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CRITICAL BOREHOLE PRESSURE FOR A VERTICAL HYDRAULIC CRACK IN THE PRESENCE OF TWO PRINCIPAL TOTAL STRESSES

by

Y. C. Hsu

ABSTRACT

In hydraulic fracturing in deep boreholes, a vertical hydraulic crack is produced by the processes of breakdown, crack propagation, shut-in, and venting (deflation). It is desired to determine whether, during repumping, the rock will sustain a given borehole pressure in the presence of two horizontal principal total stresses and a pre-existing vertical crack extending diametrically outward from a cylindrical borehole. This can be predicted by using the stress-intensity-factor fracture-mechanics approach.

I. INTRODUCTION

For a disk (penny)-shaped vertical crack, the critical borehole pressure P_{cr} required to extend a pre-existing crack can be calculated by the use of Sack's equation.¹ This equation was used by Aamodt² for predicting P_{cr} for hydraulic fractures produced in the Los Alamos Scientific Laboratory's (LASL's) Dry Hot Rock Geothermal Energy Program. If the length, a , of the pre-existing radial crack is comparable to the radius, R , of the borehole, the effect of the hole on the stress-intensity factor for the opening mode, K_I , is significant. If the radial vertical crack assumes a rectangular shape, then the problem can be approximated as the case of plane strain. The disk-shaped crack solution is, however, not valid for either of these two cases. A fracture mechanics approach will therefore be used to predict P_{cr} required to extend a pre-existing diametral crack from a cylindrical hole, subject to horizontal principal total stresses σ_2 , σ_3 , and borehole fluid pressure, P , and taking account of the pore pressure, P_o . Here P is the sum of the pumping pressure measured at the surface and the pressure resulting from the hydrostatic head of the fluid column (extending to the surface).

II. EXPERIMENTAL DATA²

A hole, located in Barley Canyon, was drilled on the western flank of the Jemez Caldera in northern New Mexico, for the LASL's Dry Hot Rock Geothermal Energy Program. The hole was drilled to a depth of 785 m and granite was encountered at a depth of 610 m. The bottom 50 m of the hole was continuously cored and was left uncased. Seven small hydraulic vertical cracks were made in this section, each from a few liters to a few tens of liters in volume. A large vertical crack was then generated with a volume of about 4400 liters over the section of the borehole between 740 and 775 m depth. The fractured interval was centered at a depth of 745 m and the borehole of 10.8-cm diam was pressurized over a section of 2.13 m between packers.

Data were obtained for pressure vs volume for initiation of the small vertical cracks and for reopening of these cracks, and pressure vs time data were obtained after shut-in of the large vertical crack. Young's modulus, E , and Poisson's ratio, ν , for the granite were determined by sonic logging in this section as 3.76×10^4 MPa (3.76×10^{10} N/m²) and 0.305, respectively. The energy required to create unit area of new surface, α , was determined

Fig. 1. Two radial vertical cracks from the borehole.

as $1.04 \times 10^2 \text{ J/m}^2$ ($1.04 \times 10^2 \text{ Nm/m}^2$). Also values of the least horizontal principal total stress, σ_3 , and the breakdown pressure, P_b , were determined as 14.7 MPa and 17.7 MPa, respectively.

III. MODEL

Consider the problem as two radial vertical cracks (a diametral vertical crack) extending radially from the borehole and subjected to principal effective stresses $(\sigma_2 - P_0)$ and $(\sigma_3 - P_0)$, and effective fluid pressure $(P - P_0)$. The crack is assumed to have uniform width, $2(a + R)$, in the direction of the axis of the borehole. This problem is approximated as the case of plane strain as shown in Fig. 1a. Define a , P_0 , R , and β as the crack length from the borehole, the pore pressure, radius of the hole, and a factor ranging from 0 to 1.

IV. SOLUTION FOR P_{cr}

By the principle of superposition, Fig. 1a is represented by the sum of two independent solutions in Fig. 1b and 1c denoted by "*" and "**." As such, K_I in Fig. 1a becomes

$$K_I = K_I^* + K_I^{**} = -(\sigma_3 - P_0) \sqrt{\pi a} F_\lambda^*(S) + (P - P_0) \sqrt{\pi a} F_\beta^{**}(S), \quad (1)$$

$$\left. \begin{aligned} \text{where } S = a/(a + R) = (a/R)/[1 + (a/R)], \\ 0 \leq \beta \leq 1 \\ \lambda = (\sigma_2 - P_0)/(\sigma_3 - P_0) > 1 \end{aligned} \right\} \quad (2)$$

$$F_\lambda^*(S) = (1 - \lambda) F_0^*(S) + \lambda F_1^*(S) \quad (3)$$

$$F_\beta^{**}(S) = (1 - \beta) F_0^{**}(S) + \beta F_1^{**}(S). \quad (4)$$

By using the conformal-mapping-function method, Bowie³ solved this problem for a uniform stress perpendicular to the crack and for biaxial stress, applied at infinite distance from the borehole. By using the same method, Hsu⁴ has just obtained solutions for K_I and K_{II} (stress-intensity factor of the sliding mode) of the problem for two radial cracks from the borehole with radius R , under inclined tension or in-plane shear, at infinity. By using the boundary collocation method, Newman⁵ solved the type of problem considered by Bowie³ and also the one where the crack was subject to $(P - P_0)$. Figures 2 and 3 show only those results which are required for the current problem and which have been summarized by Tada, Paris, and Irwin.⁶

In order to determine the value of fracture toughness K_{IC} of the rock, consider a simple problem of an internal crack of length $2a$ in a sheet, subject to uniform tensile load at infinite distance from the crack and perpendicular to the crack. Based on the Griffith theory, a crack will become unstable and begin to propagate (spread) if the elastic strain energy released by the crack extension (growth) is greater than the energy required to create the new fracture surfaces. Here, the critical tensile stress σ_{cr} required for crack extension is

$$\sigma_{cr} \sqrt{a\pi} = \sqrt{2E\alpha}.$$

Here $\sigma_{cr} \sqrt{a\pi}$ is defined as K_{IC} . Hence,

Fig. 2. Values of F_O^* , F_I^* , F_{III}^* vs S .

$$K_{IC} = \sqrt{2E\alpha} \quad (5)$$

By introducing line cracks of different length $2a$ for a given material and recording the various loads at incipient fracture, it has been found experimentally that K_{IC} indeed remains essentially constant and depends only on material constants.

The tangential stress σ_θ in the surface of the borehole corresponding to P , σ_2 , and σ_3 , is

$$\sigma_\theta = (\sigma_2 + \sigma_3 - P) - 2(\sigma_2 - \sigma_3) \cos 2\theta.$$

Here σ_θ varies from a maximum of $3\sigma_2 - \sigma_3 - P$ when $\theta = \pi/2$ to a minimum of $3\sigma_3 - \sigma_2 - P$ when $\theta = 0$,

$$\sigma_\theta|_{\max} = 3\sigma_2 - \sigma_3 - P \quad (\theta = \pi/2)$$

$$\sigma_\theta|_{\min} = 3\sigma_3 - \sigma_2 - P \quad (\theta = 0).$$

Here P , σ_2 , and σ_3 are in compression and are defined as positive ones. If $P > 3\sigma_3 - \sigma_2$ and $\sigma_2 > \sigma_3$ (with $\lambda = \sigma_2/\sigma_3$ such that $1 < \lambda < 3$), $\sigma_\theta|_{\min}$ is tensile.

Fig. 3. Values of F_O^{**} , F_I^{**} vs S .

If $P > 3\sigma_3 - \sigma_2 + T_0$, tensile failure is possible and takes place in a radial vertical plane through the direction of the greatest principal total stress σ_2 . Here T_0 is the tensile strength of the rock.

Taking account of the effect of P_0 on the rock, the effective breakdown pressure $(P_b - P_0)$ required to initiate a vertical crack is thus

$$P_b - P_0 = 3(\sigma_3 - P_0) - (\sigma_2 - P_0) + (T_0 - P_0). \quad (6)$$

By introducing λ , Eq. (6) becomes

$$\lambda = 3 - (P_b - P_0)/(\sigma_3 - P_0) + (T_0 - P_0)/(\sigma_3 - P_0). \quad (7)$$

Equations (1), (5), and (7) constitute basic equations for determining P_{cr} of the problem as shown in Fig. 1a. As $K_I = K_{IC}$, Eq. (1) yields P_{cr} as

$$P_{cr} = \frac{(K_{IC}/\sqrt{\pi a}) + (\sigma_3 - P_o) \left[(1 - \lambda F_o^* + F_1^*) \right]}{(1 - \beta) F_o^{**} + \beta F_1^{**}} + P_o, \quad (8)$$

where K_{IC} and λ are calculated based on Eqs. (5) and (7), β is such that $0 \leq \beta \leq 1$ and F_o^* to F_1^{**} are shown in Figs. 2 and 3. Since the derivation of Eq. (8) is based on a pre-existing crack with a finite radial length of a , this equation is not valid for the limiting case of $a = 0$.

V. CALCULATED P_{cr} AND DISCUSSION

Of interest is the vertical crack system centered at a depth of 745 m which was produced in the LASL's Dry Hot Rock Geothermal Energy Program. The borehole of 10.8-cm diam was pressurized over a distance of 2.13 m between packers. For competent granite, $P_o = 0$ and $T_o \approx 8 \text{ MN/m}^2$. If only water is used in the system, $\beta = 1$. Values² of σ_3 and P_b were determined as 14.7 MPa and 17.7 MPa, respectively. Substituting the above-mentioned values into Eq. (7) gives $\lambda = 2.34$. This results in $\sigma_2 = \lambda \sigma_3 = 34.4 \text{ MPa}$. The value of K_{IC} is calculated from Eq. (5) as $2.8 \text{ MN/m}^{3/2}$.

By using $K_{IC} = 2.8 \text{ MN/m}^{3/2}$, $P_o = 0$, $R = 0.054 \text{ m}$, $\sigma_3 = 14.7 \text{ MPa}$, and $\lambda = 2.34$, Eq. (8) gives P_{cr} in Table I for different values of crack length.

TABLE I

P_{cr} CALCULATED BY EQ. (8)

$a \text{ (m)}$	S	F_o^*	F_1^*	F_o^{**}	F_1^{**}	$P_{cr} \text{ (MPa)}$
2.506	0.98	1.025	1.025	0.02	1.02	15.8
0.108	0.67	1.2445	1.2158	0.25	1.20	18.4
0.054	0.5	1.4721	1.3675	0.40	1.35	18.4
0.027	0.33	1.8328	1.5638	0.60	1.55	17.6
0.0054	0.09	2.7801	1.9906	0.95	2.00	17.6

By using $K_{IC} = 2.8 \text{ MN/m}^{3/2}$, $P_o = 0$, $R = 0.054 \text{ m}$, $\sigma_3 = 14.7 \text{ MPa}$, and $\lambda = 2.34$, Eq. (8) gives P_{cr} in Table I for different values of crack length.

For very short cracks (for instance, $a = 0.0054 \text{ cm}$), $P_{cr} = 17.6 \text{ MPa}$. For longer cracks (for instance, $a = R = 0.054$), P_{cr} is increased to $P_{cr} = 18.4 \text{ MPa}$. However, when the crack is still longer (for instance, $a (= 2.66 - 0.054 = 2.506 \text{ m}) > R (= 0.054 \text{ m})$), P_{cr} is

decreased to $P_{cr} = 15.8 \text{ MPa}$. This complex phenomenon is caused by interactions between the borehole and the crack, resulting in various values of F_o^* to F_1^{**} . For very large cracks, the effect of the hole size on K_I (or P_{cr}) is negligible.

Of particular interest here is the case of the relatively large crack ($a = 2.506 \text{ m}$) produced from a borehole of radius $R = 0.054 \text{ m}$ in the LASL's Dry Hot Rock Geothermal Energy Program. It is desired to determine whether the already cracked rock will sustain, during repumping, a pressure of 15 MPa when the pre-existing crack size is $a = 2.506 \text{ m}$. It is seen that the pressure of 15 MPa will be sustained with a safety factor (S.F.) of

$$S.F. = \frac{15.8}{15} = 1.05 \text{ or } 5\%.$$

As expected for very large cracks, the value of 15.8 MPa calculated by using the present model in Fig. 1a agrees well with the one of 16.29 MPa calculated by Aamodt² using the disk-shaped crack model.

CONCLUSIONS

The fracture mechanics approach is used to predict P_{cr} required to extend a pre-existing diametral crack (either short or long) from a cylindrical borehole subject to horizontal principal total stresses σ_2 , σ_3 , and borehole fluid pressure P . For a given rock and crack size, P_{cr} can be evaluated from Eq. (8) as a function of P_o , R , σ_3 , λ , and β .

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