This document contains information which is classified "Secret" under Executive Order 11652, dated 5/22/64, and is exempt from automatic downgrading and declassification. It is the policy of the Department of Defense to release this information to the public as soon as it is determined that it is no longer of value to the national defense. It is the policy of the Department of Defense to release this information to the public as soon as it is determined that it is no longer of value to the national defense.

COMPUTED AND EXPERIMENTAL INTERACTIONS BETWEEN EDDY STRUCTURE AND DISPERSED PARTICLES IN DEVELOPING FREE-SHEAR LAYERS*

A. C. Buckingham,** W. J. Siekhaus**
University of California, Lawrence Livermore National Laboratory
Livermore, California 94550

UCRL--00936

DEC 2 015007

J. O. Keller,† J. Ellzey,† G. Hubbard,† J. W. Daily††
Department of Mechanical Engineering, University of California
Berkeley, California 94720

Abstract

We are investigating the interactive process between turbulent flow and dispersed phase particles. We are focusing on the mechanisms that appear to result in a reduction of local turbulent intensity and a corresponding reduction in wall heat transfer and subsequent wall erosion in turbulent solid propellant combustion flow. We apply computational simulations and physical experiments specialized to a developing free shear layer over a rearward facing step and over a parallel splitter plate. The flow configuration evolves in a two-dimensional, steady, combustion and non-combustion turbulent free shear mixing region, with and without particle additives. The computational simulations combine three basic components: gas phase Navier-Stokes solutions, Lagrange particle field solutions and a Monte Carlo technique for the random encounters, forces and accelerations between the two fields. We concentrate here on relatively large sized additive particles (of the order of tens of microns to 100 microns mean diameter). We examine their apparent influence in breaking up the larger, energy bearing eddy structures into smaller structures which are more readily dissipated.

1. Introduction

Particles in the Flow Field

It seems intuitively reasonable that, in a turbulent flow, particle inertial effects, fluctuating phase volume effects, and in combustion, particle enhanced acoustical coupling should affect the dissipation and transport of turbulence energy. Particle population density, size distribution and, to some extent, composition are expected to be significant parameters in particle/turbulence interaction. We also anticipate that eddy mixing rates may correspondingly diminish and that eddy diffusion-influenced phenomena such as apparent chemical reaction rates, heat, and mass transport should be appropriately altered. Our present and continuing research on reduction of solid surface erosion in contained combustion

seems to, at least qualitatively, support this view. In this paper we will review this premise in the light of our current experimental and theoretical findings.

Specifically, we discuss the measured and predicted interaction influence of controlled particle populations introduced into developing turbulent free shear layers. Most of the noteworthy earlier investigations of the influence of dust dispersal on heat, mass and momentum transport were restricted to consideration of laminar viscous flows, in contrast to the turbulent flow situations that we study here. Examples include Marble's^{1,2} theoretical research on dust-laden laminar viscous flows, particularly those affecting rocket nozzle erosion, as well as Carrier's³ original work on the attenuation of shock waves in dusty gas. These studies provide a substantial basis for the concept that particle concentrations may significantly alter both diffusion and wave propagation processes of the carrier gas.

The related questions about quantitative effects of particle mass loading on turbulence and the changes in associated flow structure are central issues in our work. The relevance to fluid dynamic combustion and erosion design problems in air breathing propulsion is apparent from experimental observations involving even moderate concentrations of a dispersed solid phase in the combustion gas stream.^{4,5,6}

Our investigation focuses on the effects of particles in free shear layer mixing combustion zones. We experimentally and theoretically study two plane two-dimensional combustion flow field configurations shown in Fig. 1. The upper figure (a) shows our older (now replaced) rearward facing step flow configuration in which a premixed vapor stream was ignited at the step and eddy enhanced mixing dominated the studied flame spread and visible features such as the eddy shedding, pairing and coalescence downstream of the step. The lower figure (b) shows the combustor configuration in which the two dimensional rectangular combustion chamber is divided by a splitter plate. The hot, premixed, pre-ignited combustion gases in the lower channel flow separately from the cold oxidizer in the upper channel. Actually both channels carry a pre-set mixture (equivalence ratio specified) of oxidizer (air) to fuel (propane). Mixture of the two streams, flame spread, and for unstable stream stratification, turbulent mixing features, are studied in this configuration.

Croze's theoretical development of the inter-phase forces of particulate suspensions⁷ and droplet suspensions,⁸ although developed for laminar viscous flows, are, generally, self-consistent and convenient for our computations; where we must, in addition, calculate the local

This paper is declared a work of the U. S. Government and therefore is in the public domain.

*Work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract

#W-7405-Eng-48 and supported by the Department of the Army, U. S. Army Research Office and ARRADCOM, Dover, N.J. under contract #15812-MS.

**Physicist, Fluid Dynamics, W-Division, Associate Fellow AIAA.

***Senior Scientist, Chemistry and Materials Science Department.

†Graduate Students, Mechanical Engineering Dept.

††Associate Professor, Mechanical Engineering Dept.

random instantaneous velocity characterizing the local turbulent field.

In our approach we simulate the back influence of the particles on the turbulent field. Following this step we revise our turbulent field calculations to describe the alteration due to the particles. The present work includes statistical approximations for the interaction between representative particle clusters and modeled random turbulent fluctuations.

In contrast to the present work, a substantial number of fluid dynamical situations exist where the complete gas-to-particle and particle-to-gas interactive coupling need not be considered. In these cases, consideration of one-way interaction is sufficient because of low particle population and the separation of size scales between the large energy-bearing gas phase turbulent structures, their production, and their dynamics as compared to the size scale of the aerosols that they drive. Examples include the meteorological scale droplet dispersal Monte-Carlo predictions of Watson and Barr,⁹ the grid scale/subgrid scale turbulent channel flow predictions of Peskin and Kuo,¹⁰ and the random sampling particle dispersion simulations of one of the present authors.¹¹⁻¹³

Dukowicz,¹⁴ by comparison, examines the turbulent gas, moderately dense droplet spray field using a partially interactive statistical method in which droplet volume and inertial influences are accounted for relative to their back influence on the mean gas phase advection. However, Dukowicz does not consider effects on the turbulent fluctuating motions.

The present numerical simulations develop from several previous stages. Initially, consideration was given to weak coupling (particle dispersal only) for very sparse particle mass loading in an unbounded turbulent field. The turbulent field was represented by a random velocity statistical sampling technique.¹¹ This was followed by simulation and analysis of the dispersed phase concentrations and mass loading influences on the turbulent wall boundary layer region within two simple geometries: plane channel flow and cylindrically bounded flow. The analysis included the effects of particles on transport properties, boundary layer entrainment and growth.¹² More recent work emphasized the effect of mixed-phase turbulent boundary layer transport on thermochemical heat transfer to the wall, subsequent gas/surface reaction energy effects and influences on both thermal and shear mechanisms in surface layer erosion.¹³ Subsequently we extended our studies to include investigation of the back influence on the developing round turbulent jet explicitly including interactive gas-particle, particle-gas influences to simulate more substantial particle loading than that considered in our previous analyses.¹⁵ Theoretical incentive for studying the round jet geometry was stimulated by the views of Danon, Wolfstein, and Hestroni on the additional dissipation apparent in particle laden turbulent jet flow.¹⁶

The Free Shear Combustion Flow Fields

Many internal combustion flows evolve, mix and burn in a free shear layer that is turbulent.

Premixed flames are stabilized by heat recirculation involving the recycling of hot combustion products which are mixed with the cold reactants in a free shear layer. Diffusion flames are controlled by the rate at which fuel and oxidizer mix in a free shear layer.

A free shear layer is formed when two streams of fluid flowing coaxially or tangentially are brought together at different axial velocities. The resulting velocity gradients cause shear forces to arise. At high Reynolds numbers the forces generated are severe enough to cause the flow to be turbulent.

The upper flow configuration, Fig. 1(a) is an experimental mock-up of the free shear layer mixing flow field that may develop in the stabilization region of pre-mixed combustors. The experimental study is concerned with lean pre-mixed flame stabilization in the turbulent free shear layer formed at the rearward facing step.

We also study pre-mixed two-channel separated flows, as seen in Fig. 1(b). These flows are dominated by large scale structures which grow by entrainment and coalescence. In combustor flows it is the entrainment process which governs the rate of combustion. For combustion to take place, the fuel and oxidizer must be brought into molecular contact by the mixing process. The rate at which this is done generally controls the rate at which products are formed, thus the flame diffusion flame. In premixed flames, one stream is premixed oxidizer and fuel, the other is hot combustion products, generally at zero axial velocity. In this case the burning rate is controlled by the rate at which the already hot products heat premixed reactants to the ignition temperature.

The splitter plate divided, two-channel flow experiments were conducted in the two-dimensional, two-stream combustion facility at the University of California at Berkeley. The same facility was formerly used for the rearward facing step configuration. The combustor was operated at fixed entrance velocities and densities and the amount of heat release within the mixing layer was varied. High speed Schlieren movies were taken of the combustor flow. Structure passing frequencies were determined from the movies at various downstream positions, and information about the pairing process was deduced. Supplemental LDA/LDV, hot film and hot wire measurements were made to obtain the fine scale turbulent features and statistical measurements.

II. Experiments

Rearward Facing Step

This configuration was equipped with a photodiode flame detector and an adjustable over-pressure switch to activate fuel shutoff. A brass foil blowout disc was placed at the bottom of the premix chamber. A 1.5 mm square cell ceramic flow straightener with a 314 blockage was used to smooth any large flow discontinuities and to prevent flame propagation into the mixing section. In addition the premixing section was packed throughout with stainless steel wool to control inlet turbulence levels. This saturated with particles and so was later replaced with a graded

glass marble bed for our particle laden combustion experiment as described later.

The test section entrance was equipped with a step which was streamlined in the upstream direction with a blockage ratio of 0.5. The step delivered a fairly uniform entrance velocity and also stabilized the flame. Optical access to the test section was provided by two 12.7 mm thick fused quartz windows (99.97% SiO₂) on the sides of the test section. Probe and optical access ports were also provided at axial positions (1 cm intervals) on the top and bottom of the test section. The products of combustion were cooled by water sprays in the exhaust system. All experiments were conducted with a test section pressure of 1 atm.

Important factors in the design of the rearward step configuration were: 1) thorough mixing of fuel and air before entering the test section, 2) a uniform velocity flow field at the entrance to the test section, and 3) safety.

Two-Channel Combustion Chamber

These experiments were carried out in the two-dimensional combustion tunnel shown in Fig. 1(b). A compressor supplied air at a maximum gauge pressure of 640 kPa and a maximum flow rate of 0.5 kg/s. Air was dried to less than 20% humidity by a desiccator drier and filtered to remove undesirable particles. The air and fuel mass flow rates were metered by sonic nozzles to an accuracy of 1%. The fuel passed through solenoid shutoff valves and flame arresters before entering the test section. Air and propane were mixed by injecting the fuel in an opposed flow mixing section upstream of the premixing section, resulting in good mixing, even at low flow rates.

The premixing section was filled with a variety of glass spheres and voids. The final 125 mm was filled with 4.32 mm diameter metal spheres. This provided an acoustic filter and reduced the free stream turbulence level. The metal spheres also acted as flash back arresters. This technique produced an integrated turbulence level of about 1% of the free stream velocity. The cross section of each channel was 173.1 mm by 53.4 mm. Each side of the mixing section was terminated by a flame holder/turbulence suppressor which consisted of three stainless steel screens separated by 8 mm gaps. The symmetric flame holder allowed the preheat flame to be stabilized on either side. The channel then contracted to 122.2 mm by 28.0 mm, a contraction area ratio of 2.7 to 1. The two channels in the contraction section were separated by a thermal stress resistant splitter plate made from space shuttle insulating tile material.

Following the contraction section was a rectangular test section of 122.2 mm by 56.9 mm and 381 mm long. The sides of the test section were quartz windows, 12.7 mm thick, mounted in a mild steel housing. The top and bottom windows, also made of quartz, were adjustable and could diffuse the flow slightly, with a bi-sected angle of about 1.5°, to minimize wall effects on the shear layer. Following the test section was a diffuser which employed boundary layer suction to eliminate acoustic noise. The combustion products were exhausted through a series of water sprays to cool the gas and reduce acoustical feedback. The pres-

sure in the test section was set to one atmosphere using a butterfly valve in the exhaust pipe.

Instrumentation

The data presented here were obtained through the use of non-intrusive optical diagnostics. This experiment used high speed Schlieren movies (5000 frames/sec) and laser Doppler velocimetry (LDV) for data acquisition. The Schlieren system was a conventional Z configuration with a xenon lamp as the light source. A Hy-CAM camera recorded the events on 16 mm film. These movies were digitized with a Vanguard Motion Analyzer.

A dual beam, real fringe interferometer and a period counter signal processor measured the entrance velocity. The output signal from the counter was read directly into a digital computer in records of sufficient length, usually 2048 points, to perform statistical calculations.

The velocity signals were processed by counter-type electronics (Macrodyne model 2096) providing a direct digital output. The mean, RMS, and the probability density distribution of each velocity signal was calculated in the laboratory and displayed in graphic form. All relevant data were stored permanently on magnetic media.

Figure 2 shows a typical unstable combustion mixing layer flow field instantaneous Schlieren exposure. The flow is running from right to left with the two-channel splitter plate on the right. Schlieren data is processed from the Hy-CAM film frames at selected intervals, digitized and subsequently replotted from the digitized, processed and analyzed digital records. Image enhancement is obtainable from the digital processing. Figure 2 is a reconstruction plot of the digital records.

Procedure for Two-Channel Experiments

The gas velocities for these experiments were 15 m/s on the top (side 1) and 5 m/s on the bottom (side 2). The general experimental schedule is shown in Table I, with the "x's" showing the actual tests run one or more times. A flat preheat flame on side 2 provided ignition energy for the shear layer combustion. The preheat flame was operated at an equivalence ratio (ϕ_2) of 0.64 which resulted in an ignition temperature of about 1770 Kelvin. Side 1 was operated at equivalence ratios of $\phi_1 = 0.0, 0.2, 0.4, 0.6$, and 0.8 . The lower edge of the shear layer was unstably stratified for $\phi_1 = 0.0, 0.2$, and 0.4 , since the density in the layer was greater than that below the layer. For $\phi_1 = 0.6$ the temperature inside the layer roughly equaled that outside and hence the bottom interface was neutrally stable. For $\phi_1 = 0.8$, the layer was less dense than the region below, resulting in a stable configuration.

Structure passing frequencies were obtained by recording the Hy-CAM film frame number every time a significant structure passed a given x location. By knowing the framing rate of the camera, a probability distribution function (as a function of frequency) was constructed. The criteria for determining a significant structure was subjective and the passing frequencies varied according to the criteria selected and the person taking the data. Qualitative trends in the PDF's, however, remained consistent. The physical interpretations

are given a firm foundation by this flow visualization data base. The fine scale structural measurements and related theoretical numerical results are used to initiate quantification of these results and to provide information for further theoretical understanding of the turbulent combustion particle interaction phenomena.

III. Numerical

General Outline

Development of the procedure for simulating sparse distributions of particles and the theoretical antecedents are described in several previous publications.¹¹⁻¹³ Results for a round turbulent jet configuration in contrast to the plant two-dimensional free shear layer flow configurations of concern here were the subject of previous discussions.^{15,20}

As an introduction, we first offer a short outline of the procedure. The first step involves modeling the continuous vapor phase including the effects of an assigned initial particle distribution by means of an averaged void fraction and finite difference full Reynolds-Favre averaged or pseudo-spectral Navier-Stokes solutions. The particle influence includes reduction of the control volume of fluid as well as the surface area available over which vapor phase stress and flux divergence (energy transfer) processes act. Hence, the stress terms are adjusted for the appropriate volumetric and area intrusion of the particles. Particles are considered inert with no interphase mass or thermal transfer and the particle sizes are assumed invariant with no agglomeration or breakup considered at this stage. The coupling forces between the particle field and the gas field are accounted for using the self-consistent control volume formulation introduced by Crowe,^{7,8} but re-written to include the turbulent (random) velocity fluctuations of the gas phase on the particle accelerations.

The particles are treated as a separate field in the calculations. Particle equations of motion are re-written in Lagrange center-of-mass form. Particle motions are treated incrementally in time, Δt . Driving forces, which develop from velocity differences between particle and gas phase, are dependent on fluctuating random components of the turbulence. Forces considered include drag, virtual mass, Basset, Magnus, and Saffman lift. These forces drive the particles in linear motions over trajectories limited, automatically, by the smallest time increment between representative particle "cluster" encounters. Ensemble averages of a statistically significant number of incremental motions produce simulated histories of particle dispersion, momenta, energy, and wall collisions.

Statistical decomposition by random selection of the velocity field produces the fluctuational vapor components and forces on the particles, randomly distributed at selected grid mesh point sites. The probability distribution integration is approximated by ensemble averaging over a finite summation with "importance sampling" techniques invoked to reduce the number of samples required for statistical significance. Sampling of the fluctuations must be repeated for a statistically significant number of gas phase trajectories in

phase space. Each trajectory represents an independent random probability density function distribution for a particle "cluster" centered at some fixed position and time in the computation mesh. Gas phase fluctuations about the mean flow speed are thereby simulated as a weighted sum of previous increments plus a currently evaluated random fluctuation sampled from a one-component probability density function (pdf) distribution. The incremental sampling procedure and summation over a finite number of events, replaces the exact integral formation. This converges to the integral value provided a sufficiently large number of samples is taken. The base distribution is taken to be the Gaussian normal distribution for weighted phase space spectrum samples of the random variables.

Following determination of the most-probable cluster trajectories, particle aggregate cloud concentrations and spatial distributions are re-evaluated for the next time interval and the void fraction distribution modified gas phase Navier-Stokes solution is updated by the ensemble averaged particle cloud simulations from the previous step.

We impose law-of-the-wall slip conditions at interior solid contours and continuous flow or periodic boundary conditions on streamwise entrance and wake. The unsteady particle field streamwise conditions imposed in the streamwise current are simply selected initial population size and a momenta distributions at inlet to represent the desired particle injection, emission or entrainment situation under study. Exhaust of particles is incrementally accounted for so that the finite array of particles selected for a computation can be replenished for re-entrance at a later time interval, if necessary. A major assumption is associated with the particle flux at the wall. Tangential flow is assumed free slip with no surface accommodation. At other than tangent incidence, either specular elastic reflection or perfect accommodation of momentum and energy are maintained as the only two options for the computation.

Continuous Phase and Mixture

The first step involves computational simulation of the unsteady Reynolds-Favre¹ averaged mixture Navier-Stokes equations. The mixture amended vapor mass density is given by,

$$\bar{\rho} = \theta \bar{\rho}_p, \quad (1)$$

while the local particle cloud mass density is given by

$$\bar{\rho}_p = (1-\theta)\bar{\rho}_p, \quad (2)$$

where $\theta \equiv$ void fraction,

$$\theta \equiv \frac{\text{vapor volume}}{\text{vapor volume} + \text{condensed phase volume}}$$

The bar overhead designates conventional Reynolds time-averaging. The particles reduce the control volume available and the corresponding surface area over which the vapor stress and flux divergence (energy transfer) processes act. Hence,

consistency demands that the stress terms must be adjusted for the volumetric intrusion of the particles, viz.

$$\bar{\tau}_p + \bar{\tau}_p^* = 0, \quad (3)$$

$$\bar{\tau}_{ij} = \theta \bar{\tau}_{ij}^*.$$

In addition, the local, instantaneous frictional drag of the condensed phase acting on the vapor phase is included in the momentum equation. The momentum associated with the particle cloud is adjusted through the mass density terms as indicated by Eq. (2). The vapor energy equation is corrected for the presence of particles by two general effects. First, all the flux divergence contributions to the work are modified by the void area as indicated by Eq. (3). Second the loss (or gain) of energy in the condensed phase from thermal diffusion or frictional work done by the particle cloud of the gas phase is represented by an additional term, the rate of change of total energy of the particle field.

Reynolds (time) and Favre' (mass) averaging are applied. For convenience, we will drop the θ product notation in subsequent developments, so that $\bar{\rho} \equiv \theta \bar{\rho}$ and $\bar{\rho}_p \equiv (1 - \theta) \bar{\rho}_p$. Time averaging is imposed in the conventional Reynolds form for the terms,

$$\phi(t) \Rightarrow \rho, p, \tau_{ij}, q_j,$$

$$\bar{\phi}(t) = \bar{\phi}_g(t) + \bar{\phi}_p(t) - \bar{\phi}_g^*(t). \quad (4)$$

Here ρ, p, τ , and q refer to mass density, scalar pressure, stress tensor and heat flux vector, respectively. The subscripts, g and p refer to gas (vapor) and particle phases, respectively. Primes denote fluctuating (random) turbulent variables while the bar overhead refers to a time-averaged mean value. Time averaging is assumed over a long enough interval, Δt , to insure cancellation of all random fluctuations, $\phi^*(t)$,

$$\bar{\phi}(t_0) \equiv \frac{1}{\Delta t} \int_{t_0}^{t_0 + \Delta t} \phi(t) dt; \quad \bar{\phi}^*(t) \equiv 0.$$

Mass (Favre') averaging applies to the specific variables shown below,

$$\psi(t) \Rightarrow u_i, h, e, I$$

$$\bar{\psi}(t) = \bar{\psi}_g(t) + \bar{\psi}_p(t) - \bar{\psi}_g^*(t). \quad (5)$$

Here u_i, h, e, I refer to the i th component of the mean plus fluctuating velocity, the total enthalpy, static enthalpy, and internal energy, respectively. The overhead tilde symbol designates a mass-averaged quantity. Note that

$$\bar{\psi}(t_0) \equiv \frac{\rho(t_0) \bar{\psi}(t_0)}{\bar{\rho}(t_0)} \quad \text{and} \quad \bar{\rho} \bar{\psi}^* = 0;$$

$$\text{but } \bar{\psi}^* = \frac{\rho^* \bar{\psi}^*}{\bar{\rho}} \neq 0.$$

Substituting the fluctuating and mean components into the vapor phase continuum equations and performing the appropriate averaging, Eqs. (4) or (5), and assuming that $\theta^*/\theta = 0$, we obtain in cartesian form:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_j) = 0, \quad \text{Continuity (6)}$$

$$\begin{aligned} \frac{\partial (\bar{\rho} \bar{u}_i)}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_j \bar{u}_i) + \theta \delta_{ij} \bar{p} \\ - \partial (\bar{\tau}_{ij} - \bar{\rho} \bar{u}_i \bar{u}_j) + \bar{F}_{D,i} = 0, \quad \text{Momentum (7)} \end{aligned}$$

$$\begin{aligned} \frac{\partial (\bar{\rho} \bar{h} - \bar{p})}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_j \bar{h} + \bar{q}_j + \bar{\rho} \bar{u}_j \bar{h}^*) \\ - \bar{\theta} \bar{u}_i (\bar{\tau}_{ij} - \bar{\rho} \bar{u}_i \bar{u}_j) - \bar{\theta} \bar{u}_j (\bar{\tau}_{ij} - \frac{\bar{\rho} \bar{u}_i^* \bar{u}_j^*}{2}) \\ + \bar{u}_{p,i} \bar{F}_{D,i} = 0. \quad \text{Energy (8)} \end{aligned}$$

In the foregoing we have used ϕ as an arbitrary variable subjected to the averaging process. We have also introduced the mean viscous (molecular) stress tensor and the mean heat flux vector, τ_{ij} and q_j , respectively. Particles are considered to contribute the gasdynamics drag, $F_{D,i}$, to the continuous phase momentum and correspondingly the drag work term $u_{p,i} F_{D,i}$ to the energy, where $u_{p,i}$ is the i th component of particle velocity.

Dispersed Particle Phase

Assuming monodisperse particle size distributions, the particle cloud equations of mass, momentum and energy conservation may be written in the form introduced by Crowe,^{8,9}

$$\frac{\partial \bar{\rho}_p}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho}_p \bar{u}_{p,j}) = 0, \quad \text{Continuity (9)}$$

$$\frac{\partial (\bar{\rho}_p \bar{u}_{p,i})}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho}_p \bar{u}_{p,i} \bar{u}_{p,j}) = \bar{F}_{D,i} \quad \text{Momentum (10)}$$

$$\begin{aligned} \frac{\partial}{\partial t} (\bar{\rho}_p \frac{u_p^2}{2}) + \frac{\partial}{\partial x_j} (\bar{\rho}_p \frac{u_p^2}{2} \bar{u}_{p,j}) \\ = \frac{d}{dt} (\frac{m}{V} \frac{u_p^2}{2}), \quad \text{Energy (11)} \end{aligned}$$

In the foregoing Eqs. (9)-(11) the subscript p denotes the particle field variables that correspond directly to the symbolically similar variables of the continuous phase. We ignore the particle fluctuations relative to the mean particle velocities in contribution to the momentum, viz.

$$\vec{u}_p = \vec{u}_p + \vec{u}_p^*$$

We have introduced the terms N , m , V representing number of particles, mass of a particle, and volume occupied by the particles, respectively. The eddy stress and scalar pressure contributions of the particle "field" are non-existent in this otherwise continuum formulation. This follows since the particles are considered sparse enough as to remain discrete rather than continuous, hence incapable of transporting and supporting momentum continuously. The particles are interactively coupled to the flow field by the forces $F_{p,i}$ which exist by virtue of the disparity between the particle velocity and the local continuous phase flow velocity that surrounds them.

Particle motions are treated incrementally in time, Δt . Driving forces, which develop from velocity differences between particle and gas phase, are dependent on fluctuating random components of the turbulence. As mentioned previously, forces considered include drag, virtual mass, Basset, Magnus, and Saffman Lift, in the order of appearance in Eq. (12). These forces drive the particles in linear motions over trajectories limited, automatically, by the smallest time increment between representative particle "cluster" encounters. Ensemble averages of a statistically significant number of incremental motions represent particle dispersion, momenta, energy, and wall collisions.

$$\begin{aligned} F_{p,j} = & \frac{\pi d^2}{8} C_D \bar{\rho} \left| \vec{u}_j - \vec{u}_{p,j} \right| \left(\vec{u}_j^* - \vec{u}_{p,j}^* \right) \\ & + C_{10} \frac{\pi d^3}{6} \left(\frac{\partial \vec{u}_j^*}{\partial t} - \frac{\partial \vec{u}_{p,j}^*}{\partial t} \right) \\ & + 3/2 d^3 \sqrt{\pi \bar{\rho} \mu} \int_0^t \frac{\partial}{\partial t} \left(\vec{u}_j^* - \vec{u}_{p,j}^* \right) (t-t')^{-1/2} dt' \\ & + \epsilon_{ijk} \frac{\pi d^2}{8} \bar{\rho} \left(\vec{u}_j^* - \vec{u}_{p,j}^* \right) \omega_k + \\ & C_2 \sqrt{\mu \bar{\rho}} d^2 \left(\vec{u}_j^* - \vec{u}_{p,j}^* \right) \left| \frac{\partial \vec{u}_j^*}{\partial x_j} \right| \end{aligned} \quad (12)$$

In the foregoing, $\vec{u}_j^* \equiv \sqrt{\vec{u}_j^* \vec{u}_j^*}$.

Continuous Phase Turbulence Influences

The present numerical techniques include some trials with our mixture modified form of the pseudo spectral large eddy simulations introduced and developed by Perzinger and his Stanford colleagues and students.^{21,22} Most of the present calculations,

however, are from continuous Favre-time averaged Navier-Stokes simulations with a modified two-equation closure scheme. The two equations differentially model the distribution of the turbulence energy K and the dissipation rate ϵ , with the explicit McCormack finite-differencing scheme.

Turbulent diffusion exchange processes other than the Reynolds stress components (i.e., turbulent heat flux, turbulent mass diffusion) are modeled as linearly proportional to the Reynolds stress through the computed eddy viscosity coefficient e_T . For example, the averaged correlation of a fluctuating scalar variable f' is approximated by the assigned proportionality coefficient N_{pf} through

$$\langle (\rho u_j)' f' \rangle = \frac{e_T}{N_{pf}} \frac{\partial \bar{f}}{\partial x_j} \quad (13)$$

where the brackets indicate the appropriate averaging (mass or time) for the enclosed fluctuating quantities and e_T represents the eddy viscosity coefficient computed from the applicable closure relations. The constitutive relationship for the Reynolds stress (R_{ij}) is assumed to have the form

$$R_{ij} = \frac{2}{3} \bar{\rho} K - 2 \bar{\rho} e_T \left\{ \bar{s}_{ij} - \frac{1}{3} \frac{\partial \bar{u}_k}{\partial x_k} \delta_{jk} \right\} \quad (14)$$

Here we have introduced $\bar{s}_{ij}$, the mean continuous-phase strain rate tensor. The eddy viscosity coefficient e_T is given in terms of the appropriate closure variables by

$$e_T = c_1 K^2 \epsilon^{-1} \quad (15)$$

where c_1 is a model constant.

Presently, we attach particular importance to the identification of the condensed phase/continuous phase coupling as it affects the moment model equations for turbulence, kinetic energy and scale. Back influence of the particles may be approximated with some consistency by consideration of the influence of the particles on turbulence energy production, transport and dissipation and some associated model of the variations induced on the turbulence structure by the particles. As illustrated by the suggestion of Danon, Wolfstein, and Bestroni,²³ one observes that the simplest turbulence kinetic energy (K) equation development must necessarily include terms for particle concentration influences. Here $\phi \equiv (\delta = 1)$.

Assume that, to a first perturbation, $\phi = \phi + \phi' + \dots$. Following a dot product of the total velocity with the momentum equation, Eq. (7), and averaging as in Eqs. (4) and (5), with some vector contraction of the components, and subtracting the time-Favre' averaged mean energy equation, one obtains the turbulence energy closure model equation,

$$\frac{\partial K}{\partial t} + \vec{u}_j \frac{\partial K}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\frac{e_T}{N_K} \frac{\partial K}{\partial x_j} \right) \quad \text{Diffusion}$$

$$+ (1+F(\lambda, \theta)) e^{-\frac{\gamma}{\tau}} \frac{\partial \tilde{u}_i}{\partial x_j} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right); \quad \text{Production}$$

$$- (1+G(\lambda, \theta)) \frac{\kappa^2}{L(\lambda)}; \quad \text{Dissipation (16)}$$

$$+ g(u, d^2, \lambda) \left(\frac{\partial \tilde{u}_i}{\partial x_j} \frac{\partial \tilde{u}_i}{\partial x_j} \right)^{\frac{\gamma}{R_{e,\phi}}} + \frac{(u'_i u'_j - \overline{u'_i u'_j})}{R_{e,\phi}} \frac{\partial \tilde{u}_i}{\partial x_j} \frac{\partial \tilde{u}_i}{\partial x_j} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)$$

$$+ \frac{(u'_i u'_j - \overline{u'_i u'_j})}{R_{e,\phi}} \frac{\partial \tilde{u}_i}{\partial x_j} \frac{\partial \tilde{u}_i}{\partial x_j} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right); \quad \text{Added Dissipation,}$$

where $R_{e,\phi} = \frac{\phi \tilde{u}_i \lambda}{\nu}$, λ is a characteristic turbulence scale, and ν is the kinematic viscosity coefficient.

In parallel, a companion equation is obtained for the dissipation function (ϵ) closure relations:

$$\frac{\partial \epsilon}{\partial t} + \tilde{u}_j \frac{\partial \epsilon}{\partial x_j} = (1+F(\lambda, \theta))$$

$$e^{-\frac{\gamma}{\tau}} \frac{\partial \tilde{u}_i}{\partial x_j} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right); \quad \text{Production}$$

$$+ \frac{\partial}{\partial x_j} \left(\frac{e^{-\frac{\gamma}{\tau}}}{N} \frac{\partial \epsilon}{\partial x_j} \right); \quad \text{Diffusion (17)}$$

$$+ (1+G(\lambda, \theta)) \frac{\kappa^2}{(L(\lambda))^2}; \quad \text{Conventional Dissipation}$$

$$+ g(u, d^2, \lambda) \frac{(u'_i u'_j - \overline{u'_i u'_j})}{R_{e,\phi}} \frac{\partial \tilde{u}_i}{\partial x_j} \frac{\partial \tilde{u}_i}{\partial x_j} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right); \quad \text{Added Dissipation}$$

modeled as in Eq. (16). Note that, by Eq. (16),

$$\lambda(\kappa, \epsilon) = C_1 C_3 \kappa^{3/2} \epsilon^{-1}.$$

In developing the form of the model equations for κ and ϵ we take advantage of the requirements of (1) consistency, (2) boundary condition compatibility, (3) moderately sparse particle concentrations.

In Eq. (16), we include an additional influence of the spectral scale of the turbulence, λ , on production, turbulence dissipation, correlation integral scale, $L(\lambda)$, and three added dissipation terms which vanish when the particle concentration is small, i.e. $g \rightarrow 0$, $G \rightarrow 0$, $F \rightarrow 0$ for $\theta \rightarrow 1$.

Statistical Particle-Turbulence Coupling

Statistical decomposition by random selection of the velocity field produces the fluctuational vapor components and forces on the particles, randomly distributed at selected point sites. The probability distribution integration is approximated by ensemble averaging over a finite summation with "importance sampling" techniques invoked to reduce the number of samples required for statistical significance.²⁴

In the following descriptions we use the brackets, $\langle \rangle$, to designate ensemble averaging of the enclosed random variable. The random selection procedure yields the expected (rms) value of the gas fluctuations about the mean velocity, which is defined,

$$u \equiv \sqrt{\langle u^2 \rangle}. \quad (18)$$

Suppressing, for simplicity, physical location vector notation; the algorithm for the phase space trajectory (where the subscript here refers to the wave number index), may be written

$$u_k = \sum_{j=1}^{k-1} A_{k,j} u_j + \delta u_k. \quad (19)$$

The second term on the RHS of Eq. (19) is a single point random sample taken from the group of independently random pdf paths chosen so as to be a statistically significant contribution to the ensemble averaged turbulent motions. The averaged mean-square sum of the independent fluctuating energy samples is constrained so that it is equivalent to the macroscopically computed continuum (Navier-Stokes) solutions of local specific turbulence energy, κ . That is,

$$\left(\frac{1}{K} \right) \sum_{k=K_0}^N \langle \delta u_k \rangle^2 = 1. \quad (20)$$

The process is repeated for all vector velocity components at selected particle "clusters", throughout the particle-occupied region of the computational grid.

Autocovariance velocity distributions are chosen from published experiments or computed by the large eddy scheme. This broadens the range of the spectrum. This is particularly important, since we are limited to a smaller than desirable range of wave numbers for velocity sampling. The autocorrelation function at some time t^n is defined as the product of the random component with itself at the previous time, t_0 .

$$a_{ii}(x_i, t^n - t_0) = \langle u_i(x_i, t^n) \times u_i(x_i, t_0) \rangle, \quad (21)$$

$$i = 1, 2.$$

Computationally, it is convenient to make use of the normalized, single component, autocorrelation coefficient,

$$ca_{ii}(t^n) = a_{ii}(x_i, t^n - t_0) / \langle u_i(x_i, t_0) \rangle^2 \quad (22)$$

where $ca_{ii}(t_0) \approx 1$ at the origin and decreases monotonically to zero away from it.

The "history" of previous trajectory motions which enters the computation of u_k as seen in Eq. (19) is weighted by the recursion coefficients, $A_{k,j}$. These introduce the influence of the accumulated spectrum of previous wave number intervals $k_m < k_i$. To illustrate Eq. (19) may be ensemble averaged, yielding

$$\langle u_k \rangle = \sum_{j=0}^{k-1} \langle A_{k,j} u_j \rangle + \langle \delta u_k \rangle. \quad (23)$$

Eq. (19) is the defining relation for the recursion coefficient $A_{k,j}$. Multiplying by $\langle u_k \rangle$, $k = k_0, k_0 + 1, \dots, k-1$, we obtain

$$a_{ii}(x_i, k-l) = \sum_{j=0}^{k-1} A_{k,j} a_{ii}(x_i, l-l-1). \quad (24)$$

Equation 24 shows the relationship between the coefficient $A_{k,j}$ and the experimentally defined correlation functions at the current interval compared to that at the previous interval.

Random velocity sampling has been applied to as many as 1100 pdf trajectory paths in wave number space (although in our experience we have found that 100-150 sample paths yield adequate accuracy). For an adequately large number of selections, the sampling procedure incrementally mimics the uniform convergence of the probability integral for the set of functions f_i of random variables x_i over the probability distribution function g .

$$\int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} [g(x_1 \dots x_p) f_1(x_1) \dots f_p(x_p) dx_1 \dots dx_p].$$

The incremental sampling procedure replaces the integral formation with a series summation. This converges to the integral value provided a sufficiently large number of samples is taken. The sampling series is represented by,

$$\frac{1}{N} \sum_{i=1}^N g(\xi_1^i \dots \xi_p^i) f_1(\xi_1^i) \dots f_p(\xi_p^i), \quad \xi_k^i = x_k^i / \Delta.$$

The base distribution is taken to be the Gaussian normal distribution for weighted w_k^i phase space spectrum samples of random variable l .

Selective weighting is applied using the "importance sampling" techniques developed primarily for chemical physics applications. Importance sampling is a method for systematic, selective sampling of the most interactive portions of the spectrum. It accelerates convergence while reducing the required number of selections and computations. Trial weighting functions are listed below. For our results we have favored the parabolic bin weighting, $w_k = 2(1 - \delta_k^2)$, which weighs the population near the origin (t_0),

$$\delta_k^2 = \frac{1}{3\sigma^2}, \quad \sigma \equiv \frac{\delta [w_k, k-1]}{\Delta}$$

$$= \frac{1/2\sigma^2}{3(1-\delta^2)}, \quad 2(1-\delta^2).$$

IV. Discussion

Experimental Observations

To examine and measure features of the interaction between particles and turbulence in a controlled situation, the experimental combustion wind tunnel facility previously described is used.¹⁷⁻¹⁹ The experiments include low speed, steady mean flow with developing turbulent eddy structure formerly at a rearward facing step, Fig. 1(a) and currently at a splitter plate, Fig. 1(b). Presampled particle concentrations with controlled size and mass distribution are injected upstream of the step or splitter plate into the low speed (10 to 20 m/s) mean flow of premixed combustion gas, for the step configuration or in one of the two channels in the splitter plate configuration. In the latter configuration the remaining channel is usually a conduit for lean oxidizer to fuel mixture which mixes with the pre-ignited combustion gases moving in the lower channel. Mixing is at the trailing edge of the splitter plate. The developing free shear layer forms at the trailing edge. The flows are run both hot (combusting) and cold. The combustion gas is propane and the combustion mixture is varied according to separately presampled equivalence ratios (ϕ) in the upper (1) and lower (2) channels as tabulated in the experimental schedule in Table I. Previously we described the recirculation and mixing regions downstream of the rearward facing step, a configuration that has since been dismantled.^{15,20} A reconstruction of the digitized Schlieren fast framing camera image of the flow for the second splitter plate divided channel configuration appears in Fig. 1(b).

To determine whether or not systematic particle influences are evident, we trace the entire eddy history through shedding and downstream convection. We ensemble averaged the observations for a large number of eddies. The results were digitized from cinematographic observations using fast framing (5000 s⁻¹) motion picture exposures of Schlieren eddy field images. For combustion flow, supplementary shadowgraph combined with fast framing films capture the flame edge and mixing region evolution.

Figure 3 illustrates histogramatically, the multi-modal breakdown of eddy shedding PDF's for passing frequencies at the rearward facing step. We compare the uni-modal distribution (pure gas) at the top to distinctly bi-modal distributions obtained with particles in the flow at the bottom figure. Adding larger particles produces a distinctly bimodal distribution of shedding frequency which corresponds to an increased population of smaller (less energetic) high frequency eddies. The tendency for breakdown of the large eddy structures increases with particle mass loading. The results suggest that particles of sufficient mass and concentration deplete the turbulent energy during the eddy production phase by breaking the eddy field into smaller, less energetic eddies which are dissipated more rapidly over a less extensive flow volume.

The passing frequency, as described previously, is the frequency at which a significant visible eddy structure passing a selected station (x) downstream of the trailing edge of the splitter plate. It is one of the principle measures of the

turbulent evolution discernable from the Schlieren/ fast framing camera flow visualization diagnostics. In particular, we are concerned with the correlations induced by particles and by combustion. We examine these as changes about a non-combusting (cold) particle-free mixing layer state. Figures 4 through 9 illustrate some of these consistent alterations. Figure 4 shows a tracer (LDV), sub-micron scale, particle laden dusty gas two channel mixing layer flow in which the letter symbols on the rows designate the equivalence ratios (ϕ) of fuel (propane) to air in the nonburning upper channel. The upper channel flow velocity is fixed at 15 m/s. The lower channel is pre-ignited and for combustion flows its equivalence ratio is fixed at 0.65 while its velocity is fixed at 5 m/s. Row letters designate the upper channel equivalence ratios: A = 0, B = 0.2, C = 0.4, D = 0.6, and E = 0.8.

The last two rows, D and E are neutrally stable and stable, respectively, since the equivalence ratio prescribes an upper channel flow mass density that is about equal to and less than that of the lower channel. Equivalence ratios define that the upper three rows are progressively more unstable in the order of more destabilizing mass density from B to C to A. Coalescence is suppressed with increased combustion (increased ϕ) so that at the more distant stations, an apparent favoring of the population of the higher frequency probabilities occurs for the combustion situations in comparison to the non-combustion situations. In all remaining cases, Figures 5 through 9, comparison of the no dust (upper figure) with the LDV dust (micron diameter and below) PDF's (lower figure) indicate a breakup of the larger eddies (lower frequencies) and a more probable eddy distribution in the higher frequencies associated with the presence of particles. The flame zone is reduced and the approach to uniform molecular mixing and completion of combustion is enhanced with the particles. Unburned fuel and flame persistence with corresponding high temperatures and heat transfer near the walls is suppressed.

Experimental LDV records of the mean two channel velocity profiles and turbulence intensity vs transverse distance in the divided channel are shown in the upper and lower pictures of Fig. 10, respectively. The corresponding computer predictions of the intensity are shown in Fig. 11 for several listed particle mass loadings, k_M , where the localized mass loading is defined

$$k_M(\vec{r}, t) = \frac{m(\text{particles})}{m(\text{particles}) + m(\text{gas})}$$

The predictions are in qualitative agreement with the experiments except near the plate axis which apparently is influenced by a lack of zonal resolution and near the walls which are subject to errors of an undetermined magnitude because of the arbitrary boundary conditions applied to the incident particles at the wall. This was discussed previously.

Third moment (skewness) distributions of the single component streamwise random velocity and fourth moment (flatness) ratio distributions developed from the LDV measurements are shown in the upper and lower pictures of Fig. 12, respectively. The computer predicted values of the streamwise auto correlations and streamwise skewness are

shown in Fig. 13 for 0.05 μm monodisperse particle populations at the designated particle loading values. Again the experimental/simulation agreement in the skewness distributions appear qualitatively accurate with the exception of the sensitive outer wall boundary (and arbitrary boundary condition) region.

More satisfactory agreement was developed in comparing the experimental Reynolds stress distributions, Fig. 14 with the computed values, Fig. 15, or various mass loadings. The companion diagonal component (turbulence energy) distributions predicted are also plotted and appear in the lower picture, Fig. 15.

The particle dispersal contours for two sizes of particles, 40 μm , and 90 μm injected with an assumed Gaussian velocity distribution about a pre-set mean of 1/10 the mean flow upper channel velocity are shown for various times in Fig. 16. From this particle concentrations were estimated for entrainment in the free shear mixing layer and for the boundary layer entrainment region over the plate or rearward facing step for later analysis.

Figure 17 shows the ensemble averaged boundary and standard deviation of measured layer thickness as a function of streamwise station on the rearward facing step. Averages were made from the multiplicity of motion picture film frames digitized and analyzed for the subsequent analysis of wall shear and heating in the particle laden flows. Figure 18 shows the estimated wall shear values developed from an assumed integral profile form based on a Stanford University, steady-state, turbulence algebraic profile model. Figure 19 shows the corresponding computed values of skin friction at designated averaged-over-the-boundary layer particle mass loadings. The agreement is promising for large particle mass loadings. The thickening of the wall boundary layer and diminishing gradients plus the predicted decrease in turbulence intensity with particle loading act, in concert, to lower the predicted shear and heat transfer. The effect is significant and promising for wall erosion reduction in particle laden internal combustion.

V. Summary

- Experimental and computational results have been generated which illustrate some of the inertial interaction features between thermally inert gas borne particles and the turbulent combustion flow that entrains them.
- The numerical predictions based on a stochastic interaction model are qualitatively reasonable representations of the experimentally observed turbulent particle laden interaction results.
- The primary effects of significant populations of fine scale particles (below 100 μm) seems to act to reduce turbulent intensity in the mixing layer, to augment break up of the layer scale persistent structures and to enhance the increase of the smaller scale inherently more dissipative structures. Mixing and combustion are increased and corresponding flame zone extent is decreased.

- The diminished flame zones and more complete mixing and combustion in the core, when particles are present, together with the thickened particle entrained wall boundary layers act to reduce wall shear, heat and mass transfer in combustors and potentially to reduce combustion wall erosion.
- The wall transport modification and free shear layer mixing and combustion enhancement dominated by the fluid dynamics as opposed to any thermal-kinetic effect of the gas borne particles in a turbulent flow. However, scaling the mass-inertial effects in estimating transport modification appears to be a much more complicated task in a turbulent flow than the corresponding scaling influences found for laminar viscous flow.

VI. References

1. F. E. Marble, "Dynamics of a Gas Containing Small Particles", in Combustion and Propulsion (5th AGARD Colloquium) (Pergamon Press, NY, 1963) p. 175.
2. F. E. Marble, "Dynamics of Dusty Gases", Ann. Reviews of Fluid Mech. **2**, 397 (1970).
3. G. F. Carrier, J. Fluid Mech. **4**, 376 (1955).
4. R. D. Fulmer and D. P. Wirtz, "Measurement of Individual Particle Velocities in a Simulated Rocket Exhaust", AIAA J. **3**, 8, 1506 (1965).
5. G. Restroni, and M. Sokolov, "Distribution of Mass, Velocity and Intensity of Turbulence in a Two Phase Turbulent Jet", J. Appl. Mech. **38**, 315 (1978).
6. S. Yuu, N. Yasukouchi and J. Tomosada, "Particle Turbulent Diffusion in a Dust Laden Round Jet", AIChE J. **24**, 3, 508 (1978).
7. C. T. Crowe and D. T. Pratt, "Gas Particle Flow in Combustion Chambers", WSCI 72-5 in Spring Meeting Western States Combustion Institute (April 24-25, 1972).
8. C. T. Crowe, "Vapor-Droplet Flow Equations", Univ. of California, Lawrence Livermore National Laboratory UCRL-51877 (August 18, 1975).
9. G. W. Watson and S. Barr, "Monte Carlo Simulation of Turbulent Transport of Airborne Contaminants", Los Alamos National Laboratory, Univ. of California LA-6103 (January 1976).
10. R. L. Peskin and C. J. Kau, "Numerical Simulation of Particulate Motion in Turbulent Gas-Solid Channel Flows", J. Fluids Engrg. **101**, 319 (Sept. 1979).
11. A. C. Buckingham, "Turbulent Dusty Gas Motion with Weak Statistical Coupling", Lawrence Livermore National Laboratory UCRL-82131, and AIAA Paper #79-1484, in AIAA 12th Fluid and Plasmadynamics Conf. (Williamsburg, VA, July 24-26, 1979).
12. A. C. Buckingham, "Dusty Gas Influences on Transport in Turbulent Erosive Propellant Flow", Lawrence Livermore National Laboratory UCRL-82876, Rev. 1, and in AIAA J. **19** (4), 501-508 (1981).
13. A. C. Buckingham, "Modeling Propellant Combustion Interacting with an Eroding Solid Surface", Lawrence Livermore National Laboratory UCRL-83724, and AIAA Paper #80-1259 in AIAA-SAE-ASME 16th Joint Propulsion Conference (Bartford, Conn., June 30-July 2, 1980).
14. J. K. Dukowicz, "A Particle-Fluid Numerical Model for Liquid Sprays", J. Comp. Phys. **33**, 229 (1980).
15. A. C. Buckingham and W. J. Siekhaus, "Simulating Interactions between Turbulence and Particles in Erosive Flow and Transport", Lawrence Livermore National Laboratory UCRL 34691 and in Numerical Methods in Laminar and Turbulent Flow, eds. C. Taylor and B. A. Schrefler, Pineridge Press, Swansea, U.K. (1981) pp. 929-940.
16. H. Danon, M. Wolfstein and G. Hestroni, "Numerical Calculations of Two-Phase Turbulent Round Jet", Int. J. Multiphase Flow **3**, 223 (1977).
17. A. R. Ganji and R. F. Sawyer, "An Experimental Study of the Flow Field and Pollutant Formation in a Two Dimensional, Premixed, Turbulent Flame", AIAA Paper #70-0017 in AIAA 17th Aerospace Sciences Meeting (New Orleans, LA, Jan. 15-17, 1979).
18. R. W. Fitz and J. W. Daily, "Experimental Studies of Combustion in a Two Dimensional Free Shear Layer", in Proc. 2nd Intl. Symp. on Turb. Shear Flows (Imperial College, London, July 2-4, 1979).
19. R. W. Fitz and J. W. Daily, "Experimental Study of Combustion in a Turbulent Free Shear Layer Formed at a Rearward Facing Step", AIAA Paper #81-0106 in AIAA 19th Aerospace Sciences Meeting (St. Louis, Mo., Jan. 12-15, 1981).
20. A. C. Buckingham and W. J. Siekhaus, "Interaction of Moderately Dense Particle Concentrations in Turbulent Flow", Lawrence Livermore National Laboratory UCRL-84559 and AIAA Paper #81-0346 in AIAA 19th Aerospace Sciences Meeting (St. Louis, Mo., Jan. 12-15, 1981).
21. D. Kwak, W. C. Reynolds, and J. H. Ferziger, "Three-Dimensional Time-Dependent Computation of Turbulent Flow", Stanford University Thermosciences Division Rept. TF-5 (May, 1975).
22. N. R. Mansour, J. Ferziger, and W. C. Reynolds, "Large Eddy Simulation of a Turbulent Mixing Layer", Stanford University Thermosciences Division Rept. TF-11 (April, 1978).
23. M. B. Faist and J. T. Muckerman, "Importance Sampling and Histogrammic Representations of Reactivity Functions and Product Distributions in Monte Carlo Quasiclassical Trajectory Calculations", J. Chem. Phys. **69**, 9 (1978)

Acknowledgement

The authors gratefully acknowledge the support and guidance of Drs. George Mayer, Phil Parrish, and Robert Singleton of the Army Research Office; Drs. Norm Banks, Jim Schmidt, Zsaba Zoltani, Richard Ward and Irv Stobie of BRL, Aberdeen Proving Ground; and Drs. Joe Lannon, Dave Downes, and Art Bracuti of ARRADCOM, Dover, N.J. We are also pleased to acknowledge the several stimulating discussions and questions developed by our colleagues Drs. Len Caveny of AFOSR, Ken Kuo of Pennsylvania State University, and Paul Gough of Paul Gough Associates. Special thanks and acknowledgement is extended to our colleagues at Lawrence Livermore National Laboratory, particularly JoAnne Levatin and John French for their programming; guidance and expertise, to Marvin Ross and Sang-Wook Kang for their theoretical insight, and to Kathy Smith and Maureen Duncan whose expert assistance in preparing and editing this manuscript was invaluable.

TABLE I.
MATRIX OF TESTS AND DIVIDED CHANNEL
FLOW CONDITIONS

	C_1					$\phi_1 = 1$
	0	0.2	0.4	0.6	0.8	
U_1	5	0		0		x
	10	0		0		x
	15	x	x	x	x	x
	20	0		0		x
	25	0		0		x
$\phi_2 = 0.65$						$\phi_2 = 1$
$U_2 = 5 \pm 5$						$U_2 = 5 \pm 5$

(a)

(b)

Fig. 1 Free shear, rectangular cross section combustor configurations; (a) rearward facing step dimensions shown in mm, (b) splitter plate divided channel, dimensions shown in cm.

Fig. 2 Splitter plate divided channel free shear mixing layer digital reconstruction; motion picture frame of Schlieren pattern.

$Re \text{ (cm}^{-1}\text{)} = 9092$, $U_{co} = 13.28 \text{ m/s}$, $T = 298 \text{ K}$

Fig. 3 PDF passing frequency distributions for rearward facing step with (a) no particles, (b) $0.05 \mu\text{m}$ mean diameter particles, (c) $90 \mu\text{m}$ mean diameter particles.

Fig. 4 Dust laden split channel combustion PDF's of passing frequency at explicitly listed stations for equivalence ratios: (A) $\phi = 0$, (B) $\phi = 0.2$, (C) $\phi = 0.4$, (D) $\phi = 0.6$, (E) $\phi = 0.8$.

Fig. 5 Dust free vs dust laden non-combustion ($\phi = 0$) PDF passing frequency distribution at listed stations.

Fig. 6 Dust free vs dust laden lean combustion ($\phi = 0.2$) PDF passing frequency distributions at listed stations.

Fig. 7 Dust free vs dust laden lean combustion ($\Phi = 0.4$) PDF passing frequency distributions at listed stations.

Fig. 9 Dust free vs dust laden lean combustion ($\Phi = 0.8$) PDF passing frequency distributions at listed stations.

Fig. 8 Dust free vs dust laden lean combustion ($\Phi = 0.6$) PDF passing frequency distributions at listed stations.

$U_1 = 15$ m/s, $U_2 = 5$ m/s; $\Phi_1 = 0.0$, $\Phi_2 = 0.0$

Fig. 10 LDV traces of mean two channel velocity profiles (above) and turbulent intensities (below) at listed stations.

Fig. 11 Computer predictions of two channel turbulent intensity profiles at listed stations for 0.5 μm particle loading.

Fig. 13 Predicted values of streamwise auto-correlations and streamwise skewness for 0.05 μm particle loading.

Fig. 12 LDV traces of streamwise skewness and streamwise flatness distributions.

Fig. 14 LDV traces of streamwise Reynolds stress distributions.

Fig. 15 Predicted values of streamwise Reynolds stress and turbulence energy distributions for 0.05 μm particle loading.

Fig. 16 Predicted particle dispersal contours near the wall boundary layer on the rearward facing step for (a) 40 μm particles, (b) 90 μm particles.

Fig. 17 RMS and standard deviation of boundary layer thickness at rearward facing step for dust laden flows.

Fig. 18 Estimated boundary layer wall shear distributions from integral form factors for rearward facing step configuration in dust laden gas.

Fig. 19 Predicted wall shear distributions for explicit variation of averaged boundary layer mass loading profiles with perfectly accommodating wall boundary conditions.