⁹	U ²³⁸		Fe		Na	
		Core	Core	Blanket	Core	Blanket	Core	Blanket
Σ_{capt}	1	-0.016	-0.038	-0.009	-0.005	-0.000	-0.000	-0.000
	2	-0.292	-1.023	-0.472	-0.070	-0.009	-0.003	-0.000
	3	-0.079	-0.210	-0.078	-0.010	-0.001	-0.005	-0.000
Σ_{fiss}	1	0.620	0.742	0.218	-	-	-	-
	2	4.642	0.033	0.013	-	-	-	-
	3	0.479	-	-	-	-	-	-
$\Sigma_{n,n}$	1	0.010	0.053	0.070	0.058	0.153	0.017	0.006
	2	0.058	0.365	0.639	0.335	0.204	0.038	0.038
	3	0.001	0.006	0.012	-0.002	0.004	-0.004	-0.001
$\Sigma_{n,n'}$	1	-0.035	-0.340	-0.057	-0.179	0.001	-0.027	0.001
	2	0.004	0.050	-0.013	-0.002	-0.004	0.006	-0.001
	3	-	-	-	-	-	-	-
ν	1	0.979	1.246	0.326	-	-	-	-
	2	6.500	0.049	0.019	-	-	-	-
	3	0.651	-	-	-	-	-	-
Material worth for 10% increase of density		5.393	-0.361	0.324	0.126	0.168	0.187	0.043

Table 4

SENSITIVITY TO INDICATED CHANGE, % $\Delta k/k$, FOR 800-LITER
Pu²³⁹-U²³⁸ METAL-FUELED REACTOR

Parameter Having 10% Increase	Coarse Group	Pu ²³⁹	U ²³⁸		Fe		Na	
		Core	Core	Blanket	Core	Blanket	Core	Blanket
Σ_{capt}	1	0.013	-0.045	-0.005	-0.005	-0.000	-0.000	-0.000
	2	0.284	-1.431	-0.308	-0.094	-0.006	-0.004	-0.000
	3	0.119	-0.459	-0.057	-0.018	-0.001	-0.009	-0.000
Σ_{fiss}	1	0.588	0.992	0.124	-	-	-	-
	2	4.638	0.041	0.008	-	-	-	-
	3	0.625	-	-	-	-	-	-
$\Sigma_{n,n}$	1	0.006	0.043	0.038	0.039	0.014	0.004	0.003
	2	0.040	0.370	0.394	0.274	0.090	0.116	0.019
	3	0.001	0.008	0.010	-0.006	0.003	-0.011	-0.001
$\Sigma_{n,n'}$	1	-0.034	-0.479	-0.041	-0.247	-0.001	-0.042	0.001
	2	-0.003	-0.039	-0.019	-0.025	-0.003	-0.012	-0.001
	3	-	-	-	-	-	-	-
ν	1	0.883	1.592	0.188	-	-	-	-
	2	6.399	0.062	0.011	-	-	-	-
	3	0.865	-	-	-	-	-	-
Material worth for 10% increase of density		5.444	-0.999	0.145	-0.080	0.095	0.042	0.021

Table 5

SENSITIVITY TO INDICATED CHANGE, % $\Delta k/k$, FOR 2500-LITER
Pu²³⁹-U²³⁸ METAL-FUELED REACTOR

Parameter Having 10% Increase	Coarse Group	Pu ²³⁹	U ²³⁸		Fe		Na	
		Core	Core	Blanket	Core	Blanket	Core	Blanket
Σ_{capt}	1	-0.011	-0.052	-0.002	-0.006	-0.000	-0.000	-0.000
	2	-0.273	-1.838	-0.156	-0.117	-0.003	-0.006	-0.000
	3	-0.105	-0.547	-0.032	-0.023	-0.000	-0.014	-0.000
Σ_{fiss}	1	0.480	1.090	0.056	-	-	-	-
	2	4.601	0.048	0.004	-	-	-	-
	3	0.863	-	-	-	-	-	-
$\Sigma_{n,n}$	1	0.003	0.029	0.017	0.017	0.006	-0.011	0.001
	2	0.023	0.288	0.188	0.131	0.041	-0.043	0.008
	3	0.001	0.010	0.005	-0.020	0.001	-0.043	-0.000
$\Sigma_{n,n'}$	1	-0.033	0.618	-0.019	-0.313	-0.001	-0.056	0.000
	2	-0.009	0.205	-0.020	-0.053	-0.002	-0.036	-0.001
	3	-	-	-	-	-	-	-
ν	1	0.727	1.770	0.086	-	-	-	-
	2	6.250	0.072	0.005	-	-	-	-
	3	1.088	-	-	-	-	-	-
Material worth for 10% increase of density		5.541	-1.795	0.041	-0.383	0.041	-0.209	0.008

Table 6

SENSITIVITY TO INDICATED CHANGE, % $\Delta k/k$, FOR 300-LITER
Pu²³⁹-U²³⁸ OXIDE-FUELED REACTOR

Parameter Having 10% Increase	Coarse Group	Pu ²³⁹	U ²³⁸		Fe		Na		O
		Core	Core	Blanket	Core	Blanket	Core	Blanket	Core
Σ_{capt}	1	-0.018	-0.014	-0.013	-0.006	-0.000	-0.000	-0.000	-0.009
	2	-0.342	-0.362	-0.672	-0.086	-0.013	-0.004	-0.000	-
	3	-0.191	-0.157	-0.162	-0.023	-0.002	-0.024	-0.000	-
Σ_{fiss}	1	0.918	0.332	0.336	-	-	-	-	-
	2	4.386	0.010	0.018	-	-	-	-	-
	3	0.799	-	-	-	-	-	-	-
$\Sigma_{n,n}$	1	0.013	0.020	0.097	0.086	0.035	0.037	0.009	0.046
	2	0.066	0.129	0.338	0.433	0.201	0.294	0.051	0.279
	3	0.005	0.011	0.044	0.027	0.014	0.019	0.000	0.005
$\Sigma_{n,n'}$	1	-0.015	-0.047	-0.074	-0.071	0.001	-0.005	0.002	0.000
	2	0.009	0.034	0.036	0.016	-0.002	0.017	-0.000	-
	3	-	-	-	-	-	-	-	-
ν	1	1.340	0.514	0.505	-	-	-	-	-
	2	6.407	0.016	0.026	-	-	-	-	-
	3	1.193	-	-	-	-	-	-	-
Material worth for 10% increase of density		5.628	-0.044	0.448	0.377	0.192	0.335	0.061	0.321

Table 7

SENSITIVITY TO INDICATED CHANGE, % $\Delta k/k$, FOR 1500-LITER
Pu239-U238 OXIDE-FUELED REACTOR

Parameter Having 10% Increase	Coarse Group	Pu239	U238			Fe		Na		O
		Core	Core	Blanket	Core	Blanket	Core	Blanket	Core	
Σ_{capt}	1	-0.013	-0.023	-0.006	-0.007	-0.000	-0.000	-0.000	-0.011	
	2	-0.321	-0.778	-0.348	-0.145	-0.007	-0.007	-0.000	-	
	3	-0.304	-0.589	-0.116	-0.059	-0.001	-0.081	-0.000	-	
Σ_{fiss}	1	0.619	0.521	0.140	-	-	-	-	-	
	2	4.106	0.018	0.008	-	-	-	-	-	
	3	1.532	-	-	-	-	-	-	-	
$\Sigma_{n,n}$	1	0.005	0.019	0.038	0.057	0.013	0.017	0.003	0.009	
	2	0.038	0.176	0.400	0.390	0.090	0.203	0.020	0.155	
	3	0.004	0.022	0.039	0.029	0.011	0.006	0.000	0.014	
$\Sigma_{n,n'}$	1	-0.018	-0.128	-0.045	-0.178	-0.002	-0.027	0.001	-0.000	
	2	0.000	0.003	-0.014	-0.016	-0.003	-0.005	-0.001	-	
	3	-	-	-	-	-	-	-	-	
ν	1	0.913	0.819	0.213	-	-	-	-	-	
	2	5.883	0.027	0.012	-	-	-	-	-	
	3	2.133	-	-	-	-	-	-	-	
Material worth for 10% increase of density		5.649	-0.759	0.096	0.072	0.101	0.105	0.022	0.139	

Table 8

SENSITIVITY TO INDICATED CHANGE, % $\Delta k/k$, FOR 1500-LITER
U233-Th OXIDE-FUELED REACTOR

Parameter Having 10% Increase	Coarse Group	U233	Th			Fe		Na		O
		Core	Core	Blanket	Core	Blanket	Core	Blanket	Core	
Σ_{capt}	1	-0.004	-0.033	-0.005	-0.007	-0.000	-0.000	-0.000	-0.011	
	2	-0.213	-0.986	-0.383	-0.146	-0.010	-0.007	-0.000	-	
	3	-0.187	-0.573	-0.109	-0.060	-0.002	-0.067	-0.001	-	
Σ_{fiss}	1	0.504	0.104	0.019	-	-	-	-	-	
	2	4.134	-	-	-	-	-	-	-	
	3	1.368	-	-	-	-	-	-	-	
$\Sigma_{n,n}$	1	0.005	0.024	0.033	0.083	0.020	0.045	0.005	0.070	
	2	0.047	0.237	0.411	0.631	0.156	0.467	0.040	0.476	
	3	0.005	0.024	0.033	0.047	0.017	0.010	0.000	0.004	
$\Sigma_{n,n'}$	1	0.008	0.050	0.031	0.079	0.014	0.023	0.002	0.000	
	2	0.011	0.071	0.030	0.046	-0.001	0.038	0.000	-	
	3	-	-	-	-	-	-	-	-	
ν	1	0.771	0.166	0.028	-	-	-	-	-	
	2	6.599	-	-	-	-	-	-	-	
	3	2.437	-	-	-	-	-	-	-	
Material worth for 10% increase of density		5.677	-1.083	0.059	0.675	0.193	0.508	0.048	0.539	

Table 9

SENSITIVITY TO INDICATED CHANGE, % $\Delta k/k$, FOR 2500-LITER
U233-Th OXIDE-FUELED REACTOR

Parameter Having 10% Increase	Coarse Group	U233	Th			Fe		Na		O
		Core	Core	Blanket	Core	Blanket	Core	Blanket	Core	
Σ_{capt}	1	-0.004	-0.036	-0.004	-0.007	-0.000	-0.000	-0.000	-0.011	
	2	-0.226	-1.286	-0.287	-0.183	-0.007	-0.009	-0.000	-	
	3	-0.205	-0.773	-0.089	-0.075	-0.002	-0.092	-0.001	-	
Σ_{fiss}	1	0.440	0.111	0.013	-	-	-	-	-	
	2	3.807	-	-	-	-	-	-	-	
	3	1.649	-	-	-	-	-	-	-	
$\Sigma_{n,n}$	1	0.004	0.021	0.023	0.069	0.013	0.036	0.004	0.055	
	2	0.037	0.226	0.296	0.562	0.110	0.399	0.027	0.396	
	3	0.004	0.025	0.028	0.037	0.014	-0.006	0.000	-0.010	
$\Sigma_{n,n'}$	1	0.003	0.027	0.020	0.048	0.009	0.015	0.002	0.000	
	2	0.006	0.058	0.016	0.031	-0.002	0.027	-0.000	-	
	3	-	-	-	-	-	-	-	-	
ν	1	0.676	0.179	0.020	-	-	-	-	-	
	2	6.321	-	-	-	-	-	-	-	
	3	2.804	-	-	-	-	-	-	-	
Material worth for 10% increase of density		5.515	-1.627	0.016	0.482	0.135	0.370	0.032	0.430	

The reactivity is also more sensitive to the capture cross section for all the materials. This is due to the previously described lower total absorption present, particularly at low energies, and to the consequent spectral changes, which are also affected by the moderating power of oxygen. Besides, the smaller volume fraction of U^{238} in these systems makes the effect of the inelastic cross sections less negative.

Similar considerations apply also to the uranium-thorium oxide systems. Here the fast fissions of thorium count very little and so, in the case of the 1500-liter reactor, next to ν and Σ_f of U^{233} and Σ_c of thorium, in order of decreasing importance, the following parameters come: $\Sigma_{n,n}$ of iron, $\Sigma_{n,n}$ of thorium, $\Sigma_{n,n}$ of sodium, Σ_c of U^{233} , Σ_c of iron, ν , $\Sigma_{n,n'}$, and Σ_f of thorium. In this system the relatively large leakage of neutrons from the core makes the reactivity very sensitive to the transport contributions of the cross sections. The effect of the inelastic scattering is generally positive because neutrons of reduced energy tend to remain in the core and die by producing fission reactions, adding to reactivity, rather than leak to the blanket where they are largely captured. This effect overcomes the reactivity losses due to the only slightly increased value of α for U^{233} with reduced neutron energy and due to the loss of the small fast fission bonus of the thorium of the core.

IV. IMPORTANCE OF PROPAGATION OF ERRORS ON THE UNCERTAINTY IN REACTIVITY

To estimate the propagation of the "errors" of the nuclear parameters into the calculation of the reactivity of a fast reactor, it is necessary to know their nature.⁽³⁾ If we assume a 16-energy-group set of parameters, it is likely that the parameters of one group are correlated with those belonging to the other groups. This correlation is expected to be mostly positive, at least for the more important parameters. All fission cross sections may be related to $\Sigma_f^{U^{235}}$ at thermal energy, for instance, and an uncertainty in this number will be reflected in all group fission cross sections. A very optimistic approach is to assume that all the errors are independent and so more likely to compensate each other.

An attempt has been made to estimate the error-correlation effect, assuming that there is a correlation for each parameter within each of the three coarse groups obtained by subdividing the 16 energy groups as shown in Table 2. For simplicity, errors will be classed as two kinds: uncorrelated and correlated errors. Later, a whole spectrum correlation will be studied. No correlation effects among different parameters were considered. Since there are cases of cross sections being measured with the same equipment, where correlations between parameters may exist, the "correlated" reactivity uncertainties obtained here will to this extent represent optimistic values.

The propagation of the errors of the nuclear parameters into the reactivity was evaluated by use of standard error theory.⁽⁴⁾ The general expressions employed for the standard deviation of the reactivity, s_K , for the uncorrelated and correlated cases are

$$s_K(\text{uncorr}) = \left[\sum_{m,z,i} \left(\kappa(m,z,i) h(m,z,\ell) \Sigma(m,z,i) \right)^2 \right]^{1/2}$$

and

$$s_K(\text{corr}) = \left[\sum_{m,z,\ell} \left(\sum_{i(\ell)} \kappa(m,z,i) h(m,z,\ell) \Sigma(m,z,i) \right)^2 \right]^{1/2},$$

where $\sum_{i(\ell)}$ means that the sum is made for all the fine groups i included in the coarse one ℓ . The other symbols have been defined previously.

The result for each system depends on the h -set input, that is, the set of percentage inaccuracies assumed for each material m , parameter z , and coarse group ℓ . A first idea has been to consider the same value of h for all the parameters and to evaluate that value of h which will make the accuracy of the reactivity within 1% for, say, 90% probability. According to the error theory, this means that the standard deviation of the reactivity should be 0.7%. The reactivity deviations resulting for the same value of h ($h = 0.1$) for all the parameters are also shown for the different systems in Tables from 3 to 9.

To perform all these calculations the program written to calculate the reactivity coefficients was extended. For all the systems considered the single value of h , applied to all materials and groups, which will produce an uncertainty of one per cent for the reactivity within 90% probability is about 0.02 in the uncorrelated case and about 0.008 for the correlated one with the sets of $\kappa(m,z,i)$ used here.

A second set of calculations have been run with the h -set shown in Table 10, which is considered to be reasonably indicative of the present accuracy of the nuclear parameters for the materials considered.⁽⁵⁾ The results for the different systems are indicated in Table 11. They do not differ much from system to system with the same fissile-fertile material, and it seems justifiable to extrapolate them to a certain extent to reactors of different size. In the very optimistic case of the uncorrelated errors, the standard deviation of the reactivity is slightly under one per cent, whereas in the more realistic case, with the errors correlated, the same deviation is about 2%. This also means that with 90% probability the calculated reactivities for the uncorrelated and correlated case will have, respectively, uncertainties of about 1.2% and 2.7% for the plutonium-uranium systems and of about 1.3% and 3.3% for the uranium-thorium ones. Many errors are actually correlated outside the three coarse groups considered as well as between various different cross sections; in the latter

case, the situation becomes worse. In the pessimistic case that the errors of a given parameter are correlated not only within the three coarse groups, but within the whole energy spectrum, the deviation of the reactivity for the plutonium-uranium systems become about 2.5%, and for the uranium-thorium systems 3.2%.

Table 10

ESTIMATED UNCERTAINTIES OF NUCLEAR PARAMETERS⁽⁵⁾

Material	Pu ²³⁹			U ²³⁸			U ²³³			Th			Fe			Na			O		
Coarse Group	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
$\Delta \Sigma_{\text{capt}} / \Sigma_{\text{capt}}$	0.1	0.08	0.1	0.05	0.05	0.05	0.05	0.08	0.1	0.1	0.05	0.05	0.25	0.15	0.05	0.15	0.15	0.1	0.2	-	-
$\Delta \Sigma_{\text{fiss}} / \Sigma_{\text{fiss}}$	0.03	0.02	0.08	0.03	0.03	-	0.03	0.03	0.08	0.05	-	-	-	-	-	-	-	-	-	-	-
$\Delta \Sigma_{n,n} / \Sigma_{n,n}$	0.1	0.1	0.1	0.08	0.08	0.08	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.05	0.05	0.05	0.15	0.1	0.1
$\Delta \Sigma_{n,n'} / \Sigma_{n,n'}$	0.15	0.15	-	0.15	0.08	-	0.15	0.15	-	0.1	0.1	-	0.2	0.15	-	0.15	0.15	-	0.2	-	-
$\Delta \nu / \nu$	0.01	0.01	0.01	0.01	0.01	-	0.01	0.01	0.01	0.1	-	-	-	-	-	-	-	-	-	-	-

Table 11

REACTIVITY UNCERTAINTIES (% $\Delta k/k$) FOR DIFFERENT
FAST REACTORS, RESULTING FROM THE ESTIMATED
INACCURACIES OF NUCLEAR PARAMETERS

Fuel	Pu ²³⁸ -U ²³⁸	Pu ²³⁹ O ₂ -U ²³⁸ O ₂	U ²³³ O ₂ -Th ²³² O ₂
Core Size, liters	800	1500	1500
Reactivity Deviation for Uncorrelated Errors	0.837	0.846	0.955
Reactivity Deviation for Errors Correlated within Coarse Groups	1.832	1.969	2.323
Reactivity Deviation for Errors Correlated within the Whole Spectrum	2.433	2.835	3.250

In Tables 12, 13 and 14 the detailed contributions of the various parameter inaccuracies within each coarse group and material are shown. They indicate the areas where improvement of experimental data would be of greatest worth. From these tables an estimate can also be made of the reactivity accuracy which will be attained with an improvement of some experimental data. If some improvements are to be considered within groups finer than the coarse ones, to produce finer estimates of this kind on a desk calculator, the coefficients $\partial k / \partial \Sigma$ obtained from the perturbation code for each region and fine group are shown for three significant systems in Tables 15, 16 and 17.

Table 12

VALUES OF THE REACTIVITY UNCERTAINTY (% $\Delta k/k$) IN THE 800-LITER Pu²³⁹-U²³⁸ METAL REACTOR RESULTING FROM THE ESTIMATED INACCURACIES OF NUCLEAR PARAMETERS (TABLE 10)

Parameter	Coarse Group	Pu239	U238	Fe	Na
Σ_{capt}	1	-0.013	-0.025	-0.013	-0.000
	2	-0.228	-0.870	-0.150	-0.007
	3	-0.119	-0.258	-0.009	-0.009
Σ_{fiss}	1	0.176	0.335	-	-
	2	0.928	0.015	-	-
	3	0.500	-	-	-
$\Sigma_{n,n}$	1	0.006	0.065	0.053	0.003
	2	0.040	0.611	0.364	0.068
	3	0.001	0.015	0.003	0.006
$\Sigma_{n,n'}$	1	-0.051	-0.780	-0.496	-0.062
	2	-0.004	-0.046	-0.042	-0.019
	3	-	-	-	-
$\Sigma_{\nu'}$	1	0.088	0.178	-	-
	2	0.640	0.007	-	-
	3	0.087	-	-	-

Table 13

VALUES OF THE REACTIVITY UNCERTAINTY ($\% \Delta k/k$) IN THE 1500-LITER
Pu²³⁹-U²³⁸ OXIDE REACTOR RESULTING FROM THE ESTIMATED
INACCURACIES OF NUCLEAR PARAMETERS (TABLE 10)

Parameter	Coarse Group	Pu239	U238	Fe	Na	O
Σ_{capt}	1	-0 013	-0 015	-0 018	-0 000	-0 022
	2	-0 257	-0 563	-0 227	0 011	-
	3	-0 304	-0 353	-0 030	0 082	-
Σ_{fiss}	1	0 186	0 198	-	-	-
	2	0 821	0 008	-	-	-
	3	1 226	-	-	-	-
$\Sigma_{\text{n n}}$	1	0 005	0 046	0 071	0 010	0 014
	2	0 038	0 461	0 480	0 111	0 155
	3	0 004	0 049	0 040	0 003	-0 014
$\Sigma_{\text{n n}'}$	1	-0 028	-0 259	-0 361	-0 040	-0 000
	2	0 001	-0 009	-0 027	0 009	-
	3	-	-	-	-	-
Σ_{ν}	1	0 091	0 103	-	-	-
	2	0 588	0 004	-	-	-
	3	0 213	-	-	-	-

Table 14

VALUES OF THE REACTIVITY UNCERTAINTY ($\% \Delta k/k$) IN THE 1500-LITER U^{233} Th OXIDE REACTOR RESULTING FROM THE ESTIMATED INACCURACIES OF NUCLEAR PARAMETERS (TABLE 10)

Parameter	Coarse Group	U233	Th	Fe	Na	O
Σ_{capt}	1	-0 002	-0 038	-0 017	-0 000	-0 021
	2	-0 171	-0 684	-0 233	-0 011	-
	3	-0 187	-0 341	0 031	-0 068	-
Σ_{fiss}	1	0 151	0 061	-	-	-
	2	1 240	-	-	-	-
	3	1 095	-	-	-	-
$\Sigma_{\text{n n}}$	1	0 005	0 057	0 103	0 025	0 105
	2	0 047	0 648	0 788	0 254	0 476
	3	0 005	0 057	0 064	0 005	0 004
$\Sigma_{\text{n n'}}$	1	0 012	0 081	0 187	0 037	0 000
	2	0 017	0 101	0 067	0 058	-
	3	-	-	-	-	-
Σ_{ν}	1	0 077	0 194	-	-	-
	2	0 660	-	-	-	-
	3	0 244	-	-	-	-

Table 15

REACTIVITY PARTIAL COEFFICIENTS FOR THE 300-LITER Pu^{239} U^{238} METAL REACTOR[illegible]

Table 16

REACTIVITY PARTIAL COEFFICIENTS FOR THE 1500-LITER $\text{Pu}^{239}\text{-U}^{238}$ OXIDE REACTOR

Group i	$\frac{\partial K}{\partial (\nu \Sigma_f)_i}$		$\frac{\partial K}{\partial \Sigma_a}$		$\frac{\partial K}{\partial \Sigma_{tr}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+1}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+2}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+3}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+4}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+5}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+6}}$	
	C	B	C	B	C	B	C	B	C	B	C	B	C	B	C	B	C	B
1	2 172	0 074	-2 357	-0 092	0 067	0 016	-0 004	-0 005	-0 137	-0 011	0 335	-0 029	-0 352	-0 031	-0 374	-0 034	-0 425	-0 039
2	4 722	0 149	-5 115	-0 173	0 113	0 026	-0 288	-0 012	-0 719	-0 047	-0 754	-0 052	-0 801	-0 058	-0 912	-0 068	-0 974	-0 080
3	8 550	0 301	-8 740	-0 327	0 161	0 040	-0 780	-0 073	-0 844	-0 082	-0 929	-0 096	-1 130	-0 116	-1 241	-0 142	-1 246	-0 167
4	11 907	0 654	-11 081	-0 551	0 153	0 049	-0 090	-0 021	-0 210	-0 056	-0 489	-0 106	-0 647	-0 167	-0 657	-0 222	-0 851	-0 247
5	17 804	1 412	-16 424	-1 139	0 152	0 059	-0 179	-0 085	0 599	-0 204	-0 840	-0 342	-0 863	-0 462	-1 153	-0 517	-1 312	-0 609
6	21 037	1 915	-19 185	-1 421	0 134	0 050	-0 497	-0 167	-0 786	-0 359	-0 818	-0 523	-1 262	-0 597	-1 352	-0 722	-1 835	-0 682
7	22 661	2 054	-20 125	-1 344	0 153	0 044	-0 315	-0 206	-0 352	-0 381	-0 722	-0 460	0 927	-0 594	-1 446	-0 551	-1 969	-0 750
8	20 167	1 963	-17 620	-1 072	0 113	0 029	-0 170	-0 166	0 367	-0 241	-0 550	-0 367	-1 010	-0 328	-1 479	-0 515		
9	14 231	1 173	-12 421	-0 553	0 045	0 012	-0 233	-0 045	-0 361	-0 121	-0 688	-0 097	-1 015	-0 210				
10	12 670	1 427	-10 830	-0 598	0 038	0 09	0 116	-0 093	-0 403	-0 065	-0 701	-0 201						
11	7 105	0 627	-6 014	-0 226	0 008	0 03	-0 162	0 013										
12	9 158	0 842	-7 539	-0 325	0 014	0 03	0 214	-0 081										
13	4 960	0 371	-3 973	-0 111	0 013	0 03	-0 120	-0 023										
14	2 859	0 171	-2 225	-0 043	0 005	0 01	-0 065	0 009										
15	2 594	0 114	-1 904	-0 024	0 003	0 01	0 087	-0 001										
16	2 052	0 046	1 763	-0 011	0 006	0 01												

Table 17

REACTIVITY COEFFICIENTS FOR THE 1500-LITER U^{233} Th OXIDE REACTOR

Group i	$\frac{\partial K}{\partial (\nu \Sigma_f)_i}$		$\frac{\partial K}{\partial \Sigma_a}$		$\frac{\partial K}{\partial \Sigma_{tr}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+1}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+2}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+3}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+4}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+5}}$		$\frac{\partial K}{\partial \Sigma_{1 \rightarrow 1+6}}$	
	C	B	C	B	C	B	C	B	C	B	C	B	C	B	C	B	C	B
1	2 217	0 076	-2 205	-0 082	0 074	0 021	0 020	-0 002	-0 006	-0 004	0 010	-0 008	0 011	-0 010	0 053	-0 012	0 072	-0 014
2	4 842	0 161	-4 859	-0 168	0 122	0 034	-0 057	-0 004	0 066	-0 012	-0 021	-0 016	0 072	-0 019	0 112	-0 025	0 233	-0 036
3	8 702	0 338	-8 630	-0 346	0 183	0 057	-0 016	-0 017	0 066	0 027	0 232	-0 035	0 305	-0 048	0 504	-0 073	0 794	0 102
4	12 634	0 849	-12 504	0 830	0 181	0 079	0 118	-0 029	0 358	-0 057	0 464	-0 102	0 749	-0 177	1 165	-0 256	1 368	-0 276
5	18 210	1 604	-18 188	-1 513	0 184	0 089	0 345	-0 061	0 497	-0 157	0 903	-0 308	1 496	-0 463	1 789	-0 502	2 291	-0 595
6	20 960	2 212	-21 326	-1 991	0 164	0 080	0 174	-0 145	0 636	-0 363	1 312	-0 581	1 648	0 637	2 224	-0 767	2 182	-0 652
7	22 098	2 382	-22 663	-1 984	0 183	0 071	0 484	-0 236	1 192	-0 470	1 546	0 531	2 152	-0 670	2 110	-0 547	2 349	-0 847
8	18 858	2 091	-19 747	-1 532	0 138	0 047	0 601	-0 206	0 903	-0 259	1 420	-0 381	1 385	-0 273	1 586	-0 536		
9	11 685	1 306	-14 183	-0 834	0 054	0 019	0 210	-0 033	0 572	-0 109	0 546	-0 041	0 688	-0 205				
10	7 794	1 406	-12 085	-0 842	0 047	0 016	0 302	-0 083	0 283	-0 011	0 398	-0 188						
11	6 710	0 618	-6 754	-0 339	0 011	0 004	-0 011	0 032										
12	4 844	0 802	-7 977	-0 483	0 016	0 004	0 074	-0 101										
13	3 590	0 342	-4 062	-0 166	0 013	0 004	-0 037	0 036										
14	1 917	0 165	-2 099	-0 064	0 004	0 001	-0 131	-0 014										
15	1 584	0 124	-1 628	-0 038	0 002	0 001	-0 019	-0 009										
16	1 175	0 115	-1 196	-0 013	0 004	0 001												

The results so far obtained may be easily used for estimating the effect of the inaccuracy of nuclear parameters on the reactivity when different materials are considered. Tables 15, 16 and 17 could eventually be used for this purpose. However, since these calculations are based on perturbation theory, the validity of this method will be limited for those materials which would not change significantly the spectrum and the neutron economy of the system considered.

V. SENSITIVITY OF BREEDING RATIO TO VARIATIONS OF NUCLEAR PARAMETERS

Another important quantity whose calculation is affected by uncertainties of the nuclear parameters is the breeding ratio. We have examined the sensitivity of two breeding ratios, whose calculation is

immediate from the initial conditions: the initial breeding ratio BR and the initial internal breeding ratio IBR, defined, respectively, as

$$BR = \frac{\text{Number of thermally fissionable atoms produced in reactor}}{\text{Number of thermally fissionable atoms destroyed in reactor}}$$

$$IBR = \frac{\text{Number of thermally fissionable atoms produced in core}}{\text{Number of thermally fissionable atoms destroyed in reactor}}$$

Both equations apply to the reactor at the start of its core and blanket loadings. For the calculation of changes in breeding ratio, a perturbation theory is not available, and so for every problem a new calculation of fluxes is necessary. This has limited the number of cases considered. The 1500-liter plutonium-uranium oxide reactor was examined more extensively, and some cases were run for the 1500-liter plutonium-uranium metal, the 300-liter plutonium-uranium oxide, and the 2500-liter uranium-thorium oxide systems. The same fractional change of the parameters $h = 0.1$ was adopted, and the results are shown in Table 18.

Table 18

SENSITIVITY STUDY OF THE BREEDING RATIO AND INTERNAL BREEDING RATIO OF FAST REACTORS

	Criticality Convergence Criteria	1500-liter Pu-U Metal Reactor		300-liter Pu-U Oxide Reactor		1500-liter Pu-U Oxide Reactor		2500-liter U ²³³ -Th Oxide Reactor	
		$\Delta BR/BR$ (x 10 ²)	$\Delta IBR/IBR$ (x 10 ²)	$\Delta BR/BR$ (x 10 ²)	$\Delta IBR/IBR$ (x 10 ²)	$\Delta BR/BR$ (x 10 ²)	$\Delta IBR/IBR$ (x 10 ²)	$\Delta BR/BR$ (x 10 ²)	$\Delta IBR/IBR$ (x 10 ²)
$n(\sum_{fiss}) - 0.1$	Radius Composition			6.2 3.7	-10.1 10.8	8.0 4.4	-9.7 6.2	4.0	-9.4
$h(\sum_{capt}^{Fertile}) - 0.1$	Radius Composition			0.6 1.1	8.9 4.4	0.9 2.0	9.3 4.4	0.5	9.5
$h(\nu^{Fissile}) - 0.1$	Radius Composition	14.82	17.11			21.3 16.1	-1.9 21.1		
$h(\nu^{Fertile}) - 0.1$	Radius Composition	4.0	4.2			4.2 3.3	-0.2 2.3		
$h(\sum_{fiss}^{Fertile}) - 0.1$	Radius Composition	2.3	2.5			2.6 2.2	-0.2 1.6		
$h(\sum_{n,n'}^{Fertile}) - 0.1$	Radius Composition	-1.6 -1.4	0.9 -0.5			-0.9 -0.8	0.4 -0.1		
$h(\sum_{n,n}^{Fertile}) - 0.1$	Radius Composition					0.2 -0.1	0.1 1.7		
$h(\sum_{n,n}^{Na}) - 0.1$	Radius Composition					-0.4 -0.5	0.7 1.4		
$h(\sum_{capt}^{Fe}) - 0.1$	Radius Composition					-0.5	-0.1		

In relation to the BR, the most "sensitive" parameters for the plutonium-uranium systems considered come in the following order of importance: ν and Σ_f of Pu²³⁹, ν and Σ_f of U²³⁸, and Σ_c and $\Sigma_{n,n'}$ of U²³⁸.

In general, it is possible to distinguish two kinds of parameters: those which appear directly in the BR (and IBR) expressions, and those which do not. A change of a parameter of the first kind, besides affecting

the BR directly, affects it indirectly, as those of the second kind also do, by changing the spectrum in the core and the blanket, by changing the transport cross section, which in turn is related to the leakage from the core, and, lastly and more important, by changing the reactivity, which has to be brought back to the initial conditions. This can be accomplished in a number of ways. In this work, the radius and the enrichment change have been chosen as criteria. In Table 19, the direct and indirect contributions to the BR (and IBR) from an increase of 10% of the first kind of parameters are shown separately. These two kinds of effects have opposite signs, and while for the BR the net result is determined mainly by the indirect effect in the case of Σ_f of Pu^{239} , the two effects almost balance in the case of Σ_c of U^{238} , with a consequent relatively small total increase of BR.

Table 19
DIRECT AND INDIRECT EFFECTS ON BREEDING RATIO AND INTERNAL
BREEDING RATIO FROM INACCURACIES OF $\Sigma_{\text{fiss}}^{\text{Fissile}}$ AND $\Sigma_{\text{capt}}^{\text{Fertile}}$

	Criticality Convergence Criteria	$\Delta \text{BR}/\text{BR} (\times 10^2)$ for $h(\Sigma_f^{\text{Fissile}}) - 0.1$		$\Delta \text{BR}/\text{BR} (\times 10^2)$ for $h(\Sigma_c^{\text{Fertile}}) - 0.1$		$\Delta \text{IBR}/\text{IBR} (\times 10^2)$ for $h(\Sigma_f^{\text{Fissile}}) - 0.1$		$\Delta \text{IBR}/\text{IBR} (\times 10^2)$ for $h(\Sigma_c^{\text{Fertile}}) - 0.1$	
		Direct Effect	Indirect Effect	Direct Effect	Indirect Effect	Direct Effect	Indirect Effect	Direct Effect	Indirect Effect
300-liter Pu-U Oxide Reactor	Radius	-8.0	14.2	10.0	-9.4	-8.2	-1.9	10.0	-1.1
	Composition	-8.0	11.7	10.0	-8.9	-8.2	19.0	10.0	-5.6
1500-liter Pu-U Oxide Reactor	Radius	-7.4	15.4	10.0	-9.1	-7.4	-2.3	10.0	-0.7
	Composition	-7.4	11.9	10.0	-8.0	-7.4	13.7	10.0	-5.6
2500-liter U-Th Oxide Reactor	Radius	-8.2	12.2	10.0	-9.5	-8.3	-1.0	10.0	-0.5
	Composition	-	-	-	-	-	-	-	-

The indirect effects behave quite similarly for all types of parameters. For those whose increases produce a positive change of the reactivity, the radius criterion shows a more positive (or less negative) effect on the BR than that of composition, whereas for those whose increases produce a negative change on the reactivity, the radius criterion shows a more negative (or less positive) effect. Similar considerations hold for the IBR, but in this case the effects are opposite, in the sense that when, for instance, the BR shows a more positive (or less negative) trend with the radius rather than with the composition criterion, the IBR shows a more negative (or less positive) one.

In Table 20, the results relative to a set of h 's for the different parameters is given. The h -set has been estimated somewhat arbitrarily for the whole energy spectrum and represents a very rough indication of the existing accuracy. If a correlation within the whole energy spectrum for each parameter is assumed, the estimated deviations of the BR and IBR for the 1500-liter Pu oxide system are as indicated. They are, respectively, around 3.5% and 5.5% if the radius criterion is applied, and 2.6% and 4% if the composition one is used. Results for similarly sized metal systems do not seem to differ very much percentagewise, as indicated in Table 18, from those obtained for the oxide core. The BR and

IBR show a slightly less sensitivity to the ν and Σ_{fiss} of Pu^{239} , but slightly more sensitivity to ν and Σ_{fiss} of U^{238} . Calculations have been run for the 300-liter Pu oxide system to study the sensitivity of the BR and IBR to Σ_f of Pu^{239} , and Σ_c of U^{238} . The results are shown in Tables 18 and 19. From them it appears that the inaccuracy of the BR should decrease a little with decreasing size, whereas that of the IBR does not seem to change much. In the uranium-thorium oxide systems the uncertainty of the IBR due to a fractionally equal uncertainty in the U^{238} fission and the thorium capture cross sections should not differ much from that of the plutonium systems, whereas that of the BR seems to be considerably less.

Table 20

EFFECT OF PROPAGATION OF ESTIMATED ERRORS OF
TABLE 10 ON BREEDING RATIO AND INTERNAL BREEDING
RATIO OF THE 1500-LITER Pu^{239} - U^{238} OXIDE REACTOR

	Criticality Convergence Criteria	$\Delta BR/BR$ ($\times 10^2$)	$\Delta IBR/IBR$ ($\times 10^2$)
$h(\Sigma_{fiss}^{Pu^{239}}) = 0.03$	Radius	2.4	-2.91
	Composition	1.32	1.83
$h(\Sigma_{capt}^{U^{238}}) = 0.05$	Radius	0.45	4.65
	Composition	1.0	2.20
$h(\nu^{Pu^{239}}) = 0.01$	Radius	2.13	-0.19
	Composition	1.61	2.11
$h(\nu^{U^{238}}) = 0.01$	Radius	0.42	-0.02
	Composition	0.33	0.23
$h(\Sigma_{fiss}^{U^{238}}) = 0.03$	Radius	0.78	-0.06
	Composition	0.66	0.48
$h(\Sigma_{n,n'}^{U^{238}}) = 0.1$	Radius	-0.9	0.4
	Composition	-0.8	-0.1
$h(\Sigma_{n,n}^{U^{238}}) = 0.1$	Radius	0.22	0.1
	Composition	-0.1	1.7
$h(\Sigma_{n,n}^{Na}) = 0.05$	Radius	-0.20	0.35
	Composition	-0.25	0.70
$h(\Sigma_{capt}^{Fe}) = 0.1$	Radius	-0.5	-0.1
	Composition	-	-
Reactivity Deviation	Radius	~3.5	~5.5
	Composition	~2.6	~4.0

VI. CONCLUSION

The final results of this study indicate that, because of inaccuracies in the values of the nuclear parameters as known within the present "state of the art," the deviations of the calculated reactivities should be about, and possibly larger than, $2\% \Delta k/k$ for all the medium and large systems considered in Table 1. This result can be extrapolated to other similar reactors. The deviation of the initial breeding ratio is about 3.5% in the 1500-liter $\text{Pu}^{239}\text{-U}^{238}$ metal and oxide systems and slightly less for smaller sizes. Greater accuracy of the breeding ratio may be expected in the U^{233} -thorium systems. Finally, the resulting deviation of the initial internal breeding ratio in the 1500-liter $\text{Pu}^{239}\text{-U}^{238}$ oxide system is about 5% and does not seem to vary considerably for different sizes and for the U^{233} -thorium systems.

As a consequence of the above-mentioned reactivity deviation of 2%, the prediction of the critical mass is also affected directly. If it be assumed that the ratio $(\Delta M/M)/(\Delta k/k)$ between the fractional fuel mass change and the corresponding reactivity averaged in all the core volume is equal to 1.8, the consequent uncertainty in the calculated critical mass will be of the order of 4%. If $(\Delta M/M)/(\Delta k/k)$ is referred to the fuel worth at the core boundary, this uncertainty becomes much bigger. As an example, for a 1500-liter metal core an error of 2% in the reactivity calculation means an error around 10% in the calculation of the critical mass.

The accuracies of the nuclear parameters for the materials considered in this paper may be used to indicate what improvements of experimental data would contribute most importantly to better predictions of reactivity and breeding ratio. Examples are given below.

Pu^{239}

1. Σ_{fiss} - Its uncertainty leads to deviations of about 1 and 1.5% of reactivity, respectively, in the metal and oxide systems, and also gives a considerable contribution to the deviations in breeding ratio and internal breeding ratio (see Table 20). The accuracy in the energy range of 1 to 25 kev is particularly important for the oxide systems.
2. ν - Its inaccuracy accounts for deviations of about 0.6% of reactivity and for about 2% of the breeding ratio for all the systems considered.

U^{233}

1. Σ_{fiss} - Its inaccuracy produces a deviation of about 1.7% of reactivity in the large oxide systems and is also a major source of deviations of the breeding ratios (see Table 18).

2. ν - Its accuracy accounts for deviations of about 0.7% of reactivity in the oxide systems.

U²³⁸

1. Σ_{capt} - Its inaccuracy is responsible for deviations of about 0.7% of reactivity for all the large systems considered. It produces also a deviation of about 1% of the breeding ratio in the 1500-liter oxide system when a composition criterion is adopted for regaining criticality. The internal breeding ratio is particularly sensitive to this parameter, and in the large oxide systems deviations from 2% to 5% are found, according to the different ways used to regain criticality.
2. $\Sigma_{n,n'}$ - Its inaccuracy alone accounts for deviations of about 1% of reactivity and 1.5% of the breeding ratio in a 1500-liter plutonium-metal system.
3. $\Sigma_{n,n}$ - In the 1500-liter metal and oxide systems considered, its inaccuracy produces a deviation of about 0.5% of reactivity. The breeding ratio is fairly sensitive to this parameter.

Th

1. Σ_{capt} - Its inaccuracy produces deviations of about 0.8% of reactivity in the large oxide systems. The sensitivity of the breeding ratio to this parameter is not as big as that in the similar Pu-U²³⁸ systems to Σ_{capt} of the fertile material.
2. $\Sigma_{n,n}$ - Its estimated inaccuracy accounts for a deviation of about 0.7% of reactivity in the large oxide systems.

Fe

1. $\Sigma_{n,n'}$ - There is a fair sensitivity to inaccuracy of this parameter, especially in the plutonium-fueled reactors. In the 800-liter plutonium-metal system examined, the assumed uncertainty determined a deviation of 0.5% of reactivity.
2. $\Sigma_{n,n}$ - The reactivity of the plutonium-uranium and uranium-thorium oxide systems is very sensitive to this parameter. In the large uranium-thorium reactors its assumed inaccuracy leads to a deviation of about 0.7% of reactivity.

O

1. $\Sigma_{n,n}$ - Uranium-thorium systems are fairly sensitive to inaccuracies in this parameter. For the reactors of this type considered its inaccuracy determines a deviation of about 0.5% of reactivity.

Na

1. $\Sigma_{n,n}$ - Its inaccuracy determines a deviation of slightly less than 0.3% of reactivity in the uranium-thorium reactors considered.

Improvements of nuclear parameters other than those mentioned above do not seem to make the theoretical prediction relatively much better in the systems considered, at least insofar as the integral quantities studied are concerned. The prediction of other quantities, like reaction rates, kinetics parameters, etc., may call for improvements different from those that could be inferred from these results, but this is beyond the scope of the present work. However, the results of an extensive analysis of this sort would eventually put more severe restrictions to the accuracies desired. Purposely it has not been specified how much the different parameters should be improved. In fact, their improvements should be suggested by considering the requirements of precision of the calculated values and the potentiality (and cost) of the several cross-section measurements. Once these two factors have been defined, it will be possible, with the results presented here, to decide which improvements are advantageous and feasible.

REFERENCES

1. S. Yiftah, D. Okrent, and P. A. Moldauer, Fast Reactor Cross Sections, Pergamon Press (1960).
2. Proceedings of the Conference on the Physics of Breeding, ANL-6122 (1959).
3. T. P. Moorhead, The Effects of Errors in Cross Section Data on Calculations for a Large Dilute Fast Reactor, Paper SM-18/15, Vienna Conference on Fast Reactors (1961).
4. Yardley Beers, Introduction to the Theory of Error, Addison-Wesley Publishing Co., Reading, Mass. (1957).
5. A. B. Smith (Private Communication).

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