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mathematical analysis of rippling of type 1 fuel plates, part I

Contract No. AT[30-1]-2639
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New York Operations Office

ALCO PRODUCTS, INC.
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AEC Research and
Development Report
UC-81, Reactors, Power
(Special Distribution)

MATHEMATICAL ANALYSIS OF RIPPLING
OF TYPE 1 FUEL PLATES, PART I

By
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Approved by:
E. W. Schrader, Acting Project Engineer

Issued: July 5, 1962

Contract No. AT(30-1)-2639
with United States Atomic Energy Commission
New York Operations Office

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ABSTRACT

Rippling phenomena due to heating in fuel plates of SM and PM type reactors are investigated analytically using small deflection theory of plates. Temperature variations across the width of the plate are accounted for. Detailed calculations are conducted for simply supported plates. It is found that within the limitations imposed by small deflection theory that the amplitude of the plate ripples induced by the heating is directly proportional to the initial amplitude.

ACKNOWLEDGMENT

The authors wish to express their sincere appreciation to Dr. F. F. Ling of Rensselaer Polytechnic Institute for his many helpful suggestions and comments and to the Alco staff members I. Segalman and J. G. Gallagher who have contributed significantly to this investigation.

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SUMMARY

Based on small deflection theory, equations have been derived for a rippled fuel plate which predict the final plate deflection as a function of initial conditions and operating temperature differences. These equations have been used to check tests which have been previously performed on single fuel plates. (5)

In the range of SM-1A operating conditions there is good agreement between the analytical solution and the test results. It appears, therefore, that the equations presented here can be used with a fair degree of confidence for predicting the plate deflections of Type 1 fuel elements. However, a more extensive comparison between the analytical model and available test results is required to substantiate this supposition. This should include a comparison with the proposed multi-plate test (5) so that the effect of the side plate restraints can be evaluated.

Additional comparisons have been made on fuel plates with various temperature distributions and show that temperature profiles which are peaked at the center of the plate are more severe than those in which the maximums occur nearer to the edge of the plate.

An outline is also presented on the use of large deflection theory for the solution of this problem. It is pointed out that for an accurate estimate of the amplitude of plate deflections resulting from rippling, the large deflection theory is required if the amplitude exceeds the plate thickness.

Appendix B reviews the existing experimental and manufacturing information relevant to the rippling problem and predicts the appearance of a lattice coolant channel which is enclosed between two rippled fuel plates. To define this "hot" channel the available information was coupled with test results and the analytical solution presented in the body of this report. Thus it has been shown that the derived equations can be used with manufacturing data to conservatively predict the channel closure resulting from rippling. A thermal analysis can then be performed to determine the effect on core performance.

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1.0 INTRODUCTION

The object of this investigation is to develop a technique to accurately predict the fuel plate ripple growth and the thermal safety of the various SM and PM type cores. This investigation is important because of the relatively thin plates (0.030 in. Types 1 and 2 and 0.040 in. Type 3) used in SM-1, SM-1A, PM-2A, SM-2 and PL-3 reactors and the observed magnitude of ripples in the SM-1 Core 1 elements. The initial investigation of thermal safety is to be performed for the SM-1A which operates at an average power density of 144 watts/cc.

This report covers the initial mathematical investigation of fuel plate rippling. A companion report⁽⁵⁾ covers the initial laboratory investigation.

Rippling of plate type fuel elements is defined as the occurrence of irregular surfaces on a flat plate when subjected to compressive forces in the plane of the plate. These stresses usually produce a sinusoidal "washboard" buckling in the longitudinal plane of the plate. Figure 2 shows the characteristic shapes resulting from rippling. This is a potentially serious problem in the thin (30 to 40 mils) fuel plates which are used in present ALCO designed reactors.

In the SM-1A brazed Type 1 and 2 fuel elements (Fig. 1) an initial ripple occurs in manufacturing because of the heating and cooling rates associated with the brazing cycle. This initial ripple is usually aggravated on the outer fuel plates due to their proximity to the furnace walls. During reactor operation these initial imperfections are then magnified because of the thermal stresses present under these conditions. This effect has been demonstrated in the laboratory as shown in Fig. 2 where the temperature differential $\Delta (T)$ indicates the difference in temperature between the side plate and active fuel region.

Need for a mathematical analysis of fuel plate rippling is dictated by the possible closure of the coolant channels resulting from the distortion of adjacent plates. In itself, rippling would only be serious if the stresses imposed were of sufficient magnitude to cause actual plate failure. However, a more likely and a more potentially serious situation could occur if the deflections induced during reactor operation obstructed the coolant flow through a given channel and lowered the index of thermal margin (DNBR) to the point where film boiling would occur. The high fuel plate temperature which would result in this case only tends to magnify the problem. This results in possible contact of adjacent fuel plates and subsequent failure of the fuel element at the point of contact causing melting and excessive release of fission products.

Fig. 1 - Fixed Element with End Box Removed

Figure 2 Fuel Plate Distortion Resulting From Thermal Stresses

In order to estimate the obstruction to coolant flow which may result during operation of the existing and replacement cores, it is necessary to accurately predict the magnitude of the rippling amplitude under actual core operating conditions. This requires, among other things, a knowledge of the initial plate geometry, the physical properties of the material, and an exact understanding of the behavior of the fuel plate (constrained by the side plate) under reactor conditions.

Some tests (see Appendix B) have been made in an effort to relate initial and final plate geometry under simulated operating conditions. However, as tests, these represent only specific points in a multi-parameter problem. The reliability of extrapolation to other situations depends on the ability to obtain similar results with a mathematical model which incorporates many of these parameters into a single expression relating initial and final conditions of a rippled fuel element.

A mathematical model is proposed which attempts to partially explain rippling phenomena. Considerations in this report are limited to elastic bending and small deflection theory.

The purpose of the mathematical analysis is two-fold:

1. To produce an expression which can be used to check existing experimental data.
2. To provide a reliable tool which can be used as the basis for further investigation into the thermal and hydraulic behavior of a rippled fuel element.

2.0 NOMENCLATURE

- a = length of fuel plate (in.)
- b = width of fuel plate (in.)
- t = thickness of fuel plate (in.)
- L = cross-sectional area of side plate region associated with a fuel plate (in.²)
- y = coordinate distance from one longitudinal edge (in.)
- z = longitudinal coordinate measured from bottom end of core (in.)
- E = modulus of elasticity (lb/in.²)
- α = coefficient of thermal expansion (°F⁻¹)
- ν = Poisson's ratio
- D = $Et^3 / 12(1 - \nu^2)$ = plate rigidity (lb-in.)
- T = temperature excess above bulk coolant temperature (°F)
- σ_1 = stress in the fuel plate (lb/in.²)
- σ_2 = stress in the side plate (lbs/in.²)
- P = $-t\sigma_1$ = stress resultant in fuel plate (lb/in.)
- Q = loading factor (lb/in.)
- Q_{cr} = critical loading factor (lb/in.)
- w = additional deflection of fuel plate (in.)
- w_0 = initial deflection of fuel plate (in.)
- A = amplitude of additional deflection (in.)
- B = amplitude of initial deflection (in.)
- n = number of half wave lengths

U = energy integral (lb-in.)

ϵ = longitudinal strain

d = channel spacing (in.)

3.0 RIPPLING ANALYSIS USING SMALL DEFLECTION THEORY

3.1 ASSUMPTIONS AND MATHEMATICAL RESULTS

The investigation of rippling in fuel plates is developed using classical plate theory. Thermal stresses are caused by temperature differences in the plate. Rippling phenomena will be examined only in terms of symmetric temperature variation across the width of the fuel plate. Plane strain considerations indicate that the hottest part of the plate will be in a state of compression.

Temperature variation through the thickness will not contribute to rippling if the distribution is symmetric with respect to the mid-plane of the plate. Temperature variation along the length of the plate is neglected; if this gradient is small, it should have no appreciable effect on rippling.

In this report, the analysis will incorporate small deflection theory. This theory is accurate only for deflections up to the order of the plate thickness. For plates with initial deflections, the theory permits calculating additional deflections as a function of temperature intensity. For an initially flat plate, the amplitude of deflection is indeterminate; only the temperature load which causes buckling can be calculated.

Figure 3. Cross-Section of Fuel Plate with its Associated Element of Side Plate

A set of fuel plates is fabricated into a box-like fuel element under a pair of side plates. Figure 1 shows a fuel assembly. Figure 3 shows a schematic of the cross-section of a fuel plate and the adjacent side plate region. It is assumed that the side plate region does not bend, has no resistance to twisting and extends elastically. The longitudinal component of strain is assumed constant in both the fuel plate and the side plate.

The plate is assumed to be an elastic, homogeneous, isotropic, medium. It is assumed to be simply supported on all four edges; with respect to the longitudinal edges. This assumption is conservative since the side plates provide some resistance to angular deflection of the plate. The short edges of the plate are free (except for a short region near the inlet comb); thus introducing a slight, optimistic error which is neglected, since the width of the plate is small compared to the length.

The thermal stress distribution is calculated by taking into account the transverse temperature distribution. Plane strain is assumed for the composite plate (i.e., the fuel plate and associated side plate region). It is assumed that the fuel plate and side plate have identical elastic properties. The temperature in the fuel plate is given by a known function, $T = T(y)$, symmetric about $y = b/2$, and in the side plate $T = 0$. The area of the side plate portion is L . The stresses in the fuel plate and side plate due to the thermal loading are given respectively by

$$\sigma_1 = E \alpha \left[\frac{t \int_0^b T dy}{b t + L} - T \right] \quad (1)$$

$$\sigma_2 = \frac{E \alpha t}{b t + L} \int_0^b T dy \quad (2)$$

Define the loading function as

$$P = - t \sigma_1 \quad (3)$$

and the loading factor

$$Q = (2/b) \int_0^b P \sin^2 (\pi y/b) dy \quad (4)$$

Define the critical loading factor as

$$Q_{cr} = \pi^2 D(a/m)^2 \left[(m/a)^2 + (1/b)^2 \right]^2 \quad (5)$$

Assume that the longitudinal mid-line of the plate deflects into a simple sine curve with n half-waves and that the deflection across the plate may be represented by a single half-wave. The deflection of the plate is then expressed by

$$w = A \sin (n \pi z/a) \sin (\pi y/b) \quad (6)$$

The temperature at which buckling occurs is found when

$$Q = Q_{cr} \quad (7)$$

where Q and Q_{cr} are given by equations (4) and (5). Note that the temperature is implicitly incorporated in Q as a consequence of (1) and (3).

If the plate has an initial deflection shape

$$w_o = B \sin (n \pi z/a) \sin (\pi y/b) \quad (8)$$

such that the deflection mode is the same as the buckling mode, then the total deflection of the plate is (as shown in Section 3.2).

$$\frac{A+B}{B} = \frac{1}{1-Q/Q_{cr}} \quad (9)$$

or

$$\frac{A}{B} = \frac{1}{Q_{cr}/Q - 1} \quad (10)$$

Table 1 shows a tabulation of $(A+B)/B$ versus Q/Q_{cr} from Equation (9).

TABLE 1
RIPPLING AMPLITUDES

<u>Q/Q_{cr}</u>	<u>$(A+B)/B$</u>	<u>Q/Q_{cr}</u>	<u>$(A+B)/B$</u>
0	1.000	0.55	2.222
0.05	1.053	0.60	2.500
0.10	1.111	0.65	2.857
0.15	1.176	0.70	3.333
0.20	1.250	0.75	4.000
0.25	1.333	0.80	5.000
0.30	1.429	0.85	6.667
0.35	1.538	0.90	10.000
0.40	1.667	0.95	20.000
0.45	1.818	1.00	∞
0.50	2.000		

Note: The temperature-amplitude relation can be found by noting that

$$T_o = T_{o\ cr} (Q/Q_{cr})$$

3.2 DERIVATION OF RESULTS FROM SMALL DEFLECTION THEORY

The deflection equation for a flat plate subjected to no load normal to the plane of the plate and to uniaxial in-plane stress is (1)

$$w = (-P/D) (w_{0,zz} + w_{,zz}) \quad (11)$$

where the comma followed by a literal subscript indicates the derivative, in this case, the second. The initial deflection, w_0 , and the compressive stress resultant $-P$ are assumed known. Equation (11) is identical with that for an initially flat plate with in-plane stress resultant, $-P$ and with loading normal to the plane of the plate $-Pw_{0,zz}$. The strain energy associated with the deflections of the plate can therefore be written in terms of the energy of the corresponding flat plate. (1)

$$U = (1/2) \int_0^a \int_0^b \left\{ -Pw_{,z}^2 + D \left[(\nabla^2 w)^2 - 2(1-\nu) (w_{,zz} w_{,yy} - w_{,zy}^2) \right] + 2wPw_{,zz} \right\} dz dy \quad (12)$$

Of all possible deflections that satisfy the boundary conditions, the one that minimizes this energy expression is the correct solution to Equation (11). The usual procedure for finding an approximate expression for the deflections is to assume a deflection shape in terms of one or more parameters and to minimize the energy in terms of these parameters.

In the present problem, we assume a simply supported plate subject to a thermal stress-resultant distribution

$$P = P(y) \quad (13)$$

The plate is assumed to have initial deflection

$$w_0 = B \sin(n\pi z/a) \sin(\pi y/b) \quad (14)$$

The additional deflection due to the thermal stress is assumed to have the same characteristic shape as the initial deflection shape

$$w = A \sin(n\pi z/a) \sin(\pi y/b) \quad (15)$$

where the parameter A is to be chosen to minimize the energy integral, Equation (12)

Substitution of Equations (13), (14), and (15) into (12) gives

$$U = \frac{ab}{8} \left(\frac{n\pi}{a} \right)^2 \left[-(A^2 + 2AB) Q + A^2 Q_{cr} \right] \quad (16)$$

where

$$Q = (2/b) \int_0^b P \sin^2 (\pi y/b) dy \quad (17)$$

and

$$Q_{cr} = \pi^2 D(a/n)^2 \left[(n/a)^2 + (1/b)^2 \right]^2 \quad (18)$$

Setting

$$\frac{du}{dA} = 0 \quad (19)$$

gives

$$A = BQ/(Q_{cr} - Q) \quad (20)$$

from which Equations (9) and (10) are immediately obtained.

The stress resultant P is found from thermal stress considerations. The stress-strain law for a plate subject to a temperature distribution which varies only in one direction may be written (2)

$$\epsilon = \sigma/E + \alpha T \quad (21)$$

Let the subscripts 1 and 2 represent the stress in the fuel plate and side plate, respectively. Then

$$\sigma_1 = E (\epsilon - \alpha T) \quad (22)$$

$$\sigma_2 = E \epsilon \quad (23)$$

The absence of a temperature term in Equation (23) comes from the assumption that the side plate temperature is zero. The total force acting on any cross section of the plate must vanish. This is expressed as:

$$L \sigma_2 + \int_0^b \sigma_1 dy = 0 \quad (24)$$

Substituting Equations (22) and (23) and noting that ϵ is a constant, we may write

$$E \left[\int_0^b (\epsilon - \alpha T) dy + L \epsilon = 0 \right] \quad (25)$$

$$\epsilon = \frac{\alpha t \int_0^b T dy}{L + bt} \quad (26)$$

$$P = - \sigma_1 t = \frac{E \alpha t}{L + bt} \left[(L + bt) T - t \int_0^b T dy \right] \quad (27)$$

Equations (20) and (27) provide the formulas necessary for small-deflection analysis.

4.0 APPLICATION OF MATHEMATICAL MODEL

Described in the previous sections are the necessary equations to mathematically predict the additional deflection of an initially distorted plate due to the effect of thermal stresses. These equations are summarized below and will be used to show how the characteristic deflection varies with temperature distribution.

4.1 SUMMARY OF EQUATIONS (Small deflection Theory)

$$\frac{A+B}{B} = \frac{1}{1 - Q/Q_{cr}} \quad (9)$$

$$Q_{cr} = \pi^2 D (a/n)^2 \left[(n/a)^2 + (1/b)^2 \right]^2 \quad (5)$$

where

$$D = \frac{E t^3}{12 (1 - \nu^2)}$$

$$Q = \frac{2}{b} \int_0^b P \sin^2 \left(\left(\frac{\pi y}{b} \right) \right) dy \quad (4)$$

where

$$P = -t \sigma_1 \quad (3)$$

and

$$\sigma_1 = E \alpha \left[\frac{t \int_0^b T dy}{bt + L} - T \right] \quad (1)$$

4.2 CRITICAL LOADING FACTOR

The critical loading factor, Q_{cr} , is mainly a function of plate properties and boundary conditions and will be identical for a given plate for all temperature distributions. Assuming that the ratio a/n is approximately equal to b (simply supported case), the equation for Q_{cr} becomes

$$Q_{cr} = 4 \pi^2 D/b^2 = \pi^2 E t^3 / \left[3b^2 (1 - \nu^2) \right] \quad (28)$$

Typical values for properties ⁽³⁾ and geometry ⁽⁴⁾ for Type 1 plates at about 400°F to 600°F are shown below:

$$a = 23 \pm .0156 \text{ in.}$$

$$b = 2.778 \pm .002 \text{ in.}$$

$$t = 0.030 \pm .001 \text{ in.}$$

$$L = 0.016 \text{ in.}^2$$

$$E = 25 \times 10^6 \text{ psi}$$

$$\alpha = 9.6 \times 10^{-6} \text{ in/in } ^\circ\text{F}$$

$$\nu = 0.3$$

the following values are obtained from the previous equations

$$D = \frac{25 \times 10^6 \text{ (lb/in}^2\text{)} (.029 \text{ in})^3}{12 (1 - 0.3^2)} = 55.84 \text{ in} - \text{lb} \quad (29)$$

$$Q_{cr} = \frac{(4) \pi^2 \frac{55.84 \text{ in} - \text{lb}}{(2.78 \text{ in})^2}}{2} = 285 \text{ lb} - \text{in} \quad (30)$$

4.3 TEMPERATURE INDUCED LOADS

Determination of the temperature induced load (Q) is made by solving the appropriate equations for a given temperature distribution. For example, consider the temperature distributions shown on the following figure:

Figure 4. Temperature Distributions Used in Comparison Study

The stress along the plate for the two cases is found from

$$\sigma_1 = E\alpha \left[\frac{t \int_0^b T dy}{tb + L} - T \right] \quad (1)$$

which becomes for the uniform case

$$\sigma_1 = -E\alpha T_0 \frac{L}{tb + L} \quad (31)$$

and for the sinusoidal distribution

$$\sigma_1 = -E\alpha T_0 \left[\frac{-2bt}{\pi(tb + L)} + \sin \frac{\pi y}{b} \right] \quad (32)$$

The resulting loading function (P) is

$$P = E\alpha T_0 \left[\frac{Lt}{tb + L} \right] \text{ for the uniform case} \quad (33)$$

and

$$P = E T_0 \alpha \left[t \sin \left(\frac{\pi y}{b} \right) - \frac{2bt^2}{(tb + L)} \right] \quad (34)$$

for the sinusoidal case.

Substitution of these values into the expression for Q

$$Q = 2/b \int_0^b P \sin^2 \left(\frac{\pi y}{b} \right) dy \quad (4)$$

and integrating with respect to y, we have for the uniform distribution

$$Q = 2/b \int_0^b \left[E\alpha T_0 \left(\frac{Lt}{tb + L} \right) \right] \sin^2 \left(\frac{\pi y}{b} \right) dy$$

$$Q = E\alpha T_0 \frac{Lt}{tb + L} \quad (35)$$

Likewise for the sinusoidal distribution, we have

$$Q = \frac{2}{b} \int_0^b E\alpha T_0 \left[t \sin \left(\frac{\pi y}{b} \right) - \frac{2bt^2}{tb + L} \right] \sin^2 \left(\frac{\pi y}{b} \right) dy$$

$$Q = \frac{2E}{\pi} \alpha T_0 \left[\frac{4t}{3} - \frac{bt^2}{(tb + L)} \right] \quad (36)$$

This can also be related to the average temperature ($\bar{T}$) by integration of the temperature function across the plate with the result

$$Q = E \alpha \bar{T} \left[\frac{4t}{3} - \frac{bt^2}{tb + L} \right] \quad (37)$$

Using the geometry and physical properties which were previously given, the value of Q for the two cases becomes

$$(\text{Uniform}) \quad Q = 1.152 T_o$$

$$(\text{Sinusoidal}) \quad Q = 2.215 T_o$$

$$\text{and} \quad Q = 3.478 \bar{T}$$

Since "Critical" buckling will occur when Q is equal to Q_{cr} the critical temperature can be found by equating these terms.

$$(\text{Uniform}) \quad Q = 1.152 T_o$$

$$Q_{cr} = 285 \text{ lb/in.}$$

$$T_{ocr} = 247.4^\circ \text{F}$$

$$(\text{Sinusoidal}) \quad Q = 2.215 T_o$$

$$T_{ocr} = 128.7^\circ \text{F or } \bar{T}_{cr} = 81.9^\circ \text{F}$$

The reason for the lower critical value for the sinusoidal temperature distribution can be seen by numerically evaluating the stress in the plate.

For the uniform temperature distribution

$$\sigma_1 = -E \alpha T_o \left[\frac{L}{tb + L} \right]$$

$$\sigma_1 = -39.75 T_o \quad (38)$$

which at the critical temperature (247.4°F) is

$$\sigma_{\text{Crit}} = -9,834 \text{ psi}$$

Likewise for the sinusoidal distribution

$$\sigma_1 = -E \alpha T_o \left[\sin \left(\frac{\pi y}{b} \right) - \frac{2bt}{(tb + L)} \right]$$

$$\sigma_1 = -240 T_o \left[\sin \frac{\pi y}{b} - .5316 \right] \quad (39)$$

the maximum compressive stress at the center of the plate is then

$$\sigma_{\max} = -112.4 T_0$$

and the critical stress at the critical temperature is

$$\sigma_{cr} = -14,468 \text{ psi}$$

The stress distributions for the two cases are shown on Fig. 5.

Although the "effective buckling span" (that portion of the plate in compression) is smaller for the sinusoidal temperature distribution, the higher compressive force in this region more than compensates for this difference in span and results in a lower buckling temperature than for the uniform case.

4.4 AMPLIFICATION FACTOR

The amplification factor on plate deflection, $(A + B)/B$, which is defined as the ratio of the total to the initial amplitude of the ripple, is shown as a function of peak temperature on Fig. 6. Using the value of 108°F ⁽⁵⁾ as the maximum temperature differential in the SM-1A Core I, the amplification factor can be determined from Equation (9).

$$\frac{A + B}{B} = \frac{1}{1 - Q/Q_{cr}}$$

which for the uniform distribution is

$$\frac{A + B}{B} = \frac{1}{1 - (1.152 T)/285} = 1.77 \quad (40)$$

and for the sinusoidal temperature distribution

$$\frac{A + B}{B} = \frac{1}{1 - (2.215 T/285)} = 6.23 \quad (41)$$

Since the actual SM-1A temperature distribution is somewhere between the two cases presented here, the value of 2.64 (see Appendix B) determined from experimental results is not unreasonable.

To apply this mathematical model to the exact form of the SM-1A temperature profile requires that the distribution be represented in a mathematical form.

Figure 5. Comparison of Stress Distributions Induced By Thermal Loading

Figure 6. Comparison of Deflection Ratios Caused By Various Temperature Distributions

Figure 7. Typical SM-1A Temperature Distribution

The loading function can then be expressed by

$$P = E \alpha t \left[T - \frac{tb A_0 T_0}{bt + L} \right] \quad (44)$$

which can be substituted in equation (4) to solve for Q

$$Q = \frac{2 E \alpha t}{b} \int_0^b \left[T - \frac{tb A_0 T_0}{bt + L} \right] \sin^2 \left(\frac{2 \pi y}{b} \right) dy \quad (45)$$

Substituting for T from Equation (42) and again integrating by parts

$$\text{for } i = 0) \int_0^b A_0 \sin^2 \left(\frac{2 \pi y}{b} \right) dy = A_0 b/2$$

$$i = 1) \int_0^b A \cos \left(\frac{2 \pi y}{b} \right) \sin^2 \left(\frac{2 \pi y}{b} \right) dy = A_1 b/2$$

$$i = 2) \int_0^b A_2 \cos \left(\frac{4 \pi y}{b} \right) \sin^2 \left(\frac{2 \pi y}{b} \right) dy = 0$$

$$\vdots$$

$$i = N) \int_0^b A_N \cos \left(\frac{2 \pi Ny}{b} \right) \sin^2 \left(\frac{2 \pi y}{b} \right) dy = 0$$

and the last term becomes

$$\int_0^b \frac{tb A_0 T_0}{tb + L} \sin^2 \left(\frac{2 \pi y}{b} \right) dy = \frac{T_0 A_0}{2} \left(\frac{tb^2}{bt + L} \right)$$

Because of the orthogonality of the function the integrals are equal to zero for values of (i) greater than one so that only those terms containing A_0 and A_1 are of significance in the evaluation of the integral.

Substituting the individual factors into the expression for Q, we have

$$Q = \frac{2 E \alpha t T_0}{b} \left[A_0 b/2 - A_1 b/2 - \frac{A_0}{2} \left(\frac{tb^2}{bt + L} \right) \right]$$

or

$$Q = E \alpha t T_0 \left[A_0 \left(1 - \frac{tb}{bt + L} \right) - A_1 \right] \quad (46)$$

An IBM-650 digital code (6) was programmed to approximate temperature distribution with a 10th order cosine fit. Results of the calculation for the SM-1A profile indicated that the slope changes associated with this distribution were too sharp even for a 10th order equation and may require several more terms in order to get the exact fit. However, since only the coefficients A_0 and A_1 are really significant in the calculation of the thermal load, it was felt that the irregularities in the equation would have little effect on the deflection characteristics of the plate. Coefficients obtained by this method were

$$A_0 = .6915$$

$$A_1 = -.1346$$

$$A_2 = 1.2044$$

$$A_3 = -5.6302$$

$$A_4 = 7.9580$$

$$A_5 = -4.7530$$

$$A_6 = -3.2615$$

$$A_7 = 8.0067$$

$$A_8 = -6.2153$$

$$A_9 = 2.1341$$

Using the values of A_0 and A_1 , and the dimensions previously given, the thermal load is found to be

$$Q = 1.733 T_0 \text{ lb/in } ^\circ\text{F} \quad (47)$$

As before, the critical value of T_1 can be found by equating Q and Q_{cr} so that

$$\text{Critical } T_o = \frac{285}{1.733} = 164.5^\circ\text{F} \quad (48)$$

As expected, this value falls between the critical temperatures calculated for the uniform and sinusoidal distribution.

The amplification factor at T_o equal to 108°F is

$$\frac{A+B}{B} = \frac{1}{1 - \frac{(1.733)(108)}{285}} = 2.91 \quad (49)$$

and is therefore very close to the 2.64 value obtained from experimental results (Appendix B).

At a temperature difference of 80°F , the amplification factor is

$$\frac{A+B}{B} = \frac{1}{1 - \frac{(80)(1.733)}{285}} = 1.95 \quad (50)$$

which is again close to the 2.22 value calculated from experimental results.

Figure 8 is a comparison of the amplification factor found from experiments and the value obtained using the analytic method described here for the SM-1A temperature distribution. Around the point of interest (80 - 100°F) the two curves agree within $\pm 10\%$. Due to the asymptotic behavior of the small deflection theory, the analytic results become very conservative as the temperature difference approaches the critical value (164.5°F). At lower temperatures ($< 75^\circ\text{F}$), the experimental data appear to be more conservative.

It may, therefore, be assumed that the analytical solution is reasonably valid at SM-1A operating conditions and can be used with a degree of confidence in this range.

Figure 8. Comparison of Analytic and Experimental Ripple Amplification Factor

5.0 FUTURE WORK

5.1 CHARACTERISTICS OF LARGE DEFLECTION THEORY

Greater accuracy for calculating rippling amplitudes is possible using large deflection theory. As pointed out earlier, small deflection theory is accurate only for amplitudes of the order of magnitude of the plate thickness. As the loads approach buckling loads, the error of small deflection theory becomes progressively worse. The main difficulty is that small deflection theory ignores the stretching of the middle plane of the plate.

Large deflection theory accounts for the finite changes of dimension and for the restraint of the stresses associated with stretching of the plate. It is necessary in large deflection theory to examine a pair of coupled non-linear differential equations and associated boundary conditions.

The application of large deflection theory of plates to the problem of plate rippling requires consideration of temperature distribution. Initial flat plates are treated by the Von Karman equations. For plates with initial curvature the Marguerre equations are used. Coan⁽⁷⁾ and Yamaki⁽⁸⁾ have examined the Marguerre equations in some detail for edge loading. Boley and Weiner⁽²⁾ have investigated simple temperature distributions in flat plates. A combination of the temperature effects and initial curvature effect is needed for the rippling problem. The Marguerre equations then will be

$$\nabla^4 F = -\alpha E \nabla^2 T + E \left[w_{,zy}^2 - w_{,zz} w_{,yy} + 2w_{o,zy} w_{o,zy} - w_{o,zz} w_{o,yy} - w_{o,yy} w_{o,yy} \right]_{,zz}$$

and

$$\nabla^4 w = (t/D) \left[F_{,yy}(w+w_o)_{,zz} + F_{,zz}(w+w_o)_{,yy} - 2F_{,zy}(w+w_o)_{,zy} \right]$$

F is the Airy stress function, related to the stresses by

$$\sigma_{zz} = F_{,yy}$$

$$\sigma_{yy} = F_{,zz}$$

$$\sigma_{yz} = -F_{,yz}$$

Suitable boundary conditions must be chosen to fit a given problem. In addition to the conditions on w, which are identical to those considered in small deflection analysis, stresses or displacements in the plane of the plate must be prescribed.

Several approximate techniques are available for solutions of these equations. The two most important of these are successive approximations and matching coefficients. In the first, the equations are linearized by a suitable approximation of w . The first equation is solved and the approximate value of F and W are substituted into the second equation which is then solved. This solution, according to Boley and Weiner is quite accurate. In the second method, F and w are represented by trigonometric series whose coefficients are found to satisfy the equations and boundary conditions.

The detailed solution of the Marguerre equations is tedious in either case but is required for an accurate solution if deflections or thermal loads exceed the limitations of small deflection theory. Since it appears (Fig. 8) that the present tests are approaching the limit of small deflection theory, more advanced testing ($T > 125^{\circ}\text{F}$) will require the large deflection analysis.

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APPENDIX B

REVIEW OF EXISTING DATA AND APPLICATION TO THERMAL ANALYSIS

B.1 GENERAL CONSIDERATIONS

In the selection of the proper parameters to be used in the thermal evaluation of the rippling problem, consideration must be given to all forms of available information. This includes the mathematical analysis presented in this report plus existing experimental results, manufacturing and post-irradiation measurements of fuel elements, and stainless steel-UO₂ property data. Compilation of this information into a form suitable for use in a revised steady state and transient analysis of SM-1A (Core I and II) requires that a degree of conservatism be maintained to account for the inherent uncertainties associated with such small quantities of data.

B.1.1 Small Deflection Theory

As previously explained in the report, the use of small deflection theory to mathematically predict the resulting channel closure is in itself a conservative approach since it would be physically impossible to obtain deflections which are much larger than the fuel plate thickness without having some appreciable changes occurring in the plate. If this theory is used to determine the amount of additional deformation resulting from thermal heating of an initially curved plate, the predicted results will, in general, be conservative as the magnitude of the deflection increases.

B.1.2 Fuel Plate Support

Likewise, the assumption that the plate is simply supported rather than fixed at the sidewalls, would be conservative if in fact significant restraint is imposed by the sidewalls. This can be seen by determining the average value of the deflection associated with the above methods of support.

1. Simply supported

$$w = (A + B) \sin(\pi y/b)$$
$$w_{avg} = 1/b \int_0^b (A + B) \sin(\pi y/b) dy = (2/\pi) (A + B)$$

2. Fixed edge

$$w = (A + B) \left[\frac{1}{2} - \frac{1}{2} \cos(2\pi y/b) \right]$$
$$w_{avg} = \frac{(1/b) \int_0^b (A + B) \left[\frac{1}{2} - \frac{1}{2} \cos(2\pi y/b) \right] dy}{b}$$
$$w_{avg} = (1/2) (A + B)$$

Since the value $2/\pi$ is greater than $1/2$, the use of a simply supported

model results in a smaller average coolant channel for the thermal analysis.

B.1.3 Mechanical Properties

The use of pre-irradiated mechanical properties also tends to be conservative since the stainless steel- UO_2 matrix becomes more brittle, and hence less susceptible to bending, with increasing irradiation⁽²⁾.

If more ductile properties are assumed to exist throughout core lifetime, the calculated deflections will be higher and thus the effect on core thermal performance greater.

B.1.4 Adjacent Fuel Plate Rippling

Another assumption which must be made for a conservative estimate of the effects of rippling on core thermal performance is in regard to the deflection or rippling of adjacent fuel plates. It has previously been shown⁽³⁾ that for internal channels the worst condition for pressure drop and flow consideration occurs when two adjacent plates are 180° out of phase. This is also true of a lattice channel between two stationary elements where the outer plates of two adjacent fuel elements are parallel. Since rippling is usually more severe on outer plates this is potentially the worst location from a thermal standpoint. Figure B.1 illustrates the possible conditions which could exist for the internal channels and external lattice passages.

For the outer channels of a control rod element, the minimum clearance is independent of the phase of an adjacent "fuel plate" since this channel is bounded on one side by the square control rod "tube". It is, however, dependent on the tolerance of the control rod tube.

If the previously described items are all used conservatively in determining the effect of rippling on the thermal performance of the core, the results should be very conservative since it is highly improbable that all would simultaneously occur in an adverse manner.

B.2 INITIAL DISTORTION

Since the final channel spacing for a rippled fuel element is the initial spacing minus the final (hot) deflection of the adjacent fuel plates, and the final deflection is directly proportional to plate temperature and initial deflection, it is extremely important to establish the magnitude of the initial rippling as well as the initial spacing for the limiting lattice passage.

B.2.1 Manufacturing Tolerances

Channel tolerance specifications for SM-1A fuel elements generally limit deviations to $\pm 10\%$ of the nominal dimension^{(4), (5), (6), (7)} (i.e. $0.133 \text{ in.} \pm .013 \text{ in.}$). Waivers⁽⁴⁾ have been given for Core II (SM-1A) in which deviations

Ripples 180° Out of Phase

Aligned Ripple: Amplitude & Phase

Figure B.1. Schematic Representation of Internal Channels and Lattice Channel Longitudinal View

of ± 0.020 in. were allowed provided no one channel had more than a 0.020 in. variation (this in effect would reduce the minimum channel clearance but would also only permit ± 0.010 in. rippling amplitudes).

Calculations ⁽⁸⁾ have also been made on the minimum obtainable external channels and indicate that it is possible to have minimum clearances of 0.094 in. However, such dimensions occur in channels bordered by one active fuel plate and a structural member. The increased channel closure resulting during "hot" operation would not be as severe as a case in which the channel is composed of two active fuel plates. This case would occur when two stationary elements are aligned so that the fuel plates in both elements are parallel (lattice passages). The minimum dimension for this situation has been shown ⁽⁸⁾ to be 0.118 in. Since the maximum dimension could by the same analysis be 0.148 in., the peak to peak amplitude of the initial ripples, assuming symmetry, is 0.015 in.

This value is in fair agreement with the results of ORNL measurements ⁽⁹⁾ taken at Fort Greely and the SM-1 measurements taken by the Army at Fort Belvoir ⁽¹⁰⁾. Approximately 65% of the readings taken on 96 SM-1A ⁽⁹⁾ fuel elements (top and bottom) had less than a 0.015 in. peak to peak variation while another 20% fell between 0.015 in. and 0.025 in. Based on these results, 13 fuel elements were recommended for specific core placement ⁽⁸⁾ in low power regions (See Table B.1) while the remaining elements were released for unrestricted core placement. Thus it can be seen that the maximum peak-to-peak initial ripple amplitude as defined in Section 1.0, can be considered to be between 0.020 in. and 0.025 in.

TABLE B.1

SPECIFIC PLACEMENT LOCATION OF SM-1A CORE 1 ELEMENTS ⁽⁸⁾

<u>Element Number</u>	<u>Maximum Ripple Amplitude, in.</u>	<u>Placement Location</u>
3S	.026	72
15S	.017	65
16S	.022	61
17S	.036	67
18S	.017	22
21S	.021	34
23S	.023	12
24S	.015	21
33S	.020	63
38S	.018	56
43S	.023	45
50S	.024	27
52S	.019	41

Average .0216

B.2.2 Number of Ripples Per Plate

The number of complete cycles which occur, that is, the number of complete ripples per plate, varies from one to six. An examination of the data taken at Fort Greely⁽⁹⁾ shows that an average of from four to five ripples will occur. Figures B.2 and B.3 show typical plots of the data taken.

As discussed within the report, the number of half sine waves which will occur in a very long, simply supported plate is approximately given by the expression

$$n = a/b$$

Using the dimensions of the SM-1A fuel element gives

$$n = 23/2.778 = 8.3$$

The number of complete waves is then approximately four, in good agreement with the observed variations.

Similar measurements are planned for SM-1A Core II during the final inspection of the core.

B.2.3 End-of-life Initial Distortion

At the end-of-life an additional factor must be considered in establishing the deflection amplitudes at this point in core lifetime. This is an increase in the "initial" distortion resulting from core operation. Although not a true initial ripple, it would represent a pseudo point from which ripple growth would occur if the reactor were shut down and then brought back to full power near the end of core life.

Although some evidence exists,⁽¹¹⁾ that this increased distortion results from an increase in the length of the fuel plate caused by fuel growth, there is also the possibility that the increase in ripple amplitude is a result of inelastic deformations resulting during core operation such as plastic flow and hardening of the matrix due to irradiation damage.

Measurements made at Oak Ridge⁽¹²⁾ on two fuel elements showed a decrease in the minimum channel dimension from 130 mils to 120 mils in the control rod element (CR-13), and from 130 mils to 124 mils on the stationary element (S-72). These measurements were made after 10.5 MWYR of operation or approximately two-thirds of design burnup.

Similar measurements⁽¹³⁾ were made on a stationary element (S-79) after 16.4 MWYR of operation. Measurements in this element indicated 42 and 57 U-235 atom percent depletion for average and maximum values of burnup. Although no pre-irradiation rippling measurements are available it can be assumed that they fell

Figure B.2. Initial Rippling of Element 17 - SM-1A

Figure B.3. Initial Rippling of Element 23 SM-1A

within the fabrication tolerances. Post-irradiation measurements indicated a maximum deviation of 0.035 in.; however, the maximum peak-to-peak amplitude of rippling was 0.040 in. to 0.042 in. This represents an increase of about 0.015 in.-0.017 in. over the 0.025 in. initial values mentioned in the previous section. It is not believed that SM-1 Core I was as bad as the SM-1A Core I.

It should be noted that while this represents a significant increase in the initial amplitude of the ripples, the maximum temperature differential at this time in life is less than at the beginning-of-life. Table B.2 shows the comparative values which distinguish these two design points.

TABLE B.2
COMPARISON OF RIPPLING PARAMETERS AT BEGINNING AND END OF
CORE LIFETIME

	<u>Point in Core Life</u>	
	<u>Beginning</u>	<u>End</u>
Initial Ripple Amplitude* (max.)	0.025 in.	0.040 in.
Initial Ripple Amplitude* (avg.)	0.015 in.	0.025 in.
Max. Nuclear Peaking Factor ** (17)	2.44	1.81
Max. Temperature Difference (T), °F	108 ⁽¹⁵⁾	80 ^{***}
Ripple Amplification Factor		
Experimental	2.64	2.22
Analytical	2.91	1.95
Estimated Hot Channel Lattice Spacing (min.)	0.045 in.	0.029 in.

-
- * Peak-to-Peak
 - ** Product of Axial and Radial Factors
 - *** Based on reduced peaks

B.2.4 Ripple Amplification Factor

The amount of additional rippling caused by heating is proportional to the initial ripple and the temperature differential between the heated and unheated portion of the fuel element. If an amplification factor is defined as the ratio of the final deflection to the initial deflection $(A + B)/B$, it becomes useful in describing the rippling problem.

Test results⁽¹⁴⁾ made on welded 0.040 in. fuel plates showed that the distortion from thermal testing resulted in a amplification factor of approximately two for a temperature differential of between 100 to 118°F. However, this value is considered unconservative since the plate thickness was 40 mils rather than 30 mils.

Recent tests⁽¹⁵⁾ made a reference brazed fuel element indicate that, for temperature differences in this range (100 - 120°F), the deflection increased at a rate of 1.52×10^{-2} in. per inch of initial deflection per °F temperature differential. The amplification factor for this case is thus

$$(A + B)/B = 1 + (1.52 \times 10^{-2} T)$$

where T is the temperature differential between the sidewall and the fuel plate. This also is in fair agreement with previous results on a solid 0.030 in. stainless plate.⁽¹⁶⁾

From the mathematical model (Section 4.5) the amplification factor was found to be

$$A + B/B = \frac{1}{1 - (1.733 T/285)}$$

The difference between the two methods is shown on Table B.1 along with the minimum estimated hot channel spacing determined by the following method:

$$D_{\min} = D_{\text{fab}} - 2 (A + B)$$

where D_{fab} = minimum fabricated dimension (0.118 in.)

A = additional deflection

B = initial deflection

Thus, for the two cases shown, the minimum channel spacing is 0.045 in. and 0.029 in. for the beginning and end of life respectively. The minimum value of 0.029 in. is still above the 0.018 in. "minimum allowable" value selected in APAE Memo-300.⁽⁴⁾

Because of the non-uniformity of the ripple amplitude, it is doubtful that all of the ripples in any one channel would result in minimum channel closures of this magnitude. A more realistic interpretation of the data would be to assume that all but one of the ripples were of average size and that one maximum ripple occurs at the most critical thermal point in the channel.

B.3 APPLICATION TO THERMAL ANALYSIS

B.3.1 Description of the Rippled Coolant Channel

Although some rippling measurements are available, (10), (12), (13), (18) they are in general only indications of what the dimensions of a particular coolant channel might be. The orientation of the adjacent element and the end-of-life condition are important factors which must also be considered in the explicit channel description at a given location.

For the beginning-of-life analysis, such information can be obtained, (8) provided allowances are made for alignment tolerances. However, at the end-of-life, the actual condition existing can only be estimated and, without the actual channel measurements, it must be assumed that the worst combination of all the adverse items previously mentioned will occur at the thermal "hot spot" in the core.

If the following assumptions are made

1. Average number of ripples per channel is four
2. Maximum ripple amplitude (hot peak to peak) is 0.073 in. at the beginning of life and 0.089 in. at the end of life
3. Average ripple amplitude (hot peak to peak) is 0.044 in. at the beginning of life and 0.056 in. at the end of life
4. Ripples on adjacent fuel elements are 180° out of phase

than a schematic representation of the "hot" rippled channel can be constructed (Fig. B.4) with the maximum ripple occurring at the location of maximum flux.

B.3.2 Thermal Analysis of the Rippled Channel

The greater magnitude of rippling at the end-of-life will probably be more than offset by the smaller nuclear factors existing. It does not appear that this condition will be more limiting than the beginning-of-life. However, to ascertain whether or not this supposition is correct, both cases should ultimately be analyzed.

A transient two-dimensional, hydrodynamic model (19) has been programmed for use on the Philco 2000 digital computer. Although the version of the code which is currently available does not contain an explicit description of variable channel spacing, it is possible to approximate the condition of a ripple channel by appropriate choice of input parameters which affect the flow redistribution, enthalpy rise and departure from nucleate boiling ratio (DNBR) calculations. This can be accomplished by defining correction factor relating the local thickness to the nominal channel thickness d_{loc}/d_{nom} . This correction factor is

Symbol	Value	
	Beginnng	End
n	8	8
d _{min-loc}	0.045	0.029
d _{min-avg}	0.074	0.063
d _{max-avg}	0.162	0.173
Length	23"	23"

Figure B.4. Schematic of Rippled Coolant Channel Under Operating Conditions

then applied to the local friction factor, local heat flux and local DNB correction factor so that the results will approximate the analysis of a rippled channel in which the geometry was described explicitly.

Using this procedure, steady state and transient analysis of the SM-1A may be made to determine the effect of rippling on core thermal performance.

B.4 SUMMARY

Sufficient experimental and manufacturing data exists and can be coupled with the mathematical solution presented in the body of this report to predict conservatively the rippled channel geometry which can be expected at beginning and end of core lifetime. A thermal analysis can then be performed using the TITE⁽¹⁹⁾ two-dimensional digital model to determine the effect of the rippling on core performance.

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