STUDIES OF HEAVY ION REACTIONS
AND TRANSURANIC NUCLEI

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STUDIES OF HEAVY ION REACTIONS
AND TRANSURANIC NUCLEI

Progress Report
for Period June 1, 1976 - August 31, 1977

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ABSTRACT

The time scale provided by the nucleon exchange mechanism in heavy-ion reactions is employed in this progress report to study kinetic energy damping. The experimental kinetic energy loss associated with a single exchanged nucleon in Kr- and Xe-induced reactions is observed to decrease linearly with total kinetic energy loss. These results, which are consistent with energy dissipation and nucleon exchange occurring on a similar time scale, are compared to existing theories. Recent studies show that nucleon exchange accounts for a smaller fraction of the total kinetic energy loss as the bombarding energy is reduced, a result which cannot be explained by a friction force which is proportional to the relative velocity. Angular-momentum-dependent interaction times are deduced giving a time scale for the evaluation of nucleon diffusion coefficients.

The disappearance rates of muons from various ls muonic atoms of actinide nuclei are reported for experiments that utilize several different detection techniques. The properties of Nilsson levels excited by direction reactions are described for some actinide nuclei.

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I. INTRODUCTION

The research activity of the group supported under United States Energy Research and Development Administration contract E(11-1)3496 is based in the Nuclear Structure Research Laboratory and in Hutchison Hall, the Chemistry-Biology building of the University of Rochester. Several active outside-user programs are currently in progress including (1) the study of heavy ion reaction mechanisms at the SuperHILAC accelerator of the Lawrence Berkeley Laboratory (2) the study of muon-induced reactions on heavy targets at the Los Alamos Meson Physics Facility LAMPF and (3) the study of proton-hole states in actinide nuclei by the (t,α) reaction.

Our research program in heavy-ion reactions at the SuperHILAC accelerator has as a principal objective to investigate the reaction mechanisms that characterize collisions between heavy projectiles ($A \geq 40$) and heavy targets ($A \geq 100$). The outstanding property of the new damped-reaction mechanism is the strong correlation of experimental features with the loss of relative kinetic energy, which signifies the stage of evolution of the reaction. During the interaction an equilibrium process occurs that can be described in terms of a nuclear diffusion mechanism. The damped reaction mechanism connects in a continuous way the domains of quasi-elastic transfer and compound nucleus formation. This feature is illustrated in Fig. 1 by a schematic classification of heavy ion reactions in terms of the degree of energy damping.

Figure 1

Schematic classification of heavy ion reactions.
The open and hatched arrows depict the reaction
flow for light and heavy ion reactions,
respectively.

Figure 1.

To date the RAM group (University of Rochester, Argonne and University of Maryland) has studied the following heavy ion reactions at the SuperHILAC:

<u>Target</u>	<u>Projectile</u>	<u>Projectile energy (Lab)</u>	<u>Measurement</u>	<u>Technique</u>
238 _U	84 _{Kr}	605	a,b,c-i	A,D
		530	b	A,D
238 _U	40 _{Ar}	340	a,c-i	A,D
		288	a,c-i	A,D
209 _{Bi}	136 _{Xe}	1140	a,c-j	A,B,D
		940	a,c-j	A,B
209 _{Bi}	84 _{Kr}	714	a-i	A,B,D
		605	a-i	A,B,D
		540	b-h	A,D
209 _{Bi}	56 _{Fe}	474	a,c-i	A,D
		403	a,c-i	A,C,D
209 _{Bi}	40 _{Ar}	340	a,c-i	A,D
		288	a,c-i	A,D
197 _{Au}	84 _{Kr}	605	b	A,D
		530	b	A,D
165 _{Ho}	136 _{Xe}	1140	a-i	A,B,D
165 _{Ho}	84 _{Kr}	714	a-i	A,B
		605	a-i	A,D
165 _{Ho}	56 _{Fe}	474	a,c-i	A,B
154 _{Sm}	136 _{Xe}	1140	a,c-j	A,B
		746	a,c-j	A,B
154 _{Sm}	84 _{Kr}	712	a,c-i	A,B
		605	a,c-i	A,B
		494	a,c-i	A,B
144 _{Sm}	136 _{Xe}	1140	a,c-j	A,B
		746	a,c-j	A,B
144 _{Sm}	84 _{Kr}	712	a,c-i	A,B
		605	a,c-i	A,B
		494	a,c-i	A,B
109 _{Ag}	136 _{Xe}	1140	a-i	A,B,D
		605	a,c-i	A,B

a σ_R b σ_{CF}/σ_R c $d\sigma/d\Omega$ d $d\sigma/dE$ e $d^2\sigma/d\Omega dE$ f $d\sigma/dA, d\sigma/dZ$ g $d^2\sigma/d\Omega dA, d^2\sigma/d\Omega dZ$ h $d^2\sigma/dEdA, d^2\sigma/dEdZ$ i $d\sigma_{El}/d\sigma_{Ruth}$

j light particles

A single counter

B ΔE -E telescope

C time-of-flight

D correlated fragments

Three postdoctoral research fellows have devoted essentially full time to the heavy-ion program over the past year. All of these experiments with projectiles $A \geq 40$ are performed with the SuperHILAC accelerator of the Lawrence Berkeley Laboratory.

The study of muon-induced reactions on heavy targets at the Los Alamos Meson Physics Facility LAMPF is continuing (experiments 142 and 292). Measurements of the muon-capture rates by detection of the electrons from the decay of muons in the 1s orbit of muonic ^{232}Th , ^{235}U , ^{238}U and ^{239}Pu have been completed. In addition, similar measurements by detection of neutrons and gamma rays from the nuclear capture of muons (and nuclear excitations produced by muonic radiationless processes) have been performed and the data analyses are near completion. The latter measurements include the nuclei ^{232}Th , ^{233}U , ^{235}U , ^{238}U , ^{237}Np , ^{239}Pu and ^{242}Pu . Future measurements will emphasize the detection of fission fragments. One graduate student is in residence at Los Alamos and during runs assistance is given by the team members from the University of Rochester, Los Alamos and Livermore.

The direct reaction studies on actinide nuclei are performed with the Emperor tandem Van de Graaff in the Nuclear Structure Research Laboratory and with the unique triton beam of the Los Alamos tandem accelerator. These experiments are performed mostly by visitors and collaborators from other institutions.

We wish to acknowledge the direct support of our research programs by the U. S. Energy Research Development Administration. In addition, we are grateful to the National Science Foundation for providing indirect support through the use of the facilities at the Nuclear Structure Research Laboratory.

II. RESEARCH PROGRAM

A. Direct Reaction Studies on Actinide nuclei

1. A Study of the Isotopes $^{233,235,237}\text{Pa}$ and $^{229,231}\text{Ac}$ by the (t, α) Reaction--R. C. Thompson*, W. Wilcke, J. R. Huizenga, W. K. Hensley**, D. G. Perry**

Recently a program was started to study the level schemes of the nuclei ^{233}Pa , ^{235}Pa , ^{237}Pa and $^{229,231}\text{Ac}$. Except for ^{233}Pa , little is known about these nuclei since they are difficult to investigate by radioactive decay and most single-nucleon transfer reactions due to the short lifetimes of the parent and target nuclei, respectively. Only the proton pickup (t, α) reaction provides a convenient tool for the study of these isotopes, and the 15-MeV triton beam of the Los Alamos Scientific Laboratory tandem facility was employed to generate the appropriate reactions. As it has been outlined qualitatively in last years annual report (COO-3496-56) members of the previously unobserved $1/2[400]$ and $3/2[402]$ proton configurations are the most strongly populated levels of the (t, α) spectra. In the meantime, additional levels have been identified, including firm assignments of the $1/2[503]$ and $3/2[651]$ rotational bands.

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A total of 51 levels have been assigned in the above isotopes. The analysis was made with the aid of the DWBA theory and a blocked BCS calculation, including up to 30 Nilsson states, which gives the Fermi surface and the pairing energies. The pairing interaction strength $G = -138$ keV was determined from the observed pairing gap of the adjacent even-even nuclei. The resulting quasi-particle energies were used in a subsequent diagonalization of the Coriolis operator, which was multiplied by an attenuation factor $\eta < 1$. This parameter and the two deformation parameters ϵ_2 and ϵ_4 of the Nilsson model are essentially the only free parameters of the present analysis. The deformation parameters, however, are strongly constrained by the measurement of the deformation in the adjacent isotopes of Th and U since it has been shown¹⁾ that the quadrupole deformation is monotonically increasing with increasing neutron number. This change of the deformation is assumed in the Pa and Ac isotopes, too, and explains the observed change in the ground state configuration from $J=3/2 \ 1/2[530]$ in ^{233}Pa to $J=1/2 \ 1/2[400]$ in ^{237}Pa and from $J=3/2 \ 3/2[651]$ to $J=1/2 \ 1/2[400]$ in the actinide isotopes: the deformation increase induces a level crossing between the $1/2[400]$ level, which has a positive slope with respect to ϵ_2 , and the $1/2[530]$ and $3/2[651]$ levels, respectively, which show negative slopes.

Alpha spectra from the $^{234}\text{U} (t, \alpha)$ ^{233}Pa , $^{236}\text{U} (t, \alpha)$ ^{235}Pa and $^{238}\text{U} (t, \alpha)$ ^{237}Pa reactions are shown in Fig. 2 and alpha spectra from the $^{230}\text{Th} (t, \alpha)$ ^{229}Ac and $^{232}\text{Th} (t, \alpha)$ ^{231}Ac

Figure 2

Alpha spectra from the reaction $^{234}\text{U}(t,\alpha)^{233}\text{Pa}$,
 $^{236}\text{U}(t,\alpha)^{235}\text{Pa}$ and $^{238}\text{U}(t,\alpha)^{237}\text{Pa}$. The
ordinates are calibrated approximately in
 $\mu\text{b}/\text{sr}$ to facilitate a comparison of the spectra.

Figure 2

reactions are shown in Fig. 3. In Tables 1-5 are shown the spectroscopic information of the isotopes under study, along with the theoretical values of the cross sections and excitation energies.

The most prominent level populated by the (t, α) reaction in these nuclei is the $J=1/2$ member of the $1/2[400]$ band which previously was only tentatively assigned. In Fig. 4 it is shown that this band has a positive slope with respect to ϵ_2 , contrary to the $1/2[530]$ band. Therefore, as a hole state, the excitation energy of the $1/2[400]$ band decreases monotonically with increasing ϵ_2 , i.e. with increasing atomic mass, and it becomes the ground state band in ^{237}Pa and ^{231}Ac . Another very large peak in each spectrum is formed by the bandhead of the $3/2[402]$ configuration which has a positive slope also and is therefore shifted in a similar manner as the $1/2[400]$ band.

At excitation energies above about 600 keV two fairly strong lines are seen in all spectra which separate in energy from each other with increasing ϵ_2 deformation. As seen in Fig. 4 the $1/2[541]$ and the $9/2[514]$ states are the only proton-hole states found at this excitation energy in the Nilsson diagram and they show the required behavior with respect to changes of ϵ_2 . The strongly populated members of these bands, the $J=11/2$ and $J=5/2$ levels of the $9/2[514]$ and $1/2[541]$ bands, respectively, have the expected relative cross sections. It should be noted, however, that in the $1/2[541]$

Figure 3

Alpha spectra from the reactions $^{230}\text{Th}(t,\alpha)$
 ^{229}Ac and $^{232}\text{Th}(t,\alpha)^{231}\text{Ac}$.

Figure 3

Table 1
Spectroscopic Information for ^{233}Pa

Line	E	(keV)	J	$K^\pi [Nn_z A]$	$\frac{d\sigma}{d\Omega}^{\text{exp}}_{60^\circ}$ [$\mu\text{b/sr}$]	$\frac{d\sigma}{d\Omega}^{\text{theor}}$	E theor
		*					
0	0	0	3/2	$1/2^- [530]$	79	63	0 ^{b)}
1	57	2	7/2	$1/2^- [530]$	80	107	46
2	104	6	9/2	$3/2^+ [651]$	58	28	99
			+7/2	$3/2^+ [651]$			
3	163 ^{a)}	1	13/2	$3/2^+ [651]$	321	77	203
4	169 ^{a)}	1	1/2	$1/2^+ [400]$		247	216
5	202 ^{a)}	1	3/2	$1/2^+ [400]$		39	243
6	290	6			8		
7	347	6			19		
8	411	10			3		
9	444	5	3/2	$3/2^+ [402]$	143	166	375
10	474	10	5/2	$3/2^+ [402]$	27	10	405
11	523	10			4		
12	543	10			20		
13	569	10			36		
14	690	12			9		
15	736	8			21		
16	800	12			2		
17	833	12			5		
18	863	12			6		
19	942	12			7		
20	980	8	(11/2	$9/2^- [514])$	74	51	1051
21	1044	12			8		
22	1065	8	(5/2	$1/2^- [541])$	38	28	1143

a) Level energies adopted from ref. 16.

b) The theoretical level energies have been shifted by a small additive constant to set the energy of the ground state equal to zero. The value of this constant ranges from 4 to 13 keV in the isotopes under study.

Table 2
Spectroscopic Information for ^{235}Pa

Line	E	(keV) $\pm$	J	$K^\pi(Nn_2\Lambda)$	$d\sigma/d\Omega_{\text{exp}}$ 60° [$\mu\text{b/sr}$]	$d\sigma/d\Omega_{\text{theor}}$	E theor
0	0	0	3/2	1/2 ⁻ [530]	87	73	0
1	19	3	1/2	1/2 ⁺ [400]	226	219	69
2	55	4	7/2	1/2 ⁻ [530]	107	103	43 ^{a)}
3	65	6	9/2	3/2 ⁺ [651]	65	31	83
4	100	4			52		
5	132	4	13/2	3/2 ⁺ [651]	99	82	172
6	162	6			7		
7	192	8			13		
8	252	8			17		
9	345	6	3/2	3/2 ⁺ [402]	132	143	197
10	378	10	5/2	3/2 ⁺ [402]	17	10	227
11	484	12			5		
12	525	12			21		
13 b)	570	12			31 ^{b)}		
14	630	12			26		
15	649	12			12		
16	682	12			11		
17	751	8	(11/2	9/2 ⁻ [514])	77	38	1009
18	965	8	(5/2	1/2 ⁻ [541])	59	25	1040

a) The 3/2 1/2[400] state is expected at 50-60 keV excitation energy. Additional strength in the doublet (2,3) is therefore probably due to this level.

b) Probably doublet.

Table 3
Spectroscopic Information for ^{237}Pa

Line	E	(keV)	J	$K^\pi [Nn_z \Lambda]$	$\frac{d\sigma}{d\Omega}^{\text{exp}}_{60^\circ}$ [$\mu\text{b/sr}$]	$\frac{d\sigma}{d\Omega}^{\text{theor}}$	E theor
0	0		1/2	$1/2^+ [400]$	210	192	0
1	35	2	3/2	$1/2^+ [400]$	51	35	26
2	90	3	3/2	$1/2^- [530]$	121	87	101
3	105	5	9/2	$3/2^+ [651]$	47	38	78
4	147	6	7/2	$1/2^- [530]$	82	98	144
5	158	6	13/2	$3/2^+ [651]$	145	71	119
6	202	6			7		
7	258	4			30		
8	319	4			9		
9	364	4	3/2	$3/2^+ [402]$	100	98	95
10	393	6	5/2	$3/2^+ [402]$	22	13	124
11	491	4			9		
12	554	8			16		
13	577	8			23		
14	624	4	(11/2	$9/2^- [514])$	82	25	919
15	686	8			11		
16	714	8			20		
17	741	8			8		
18	972	8			30		
19	1025	6	(5/2	$1/2^- [541])$	66	a)	
20	1112	4			32		

a) The theoretical level energy is less than $6\hbar\omega_0$ and therefore not included in the BCS calculation.

Table 4
Spectroscopic Information for ^{229}Ac

Line	E	(keV)	J	$K^\pi [Nn_z \Lambda]$	$d\sigma/d\Omega_{\text{exp}}$ 60° [$\mu\text{b/sr}$]	$d\sigma/d\Omega_{\text{theor}}$	E theor
0	0		3/2	3/2 [651] +	} 6	0.6	0
	5 ^{a)}		5/2	3/2 [651]		6	5
1	26	5	9/2	3/2 [651]	46	31	32
2	70	5	(5/2	3/2 [532])	12	15	168
3	91	5	13/2	3/2 [651]	92	75	88
4	102	8	3/2	1/2 ⁻ [530]	39	61	1
5	120	5	(7/2	3/2 ⁻ [532])	22	22	211
6	151	5	1/2	1/2 [400]	206	248	217
7	165	7	7/2	1/2 ⁻ [530]	104	96	42
8	191	4	(3/2	1/2 [400])	72	40	244
9	283	6			8		
10	336	4	3/2	3/2 [402]	130	153	375
11	366	4	(5/2	3/2 [402])	71	10	406
12	422	4			13		
13	464	4			14		
14	489	4			12		
15	598	4			74		
16	623	6			21		
17	654	4			17		
18	825	6			16		
19	930	4	(11/2	9/2 ⁻ [514])	85	35	1080
20	1004	4	(5/2	1/2 ⁻ [541])	51	24	1155
21	1021	8			15		
22	1111	8			7		
23	1167	6			20		
24	1249	6			48		

a) Theoretical value

Table 5
Spectroscopic Information for ^{231}Ac

Line	E	(keV)	J	$K^\pi [Nn_z \Lambda]$	$\frac{d\sigma}{d\Omega}_{\text{exp}}$ 60° [$\mu\text{b/sr}$]	$\frac{d\sigma}{d\Omega}_{\text{theor}}$	E theor
0 ^{a)}	0		1/2	1/2 ⁺ [400]	258	181	13
1	38	4	3/2	1/2 ⁺ [400]	122	63	39
			+3/2	1/2 ⁻ [530]		61	0
2	76	5	9/2	3/2[651]	36	29	41
3	94	3	7/2	1/2 ⁻ [530]	146	96	43
4	135	3	13/2	3/2[651]	76	42	102
5	235	4	3/2	3/2[402]	183	112	78
6	257	10	5/2	3/2[402]	28	10	109
7	305	4			30		
8	350	4			28		
9	420	6			11		
10	469	8			17		
11	647	10			15		
12	671	4	(11/2	9/2 ⁻ [514])	88	26	832
13	797	4	(5/2	1/2 ⁻ [541])	46	24	1155
14	1021	7			13		
15	1100	6			51		
16	1126	6			95		
17	1288	5			35		

a) Note remark in text about the ground state.

Figure 4

Part of the Nilsson diagram calculated for
proton states at $A = 233$. Parameters are:
 $\chi = 0.058$, $\mu = 0.646$, $\epsilon_4 = -0.04$.

Figure 4

band the $J=1/2$ and $J=3/2$ members should be excited with about half the cross section of the $J=5/2$ state, whereas such peaks are not visible. Moreover, the energy spacing between the $J=3/2$ $3/2[402]$ level and the peak proposed to be the $J=5/2$ $1/2[541]$ level remains nearly constant in the Pa isotopes, whereas these Nilsson configurations have very different slopes with respect to ε_2 . Therefore the assignment of the $1/2[541]$ and $9/2[514]$ bands is only tentative.

1. C. E. Bemis, Jr., F. K. McGowan, J. L. C. Ford, Jr.,
W. T. Milner, P. H. Stelson and R. L. Robinson, Phys. Rev.
C8(1973)1466.

2. Influence of Form Factors and Multistep Effects on the
 $^{232}\text{Th}(d,t)$ ^{231}Th and $^{230}\text{Th}(d,p)$ ^{231}Th Reactions--W. Wilcke,
W. Felix*, Th. W. Elze*, J. R. Huizenga, R. C. Thompson**
and R. M. Dreisler***

For a number of years single-nucleon transfer reactions have provided information on the structure of heavy deformed nuclei, such as rare-earth and actinide nuclei. The experiments were usually analyzed with the framework of the Distorted Wave Born Approximation (DWBA), with the Nilsson model used to calculate the amplitude of the radial form factors. This fairly simply approach turned out to describe the gross features of the observed angular distributions and cross sections rather well. However, finer details were not reproduced by the theory. Therefore, in order to investigate possible reasons for observed discrepancies, a series of experiments were performed,

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including measurements of angular distributions of the reactions $^{232}\text{Th}(d,t)^{231}\text{Th}$ and $^{230}\text{Th}(d,p)^{231}\text{Th}$. Figure 5 shows a triton spectrum obtained by bombarding a $50 \mu\text{g}/\text{cm}^2$ ^{232}Th target with 17-MeV deuterons and analyzing the outgoing tritons by means of an Enge split-pole magnetic spectrograph. In the analysis of the data, emphasis was placed on a study of the influence of multistep processes and improved radial form factors on the differential cross sections. In Fig. 6 are shown as dashed lines the results of normalized DWBA calculations employing form factors obtained from solving the Schrödinger equation for a spherical Saxon-Woods potential. Another set of form factors was calculated by solving a deformed Saxon-Woods potential eigenvalue problem. The projections of the corresponding eigenfunctions on the subspace of definite angular momentum serves then as form factors. The solid lines in Fig. 6 are the results of DWBA calculations using the "projected" form factors. The differences in the results of the two calculations are very small. This is due to the fact that the reaction under study is localized outside the nucleus even though the energy of the incident beam is well above the Coulomb barrier. Hence, the model-independent tails of the form factors determine completely the shape of the angular distributions.

The different radial shapes of the form factors and their normalizations, however, strongly influence the spectroscopic factors. The ratio R of experimental-to-theoretical cross sections taken at the maximum of the angular distributions of all unambiguously identified levels in ^{231}Th are shown in

Figure 5

Triton spectrum from the $^{232}\text{Th}(\text{d},\text{t})\ ^{231}\text{Th}$ reaction.

Figure 6

Differential cross sections for the $^{232}\text{Th}(d,t)^{231}\text{Th}$ reaction. The dashed lines are normalized DWBA calculations employing form factors obtained from solving the Schrödinger equation for a spherical Saxon-Woods potential. The solid lines are the results of DWBA calculations using "projected" form factors (see text).

Figure 6

Fig. 7. The dotted line (I) connects the results of DWBA calculation using Nilsson coefficients and spherical form factors.

The results using the projected form factors are shown by line III. The obvious improvement is primarily due to differences in the radial behavior between spherical and projected form factors and less due to changes in the normalization of the form factors. This is illustrated by line II in Fig. 7, which represents DWBA calculations employing spherical form factors as line I, but with Nilsson coefficients replaced by the normalization integrals of the projected form factors.

Although the DWBA reaction theory with projected form factors describes the experimental results very well, the influence of multistep processes was studied by performing CCBA calculations using the code MARS¹) and projected form factors. The results are plotted in Fig. 8 and show that the influence of higher order processes is very small, even for the weakly populated members of the ground state rotational band. A detailed study showed that the contributions of different higher order effects (i.e. various paths, deexcitation effects) cancel each other in the reaction under study. Hence, it cannot be excluded that even rather similar reactions will show a stronger influence of multistep processes.

Some additional measurements of the stripping reaction $^{230}\text{Th}(d,p)^{231}\text{Th}$ were made because this reaction populates the same residual nucleus and, therefore, the same form factors describe the neutron transfer (apart from small binding energy

Figure 7

Ratio of experimental to theoretical cross sections
taken at the maximum of the angular distributions.
The explanation of the different lines is given in
the text.

Figure 7

Figure 8

The same data which are shown in Figure 6 compared to CCBA calculations to study the influence of multistep processes. The solid and broken line refer to results with the spherical and coupled channel optical potentials, respectively.

effects). In Fig. 9 the R values for the (d,p) reaction are shown and compared with the equivalent values from the (d,t) reaction. It is noticeable that the R values are fluctuating in a rather characteristic manner around their mean values, independent of the reaction type. This may indicate that the single-particle wave functions, i.e., the form factors, are responsible for the remaining small deviations of the relative cross sections between experiment and theory and not the reaction theory. Although the measured angular distributions for the (d,p) reaction are incomplete, slight hints were found that CCBA effects are more important in this case.

At the present, the reactions $^{238}\text{U}(\text{d},\text{t})$, ^{237}U and $^{240}\text{Pu}(\text{d},\text{t})$, ^{239}Pu are under study.

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Technical Report No. 30 (unpublished.)
3. Study of Nuclear-Coulomb Interference Effects in Inelastic Deuteron Scattering on ^{238}U --W. Feix*, W. Wilcke, Th. W. Elze*, H. Rebel**, J. R. Huizenga, R. C. Thompson***, R. M. Dreizler****

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Figure 9

The ratios of the experimental to the theoretical cross sections for the $^{230}\text{Th}(d,p)^{231}\text{Th}$ reaction (solid line) are compared to the equivalent ratios for the $^{230}\text{Th}(d,t)^{231}\text{Th}$ reaction (broken lines).

$$R = d\sigma(\text{exp}) / d\sigma(\text{theor})$$

During a study of deuteron-induced reactions on actinide nuclei, the elastic and inelastic scattering of 17-MeV deuterons on ^{238}U have been investigated. It was found that coupled-channel calculations using a Coulomb potential derived from a deformed homogeneous charge distribution fail to reproduce the measured angular distribution for the population of the first 2^+ state. However, the introduction of a diffuse charge distribution for both the target and projectile leads to good agreement with experiment and allows the determination of deformation parameters. Studies of the influence of non-homogeneous charge distributions on the scattering of α -particles and ^{16}O ions were reported earlier [1-3].

The measurements were performed by bombarding a thin ^{238}U target with a 17-MeV deuteron beam from the University of Rochester MP tandem Van De Graaff accelerator. The target was prepared by evaporation of approximately $80\text{ }\mu\text{g}/\text{cm}^2$ of metallic uranium onto a $20\text{ }\mu\text{g}/\text{cm}^2$ carbon foil. The scattered deuterons were recorded by a position sensitive silicon detector, located in the focal plane of an Enge split-pole magnetic spectrograph, with an energy resolution of approximately 9-keV FWHM. A spectrum is shown in Fig. 10. The energy resolution obtained is more than sufficient to resolve the 2^+ state at 48-keV excitation energy at angles larger than about 45° . However, at smaller angles, where the peak-to-valley ratio of the 2^+ level approaches 3, the low energy tail of the strong elastic scattering peak has to be taken into account. A background fit using a fourth-order polynomial

Figure 10

Deuteron scattering from ^{238}U at a backward angle.

Figure 10

expression did not give consistent results and, therefore, a method was used which employs the fact that both peaks have the same shape since their energy difference is very small compared to the absolute energy of the deuterons. If the line shape of the larger peak is given by the function $f(x)$, then the superposition of the two peaks separated by the known distance d and an unknown intensity ratio $v < 1$ can be written as

$$r(x) = f(x) + v \cdot f(x-d),$$

where $r(x)$ is the measured spectrum. Solving this equation for $f(x)$ yields for the peak shape

$$f(x,v) = \sum_{n=0}^{\infty} (-v)^n r(x-nd).$$

If $v \ll 1$, as in the present problem, the calculated peak shape on the low energy side of the peak depends very sensitively upon the value of the parameter v . Small deviations from an optimum value v_0 generate unphysical bumps on the tail of $f(x,v)$, and this fact can be used to determine v_0 . Integration of $f(x,v_0)$ finally leads to the peak area. This method was used to extract the cross sections of the 0^+ and the 2^+ peak for all angles smaller than 45 degrees.

The starting point for the choice of the optical potential was provided by the hypothesis [4] that the same optical potential can be used in neighboring spherical and deformed nuclei, provided the collective states are taken into account explicitly by performing a coupled channel analysis. Therefore, a set [5] of optical parameters (named SP in Table 6) was

Table 6

Deuteron optical model parameters

	V	r_o	a_o	W_D	r_D	a_D	r_c
SP	103.5	1.10	0.91	11.44	1.39	0.81	1.3
CCP	80	1.10	0.91	20.44	1.33	0.85	1.3

Units: MeV or fm

For the definition of the optical potential, see ref. 5

selected, which fits the elastic scattering of 18.7-MeV deuterons on the spherical nucleus ^{208}Pb . It was found, however, that these parameters did not fit the present elastic scattering data at larger angles when used in a coupled channel* calculation including the 0^+ and 2^+ states (broken line in Fig. 11). Hence, the optical model parameters were adjusted, giving the parameter set CCP in Table 6. The solid line in Fig. 11 is obtained with these parameters, which are used in all subsequent calculations.

The reduction of the elastic cross section due to the population of the 2^+ state is illustrated in Fig. 11 by comparing the dotted and the solid lines. The dotted line is the result of a calculation using the CCP potential, but without coupling to the 2^+ state.

Contrary to the elastic scattering results, the inelastic scattering was found to be very sensitive to the shape of the Coulomb potential. Two potentials were compared, both of which are written

$$V_c(\vec{r}) = \int \frac{\rho_t(\vec{r}_t) \cdot \rho_D(\vec{r}_D)}{|\vec{r} + \vec{r}_D - \vec{r}_t|} \cdot d^3r_t \, d^3r_D$$

(i) The first model [6] assumes a homogeneous charge distribution for the target and a point charge for the projectile

$$\rho_t(\vec{r}_t) = \begin{cases} \rho_0 & r_t \leq R(\hat{r}_t) \\ 0 & r_t > R(\hat{r}_t) \end{cases}$$

*The calculations were performed with the computer code JUPITOR [6] and a modified version of ECIS [7]

Figure 11

Elastic deuteron scattering from ^{238}U . The solid and dashed lines refer to two coupled channel calculations using the CCP potential and the SP potential, respectively, and the dotted line to an uncoupled calculation employing the CCP potential (see Table 6).

Figure 11

with

$$R(\hat{r}_t) = R_0 [1 + \beta_2^C Y_{20}(\hat{r}_t) + \beta_4^C Y_{40}(\hat{r}_t)]$$

$$R_0 = c_0 A^{1/3}$$

and

$$\rho_D(\vec{r}_D) = Z_D e \cdot \delta(\vec{r}_D).$$

(ii) The second model [1] assumes a deformed Fermi distribution for the target and a spherical Gaussian for the deuteron charge distribution.

$$\rho_t(\vec{r}_t) = \rho_0' (1 + \exp \frac{r_t - c(\hat{r}_t)}{a})^{-1}$$

$$c(\hat{r}_t) = c_0 [1 + \beta_2^C Y_{20}(\hat{r}_t) + \beta_4^C Y_{40}(\hat{r}_t)]$$

and

$$\rho_D(\vec{r}_D) = \rho_0'' \exp \left(-\frac{|\vec{r}_D|^2}{\alpha^2} \right)$$

where ρ_0' and ρ_0'' are defined by the normalization to $Z_t e$ and $Z_D e$, respectively. Since the values [8] of $c_0 = 1.1$ fm and $a = 0.6$ fm were taken from electron scattering data, no free parameters are introduced into the second model. The width α^2 of the Gaussian was set equal to the mean square radius [9] $\langle r^2 \rangle = 4.31$ fm² of the deuteron. The charge deformation parameters $\beta_2^C = 0.261$ and $\beta_4^C = 0.106$ were determined [8] from Coulomb excitation experiments, assuming a deformed Fermi* distribution. The dot-dashed line shown in Fig. 3 is

*actually a 'modified-c-distribution'

the result of a two-state coupled channel calculation, assuming the homogeneous charge distribution, while the thick solid line was calculated using the deformed Fermi distribution. The very good agreement with experiment is entirely due to the introduction of the Fermi charge distribution. Before this point was recognized, extensive trial runs using different optical model parameters were made to search for a satisfactory agreement with experiment, but all these attempts failed. The observed angular distribution is determined by rather pure nuclear excitation at backward angles and pronounced nuclear-Coulomb interference effects at angles forward of 45 degrees. This is indicated by the fact that the differential cross section at backward angles, to good approximation, is proportional to $(\beta_2^N)^2$, where β_2^N is the optical potential deformation parameter. The shape of the angular distribution at forward angles on the other hand is very sensitive to the ratio of the β_2^N to β_2^C . This is illustrated by the dashed line in Fig. 12, which represents the result of a calculation where β_2^C is increased by only 8% to 0.28. Therefore, it is possible to determine β_2^N and β_2^C separately by measuring the angular distribution at both forward and backward angles. The value of β_2^C was found to be consistent with the result of a previous Coulomb excitation measurement [8]*.

*The larger value of β_1^C given by Bemis et al. [8] seems to result from the notably smaller value of β_2^C and the lower value of the diffuseness $a = 0.5$ fm used by those authors in their interpretation of the measured $B(E2)$ value on the basis of a deformed Fermi distribution. Increasing β_2^C and the diffuseness to $a = 0.6$ fm as suggested by electron scattering results [12] requires a smaller value of β_1^C which is consistent with our choice.

Figure 12

Inelastic deuteron scattering from ^{238}U . The solid and dot-dashed lines refer to coupled channel calculations using diffuse and hard-edge charge distributions, respectively. The sensitivity of the calculations to a change of the deformation parameters is illustrated by the dashed line (see text). The dotted line is the result of a $0^+ - 2^+ - 4^+$ coupling without inclusion of Coulomb excitation.

Figure 12

The numerical value of β_2^N for the optical potential of the system ^{238}U -deuteron determined by the present experiment is 0.22 and proves to be significantly smaller than the value of the charge deformation parameter. Such a result is not unexpected, even if one assume that the deformation of the mass distribution in the ^{238}U nucleus follows the deformation of the proton distribution. The size and shape of the optical potential are the result of a convolution of projectile and target properties, and the reduced value of β_2^N may reflect the finite size of the projectile smearing out the deformation of the target nucleus. Indeed, recent folding model analyses of α -particle and heavy-ion scattering have revealed such effects [10,11] which must be considered when results from different types of experiments are compared.

The inclusion of the 4^+ level in the calculations would increase the quoted value of β_2^N by approximately 5%, since the measured cross section of the 4^+ level is 10% of the cross section for the 2^+ level at backward angles. The dotted line in Fig. 12 indicates the calculated cross section of the 4^+ level without inclusion of the Coulomb excitation because of the limited computer core space available.

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4. Levels of ^{235}Np Excited by the $^{234}\text{U}(^3\text{He},d)$ and $^{234}\text{U}(\alpha,t)$ Reactions--R. D. Griffioen*, R. C. Thompson** and J. R. Huizenga

During the past several years studies of proton transfer reactions in the actinide region have provided considerable new information on the systematics of single-proton states in this region.¹⁻⁵ The information obtained by means of these reactions is usually complementary to that obtained by α - and β -decay studies. Two of the odd-A isotopes of neptunium, namely ^{237}Np and ^{239}Np , have been studied previously with proton transfer reactions.^{1,2}

Here, we report results of measurements of proton states in ^{235}Np excited by the $^{234}\text{U}(^3\text{He},d)$ and $^{234}\text{U}(\alpha,t)$ reactions. The level scheme of ^{235}Np has been studied previously by α decay⁶⁻⁸ and β decay^{9,10} leading to the identification of the ground state $5/2^+[642]$ rotational band and the $5/2^- [523]$ band beginning at 49.3 keV. This latter band is the ground state band in ^{239}Am and more than 99% of the α decay of ^{239}Am goes to the three lowest members of this band in ^{235}Np . Three levels at 756.4, 799.6 and 819.0 keV have been identified in the β decay of ^{235}Pu as belonging to the $3/2^- [521]$ band.¹⁰ In this study we find that this assignment is incorrect and that this band begins at 566 keV. This new assignment is also more consistent with the location of this band in ^{237}Np and ^{239}Np .

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Additional levels in ^{235}Np have been observed by means of the $^{237}\text{Np}(p,t)$ reaction.¹¹ Two new low-lying levels, tentatively identified as the $11/2^+$ and $13/2^+$ members of the ground state rotational band, were reported, as well as seven new levels between 834 and 1818 keV. No new band assignments were suggested as a result of this (p,t) reaction work. In this study we substantiate the assignment of the $13/2^+$ member of the ground state band.

A fourth band which we identify in this study is the $1/2^-$ [530] band. The systematics of the variation of each of the four bands mentioned above in the three isotopes of neptunium, ^{235}Np , ^{237}Np , and ^{239}Np , is shown. Some tentative assignments to higher lying bands are also made.

The experimental techniques and methods of data analysis used in this work are similar to those which have been described previously.^{1,3} The target of ^{234}U had dimensions $1 \times 5 \text{ mm}^2$ and thickness $\sim 25 \mu\text{g}/\text{cm}^2$ and was deposited on double $20 \mu\text{g}/\text{cm}^2$ carbon backing. The target was bombarded with 30-MeV ^3He and α particles from the University of Rochester Emperor tandem Van de Graaff accelerator. The deuterons from the $(^3\text{He},d)$ reaction and the tritons from the (α,t) reaction were analyzed with an Enge split-pole magnetic spectrograph and recorded on photographic plates (Kodak type NTB, 50 μm). The plates were covered with thin aluminum absorbers to enhance the track density. The overall energy resolution was 16-20 keV for the (α,t) spectra and 18-20 keV for the $(^3\text{He},d)$ spectra. Average exposure times were about 18 hours for both reactions. The plates were developed and scanned in 1/4 mm strips.

A NaI monitor counter located at 45° relative to the beam was used to detect elastically scattered ^3He and α particles and thus to determine absolute differential cross sections. The elastic scattering cross section was assumed to be equal to the Rutherford cross section at 45° for ^3He and equal to 1.15 times the Rutherford cross section for α particles, as indicated by optical model calculations. Short elastic scattering runs with the spectrograph at 30° confirmed these cross sections to within 10%, assuming that the cross section at 30° is equal to the Rutherford cross section.

Distorted Wave Born Approximation (DWBA) calculations using computer code DWUCK¹² indicate that the differential cross sections for the $(^3\text{He},d)$ reaction have broad maxima between 30° and 60° and those for the (α,t) reaction have broad maxima between 50° and 80° . The $(^3\text{He},d)$ spectra were measured at 30° , 45° , 60° and 75° and the (α,t) spectra were measured at 60° and 75° . The spectra were analyzed with a computer fitting program which fit the peaks in each spectrum to a standard shape, which was picked from one of the most intense and best peaks in the spectrum. Energies and intensities for each of the peaks in the spectrum were determined by this program.

Table 7 lists the excitation energies of, the absolute differential cross sections for, and the assignments given to each of the levels. Since the angular distributions for these reactions with actinide nuclei are insensitive to the transferred angular momentum, the technique^{1,3} of using the ratio of the cross sections of the (α,t) and $(^3\text{He},d)$ reactions as an indication of the transferred angular momentum was investigated.

Table 7

Energies and cross sections of levels excited in the $^{234}\text{U}(^3\text{He},\text{d})$ and $^{234}(\alpha,\text{t})$ Reactions

Level	Excitation Energy (keV)		$\frac{d\sigma}{d\Omega}$ ($^3\text{He},\text{d}$) ($\mu\text{b}/\text{sr}$)				$\frac{d\sigma}{d\Omega}$ (α,t) ($\mu\text{b}/\text{sr}$)		Assignment
	from γ decay	this work	30°	45°	60°	75°	60°	75°	
1	0	3+3				0.56 $\pm$.12	1.16 $\pm$ 0.16	0.76 $\pm$ 0.12	5/2 $^+$ [642+]
2	34.3								7/2 $^+$ [642+]
3	49.3	46 $\pm$ 3	1.48 $\pm$ 0.21		0.79 $\pm$.11		1.60 $\pm$ 0.19	2.07 $\pm$.20	5/2 $^-$ [523+]
4	79.0	82 $\pm$ 1	3.79 $\pm$.39	7.31 $\pm$ 2	4.40 $\pm$.26	2.56 $\pm$.25	9.62 $\pm$.45	6.86 $\pm$.36	9/2 $^+$ [642+]
5	91.6								7/2 $^-$ [523+]
6	113 ^a								11/2 $^+$ [642+]
7	146.8	146.8 ^b	5.43 $\pm$.44	8.4 $\pm$ 1.3	6.10 $\pm$.31	5.00 $\pm$.35	36.6 $\pm$.9	32.8 $\pm$.8	9/2 $^-$ [523+]
8	201 ^a	200 $\pm$ 1	6.82 $\pm$.50	7.02 $\pm$.6	11.2 $\pm$.4	6.89 $\pm$.41	54.5 $\pm$ 1.1	49.2 $\pm$ 9	13/2 $^+$ [642+]
9		352 $\pm$ 2	10.6 $\pm$.6	10.0 $\pm$.5	7.52 $\pm$.35	4.40 $\pm$.33	4.71 $\pm$.32	7.95 $\pm$.38	3/2 $^-$ [530+]
10		(371 $\pm$ 3)					2.20 $\pm$.22		(1/2 $^-$ [530+])
11		408 $\pm$ 2	4.95 $\pm$.42	8.04 $\pm$.47	5.70 $\pm$.30	2.90 $\pm$.26	12.7 $\pm$.53	11.9 $\pm$.47	7/2 $^-$ [530+]
12		441 $\pm$ 3	0.73 $\pm$.18				2.38 $\pm$.23	2.29 $\pm$.21	5/2 $^-$ [530+]
13		520 $\pm$ 1	(~2.5)	3.58 $\pm$.31	3.04 $\pm$.27	2.20 $\pm$.23	3.09 $\pm$.26	4.89 $\pm$.30	
14		565 $\pm$ 1		7.45 $\pm$.45	5.96 $\pm$.31	4.42 $\pm$.33	3.87 $\pm$.29	6.80 $\pm$.35	3/2 $^-$ [521+]
15		602 $\pm$ 3	(9 $\pm$ 6)	2.34 $\pm$.25	1.61 $\pm$.16		4.24 $\pm$.31	3.74 $\pm$.26	5/2 $^-$ [521+]

Table 7 (continued)

Level	Excitation Energy (keV)		$\frac{d\sigma}{d\Omega}$ ($^3\text{He}, d$) ($\mu\text{b/sr}$)				$\frac{d\sigma}{d\Omega}$ (α, t) ($\mu\text{b/sr}$)		Assignment
	from γ decay	this work	30°	45°	60°	75°	60°	75°	
16		644 $\pm$ 1	37.3 $\pm$ 4.0	42.3 $\pm$ 1.1	32.4 $\pm$.7	17.9 $\pm$.7	51.7 $\pm$ 1.1	54.5 $\pm$ 1.0	7/2 $^-$ [521 $+$]
17		681					0.58 $\pm$.11		
18		700 $\pm$ 3			1.22 $\pm$.15	1.47 $\pm$.19	2.31 $\pm$.23	3.96 $\pm$.27	9/2 $^-$ [521 $+$]
19	756.4	761 $\pm$ 2		4.17 $\pm$.35	3.60 $\pm$.28	4.42 $\pm$.33	5.20 $\pm$.34	8.02 $\pm$.38	(11/2 $^-$ [521 $+$])) ^c
20	779.6								(doublet?)
21	819.0	820 $\pm$ 3	4.3 $\pm$ 4	3.90 $\pm$.34	3.21 $\pm$.26		3.27 $\pm$.27	3.17 $\pm$.24	
22		870 $\pm$ 3	6.6 $\pm$ 3	3.68 $\pm$.33	3.05 $\pm$.26		4.89 $\pm$.33	4.92 $\pm$.30	
23		922 $\pm$ 1	4.6 $\pm$ 2.0	3.06 $\pm$.31	1.98 $\pm$.24		3.13 $\pm$.26	3.49 $\pm$.25	(7/2 $^-$ [514 $+$])) ^c
24	936.7								
25	944.7								
26		978 $\pm$ 1	4.61 $\pm$.49	6.32 $\pm$.43	4.61 $\pm$.32	3.58 $\pm$.29	8.71 $\pm$.44	7.93 $\pm$.38	(9/2 $^-$ [514 $+$])) ^c
27		1024 $\pm$ 1	4.32 $\pm$.48	6.64 $\pm$.45	4.21 $\pm$.32	3.00 $\pm$.27	3.98 $\pm$.30	4.41 $\pm$.29	
28		1064 $\pm$ 2	2.40 $\pm$ 0.40	6.69 $\pm$.45	4.16 $\pm$.32	3.55 $\pm$.29	8.27 $\pm$.43	8.32 $\pm$.39	(9/2 $^-$ [514 $+$])) ^c
29		1117 $\pm$ 3	3.85 $\pm$.45	3.58 $\pm$.35	1.90 $\pm$.27		3.91 $\pm$.29	5.09 $\pm$.31	
30		1160 $\pm$ 2	6.28 $\pm$.53	8.44 $\pm$.50	6.33 $\pm$.39	5.82 $\pm$.37	19.2 $\pm$.7	20.9 $\pm$ 6	(13/2 $^+$ [633 $+$])) ^c

Table 7 (continued)

Level	Excitation Energy (keV) from γ decay this work	$\frac{d\sigma}{d\Omega}$ ($^3\text{He}, d$) ($\mu\text{b/sr}$)				$\frac{d\sigma}{d\Omega}$ (α, t) ($\mu\text{b/sr}$)		Assignment
		30°	45°	60°	75°	60°	75°	
31	1227 $\pm$ 3	4.99 $\pm$.49	3.52 $\pm$.35	2.91 $\pm$.32	1.81 $\pm$.21	4.31 $\pm$.31	9.90 $\pm$.43	
32	1262 $\pm$ 2	4.73 $\pm$.47	5.30 $\pm$.41	3.05 $\pm$.33	3.16 $\pm$.28	2.82 $\pm$.25		
33	1310 $\pm$ 2	5.62 $\pm$.50	6.64 $\pm$.46	3.47 $\pm$.34	3.53 $\pm$.29	3.78 $\pm$.29		
34	1364 $\pm$ 2	5.40 $\pm$.49	7.69 $\pm$.49	3.69 $\pm$.35	3.50 $\pm$.29	3.13 $\pm$.26	5.37 $\pm$.31	
35	1510 $\pm$ 2	3.85 $\pm$.43	5.56 $\pm$.44	2.94 $\pm$.36		2.64 $\pm$.24		
36	1607 $\pm$ 2		4.27 $\pm$.43	4.10 $\pm$.40	3.29 $\pm$.28			
37	1675 $\pm$ 4	4.61 $\pm$.45	6.72 $\pm$.51	3.18 $\pm$.50	3.8 $\pm$ 2	4.16 $\pm$.30	7.80 $\pm$.38	
38	1696 $\pm$ 2			2.7 $\pm$.7		2.49 $\pm$.23		
39	1758 $\pm$ 3		5.40 $\pm$.43	2.1 $\pm$.8	3.79 $\pm$ 2	5.02 $\pm$.34	6.12 $\pm$.34	(9/2 $^-$ {505+}) ^c
40	1845 $\pm$ 3	10.9 $\pm$.7	14.9 $\pm$ 0.7	11.2 $\pm$ 1.1	12.6 $\pm$ 6.	7.09 $\pm$.40	10.9 $\pm$.5	(7/2 $^-$ {512+}) ^c
41	1918 $\pm$ 4	9.34 $\pm$.65	7.9 $\pm$ 1.6	2.6 $\pm$ 1.3		3.47 $\pm$.28		
42	2050 $\pm$ 5		5.9 $\pm$ 2.8	4.1 $\pm$ 1.1	10.3 $\pm$ 5.	5.53 $\pm$.35	9.48 $\pm$.42	

a) from $^{237}\text{Np}(p, t) ^{235}\text{Np}$ (ref.11)

b) Reference State whose excitation energy is assumed to be 146.8 keV

c) parentheses indicate uncertain assignments

Because the reactions were run at several angles, the validity of this technique, which has been questioned by Erskine et al⁵, could be checked. The cross sections for the two reactions were calculated with computer code DWUCK¹² using the optical model parameters given by Elze and Huizenga.¹ These computations were performed for three different reaction Q-values, corresponding to excitation energies in ²³⁵Np of 0, 1 and 2 MeV. The cross sections for other energies were obtained by linear interpolation. The proposed energy level diagram for ²³⁵Np showing the assignments to rotational bands (see discussion below) is shown in Fig. 13.

The differential cross section for a proton stripping reaction on an even deformed target nucleus may be written:

$$(d\sigma/d\Omega)_{0+J=j} = 2 N_0 c_{j\ell}^2 U^2 \sigma_{\ell DW}(\theta, Q)$$

where N_0 is a normalization factor, $\sigma_{\ell DW}(\theta, Q)$ is the single-proton transfer cross section, $c_{j\ell}$ is the amplitude in the expansion of the Nilsson state under consideration in terms of spherical limit functions, and U^2 is the probability that the level into which the proton is transferred is empty. In cases where Coriolis coupling is appreciable and $\Delta N=2$ mixing is taken into account, the expression for the differential cross section becomes:

$$(d\sigma/d\Omega)_{0+J=j} = 2N_0 \sum_N \left(\sum_i a_i c_{j\ell}^i U_i \right)^2 \sigma_{\ell DW}(N, \theta, Q)$$

where a_i is the amplitude of the i -th single-proton configuration and N is the number of the oscillator shell. The $\sigma_{\ell DW}(N, \theta, Q)$ were calculated, as mentioned above, using computer code DWUCK.¹²

Figure 13

Partial level scheme of ^{235}Np showing levels excited in the $^{234}\text{U}(^3\text{He},d)$ reaction and/or the $^{234}\text{U}(\alpha,t)$ reaction. Parentheses indicate uncertain assignments. On the left are all the levels seen below 1250 keV. Dotted lines in the rotational bands indicate levels seen in other work. Energies are in keV.

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Figure 13

The differential cross sections were then calculated including Coriolis and $\Delta N=2$ mixing. Theoretical calculations of equilibrium deformations for actinides using the Strutinsky shell-correction method have been made^{13,14} and Coulomb-excitation experiments in the actinide region have been performed which allow for model-dependent deformation parameters to be extracted.¹⁵ All of these indicate that ϵ_2 should lie between 0.18 and 0.22 and ϵ_4 between -0.04 and -0.08 in the vicinity of ^{235}Np . We varied these two parameters, as well as the Coriolis attenuation factor, in order to obtain the best energy fit for the $5/2^+[624]$ and $5/2^-[523]$ bands. This best fit was obtained with $\epsilon_2 = 0.19$, $\epsilon_4 = -0.06$ and a Coriolis attenuation factor of 0.6, and these parameters were used in all of the subsequent calculations. Actually the calculated cross sections for most levels are very insensitive to variations of the deformation parameters over the range indicated. Predicted excitation energies are, of course, strongly effected. Values of κ and μ were taken from the work of Nilsson et al.¹³ The rotational band A-parameter was chosen to be 6.0 keV from the known energy spacings of the $5/2^-[523]$ band and the fact that in ^{237}Np and ^{239}Np there is very weak Coriolis coupling for this band.^{1,2} Comparison of these calculations with the experimental cross sections was also utilized in making assignments, as discussed below, to various rotational bands.

Previous studies^{1,2} of proton transfer reactions to levels in ^{237}Np and ^{239}Np , as well as calculations of the $c_{j\lambda}$ coefficients, indicate that the largest peak in the $(^3\text{He},d)$ reaction

should be due to the $7/2^-$ [521] level and the largest peaks in the (α, t) reaction should be due to the $9/2^-$ [523], $13/2^+$ [642] and $7/2^-$ [521] levels. Of these three levels the $9/2^-$ [523] level is most well known. Its energy, as determined in the α decay of ^{239}Am , is 146.8 ± 0.7 keV.⁸ Since the proton transfer reaction to the ground state of ^{235}Np is very weak, the energies of all of the spectra taken in this study were normalized to the 146.8 keV level.

In order to check the usefulness of the ration (R) of the cross sections for determining l values, we made use of several peaks of known l transfers. Comparing the experimental cross section ratio with the calculated one for various pairs of angles one obtains the relative normalization $[N_0(\alpha, t)/N_0(^3\text{He}, d)]$ for the two reactions. We found that this relative normalization varied by about a factor of 2. This indicates that if the normalization factors are fixed, the cross section ratios will vary by about a factor of 2 depending on the angles chosen. This variation is much larger than the errors involved in the experimental cross sections. In the case of the (d, t) and $(^3\text{He}, \alpha)$ reactions, R varies over two to three orders of magnitude as l varies from 0 to 6. (see e.g. references 16 and 17.) In these cases, a variation in R of a factor of 2 would still allow for a rather conclusive assignment for the l -transfer values. In the case of the $(^3\text{He}, d)$ and (α, t) reactions R varies much less as l changes from 0 to 6. For example, in the paper of Erskine et al.⁴ in which the question of the validity of using R is raised, the calculated value of R varies by less than a factor of 2 as l changes from 0 to 6 (at 0 excitation energy).

If the experimental R's have a variation of a factor of 2, then indeed very little reliable information can be obtained in this case regarding the ℓ -transfer values. Part of the problem was the choice of angles for the two reactions which gave rise to such a small theoretical variation in R.

In this study we pick the angles for the two reactions which give the greatest expected variation of R with ℓ . These angles are 45° for the $(^3\text{He}, d)$ reaction and 60° for the (α, t) reaction. This gave a variation in the calculated R as ℓ varies from 0 to 6, of a factor of 7.3 at 0 MeV excitation and 18.3 at 1 MeV excitation. Even using this optimum case, fluctuations in R of a factor of 2 enable us to determine ℓ to within about ± 1 unit.

The normalization constant N_0 for the $(^3\text{He}, d)$ reaction was taken to be the usual theoretical value of 4.42. This value gives a reasonable agreement between the calculated and experimental cross sections for the $7/2^-$ [521] level which is the strongest level seen in this reaction. The normalization constant of the (α, t) reaction is much less well known and empirical values ranging from 71-160 have been used.¹⁻⁵ We have used the value which Erskine et al.⁴ used to give the best overall agreement with ratios of cross sections, $N_0 = 120$. This value gives reasonable agreement between the theoretical and experimental cross sections for the three most intense peaks seen in the (α, t) reaction. It also gives reasonable agreement for the cross section ratios for levels of known spin, including the $13/2^+$ [642] and $7/2^-$ [521] levels which are confirmed or identified in this study.

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B. Heavy Ion Reactions

1. Internuclear Potentials from Heavy Ion Fusion Excitation Functions--J. R. Birkelund and J. R. Huizenga

In a recent paper¹ Scobel et al have deduced fusion barrier parameters from a considerable number of well determined complete fusion excitation functions measured for ^{35}Cl induced reactions on a series of targets with masses between ^{27}Al and ^{124}Sn . In principle, the barrier parameters determined from complete fusion reactions may lead to a measurement of the internuclear potential at radial separations smaller than the separation region tested by elastic scattering measurements. However, because of the depth of interpenetration of the target and projectile necessary to reach the fusion barrier, the effect of nuclear friction cannot be ignored if the nuclear potential is to be extracted from fusion barrier parameters. In addition, since the nuclear potential is determined from the difference between the barrier height and the Coulomb potential, the deduced nuclear potential is very seriously affected by errors in the radial position of the barrier.

If friction is neglected in the analysis of fusion excitation functions, the fusion cross section $\sigma_f(E)$ at incident projectile energy E is given by¹

$$\sigma_f(E) = \pi R_B^2 \left(1 - \frac{V_B}{E}\right) \quad (1)$$

where R_B is the s-wave barrier radius and V_B is the barrier height obtained from the combination of the Coulomb potential V_C and nuclear potential V_N .

$$V_N(R_B) = V_B(R_B) - V_C(R_B) \quad (2)$$

Barrier parameters are obtained from the data by fitting a straight line through a plot of σ_f vs $1/E$. The slope and intercept of this line with the $1/E$ axis lead to the s-wave barrier radius and height, respectively. The validity of this analysis requires that all l waves contributing to the fusion cross section have the same barrier radius R_B , a condition which is probably not fulfilled for most reactions.

If the friction is important and included in the derivation of $\sigma_f(E)$, again for the case of constant orbital angular momentum, Eq. (1) is modified to first order to

$$\sigma_f(E) = \pi R_B^2 \left(1 - \frac{V_B + E_F}{E} \right) \quad (3)$$

where E_F is the energy loss due to friction on that portion of the trajectory in the entrance channel leading up to the barrier. The magnitude of the energy loss due to friction depends on the radial dependence of the friction forces near the barrier. For some systems it may be that the s-wave barrier lies close to or outside the range of the friction where E_F is zero, and the friction correction included in Eq. (3) may be neglected. However, in most cases as the projectile energy increases, the l -dependent barriers move to smaller radii. Hence, the friction loss is expected to become more important than at the s-wave barrier radius, and, in general, E_F will be a function of the projectile energy. Qualitatively, the effects of friction on the derived nuclear potential may be seen by the assumption that E_F is constant for the bombarding energies used in the data analysis. If R_B is assumed constant also, then Eq. (3) represents a straight line with

a $1/E$ axis intercept which leads to a value of $V_B + E_f$, and a slope related to the s-wave barrier radius by

$$\frac{d\sigma_f}{d(1/E)} = -\pi R_B^2 (V_B + E_f) \quad (4)$$

Thus, the effect of a neglect of friction in the analysis of fusion cross section data is to produce a value for the barrier which is too high and, hence, an underestimate of the absolute magnitude of the nuclear potential, if the barrier radius is independent of l . Further, in most cases the slope of the σ_f vs $1/E$ curve is less well determined than the intercept on the $1/E$ axis, and this leads to an uncertainty in the barrier radius R_B . This uncertainty is further increased by the general dependence of the barrier distance on l , and hence projectile energy, which causes the σ_f vs $1/E$ curve to deviate from a linear function. An error in the barrier radius is reflected as error in the nuclear potential through Eq. (2), since an accurate knowledge of the radius is necessary to evaluate the large and strongly radial dependent Coulomb potential.

Scobel et al.¹ have compared the nuclear potential derived from their fusion data with various theoretical potentials. In Fig. 14 this comparison is extended, using the proximity potential of Blocki et al.² and nuclear potentials derived from fusion excitation functions listed in Table 8. The fusion data for Fig. 14 were analyzed neglecting nuclear friction by the use of Eq. (1). The proximity potential is expressed as a function of the surface separation ζ and given by²

Figure 14

A comparison of the universal proximity potential² $\phi(\zeta)$ as a function of the dimensionless quantity ζ (solid line) with experimental data (points). The analysis of elastic scattering data³ produces the circles and the analysis of inelastic reaction data produces the triangles. The open triangles are based on excitation function measurements of fusion cross sections by counter-telescope measurements of evaporation residues and/or fission fragments, while the filled triangles rely on excitation functions based on summing measured partial fusion cross sections. The latter values are subject to the largest errors. The region enclosed by the dashed line in the insert shows the part of the proximity potential tested by experimental data. Errors in ζ are reflected also in ϕ (see text) as illustrated for two fusion points.

Figure 14

Table 8

Barrier parameters measured from experimental fusion excitation functions by use of Eq. (1)

REACTION	R_B (EXP) fm	$V_B(R_B)$ (EXP) MeV	$-V_N(R_B)$ (EXP) MeV	C_P fm	C_T fm	γ MeV fm ⁻²	$\frac{4\pi\gamma C_T C_P}{C_T + C_P}$ MeV fm ⁻¹	ζ_B (EXP.) fm	$-\dot{\phi}(\zeta_B)$ (EXP.)	R_B Theory fm	Ref.
$^4\text{He} + ^{152}\text{Dy}$	9.9±0.2	17.1±0.1	2.1±0.1	1.21	6.06	0.9238	11.704	2.63±0.2	0.179±0.008	9.74	10
$^4\text{He} + ^{233}\text{U}$	9.8±0.6	24.0±0.2	5.6±0.2	1.21	7.11	0.8791	11.420	1.51±0.6	0.495±0.02	10.66	14
$^7\text{Li} + ^{159}\text{Tb}$	8.6±0.2	23.7±0.1	9.0±0.1	1.63	6.16	0.8963	14.518	0.79±0.2	0.622±0.007	10.20	10
$^{12}\text{C} + ^{12}\text{C}$	6.65	6.17	1.63	2.12	2.12	0.9517	13.996	2.41	0.116	7.32	11
$^{12}\text{C} + ^{152}\text{Sm}$	10.9±0.3	46.8±0.2	2.3±0.2	2.12	6.06	0.9022	17.806	2.73±0.3	0.129±0.02	10.27	10
$^{16}\text{O} + ^{12}\text{C}$	7.55	7.69	1.46	2.42	2.12	0.9517	13.515	3.01	0.108	7.49	11
$^{16}\text{O} + ^{27}\text{Al}$	8.1±0.15	16.1±0.6	2.4±0.6	2.42	3.05	0.9508	16.135	2.63±0.15	0.148±0.04	8.07	15
$^{16}\text{O} + ^{148}\text{Nd}$	9.8±0.2	57.8±0.5	12.8±0.5	2.42	6.00	0.9240	20.023	1.37±0.2	0.64±0.03	10.41	10
$^{16}\text{O} + ^{208}\text{Pb}$	11.4±0.8	74.8±2	8.2±2	2.42	6.82	0.8830	22.446	2.14±0.8	0.36±0.09	11.10	12
$^{18}\text{O} + ^{12}\text{C}$	9.3±0.2	7.51	1.28	2.55	2.12	0.9442	13.735	3.19	0.093	7.66	11
$^{18}\text{O} + ^{148}\text{Nd}$	7.86	58.3±0.8	16.3±0.8	2.55	6.00	0.8963	20.155	0.71±0.2	0.81±0.04	10.59	10
$^{19}\text{F} + ^{12}\text{C}$	7.64	8.08	2.1	2.62	2.12	0.9499	13.988	2.90	0.150	7.65	11
$^{32}\text{S} + ^{24}\text{Mg}$	8.5±0.3	27.8±0.3	4.7±0.3	3.27	2.90	0.9495	18.339	2.3±0.3	0.26±0.02	8.49	9
$^{32}\text{S} + ^{27}\text{Al}$	8.3±0.3	29.2±0.2	6.9±0.2	3.27	3.05	0.9512	18.863	1.98±0.3	0.37±0.01	8.62	9
$^{32}\text{S} + ^{40}\text{Ca}$	9.0±0.3	43.2±0.2	8.0±0.2	3.27	3.59	0.9504	20.438	2.14±0.3	0.39±0.01	8.96	9
$^{35}\text{Cl} + ^{27}\text{Al}$	8.4±0.2	30.7±0.4	7.2±0.4	3.40	3.05	0.9499	19.191	1.95±0.2	0.38±0.02	8.73	1
$^{35}\text{Cl} + ^{48}\text{Ti}$	8.7±0.3	49.2±0.6	12.7±0.6	3.40	3.87	0.9455	21.504	1.43±0.3	0.59±0.03	9.35	1
$^{35}\text{Cl} + ^{56}\text{Fe}$	8.5±0.4	50.7±1.1	7.4±1.0	3.40	4.12	0.9466	22.150	2.0±0.4	0.74±0.05	9.40	1

$^{35}\text{Cl} + ^{58}\text{Ni}$	9.0 ± 0.2	61.3 ± 0.3	14.9 ± 0.3	3.40	4.17	0.9499	22.357	1.43 ± 0.2	0.67 ± 0.01	9.53	1
$^{35}\text{Cl} + ^{60}\text{Ni}$	9.2 ± 0.2	61.0 ± 0.3	13.5 ± 0.3	3.40	4.23	0.9470	22.431	1.57 ± 0.2	0.60 ± 0.01	9.60	1
$^{35}\text{Cl} + ^{62}\text{Ni}$	9.6 ± 0.2	60.8 ± 0.3	10.6 ± 0.3	3.40	4.29	0.9429	22.472	1.91 ± 0.2	0.47 ± 0.01	9.67	1
$^{35}\text{Cl} + ^{64}\text{Ni}$	9.7 ± 0.2	60.3 ± 0.3	10.4 ± 0.3	3.40	4.34	0.9377	22.564	1.96 ± 0.2	0.46 ± 0.01	9.73	1
$^{35}\text{Cl} + ^{90}\text{Zr}$	9.8 ± 0.2	84.0 ± 0.5	15.9 ± 0.5	3.40	4.96	0.9386	23.793	1.44 ± 0.3	0.67 ± 0.01	10.20	1
$^{35}\text{Cl} + ^{116}\text{Sn}$	9.8 ± 0.4	102.3 ± 0.9	22.6 ± 0.9	3.40	5.47	0.9302	24.509	0.93 ± 0.4	0.92 ± 0.04	10.62	1
$^{35}\text{Cl} + ^{124}\text{Sn}$	10.1 ± 0.4	104.0 ± 0.8	17.2 ± 0.8	3.40	5.61	0.9098	24.203	1.09 ± 0.4	0.71 ± 0.03	10.77	1
$^{40}\text{Ar} + ^{109}\text{Ag}$	$10.2^{\text{a})}$	$110.6^{\text{a})}$	8.8	3.59	5.34	0.9241	24.929	1.27	0.35	10.72	13

^{a)} These values of R_B and $V_B(R_B)$ are obtained from the two lowest energy data points.¹³ With the reported errors in these two points, the values of $[R_B, V_B(R_B)]$ range from (13.0 fm, 115 MeV) to (8.9 fm, 94 MeV). The equivalent values of $[\zeta_B, -\phi(\zeta_B)]$ are (4.0 fm, -0.86) and (-0.04 fm, 1.71), respectively. Although the first limit is meaningless, the wide range of allowed values of ζ_B and $\phi(\zeta_B)$ for this reaction illustrate the importance of precise data in fixing R_B before a meaningful comparison can be made between the experimental and theoretical nuclear potential.

$$V_N(\zeta) = 4\pi\gamma [C_T C_P / (C_T + C_P)] b \phi(\zeta) \quad (5)$$

where

$$\gamma = 0.9517 \{ 1 - 1.7826 [(N_T + N_P - Z_T - Z_P) / (A_T + A_P)]^2 \}$$

$$C_i = R_i [1 - (b/R_i)^2 + \dots]$$

$$R_i = 1.28 A_i^{1/3} - 0.76 + 0.8 A_i^{-1/3}$$

$$\zeta = r - C_T - C_P$$

$$b = 1 \text{ fm}$$

The quantity $\phi(\zeta)$ is a universal function given by Blocki et al.² Thus an experimental value for $\phi(\zeta_B)$ at the fusion barrier is given by

$$\phi_{\text{exp}}(\zeta_B) = V_N(R_B) \{ 4\pi\gamma [C_T C_P / (C_T + C_P)] b \}^{-1} \quad (6)$$

where

$$\zeta_B = R_B - C_T - C_P$$

The experimental values of $\phi(\zeta_B)$ are compared with the theoretical curve in Fig. 14. The triangles represent values obtained from fusion data, and the full dots indicate values obtained from elastic scattering using the method of Christensen et al.³ In addition, the open triangles in Fig. 14 are based on excitation function measurements of fusion cross sections by counter telescope measurements of evaporation residues and/or fission fragments, while the filled triangles rely on excitation functions based on summing measured partial fusion cross sections. The latter values are subject to the largest errors. Fig. 14 shows the smaller surface separation region tested by the fusion data. There is a qualitative agreement

between the data and the theoretical proximity potential down to surface separations of 1 fm. However, the experimental potentials are systematically deeper than the proximity potential.

The errors shown in Table 8 arise from the uncertainty in the slope of the σ_f vs $1/E$ line which causes an error in R_B , and in the intercept on the $1/E$ axis which causes an error in $V_N(R_B)$ and, hence, in $\Phi(\zeta_B)$. Thus the error in $V_N(R_B)$ produced by an error in R_B is not included in Table 8. Errors in the barrier radius alter both $\zeta(R_B)$ and $V_N(R_B)$ through the Coulomb potential in Eq. (2) and cause the fusion data points in Fig. 14 to move roughly parallel to the theoretical curve as shown by the slanted error bars in Fig. 14. Thus the agreement between the proximity potential and the measured values of V_N from fusion is not altered greatly by errors in R_B . However, there remains considerable uncertainty concerning the actual magnitude of $V_N(R_B)$, because of uncertainty in R_B , even if a friction free analysis is accepted. The effect of including friction in the analysis is to increase the magnitude of the derived nuclear potential, thus moving the fusion data point below the theoretical curve in Fig. 14. However, there is at present no simple method for the estimation of the energy loss due to friction. Seglie et al⁴ have used the proximity nuclear potential together with friction described by a form factor of range to $\zeta = 4.0$ fm in a classical trajectory calculation of the critical angular momentum for fusion. These calculations give reasonable agreement with experimental critical angular momenta for some heavy ion reactions. Randrup⁵ in a proximity

formulation of nuclear friction suggests that the range of the friction should be to $\zeta \approx 3.2$ fm. In both of these cases, the friction is effective before the projectile reaches the barrier, and cannot be ignored in analysis of fusion excitation functions.

The experimentally determined values of R_B may be compared with the values expected from the proximity potential by use of an approximate expression for $\phi(\zeta)$ given by Blocki et al.² For values of $\zeta > 1.2511$ the position of the barrier is given by

$$R_B \exp\left(-\frac{R_B}{1.5}\right) = K \left[\frac{Z_T Z_P (C_T + C_P)}{Y C_T C_P} \right]^{1/2} \exp\left[-\frac{C_T + C_P}{1.5}\right] \quad (7)$$

where K is a universal constant. The experimental barrier radii are compared with the values derived from the proximity potential in Table 8. As can be seen from the data in this table, the proximity potential, without including friction, gives qualitative agreement with the experimental radii.

A method for fusion excitation function analysis has been recently suggested by Bass⁶ in which the nuclear potential is extracted at distances inside the $l = 0$ barrier by measurements of the slope of the σ_f vs $1/E$ curve at energies where this function is no longer a straight line. In the Bass analysis, the slope of a tangent drawn to the σ_f vs $1/E$ curve gives the barrier position and the intercept of the tangent on the $1/E$ axis gives the barrier height. This method is greatly affected in accuracy by uncertainty in the barrier position since the slope of the tangent is not well determined by data currently available in the higher energy range. The Bass analysis leads to estimates of the nuclear potential which lie on curves roughly parallel

to the theoretical proximity potential curve extending over a range of ζ values. However, with currently available data no precise value for the ζ value tested is obtained. Of more fundamental significance, Bass purports to sample ζ values smaller than the s-wave barrier where the effects of friction are even more important than for measurements of the s-wave barrier.

The effect of friction is also important for the analysis of fusion cross sections by use of the method suggested by Glas and Mosel^{7,8} which depends upon the concept of a critical separation necessary for fusion. In the Glas and Mosel analysis the fusion excitation function near the s-wave barrier is given by Eq. (1), while at higher energies the asymptotic expression for the excitation function is given by

$$\sigma_f(E) = \pi R_{CR}^2 \left(1 - \frac{V_{CR}}{E}\right) \quad (8)$$

where R_{CR} is the critical radius and V_{CR} is the conservative potential at the critical radius ($R_{CR} < R_B$). In view of the important role of friction and energy dissipation at higher energies and smaller separation distances, the validity of Eq. (8) is questionable.

In summary, the analysis of fusion data for the nuclear potential at the barrier is complicated in principle by the difficulties involved in a calculation of the frictional energy loss, and in practice by the problem of measurement of excitation functions with sufficient accuracy to closely define the barrier radius. Until these two problems are adequately solved, it is not possible to make quantitative comparisons between the nuclear potential derived from fusion data and theoretical nuclear potentials.

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2. Interaction Times for Damped Heavy-Ion Collisions--

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The important independent variable in theories of heavy ion reactions is the interaction time. However, this variable is not directly observable and its value must be inferred from more directly observable parameters such as the total kinetic energy loss. A knowledge of the angular-momentum dependent interaction times is essential for understanding of the mass, kinetic energy loss and angular distributions of reaction products from heavy-ion collisions. Since the equilibration processes are not completed during the short interaction times encountered in collisions between the very heavy ions under consideration, the amount of kinetic energy loss signifies the stage of evolution of the system, and hence, the total interaction time experienced. This view is supported by classical dynamical calculations^{1,2} which show that the energy loss is a monotonic function of the initial angular momentum and the total interaction time. Associated with each interaction time is a Z distribution which is characterized by a variance σ_Z^2 .

Figure 15 shows the ~~differential cross section in~~ millibarns per MeV of total kinetic energy loss, $d\sigma/d(\text{TKE})_{\text{loss}}$,

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plotted versus the total kinetic energy loss. The continuous range of kinetic energy losses observed in heavy-ion collisions is illustrated in this figure. In order to calculate angular-momentum dependent interaction times, the assumption is made that there is a monotonic increase in the total kinetic energy (TKE) loss* with decreasing values of the impact parameter. For simplicity, we employ a sharp cutoff model where the cross section for angular momenta up to l_j is given by $\sigma_j = \pi \lambda^2 (l_j + 1)^2$. Using experimental results on the heavy ion reaction cross section as a function of TKE loss, $d\sigma/d(\text{TKE loss})$, the angular momentum is related to the TKE loss by

$$l_j = \left\{ (l_j + 1)^2 - \frac{\Delta\sigma_{ij}}{\pi \lambda^2} \right\}^{1/2} - 1 \quad (1)$$

where $\Delta\sigma_{ij} = \sigma_j - \sigma_i$ is the cross section in a TKE window $E_i \leq \text{TKE} \leq E_j$.

The procedure for converting energy loss to l is illustrated in Fig. 16. Starting with l_{max} and zero TKE loss, a deflection function is constructed from the experimental data, for a range of the higher l values. Examples of such deflection

*An alternate assumption would relate the total energy dissipated to the impact parameter. The TKE loss is related to the energy dissipated by, $E_{\text{diss}} = \text{TKE}_{\text{loss}} + [E_C(\text{final}) - E_C(\text{initial})]$. When the difference between Coulomb energies of the initial and final fragments is small, the two quantities are nearly the same.

Figure 15

Differential cross section, $d\sigma/d(\text{TKE})_{\text{loss}}$, for the events of various degrees of kinetic energy damping. Each arrow corresponds to the kinetic energy loss for an event with final kinetic energy equal to that of the Coulomb energy at the separation distance of the strong absorption radius. For the Xe-induced reactions, the integral of the plotted differential cross section gives nearly the total reaction cross section. However, in the case of the $^{165}\text{Ho} + ^{84}\text{Kr}$ reaction some 700 mb of cross section is in the range $\theta_{\text{c.m.}} < 20^\circ$ and missing in the figure.

007-3611

Figure 15

Figure 16

Schematic illustration of the procedure for converting kinetic energy loss to angular momentum l .

007-3813

Figure 16

functions are shown as solid lines in Figs. 17 and 18 for the $^{165}\text{Ho} + ^{84}\text{Kr}$ ($E_{\text{Lab}} = 714 \text{ MeV}$) and $^{209}\text{Bi} + ^{136}\text{Xe}$ ($E_{\text{Lab}} = 1130 \text{ MeV}$) reactions, respectively. The plotted angles represent the angles where the cross section is at a maximum for a particular kinetic energy loss. For the first reaction the emission angle of the lighter fragment decreases as l decreases, whereas for the second reaction the emission angle is almost independent of l . For each reaction the energy damping and the variance in the l distribution both increase as l decreases.

The angular-momentum-dependent interaction time is calculated from the deflection functions with the expression^{3,4}

$$\tau(l) = \Delta\theta(l) \mathcal{I}(l) / \hbar l \quad (2)$$

where $\Delta\theta(l)$ is the difference between the Coulomb deflection angle $\theta_C(l)$ and the actual reaction angle $\theta_{\text{exp}}(l)$ and $\mathcal{I}(l)$ is the moment-of-inertia of the double-nucleus system. The Coulomb deflection angle is estimated by

$$\theta_C(l) = 180^\circ - \theta_1 - \theta_3 \quad (3)$$

where the subscripts refer to the entrance and exit channels, respectively, and

$$\theta_j = + \arccos \frac{1}{\epsilon_j} - \arccos \frac{K_j + R_j}{\epsilon_j R_j} \quad (\text{for } j = 1 \text{ and } 3) \quad (4)$$

In Eq. (4) the parameters ϵ and K are determined by

$$\epsilon = \left[1 + \frac{2El^2\hbar^2}{\mu(Z_p Z_T e^2)^2} \right]^{1/2} \quad (5)$$

$$K = \frac{l^2 \hbar^2}{\mu Z_p Z_T e^2} \quad (6)$$

Figure 17

Deflection functions for the reaction $^{165}\text{Ho} + ^{84}\text{Kr}$. The dashed curve is the Coulomb deflection function, calculated for the NS condition. The curve $\theta_{\text{exp}}(\ell)$ is the deflection function generated from experimental data (solid line) and extrapolated into an angular region where no data exists (long-short dashed line). $\Delta\theta(\ell)$ is the angle through which the intermediate double-nucleus system rotates during the interaction.

Figure 17

Figure 18

Same as Figure 17 but for the reaction $^{209}\text{Bi} + ^{136}\text{Xe}$.

Figure 18

The above procedure for estimation of $\Delta\theta(\ell)$ is illustrated in Fig. 19. The evaluation of $\tau(\ell)$ requires the adoption of a collision model. Here we present the results labeled as "nonsticking" (NS) and "sticking" (S) collisions. A sticking collision is defined by rigid rotation of the double-nucleus system as a whole. By a NS collision we specify that the entrance and exit channel orbital angular momenta are the same ($\ell_f = \ell_i$) and the moment-of-inertia $\mathcal{I}_{NS} = \mu R^2$ where $\mu = M_1 M_2 / (M_1 + M_2)$ and R is the contact radius of the double-nucleus system (in the calculation the strong absorption radius⁵ is used). In contrast to a nonsticking collision, the interaction time for a sticking collision is calculated with $\mathcal{I}_{NS}$ and the final orbital angular momentum where $\ell_f = (\mathcal{I}_{NS} / \mathcal{I}_S) \ell_i$ and $\mathcal{I}_S = \mathcal{I}_{NS} + (2/5)(M_1 R_1^2 + M_2 R_2^2)$. In the calculations presented, any variation in $\mathcal{I}_{NS}$ with angular momentum or time is neglected. In Figs. 17 and 18, Coulomb deflection functions are shown for the "nonsticking" (NS) model.

The above definition of the deflection during the reaction implies that no kinetic-energy loss due to dissipation or dynamical deformation effects occurs at separation distances larger than the strong-absorption radius R_{SA} . The agreement of realistic deflection function calculations with experimental angular distributions for the heavy systems considered here suggests that the deviation of the trajectory with $\ell = \ell_{\max}$ from a pure Coulomb trajectory is small. Hence, for $\ell < \ell_{\max}$ the difference between Coulomb and observed deflection is used

Figure 19

Schematic illustration of the procedure used to determine $\Delta\theta(l)$.

Figure 19

$$\Delta\theta(l) = \theta_c(l) - \theta_{exp}(l)$$

$$\theta_c = \pi - (\theta_c^i - \phi_c^i) - (\theta_c^f - \phi_c^f)$$

$$\tau(l) = \Delta\theta(l) \mathcal{I}(l) / \hbar l$$

$$\mathcal{I}_{NS} = \mu R_{SA}^2$$

$$\mathcal{I}_S = \mathcal{I}_{NS} + 2/5 (M_1 R_1^2 + M_2 R_2^2)$$

to evaluate the angle through which the intermediate system rotates during the nuclear interaction. (See Eq. 3 and Fig. 19).

It should, however, be realized that it is, in principle, impossible to characterize the reaction between very heavy ions by a single deflection function. For such systems there are many intrinsic degrees-of-freedom coupled to the collective motion, and many different reaction paths may lead to similar values of a given experimental observable. Therefore, selecting a certain value of one experimental variable leads to a distribution of values of another variable fluctuating around its mean value. In this sense, the deflection functions derived above represent only average experimental deflection functions.

The angular-momentum-dependent interaction times calculated for the above procedure (see Eq. (2)) are displayed in Fig. 20 for the nonsticking model. Multiplication of the angular momentum ℓ by the factor $[10^6 (A_T + A_P)^{1/3} / (Z_T + Z_P)^2] (1/\ell_{\max})$ produces straight lines on a semilog plot of approximately the same slope for the three reactions. The angular momentum range for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction shown in Fig. 20 is $120 \leq \ell \leq 430$.

For both the sticking and nonsticking models the interaction time is given empirically by the relation

$$\tau(\ell) = \tau_0 \exp(-\alpha\ell) \quad (7)$$

The constant α for the reactions displayed in Fig. 20 ($E_{\text{lab}} = 8.5 \text{ MeV/amu}$) is given by

Figure 20

Interaction times as a function of angular momentum l for the nonsticking model for three reactions.

Figure 20

$$\alpha = K \frac{(A_T + A_P)^{1/3}}{(Z_T + Z_P)^2} \frac{1}{l_{\max}} \quad (8)$$

Values of K and τ_0 for the sticking and nonsticking models are listed in Table 9. The value of $l_{\max}$ is determined from elastic scattering using the one-quarter point technique. The value of K is unique for each model, and independent of the reaction examined. As can be seen from Table 9, the value of K is slightly smaller for the sticking model as compared to the nonsticking model. This reflects the slower increase in reaction time as l decreases for the sticking model. Values of τ_0 are somewhat larger for the sticking model. However, the observed difference in τ_0 between the two models is probably not greater than the error in the determination of the quantity.

Table 9

Values of the coefficients τ_0 , α and K to be used in Eqs. (8) and (9) for calculation of the angular-momentum-dependent interaction times. The laboratory bombarding energies are approximately 8.5 MeV/amu.

Reaction	Model	τ_0 (sec)	α	K
$^{165}\text{Ho} + ^{84}\text{Kr}$	S	1.65×10^{-19}	0.0224	12610
	NS	1.62×10^{-19}	0.0244	15050
$^{209}\text{Bi} + ^{84}\text{Kr}$	S	5.16×10^{-20}	0.0171	12610
	NS	6.62×10^{-20}	0.0205	15050
$^{209}\text{Bi} + ^{136}\text{Xe}$	S	1.86×10^{-20}	0.00974	12610
	NS	2.18×10^{-20}	0.0118	15050

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3. Diffusion Coefficients in Heavy Ion Reactions--J. R. Birkelund, W. U. Schröder, J. R. Huizenga, K. L. Wolf* and V. E. Viola, Jr.**

The (experimental charge distributions) of the type shown in Fig. 21 are suggestive of a diffusion process and have been analyzed with a Fokker-Planck equation in terms of the variance σ_z^2 . In its simplest form with one observable x and constant drift v and diffusion D coefficients, the Fokker-Planck equation has Gaussian solutions.¹

$$P(x,t) = (4\pi Dt)^{-1/2} \exp(-(x-vt)^2/4 Dt) \quad (1)$$

The centroid $x_0 = vt$ and the variance $\sigma^2 = 2Dt$ are linear functions of the interaction time. The drift coefficient v_A is related to the diffusion coefficient through the Einstein relation

$$v_A = - \frac{1}{T} D_A \frac{\partial}{\partial A_1} U_\ell(A_1) \quad (2)$$

where U_ℓ denotes the ground-state energy of the combined system with relative angular momentum ℓ and fragmentation A_1 . The local temperature T is determined by the excitation energy.

A relationship between experimental values of the total kinetic energy loss and the variance σ_z^2 of the fragment charge distributions for very heavy damped collisions is shown in Fig. 22. It is important to emphasize that the data in this figure are all taken at a bombarding energy of 8.5 MeV/amu.

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Figure 21

Plot of the differential cross section $d^2\sigma/dE dZ$ as a function of atomic number for various final kinetic energy bins of 50-MeV width measured for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction. The solid lines are Gaussian fits to the data.

Figure 21

Figure 22

Total kinetic energy loss as a function of the variance of the charge distribution σ_z^2 of the projectile-like fragments for four heavy-ion reactions. The data displayed in this figure were all taken at a laboratory bombarding energy of 8.5 MeV/amu.

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Figure 22

This relationship displayed in Fig. 22 in conjunction with the experimental cross section as a function of the total kinetic energy loss is used to calculate angular-momentum-dependent values of the variance $\sigma_Z^2(l)$. In the above theory of nucleon diffusion, the value of $\sigma_Z^2(l)$ is related to the interaction time $\tau(l)$ by

$$\sigma_Z^2(l) = 2D_Z(l)\tau(l) \quad (3)$$

The interpretation of the experimental fragment Z distributions in terms of Eq. (3) is subject to similar observations as made for the construction of an experimental deflection function (see Section II B2). The experimental variance σ_Z^2 of the Z distribution is an average value determined by the range of l waves contributing to a given TKE window. A factorization of σ_Z^2 according to Eq. (3) into mean values of D_Z and τ applies only if $D_Z(l)$ is a slowly varying function of both Z and l because the above analysis suggests that the total interaction time $\tau(l)$ is a rapidly varying function decreasing exponentially with increasing l . This requirement on $D_Z(l)$ seems, indeed, to be fulfilled as indicated by model calculations and an experimentally observed small drift coefficient v_Z .

The value $\tau(l)$ entering Eq. (3) is the mean value of the time during which the nucleon diffusion mechanism operates. In this analysis it is assumed that $\tau(l)$ is the total interaction time evaluated by the procedure outlined above. However, it is conceivable that nucleon diffusion occurs only during a part of this time, although there is presently no experimental evidence for such a division of the total interaction time.

Values of $\tau(l)$ and $\sigma_Z^2(l)$ determined for the $^{165}\text{Ho} + ^{84}\text{Kr}$ ($E_{\text{Lab}} = 714 \text{ MeV}$) and the $^{209}\text{Bi} + ^{136}\text{Xe}$ ($E_{\text{Lab}} = 1130 \text{ MeV}$) reactions are plotted in Figs. 23 and 24, respectively. The two different sets of values of $\tau(l)$ for each reaction in this figure are based on the above NS and S models. The values of the diffusion coefficients for three heavy ion reactions determined by the slope of the line fitted to the $\sigma_Z^2(l)$ vs $\tau(l)$ data are given in Table 10 for each of the above two models. The proton number diffusion coefficient does not refer to proton diffusion alone, but to mass diffusion measured by the number of transferred protons. Assuming a conservation of the equilibrium Z/A ratio, the proton number (D_Z) and mass number (D_A) diffusion coefficients are related by $D_Z = (Z/A)^2 D_A$.

Individual values of D_Z for the nonsticking (NS) and sticking (S) models are plotted as a function of the initial angular momentum l in Fig. 25. The values for the sticking model are smaller than the result derived from the slope technique since the line fitted to the data does not pass through the zero point.

The present diffusion coefficients, based on an angular momentum dependent interaction time, are significantly smaller than those reported by Nörenberg.² However, the difference is understood insofar that deflection functions were not employed in ref. 2 and their resulting angle $\Delta\theta$ is too small. The value of D_Z for an average l value of 160 $\hbar$ for the $^{197}\text{Au} + ^{86}\text{Kr}$ reaction deduced by Moretto and Schmitt³ at a laboratory energy of 620 MeV is comparable to our values for ^{84}Kr projectiles. The latter authors assume a sticking model and a

Figure 23

The variance of σ_z^2 of the experimental fragment-charge distribution plotted as a function of the interaction time τ for the reaction $^{165}\text{Ho} + ^{84}\text{Kr}$. The two sets of results correspond to the assumption of nonsticking (NS) or sticking (S) of the two ions during the interaction.

Figure 23

Figure 24

Same as Figure 23 but for the reaction $^{209}\text{Bi} + ^{136}\text{Xe}$.

Figure 24

Table 10

Proton number (D_Z) and mass number (D_A) diffusion coefficients in units of $10^{22} \text{ sec.}^{-1}$ for Kr- and Xe-induced reactions. The proton number diffusion coefficient does not refer to proton diffusion alone, but to mass diffusion measured by the number of transferred protons; hence, $D_Z = (Z/A)^2 D_A$ for a constant Z/A ratio. The diffusion coefficients listed in this table are calculated from the slopes of lines drawn through plots of $\sigma_Z^2(\ell)$ vs $\tau(\ell)$ over a range of ℓ values. In the case of the sticking model, for example, the points for the highest ℓ waves do not lie on a line which passes through the origin (see Figs. (23) and (24)). Individual values of $D_Z(\ell)$ for the sticking model are ℓ dependent and increase initially as ℓ decreases. The Kr- and Xe-projectile energies (lab) are 714 and 1130 MeV, respectively. The errors in the diffusion coefficients are of the order of 30%. However, the values scale with the contact radius which for the reported values is assumed to be the strong absorption radius, R_{SA} .

Reaction	Sticking Model		Nonsticking Model	
	D_Z	D_A	D_Z	D_A
$^{209}\text{Bi} + ^{136}\text{Xe}$	0.75	4.8	1.1	7.0
$^{209}\text{Bi} + ^{84}\text{Kr}$	0.62	3.7	0.87	5.3
$^{165}\text{Ho} + ^{84}\text{Kr}$	0.55	3.2	0.74	4.3

Figure 25

Plot of $D_z(l)$ as a function of the initial angular momentum for the $^{165}\text{Ho} + ^{84}\text{Kr}$ and $^{209}\text{Bi} + ^{136}\text{Xe}$ reactions. The bombarding energy in each case is 8.5 MeV/amu. The diffusion coefficients for both the nonsticking (NS) and sticking (S) models are included.

007-3856

Figure 25

Gaussian distribution of interaction times, the centroid of which varies linearly with ℓ .

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4. Mechanisms of Kinetic Energy Dissipation--J. R. Huizenga, J. R. Birkelund, W. U. Schröder, K. L. Wolf* and V. E. Viola, Jr.**

In previous sections evidence has been presented to show that kinetic energy loss and mass exchange are strongly correlated. This result is consistent with the view that nucleon diffusion evolves in a continuous fashion as energy is dissipated and supports a close correspondence of the time scales governing energy dissipation and nucleon exchange. In this section we employ the microscopic time scale provided by the nucleon exchange process to study the energy dissipation mechanism.

At the heavy-ion bombarding energies discussed here and moderate excitation energies, one expects that nucleon exchange and particle-hole excitation without nucleon exchange are mediated by the one-body interaction given by the single particle field. Such a mechanism gives rise to classical friction forces proportional to the relative velocity v of the two ions¹⁻³

$$\vec{F} = -k\vec{v} \quad (1)$$

In general, k is an anisotropic tensor dependent on the separation distance of the two ions, which is equivalent to an implicit time dependence of k . The importance of one-body

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friction in nuclear fission and heavy-ion reactions has been discussed also by other authors.⁴⁻⁸ Two-body friction induced by inelastic scattering of individual nucleons of target and projectile, which may also play a role in energy dissipation, has in general a velocity dependence more complicated⁹ than that implied by Eq. (1).

The energy loss rate associated with Eq. (1) is

$$-dE/dt = 2(k/\mu) E \quad (2)$$

where μ is the reduced mass. Integration of Eq. (2) with a constant coefficient k gives

$$\ln(E_0/E) = 2(k/\mu)t \quad (3)$$

where $E_0 = E_{cm} - E_B$, $E = E_0 - E_{loss}$ and E_{cm} and E_B are the incident c.m. kinetic and Coulomb energies, respectively.

Employing the microscopic time scale provided by the nucleon exchange process, namely

$$d\sigma_z^2/dt = 2D_z(t) \quad (4)$$

and substituting the relation into Eq. (2) and integrating leads to a relation

$$\ln(E_0/E) = (k/\mu D_z) \sigma_z^2 \quad (5)$$

provided that the ratio (k/D_z) is independent of time. Hence, a linear relation is predicted between $\ln(E_0/E)$ and σ_z^2 . Such a plot is shown in Fig. 26 where E_B is calculated at the strong absorption radius. Good agreement between experiment and this simple theory is obtained for energy losses up to 200 MeV. If k/D_z is the same for Kr- and Xe-induced reactions at a bombarding energy of 8.5 MeV/amu, the slopes of Fig. 26 lead to

Figure 26

Plot of $\ln E_0/E$ as a function of the variance σ_z^2 , of the charge distribution of the projectile-like fragment. The solid line is drawn through the Xe data. The dashed line is calculated relative to the solid line for the Kr-induced reactions by Eq. (5) taking into account the difference in reduced mass. See text for definition of E and E_0 .

007-3610

Figure 26

$$k/D_z = (0.9 \pm 0.3) \times 10^{-43} \text{ MeV sec}^2 \text{ fm}^{-2} \quad (6)$$

The result of a constant friction coefficient rather than a strongly varying form factor as suggested by classical dynamical calculations^{2,3,10} can be understood as being due to the fact that in the analysis reference was made to a nucleon diffusion mechanism in order to derive a microscopic time scale.¹

Although it is presently not quite clear to what extent the one-body dissipation mechanism applies, the experimental results are consistent with the view that energy dissipation mediated by nucleon exchange and the other possible dissipation mechanisms proceed on a similar microscopic time-scale and have a similar dependence on the separation distance of the two ions and the angular momentum. In principle, a fast dissipation mechanism¹² operating only during a small initial part of the total interaction time cannot be excluded. However, it has to lead to a friction-like relation between the energy loss and the total interaction time which is similar to Eq. (2) in order to fit the data.

An interesting new experimental observation^{13,14} is that k/D_z increases as the bombarding energy decreases. For example, as the bombarding energy is reduced from 1130 to 900 MeV for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction, the value of k/D_z increases¹³ by a factor of about 2 (see Table 12).

Kinetic energy is known to be lost in nucleon exchange between target and projectile¹⁵. In a simple version of a one-body nucleon exchange process,⁴ where a nucleon of mass m is assumed to be loosely bound and at rest with respect to the

donor nucleus, its relative momentum $\Delta p = mv$ is dissipated in the transfer. The resulting loss of total kinetic energy per nucleon exchange δE_{ex} is then proportional to the kinetic energy available prior to exchange¹⁶

$$\delta E_{ex} = (m/\mu)(E_{cm} - E_B - E_{loss}) = (m/\mu)E \quad (7)$$

where $E_{ex} = (-dE_{ex}/dt)/(dN/dt)$ and dN is the number of exchanged nucleons per unit time dt . Consequently, the energy loss rate for nucleon exchange alone is

$$-dE_{ex}/dt = (m/\mu)E(dN/dt) = k_{ex} v^2 \quad (8)$$

This energy loss rate is equivalent to that due a friction force $F_{ex} = -k_{ex} v$ (see Eq. (1)) with a frictional coefficient

$$k_{ex} = (m/2) (dN/dt) \quad (9)$$

where $(dN/dt) = 2(A/Z)D_Z$. The total number N of nucleon exchanges is obtained from the number $N_Z = \sigma_Z^2$ of protons exchanged by scaling N_Z according to the mass-to-charge ratio, $N = (A/Z)\sigma_Z^2$. This is justified by the experimental observation of a fast equilibration¹⁷ of the mass-to-charge asymmetry degree of freedom in damped heavy-ion reactions which indicates that neutron and proton exchange rates are similar. Substitution of dN/dt into Eq. (9) gives

$$(k_{ex}/D_Z) = m (A/Z) \quad (10)$$

where m is the nucleon mass. Converting the units of $\text{erg sec}^2 \text{cm}^{-2}$ to $\text{MeV sec}^2 \text{fm}^{-2}$

$$(k_{ex}/D_Z) = 0.1044 \times 10^{-43} (A/Z) \text{ MeV sec}^2 \text{fm}^{-2} \quad (11)$$

For the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction at a bombarding energy of 1130 MeV, it follows that k_{ex}/D_z is approximately 30% of k/D_z (see Eq. (6)). Hence, energy dissipation caused directly by the nucleon exchange process represents only a fraction of the total energy dissipation. Furthermore, this fraction decreases as the bombarding energy is reduced.

If one denotes the friction force coefficient due to one-body dissipation processes such as particle-hole excitation without nucleon exchange by k_{nex} , the resulting total energy dissipation rate (in analogy to Eq. (2)) is

$$-dE/dt = (2/\mu)(k_{\text{nex}} + k_{\text{ex}})E \quad (12)$$

It may be expected that the rates of one-body processes without and with nucleon exchange represented by the friction coefficients k_{nex} and k_{ex} , respectively, are somewhat different in magnitude. However, since both processes are induced by the same time-dependent single-particle potential, the two friction coefficients are expected to have the same time dependence. This is equivalent to both friction forces having the same spatial form factor. Under this condition, the energy loss rates due to particle-hole excitation and nucleon exchange are proportional to each other. The microscopic time scale corresponding to the nucleon exchange mechanism may then be used to calculate the total energy lost during the time necessary for one nucleon exchange. Consequently, this total energy loss associated with a single exchanged nucleon, δE , is given by

$$\delta E = - d(E_{nex} + E_{ex})/dN = (m/\mu)[1 + (k_{nex}/k_{ex})]E \quad (13)$$

Similar arguments apply to the case of two-body friction and exchange which, in general, give rise to a more complicated energy dependence.⁹

Experimental results for the energy loss per nucleon exchanged for the reactions¹¹ $^{165}\text{Ho} + ^{84}\text{Kr}$, $^{165}\text{Ho} + ^{136}\text{Xe}$ and $^{209}\text{Bi} + ^{136}\text{Xe}$ at a bombarding energy of 8.5 MeV/amu and for the reaction¹⁸ $^{197}\text{Au} + ^{86}\text{Kr}$ at 7.2 MeV/amu are presented in Fig. 27. The values of δE were obtained by differentiating curves of E_{loss} vs N fitted to the experimental data points. The estimated errors are, therefore, correlated. The $^{197}\text{Au} + ^{86}\text{Kr}$ data were transformed into the center-of-mass system assuming ^{86}Kr -kinematics for all fragments.

The most important observation to be made from the data is that the energy δE dissipated during the time necessary for the exchange of a single nucleon decreases with decreasing available kinetic energy E (i.e. increasing energy loss and total excitation energy of the system). The data points follows a straight line intercepting the abscissa at $(m/\mu) E = 0$ as predicted by Eq. (13) over an unexpectedly wide range of available energies. For small available energies the data points deviate from these lines. This indicates that large energy losses may lead to a highly deformed intermediate system. Hence, the Coulomb energy may be much lower than the one in the entrance channel, which is used to calculate the available energy. On the other hand, the one-body mechanism may also

Figure 27

Total kinetic energy δE dissipated per nucleon exchanged as a function of the available kinetic energy per nucleon. The solid lines are drawn through the data points according to Eq. (13). The dashed line represents the amount of energy dissipated by the exchange process itself ($k_{nex} = 0$ in Eq. (13) or Eq. (7)).

Figure 27

lose importance at high excitation energies, where the Pauli principle is less inhibitive for two-body nucleon collisions.

As is indicated by the dashed line in Fig. 27 corresponding to Eq. (7) (or to Eq. (13) with $k_{nex} = 0$), energy dissipation by nucleon exchange alone can account for ~30% of the total energy loss for the 8.5 MeV/amu data and for ~15% for those at 7.2 MeV/amu. The good description of the data by the linear relation predicted by the one-body mechanism suggests that the ratio k_{nex}/k_{ex} is, indeed, constant over a wide range of excitation energies and l values. Although the $^{197}\text{Au} + ^{86}\text{Kr}$ data⁸ exhibit a linear dependence on the available energy, they do not lie on the line determined by the 8.5 MeV/amu reaction data. Studies^{13,14} of similar reactions of various bombarding energies indicate that nucleon exchange accounts for a smaller fraction of the total energy loss as the bombarding energy is reduced.

The diffusion model proposed by Nörenberg and collaborators¹⁹⁻²¹ treats nucleon exchange and energy loss on the same basis accounting for the dependence of the transport coefficients on excitation energy or energy loss. Noticing that the rates of energy loss and nucleon exchange are determined by the energy drift coefficient v_E and the mass diffusion coefficient D_A according to $dE/dt = -v_E$ and $dN/dt = (2Z/A)D_A$, one obtains²

$$\delta E = \frac{A}{Z} \frac{v_E}{2D_A} = \frac{A}{Z} \frac{A^2}{8} \frac{(g_1 + g_2)^{1/2}}{(g_1 g_2)^{1/3}} (g_1^{1/3} + g_2^{1/3})^2 E_{\text{loss}}^{-1/2} \quad (14)$$

for the total energy dissipated during the exchange of one nucleon. Here, $\Delta \approx 2.5$ MeV is a mean value of the energy dissipated in one nucleon exchange or particle-hole excitation process, and $g_k = A_k/12$ are the single-particle level density parameters of the two ions.

Theoretical curves represented by Eq. (14) are compared in Fig. 28 with the experimental energy loss per nucleon as a function of energy loss. The level density parameters were calculated for the projectile-target combination of fragments. As can be seen in Fig. 28, the diffusion model can account for the high energy loss observed per nucleon exchange for low values of kinetic energy loss corresponding to small interaction times.²² However, the predicted functional dependence (Eq. (14)) is not supported by the data. It is interesting to notice, that on this plot also the $^{197}\text{Au} + ^{86}\text{Kr}$ data follow the experimental systematics for small values of E_{loss} . If the diffusion model is modified to account for the reaction dynamics employing velocity-proportional friction forces, the result is quite similar to one-body dissipation.

In conclusion, the observed correlation between nucleon exchange and energy loss suggests that the microscopic time scale provided by the exchange mechanism can be used to study the energy dissipation mechanism in heavy-ion reactions. The experimental energy loss associated with a single nucleon exchange exhibits a linear dependence on the available kinetic energy, as predicted by a one-body dissipation mechanism. The diffusion model can account for high amounts of energy

Figure 28

Comparison of the experimental energy dissipated per nucleon exchange with the predictions of the diffusion model (solid curves) according to Eq. (14).

Figure 28

dissipated per nucleon, δE , for low energy losses, however, does not predict correctly the decrease of δE with increasing energy loss. Neither model accounts directly for the increase of $k_{\text{nex}}/k_{\text{ex}}$ (or k/D_2) with decreasing bombarding energy.

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5. The Dependence of Nuclear Orbiting and Charge Exchange on Bombarding Energy in the $^{144}\text{Sm} + ^{84}\text{Kr}$ Reaction--

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The dependence of nuclear orbiting and charge exchange on bombarding energy is studied using beams of ^{84}Kr ions at laboratory energies of 470, 595 and 720 MeV on a ^{144}Sm target. These energies correspond to 1.2, 1.5 and 1.8 times the Coulomb barrier, respectively. [The angular distributions of the projectile-like products show large cross sections extending to small angles. This is consistent with orbiting or "negative angle scattering" in the 720 MeV case. But the nearly constant, large cross sections observed at low angles for the 595 MeV and 470 MeV cases are difficult to explain using the simple orbiting picture of Wilczynski.

The variances of the charge distributions σ_z^2 increase smoothly as a function of the energy dissipated in the reaction (the total kinetic energy loss). For exit channel energies above the Coulomb barrier, σ_z^2 scales only with dissipated energy, and is independent of bombarding energy (and of relative ion velocity in the entrance channel). Using a statistical model, σ_z^2 is a measure of the number of protons

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exchanged in the interaction. Assuming the number of nucleons exchanged, N_A , is given by $(A/Z)\sigma_Z^2$, values of approximately 9.3 to 10.7 MeV are derived for (the energy loss associated with each nucleon exchanged for total dissipated energies from 10 to 70 MeV.)

6. Possible Static Deformation Effects in ^{84}Kr - and ^{86}Kr -Induced Reactions on Targets of ^{144}Sm and ^{154}Sm --D. G. Raich*, A. C. Mignerey*, K. L. Wolf*, R. L. Boudrie*, C. T. Roche*, W. U. Schröder, J. R. Birkelund, J. R. Huizenga and V. E. Viola, Jr.**

As previously reported¹ there is an appreciable difference between angular distributions of products from Kr-ion induced reactions on ^{144}Sm and ^{154}Sm . We have studied this apparent static deformation effect at laboratory bombarding energies of 470, 600, and 720 Mev, corresponding to 1.2, 1.5, and 1.8 times the interaction barrier. The difference is most pronounced at the lowest projectile energy, where the differential cross section $d\sigma/d\theta_{\text{cm}}$ of krypton-like products at angles forward of the grazing angle is nearly twice as large from reactions on highly deformed ^{154}Sm as from those on spherical ^{144}Sm . The enhanced cross section is associated primarily with reactions from which the products are emitted at the relatively low (fully relaxed) total kinetic energies typical of strongly damped collisions. We can rule out the possibility that this effect is due to a fusion-fission mechanism because the yield at very backward angles is, with both targets, very small ($\sigma_{\text{cf}} < 300 \text{ mb}$). Despite the difference in angular distributions, the widths of the product charge distributions are the same at a given center-of-mass deflection angle θ_{cm} .

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7. The Systematics of Interaction and Strong Absorption
Radii Determined from Heavy-Ion Elastic Scattering --

J.R. Birkelund and J.R. Huizenga

A considerable number of heavy ion elastic scattering angular distributions have been reported for a range of projectiles from ${}^7\text{Li}$ to ${}^{136}\text{Xe}$ on a variety of targets. These data are generally fitted by use of an optical model or a Fresnel scattering model. Such fits to the data lead to a wide range of nuclear optical potential parameters which may be understood by the introduction of the concept of a strong absorption radius. It is seen empirically that for a given target-projectile pair, the optical potentials which lead to a fit to elastic scattering angular distributions have nearly the same values at a particular radius. This radius is at or very close to the apsidal distance for the classical Coulomb trajectory of the partial wave for which the optical model transmission coefficient T_ℓ is 0.5. This radius is defined as the strong absorption radius¹⁾ and is given by

$$R_{SA} = (1/k) \{ \eta + [\eta^2 + \ell_{1/2}(\ell_{1/2} + 1)]^{1/2} \} \quad (1)$$

where k is the wave number in the entrance channel, η is the Coulomb parameter, and $\ell_{1/2}$ is the angular momentum of the partial wave which determines the strong absorption radius.

We have

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} \quad (2)$$

where v is the asymptotic velocity in the entrance channel.

The Fresnel model²⁾ leads to a measurement of an interaction radius defined from the quarter-point angle $\theta_{1/4}$ at which the

ratio of elastic to Rutherford cross section $\frac{\sigma_{EL}}{\sigma_{RUTH}}$ is 0.25.

This interaction radius is given by

$$R_{INT} = (\eta/k) [\operatorname{cosec}(\theta_{1/4}/2) + 1] \quad (3)$$

It is found empirically that the Fresnel R_{INT} is very close to the optical model R_{SA} for heavy ion reactions.³⁾

Recently Christensen and Winther⁴⁾ have suggested a method for determination of the radius at which the nuclear potential is tested by elastic scattering. The Christensen-Winther method determines a radius slightly inside the apsidal distance for a classical trajectory which leads to scattering at the rainbow angle in the combined Coulomb potential and nuclear potential determined from an optical model analysis. The radius found by the Christensen-Winther method is close to R_{SA} or R_{INT} for light heavy ion reactions.

Investigations of the systematics of strong absorption radii have been described in the literature^{3,5)}. Perhaps the most accurate method for parameterization of the strong absorption radii has been based on the separation distance τ of the nuclear matter half-density radii³⁾. The half-density radii of the target C_T and projectile C_P are determined from the formulae of Süssman⁶⁾, which are:

$$R_i = 1.28A^{1/3} - 0.76 + 0.8/A^{1/3} \quad (4)$$

$$C_i = R_i [1 - (b^2/R_i^2) + \dots]$$

where a value of $b = 1 \text{ fm}$ is generally used.

The first systematic treatment of R_{SA} based on the matter radii gave

$$R_{SA} \approx C_T + C_P + 3.2 \text{ fm} \quad (5)$$

However, the constant of 3.2 fm given in Eq. (5) varies slightly with $C_T + C_P$, with smaller values as $C_T + C_P$ increases. The elastic scattering data tested in Table 11 have been plotted in Fig. 29 and fitted to the relation:

$$\zeta = 4.49 - (C_T + C_P)/6.35 \quad (6)$$

where the value of ζ is defined by

$$\zeta = R_{SA} - C_T - C_P \quad (7)$$

For the data of Table 11, where the quarter point is known from the literature, R_{INT} has been used in Eq. (7). In cases where the quarter point is not published, either R_{SA} is used if known or the method of Christensen and Winther is used to determine the radii for cases where only optical model potential parameters are available. Eq. (6) represents a least-squares fit to the data plotted in Fig. 29. The root-mean-square deviation in R_{SA} is defined by

$$\overline{\Delta R}_{SA} = \left[(1/n) \sum_i (R_{SA}^{Th} - R_{SA}^{exp.})^2 \right]^{1/2} \quad (8)$$

The formula of Eq. (6) leads to a value of $\overline{\Delta R}_{SA}$ of 0.33 fm. If the data of Fig. 29 are fitted by

$$R_{SA} = (1.587 - 0.0000858 Z_T Z_P) (A_T^{1/3} + A_P^{1/3}) \quad (9)$$

the value of $\overline{\Delta R}_{SA}$ is 0.94 fm, and if the expression for R_{SA} is taken to be

$$R_{SA} = (2.04 - 0.0861 \ln Z_T Z_P) (A_T^{1/3} + A_P^{1/3}) \quad (10)$$

then the value of $\overline{\Delta R}_{SA}$ is 1.91. The values of $\overline{\Delta R}_{SA}$ suggest that

Table 11. Elastic scattering data used in the systematics of strong absorption radii.

Reaction	Lab. Energy (MeV)	R_{SA} (fm)	Refs.
$^{10}\text{B} + ^{12}\text{C}$	28	8.64	7
$^{11}\text{B} + ^{11}\text{B}$	11	8.72	8
^{12}C	28	8.70	7
^{208}Pb	72.2	12.41	9
^{209}Bi	74.6	12.26	10
$^{12}\text{C} + ^{12}\text{C}$	14.0	7.97	11
^{13}C	21.0	8.45	12
^{16}O	15.0	8.53	11
^{24}Mg	24	8.28	49
^{58}Ni	124.5	9.29	53
^{64}Ni	48	10.29	19
^{96}Zr	38	10.83	23
^{107}Ag	124.5	10.53	53
^{115}In	124.5	10.62	53
^{142}Nd	70.4	11.32	56
^{144}Nd	70.4	11.41	56
^{146}Nd	70.4	11.42	56
^{148}Nd	70.4	11.73	56
^{150}Nd	70.4	11.76	56
^{181}Ta	124.5	11.68	53
^{194}Pt	78	11.92	57
^{206}Pb	95	12.26	58

Reaction	Lab. Energy (MeV)	R_{SA} (fm)	Refs.
^{208}Pb	96,116	12.35-12.22	29
$^{13}\text{C} + ^{40}\text{Ca}$	40,68	9.73-9.91	15
^{94}Mo	51	10.83	24
^{96}Mo	54.5	10.91	24
^{144}Sm	66	11.83	28
^{207}Pb	86.1	12.35	10
$^{14}\text{N} + ^{14}\text{N}$	26	8.64	13
^{28}Si	53,63	8.64-8.65	46
^{40}Ca	30,36	9.43-9.32	16
^{48}Ca	50,60	10.11-10.17	14,17
^{56}Fe	36,40	9.49-9.61	16
^{62}Ni	36,42	9.52-10.05	16
^{70}Ge	42,45	10.21-10.39	16
^{74}Ge	38,45	9.71-10.26	16
^{90}Zr	45,50	10.15-10.47	16
^{118}Sn	54	10.29	16
$^{15}\text{N} + ^{28}\text{Si}$	55,60	8.77-8.61	46
^{89}Y	49.5	10.72	25
$^{16}\text{O} + ^{16}\text{O}$	27	8.85	11
^{24}Mg	29,30,33	8.82-8.78	46,49
^{26}Mg	45,60,67.9	8.56-9.65	14,45,46
^{27}Al	42,47,60,66.9	8.96-8.58	14,46
^{28}Si	33,36,38,50,53,55,60 66,72,81,142.5,215	8.94-8.10	14,45
^{30}Si	60	9.73	14
^{48}Ca	60	10.17	14
^{54}Fe	46,48,52	9.82-9.87	34

Reaction	Lab. Energy (MeV)	R_{SA} (fm)	Refs.
$^{16}\text{O} + ^{56}\text{Fe}$	43,44,60	10.08-9.62	20,21
^{58}Ni	44,60	9.93-10.04	22
^{59}Co	141.7	9.72	55
^{60}Ni	141.7	9.78	55
^{64}Ni	56	10.54-10.21	19,44
^{65}Cu	41,46	10.42-10.35	47
^{70}Ge	41,50	10.61-10.58	21
^{72}Ge	56	10.53	44
^{74}Ge	56	10.49	44
^{76}Ge	56	10.53	44
^{88}Sr	52,60	10.72-10.80	22
^{90}Zr	54,60	10.68-10.86	21
^{116}Sn	66	11.35	27
^{120}Sn	65.8	11.48	27
^{142}Nd	70	11.53	22
^{181}Ta	83,90,96	12.09-12.18	52
^{208}Pb	83,88,90,94,96,102 104,129.5,192	12.78-12.31	29,30,52,60
^{235}U	140	12.58	40
$^{17}\text{O} + ^{28}\text{Si}$	45,50,60,67.5,72	8.93-8.66	46
$^{18}\text{O} + ^{28}\text{Si}$	36,69	9.01-8.97	46
^{72}Ge	56	10.80	44
^{74}Ge	56	10.81	44
^{76}Ge	56	10.98	44

Reaction	Energy	R_{SA} (fm)	Refs.
$^{20}\text{Ne} + ^{94}\text{Zr}$	166.3, 177	11.03-10.95	60
^{150}Nd	175.3	12.27	61
^{197}Au	208	12.15	38
^{206}Pb	208	12.22	39
^{207}Pb	208	12.22	39
^{208}Pb	160.7, 161.2, 208	13.05-12.32	29, 39, 60
^{209}Bi	208	12.35	38
^{235}U	175, 252	12.83-12.66	40
$^{22}\text{Ne} + ^{88}\text{Sr}$	65.4	11.07	26
^{126}Te	93.5	11.87	59
$^{32}\text{S} + ^{27}\text{Al}$	73, 86.2, 100, 110	9.96-9.42	18, 35
^{24}Mg	75, 90, 110, 120	9.16-9.55	35
^{40}Ca	82.5, 85, 90	10.47-10.54	35
$^{35}\text{Cl} + ^{27}\text{Al}$	119.5, 129.5	9.96-9.91	50
^{58}Ni	119.6, 129.6, 164.4	11.00-11.13	50
^{62}Ni	99.4, 119.5, 129.5, 159.5 164.5, 169.5	11.08-11.23	50, 51
^{116}Sn	159.5, 164.5	12.11-12.32	50
^{120}Sn	164.3	12.34	50
^{124}Sn	164.6	12.37	50
^{141}Pr	159.4, 164.4	12.60-12.56	50
$^{40}\text{Ar} + ^{109}\text{Ag}$	169, 197, 236, 288, 337	12.02-11.30	41
^{121}Sb	282, 340	12.02-11.97	41
^{209}Bi	286, 340	13.44-13.22	3

Reaction	Energy	R_{SA} (fm)	Refs.
^{238}U	286,340	13.58-13.61	3
$^{84}\text{Kr} + ^{65}\text{Cu}$	494,604	12.06-12.08	41
^{181}Ta	457	13.33	36
^{197}Au	457	13.41	36
^{208}Pb	494,500,510,718	13.70-14.27	31,32,43
^{209}Bi	600,712	14.25-14.17	3
^{232}Th	500	13.52	32
^{238}U	456	14.19	33
$^{136}\text{Xe} + ^{181}\text{Ta}$	1120	14.56	48
^{208}Pb	1120	14.83	48
^{209}Bi	1130	15.10	54

Figure 29

Plot of $\zeta(R_{SA})$, where $\zeta(R_{SA}) = R_{Int} - C_T - C_P$, as a function of $C_T + C_P$. The interaction radius R_{Int} is deduced from elastic scattering data.

Figure 29

the analysis of the R_{SA} systematics in terms of $C_T + C_P$ is more accurate over the whole range of available data than simple parameterizations of the forms given by Eqs. (9) and (10).

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8. Bombarding Energy Dependence of the Reaction

$^{209}\text{Bi} + ^{136}\text{Xe}$ -- W. Wilcke, A.D. Hoover, J.R. Birkelund,
W.U. Schroder, J.R. Huizenga, A. Mignerey*, D.G. Raich*,
K.L. Wolf* and V.E. Viola**

In the past, much information about nuclear reaction mechanisms has been gained by varying the bombarding energy of the projectile. Since such an energy dependence is also expected for reactions between very heavy ions, the system ^{136}Xe on ^{209}Bi , which has been thoroughly studied at $E_L = 1130$ MeV, was reexamined at a lower energy of $E_L = 900$ MeV. This drop in energy reduces the available energy above the Coulomb barrier, in the center-of-mass system, by a factor of 2.

The experimental setup was similar to the one described previously and consisted of three ΔE -E solid state detector telescopes and an additional stop-detector. Fig. 30 shows the elastic scattering cross section in the laboratory frame, from which an experimental quarter-point of 43.5° is determined. With the aid of the Fresnel model a strong absorption radius R_{SA} of 16.1 fm is deduced, which is 0.9 fm larger than the one determined for the higher bombarding energy. In Table 12 a direct comparison of the relevant parameters is shown. The angular distribution of the light-strongly-damped fragments in the laboratory frame is shown in Fig. 31. The differential cross section is strongly focussed at a laboratory angle of 39.5° , which is 4° inside of the measured quarter point angle.

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Figure 30

Ratios of experimental elastic scattering cross section to Rutherford cross section as a function of laboratory angle for the reaction $^{209}\text{Bi} + ^{136}\text{Xe}$ at 900 MeV (laboratory energy). The line is drawn through the experimental data to guide the eye.

Figure 30

Table 12. Comparison of relevant parameters for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction at two laboratory bombarding energies.

Quantity	900 MeV	1130 MeV
E/A	6.62 MeV/amu	8.31 MeV/amu
$E_{\text{cm}} - V_{\text{Coul}}$	121 MeV	260 MeV
R_{SA}	16.1 fm	15.2 fm
l_{max}	384 $\hbar$	484 $\hbar$
$\sigma_{\text{R}}(\text{exp})$	2.28 b	2.8 b
$\sigma_{\text{R}}(\text{Fresnel})$	2.17 b	2.7 b
Peak in ang. dis. (lab)	43.5 deg	33 deg
Peak in ang. dis. (c.m.)	70.2 deg	54 deg
k/D_2	17.8 amu $1.8 \times 10^{-43} \text{ MeVsec}^2 \text{ fm}^{-2}$	8.9 amu $0.9 \times 10^{-43} \text{ MeVsec}^2 \text{ fm}^{-2}$
$D_2(\text{NS})$	$1.1 \times 10^{-22} \text{ sec}^{-1}$	$1.1 \times 10^{-22} \text{ sec}^{-1}$
$D_2(\text{S})$	$0.8 \times 10^{-22} \text{ sec}^{-1}$	$0.75 \times 10^{-22} \text{ sec}^{-1}$

Figure 31

Angular distribution of all projectile-like fragments
in the laboratory frame for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction
at a bombarding energy of 900 MeV (laboratory).

Figure 31

No indication of orbiting (i.e., non vanishing cross section at 0°) is visible. These results are consistent with the observations made for other very heavy systems. The total reaction cross section obtained by integrating the cross section in Fig. 31 is equal to 2.28 b which agrees rather well with the result of the Fresnel model (2.17 b), if the experimental value of the quarterpoint angle is used to calculate the cross section.

Due to the large projectile mass, the recoiling target nuclei and the heavy fragments, respectively, have sufficient kinetic energy to be detected easily in the counter telescope. The angular distribution of heavy (Bismuth like) fragments is shown in Fig. 32. The observed asymmetry of the angular distribution is mainly due to a kinematic effect, which compresses the angular distribution of all events with negative Q-values. The integrated cross section in Fig. 32 is 1.6 barn, which is to be compared to 2.3 b for the light fragments. The difference is probably due to subsequent fission of a percentage of the heavy fragments. If this interpretation is correct, approximately 30% of all damped events lead to the subsequent fission of the Bi-like fragments at a bombarding energy of 900 MeV.

Assuming a constant ratio of mass to charge for all fragments a center-of-mass transformation was performed. In Fig. 33 is shown the charge distribution of the light fragments as a function of the final kinetic energy. Similar to the results at the higher bombarding energy, the distributions remain gaussian even at very low energies, and the centroid shifts only by two Z-units from Xe to Ba, indicating a small drift coefficient

Figure 32

Angular distribution of the heavy (Bismuth-like) fragments in the laboratory frame for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction at a bombarding energy of 900 MeV (laboratory).

Figure 33

Plot of the differential cross section $d^2\sigma/dEdZ$ as a function of atomic number for various final kinetic energy bins of 50-MeV width measured for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction at a bombarding energy of 900 MeV (laboratory). The solid lines are Gaussian fits to the data.

Figure 33

for the diffusion process.

If a velocity-dependent friction force

$$\vec{F} = -k \vec{v} \quad (1)$$

is assumed, the rate of energy loss is given by

$$- \frac{dE}{dt} = 2(k/\mu)E \quad (2)$$

where μ is the reduced mass. Using the general result of the diffusion theory, namely that the variance σ_z^2 of a distribution is proportional to the diffusion time t ,

$$\sigma_z^2 = 2D_z \cdot t \quad (3)$$

it is possible to eliminate dt from eq. (2). The resulting differential equation is easily integrated. Insertion of boundary conditions leads to

$$\ln(E_0/E_{av}) = (k/\mu D_z) \sigma_z^2 \quad (4)$$

where E_0 and E_{av} are the energies above the Coulomb barrier in the incident and exit channel, respectively. Thus it is possible to determine the ratio of the friction constant k to the diffusion constant D , very straightforward from measurable quantities. In Fig. 34 it is shown that this ratio k/D increases by a factor of 2 for the lower bombarding energy. To determine D (and hence k) independently, use is made of a classical model. For a given angular momentum ℓ , the difference between the measured scattering angle θ_{exp} and the calculated angle¹⁾ θ_c is ascribed to a rotation $\Delta\theta$ of the double-nucleus system as illustrated in Fig. 35. For a given moment-of-inertia J of the system, the angle of

Figure 34

Plot of $\ln(E_0/E_{av})$ as a function of variance σ_z^2 , of the charge distribution of the projectile-like fragments. Comparisons for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction at bombarding energies of 900 and 1130 MeV are made.

Figure 35

Deflection functions for the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction at laboratory bombarding energies of 900 and 1130 MeV. The procedures for calculating the different deflection functions are given elsewhere^{1,2)}.

Figure 35

rotation $\Delta\theta$ is related to the interaction time t by

$$\Delta\theta = t \cdot \hbar l / \sqrt{J} \quad (5)$$

In the framework of this model, the interaction time t is known. The diffusion constant D_z is determined by plotting t vs. σ_z^2 (see Eq. 3) as shown in Fig. 36. The straight lines are fitted to the data points, and indicate that the diffusion constant is approximately independent of interaction time (and angular momentum l). In addition, the diffusion constant is approximately independent of bombarding energy (see Table 12). Therefore, the friction constant k increases by a factor of 2 if the energy above the Coulomb barrier is reduced by this same factor. At present, we are trying to understand the reasons for the observed dependence of the friction constant k on bombarding energy.

In Fig. 37 is shown a Wilczynski plot of the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction at a laboratory bombarding energy of 900 MeV. It is interesting to note that the cross section ridge moves slightly to more backward angles with increasing kinetic energy loss, whereas for $E_{\text{Lab}} = 1130$ MeV, the corresponding ridge remains at approximately the same angle for all energy losses. This behavior has been predicted by a model of interacting nuclei undergoing a rotation during the contact time. At the lower bombarding energy, the angular momentum corresponding to a given impact parameter is smaller. Therefore, the angle of rotation $\Delta\theta$ is diminished and not large enough to compensate the increasing Coulomb deflection in the exit channel for high energy losses. At $E_L = 1130$ MeV, the two contributions to the scattering angle

Figure 36

The variance $\sigma_z^2(\ell)$ of the experimental fragment-charge distributions are plotted as a function of the interaction time $\tau(\ell)$ for the reaction $^{209}\text{Bi} + ^{136}\text{Xe}$ ($E_{\text{lab}} = 900 \text{ MeV}$). The two sets of results correspond to the assumption of nonsticking (NS) and sticking (S) of the two ions during the interaction. When final corrections are applied to $\Delta\theta(\ell)$, it is expected that the curve for the nonsticking (NS) model will pass through zero.

Figure 36

Figure 37

Contour diagram of the double differential cross section $d^2\sigma/d\Omega d(\text{TKE})$ for projectile-like products from the $^{209}\text{Bi} + ^{136}\text{Xe}$ reaction at a laboratory energy of 900 MeV. Such TKE vs. angle plots are known as Wilczyński diagrams.

are known to compensate each other. At even higher bombarding energy, (one predicts first a forward moving ridge and finally some orbiting of the system.)

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C. Muon-Induced Reactions

1. Total Muon-Capture Rates in ^{232}Th , ^{235}U , ^{238}U and ^{239}Pu
by Electron Measurements--M. W. Johnson, W. U. Schröder
J. R. Huizenga, W. K. Hensley*, D. G. Perry*, and
J. C. Browne**

Investigating the muon-capture process in the actinide region is of considerable interest from several points of view: very little experimental data¹ on total muon-capture rates are available for this region of nuclei, where the simple Primakoff theory² which successfully predicts experimental capture rates for a wide range of other target nuclei,³ is expected to fail. Hence, accurate measurements of muon-capture rates in actinides provide a sensitive test of recently developed, improved theories⁴ on muon capture. Another interesting aspect of such studies has been pointed out by Blocki⁵ and Bloom,⁶ who suggested that an actinide nucleus may be excited to states in the second well of the double-humped fission barrier by a radiationless muonic cascade transition. Bloom⁶ predicts that, due to isomeric fission of the nucleus in the second well, the rate λ_f of muon-induced fission should be higher than the rate λ_e of muon decay by an amount

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$$\Delta\lambda = \lambda_f - \lambda_e = \frac{\epsilon\lambda_{if}}{\alpha\lambda_f} (\lambda_{if} + \lambda_{bt}) \quad (1)$$

Here ϵ is the relative population of states in the second well as compared to those of the first well, α is the branching ratio of fission following muon capture, λ_{if} is the isomeric-fission rate, and λ_{bt} is the rate for the system in the second well to tunnel back into the first well. So far, available data on muon-induced fission and muon-capture¹ in actinides have not conclusively confirmed this effect.

The present study aims at an accurate determination of muon-capture rates by a measurement of the electrons from the decay of muons in the 1s orbit of muonic ^{232}Th , ^{235}U , ^{238}U and ^{239}Pu . The experiments were performed at the LAMPF stopped-muon channel of the Los Alamos Scientific Laboratory. A schematic diagram of the experimental setup is shown in Fig. 38. Muons and electrons were measured with separate conventional plastic counter telescopes. Targets were metallic sheets of 5-10 g/cm² thickness. A time-to-amplitude converter was started with a fast 1234 coincidence signal indicating a muon stop in the target and stopped by a 567 coincidence signal signifying an electron.

Decay electrons were accepted during a time interval from -200 nsec to 3.8 μsec with respect to a muon stop. In Fig. 39 a part of such an electron-time distribution is shown, together with a least-squares fit indicated by the full curve. The total background resulted mainly from events random in time but had a 20% component decreasing exponentially in time with a decay constant τ_B between 1 and 2 μsec , characteristic of

Figure 38

Experimental arrangement of the muon telescope (1,2,3,4)
and the electron telescope (5,6,7).

Figure 39

Part of the experimental electron time distribution of muonic ^{235}U . The solid line represents the fit to the data points. The total background (dashed) is a sum of a random background (dotted) and two exponentials corresponding to muon lifetimes in copper and light materials, respectively.

Figure 39

muon lifetimes in light materials near the target. The experimental muon lifetimes are (79.2 ± 2.0) , (75.4 ± 1.9) , (73.5 ± 2.0) , and (73.4 ± 2.8) nsec for muonic ^{232}Th , ^{235}U , ^{238}U , and ^{239}Pu , respectively.

From the experimental muon lifetimes τ_e , the total muon-capture rates λ_c^{exp} can be evaluated according to $\lambda_c^{\text{exp}} = 1/\tau_e - R\lambda_0$, where λ_0 is the decay rate of the free muon and R is the Huff factor.⁷ The resulting experimental capture rates are summarized in Table 13 and compared to two theoretical predictions. Eckhause et al.³ were able to fit experimental capture rates for a wide range of nuclei with the formula

$$\lambda_c^{\text{th } 1}(A, Z) = 168.44 Z_{\text{eff}}^4 (1 - 3.14 \frac{A-Z}{2A}) \text{ sec}^{-1} \quad (2)$$

the structure of which was predicted by the Primakoff theory.² Here, Z_{eff}^4 represents the overlap integral of the 1s muonic wave function with the nuclear charge distribution. Goulard and Primakoff⁴ have determined unknown matrix elements of their improved model by a fit to experimental data yielding

$$\lambda_c^{\text{th } 2}(A, Z) = 218.41 Z_{\text{eff}}^4 (1 - \frac{\epsilon_\mu}{m_\mu})^2 [1 - 2(\frac{m_\mu - \epsilon_\mu}{2m_N})] \cdot \{1 - 0.03 \frac{A}{2Z} + 0.25 \frac{A-2Z}{2Z} - 3.24 (\frac{A-Z}{2A} + \frac{|A-2Z|}{82A})\} \quad (3)$$

where ϵ_μ is the 1s muon binding energy of approximately 12 MeV, and m_μ and m_N are the muon and nucleon masses, respectively. The values Z_{eff}^4 were calculated accounting for nuclear deformation,⁸ which results in a non-negligible effect on the capture rates (cf. Table 13). One notices that the general trend of increasing experimental capture rate with increasing

Table 13

Experimental muon-capture rates λ_c^{exp} compared to the predictions of the Primakoff Formula ($\lambda_c^{\text{th } 1}$) and of the Goulard-Primakoff theory ($\lambda_c^{\text{th } 2}$). Capture rates are given in units 10^7 sec^{-1} . $z_{\text{eff}}^4(\beta)/z_{\text{eff}}^4(0)$ is the ratio of capture rates evaluated with a deformed and spherical nuclear charge distribution.

Target	λ_c^{exp}	$z_{\text{eff}}^4(\beta)/z_{\text{eff}}^4(0)$	$\lambda_c^{\text{th } 1}$	$\lambda_c^{\text{th } 2}$
^{232}Th	1.22 ± 0.03	0.966	0.94	1.11
^{235}U	1.29 ± 0.03	0.959	1.08	1.20
^{238}U	1.32 ± 0.04	0.955	0.89	1.08
^{239}Pu	1.33 ± 0.04	0.952	1.18	1.28

Z is reproduced by theory. However, the theoretical values are generally too low, and the theoretical dependence of the capture rate on A and Z appears to be somewhat stronger than experimentally observed. This applies in particular to the simpler Primakoff theory. Whereas in the present experiment no significant isotopic effect of the capture rate in ^{235}U and ^{238}U was found, theories predict an effect of 10-20%. A similar overestimation of the isotopic effect by theory has been observed also in the cases of muon capture in $^{203,205}\text{Tl}$ ⁹ and $^{206,207,208}\text{Pb}$.¹⁰ It appears, however, that the Goulard-Primakoff formula Eq. (3) represents a definite improvement over the older Primakoff formula Eq. (2).

The present results suggest that, within the limits of experimental accuracy, there is no difference $\Delta\lambda = \lambda_f - \lambda_e$ between the disappearance rate $\lambda_e = 1/\tau_e$ and the rate λ_f of muon-induced fission except possibly in the case of ^{235}U . As an example, for ^{238}U a value of $\Delta\lambda \leq 4 \cdot 10^4 \text{ sec}^{-1}$ is deduced, which suggests an upper limit of $\epsilon \leq 3 \cdot 10^{-2}$ for the population of second-well states during the muon cascade. However, the quantities λ_{if} and λ_{pt} entering Eq. (1) are not well-known in the presence of the 1s muon. An extension of Bloom's calculation in order to account for differences in the capture rates and the capture-induced fission branching ratio for the first and second well indicates a high sensitivity of $\Delta\lambda$ on these differences. This implies that the present results set an even lower limit on ϵ than the above value, which is consistent with known systematics for the population of shape-isomeric states by nuclear reactions.

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2. Precision Measurements of Muon Lifetimes on
 ^{232}Th , ^{235}U , ^{238}U , ^{237}Np , ^{239}Pu and ^{242}Pu
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Attempts to measure the lifetimes of heavy muonic atoms either by observing muon decay electrons, as described in Section C. 1, or by observing fission fragments produced by μ -capture, as done by many other groups, suffer from a fundamental experimental difficulty: the relatively low branching ratios for these processes, combined with the difficulty in obtaining a satisfactory counting efficiency, lead to counting rates too low to produce lifetimes accurate to ≤ 1 nanosecond in a reasonable amount of time. [It is necessary, however, to obtain ^{for 20% work} this degree of accuracy in order to prove or disprove the capture-rate anomalies suggested by Bloom¹ or by Hadermann².] To remedy these deficiencies, we have performed experiments to determine the muon capture rates on six representative actinide isotopes using much more efficient observational techniques, i.e. the detection either of neutrons or of nuclear γ -rays emitted following μ -capture.]

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The experimental arrangement for these studies was similar to that described in Section C. 1 except that the electron telescope was replaced by a liquid scintillator capable of detecting both γ -rays and neutrons. The signals produced by this detector were sorted using a pulse-shape-discrimination (PSD) system and used to produce time spectra in the same fashion as in the electron study. Routing bits generated by the PSD system were used to distinguish neutrons from γ -rays. Two features of the neutron spectra should be noted: the spectra have far better peak-to-background ratios than in electron study (typically 30:1 compared to 10:1), and the prominence of the "prompt" neutron peak clearly varies from isotope to isotope.

The measurements on ^{237}Np and ^{242}Pu were made on much smaller quantities of target material than the others; however the quality of the data remains good in these cases. This would suggest that, with the excellent muon beam available at LAMPF, it may be possible to extend these measurements to rarer targets such as ^{233}U or ^{234}U . These measurements should help to establish the accuracy of the Primakoff formalism³ in the actinide region.

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PROTON-HOLE STATES POPULATED IN ACTINIDE NUCLEI BY THE
(t, α) REACTION
173 Meeting of the American Chemical Society at New Orleans,
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March 21-25, 1977.

J. R. Huizenga, W. U. Schröder, J. R. Birkelund, K. L.
Wolf and V. E. Viola
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A. C. Mignerey, D. G. Raich, K. L. Wolf, R. G. Boudrie,
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THE DEPENDENCE OF NUCLEAR ORBITING AND CHARGE EXCHANGE
ON BOMBARDING ENERGY IN THE $^{144}\text{Sm} + ^{84}\text{Kr}$ REACTION
Fall Creek Falls Meeting on Heavy-Ion Collisions,
Pikeville, Tenn. June 13-17, 1977.

D. G. Raich, A. C. Mignerey, K. L. Wolf, R. L. Boudrie,
C. T. Roche, W. U. Schröder, J. R. Birkelund, J. R.
Huizenga and V. E. Viola
POSSIBLE STATIC DEFORMATION EFFECTS IN ^{84}Kr - AND ^{86}Kr -
INDUCED REACTIONS ON TARGETS OF ^{144}Sm AND ^{154}Sm
Fall Creek Falls Meeting on Heavy-Ion Collisions,
Pikeville, Tenn. June 13-17, 1977.

Th. W. Elze, W. Feix, W. Wilcke and J. R. Huizenga
MULTISTEP PROCESSES AND FORM FACTORS FOR SINGLE NUCLEON
TRANSFER
International Conference on Nuclear Structure, Tokyo,
Japan, September 5-10, 1977.

C. Invited Lectures

"STRONGLY DAMPED HEAVY-ION COLLISIONS", Nuclear Chemistry
Gordon Research Conference, New London, New Hampshire,
June 21-25, 1976.

"NUCLEON TRANSFER AND ENERGY DISSIPATION IN HEAVY ION
REACTIONS", Physics Colloquium, Chalk River Nuclear Lab-
oratories, Chalk River, Ontario, Canada, August 5, 1976.

"MECHANISM OF VERY HEAVY ION REACTIONS", Gesellschaft
für Schwerionenforschung (GSI), Darmstadt, Germany,
September 1, 1976.

"INTERACTION TIMES AND DIFFUSION COEFFICIENTS FOR HEAVY ION COLLISIONS", Max-Planck Institut für Kernphysik, Heidelberg, Germany, September 2, 1976.

"NUCLEAR ORBITING AND CHARGE DIFFUSION IN ^{84}Kr - AND ^{136}Xe -INDUCED REACTIONS", European Conference on Nuclear Physics, Caen, France, September 6-11, 1976.

"HEAVY ION REACTIONS: A NEW FRONTIER IN NUCLEAR SCIENCE", Wednesday Evenings at the University Lecture Series, University of Rochester, Rochester, New York December 1, 1976.

"HEAVY ION EXPERIMENTS AT THE SUPER HILAC", Bevatron/Bevalac - Super Hilac, Users' Meeting, University of California, Berkeley, California, February 17-19, 1977.

"ENERGY DISSIPATION, MASS DIFFUSION AND INTERACTION TIMES FOR HEAVY ION COLLISIONS", Physics Seminar, McGill University, Montreal, February 22, 1977.

"ENERGY DISSIPATION, MASS DIFFUSION AND INTERACTION TIMES FOR HEAVY ION COLLISIONS", Physics Colloquium, University of Texas, Austin, Texas, March 30, 1977.

"ENERGY DISSIPATION, MASS DIFFUSION AND INTERACTION TIMES IN HEAVY ION COLLISIONS", Chemistry Research Conference, University of Maryland, Department of Chemistry, April 7, 1977.

"HEAVY ION COLLISIONS: A NEW REACTION MECHANISM", Chemistry Colloquium, Purdue University, Lafayette, Indiana, April 14, 1977.

"DAMPED HEAVY ION COLLISIONS", Physics Colloquium, Indiana University, Bloomington, Indiana, April 15, 1977.

"DAMPED HEAVY ION REACTION", Workshop on Deep Inelastic Processes in Heavy Ion Reactions, State University of New York at Stony Brook, Stony Brook, New York, April 29 and 30, 1977; Paper given by W. U. Schröder.

"ENERGY DISSIPATION, MASS DIFFUSION, AND INTERACTION TIMES FOR HEAVY ION COLLISIONS", Meeting on Heavy Ion Collisions, Fall Creek Falls State Park, Pikeville, Tenn. 37367, June 13-17, 1977.

D. Professional Activities

Chairman of Chemistry Graduate Student Admission Committee

Chairman of NRC-NAS Committee on Nuclear Science

Member of Search Committee for Chairman of the Department of Radiation Biology and Biophysics

Member of the Curriculum Review Committee, College of Arts and Sciences

Member of Los Alamos Meson Physics Faculty Program Advisory Committee

Chairperson of Nuclear Science Division Visiting Committee, Lawrence Berkeley Laboratory, University of California

Member of Advisory Panel for the Cyclotron Institute, Texas A and M University

IV. PERSONNEL

Dr. J. R. Huizenga, Professor, Departments of Chemistry and Physics

Dr. J. R. Birkelund, Postdoctoral Research Associate

Dr. D. Hilscher*, Postdoctoral Research Associate

Dr. W. U. Schröder**, Postdoctoral Research Associate

Dr. W. W. Wilcke, Postdoctoral Research Associate

Mr. R. A. Atcher***, Graduate Student, Department of Chemistry

Ms. A. Hoover****, Graduate Student, Department of Chemistry

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**Supported in part up to May 31, 1977 by a grant from the German Academic Exchange Service, DAAD.

***In residence at the Chemistry Division, Argonne National Laboratory; Supported by an Argonne Laboratory - Graduate appointment.

****Supported September 1976 to May 1977 by teaching assistantship in Department of Chemistry.

Mr. M. W. Johnson*****, Graduate Student, Department of Chemistry

Mr. P. R. Romano, Undergraduate Student, Department of Chemistry

Dr. R. D. Griffioen*****, Professor, Calvin College
Grand Rapids, Michigan

*****In residence at LAMPF.

*****Summer Visitor (1976).