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MATHEMATICAL MODELS AND ILLUSTRATIVE RESULTS FOR
THE RINGBEARER II MONOPOLE/DIPOLE
BEAM-PROPAGATION CODE

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Background:

Based on the success of the RINGBEARER beam propagation code¹ wherein beam saussaging² and collision driven erosion³ were first observed, we have developed a linearized monopole/dipole particle simulation code, RINGBEARER II. This code allows for a transverse displacement of the beam and hence is useful for studying the hose instability⁴ and other non-axisymmetric propagation phenomena. In part I we enumerate the mathematical models used in constructing RINGBEARER II. In part II several fundamental difficulties encountered in the particle simulation are discussed along with some remedies. In part III we present output for test cases illustrating the nature of the hose instability and the approach taken to diagnose code results.

The modeling presented in this report may be significantly modified in the future to overcome the aforementioned difficulties. The hose results should be regarded as illustrative and subject to change due to code improvements and variations of system parameters.

1. MATHEMATICAL MODELS

Code Components and Flow:

The main components and flow of the RINGBEARER II code are indicated in Figure 1. The beam current is divided into a zero order and a first order component. The zero order component is assumed to be cylindrically symmetric so only monopole ($m = 0$) fields are driven. The first order current is not cylindrically symmetric; it has a dipole component ($m = 1$) and drives dipole fields. The first order component of the beam current responds to both the monopole and the dipole fields. Monopole and dipole components of plasma quantities are generated by direct ionization by beam electrons, and by avalanche ionization due to the ohmically heated plasma. The effect of radially nonuniform gas density is included. Monopole and dipole conductivities are determined from the plasma densities assuming the plasma is

collision dominated. Field equations are then solved to determine the pinch forces. Particle variables (zero order and first order) are advanced under the self-consistent fields. The evolution of zero order quantities also includes the effects of energy loss and scattering whereas the evolution of the first order quantities includes only energy loss by the monopole electric field -- this is required to preserve the translational symmetry of the system. Monopole and dipole radial current density profiles are then computed from the zero and first order particle positions. The major modeling areas to be described are 1) the pinch field equations, 2) the plasma density and conductivity calculation, 3) the zero and first order particle pushing and 4) the computation of current densities from particle quantities.

Coordinate System:

The coordinates used to describe the propagating beam are Z , X , r , and θ ; where Z is the propagation distance for a beam segment from its point of injection, X is distance measured backwards from the beam head, and r and θ are the transverse coordinates. The longitudinal coordinate X is related to the time t by $X = ct - Z$. A multipole expansion is performed on continuum quantities such that $f(r, \theta) = f_0(r) + f_{1s}(r) \sin \theta + f_{1c}(r) \cos \theta + \dots$. We assume for the beam current density component J_z that $J_{z1c} = 0$ and only the sine component, J_{z1s} , is present. This corresponds to the beam having a displacement in the y direction only in a cartesian coordinate system. In this case all dipole quantities are pure sine or cosine quantities and they will be denoted only by the subscript "1" without the "s" or "c". The particles are described in a cartesian coordinate system, x , y ; where lower case letters indicate the transverse plane. An individual particle is denoted by its zero order quantities; γ_i , x_{0i} , y_{0i} , v_{x0i} , v_{y0i} , r_{0i} ; and its first order quantities x_{1i} , y_{1i} , v_{x1i} , v_{y1i} . Here γ_i is the relativistic Lorentz factor characterizing the energy of the i^{th} particle and $r_{0i}^2 = x_{0i}^2 + y_{0i}^2$. NOTE that for field quantities, the subscript "0/1" indicate the monopole/dipole components of the quantity. For particles, the "0/1" subscript indicates zeroth or first order quantities.

Field Equations:

The equations describing the (r, X) evolution of the monopole and dipole pinch fields, A_0 and A_1 , in a conducting medium characterized by the scalar conductivities σ_0 and σ_1 , driven by the Z components of the beam current densities J_{B0} and J_{B1} have been discussed at length elsewhere.^{1,5} We enumerate the equations below; note Equation (3) has been recast in a form to make translational invariance manifest. Here A (denoted in previous references as $\mathcal{A}$) is the pinch potential and ϕ the electrostatic potential.

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} (A_0 + \phi_0) = -\frac{4\pi}{c} \left(J_{B0} - \sigma_0 \frac{\partial A_0}{\partial X} \right), \quad (1)$$

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial^2 A_0}{\partial r \partial X} = \frac{1}{r} \frac{\partial}{\partial r} r \frac{4\pi\sigma_0}{c} \frac{\partial \phi_0}{\partial r}, \quad (2)$$

$$\begin{aligned} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} (rA_1 + r\phi_1) &= -\frac{4\pi}{c} J_{B1} + \frac{4\pi\sigma_0}{c} \frac{\partial A_1}{\partial X} \\ &+ \frac{4\pi\sigma_1}{c} \frac{\partial A_0}{\partial X}, \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \frac{\partial}{\partial X} (rA_1) &= \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial}{\partial r} \frac{4\pi\sigma_0}{c} (r\phi_1) \\ &+ \frac{4\pi}{c} \left(\frac{1}{r} + \frac{\partial}{\partial r} \right) \sigma_1 \left(\frac{\partial \phi_0}{\partial r} - \phi_1 \frac{\partial \sigma_0}{\partial r} \right). \end{aligned} \quad (4)$$

Boundary conditions at $r = 0$ are:

$$\frac{\partial A_0}{\partial r} (r = 0) = \frac{\partial \phi_0}{\partial r} (r = 0) = 0, \quad (5)$$

$$A_1(r = 0) = \phi_1(r = 0) = 0. \quad (6)$$

Monopole boundary conditions are applied at $r = b$ where b is the maximum radius considered:

$$A_0(r = b) = \phi_0(r = b) = 0 \quad . \quad (7)$$

To insure translational invariance dipole "boundary conditions" are applied by demanding that:

$$A_1 \propto 1/r, \quad \phi_1 \propto 1/r; \quad r > b \quad . \quad (8)$$

A crucial test case for the fieldsolver is the uniform translation of the beam by Y (such that $J_{B1} = -Y \partial J_{B0} / \partial r$). The field solver must yield $A_1 = -Y \partial A_0 / \partial r$ and the net force on the beam must vanish exactly. This is accomplished in RINGBEARER II on the nonuniform radial mesh by utilizing interlaced monopole and dipole grids such that dipole quantities are known at exactly the midpoints on the monopole grid. Using this gridding along with judicious differencing in the field solver and conductivity calculation, exact translational invariance has been achieved.

Conductivity Generation:

Monopole air physics models have been presented in detail elsewhere^{1,5} and will only be briefly delineated. Plasma density n_e is generated by direct deposition and avalanche ionization:

$$\frac{\partial n_e}{\partial X} = \kappa J_B + \frac{v_{i0}}{c} n_e \quad . \quad (9)$$

Here κ is the direct deposition rate,

$$\kappa = \frac{\rho E'}{\Delta E e c} \quad . \quad (10)$$

Here E' is the usual stopping power, ρ is the gas density, ΔE is the energy per ion pair, and e the charge on an electron. Avalanche ionization is written as a function of $(E^2/\bar{p}^2)$ where E is the magnitude of the electric field and $\bar{p}$ is the gas density relative to its value at 760 torr.

$$v_{i0} = \frac{A \bar{p} S^3}{1 + BS + CS^2 + DS^3}, \quad (11)$$

$$S = E^2 / \bar{p}^2,$$

$$A = 1.423 \cdot 10^{-4}, \quad B = 9.179 \cdot 10^{-6},$$

$$C = 2.656 \cdot 10^{-10}, \quad D = 2.820 \cdot 10^{-17}.$$

Conductivity σ is related to n_e by:

$$\sigma = \frac{n_e e^2}{m_e \nu_m}. \quad (12)$$

Here m_e is the mass of the electron and $\nu_m = (4.86 \times 10^{12}) \bar{p}$ is the momentum transfer collision frequency.

Dipole conductivity is generated by the same mechanisms:

$$\frac{\partial n_{e1}}{\partial X} = \kappa j_{B1} + \frac{\nu_{i0}}{c} n_{e1} + \frac{\nu_{i1}}{c} n_{e0} \quad (13)$$

$$\sigma_1 = \frac{n_{e1} e^2}{m_e \nu_m}. \quad (14)$$

The quantity ν_{i1} is the distortion of the avalanche rate due to the dipole electric fields. It is the second term of the Taylor expansion of $\nu_i(|\underline{E}|)$:

$$\nu_{i1} = \frac{\partial \nu_{i0}}{\partial E^2} 2 \vec{E}_0 \cdot \vec{E}_1. \quad (15)$$

Although Equation 15 is formally correct, it is not used to calculate ν_{i1} ; a numerical derivative of $\nu_i(|\underline{E}|)$ is taken, and the evaluation of E_0 and E_1 is done so as to preserve translational invariance.

The air chemistry model in RINGBEARER II is deliberately chosen to be simple in order to facilitate an understanding of code results. Recombination, which is easy to incorporate, is omitted to extend the region over which hose growth should obey a power law. Since no adequate simple models for delta ray ionization exists, no attempt has been made to incorporate these effects. At present RINGBEARER II is being used to study dipole phenomena with emphasis on particle effects, hence the use of a simplified air chemistry is justified. For actual detailed propagation predictions, a better air chemistry model such as that employed in EMPULSE III or EPIC⁶ is needed.

Particle Pushing:

Particles are advanced in Z using the paraxial approximation with appropriate modifications to incorporate scattering and energy loss in the zero order trajectory. The advance is made in cartesian coordinates; first in zeroth order and then in first order. First order forces result both from the first order displacement interacting with the monopole fields and the forces due to the dipole fields. In the following equations, the subscript "i" (indicating particle number) has been omitted.

Energy loss is due to scattering and the "stopping" power:

$$\frac{d\gamma}{dZ} = \left. \frac{d\gamma}{dZ} \right|_{E_Z} + \left. \frac{d\gamma}{dZ} \right|_{STOP} = \frac{-eE_Z}{mc^2} - \frac{\rho E^2}{mc^2} \quad (16)$$

One usually assumes $d(\gamma \vec{v}_\perp)/dt = \vec{F}_\perp$. However, this introduces a subtle error in the paraxial approximation. Considering a particle with $\vec{F}_\perp = 0$, $E_Z \neq 0$; one would find an increasing $v_\perp$ due to "stopping power" energy losses. To avoid this unphysical expansion, we eliminate the "stopping power" contribution to $d\gamma/dZ$ when computing $d(\gamma \vec{v}_\perp)/dZ$, to arrive at:

$$\gamma \frac{dv_{x0}}{dZ} + v_{x0} \left. \frac{d\gamma}{dZ} \right|_{E_Z} = \frac{e}{mc} \frac{\partial A_0}{\partial r} \frac{x_0}{r_0} + \frac{cF_{sx}}{mc} \quad (17)$$

$$\gamma \frac{dv_{y0}}{dZ} + v_{y0} \left. \frac{d\gamma}{dZ} \right|_{E_Z} = \frac{e}{mc} \frac{\partial A_0}{\partial r} \frac{y_0}{r_0} + \frac{\delta F_{sy}}{mc} \quad (18)$$

The scattering force δF_s is shown in detail in reference 1.

The first order particle quantities are advanced using expansions of the force equations and assuming constant γ during the advance.

$$\frac{dv_{x1}}{dz} = \frac{e}{\gamma mc} \left[\frac{\partial A_0}{\partial r} \frac{x_1}{r_0} + \left(r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial A_0}{\partial r} \right) \left(\frac{x_0^2 x_1 + x_0 y_0 y_1}{r_0^2} \right) + \left(\frac{\partial A_1}{\partial r} - \frac{A_1}{r} \right) \left(\frac{x_0 y_0}{r_0^2} \right) \right] , \quad (19)$$

$$\frac{dv_{y1}}{dz} = \frac{e}{\gamma mc} \left[\frac{\partial A_0}{\partial r} \frac{y_1}{r_0} + \left(r \frac{\partial}{\partial r} \frac{1}{r} \frac{\partial A_0}{\partial r} \right) \left(\frac{x_0 y_0 x_1 + y_0^2 y_1}{r_0^2} \right) + \frac{\partial A_1}{\partial r} \frac{y_0^2}{r_0^2} + \frac{A_1}{r_0} \frac{x_0^2}{r_0^2} \right] . \quad (20)$$

Diagnostics:

The RINGBEARER II code has many diagnostic capabilities useful for code debugging and developing an understanding of the physical processes involved. Two diagnostics are particularly useful and are output to a print file for postprocessing with the RINGOUT2 postprocessor code. The beam radius is diagnosed with $R_{\frac{1}{2}}$, the radius within which half of the beam current is contained:

$$\frac{I_B}{2} \equiv \int_0^{R_{\frac{1}{2}}} 2\pi r j_B(r) dr . \quad (21)$$

This half radius is used in lieu of an RMS radius since RMS quantities tend to be dominated by a few particles which escape the beam and freely expand. The principal dipole diagnostic is $\bar{Y}$, the average dipole displacement

$$\bar{Y} = (1/N) \sum_{i=1}^N y_{1i} \quad (22)$$

These two diagnostics are output at selected intervals in X and Z. The RINGOUT2 postprocessor uses the R_z information to determine the beam erosion rate dx/dZ , and Nordsieck length L_N . The $\bar{Y}$ information is used to study hose evolution; growth rates and convective velocities are computed and displayed. These diagnostics will be discussed in the sample output section; they are introduced here since they will be used in the discussion of the computation of J_{B0} and J_{B1} .

Calculation of J_{B0} , J_{B1} :

Several algorithms exist for the computation of $J_{B0}(r)$ and $J_{B1}(r)$. For N point particles one has

$$J_B(\vec{r}) = \frac{I_B}{N} \sum_{i=1}^N \frac{\delta(r - r_i)}{r} \delta(\theta - \theta_i) \quad (23)$$

Formally one notes:

$$J_{B0}(r) = \frac{1}{2\pi} \int_0^{2\pi} d\theta J_B(\vec{r}) \quad , \quad (24)$$

$$J_{B1}(r) = \frac{1}{\pi} \int_0^{2\pi} d\theta \sin \theta J_B(\vec{r}) \quad (25)$$

Combining Equations (23), (24) and (25) is of little use. One wishes to compute J_{B0} and J_{B1} values on a radial grid such that using the J_{B0} , J_{B1} to compute A_0 , A_1 on the grid one would find the same field values as if one had used the J_B in Equation (23).

As a guide to an appropriate particle laydown for J_{B0} , we consider Equation (1) with $\phi_0 = \partial/\partial x = 0$:

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial A_0}{\partial r} = - \frac{4\pi}{c} J_{B0} \quad (26)$$

This equation can be formally integrated for point particles to compute $r(\partial A_0/\partial r)$. The resultant equation can be numerically differenced to compute J_{B0} on the grid, to obtain

$$\begin{array}{c} \text{Grid } J_{B0} \\ \downarrow \\ \frac{-4\pi}{c} J_{B0} \end{array} = \underbrace{\frac{1}{r} \frac{\partial}{\partial r}}_{\text{Difference numerically on the grid}} \left[\underbrace{\int_0^r \frac{-4\pi}{c} J_{B0} r' dr'}_{\substack{\text{Evaluate at each} \\ \text{grid point with } J_{B0} \\ \text{from particles}}} \right] \quad (27)$$

A similar solution for the dipole current can be obtained from

$$\frac{d}{dr} \frac{1}{r} \frac{d}{dr} r A_1 = - \frac{4\pi}{c} J_{B1} \quad (28)$$

Integrating twice while applying boundary conditions one finds:

$$r A_1 = - \int_0^r r' dr' \int_{\infty}^{r'} \frac{4\pi}{c} J_{B1} dr'' \quad (29)$$

$$r A_1 = \frac{r^2}{2} \int_r^{\infty} \frac{4\pi}{c} J_{B1} dr'' + \int_0^r \frac{r'^2}{2} \frac{4\pi}{c} J_{B1} dr' \quad (30)$$

Equation (30) is obtained by a partial integration of Equation (29). The first integral in Equation (30) is zero for a displaced ring of particles. It has been omitted in RINGBEARER II runs made to date. Effects of its inclusion are now being studied. We find

Grid J_{B1}

$$\frac{-4\pi}{c} J_{B1} = \frac{\partial}{\partial r} \left[\frac{1}{r} \frac{\partial}{\partial r} \left[\int_0^r \left(\frac{r'^2}{2} \frac{4\pi}{c} J_{B1} \right) dr' \right] \right] + \frac{r^2}{2} \int_r^\infty \frac{4\pi}{c} J_{B1} dr''$$

Difference numerically on the grid Evaluate at each grid point with J_{B1} from particles

Particle J_{B1} (31)

The use of Equations (27) and (31) to compute J_{B0} and J_{B1} is somewhat complicated and noisy. It does have the advantage that, with the interlaced gridding in RINGBEARER II, one has for a beam rigidly displaced by $\bar{Y}$ that $J_{B1} = -\bar{Y} \partial J_{B0} / \partial r$, and translational invariance can be achieved.

To achieve some smoothing and a point of contact with the rigid beam code work RINGBEARER II has a hierarchy of possible current density calculation schemes. These schemes use the average particle quantities $R_{\frac{1}{2}}$ and $\bar{Y}$ in computing J_{B0} , J_{B1} . They are:

BR: Bennett/Rigid - the monopole beam profile is forced to be a Bennett profile; the dipole current is computed assuming a rigid displacement

$$J_{B0} = \frac{I_B}{\pi R_{\frac{1}{2}}^2} \frac{1}{(1 + r^2/R_{\frac{1}{2}}^2)^2} \quad (32)$$

$$J_{B1} = -\bar{Y} \frac{\partial J_{B0}}{\partial r} \quad (33)$$

Here $R_{\frac{1}{2}}$ and $\bar{Y}$ are determined from the particles (see Eqn. 21, 22). This model should have the greatest overlap with the EMPULSE code results. Note the monopole current is always monotonic (no hollowing etc.) and the dipole current is always of one sign. This option represents a very severe smoothing of the particle information.

BN: Bennett/Nominal - the monopole beam profile is again forced to be a Bennett but J_{B1} is computed using the radially varying $\bar{Y}(r)$.

$$J_{B0} = \frac{I_B}{\pi R_{\frac{1}{2}}^2} \frac{1}{(1 + r^2/R_{\frac{1}{2}}^2)^2} , \quad (34)$$

$$J_{B1} = - \frac{\partial}{\partial r} \left(\bar{Y}(r) J_{B0}(r) \right) . \quad (35)$$

NN: Nominal/Nominal - the monopole beam profile, $J_{B0}(r)$ is allowed to be arbitrary. $J_{B1}(r)$ is calculated knowing $J_{B0}(r)$ and $\bar{Y}(r)$ using Equation 35. This approach should be the most nearly correct; it is the noisiest.

Startup:

The monopole portion of RINGBEARER II uses a conventional initialization where the beam is injected with a fixed radius, a Bennett profile, and a Gaussian velocity distribution. The injection conditions are uniform in X unless emittance tailoring is being studied.

Dipole perturbations (usually a half sine wave in X) are applied locally in X after the beam has travelled some distance in Z . The perturbation is a rigid displacement of specified beam segments. First order velocities are started at zero.

II. PATHOLOGIES

Continuing Code Development - Problem Areas:

When RINGBEARER II first began operation in September 1981, several subtle difficulties and unforeseen effects were discovered.⁷ Much time and effort have gone into understanding and counteracting these effects. The primary manifestation of the problems has been the failure of the code to operate properly at large Z as illustrated in the next section. Other questions on the implementation and interpretation of the linearized code have arisen.

The deleterious effects arise from single particle orbit phenomena and statistical phenomena which can be studied using a single disk code. Effects are due to the interaction of the monopole field with the first order particle quantities so one can set $A_1 = 0$ for this work. For a single disk with an initial displacement, one expects to see a continued oscillation and damping of $\bar{Y}(Z)$ as particles phase mix. One predicts $\bar{Y}(Z \rightarrow \infty) \rightarrow 0$ but this is not observed. The frustration of this damping has been traced to several sources which are now discussed.

Single Particle Orbit Pathology - Linear Growth:

The linearized particle quantities may be regarded as describing the separation between two initially infinitesimally separated particles. In an anharmonic but fixed potential (no self consistency), the two particles will have different orbital periods and will separate linearly in Z . Thus, $Y_1(t) \sim (1 + \delta\omega Z) \cos(\omega_0 Z)$ where $\delta\omega$ characterizes the frequency shift due to the anharmonic potential. The shift $\delta\omega$ is determined from the zero order orbit energy E and angular momentum L and from the first order perturbed quantities. Special starts can be constructed with $\delta\omega = 0$ but they are of little practical application. The linear first order orbit growth is a physical effect which should be retained; the growth will place serious constraints on the number of particles required to perform the simulation.

Single Particle Orbit Pathology - Numerical Exponential Growth:

It has been shown⁸ that in a fixed potential with a sufficiently gentle falloff the first order particle quantities will grow at worst linearly, and not exponentially in time. However, exponential orbits are often observed in practice due to numerical effects. Considering a Bennett potential, particles with high energy E and low angular momentum L can cross the origin ($r = 0$) with arbitrarily large radial velocities necessitating extremely small time steps in a leapfrog pusher. For particles with maximum radius within the Bennett radius, ten Z steps per betatron wavelength will be adequate. However, at several Bennett radii, thirty Z steps may prove inadequate.

Several avenues have been investigated to overcome this problem. In one dimension a staggered (zero order and first order quantities known at different Z location.) energy conserving pusher have been developed which always reproduces the linear growth. Implementation of this pusher in the full code appears impractical. Force quantity interpolation is via a cubic spline to smooth out discontinuity in force variation with radius. A relatively coarse radial grid is used in order to reduce field noise levels. Note that such noise levels are intrinsically high due to the high order derivatives appearing in Equations (19, 20).

Particle pruning; that is, the procedure of not counting particles with y_1 some number of times larger than the computed $\bar{Y}$, has been implemented. Pruning eliminated most of the noise problems, but it altered the location of the hose saturation. While the pruned results had considerably greater visual appeal, we appear to have lost the proper description of the mode. This approach will; however, be investigated further.

Single Particle Orbit Pathology - Physical Exponential Growth:

When the self consistent monopole potential is used, one can have particle orbits which exponentiate due to parametric coupling with the fluctuations in the field. In particular, if the entire beam is sausageing at a frequency ω_0 , particles with commensurate orbital wavelengths (e.g., $\omega_0/2$) will exhibit exponentially growing first order orbits. Analytically the exponentiation can be understood by observing Equations (19) and (20) which are in the form of Hill's equations if $A_1 = 0$ and A_0 has an oscillatory component. In a Hill's equation, there are parameters for which there exist unstable solutions. (Mathieu's equation is a special case of Hill's equation.) Since the monopole oscillation frequency is relatively broadband, one actually observes particles driven to large but not exponentially large amplitudes. However, one can still find $\bar{Y}$ being dominated by a very few particles.

Particle Statistics:

In the absence of dipole fields, one anticipates a dipole beam perturbation will relax by phase mix damping due to the spread in orbit frequencies in the anharmonic potential. Formally one expects approximately

$$\bar{\gamma}(Z) = \int_0^{\omega_{\beta 0}} f(\omega) \cos(\omega Z) d\omega, \quad (36)$$

where $f(\omega)$ is the distribution of oscillator frequencies and $\omega_{\beta 0}$ the maximum frequency encountered. In a conventional multicomponent modeling N_{osc} oscillators are placed at fixed frequency intervals and the evaluation of the integral is effectively done using Simpson's rule. The maximum amplitude achieved will scale as $1/N_{osc}$ and the long-time failure mode will be a "regurgitation."⁹ When equation is modeled with N_p particles which have essentially random frequencies one is performing a Monte Carlo evaluation of the integration. Now the minimum amplitude achieved scales as $1/N_p^{1/2}$ and the failure mode is a "random phase approximation" where $\bar{\gamma} \sim 1/N_p^{1/2}$ for large Z . One might thus expect that a particle code with $N_p = N_{osc}^2$ could reproduce the component model results. Unfortunately due to linear growth problems, one actually would require $N_p = (1 + \bar{\delta\omega}Z) N_{osc}^2$ where $\bar{\delta\omega}$ is an appropriate average of the frequency shifts. For typical cases of interest, this $\bar{\delta\omega}Z$ term is of order 10.

Symmetries and Starts:

In a fixed potential, one characterizes the zero order orbit of the i^{th} particle by E_i , L_i , ψ_i and ϕ_i where E_i and L_i are the constant energy and angular momentum. The quantity $\phi_i(Z)$ is the azimuthal angle and $\psi_i(Z)$ characterizes the phase angle along the particle orbit. One would like to load densely in this four-dimensional space. However, this would require a prohibitive number of particles so reductions in dimensionality are in order. It has been shown by others¹⁰ that by following a single particle using complex first order quantities, one can characterize an entire ring of

particles with all ϕ_i . In our scheme we find that following two particles initially loaded 90° apart is equivalent to following an infinite number of particles uniformly distributed in ϕ . Thus the dimensionality has been reduced by 1.

For the remaining three dimensions (E, L, and ψ), we have considered several further reductions. One can limit to purely circular orbits, so $L = L(E)$. One finds considerably improved damping but the damping rate does not agree with that obtained for the usual initial conditions. The full simulation run with circular orbits was also sausage mode unstable. Straight line orbits with $L = \psi(Z = 0) = 0$ also give good damping in a fixed field calculation but are not useful in a full simulation. A reduction in the ψ dimension may be possible in fixed fields for a fixed frequency disturbance. We are still seeking some form of "quiet" or ordered start.

Alternative Approaches:

The difficulties encountered in the operation of RINGBEARER II have led us to investigate several alternative approaches suggested by or used by others.^{11,12} The linearization and use of two separating particles to describe the system response to a perturbation has a fundamental flaw. As seen in Equations (19) and (20), the first order quantities are driven by zero order fields. By way of contrast in the Vlasov model of W. Sharp et al.,¹⁰ the perturbed distribution function is driven only by the perturbed field. There may be a more optimal basis than single particle orbits for describing the system response to a perturbation. Monopole and dipole field quantities appear in the first order force equation (Eqn. 19, 20) to insure translational invariance when $A_1 = -\bar{\gamma}a_0/\partial r$. Any alternative scheme should also preserve this invariance.

III. ILLUSTRATIVE OUTPUT

Output of the RINGBEARER II Code - Examples:

The large beam propagation codes produce prodigious amounts of output which must be organized and analyzed in order to obtain an understanding of propagation phenomena. Much analyses and data presentation for the RINGBEARER II code are accomplished in the RINGOUT2 postprocessor. In this section, we will present code results for several cases. Our purpose is twofold; first to demonstrate the operation of the code, and; second, to illustrate how code results are analyzed.

Code results are presented as functions of X , Z , or both. The propagation codes compute results in the X , Z plane illustrated in Figure 2. The coordinate X measures distance back from the beam head toward the beam tail. The pinch field develops in X as seen in the partial differential Equations 1, 2, 3 and 4. In these equations Z is only a parameter. A diagnostic which is fixed in the laboratory (e.g., at a port on a pipe) records the evolution in X of the beam. The coordinate Z measures the propagation distance of a beam segment from the injection point. If one were riding on the beam Z would advance (while X remained fixed). Under the paraxial approximation the beam segments are assumed to move at c in the Z direction. Then for the dynamic evolution of the beam particles Z is the "time" coordinate in the ordinary differential equations and X is a parameter. We generally present contour plots of the principal diagnostics, $R(X,Z)$ and $Y(X,Z)$ in the X,Z plane. In this guided tour section we will present the contour plots along with several cuts to facilitate understanding. The method for determining principal measured propagation parameters will also be delineated.

Two cases are presented in this example. Both cases utilized beam parameters appropriate to the ATA accelerator. In the first case where monopole phenomena are considered (OR0309A), the effects of energy loss and collisions are included. In the second case where a hose perturbation is followed (OR0424D) energy loss and collisions are omitted.

Figure 3 displays the beam parameters and the predicted and observed erosion rate and Nordsieck length for run (OR0309A). Figure 4 presents a contour plot of $R_{\frac{1}{2}}(X,Z)$; contours are labelled. To aid in understanding this plot, slices of $R_{\frac{1}{2}}(Z)$ at several X are presented in Figure 5; slices of $R_{\frac{1}{2}}(X)$ at several Z are given in Figure 6.

Referring to Figure 5A, one sees the beam head injected with $R_{\frac{1}{2}} = .5$ undergoing free expansion since no pinch field can develop at $X = 0$. In Figure 5B, the segment at $X = 20.5$ cm pinches initially; however, around $Z = 500.0$ cm the eroding head has fallen back to $X = 20.0$ cm and this segment expands. In Figure 5C, 5D the pinched beam slowly (albeit exponentially) expands due to Nordsiecking. The beam radius $R_{\frac{1}{2}}(Z, X = 100.0 \text{ cm})$ has been replotted in Figure 7 on a semi-log scale making the exponential radius growth manifestly obvious. Doing a least squares fit to this line RINGOUT2 computes both the equilibrium radius at $Z = 0$ (used to compute initial the net current), and the Nordsieck length, L_N . Note the observed Nordsieck length is 15.6m whereas the predicted value is 17.3m.

Figure 6 gives the X history of the radius $R_{\frac{1}{2}}$ at several Z values. In Figure 6A at $Z = 0.0$ cm, one sees the uniform injection condition, $R_{\frac{1}{2}} = .5$. After propagation in Figures 6B,C,D the beam head and neck are clearly visible. The erosion of the beam head causes the neck to move back in X as Z advances. The expansion of the beam tail is also apparent.

Returning to Figure 4, one can now observe the effects of Nordsiecking which causes two contours, $R_{\frac{1}{2}} = .5$ cm and $R_{\frac{1}{2}} = .75$ cm, to "peel off" as Z advances. Erosion causes the neck region, where the $R_{\frac{1}{2}} = 5.0$ to $R_{\frac{1}{2}} = 1.0$ contours are parallel and close, to move back in X . This erosion rate is determined in RINGOUT2 by doing at least squares fit to the $R_{\frac{1}{2}} = 2.0$ contour. The observed erosion rate was $dX/dZ = .031$ whereas the analytically predicted rate was .033.

For the hose mode analysis we consider case OR0424D where collisions and energy loss have been removed. Figure 8 is the equivalent of Figure 3, Figure 9 corresponds to Figure 4. Note in Figure 9 the erosion rate is very small due to the absence of collisions and energy loss. Also the beam no longer Nordsiecks.

Figure 10 gives the full $\bar{Y}(X,Z)$ contour plot. Contours are spaced logarithmically; negative contours are dotted. The oscillations in Z are due to betatron motion; they are highlighted in the $\bar{Y}(Z)$ slices in Figure 11. The X slices for this figure are given in Figure 12. The slice at $Z = 68.0$ cm is nearly the initial perturbation. Subsequent slices show the X varying wavelength of the mode and the convection of individual peaks back in the beam.

Figures 10, 11 and 12 reveal a major difficulty in the hose code. Beyond $Z = 500.0$ cm the contour plot becomes jagged and ill behaved. The slices versus Z reveal an oscillation which grows, decays, and then around $Z = 500.0$ cm begins to grow again. The slices versus X show a very jagged behavior beyond 500 cm. This failure of the code (these phenomena are unphysical) has been the subject of intense study as discussed in Part II and reported elsewhere.⁶

Primary interest in hose results centers not on $\bar{Y}(X,Z)$ but on the "magnitude" $|\bar{Y}(X,Z)|$ when the "carrier" betatron oscillations have been removed (i.e., the signal is detected). The plot of $|\bar{Y}(X,Z)|$ is given in Figure 13. Disregarding the results beyond $Z = 500.0$ cm, the convective nature of the instability can be seen. At a given X there is a $Z = Z_{SAT}$ at which saturation occurs at $\bar{Y} = \bar{Y}_{SAT}(X)$. Beyond this Z value $\bar{Y}$ decreases in amplitude (at least for a while). This convective nature of the hose instability is a key concept to be analyzed and understood with the RINGBEARER II code.

The location of the saturation points in X and Z is given in Fig. 14. The inverse slope of this line is the convection or group velocity of the instability. Note this velocity is low near the beam head and higher in the tail. This group velocity is to be distinguished from the phase velocity of a

disturbance which can be determined from the inverse of the slope following a constant phase in Figure 10.

Analytic predictions¹¹ of the saturated amplitude for the hose instability state that

$$\bar{Y}_{SAT}(X) = \left(\frac{X}{X_0} \right)^p . \quad (37)$$

Thus, $\bar{Y}_{SAT}(X)$ should be a straight line on a log-log plot. This is nearly so as seen in Figure 15. The "+" signs indicate observed saturation values; the straight line the least squares fit by RINGOUT2. For this calculation, the actual X value used in the fit is computed by subtracting off the X value of the $R = 2.0$ cm contour at the particular Z at which saturation occurs. For this case, we find $p = 4.39$ and $X_0 = 7.1$ cm.

A final crucial diagnostic is the wavelength in Z which characterizes the instability. This wavelength, which is usually several times the betatron wavelength, is computed from the oscillations of the last disk in the beam tail. We observe this wavelength is a sensitive function of where the initial perturbation is placed and what current laydown algorithm is utilized.

Conclusions:

The mathematical models and results for the RINGBEARER II particle simulation code have been presented. The code is under active development and substantive changes may be made in the future. Hence this document represents an interim report.

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Figure 1. Flow chart for RINGBEARER II illustrating the main computational components and program loop structure in X, Z.

COORDINATES:

Z — Propagation distance
from accelerator

X — Distance back from
beam head

DIAGNOSTICS:

$$R_{1/2} = \frac{I_B}{2} = \int_0^{R_{1/2}} 2\pi r J_{B0} dr$$

$$\bar{Y} = \sum_{i=1}^N Y_{1i} / N$$

Figure 2. Illustration of coordinate system, principal diagnostics, and physical meaning of the X, Z display used by the RINGOUT2 postprocessor for interpreting RINGBEARER II results.

OR0309A DATE 03/09/82 TIME 18:22:29 RINGOUT2

50 MEV BEAM 14 FEET, 90CM IN Z 102CM IN X JOB, JOB1 BA

RUN PARAMETERS:	IBEAM	=	1.0000E+24 (AMPS)
	RISE LENGTH	=	1.5200E+01 (CM)
	GRYMA	=	1.0000E+00
	ABEAM	=	5.0200E-21 (CM)
	MINI	=	1.0000E+04 (CM) (0)
	PRESSURE	=	2.5000E+20 (TOP)
	ETAP	=	6.0000E-01
COMPUTED VALUES:	NET CURRENT	=	5.3410E+20 (AMP)
	INITIAL TEMPA	=	2.9200E+21 (CM)
	ACCELERATION	=	2.9200E+21 (CM)
	RECO	=	4.0000E-21 (CM)
	X INC A-2 CONTOUR	=	4.0000E+21 (CM)
PREDICTED:	ENERGY LOSS	=	2.5000E+22
	COLLISION DRIVEN	=	2.0000E+23
	PREDICTED TOTAL	=	3.0000E+23
	OBSERVED	=	3.0000E+23
ACCELERATION:	PREDICTED	=	1.7000E+21 (CM)
	OBSERVED	=	1.5000E+20 (CM)

Figure 3. RINGOUT2 tabular output for run OR0309A presenting beam parameters with several predicted and observed diagnostic parameters.

Figure 5. Plots of $R_1(Z)$ at selected X locations for run 0R0309A.

Figure 6. Plots of $R_2(X)$ at selected Z locations for run OR0309A.

ROTATION PINCHER II OUTPUT 0000000
TAIL RADIUS VERSUS Z

TIME, DATE:

13:47:04

P5216782

LNORD =

1.081E-20

RECP =

0.175E-01

Y
R
A

$L_N = 15.6m$

$R_t(X=100cm, Z=0)$

$= .46cm$

Z (cm)

Figure 7. Plot of beam tail radius $R_t(X = 99.5 \text{ cm}, Z)$ on a semilog scale illustrating the exponential nature of the Nordstreck expansion.

0024240

DATE 04/04/90 TIME 18:12:56

RINGOUT2

50 MEV BEAM W/ PERT. 820CM IN Z 100CM IN X LB0. J91 NN

RUN PARAMETERS:	IBEAM	•	1.0000E+04 (AMPS)
	RISE LENGTH	•	1.5000E+01 (CM)
	GAMMA	•	1.0000E+22
	ABEAM	•	5.0000E-01 (CM)
	VINJ	•	1.9200E+29 (CM/SEC)
	PRESSURE	•	7.6000E+02 (TOAR)
	ETA2	•	6.9837E-01
COMPUTED VALUES:	NET CURRENT	•	8.4505E+03 (AMPS)
	INITIAL LBETA	•	2.9007E+01 (CM)
	ACTUAL LBETA	•	2.8686E+01 (CM)
	REQ0	•	4.5454E-01 (CM)
	X INC R-2 CONTOUR	•	1.0000E+01 (CM)
EROSION:	ENERGY LOSS	•	2.4507E-02
	COLLISION DRIVEN	•	9.1791E-03
	PREDICTED TOTAL	•	3.3686E-02
	OBSERVED	•	0.
NORDSIECKING:	PREDICTED	•	1.7493E+03 (CM)
	OBSERVED	•	1.5215E+04 (CM)
HOSE:	YBAR MINIMUM	•	-1.7528E+06 (CM)
	YBAR MAXIMUM	•	2.3597E+06 (CM)
	P(Y-(X/X0)=-P)	•	4.3851E+02
	X0	•	7.1395E+02 (CM)
	YSAT(X=100.0)	•	1.0637E+05
	XLASTSAT	•	9.9500E+01 (CM)
	AVERAGE WAVELENGTH	•	7.8606E+01 (CM)
	MEDIAN WAVELENGTH	•	7.9486E+01 (CM)

Figure 8. RINGOUT2 tabular output for run 0024240 presenting beam parameters. Beam electron collisions and energy loss have been toggled off; hence the disagreement between predicted and observed erosion and Nordsieck values. Hose diagnostics are also included.

ROIIC429

RINGBEARER II OUTPUT OR0424D

CONTOUR PLOT OF RHALF (X,Z)

TIME, DATE:

17:02:53

05/25/82

LNCR0 -

1.522E+04

NE00 -

4.645E-01

RHALF

C1 11 - 2.000E-01

C1 21 - 3.000E-01

C1 31 - 4.000E-01

C1 41 - 5.000E-01

C1 51 - 7.000E-01

C1 61 - 1.000E+00

C1 71 - 2.000E+00

C1 81 - 3.000E+00

C1 91 - 4.000E+00

C1101 - 5.000E+00

Figure 9. Contour plot of $R_1(X,Z)$ for run OR0424D. Contrast with Figure 4, note the reduction in erosion and absence of Nordsiecking.

ROI10429 RINGREORDER II OUTPUT OR04240
CONTOUR PLOT OF YBAR(X,Z)

TIME, DATE:

17:02:53

05/25/82

LNORD =
1.522E+04
REGB =
4.645E-01

0.
0.
T0MIN =
-1.753E+05
T0MAX =
2.590E+00

T0BAR

C1 11 = -1.000E+10
C1 21 = -1.000E+00
C1 31 = -1.000E+05
C1 41 = -1.000E+04
C1 51 = -1.000E+00
C1 61 = -1.000E+05
C1 71 = -1.000E+02
C1 81 = -1.000E+04
C1 91 = -1.000E+05
C1 101 = -1.000E+00
C1 111 = -1.000E+10
C1

Figure 10. Contour plot of $\bar{Y}(X,Z)$ for run OR04240 illustrating the oscillatory, growing, and convective nature of the hose mode. Solid contour lines represent positive $\bar{Y}$ values; dotted lines are for negative $\bar{Y}$. Lettered arrows indicate slices in $Y(Z)$ and $Y(X)$ presented in Figures 11, 12.

Figure 11. Plots of $\bar{V}(Z)$ at selected X locations for run ORC424D.

Figure 12. Plots of $\bar{Y}(X)$ at selected Z locations for run OR0424D.

ROI10429

RINGBEARER II OUTPUT OR0424D

CONTOUR PLOT - AMP(YBAR(X,Z))

TIME, DATE:

17:02:53

05/25/82

LNORD =

1.522E+04

REQ0 =

4.645E-01

0.

0.

ZDWIN =

-1.753E+06

YBARX =

2.359E+06

AMP YBAR

C1 1) 1.000E+00

C1 2) 1.000E+01

C1 3) 1.000E+02

C1 4) 1.000E+03

C1 5) 1.000E+04

C1 6) 1.000E+05

C1 7) 1.000E+06

C1 8) 1.000E+07

C1 9) 1.000E+08

C1 10) 1.000E+09

Z (CM)

X (CM)

Figure 13. Contour plot of $|Y(x,Z)|$ for run OR0424D where the "carrier signal" betatron oscillation in Z has been removed. The convective nature of the mode is apparent.

RO110429

RINGBEARER II OUTPUT OR0424D
PLOT OF SATURATION POINT IN Z

TIME, DATE:
17:22:53
05/25/82

LNORD .
1.522E+04
REQ0 .
4.545E-01
0.
0.
YBMIN .
-1.753E+06
YBMAX .
2.359E+06
P .
4.385E+00

Figure 14. Trajectory plot of the location of saturation points in X,Z of the hose mode for run OR0424D. This trajectory traces the first maximum in Z (hilltop) from Figure 13.

ROI10429

RINGBEARED II OUTPUT OR0424D

GAIN YMAX AT SATURATION VS. X

TIME. DATE:

17:02:53

05/25/82

LNORD .

1.522E+04

RE00 .

4.545E-01

.

0.

.

0.

YBMIN .

-1.753E+06

YBMAX .

2.355E+05

P

4.395E+02

X-XN(CM)

Figure 15. Plot of saturated gain $\bar{Y}$ versus X on a log-log scale to verify the power law growth prediction for run OR0424D.

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