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MEASUREMENT AND ANALYSIS OF FLOW WALL SHEAR
STRESS IN AN INTERIOR SUBCHANNEL OF
TRIANGULAR ARRAY RODS

by

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ABSTRACT

A simulated model of triangular array rods with pitch to diameter ratio of 1.10 (as a test section) and air as the fluid flow was used to study the LMFBR hydraulic parameters. The wall shear stress distribution around the rod periphery, friction factors, static pressure distributions and turbulence intensity corresponding to various Reynolds numbers ranging from 4140 to 36170 in the central subchannel were measured. Various approaches for measurement of wall shear stress were compared. The measurement was performed using the Preston tube technique with the probe outside diameter equal to 0.014".

Our results showed that the nondimensional wall shear stress distribution is a monotonical function of the azimuthal angle, and its maximum value occurs at the largest flow area ($\theta = 30$ degrees). The friction factors calculated from the wall shear stress distribution results are less than the friction factors calculated from the pressure drop data measured at the rod gap. The former friction factor results agree with the results of pipe flow multiplied by a coefficient or the results of a calculation on an equivalent annular zone. The static pressure around the rod periphery is a function of the azimuthal angle, and decreases as the point moves away from the rod gap. The distribution becomes steeper as the Reynolds number increases. There were no detectable effects of secondary flow in the region very close to the rod surface.

A comparison of the results of this work and previous experimental and analytical investigations is presented.

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NOMENCLATURE

- A = channel flow area
- A_s = subchannel area; ($0 \leq \theta \leq 30$)
- a = profile distance between wetted wall and point of maximum velocity
- B = absolute flow static pressure at measurement plan
- c = constant in surface fence measurement
- C_p = specific heat at constant pressure
- C_s = constant in "law of the wall"
- C_T = constant in "law of the wall"
- C_v = specific heat at constant volume
- C_w = constant in the "law of the wall"
- D = rod diameter
- D = diameter of static tap hole connection
- $\bar{D}$ = diffusion coefficient
- d = outside diameter of Preston tube
- d = diameter of pressure tap hole
- D_n = hydraulic diameter of subchannel
- D_p = pipe inside diameter
- D_1 = pipe inside diameter
- D_2 = orifice plate hole diameter
- d_{in} = inside diameter of Preston tube
- e = absolute pressure of air moisture
- e = eccentric distance of Preston tube
- e = natural logarithmic base
- e = one half of rod gap distance ($\approx g/2$)

- f = subchannel friction factor resulting from pressure drop data
- f = symbol for function
- f_o = pipe flow friction factor
- f_s = subchannel friction factor calculated from wall shear stress distribution
- G = specific gravity of gas
- g = gap distance between rods ($\theta=0$)
- g_c = standard gravity acceleration
- h = surface fence height
- K = flow coefficient (orifice plate)
- K = radius of r_m to the local radius of curvature of the wetted perimeter
- K = constant in surface fence measurement
- $\bar{K}$ = average mass transfer over the electrode surface
- k = ratio of specific heat at constant pressure to specific heat at constant volume
- K_e = defined geometrical parameters in a subchannel
- $K(x^*)$ = defined coefficient in calculation of wall shear stress measurement uncertainty
- L = distance between two tap holes along rod axis
- L = distance between measurement plane and test section entrance
- $\bar{L}$ = electrode length
- l = coordinate along rod periphery
- l = length of tap hole

- M = Mach number
 $\hat{n}$ = unit vector perpendicular to the surface fence
 P = pitch of rod array
 p = time averaged flow pressure
 p_1 = static pressure before orifice
 $p(R)$ = reservoir pressure ($\equiv p_1$)
 p_w = subchannel wetted perimeter
 p_s = static pressure at the channel wall
 p_{xA} = defined pressure drop in an element of flow cross section (ΔA)
 R = arbitrary final measured results
 Re_d = Preston tube Reynolds number ($\rho \bar{V} d / 2\mu$)
 Re = Reynolds number
 R_c = general radius of curvature of wetted surface
 $RMS(x)$ = true root mean square of the variable x
 r = radial coordinate measured from center of curvature of wetted wall
 r_m = radial coordinate of the point which is located on the maximum velocity profile
 r_1, r_2 = radius of internal and external tube of an annular channel
 r_o = radius of pipe
 r_o^+ = dimensionless radius of pipe
 $\bar{S}$ = average velocity gradient
 T = temperature
 T = integration time

τ	= constant in "law of the wall"
τ_o	= defined wall shear stress in an element of flow cross section (ΔA)
T_1	= flow temperature before the orifice plate
u, v, w	= time averaged velocity in the direction a) Cartesian coordinates: x, y , or z b) cylindrical coordinates: x, r or θ
u', v', w'	= fluctuation velocity in the direction a) Cartesian coordinates: x, y , or z b) cylindrical coordinates: x, r or θ
$\bar{u}', \bar{v}', \bar{w}', \bar{V}'$	= time averaged values of u', v', w', V' respectively
u^+	= dimensionless velocity [$\equiv u/(\tau_w/\rho)^{1/2}$]
u^*	= friction velocity [$\equiv (\tau_w/\rho)^{1/2}$]
u_n	= component of flow velocity perpendicular to surface fence
u_t	= flow resultant velocity
u_θ	= secondary flow velocity
$\hat{u}_t$	= unit vector in the direction of resultant flow
$\bar{U}$	= averaged flow velocity over the channel flow area
$\bar{V}$	= averaged flow velocity over the channel flow area
$\bar{V}'_\theta$	= velocity of large scale motion of eddies in the θ direction
w	= total mass flow rate

- w_s = subchannel mass flow rate
- x = coordinate along rod axis
- x = parameter in equivalent annular zone friction pressure drop calculation
- x_i = arbitrary variable
- x^* = dimensionless parameter of Preston tube
- y = distance from rod wall to point of interest in radial direction or Cartesian coordinate in plane of cross-section
- y^+ = dimensionless distance from wall $[= y/v(\tau_w/\rho)^{1/2}]$
- $\hat{y}$ = distance along normal to rod surface, from wall to isotach line
- $\hat{y}_o$ = maximum value of y in a subchannel
- y_d^+ = dimensionless characteristic of Preston tube

$$= \frac{d(\tau_w/\rho)^{1/2}}{2v}$$
- y^* = Preston tube Reynolds number
- y = flow expansion factor
- Y = supercompressibility factor
- z = coordinate perpendicular to x and y
- α = angle between the main flow direction and the normal to surface fence
- β = angle between the normal to the surface fence and the main flow direction
- β = constant in "law of the wall"
- γ = angle between the resultant flow direction and the main flow

- δ = width of flow element corresponding to tap hole diameter
- Δp_{acc} = acceleration pressure drop
- Δp_u = orifice plate pressure drop
- Δp_x = static pressure drop along coordinate x
- Δp_d = time averaged of value of Δp_{di}
- Δp_{di} = difference between the Preston tube reading and the local flow static pressure
- Δx = element of variable x
- Δx = uncertainty included in variable x
- ϵ_{mr} = eddy diffusivity of momentum in r direction
- $\epsilon_{m\theta}$ = eddy diffusivity of momentum in θ direction
- θ = azimuthal angular coordinate
- κ = constant in "law of the wall"
- μ = viscosity
- ν = kinematic viscosity
- ρ = density
- $\Sigma \Delta \tau_w$ = total uncertainty included in wall shear stress measurement
- σ = standard deviation
- τ_d = local wall shear stress measured in an arbitrary cross section channel using a Preston tube whose outside diameter is equal to d
- $\tau_{d,K=0}$ = local wall shear stress measured in round tube using a Preston tube whose outside diameter is equal to d

- τ_w = local wall shear stress
- $\overline{\tau_w}$ = average value of wall shear stress distribution in a subchannel .
- τ_{wf} = local wall shear stress resulting from pressure drop data
- $\tau_{\theta x}$ = flow shear stress acting along x direction and located in a plane perpendicular to direction
- φ = angle between the Preston tube axes and rod axes
- ϕ_s = defined geometrical parameter in a subchannel cross-section
- ψ = stream function
- ω = specific humidity of air
- ω_R = uncertainty interval of result R

CHAPTER 1

INTRODUCTION

1. Foreword

Nuclear power reactors are based on the production of kinetic energy during fission interactions in fuel rods. Each rod consists of fissile material clad within a suitable metal. The next process is to remove this generated energy and convert it to available work or energy. The rate of heat release and consequent power generation in a given reactor core is limited by thermal rather than by nuclear considerations. Most of the generated energy is removed due to conduction heat transfer (a few percent by radiation) within the fuel pin and then by convection heat transfer between clad and coolant which is passing through the spaces between the rods. In some reactors the surface temperature of the fuel clad limits the reactor power. The clad temperature can be calculated and controlled provided that the heat transfer coefficient is known. The most frequently used materials for moderators and coolants in nuclear power reactors are listed in Table 1.1.

1.2 The Wall Shear Stress Importance

In the case of the liquid fast breeder reactor, with which this work is mostly concerned, it is common to use liquid sodium as coolant with a triangular array of rods. It is obvious that in order to have better heat transfer,

we should have turbulent flow. At the present time, in order to find heat transfer coefficients, it is necessary to know the velocity distribution in the core or in individual channels. If we know the wall shear stress distribution around the channel periphery, by assuming a universal velocity law ($u^+ = f(y^+)$) we can find the flow velocity distribution through the channel.

The results of the wall shear stress also enable us to find the coefficients of turbulent heat transfer and mass transfer depending on the model that we choose. For example, following the model of reference (18), we can find the eddy diffusivities by equations (1.1) and (1.2).

$$\epsilon_{m,\theta} = 0.154 y^4 \sqrt{\frac{\tau_w}{\rho}} \quad (1.1)$$

$$\epsilon_{m,r} = c_r y^4 \sqrt{\frac{\tau_w}{\rho}} \quad (1.2)$$

However, no matter what kind of model for turbulent transfer we choose, the results of wall shear stress distribution in most cases can be used to check the accuracy of the model and calculations. The wall shear stress distribution is also required to find the pressure gradient $\frac{\partial p}{\partial x}$, which is necessary to solve the axial momentum equation (1.3).

$$\frac{1}{r} \frac{\partial}{\partial r} r u \left(\frac{\partial \psi}{r \partial \theta} \right) - \frac{\partial}{r \partial \theta} \bar{u} \left(\frac{\partial \psi}{\partial r} \right) - \frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{\partial u}{\partial r} - \frac{\overline{v' u'}}{v} \right) = - \frac{\partial}{r \partial \theta} \left(\frac{\partial u}{r \partial \theta} - \frac{\overline{u' w'}}{v} \right) = - \frac{1}{\rho v} \frac{\partial p}{\partial x} \quad (1.3)$$

where:

$$w = -v \frac{\partial \psi}{\partial r} \quad (1.4.a)$$

$$v = v \frac{\partial \psi}{r \partial \theta} \quad (1.4.b)$$

and ψ is the stream function.

Up to this time there is not any reliable analytical or semi-analytical formula to calculate the wall shear stress distribution around the periphery of a non-uniform cross section, especially for compacted rod bundles (see Chapter 5). Therefore it is necessary to have more accurate experimental results. Also, the experimental results check the accuracy of the analytical models and help to formulate more accurate and reliable methods to predict the wall shear stress distribution.

<u>Reactor Type</u>	<u>Moderator</u>	<u>Coolant</u>	<u>Cladding</u>
PWR	Light Water	Light Water	Zircaloy
BWR	Light Water	Light Water	Zircaloy
CANDU	Heavy Water	Heavy Water	Zircaloy
HTGR	Helium Gas	Helium Gas	Graphite
LMFBR	None	Liquid Metal	Stainless Steel

TABLE 1.1
Clad Material of Different Types of Reactors

CHAPTER 2

BACKGROUND INFORMATION

2.1 Introduction

The importance of the wall region in the generation and maintenance of turbulence is well known. Since the kinetic energy of turbulence is continuously dissipated to internal thermal energy through viscous effects, a continuous supply of new turbulence must be created within the flow if the quasi-steady-state character of the turbulent flow is to be maintained. Up until this time it has been assumed that the source of energy is the mean flow, but this assumption has not been confirmed because of the mechanics of turbulent transfer is not yet known (13).

Based on the understanding of the nature of the wall region it will be possible to formulate a mechanism which will allow prediction of the heat and mass transfer rates without the need of adjustable constants from heat and mass transfer data, considering only the turbulence parameters themselves.

2.2 Universal Velocity Law

In the region close to the wall in a fully turbulent flow it is possible to write (see Fig. (2.1)):

$$u = f(y, \tau_w, \nu, \rho, \text{roughness}) \quad . \quad (2.1)$$

In the case of a smooth wall we can neglect the roughness effect. We then apply dimensional analysis and write

equation (2.1) in the form of two dimensional groups as follows:

$$\frac{u}{\sqrt{g_c \tau_w / \rho}} = f \left(\frac{y}{v} \sqrt{g_c \tau_w / \rho} \right) \quad (2.2)$$

or eventually:

$$u^+ = f(y^+) \quad (2.3)$$

Equation (2.3) is called the universal velocity law. Regardless of whether the flow is an internal or external flow, but specially for incompressible flow, this relationship between velocity and the wall shear stress has been experimentally confirmed by many investigators (e.g. 14,15) for fully turbulent flow in a channel in which the radius of curvature around the periphery is uniform, i.e. a geometry in which flow conditions do not vary around the periphery.

2.3 Behavior of the Selected Flow

The selected flow in this work is air and as we know it is compressible. However, if in turbulent flow the fluctuating Mach number, $u / (\text{speed of sound})$, is small or the pressure fluctuations compared to the absolute pressure in the medium are small, then the general behavior of turbulent flow will be unaffected by the compressibility properties of flow. Since the velocity fluctuations are usually

a small percentage of the mean velocity, therefore the above conditions exists if the mean stream Mach number is $< \frac{1}{5}$ (3, p.17,18). Based on the same reasons, if the change in absolute temperature across the flow is less than a factor of 5 or 6 then density fluctuations resulting from temperature differences through the compressible flow can be ignored. Of course, mean density changes affect quantitative behaviors of flow because of their effect on mean flow, but dimensionless turbulence quantities (e.g. typical eddy wavelength to the distance from the wall) will be very nearly independent of the compressibility property. For the conditions of this work all of the formerly mentioned conditions exist. Therefore from now on, we can consider the flow as locally incompressible and use the proper equations. Of course in calculations of pressure drop we should consider flow acceleration due to compressibility effects if the density variation is significant.

2.4 Nuclear Reactors Flow Channel Geometries

Most nuclear reactor cores are composed of groups of rod bundles. These rod bundles are mostly in the form of squares or hexagons, although sometimes a circular cylindrical form is employed. Each rod bundle consists of a cluster of fuel rods arranged in either a square array (generally LWRs, Fig. (2.2)) or a triangular array (LMFBRs, Fig. (2.3)).

2.5 Analysis of Flow Behavior in Subchannels

To solve the previously mentioned problems of heat transfer in reactor bundles, it is common to divide the bundle into several subchannels as shown in Figures (2.2) and (2.3), and then analyze the flow behavior within the subchannels. From these figures it is apparent that subchannel geometries are not peripherally uniform and in practice the ratio of P/D , especially for compacted triangular arrays which this work mostly concerns itself with, is very close to 1. Therefore the subchannel geometry is far from peripherally uniform. The variation of the wall shear stress (τ_w) over the perimeter of a nonuniform channel is closely connected to the transfer of momentum around the perimeter (θ direction). In the following paragraphs we discuss several models which have been utilized to describe the transfer of momentum in turbulent flow.

In the analysis of turbulent flow structure it is possible to show that total transfer of momentum is due to two processes: molecular viscosity (Newton's viscosity) and turbulent transport of energy. By making an assumption that the total transport of turbulence energy is brought about by a gradient-type diffusion caused by small scale motion of eddies plus convection by large scale motions of eddies, then the total transfer momentum along the θ direction in the form of stresses can be written as follows (43, pg. 288):

$$\tau_{\theta} = - \rho \nu \frac{\partial u}{\partial y} - \rho \epsilon_{m\theta} \frac{\partial u}{r \partial \theta} + \rho \overline{u'v'_{\theta}}, \quad (2.4)$$

where the coordinates are shown in Figure (2.4) u is velocity along the rod axis (x) and v'_{θ} denotes the velocity of the large-scale motion in the θ direction. A similar equation can be derived for the transfer of heat. In equation (2.4) the first and second terms are due to molecular effects and small scale turbulent eddies respectively. The third term is due to the convective transfer of momentum resulting from the large-scale motion of eddies. Therefore both the small and large scale eddy fluxes are responsible for the increased transport rates which characterize turbulent flow. These eddies are modeled as patches of fluid which have randomly fluctuating velocity distributions superimposed upon the local average axial velocity (u). As Figure (2.5) shows (for two dimensional main flow), the larger the axial velocity gradient ($\frac{\partial u}{\partial y}$ or $\frac{\partial u}{\partial z}$), the greater will be the u' (absolute value) induced by a v' (or w'), and hence the larger will be the product $\overline{u'v'}$ (or $\overline{u'w'}$). Similar phenomena exist for the transfer of heat flux as Figure (2.5-c) shows. The convective transfer of momentum should appear more strongly in channels with a sharply varying cross-sectional shape, i.e. small pitch to rod diameter ratio (P/D close to 1), where the effect of large-scale eddies on the velocity field appear most strongly in the θ

direction (in the direction in which the velocity varies slowly), although transfer of eddies through the gap clearance for very small P/D due to flow laminarization will be very small (16). However along the y axis, convective momentum transfer plays a small part, since the velocity gradient is very high and therefore this region is in the domain of gradient transfer of momentum. In a channel with slowly varying shape (e.g. large P/D or eccentric cylinders) the effect of convective transfer term is small. Neglecting the effect of convective momentum transfer in the case of sharply varying cross section may cause considerable error.

Reference (25) used this latter method and consequently utilized equation (2.5) to calculate the wall shear stress. This equation is the results of a force balance on a sub-channel fluid element (see Figure (2.6))

$$\tau_w \Delta l = \frac{\partial p}{\partial x} \Delta A \quad , \quad (2.5)$$

where ΔA is the element area between two constant velocity gradient lines (see Figure (2.8)). Equation (2.5) considers no shear forces acting on the velocity gradient lines because the normal velocity derivatives are zero along those lines ($\frac{\partial u}{\partial y} = 0$ and no effect of large scale motion of eddies). Equation (2.5) predicts a wall shear stress distribution very far from the measured distribution as will be shown in Chapter 5.

Reference (5) (Ibragimov et al.) used the idea of equation (2.4) and derived a model for the exchange of large-scale momentum between the cells. This led the authors to propose a semi-empirical relation for the wall shear stress distribution, equation (2.6):

$$\frac{\tau_w(\theta)}{\tau_w} = c \left[1 - \exp \left(- \frac{7.7}{\phi_s^{0.8}} \cdot \frac{\hat{y}(\theta)}{\hat{y}_{av}} \right) \right] \quad (2.6)$$

where:

$$\phi_s = \frac{A_s}{\hat{y}_{av}^2}$$

A_s = subchannel flow area

$\hat{y}(\theta)$ = distance from wall to maximum velocity line
along normal to the wall

$\hat{y}_{av}$ = average value of $y(\theta)$ over the subchannel area

and c is a normalization constant given by

$$\frac{1}{\theta_{max}} \int_0^{\theta_{max}} \frac{\tau_w(\theta)}{\tau_w} d\theta = 1 \quad (2.7)$$

where the $\theta=0$ and $\theta=\theta_{max}$ correspond to isotachs. Equation (2.6) determines the wall shear stress distribution utilizing only

geometric parameters of the channel. In order to find the average wall shear stress ($\bar{\tau}_w$), the authors of (5) used the Blasius formula for circular tubes, equation (2.8) and then found $\bar{\tau}_w$ using equation (2.9).

$$f_o = \frac{0.0791}{Re^{0.25}} \quad , \quad (2.8)$$

$$\tau_w = \frac{A_s (4/D_h) \rho / 2 \bar{V}_s^2 f_o}{R_c \theta_{max}} \quad , \quad (2.9)$$

where:

$\bar{V}_s$ = average of flow velocity over the subchannel area

R_c = radius of curvature of the wall surface.

The same authors in a subsequent paper (9) proposed a new expression for the friction factor, i.e.,

$$f/f_o = [0.58 + 0.42 \exp(-0.021 K_e^3)] \left\{ 1 + 0.1 \left[\frac{\hat{y}_{av}}{R_c} + 1 \right]^{4/3} \right\} \quad , \quad (2.10)$$

where f_o = smooth tube friction factor,

$$K_e = \phi_s^{0.25} \left[\frac{\hat{y}_{max} \hat{y}_{min}}{\hat{y}_{av}} \right] \quad ,$$

and then f from equation (2.10) is replaced by f_o in

equation (2.9). The generality of this approach (5) makes it very attractive but the results do not agree very well with the experimental results (see Chapter 5). This is probably due to the model which has been chosen for the exchange of large scale motion of eddies. This method predicts the ratio $\tau_w/\bar{\tau}_w$ independent of the Reynolds number which is not quite correct. However the results of the prediction of $\tau_w/\bar{\tau}_w$ by this method is much closer to the experimental results than the results of the method proposed by reference (25).

2.6 Universal Velocity Law in Subchannel

Several investigators theoretically and experimentally have shown that the universal velocity law is applicable to subchannel geometries for describing velocity distribution along normals to the surface (e.g. 5,6). If we assume a definite form of the relationship for $u^+ = f(y^+)$, then we are able to find the velocity distribution along the normal to the wall provided that we know the wall shear stress.

The simplest assumption is to take the universal velocity distribution law for a circular pipe. There are several mathematical forms for the universal velocity law and we now list several of the most current and well known forms.

Prandtl-Karman (7, pp. 556):

$$u^+ = \frac{1}{\kappa} \left(\ln \frac{y^+}{\nu} - \ln \beta \right) \quad , \quad (2.11)$$

where ν is kinematic viscosity and κ , β are constants. κ is independent of the wall nature ($\kappa \approx 0.4$) and β should be determined by the wall nature. In the case of turbulent flow in a smooth pipe with its radius equal to r_0 , the above equation can be written (8, p. 73):

$$u^+ = y^+ \quad 0 < y^+ < 5 \quad (2.12)$$

$$u^+ = 5.0 \ln y^+ - 3.05 \quad 5 < y^+ < 30 \quad (2.13)$$

$$u^+ = 2.5 \ln y^+ + 5.5 \quad 30 < y^+ < r_0^+/2; \quad r_0^+ = \frac{r_0}{\nu} \sqrt{\frac{\tau_w}{\rho}} \quad (2.14)$$

where r is measured from the pipe centerline. In the case of the boundary layer on a plane wall we have (9):

$$u^+ = 2.44 \ln y^+ + 4.9 \quad (2.15)$$

Reichardt's universal velocity distribution for turbulent flow in pipes for $0 < y^+$ is (6):

$$u^+ = 2.5 \ln(1 + 0.4y^+) + 7.8 \left[1 - e^{-1/11y^+} - 1/11y^+ e^{-0.33y^+} \right] + f_1(y/y_0) \quad (2.16)$$

where

$$f_1(\hat{y}/\hat{y}_0) = 2.5 \ln \frac{1.5[2 - \hat{y}/\hat{y}_0]}{1 - 2[\hat{y}/\hat{y}_0 - 1]^2} \quad (2.17)$$

where $\hat{y}_0$ for subchannel can be interpreted as the distance between the rod surface and the line of symmetry (taken perpendicular to the surface).

Sherwood and Smith (11) from flow visualization in

turbulent pipe flow have found the following equation:

$$\begin{aligned} y^+ = & u^+ + 5.32 \times 10^{-2} (u^+)^2 - 7.68 \times 10^{-3} (u^+)^3 + 2.19 \times 10^{-4} (u^+)^4 + \\ & + 1.64 \times 10^{-4} (u^+)^5 - 2.16 \times 10^{-5} (u^+)^6 + 9.12 \times 10^{-7} (u^+)^7. \end{aligned} \quad (2.18)$$

This equation matches equation (2.11) at $y^+=150$ and these investigators indicate that equation (2.18) shows less deviation from experimental pipe flow data than others especially for the range $0 < y^+ < 20$ as figures (2.9) and (2.10) show.

Investigators of reference (10) have shown that for subchannels with $P/D=1$, experimental data for u^+ within the range $80 < y^+ < 1000$ are slightly less than the equation (2.14) prediction.

Reference (6) shows that the velocity profiles predicted by equation (2.16) agree well with experimental results obtained from a triangular array subchannel model. Therefore one can conclude that it is plausible to use equations (2.18) and (2.16) ($y^+ < 20$ in equation (2.18), $y^+ > 20$ in equation (2.16)) and the results of wall shear stress distribution to calculate the flow velocity profile.

It is of importance to mention that authors of equations (2.11) to (2.14) believe that a laminar sublayer zone exists close to the wall ($0 < y^+ < 5$), but recent visual investigations have shown that the region is not laminar (12,13), since

this region exhibits a nonregular periodicity of flow properties in time. However, this periodicity within the sublayer is much less pronounced than the fluctuation of the main flow due to attenuation and averaging effects. In fact the relatively small amount of mixing within the sublayer compared to that of the core causes the transport in this region to proceed in part by molecular mechanisms and consequently more slowly here than elsewhere. This region is disturbed by velocity fluctuations of small magnitude, and by insertion of bulk flow elements from the adjacent region.

FIGURE (2.1)

Turbulent Flow Close to Wall of a Channel (after reference (2)).

Subchannels: (1) Central
(2) Side
(3) Corner

$AB = \text{Pitch} = P$

FIGURE (2.2)

Square Array Rod Bundle

FIGURE (2.3)
Triangular Array Rod Bundle

FIGURE (2.4)
Coordinates Identification

FIGURE (2.5)

Two Dimensional Turbulent Momentum and Heat Transfer

FIGURE (2.6)
Constant Velocity Gradient Curves and
Definition of flow element

FIGURE (2.7)
Subchannel Isotach and Diagram of Secondary Flow

FIGURE (2.9)
Typical Velocity Gradient Lines (curves)
in a Subchannel (after reference (5))

FIGURE (2.9) The universal velocity profile.
(after reference (11))

FIGURE (2.10)
Universal Velocity Profile in the Wall Region
(after reference (11))

CHAPTER 3
LITERATURE SURVEY AND THE EXPERIMENTAL
METHOD SELECTION

3.1 Introduction

Before selecting a method for conducting the experiment, we reviewed the literature which has dealt with the measurement of wall shear stress. The first part of this chapter describes and compares the methods which have been used by other investigators to find the local wall shear stress. The second part presents the selected method for our investigation and the related correlations.

3.2 Literature Survey

3.2.1 Comparison of Different Methods

The literature survey indicated the existence of the following methods:

1. (Ref. 45) Electrochemical Wall Shear Stress Meter

This method is based on the principal that the transfer of momentum by turbulent velocity fluctuations in the viscous sub-layer are negligible compared with transfer by molecular viscosity. The technique uses an electrode mounted flush with a solid wall at high enough voltages to reduce the concentration of the reacting species at the surface to zero. The flow acts as an electrolyte for conducting the electric current. Under these conditions the current is proportional to the rate of mass transfer over the electrode surface. These mass transfer measurements

are related to the velocity gradient of the wall in which the electrode is mounted flush. The relation between the average mass transfer over the electrode surface $\bar{K}$ and the average velocity gradient ($\bar{S}$) can be written as:

$$\bar{K} = \frac{1.5}{\sqrt[3]{9} \Gamma(4/3)} \left(\frac{\bar{D}^2 \bar{S}}{\bar{L}} \right)^{1/3}, \quad (3.1)$$

where

$\bar{D}$ =diffusion coefficient

$\bar{L}$ =electrode length (in the flow direction).

The authors mention that the method has advantages over the hot wire anemometer (which will be described later) in that it introduces no disturbances and yields measurements of the velocity field much closer to the wall. One of the disadvantages of this method is that its electrodes vibrate at high Reynolds numbers. Another is that it is difficult to accurately control the electrode distance from the wall (if one is interested in studying the flow behavior close to the wall). Also, it is necessary to use a special fluid (potassium ferri- and ferrocyanide and sodium hydroxide).

2. (Refs. 42&46) Razor Blade Technique

This method is very similar to the Preston tube method and is based on the assumption that a universal velocity law is applicable in the region close to the wall ($u^+ = f(y^+)$). The technique involves soldering a segment of a razor

blade (with cut edge) on the wall of a channel such that it is located over a static pressure tap hole. The razor blade with its cutting edge provides a chamber (under the edge) which acts as a fine pitot tube and yields a flow stagnant pressure. The readings of the razor blade tap hole, together with the local static pressure will yield a value of the velocity gradient at the region very close to the wall. The investigators of reference (46) have used this method to measure the wall shear stress in a turbulent boundary layer with tangential, secondary injection. The validity of the method requires that the razor blade be embedded in the flow sublayer. There is not a general calibration curve for this technique and in practice, the geometry of every individual probe necessitates its calibration.

3. (Ref. 47) Direct Instrument (Floating Element)

This technique is based on the movement of a floating element under the tension of the flow wall shear stresses. The element movements are converted and amplified via an electric transformer to a current. The electric current is proportional to the movement of the floating element and consequently to the flow wall shear stress. The investigator of reference (47) used this technique to measure the wall shear stress on the outside surface of a 3.5" tube in an air flow parallel to the tube axes. The width of floater (in the direction perpendicular to the

main flow) in the experiment was 3.18 mm. One of the disadvantages of this method is that it can only measure the wall shear stress based on the average value within the width of floater. Another is the design difficulty of inserting the necessary electric transformer inside the present test section rods. Also in practice, each individual device needs to be calibrated. The advantage of this technique with respect to the others is that it is capable of measuring the wall shear stress directly.

4. (Refs. 14; 51; 15, 49, 26, 52)

The Preston Tube Method

This method is based on the assumption of a universal inner law (or the law of the wall) common to boundary layers and fully developed turbulent pipe flow. According to this assumption the velocity distributions in a turbulent boundary layer near the wall, up to about the first tenth of the boundary layer thickness, can be represented by

$$\frac{u}{u^*} = g\left(\frac{yu^*}{\nu}\right) \quad \text{or}$$

$$\frac{\tau_w d^2}{4\rho\nu^2} = f\left(\frac{d^2 \Delta P_d}{4\rho\nu^2}\right) \quad , \quad (3.2)$$

where

u = main flow velocity,,

$$u^* = \sqrt{\tau_w / \rho} \quad , ,$$

d =Preston tube outside diameter ,

Δp =difference between Preston tube reading and local static pressure ,

τ_w =wall shear stress ,

ρ =mass density ,

ν =kinematic viscosity .

One type of Preston tube is the Cobra type (two-tap) Preston tube. The investigators of references (49) and (52) used the latter kind of probe to measure the effects of secondary flow in a noncircular (flat wall) channel. Reference (52) assumed that the distribution of secondary flow velocities in an interior channel of a triangular array rods with the pitch to diameter ratio equal to one ($P/D=1$) is the same as in the equilateral triangle inscribed in it (where they had only experimental data for the latter case). In order to measure the wall shear stress by direct utilization of a Preston tube, the skewed flow profile (by emersing the probe in the flow wall region) should be approximately and preferably smaller than the flow sublayer (collateral region). This condition requires that the probe diameter satisfies the condition of $y_d^+ \approx 30$

where $y_d^+ = \frac{d \sqrt{\tau_w / \rho}}{2\nu}$.

5. (Ref. 49) Hot Film Probe

This method is based on the Reynolds analogy between the transfer of momentum and the transfer of energy. The

method is analogous to the electrochemical technique (mentioned before). It is possible to design a heated element embedded in the wall to give a determination of skin friction. There are some assumptions in using this technique to find wall shear stress. One of them is that the Prandtl number of the flowing fluid is constant, which is reasonable for air but is not for water.

This diagnostic device is not useful to measure the wall shear stress distribution when the peripheral variation of the wall shear stress (τ_w) is small (P/D large) (38).

6. (Refs. 48&51) Surface Fence

Basically this consists of a projection of very small height positioned normal to the wall and the flow direction. By means of pressure tapping at the front and rear of the projection, a pressure difference, Δp , is measured which is linearly related to the wall shear stress, τ_w , provided that the height is sufficiently small,

$$\Delta p = k\tau_w \quad . \quad (3.3)$$

This linear relation is actually based on the law of the velocity distribution in the sublayer of the boundary layer on which the Preston tube technique and the hot film probe are also based,

$$\frac{u}{u^*} = \frac{u^*y}{\nu} \quad .$$

Investigators of reference (49) say that the sublayer fence cannot be used for determination of the direction of the wall shear stress vector, but J. D. Vagt (ref. 48) says if the height of the fence is 0.1 mm or smaller and collateral flow can be assumed in the viscous sublayers, then the surface fence should give the true flow direction. Advantages of sublayer fence over Preston tube are that the pressure signal obtained by a surface fence is approximately twice as large as that from a Preston tube, and there is no need for an additional static pressure tapping. The disadvantage of this device is that it can only measure an integrated value over the height of the fence and thus gives an average value of the direction of the shear stress close to the wall. Also the calibration and installation of the device is difficult.

7. (Refs. 51&49) Stanton Tube

This technique is much simpler than the Preston tube and surface fence. It consists of a small tube that protrudes through the wall of the channel and lies perpendicular to the flow direction. The technique is based on pressure difference between the pressure of this tube and local static pressure. Therefore this technique cannot be used for detecting the direction of the flow.

8. (Ref. 53) Slot Type Gage

This gage was designed and used to measure the wall

shear stress distribution in a model of interior subchannel of triangular array rods. Figure 3.1 shows the related design for the calibration of the device in pipe flow. The technique is based on the difference between pressure readings of the two slots. As Figure 3.1 shows, the apparatus consists of two slots where the downstream edge of one of these slots is inclined, while the edges of the other slot are sharp. Therefore the difference between the pressure readings of these two slots is proportional to the difference between the flow dynamic pressure and static pressure (like the Preston method). A slightly modified device was used by the investigators of (53) to measure the wall shear stress distribution in a model rod's subchannel (Fig. 3.2). Figure 3.3 shows the results for the case of triangular array wire wrapped rods with $P/D=1.3$. The investigators mention that the Preston tube technique is not a good method for the wall shear stress measurement in a subchannel with very small pitch to diameter ratio ($P/D \rightarrow 1$, the effect of the probe in a small subchannel). Therefore the slot type gage is an applicable method to the bundles with P/D very close to 1. However as the authors assert, there are some difficulties in application of this technique for measurement of the wall shear stress distribution around the rod periphery. These include maintaining the geometric similarity between the

calibrated gage and the one which is made on the actual test section and the present lack of a general calibration curve. Also the width of the gages should be made very small if one is looking for very accurate data. The same investigators (53) also measured the wall shear stress distribution in a model of interior subchannel of triangular array rods using the Preston tube method. Their design was such that they could only measure the wall shear stress in some selected points around the subchannel periphery. Figure 3.4 shows their results for the case of triangular bar rods using the Preston tube method.

Table 3.1 summarizes the available references which have presented wall shear stress distributions results from measurement.

3.2.2 Conclusions for the Methods Comparison

From the survey, it was concluded that the Preston tube and surface fence are the most favorable devices. In the case of the surface fence, reference (15) suggests that the following relations between the fence pressure reading difference (Δp) and the flow velocity exist:

$$\frac{\Delta p}{c \cdot u_n^{1.5}} = 1 + 0.75 \frac{h}{\tau_w} \frac{dp}{dx} \quad , \quad (3.4)$$

$$\tau_w \sim u_n^{1.5} \quad , \quad (3.5)$$

where

$c = \text{constant}$

$h = \text{fence height (from the channel wall)}$

$\tau_w = \text{wall shear stress}$

$\frac{dp}{dx} = \text{pressure gradient}$

$u_n = \text{flow velocity perpendicular to the fence at the distance } h \text{ from the wall.}$

Reference (48) suggests that the height of the fence (h) be equal or smaller than 0.10 mm. If we assume $h = 0.1$ mm and use the data of the present work corresponding to $Re = 18890$, then the magnitude of the second term on the right hand side of equation (3.4) will be 0.0022 which is negligible relative to one. Therefore we will get

$$\Delta p \sim u_n^{1.5} \sim \tau_w^{1.5}, \quad (3.5)$$

where

$u_n = \text{component of the flow velocity in the direction perpendicular to the surface fence (see the sketch).}$

The component of flow velocity perpendicular to the surface fence (u_n) can be described as:

$$u_n = u_t \cos \alpha = u_t \cos(\beta - \gamma), \quad (3.6)$$

where the nomenclatures are shown in the sketch and are determined as follows:

$u_t = \text{total resultant flow velocity } (\vec{u}_t = \vec{u} + \vec{u}_\theta)$

$$\alpha = (\beta - \gamma)$$

α = angle between the normal to the surface fence and the main flow direction

γ = angle between the resultant flow direction and the main flow

u = main flow velocity

u_θ = secondary flow velocity

$\hat{n}$ = unit vector perpendicular to the surface fence

$\hat{u}_t$ = unit vector in the flow resultant direction.

Substituting the u_n from (3.6) in (3.5), we get

$$\Delta p \sim u_t^{1.5} [\cos \alpha]^{1.5} \sim u_t^{1.5} [\cos(\beta - \gamma)]^{1.5} \quad (3.7)$$

Figures 3.5 and 3.6 show the plot of $\Delta p/\Delta p(\alpha=0)$, as a function of magnitude of secondary flow (in a flat wall channel) and the angle β based on Eq. (3.7).

In Figures 3.7 and 3.8 these graphs are plotted based on the experimental correlation of reference 48.

$$\frac{\Delta p_{fence}(\alpha^\circ)}{\Delta p_{fence}(0^\circ)} = e^{-(1.025 \frac{\pi}{180} \alpha^\circ)^2} \quad (3.8)$$

It can be seen from these figures (3.5 to 3.8) that surface fence cannot determine flow direction within 6°. Also it is possible to use a Pitot tube to find direction of flow, but this will be within ±5° in error (50, p. 248) of the pitot tube with the best shape. Therefore neither the surface fence nor the Preston tube has a relative advantage regarding measurement of flow direction.

3.3 The Selected Measurement Method

The method comparison in the previous section demonstrated that neither the surface fence nor the Preston tube has relative advantages. However the Preston tube method is much better established by previous investigators for the measurement of wall shear stress. At the present time many calibration correlations for Preston tube method exist. Therefore, we decided to measure the wall shear stress distribution in the present test section using the Preston tube method.

Several investigators, e.g. (14, 51, & 15) have calibrated the local wall shear stress (τ_w) versus the difference between the fluid stagnant pressure registered by the Preston

tube and the local static pressure in a turbulent pipe flow. The calibrations established by the experimental results of Patel (15) are more accurate and reliable than others. The calibration correlations established by Patel are as follows:

The calibration equation in the range of $0 < x^* < 2.9$ or $0 < y^* < 1.5$ is

$$y^* = 1/2x^* + 0.037 \quad , \quad (3.9)$$

in the range of $2.9 < x^* < 5.6$ or $1.5 < y^* < 3.5$

$$y^* = 0.8287 - 0.1381x^* + 0.1437x^{*2} - 0.006x^{*3} \quad , \quad (3.10)$$

in the range of $5.6 < x^* < 7.6$ or $3.5 < y^* < 5.3$

$$x^* = y^* + 2 \log_{10}(1.95y^* + 4.10) \quad , \quad (3.11)$$

where

$$x^* = \log_{10} \left(\frac{\Delta p_d d^2}{4\rho v^2} \right) \quad \text{and} \quad y^* = \log_{10} \left(\frac{\tau_w d^2}{4\rho v^2} \right) \quad ,$$

d = outside diameter of the Preston tube

Δp_d = difference between the Preston tube reading and
the local flow static pressure

ρ = flow density

v = kinematic viscosity

τ_w = wall shear stress

Patel also confirmed the results of Rechenberg (51) and mentioned that for a round Preston tube the ratio of inside to outside diameter (d_{in}/d) has a negligible effect on the calibration. This method, under some limitation, can be used for measurement of the wall shear stress distribution in a non-circular channel. Dr. Eifler (26) has extensively studied the Preston tube method limitations and the design specifications of our Preston tubes conform to the limits recommended by Eifler (see Appendix B).

TABLE 3.1

Summary Table of Measurements of the Wall Shear
Stress Distribution in Triangular Array Rods

<u>Investigators</u>	<u>p/D*</u>	<u>L/D_h</u>	<u>Re x10⁻³</u>
Eifler (54)	1.08	137	10-152
Kjellstrom (55)	1.217	81	149-373
Hall and Svenniggsson (57)	1.217	81	270
Subbotin et al. (10, 37)	1.0 1.05 1.1 1.2	106 154 100 72	14.-95 15.-63.3 18.8-81 11.5-81.9
Trupp and Azad (38)	1.2 1.35 1.5	51 30 20	12-84 12-84 12-84
Simonek and Mrakva (53)	1.615 2.14 1.3#		309.4-546.5 309.4-546.5 273.1-362.5
Ibragimov et al.	+	--	40
Rehme (36)	1.071	176	61-87
This Work	1.1	182	4.14-36.17

Wire wrapped rods.

* All data for triangular array.

+ See Figure 2.8 for test section cross section.

FIGURE (3.1) Details of Calibration
Design of Slot-Type Gage
(after reference (53))
(dimensions in millimeters)

FIGURE (3.2) Application Scheme of Slot-Type
Gage in a Subchannel of Triangular Array Rods
(after reference (53))

FIGURE (3.3)
Experimental Results of Reference (53) for Nondimensional Wall Shear Stress
Distribution in an Interior Subchannel of a Model of Triangular
Array Wire Wrapped Rods

FIGURE (3.4)

Experimental Results of Reference (53) for Nondimensional Wall Shear Stress Distributions in an Interior Sub-Channel of a Model of Triangular Array Bare Rods

FIGURE 3.5 RESULTS OF THEORETICAL FORMULA FOR SURFACE FENCE READING VERSUS FLOW IMPACT ANGLE

FIGURE 3.6 RESULTS OF THEORETICAL FORMULA FOR SURFACE FENCE READING VERSUS FLOW IMPACT ANGLE

FIGURE 3.7 RESULTS OF EXPERIMENTAL FORMULA FOR SURFACE FENCE READING VERSUS FLOW IMPACT ANGLE

FIGURE 3.8 RESULTS OF EXPERIMENTAL FORMULA FOR SURFACE FENCE READING VERSUS FLOW IMPACT ANGLE

CHAPTER 4
DESCRIPTION OF THE EXPERIMENTAL APPARATUS
AND TEST PROCEDURE

4.1 Introduction

This chapter deals with the description of the flow delivery system, test section, measurement facilities (probe positioner, Preston tube), measurement instruments and the measurement procedure.

During the course of the present investigation, the experimental approach was refined in the following steps:

First, modification of the test section:

- a) from the original to an intermediate design
- b) from the intermediate design to the final design.

Second, changes of flow delivery system:

- a) two regulator valves in line with the air filter immediately before them
- b) no regulator valve in line but the filter immediately before the diffuser.

Third, changes of air source

- a) the compressor as the air source with regulator valves in line
- b) the blower as the air source
- c) the compressor as the air source without any regulator valve in line.

Fourth, modification of the Preston tube. (Figure 4.6 shows the models of designed probes.)

4.2 Flow Delivery System and Test Section

A schematic of the test rig is given in Figure (4.1), and its components are discussed in the following sections.

4.2.1 Test Section

The original test section was designed by the author of reference (16), then modified by adding rotating parts and pressure tap holes such that we could measure the wall shear stresses distribution.

The test section is a six channel design shown in Figure (4.2). (See Figure (5.9) for subchannel identification.) Longitudinally it consists of four separated similar partial rod segments each 24 inches long. The dimensions of the test section are shown in Figure (4.2) and Table (4.1). The rods are centerless ground to the correct diameter, then milled to create a reference surface and finally bolted and pinned into place, as Figure (4.2) shows. Some characteristics of the test section are as follows:

1. made of brass;
2. bare rods;
3. Fluid boundary conditions are approximately the same as that of a central subchannel in a rod bundle. (As Figure (4.2) shows, two rods are more than half a cylinder.);
4. The design enhances machining and maintenance simplicity and provides the ability to change P/D

ratio and accept design modification on each part of the test section.

4.2.2 Air Sources

4.2.2.1 Compressor

The source of pressurized air in the present work was the lab.compressor. The characteristics of the compressor are as follows.

The maximum capacity of the compressor is 170 c.f.m. at a working pressure of 95 psig. The compressor is equipped with a controller switch. When air consumption is less than 170 c.f.m., this controller provides off and on periods for the compressor. During the on period the static pressure goes up to 115 psig. During the off period the pressure goes down to 95 psig. The storage tank (A) (see Figure (4.1)) accumulates the air during the compressor on period and supplies air during the off period. Tank (B) also acts as an accumulator as noted in Section 4.2.3.

4.2.2.2 Blower

Figure (4.1) shows the position of the blower, which functions to draw air through the test section. In order to provide the capability to use either the compressor or blower as air sources, we installed the blower after the flow meter. A flexible tube connects the blower to the flow meter to isolate the test section from the blower mechanical vibration. The flexible tube also provides a good convenient

means to disconnect the blower from the flow line when the head compressor is used as the air source. The maximum head pressure of the blower is equal to 1.27" Hg.

4.2.3 Flow Delivery System

4.2.3.1 Using the Compressor as the Air Source

The compressor is connected to the regulator valves via a 1" black iron pipe. The compressor is 120 feet from the test section. The details of the flow line are as follows:

- 1) up to point (b), 1" standard black iron pipe;
- 2) from point (b) to (e), 3/4" standard black iron pipe, 108" in length;
- 3) from point (e) to (f), 3/8" standard black iron pipe. The diameter of this section has an important effect on promoting steady flow because this section is immediately downstream of the regulator valve. Flow behavior was tested utilizing 3/4", 1/2" and 3/8" diameter section and it was observed that 3/8" line yielded the optimum flow condition, 130" in length.
- 4) from point (f) to (h), 3/4" standard black iron pipe, 45" in length.
- 5) from point (h) to (i), 3/4" brass fittings, 5" in length.

Flow line apparatus:

- 1) Pressure regulator valve (A). (PRV-A) reduces pressure to 50 psig and pressure regulator valve (PRV-B) acts to reduce the pressure from 50 psig to

5 psig. Both of the regulator valves are adjustable within their ranges.

- 2) Gate valve (B) bleeds air. This bleeding removes air moisture, oils and dusts. The bleeding also helps to keep the flow rate steady.
- 3) Globe valve (D) was used for fine bleeding.
- 4) Storage tank (B) was added to compensate the line pressure variations. However, the effectiveness of compensation was found to depend upon the flow circumstances and some times tank (B) did not help at all. Under these conditions valve (C) was closed to isolate tank (B) from the flow line.
- 5) Filter (A) removes air moisture, oils and dusts.
- 6) The diffuser downstream of point (i) expands the flow to match the test section cross-section. It is 13 inches in length with a 3 degree expansion angle. The diffuser can be removed easily which disconnects the test section from pipe line. We removed the diffuser from the flow line when we wanted to use the blower as an air source.

4.2.3.2 Using the Blower as the Air Source

In order to use the blower as an air source we disconnected the flow line at position i before the diffuser. Then the remaining parts of the flow line were as we will demonstrate in the following sections.

4.2.4 Prechannel Section

The prechannel section is designed to enhance the flow

development. It is made of steel plates which are 24 inches in length and 2"x2 19/32" inside cross-section. At the beginning of the section three perforated plates are installed (see Figure (4.3)).

The plates were designed and installed according to reference (44). A few filter papers and a fine screen are attached to the upstream face of the first plate. These filter papers remove fine particles, oils and dusts which have passed through the filter (A).

4.2.5 Flow Meter

The flow of air after passing through the test section enters a 2" standard black iron pipe. A suitable flange for installation of orifice plates is fixed at the end of the pipe. The orifice plates are square edged (they are made according to ASME codes). A cluster of steel tubes were installed upstream of the orifice plate to straighten the flow. Figure (4.4) shows the design. The flow temperature was measured at the location shown in Figure (4.5) (according to reference (20)) using a 1" immersion thermometer.

4.3 Measurement Facilities

4.3.1 Pressure Drop Tap Holes

The pressure drop in the test section can be measured through 12 pressure tap holes. Figure (4.5) shows the location of these tap holes. The holes on sides B and D had been made during previous investigation and they are 1/8" in

diameter. The Holes on sides A and C were drilled in the present work. In order to prevent any disturbance in static pressure measurement and to make the measurement more accurate these new holes were chosen to be $1/16"$ in diameter (according to reference (21)). The axes of all of these tap holes coincide with $\theta=0$ which is defined in Figure (5.1). The tap holes were completely deburred using very fine sand paper.

4.3.2 Probes (Preston Tube)

The stem of probes are made of stainless steel tubing with an outside diameter of $1/8"$ and the wall thickness around $0.020"$. The tip of the probes (Preston tubes) are made of medical injection needles. The probes were custom made by United Sensor Co. in Watertown, Massachusetts. During the course of this work we designed and used three different models of probes having various outside diameters. Figure (4.6) shows the models and Table (4.2) the dimensions of the designed probes. Table (5.1) lists the test conditions and the corresponding Reynolds no. for each probe. For probe model (1) because of the offset (dimension e), the assurance that its tip is at the wall is higher than two others. Unfortunately, the probe model (1) became blocked after a short time running. These blockages were probably due to the eccentric bend in the probe ip (see Figure (4.6)). Therefore the probe model (2) was designed and used to complete

and used to complete three measurements. Unfortunately this probe was broken. In order to protect the Preston tube against the wall the probe model (3) was designed. However this probe became blocked before any data could be taken. We tried to open this probe by inserting a very thin wire, but we couldn't open the probe completely even though the wire could pass through the Preston tube easily. It was suspected that deposits from the soldering gas had blocked the Preston tube. Therefore a probe of model (3)', named probe 7, was built and with the last probe we completed the remaining five measurements. In this latter probe, the needle was connected to the probe stem using a special welding liquid rather than using a torch and silver solder. In all the cases the ratio of Preston tube inside diameter (d_{in}) to outside diameter (d), is 1/2.

Reference (15) established the wall shear stress measurement correlation using Preston tubes with varying inside diameters (d_{in}) to outside diameter (d) ratios within the range of $0.17 < d_{in}/d < 0.83$. Therefore the results of the present work should be covered by the calibration correlations since d_{in}/d is equal to 0.5.

4.3.3 Probe Positioners

4.3.3.1 Azimuthal Probe Positioner Facilities

Measurement of the wall shear stress distribution in the present subchannel model required that the probe be capable

of being rotated around the rod periphery. In order to facilitate this rotation we cut and milled a part of the rod A. The milled section length is $5/16$ " (along rod axis) and we replaced this part by a complete cylinder (see Figure (4.7)). The position of this modification was chosen so that the probe stem axis is 4.5 inches up-stream of the test section exit (see Figure (4.8)). The probe passes through the center of the cylinder. The probe is covered by a Teflon tube such that it is electrically insulated from the cylinder. We milled a window on the subchannel wall (A) so that the probe stem can pass through this window (see Figure (4.9)). An O ring was placed around the window to seal it and the window was capped by a straight plate (see Figure (4.10)). The plate is able to move in the z direction inside of two guiders (bridges). The probe stem passes through the plate. To protect the probe stem against bending, it was passed through a steel tube and to prevent the leakage of air between the steel tube and the plate, a gromet was installed on the plate. An azimuthal probe positioner fixes the probe azimuthal positions at each desired angle θ (see Figure (4.11)). This positioner was installed on the back of the side (A) of the test section.

4.3.3.2 Radial Positioner

When we measure the wall shear stresses the probe should be in contact with the wall, but when we want to change the azimuthal position of the probe it should be

clearly separated from the rod surface. Therefore we designed a system to allow the changing of the probe radial position. Figure (4.12) shows the designed system (the frame was custom made by United Sensor Co.). The micrometer moves the probe in the radial direction and enables determination of the radial position of the probe. However the micrometer and the positioner are electrically insulated from the probe. Therefore we designed a low voltage circuit including the probe, test section and a small bulb (see Figure (4.16)). This circuit lights the small bulb when the probe touches the wall.

4.3.4 Static Pressure Measurement Facilities

In the intermediate designed test section we drilled two tap holes on the rod C. One of these tap holes was drilled at $\theta=0$ and the other at $\theta=30$ degrees (see Figure (4.13a) for design and Figure (5.1) for the angle identification). The plane of the axes of these holes was $1/4$ inch up stream of the probe stem axis. During the experiment it appeared that the static pressure at $\theta=30$ was less than the static pressure at $\theta = 0$ degrees.

In order to be sure that this pressure difference was not due to the machining of the tap holes, we decided to design a means such that we could measure the static pressure at each desired orientation around the rod periphery using the same tap hole for all the measurements. In this new design we milled a part of the rod C, and replaced that part with

a complete cylinder (see Figure (5.13b)). This replaced cylinder was installed such that it could rotate around the rod axis. A tap hole with 1/16 inch diameter was drilled in this cylinder. The tap hole was drilled 4 7/8 inches downstream of the cylinder upstream boundary. The cylinder was centered and leveled with the rod with extensive accuracy. The maximum value of the measured difference between the cylinder and rod level was less than 10^{-4} inch. This latter situation and the fact that the static tap hole is 4 7/8 inches $\approx 5 \times D_h$ downstream of the cylinder upstream boundary insures that there is not significant effect of cylinder boundary separation on the static pressure measurement (effects of flow perturbation). The subchannel wall on side C was milled such that the cylinder can rotate (see Figure (4.14)). In order to seal the window we provided a suitable O ring (see Figure (4.15)) and capped the window with a plexiglass plate (The design is similar to that which we used to cap the probe window). Brass tubing passes through the plexiglass plate and connects the tap hole to Poly-flo tube which eventually connects the tap hole to the measurement instruments. We made a housing and installed a gromet on the plexiglass at the point where the brass tubing passes through the plexiglass. The gromet prevents the leakage of air through the space between the brass tubing and the plexiglass. The plexiglass

plate can slide on the back of the subchannel wall such that the tap hole rotates around the rod periphery. A graded plate was added to the back of subchannel to show the azimuthal position of the tap hole.

4.4 Measurement Instruments

4.4.1 Wall Shear Stress Measurement Instruments

According to our selected method the total wall shear stress is found by measurement of the difference between the flow stagnant pressure which is registered by the Preston tube (while touching the wall) and the local static pressure. The difference between the Preston tube pressure and static pressure was measured by the following devices as shown in Figure (4.16):

- 1) Barocel pressure sensor model 570 D made by Datametric (one with the upper limit scale equal to 10 mm Hg and another with the upper limit scale equal to 100 mm Hg).
- 2) Electronic manometer signal conditioner model 1173 made by Datametric for visual monitoring of the pressure difference by means of the pressure sensor DC voltage output. This instrument has seven scale ranges from 1 to 0.001 x Barocel full scale.
- 3) Digital voltmeter to read the signal output with more accuracy (Δp_{di}).
- 4) Integrator voltmeter to find the mean value (Δp_d) of the fluctuating pressure (Δp_{di}).

- 5) RMS voltmeter to find the true root mean square value of the measured pressure difference ($\text{RMS}(\Delta p_d)$) and of the turbulent flow intensity.

4.4.2 Pressure Drop Measurement Instruments

The static pressure drop at the wall (Δp_x) was measured using:

- 1) Barocel pressure sensor.
- 2) Electronic manometer signal conditioner.
- 3) Digital voltmeter.

These instruments were described in section (4.4.1).

4.4.3 Flow Rate Measurement Instruments

The flow rate is determined from measurement of the orifice plate pressure drop, the flow static pressure at the upstream side of orifice plate and the flow temperature.

The flow temperature was measured using the immersion thermometer at the location shown in Figure (4.4). The orifice plate pressure drop was measured using either the same instruments which were mentioned in section (4.4.1) (except the RMS voltmeter) or the mercury manometer (see Figure (4.5)). The first set of instruments were used when the orifice pressure drop was less than 100 mm Hg (This is the upper limit of the large scale Barocel pressure sensor.) and the second instrument when the pressure drop was more than 100 mm Hg.

4.4.4 Static Pressure Measurement Instruments

The static pressure at high flow pressure (more than 100 mm Hg) was measured by a Helicoid pressure gage (0.1 psig subdivision). For low flow, pressure was measured by the Barocel pressure sensor.

4.5 Measurement Procedure

Before performing initial measurement, the flow of the test section was inspected for flow leakage. Whenever the test section was disassembled we reinspected for the flow leakage before starting any measurement using the Leak-Tec liquid. Elimination of flow leakage is very important, especially when the value of the fluid leaking is so large that the flow boundary layer cannot fully develop before it reaches the measurement plane.

The wall shear stress distribution was measured at different flow rates using different flow delivery system operating procedures (a,b,c) which are the procedures described in the following section (4.5.1).

4.5.1 Wall Shear Stress Measurement

Procedure (a):

The local wall shear stress was to be found through measurement of the difference between the local flow stagnant pressure and the static pressure. We measured the pressure difference (Δp_s) with the Barocel pressure sensor and accessory instruments as mentioned in section (4.4.1).

In the preliminary flow line design the two pressure regulator valves were in line and the source of air was

the main laboratory system which had a pressure fluctuation corresponding to the compressor on-off cycle. Therefore the maximum line pressure corresponding to the beginning of the compressor off period (110 psig) was different than that at the end of compressor off period (90 psig). This difference in flow rate was very small (less than 0.05" Hg change on the orifice pressure difference manometer) and could not be suppressed by the pressure regulator values even by adjusting the regulator valves at their optimum positions. However because the Barocel pressure sensor is very sensitive, these changes could be seen clearly on the voltmeter (30% of registered Δp_d). Therefore we measured the (Δp_d) at a fixed time after the beginning of a compressor off period. This time was selected based on operating experience as that time at which the rate of change in pressure was lowest. Experiment number 4 (see Chapter 5) demonstrates the set of measured data resulting from the preliminary designed flow line.

Procedure (b):

In order to eliminate the flow rate fluctuations, we used the blower as an air source. The blower was installed after the flow meter and drew air through the test section. The blower is driven by a 3 phase electric motor. We usually performed the experiments late night and early morning. At this time the changes in the laboratory voltage

are very small. Therefore the blower PRM and consequently the flow rate is steady at this time. We measured the orifice plate pressure drop with the Barocel pressure sensor several times during each measurement. The results showed that the flow rate was constant.

During the experiments it appeared that the difference between the flow stagnation pressure at the wall and the static pressure (Δp_{di}) fluctuates. The fluctuation appeared even when the flow source was the blower, the Preston tube was moved into contact with the wall, and the tube outside diameter was only 0.008". Therefore one can conclude that the flow sublayer is turbulent. We then integrated the instantaneous pressure difference readings (Δp_{di}) and found the mean value using the integrator voltmeter, according to the following equation.

$$\Delta p_d = \frac{1}{T} \int_0^T [\Delta p_{di}] dT$$

The period of integration was chosen depending upon the case. In most of the cases the period was 10 seconds, but sometimes it was 100 seconds. The time period of integration was taken such that the recorded fluctuation in the integrated reading (Δp_d) was less than the last digit number of the voltmeter.

Before starting the measurement we let the blower run for 3 to 4 hours. This prerunning enhanced the steadiness

of the flow regime. Experiments 1,2,3 and 5 utilized this procedure.

Procedure (c):

The maximum flow Reynolds number that we could approach by utilization of the blower was equal to 10475 based on D_h of the test section. In order to have flow with Reynolds number greater than 10475 we used the main laboratory compressor as the source of air but bypassed the pressure regulator valves and bleed through valves B, G and F. In this manner the compressor was made to operate continuously thereby eliminating the pressure cycling described earlier. The combination of the compressor capacity and the bleed flow necessary to keep the pressure below 95 psig and thereby keep the compressor operating continuously yielded a maximum test section of Reynolds number of about 37,000.

This mode of operation was abnormal for the compressor and led to the introduction of oil in the line from the compressor lubrication system. The oil contamination persisted for about the first hour of compressor running time.

The bleed through valve (B) removed the air dusts, extra moisture and oils. The dusts and extra moisture come with air when air passes through the black iron pipes. We never used the gate valve (E) to control the flow rate. The reason is that a partially closed gate valve causes

shock wave propagation downstream of the flow. (The effect is dependent on how many turns the valve is closed and also on the size of the valve; e.g. see (55).) Therefore, for approaching the desired flow rate we controlled the flow rate using the bleeding valve (E). The bleeding of air provided a good means for producing a steady state flow regime during the measurements as bleeding of air degrades the propagation of upstream flow fluctuations.

The flow rate became constant after the compressor warmed up (one hour). We did experiments 6 and 7 using the procedure (c):

In all of the experiments, we turned on the electronic instruments at least three hours before taking data. The maximum environment and the flow temperature changes during each experiment was less than 0.5°C . For each one degree change in the azimuthal position of the Preston tube we waited between 4 and 9 seconds, depending on the case. The waiting interval allowed the pressure difference across the Barocel pressure sensor to approach the actual value and the integrator voltmeter could show the true Δp_{d1} mean value (Δp_d).

The wall shear stress distributions were measured in the subchannel number (1) (see Figure (5.9) for subchannel and Figure (5.1) for azimuthal direction identification) and within $\theta=0$ to $\theta=34$ degrees. For this measurement the Preston tube was moved in the subchannel (1) while the static pressure tap hole was moved in the subchannel (2). In order to detect the subchannel symmetry, we measured the wall shear

stresses beyond 30 degrees, up to 34 degrees and also in the negative θ direction (in the subchannel (3)) at -33 degrees.

4.5.2 Static Pressure Measurement Procedure

The flow static pressure was measured through the rotational or local tap hole at θ equal to zero. (see Figure (5.1)). The pressure was measured relative to the atmospheric or the orifice plate static pressure. At low flow, the pressure was measured using the Barocel pressure sensors. At high flow pressure, the pressure was measured using the Helicoid pressure gage. The absolute flow pressure was calculated by adding the orifice plate static pressure and the local atmospheric pressure. The local atmospheric pressure was measured using the laboratory barometer.

4.5.3 Root Mean Square Measurement Procedure

The true root mean square (RMS) of the instantaneous pressure difference between the Preston tube pressure and the static pressure (Δp_{di}) was measured using the RMS voltmeter. The RMS of the Δp_{di} was recorded at the same time that we registered the mean value of the pressure difference (Δp_d).

4.5.4 Pressure Drop Measurement Procedure

The axial static pressure drop (Δp_x) was measured on sides A and C through tap holes 1, 2 and 3, on side B through tap holes 2 and 3 (see Figure (4.5) for tap hole identification).

4.5.5 Flow Rate Measurement Procedure

The pressure drop across the orifice plate was measured using the Barocel pressure sensor at low orifice plate pressure drop and the mercury manometer at high pressure drop. The flow static pressure upstream of the orifice plate was measured in the same way as mentioned in section (4.5.2).

The humidity of air was measured using the wet dry temperature data and the psychometric chart. The wet bulb temperature was measured at the exit of flow meter when we used the compressor as the air source, but when the blower was the source of air the wet bulb temperature was measured at the blower entrance.

TABLE (4.1)

Test Section Specifications

Triangular array (bare rods)	60°	
Pitch-to-diameter ratio (P/D)	1.10	
Pitch (P)	1.65"	
Rod Diameter (D)	1.5"	
Gap (g)	0.15"	
Hydraulic Diameter (D_h)	0.5013"	
Subchannel Flow Area (A_g)	0.049217 in ²	
Total Flow Area (A)	1.595 in ²	
	inches	L/D_h
Total Length	96.2*	192
Probe Steam Axis	91.7	183
Measurement Position	91.3	182

*A flow split plate with 48 inches in length is installed at the beginning of the test section.

TABLE (4.2)
Probe Models Specifications

Probe No.	Model	Preston tube diameter		e mm	Connection Material
		inside inches	outside		
1	1	0.004	0.008	1	silver
2	1	0.006	0.012	1.5	silver
3	1	0.007	0.014	2	silver
4	2	0.007	0.014	--	silver
5	3	0.007	0.014	--	silver
6	3	0.007	0.014	--	silver
7	3	0.007	0.014	--	welding liquid

FIGURE (4.1) Schematic Diagram of Flow Delivery System

FIGURE (4.3) Details of Perforated Plates

FIGURE (4.4) Details of Flow Meter

(i): TAP HOLE NO. i

MERCURY
MANOMETER

THERMOMETER

PROBE
AXIS

ORIFICE
PLATE

Pressure Drop Tap Holes

FIGURE (4.6)

Probes Model

FIGURE (4.7)

The Probe Rotation and Insulation Facility

FIGURE (4.8)

Axial Positions of the Rotating Static
Pressure Tap Hole and the Probe Stem

FIGURE (4.9)

Probe Window

FIGURE (4.10).
Details of Window Capping Plate

FIGURE (4.11) Probe Azimuthal Positioner

FIGURE (4.12) Probe Radial Positioner

FIGURE (4.13)

Static Pressure Tap Holes

FIGURE (4.14)

Static Pressure Tap Hole Rotation Facilities

FIGURE (4.15) Static Pressure Tap Hole Window

FIGURE (4.16)

Diagram of Electronic Instruments Utilized for Measurement

CHAPTER 5

RESULTS AND DISCUSSION

5.1 Introduction

In doing the experiment, the following objectives were established:

First - measure the wall shear stress and pressure drop as much as possible in the fully developed flow region.

Second - locate the measurement region far enough from the end of the test section to avoid any perturbations from the change in cross section area at the end of the test section.

Third - perturb the flow at measurement point as little as possible.

Fourth - measure the symmetry of the test section and subchannels.

Considering the above points, we analyzed the conditions and improved test section as follows:

1) Since the total length of the available test section was limited (96.2 inches), flow development was enhanced insofar as possible by installation of the entrance section, as mentioned in Chapter 4. The measurement plane is located at $L/D_h = 182$.

In reference (16) it is shown that for this test section, cross flow at $L/D_h = 44.5$ and $L/D_h = 92.5$ are the same. The friction pressure drops measured in the present work show

that the flow is fully developed. Of course the latter criteria seems to be a little loose, as investigators of reference (38) registered a linear static pressure distribution ($p_s(x)$) along the channel length (around the measurement plane), but based on their results the flow had probably not been fully developed.

Experimental results of reference (10) showed that for subchannels of triangular array rods with $P/D=1$, the velocity distributions for planes at $85 D_h$ and $106 D_h$ from the entrance plane are the same, so the flow between $90 D_h$ to $95 D_h$ from the entrance cross section is fully developed. Reference (37) mentions that this distance for larger P/D will be even shorter than for $P/D=1$. Investigators of reference (23) measured the axial velocity profile at $L/D_h=77$ and $L/D_h=113$. The test section geometry dimensions were $P/D=1.123$, $D=2.014$ inches, $D_h=0.787$ inches, which are relatively close to the dimensions of present test section. They also used an entrance section. The results of their experiment showed that the flow between $L/D_h=77$ and $L/D_h=113$ was still developing, but because the flow development in this distance was of the order of experimental error, they accepted the flow at $L/D_h=113$ as fully developed. The same investigators (23) also concluded that the mean velocity profile is the most reliable criterion of the onset of fully developed flow.

Following the above discussions, it is plausible to

assert that in the case of this study where $L/D_h=182$, the flow is completely fully developed.

2) The measurement location was $10 D_h$ upstream of the test section exit. According to reference (22) partial blockage of flow does not have an effect at a point located at $5 D_h$ upstream of the blocked plane.

3) In order to perturb the flow as little as possible and keep the boundary layer similarity ($y_d^+ \approx 30$), the probes were made as small as possible (see Chapter 4 for probe details).

4) We tried to measure the subchannels symmetry by measuring the wall shear stresses over $\theta=0$ to $\theta=32$ or 34 degrees, and by measuring the pressure drop on the sides (A), (B), (C) at $\theta=0$. (See Figure 5.1)

Table (5.1) summarizes the measurements and the conditions under which they were performed. As mentioned in Chapter 4 the static pressure in the intermediate test section design was able to be measured only at $\theta=0$, $\theta=30$ degrees, while in the final design it was possible to measure the static pressure (p_s) at each desired degree within 0 to 34° . The probe outside diameter (d) for all of the results is 0.014 ", with the ratio of inside diameter to outside diameter equal to 0.5 . Although we tried to use the probes with the outside diameter less than 0.014 inches, like 0.012 , 0.010 and 0.008 inches, to measure the shear stresses, unfortunately these smaller probes (Preston tubes) became

Experiment Number	Test Section	Flow Source	Probe* Model	$Re \times 10^{-3}$	Measured Parameters
1	Final design	Blower	3	4.15	$\tau_w(\theta), \bar{\tau}_w, \frac{\tau_w(\theta)}{\bar{\tau}_w}, f_s, f, \text{RMS}$
2	Final design	Blower	3	4.14	$\tau_w(\theta), \bar{\tau}_w, \frac{\tau_w(\theta)}{\bar{\tau}_w}, f_s, f, p_s(\theta), \text{RMS}$
3	Final design	Blower	2	9.11	$\tau_w(\theta), \bar{\tau}_w, \frac{\tau_w(\theta)}{\bar{\tau}_w}, f_s, f, p_s(\theta), \text{RMS}$
4	* Intermediate design	Compressor (Regulator valves)	2	9.52	$\tau_w(\theta), \bar{\tau}_w, \frac{\tau_w(\theta)}{\bar{\tau}_w}, f_s, f, p_s(0), p_s(30)$
5	+ Final design	Blower	2	10.47	$\tau_w(\theta), \bar{\tau}_w, \frac{\tau_w(\theta)}{\bar{\tau}_w}, f_s, f$
6	Final design	Compressor (Continuously)	3	18.89	$\tau_w(\theta), \bar{\tau}_w, \frac{\tau_w(\theta)}{\bar{\tau}_w}, f_s, f, \text{RMS}$
7	Final design	Compressor (Continuously)	3	36.17	$\tau_w(\theta), \bar{\tau}_w, \frac{\tau_w(\theta)}{\bar{\tau}_w}, f_s, f, p_s(b), \text{RMS}$

TABLE (5.1)
List of Test Conditions and Measured Parameters

Legend * For probes model, see Chapter 4.
 + These results are unreliable.

blocked after a short time running. However the 0.014 inch Preston tube was satisfactory, since it lies within $y^+ = 30$ (see Appendix B).

Table (5.1) lists seven experimental runs. In the following presentation of the data, runs 4 and 5 are not included since these tests are not considered reliable. In run 4 the air source was compressor and the pressure regulator valves were in line. The pressure regulator valves could not keep the flow rate completely constant and due to the fluctuation of these regulators, we did not succeed in having a steady state flow regime. In fact we ran the test section 4 times to measure the wall shear stress under the above condition, but because the fluctuation of the flow rate was even more than run number 4 (consequently more unreliability), we do not show the other results in this report.

The results of run number 5 are not reliable, because after the experiment number 5 was finished another measurement but unfortunately it appeared that the probe was already broken. Therefore the results of experiment number five are suspected to be erroneous.

5.2 Results

5.2.1 Results - Wall Shear Stress Distribution ($\tau_w(\theta)$)

Table (5.1) shows that the wall shear stress distributions were measured for different Reynolds numbers. The numerical values for all wall shear stress results are given in Appendix A, and plots of final results for

cases 1, 2, 3, 6 and 7 for τ_w versus θ are shown in the lower portion of Figures (5.2) to (5.8).

As these figures demonstrate, regardless of Reynolds number value, the maximum wall shear stress occurs at $\theta=30$ degrees. This agrees with the prediction of Ibragimov (5), reference (23) and the experimental results of reference (37).

The wall shear stress at $\theta=0$ ($\tau_w(0)$) was calculated in an alternative way using pressure drop data. At $\theta=0$, the subchannel has symmetry in both the r and θ directions. Consider an element of fluid along the x axis whose cross section is shown in the following sketch (abcd). The side bc of element cross section coincides with the pressure tap hole diameter. The width of the element is established no longer than bc to minimize the shear stresses acting on faces ab and dc and no smaller than the pressure tap hole diameter since this is the length over which the measured axial pressure gradient applies. The length of element (L) is equal to the distance between two pressure tap holes (17 inches) with which we measure the pressure drop (Δp_x).

Writing an axial momentum balance we obtain

$$\Delta p_x \cdot \delta \cdot e = \tau_w(0) \cdot \delta \cdot L - 2\tau_{\theta x} \cdot L \cdot e, \quad (5.1)$$

where δ , e and L are dimensions defined in the sketch shown on the following page.

Now, if we ignore the shear stress $\tau_{\theta x}$ (δ approaches zero), equation (5.1) becomes

$$\tau_w(0) = \frac{\Delta p_x e}{L} \quad (\delta \rightarrow 0) \quad , \quad (5.2)$$

where in this case $\tau_w(0) \equiv \tau_{wf}(0)$ to distinguish between wall shear stress measured using the Preston tube method ($\tau_w(0)$) and the latter method. Table (5.2) shows the resultant ($\tau_{wf}(0)$). Table (5.2) also shows that for all cases,

$\tau_{wf}(0)$ is less than $\tau_w(0)$ which is measured by Preston tube. This is probably due to the neglect of side shear stresses on sides (ab) and (cd) as discussed above.

The wall shear stress was measured beyond 30 degrees, up to the maximum $\theta=34$ degrees to check the symmetry of the subchannels around $\theta=30$ degrees. The lower part of Figures (5.2) to (5.8) also show these results. Also at some selected points, especially around $\theta=-30$ degrees and $\theta=0$ degrees in subchannel 3, the wall shear stresses were measured and symmetry confirmed within experimental error between subchannels 1 and 3 (see Figure (5.2) for channel identification and Appendix A for the data).

The true root mean square (RMS) of the Δp_d fluctuations were measured (according to equation (5.3)) as reported in Appendix A. The fluctuation range changes as Reynolds number changes, but it is almost constant for different angles (θ) for fixed Reynolds number.

$$[\text{RMS } (\Delta p_d)]^2 = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} [\Delta p_{di} - \Delta p_d]^2 dt \quad (5.3)$$

Because of the Preston tube pressure fluctuations, the instantaneous Δp_d (or Δp_{di}) was integrated and averaged for 10 or 100 seconds, according to the following formula:

$$\Delta p_d = \frac{1}{T} \int_{-T/2}^{+T/2} [\Delta p_{di}] dt$$

In fact this integrated Δp_d was used in the Patel correlation (see Chapter 3) to calculate the $\tau_w(\theta)$.

5.2.2 Results - Average Wall Shear Stress ($\bar{\tau}_w$)

The average wall shear stress was calculated using a curve fitted to the shear stress distribution results. These fitted curves are shown in the lower portion of Figures (5.2) to (5.8). The fitted curves were integrated from zero to $\pi/6$ and then divided by $\pi/6$ to obtain $\bar{\tau}_w$. In performing the integration

$$\bar{\tau}_w = \frac{\int_0^{\pi/6} \tau_w(\theta) d\theta}{\pi/6} = \frac{\frac{1}{2}[\tau_w(0) + \tau_w(30)] + \sum_{n=1}^{29} \tau_w(n)}{30}, \quad (5.4)$$

the Sympon rule was used as shown by equation (5.4). The results of the $\bar{\tau}_w$ are also noted on Figures (5.2) to (5.8) and Table (5.2).

5.2.3 Results - Non-Dimensional Wall Shear Stress $\left(\frac{\tau_w(\theta)}{\bar{\tau}_w}\right)$

The wall shear stress at every point on the curve fitted to the experimental data of $\tau_w(\theta)$ was divided by $\bar{\tau}_w$ calculated from equation (5.4). The upper portion of Figures (5.2) to (5.8) show the results $(\tau_w(\theta)/\bar{\tau}_w)$.

For comparison the results of experiment numbers

2, 3, 6 and 7 of Table 5.1 are depicted in Figure 5.10.

This figure shows the effect of Reynolds number variations.

For Reynolds number higher than 9000, the curves $\tau_w(\theta)/\bar{\tau}_w$ for different Reynolds numbers are very close together, except at θ near zero and 30 degrees (see Figure 5.10). For increasing Reynolds number, the value of non-dimensional wall shear stress $\tau_w/\bar{\tau}_w$ at $\theta=0$ and $\theta=30$ degrees declines (very little at $\theta=0$ degrees). Of course this movement is small and almost within the experimental error. Since the curves for $Re=4150$ and $Re=36170$ are clearly separate and different at $\theta=0$ and $\theta=30$ (larger than measurement error), it is plausible to assert that the $\tau_w(\theta)/\bar{\tau}_w$ curve at $\theta=0$ and $\theta=30$ changes as the Reynolds number climbs from $Re=4000$ to $Re=36000$, and the pattern of this movement is according to Figure (5.10). However, very accurate and sensitive experiments for measurement and flow sources are necessary to clearly show the pattern of these movements, especially for small changes of Reynolds number.

Figure (5.10) also shows that the form of the $\tau_w(\theta)/\bar{\tau}_w$ distribution corresponding to $Re=4150$ is very different than the curves corresponding to $Re=9110$ and higher. This is probably due to the fact that the flow regime at this condition ($Re=4150$) is between the laminar and stabilized turbulence. The wide range of fluctuation in the Δp_{di} reading and the ratio of the root mean square of Δp_{di} (RMS) to the value of Δp_d (an average quantity) confirms the instability of the regime.

5.2.4 Secondary Flow

The range of fluctuation and the values of RMS for each Reynolds number within $\theta=0$ degrees to $\theta=30$ degrees were the same. Therefore the intensity of turbulence close to the wall for constant flow rate around the periphery of the subchannel is uniform. From these observations it appears that even if there is any secondary flow in the subchannel, there is no detectable current in the region very close to the wall, i.e., $y^+ < 30(36)$. Therefore the wall shear stress is not skewed from the rod axis by the effect of secondary flow.

5.2.5 Results - Friction Factors (f, f_s)

The friction factor (f) was calculated using the pressure drop along rod axis as defined in Eq. (5.5)

$$f = \frac{\Delta P_x}{4(L/D_h)\rho/2 \bar{V}^2} \quad , \quad (5.5)$$

where $\bar{V}$ is the averaged velocity of the flow over the whole channel cross-section area. In deriving Eq. (5.5) we ignored the effect of acceleration pressure drop (Δp_{acc}) due to density change. ΔP_x is measured at $\theta=0$ either on side A or C (see Figure 5.9). The results of pressure drop measurements on sides A, C are exactly equal (within the sensitivity of the instruments, 1×10^{-5} mm Hg), and the difference between pressure drops on sides A and B

is less than 0.3%. Of course, the results from sides A and C are more accurate because their pressure tap hole's diameter are equal to 1/16 inch (see Chapter 4).

Figure (5.11) shows the results of f versus the Reynolds number. The governing equation for the fitted line in the logarithmic plane is

$$f = \frac{0.093}{Re^{0.254}} \quad (5.6)$$

The fluctuation in the friction pressure drop was less than 0.3% $\{\Delta(\Delta p_x)/\Delta p_x\}$, and even when we changed the electronic manometer scale to one higher scale (less sensitive), this small fluctuation was eliminated.

The friction factor (f_s) was calculated in an alternative way using average wall shear stress distribution ($\bar{\tau}_w$) and equations (5.7) to (5.9). In deriving (5.7) to (5.9) the following situations were considered.

The static pressure measurements around the subchannel periphery, friction pressure drops on sides A and C and the wall shear stresses around $\theta=30$ degrees showed that there is good symmetry between subchannels 1, 2, 3, and 4 and also between 3 and 5 and between 1 and 6. Therefore AB, Aa and Bb (shown in Figure (5.9)) are isotach lines. If we further assume the pressure across the subchannel cross area is uniform, then a momentum balance along the x direction

on subchannel yields:

$$\frac{\Delta P_{xs}}{L} = \frac{R \int_0^{\pi/6} \tau_w(\theta) d\theta}{A_s} \quad (5.7)$$

where ΔP_{xs} has been introduced to distinguish between pressure drop measured using the results of the pressure tap holes (ΔP_x) and this latter method (Eq. (5.7)). By definition

$$\int_0^{\pi/6} \tau_w(\theta) d\theta \equiv \pi/6 \cdot \bar{\tau}_w \quad (5.8)$$

Now using Equations (5.7) and (5.8) and the definition of friction factor we obtain:

$$f_s = \frac{\Delta P_{xs}/L}{\frac{4}{D_h} \frac{\rho}{2} \bar{V}^2} = \frac{R \frac{\pi}{6} \bar{\tau}_w}{A_s \frac{4}{D_h} \frac{\rho}{2} \bar{V}^2} \quad (5.9)$$

Figure (5.11) shows the results and equation (5.10) governs the fitted line.

$$f_s = \frac{0.1232}{Re} \quad (5.10)$$

As Figure (5.11) shows, for all cases the friction factor resulted from integration of the wall shear stresses

(f_s) is smaller than (f) (calculated from Eq. (5.5)). This agrees with the experimental results of reference (37).

5.2.6 Results - Peripheral Static Pressure Distribution

In the intermediate test section we could measure the static pressure only at $\theta=0$ degrees and $\theta=30$ degrees. The results of experiments at Reynolds equal to 9520 showed that there is a small difference between $p_s(30)$ and $p_s(0)$, i.e. $p_s(0)-p_s(30)=0.010$ mmHg. In this case the tap holes were two separate holes with 1/16 inch diameters.

In the final test section the static pressure around the rod periphery at different Reynolds numbers was measured. These new results confirmed the previous results and showed that p_s is a function of θ and decreases as θ increases. In order to have better accuracy the pressure distribution was measured relative to the orifice plate static pressure, and then it was converted to the absolute pressure. Of course the difference in relative pressure scale is much more; for example, at $Re=4150$ we have:

$$\frac{p_s(30)-p_s(R)}{p_s(0)-p_s(R)} = 92.5\%$$

where $p_s(R)$ is reservoir pressure or static pressure before the orifice plate. The upper part of Figures (5.3), (5.4), & (5.8) show the static pressure distribution.

The static pressure was also measured in negative θ direction and results showed that there is symmetry between

distribution of static pressure distribution in the sub-channels (1) and (3) (within measurement accuracy).

From the above discussion we come to the following conclusion:

If we assume that at a plane located at some point up stream of the measurement plane, the pressure around the subchannel periphery is uniform, then the fact that at the measurement plane $p_s(30) < p_s(0)$ indicates that

$$\left. \frac{dp}{dx} \right|_{30} > \left. \frac{dp}{dx} \right|_0 .$$

Therefore the pressure drop around the subchannel periphery is not uniform, but rather is a function of θ .

This latter result appears implicitly in reference (5), but by considering the given data and flow area between the two velocity gradient curves (see Figure 2.8), it is possible to find the amount of dp/dx for different θ .

The condition

$$\left. \frac{dp}{dx} \right|_{30} > \left. \frac{dp}{dx} \right|_0$$

disagrees with the assumption of uniform dp/dx over the subchannel area included in the Deissler method (25) and many other analytical methods.

Note: We made sure that the $p_s(30) < p_s(0)$ is not affected by the flow leakage. This was proven by the fact

that when we used the blower as the air source (draw the air through the test section) any leakage caused the static pressure to increase. Further from the test section design the probability of the flow leakage at $\theta=30$ was more than $\theta=0$ degrees.

5.3 Error Analysis

In this work we have two major measurement uncertainties. One is associated with the wall shear stress measurement and the other is associated with the flow rate or Reynolds number measurement. We can break down each major uncertainty category into particular parameter error sources as follows:

A) Uncertainty included in the wall shear stress measurement:

- 1) Preston tube error
 - a) Deviation from calibration correlations
 - b) Error due to alignment of the Preston tube in the flow
- 2) Patel calibration correlation error
- 3) Error in density measurement
 - a) Temperature measurement error
 - b) Pressure measurement uncertainty
 - c) Humidity measurement uncertainty
- 4) Error due to pressure tap hole dimensions
- 5) Uncertainty due to viscosity
- 6) Uncertainty due to pressure measurement (Δp_d) apparatus

B) Uncertainty included in the Reynolds number measurement:

The Reynolds number is defined as:

$$Re = \frac{\rho \bar{V} D_h}{\mu} = \frac{w D_h}{A \mu} \quad (5.11)$$

where:

w = flow rate

A = test section area or channel flow area.

Then the Reynolds number uncertainty components are:

- 1) Flow rate measurement uncertainty
- 2) Error due to test section dimension geometry
- 3) Error due to temperature reading.

Appendix B analyzes in full the uncertainty of each particular parameter, which effects either the wall shear stress or Reynolds number measurement. Also in Appendix B the total uncertainty in measured wall shear stress and Reynolds number for each individual case is calculated.

Table (5.3) shows the results of uncertainties for different Reynolds numbers. The results of uncertainty of wall shear stress at $\theta=0$ and $\theta=30$ degrees are shown in Figures (5.2) to (5.4) & (5.7) to (5.8) by error bars. As we see, at different Reynolds numbers the error bar has significant variation and the percent error at $\theta=30$ degrees is different than that at $\theta=0$ degrees. The absolute amount of errors as θ changes is about the same, so we can assume

that one value characterizes the error for all data at a given Reynolds number.

In all cases except $Re=18890$, the maximum uncertainty of Reynolds number is 0.8%. At $Re=18890$ this uncertainty is 2.2%. As Appendix B shows this large difference between these two Reynolds number uncertainties comes from the orifice hole and flow condition. This difference also shows how some small changes in flow circumstances can cause a large amount of error in the flow rate measurement.

5.4 Comparison of Results with Previous Experimental and Analytical Investigations

5.4.1 Comparison of Dimensionless Wall Shear Stress Distribution (τ_w/τ_w) with Previous Experimental Results

As discussed in Chapter 3, many measurements of wall shear stress have been made over a wide geometry range. Only the results of Subbotin et al. (37) are at a comparable P/D to this experiment. Subbotin et al. did a series of experiments on a subchannel model with $P/D=1.1$ and within the range of $18.8 \times 10^3 < Re < 81 \times 10^3$. Figure (5.16) shows the cross section of their test section. As we see, there is not a good symmetry between subchannels in that test section. This asymmetry is due to the presence of separator walls. Therefore the constant velocity curves are not perpendicular to the subchannel isotachs and a strong

secondary flow in the central subchannels exist. Because of these effects their results at θ around 30 degrees showed a little decline. Their results are widely spread out and the results which are shown in Figures (5.13) to (5.15) are determined from the middle of their experimental result points. The authors have noted the decline at $\theta=30$ degrees and tried to determine the actual $\tau_w/\bar{\tau}_w$ distribution at around 30 degrees by extrapolation from a lower θ while meeting the following constraint:

$$\frac{6}{\pi} \int_0^{30} \frac{\tau_w(\theta)}{\bar{\tau}_w} d\theta = 1$$

The extrapolated curve shows that $(\tau_w/\bar{\tau}_w)_{\max}$ occurs at $\theta=30$ degrees and its value is 1.135 ($\tau_w(30)/\bar{\tau}_w=1.135$), which is higher than the results of the present work. This extrapolation however still does not compensate the deviation of their results for $\tau_w/\bar{\tau}_w$ distribution around $\theta=0$ degrees. The reason is that, in the calculation of $\tau_w/\bar{\tau}_w$ distribution around $\theta=0$ degrees, they used the average wall shear stress ($\bar{\tau}_w$) which they had found from the unextrapolated result points and consequently used smaller $\bar{\tau}_w$ than the actual. (Be reminded that their result points for θ around 30 degrees are lower than the extrapolated curve.) Therefore one can conclude that their result points for $\tau_w/\bar{\tau}_w$

distribution around $\theta=0$ degrees is still higher than the actual. For the same reasons as above one can describe why their extrapolated curve at $\theta=30$ degrees predicts $\tau_w/\bar{\tau}_w$ greater than the results of the present work.

Trupp and Azad⁽³⁸⁾ did a series of experiments on triangular subchannel models with variational pitch to diameter ratio (P/D). The closest one to the present work is one with $P/D=1.2$. Their results show that $\tau_{w\max}$ does not occur at $\theta=30$ degrees, but rather it occurs at $\theta=24$ degrees. This shift in $\tau_{w\max}$ is either due to asymmetry between subchannels (as was discussed in the previous paragraph) or due to the fact that at the measurement point ($L/D_h=51.2$) the flow was not a fully developed flow but rather it was a developing flow. (This point has been hinted at in reference (38)).

5.4.2 Comparison of Dimensionless Wall Shear Stress Distribution ($\tau_w/\bar{\tau}_w$) with Analytical Predictions

Figures (5.12) to (5.15) compare the present results with the analytical results of Ibragimov (5), Bartzis and Todreas (23), Carajilescov and Todreas (17), Deissler and Taylor (25) and Sparrow and Loeffler (40) (laminar flow; shown only in Figure (5.12)).

The dimensionless wall shear stress distribution ($\tau_w/\bar{\tau}_w$) calculated from Ibragimov et al. method is not a function of Reynolds number. This method predicts that the $\tau_w/\bar{\tau}_w$ distribution for small P/D is flatter than actual. This is

probably due to the form of model which has been chosen to include motion of large scale eddies.

The results of the Bartzis method for the τ_w/τ_w agrees with the present results better than the results of the Ibragimov method.

Carajilescov's method agrees with the results of the present work better than Ibragimov, especially at the range of $0 < \theta < 16$ degrees, but outside this range the agreement is poor. Further, as reference (17) shows, this method at high Reynolds number predicts a non-monotical τ_w/τ_w distribution, i.e. the maximum τ_w shifts to a lower θ angle. This probably happens because the author considers very strong effects of secondary flow in the related equation of the model.

Some of the differences between the methods of the references (23) and (17) are that first, in (23) unisotropic eddy viscosities ($\epsilon_{mr}/\epsilon_{m\theta} = n \neq 1$) are considered, while in (17) isotropic ($\epsilon_{mr} = \epsilon_{m\theta}$) are used; second, in (17) the turbulent kinetic energy and turbulent transport equations are solved based on different length scale in the radial and tangential directions while the method of (17) is based on only one length scale (radial); third, (17) considers secondary flow while (23), in order to find τ_w/τ_w , doesn't consider any secondary flow.

The results of Deissler and Taylor (25) also have been compared. The results of this method are not functions of Reynolds number (20,000 to 900,000). As Figures (5.12) to (5.14)

show these results are far from the actual $\tau_w/\bar{\tau}_w$ distribution. This is due to the form of their assumed momentum balance which as discussed in Chapter 2 was

$$\tau_w = - \frac{\Delta A}{\Delta l} \frac{dp}{dx} ,$$

where ΔA = flow element area (between two constant velocity-gradient lines) and

Δl = element of peripheral distance along the wall, so they assumed there were no shear forces acting on the constant velocity gradient lines.

5.4.3 Comparison of Friction Factors (f, f_s)

Many investigators have measured or calculated the friction factor in infinite triangular arrays using different experimental or analytical methods. Despite the large number of results, there is still a large amount of uncertainty connected with this parameter. Here we compare some of the most well known or recent results with the present work.

K. Rehme (24) measured the pressure drop by doing a series of systematic measurements for different P/D and within a wide range of Reynolds numbers on bundles with different numbers of rods. He also compared most of the results of other investigators. In many cases, and specially at low Reynolds numbers, the friction factor for equivalent annular zone could match the upper limit. For equivalent

annular zone we can get:

$$\sqrt{\frac{2}{f}} = 2.5 \ln \operatorname{Re} \sqrt{\frac{2}{f}} + 5.5 - \frac{3.966 + 1.25x}{1+x} - 2.5 \ln 2(1+x), \quad (5.12)$$

where for triangular array rods the "x" turns out to be

$$x = \sqrt{\frac{2\sqrt{3}}{\pi} P/D}. \quad \text{For the present case, equation (5.12) reduces to:}$$

$$\sqrt{\frac{2}{f}} = 2.5 \ln \operatorname{Re} \sqrt{\frac{f}{2}} - 0.063181; \operatorname{Re} = \frac{\rho \bar{V} D_h}{\mu}, \quad (5.13)$$

where D_h is the subchannel hydraulic diameter as before. This formula is based on the equation of friction factor for annular pipe flow.

The equivalent annular zone can be defined if the hexagonal elementary cell around the interior rod of an infinite triangular array rods are replaced by the annular zone of the same area (see Figure (5.17)).

Figure (5.18) compares the results of reference (24) for $P/D=1.084$ and $P/D=1.125$ and the results of equation (5.13) with the present results (f_s). As we can see, the results of equation (5.13) are very close to the friction factors deduced from the shear stress distribution (f_s).

Reference (37) has fit an analytical expression to the friction factors from a series of experimental measurements. In that method the wall shear stress was integrated over the subchannel perimeter (like equation (5.4)). The resultant expression was as follows:

$$f_s/f_o = 0.57 + P/D [1 - e^{-112(P/D-1)}] (\log P/D)^{0.27P/D} ;$$
$$1 < P/D < 1.2 \quad , \quad (5.14)$$

where f_o is the friction factor for a round tube when the flow is passing through the tube at the same Reynolds number as f_s is calculated. For $P/D=1.1$, equation (5.11) gives $f_s/f_o=0.995$, which is a little less than the factor that can fit the results of the present work. In the case of the present data we get $f/f_o=0.997$. At Reynolds number around 10,000, the friction factor resulting from either the present work or other investigators lies below the fitted line in the logarithmic coordinates. Therefore the form of $f = k/Re^n$ is not an accurate expression for friction factor when the Reynolds number varies widely.

Experiment Number	Flow Source	$Re \times 10^3$	$\tau_{wf}(0) \times 10^2$ (lb_f/ft^2)	$\tau_w(0) \times 10^2$ (lb_f/ft^2)	$\bar{\tau}_w \times 10^2$ (lb_f/ft^2)	f	f_s
1	Blower	4.14	0.2137	0.282	0.327	0.0112	0.01049
2	Blower	4.15	0.2174	0.29	0.337	0.011	0.01039
3	Blower	9.11	0.78	1.0558	1.2498	0.00896	0.008032
4	⁺ Compressor	9.52	0.7285	1.087	1.3	0.009	0.009055
5	⁺ Blower	10.45	0.847	1.325	1.614	0.00863	0.007562
6	Compressor	18.89	2.0638	2.71	3.22	0.008095	0.0075
7	Compressor	36.17	9.25	12.2	14.4798	0.006468	0.006054

TABLE (5.2)

Experimental Results for: $\tau_{wf}(0)$, $\tau_w(0)$, $\bar{\tau}_w$, f , f_s .

Legend: ⁺=the results of these experiments are not reliable.

TABLE (5.3)

The Wall Shear Stresses and Reynolds Numbers' Uncertainties.

Exp. No.	Re No.	Flow Source	Orifice Hole Diameter (D'' ₂)	$\tau_w(0) \times 10^2$ (lb/ft ²) $\times 10^2$	$\tau_w(30) \times 10^2$ (lb/ft ²) $\times 10^2$
1	4140	Blower	0.5	0.282	0.369
2	4150	Blower	0.5	0.29	0.386
3	9110	Blower	1.45	1.0558	1.407
6	19890	Compressor	0.5	2.71	3.63
7	36170	Compressor	1.45	12.2	16.09

continued on next page

TABLE (5.3) (continued)

Exp. No.	Measurement Uncertainty		Total Uncertainty (Measurement Uncertainty Plus Correlation Uncertainty)				Re No. Uncertainty $\Delta Re/Re\%$
	$\Delta \tau_w / \tau_w\%$ $\theta=0$	$\Delta \tau_w / \tau_w\%$ $\theta=30$	$\Sigma \tau_w / \tau_w\%$ $\theta=0$	$\Sigma \tau_w / \tau_w\%$ $\theta=30$	$\Sigma \Delta \tau_w$ lb/ft^2 $\theta=0$	$\Sigma \Delta \tau_w$ lb/ft^2 $\theta=30$	
1	0.88	0.6	1.88	1.6	5.3×10^{-5}	5.9×10^{-5}	0.8
2	0.88	0.6	1.88	1.6	5.45×10^{-5}	6.2×10^{-5}	0.8
3	0.78	0.7	2.28	2.2	2.4×10^{-4}	3.09×10^{-4}	0.8
6	0.98	0.79	2.48	2.29	6.7×10^{-4}	8.3×10^{-4}	2.2
7	0.6	0.56	2.08	2.06	2.5×10^{-3}	3.3×10^{-3}	0.8

Pitch to Diameter Ratio	1.1
Rod Diameter	1.5"
Hydraulic Diameter	0.5013"

FIGURE (5.1)
Subchannel Boundaries Identification

FIGURE (5.2)

Wall Shear Stress Distribution

FIGURE (5.3)
 Wall Shear Stress and Static Pressure Distribution

FIGURE (5.4)

Wall Shear Stress and Static Pressure Distribution

FIGURE (5.5)

Wall Shear Stress Distribution

FIGURE (5.6)

Wall Shear Stress Distribution

FIGURE (5.7)

Wall Shear Stress Distribution

FIGURE (5.8)

Wall Shear Stress and Static Pressure Distribution

FIGURE (5.9)

Subchannels Identification

FIGURE (5.10)
Comparison of the Present Nondimensional Wall
Shear Stress Distributions

FIGURE (5.11)
Friction Factor

FIGURE (5.12)

Comparison of the Wall Shear Stress Distribution
 Results with Other Investigators

FIGURE (5.13)

Comparison of the Present Nondimensional
 Wall Shear Stress Results with Other investigators

FIGURE (5.14)

Comparison of the Present Nondimensional
 Wall Shear Stress Results with Other Investigators

FIGURE (S.15)

Comparison of the Present Nondimensional
 Wall Shear Stress Results with Other Investigators

FIGURE (5.16)

Diagram of Subbotin et al. Channel Cross Section

FIGURE (5.17)

Equivalent Annular Zone Configuration

FIGURE (5.18)

Comparison of the Present Friction Factor Results with Other Investigators

CHAPTER 6

CONCLUSION AND FUTURE WORK

This chapter describes the conclusion of the present investigation and reveals future work recommendations.

6.1 Conclusion

The wall shear stress is a monotonical function of θ , within $0 < \theta < 30$ degrees, and the maximum value of the wall shear stress occurs at $\theta = 30$ degrees.

The distribution of τ_w is symmetrical with respect to $\theta = 0$ and $\theta = 30$ degrees, provided that there is good symmetry between the neighboring subchannels.

There were no detectable effects of secondary flow in the region very close to the wall. Therefore the wall shear stress is not skewed from the rod axes by the effect of secondary flow.

The friction factor calculated from the average wall shear stress distribution (f_s) is less than the friction factor resulting from the measured pressure drop at $\theta = 0$ degrees (f). The f_s can be calculated using the equivalent annular zone formula (Eq. (5.13)) or the round tube friction factor multiplied by a coefficient.

The static pressure is not uniform around the subchannel periphery and decreases as θ increases from 0 to 30 degrees.

At $Re = 4000$ the flow regime is not a stabilized turbulence, but, however, it is not laminar. The shape of the dimensionless wall shear stress distribution curve ($\tau_w / \bar{\tau}_w$)

at this Reynolds number is very different from when the Reynolds number is greater than 9000.

6.2 Future Work

1) During the course of the present experiments, we could only achieve a maximum flow rate corresponding to a Reynolds number around 37000. It is expected that the effects of secondary flow at high Reynolds numbers are more pronounced than at lower Reynolds numbers (e.g. 17). The experimental results of wall shear stress distributions corresponding to higher Reynolds numbers will provide a better basis for determining the behavior of secondary flow and its effects. In order to establish a reliable analytical model for the calculation of the velocity field and the wall shear stress distribution in a wide range of flow rates, the high Reynolds number results ($100-300 \times 10^3$) are necessary.

2) Three different designs of flow delivery systems were used to supply air through the test section (see Chapter 4). The investigation showed that the flow delivery system with the blower as the air source yields a cleaner air flow and more steady flow rate than the others. Therefore for future work the utilization of a blower as the air source is recommended.

3) The results of the experiments showed that the non-dimensional wall shear stress distribution (τ_w/τ_w) at flow rates corresponding to Reynolds numbers greater than 9000 is a weak function of the Reynolds number. However, a very

accurate and sensitive source of flow, e.g. a blower equipped with an electric voltage regulator and temperature controller is necessary if one is interested in studying the function $(\tau_w/\bar{\tau}_w = f(Re))$.

4) The data reading and measurement instruments are very sensitive to environmental temperature variation during each individual experiment. Therefore, in order to obtain more accurate data, it is necessary that the room temperature be kept constant (less than 0.5 °C variation). In this thesis this control was achieved by running all experiments late at night. If more convenient testing times are desired, the Barocel should be equipped with a thermal bas.

5) This work was devoted to a fixed pitch to diameter ratio ($P/D=1.1$). However, as mentioned in Chapter 4, the test section is capable of being modified to various pitch to diameter ratios. Measurements at different pitch to diameter ratios are necessary to establish an analytical expression for the friction factor variation with P/D .

The experimental measurement at smaller pitch to diameter ratios is also recommended. The reason is that by comparison between the flow patterns in a triangular array subchannel and a triangular duct, one can conclude that the effects of secondary flow are more pronounced for smaller P/D ratios.

6) The probes which were utilized in this work were not calibrated specifically. Rather, we accepted the previous calibration correlations. An error analysis shows a large

percentage of uncertainties in results are due to calibration correlation uncertainties. Therefore a need for establishing a new set of accurate calibration correlations is apparent.

7) The turbulence intensity of the flow in the region very close to the wall was measured which gives some knowledge of the effect of secondary flow in the region. However detection of secondary flow loop (or loops) in subchannel using the Preston tube is qualitatively and may be quantitatively possible. For this purpose the flow turbulence intensities should be measured at the bulk of the subchannel flow and along the isotachs. In fact the present designed test section and probe positioner provide one of the best design facilities to handle this project. The only necessary modification is to change some parts of the probe radial positioner, which is custom-made by the United Sensor Co., by parts with closer tolerances such that the Preston tube location can be very accurately defined.

APPENDIX A

TABLES OF DATA AND RESULTS

This section presents the tables of resulting data corresponding to each Reynolds number.

Re=4140

TABLE A.1(a)

Wall Shear Stress Distribution Data and
RMS of the Preston Tube Pressure Difference

θ deg	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{dt})}{\tau_w(30)}$ %	θ deg	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{dt})}{\tau_w(30)}$ %
0	0.2815	0.861	63	23	0.35	1.0688	
1		0.864		24		1.091	
2		0.865		25	0.363	1.107	62.95
3	0.284	0.8685		26		1.116	
4		0.8731		27		1.122	
5	0.29	0.881	62.8	28	0.369	1.269	62.95
6		0.8899		29		1.1928	
7		0.9021		30	0.369	1.1928	
8		0.9144		32	0.369		
9		0.9235		33	0.3611		
10	0.307	0.9327	62.9	34.5	0.361		
11		0.945		-5	0.29		
12		0.9572		-10	0.307		
13		0.9664		-15	0.328		
14		0.9755		-20	0.35		
15	0.322	0.9847		-25	0.363		
16		0.9969		-30	0.3662		
17		1.006		-34.5	0.363		
18		1.0153					
19		1.0245					
20	0.336	1.0306	62.95				
21		1.0428					
22		1.0428					

TABLE A.1(b)
Pressure Drop

Tap Holes	A ₁ -A ₂	C ₁ -C ₂	A ₁ -A ₃	B ₁ -B ₂	B ₂ -B ₃	D ₁ -D ₂
$\frac{\Delta P_x}{L}$ lb _f /ft ² ·in	0.0285	0.0285	0.0286	0.0281	0.02767	0.02698

TABLE A.1(c)
Flow Properties

$\bar{\tau}_w \times 10^2$ lb _f /ft ²	T _{dry} °C	T _{wet} °F	$\rho \times 10^2$ lb _m /ft ³	p _s (0) mm Hg	(p _{orifice}) ^{p₁} mm Hg	Δp _u mm Hg
3.27×10 ⁻³	22.8	57	7.26376	743.9496	743.2896	21.12

Re=4150

TABLE A.2(a)

Wall Shear Stress Distribution Data and
RMS of the Preston Tube Pressure Difference

θ deg	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{dt})}{\tau_w(30)}$ %	θ deg	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{dt})}{\tau_w(30)}$ %
0	0.28978	0.861	63.3	17		1.01395	
1		0.8625		18		1.0258	
2		0.8625		19		1.0377	
3		0.8729		20	0.36289	1.0511	62.82
4		0.8803		21		1.0659	
5	0.299496	0.8892		22		1.08075	
6		0.8996		23		1.0956	
7		0.91		24		1.11044	
8	0.3119865	0.9204	62.7	25	0.380863	1.1268	62.95
9		0.9293		26		1.1327	
10		0.9382		27		1.1395	
11		0.95012		28	0.3858	1.1446	
12	0.323989	0.9605		29		1.14461	
13		0.9709		30	0.38583	1.1461	62.95
14		0.98128		32	0.3858		
15	3.2983	0.99168	62.82				
16		1.0021					

TABLE A.2(b)

Static Pressure Distribution

θ	$\frac{P_s(\theta)}{P_s(0)} \%$	$\frac{P_s(\theta) - P_s(R)}{P_s(0) - P_s(R)} \%$
0	1	1
4	99.99995	99.94
8	99.999866	99.85
15	99.9995972	99.552
20	99.999463	99.403
25	99.9993958	99.328
30	99.9993287	99.254

TABLE A.2(c)
Pressure Drop

Tap Holes

	A_1-A_2	C_1-C_2	A_1-A_3	B_1-B_2	B_2-B_3	D_1-D_2
$\Delta P_x/L$ ($lb_f/ft^2 \cdot in$)	0.02899	0.02899	0.0291	0.0286	0.02814	0.028

TABLE A.2(d)
Flow Properties

$\bar{\tau}_w \times 10^3$ lb_f/ft^2	T_{dry} $^{\circ}C$	T_{wet} $^{\circ}F$	$\rho \times 10^2$ lb_m/ft^3	$P_s(0)$ mm Hg	P_1 ($P_{orifice}$) mm Hg	ΔP_u mm Hg
3.368	24.5	60	7.2233	744.55	743.83	21.51

Re=9110

TABLE A.3(a)

Wall Shear Stress Distribution Data and
RMS of the Preston Tube Pressure Difference

θ deg	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{d1})}{\tau_w(30)} \times$ %	θ deg	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{d1})}{\tau_w(30)} \times$ %
0	1.0558	0.8447	13.51	23		1.1	
1	1.0625	0.8501		24	1.3855	1.1085	13.51
2		0.848		25		1.0793	
3	1.069	0.8553	13.51	26	1.395	1.116	13.51
4		0.8625		27		1.1193	
5	1.0955	0.8765	12.84	28	1.4011	1.121	13.51
6	1.0981	0.8786	13.51	29		1.122	
7		0.8889		30	1.4069	1.1256	13.51
8	1.1384	0.9109	13.51	32	1.4069		13.51
9		0.92					
10	1.1732	0.9387	13.51	-1	1.0625		
11		0.9497		-2	1.063		
12	1.205	0.9681	13.51	-5	1.09		
13		0.9841		-10	1.16		
14		1.00		-28	1.388		
15	1.2736	1.019	13.51	-30	1.4		
16		1.0321		-32	1.395		
17		1.0441					
18	1.32875	1.0625	13.51				
19		1.0705					
20	1.347	1.0777	13.51				
21		1.0865					
22		1.0921					

*Absolute value of the RMS is not
correct in this measurement.

TABLE A.3(b)

Static Pressure Distribution

θ deg.	$\frac{p_s(\theta)}{p_s(0)} \%$	$\frac{p_s(\theta) - p_s(R)}{p_s(0) - p_s(R)} \%$
0	1	1
5	99.8765	99.999618
10	99.6296	99.998855
15	99.531	99.998549
20	99.3827	99.99809
25	99.3209	99.9979
30	99.2593	99.9977
33	99.38	99.998

TABLE A.3(c)
Pressure Drop

Tap Holes						
	A ₁ -A ₂	C ₁ -C ₂	A ₁ -A ₃	B ₁ -B ₂	B ₂ -B ₃	D ₁ -D ₂
$\Delta p_x/L$ lb _f /ft ² ·in	0.104	0.104	0.1045	0.1095	0.1067	0.1077

TABLE A.3(d)
Flow Properties

$\bar{\tau}_w \times 10^2$ lb _f /ft ²	T _{dry} °C	T _{wet} °F	$\rho \times 10^2$ lb _m /ft ³	P _s (0) mm Hg	P ₁ (P _{orifice}) mm Hg	ΔP _u mm Hg
1.2499	23	57	7.2	734.43	732.0	1.01

Re=9525

TABLE A.4 (a)

Wall Shear Stress Distribution Data

θ deg.	$\tau_w \times 10^2$ lb/ft ²	$\tau_w/\bar{\tau}_w$ (fitted curve)	θ deg.	$\tau_w \times 10^2$ lb/ft ²	$\tau_w/\bar{\tau}_w$ (fitted curve)
0	1.087	0.8362	18		1.0596
1		0.8366	19		1.0727
2		0.8374	20	1.406	1.085
3		0.8382	21		1.0988
4		0.846	22		1.1073
5	1.116	0.8574	23		1.1211
6		0.8674	24		1.1296
7		0.8766	25	1.47	1.1365
8		0.8881	26		1.144
9		0.9027	27	1.498	1.1496
10	1.199	0.922	28		1.1511
11		0.9382	29	1.502	1.155
12		0.9535	30	1.504	1.155
13		0.9712	32	1.501	
14		0.9904	34	1.501	
15	1.31	1.006	35	1.48	
16		1.0227			
17		1.0396			

TABLE A.4(b)
Pressure Drop

Tap Holes		
	$A_1 - A_2$	$C_1 - C_2$
$\Delta p_x / L$ $lb_f / ft^2 \cdot in$	0.097	0.097

TABLE A.4(c)
Flow Properties

$\bar{\tau}_w \times 10^2$ lb_f / ft^2	T_{dry} $^{\circ}C$	T_{wet} $^{\circ}F$	$\rho \times 10^2$ lb_m / ft^3	$p_s(0)$ in Hg	p_1 (P _{orifice}) in Hg	Δp_u in Hg
1.3005	21	49	8.53	31.72	31.72	4.3

Re=10475

TABLE A.5(a)

Wall Shear Stress Distribution Data

θ	$\tau_w \times 10^2$	$\tau_w / \bar{\tau}_w$	θ	$\tau_w \times 10^2$	$\tau_w / \bar{\tau}_w$
	(lb _f /ft ²)			(lb _f /ft ²)	
0	1.313	0.824	18	1.68	1.0654
2	1.33	0.824	20	1.757	1.0902
3	1.332	0.825	22		1.1088
4	1.35	0.836	24		1.123
5	1.374	0.851	25	1.817	1.1255
6		0.844	26		1.1317
8		0.8982	27	1.828	1.1323
10	1.515	0.9353	28	1.822	1.13
12		0.9694	29	1.855	1.13
14		1.0053	30	1.855	1.13
15	1.651	1.0227	32	1.855	
16		1.0363			

TABLE A.5(b)
Pressure Drop

Tap Holes	A_1-A_2	C_1-C_2	A_1-A_3	B_1-B_2	B_2-B_3
$\frac{\Delta p_x}{L}$ $lb_f/ft^2 \cdot in$	0.113	0.113	0.1183	0.119	0.116

TABLE A.5(c)
Flow Properties

$\bar{T}_w \times 10^2$ lb_f/ft^2	T_{dry} $^{\circ}F$	T_{wet} $^{\circ}C$	$\rho \times 10^2$ lb_m/ft^3	$p_s(0)$ in Hg	p_1 ($p_{orifice}$) in Hg	Δp_u in Hg
1.6144	21	50	7.2	28.7	28.62	0.055

Re=18890

TABLE A.6(a)

Wall Shear Stress Distribution Data and
RMS of the Preston Tube Pressure Difference

θ deg.	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{di})}{\tau_w(30)}$ %	θ deg.	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{di})}{\tau_w(30)}$ %
0	2.71	0.83992	17.47	22		1.1158	
1	2.7	0.8368		23		1.1126	
2	2.714	0.84116		24		1.1188	
3	2.719	0.8427		25	3.62	1.122	17.2
4	2.743	0.8501		26		1.123	
5	2.772	0.859	17.09	27		1.124	
6		0.8678		28	3.63	1.125	16.7
7		0.8833		29			
8		0.8942		30	3.63	1.125	16.6
9		0.9081		32	3.63		16.7
10	2.99	0.9267	16.7	35	3.62		
11		0.9391					
12	3.09	0.9577	16.7	-2	2.71		
13		0.9732		-20	3.51		
14		0.9918		-30	3.64		
15	3.259	1.01	16.8	-32	3.64		
16		1.0321					
17		1.0383					
18		1.0662					
19		1.085					
20	3.483	1.091	17.				
21		1.1065					

TABLE A.6(b)

Pressure Drop

Tap Hole	$A_1 - A_2$	$C_1 - C_2$
$\Delta p_x / L$ $\text{lb}_f / \text{ft}^2 \cdot \text{in}$	0.27517	0.27517

TABLE A.6(c)

Flow Properties

$\bar{r}_w \times 10^2$ $\text{lb}_f / \text{ft}^2$	T_{dry} $^{\circ}\text{C}$	T_{wet} $^{\circ}\text{F}$	$\rho \times 10^2$ $\text{lb}_m / \text{ft}^3$	$p_s(0)$ in Hg	p_1 (P _{orifice}) in Hg	Δp_u in Hg
3.2265	30	66	10.48	43.3	36.2	13.9

Re=36170

TABLE A.7(a)

Wall Shear Stress Distribution Data and
RMS of the Preston Tube Pressure Difference

θ deg.	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{d1})}{\tau_w(30)}$ %	θ deg.	$\tau_w \times 10^2$ (lb _f /ft ²)	$\tau_w / \bar{\tau}_w$ (fitted curve)	$\frac{RMS(\Delta p_{d1})}{\tau_w(30)}$ %
0	12.207	0.8431	23.8	17		1.053	
1		0.845		18		1.0641	
2		0.849		19		1.0724	
3	12.42	0.8568		20	15.578	1.0807	22.8
4		0.8637		21		1.0869	
5	12.66	0.8741	22.9	22		1.09106	
6		0.8845		23		1.0966	
7		0.898		24		1.09867	
8		0.9114		25	15.934	1.1007	23.1
9		0.9259		26		1.1021	
10	13.737	0.9461	23.5	27	16.026	1.10678	22.9
11		0.9604		28		1.10844	
12	14.191	0.9805		29	16.083	1.11073	
13		0.9943		30	16.095	1.1115	22.9
14		1.0088		33	16.072		
15	14.93	1.0247	23.6				
16		1.0399					

TABLE A.7(b)
Static Pressure Distribution

θ	$\frac{p_s(\theta)}{p(0)}$	$\frac{p_s(\theta) - p_s(R)}{p_s(0) - p_s(R)}$
0	1	1
5	99.998126	99.9459
10	99.99623	99.8918
15	99.99438	99.8377
20	99.99250	99.7835
25	99.98875	99.6753
30	99.9878	99.6212
-5	99.602	99.891

TABLE A.7(c)

Pressure Drop

Tap hole	$A_1 - A_2$	$C_1 - C_2$
$\Delta p_x / L$ $lb_f / ft^2 \cdot in$	1.234	1.234

TABLE A.7(d)

Flow Properties

$\bar{\tau}_w \times 10^2$ lb_f / ft^2	T_{dry} $^{\circ}C$	T_{wet} $^{\circ}F$	$\rho \times 10^2$ lb_m / ft^3	$p_s(0)$ $mm\ Hg$	p_1 ($p_{orifice}$) $in\ Hg$	Δp_u $mm\ Hg$
14.479	26.5	56	7.476	771.5	30.4	16.5

APPENDIX B
ERROR ANALYSIS

B.1 General

In this appendix we try to evaluate the existing errors in the measurement of wall shear stress and Reynolds number. There are many types of errors which can contribute to the error in each variable. We can divide the total error into major classified errors. There are two separate sets of definitions which define these major errors. The first definition categorizes the total error into: random and systematic (e.g. 59, p. 10 and 60, p. 7); and the second definition categorizes into: accidental error, fixed error and mistakes (41). In the present work, we use the second definition because the sources of errors are defined more precisely. If one observer uses certain instruments to measure a particular parameter at a fixed point, then the resulting errors will not be due to accident. Mistakes are completely erroneous reading of scales, watches, and so on (41). In our final analysis, we ruled out those results that we thought might be completely erroneous (mistakes). Therefore, in the present work we have only fixed errors. In reference (41) the term uncertainty is defined as the possible value of error which the data might have, while the error is as the actual difference between the true and observed values.

Suppose that the final result, R , is a function of n independent variables, $x_1, x_2, \dots, x_n$,

$$R = R(x_1, x_2, \dots, x_n) \quad . \quad (B.1)$$

In the following text we will discuss methods which we can use to evaluate the uncertainty included in the final result, R :

First: If R is a linear function of n independent variables, and if the uncertainty interval of the i^{th} variable from its mean is $(\pm \Delta x_i)$, then the uncertainty interval of R , from its mean value, is given by

$$\Delta R = \left| \frac{\partial R}{\partial x_1} \Delta x_1 \right| + \left| \frac{\partial R}{\partial x_2} \Delta x_2 \right| + \dots \quad (B.2)$$

Equation (B.2) is called the linear equation. If it is used the uncertainty in final result (R) will be calculated higher than the most probable. This is because of the fact that the error in each variable can have a range of values, and it is unlikely that all of them will have the most adverse values at the same time.

Second: If R is a linear function of n independent variables, x_i , each of which is distributed with a standard deviation σ_i , then the standard deviation of R is given by

$$\sigma_R = \left[\left(\frac{\partial R}{\partial x_1} \right)^2 \sigma_1^2 + \left(\frac{\partial R}{\partial x_2} \right)^2 \sigma_2^2 + \dots + \left(\frac{\partial R}{\partial x_n} \right)^2 \sigma_n^2 \right]^{\frac{1}{2}} \quad (B.3)$$

Third: If R is a linear function of n independent variables, x_i , each of which is normally distributed, then the uncertainty interval of the final result of R , ω_R , is

$$\omega_R = \left[\left(\frac{\partial R}{\partial x_1} \right)^2 \Delta x_1^2 + \left(\frac{\partial R}{\partial x_2} \right)^2 \Delta x_2^2 + \dots + \left(\frac{\partial R}{\partial x_n} \right)^2 \Delta x_n^2 \right]^{\frac{1}{2}} \quad (B.4) ,$$

where Δx_i is the uncertainty interval for the variable x_i th .

When the result R is not a linear function of n variables equations (B.2) to (B.4) with approximation still can be used. In engineering experiments it is normal to use equation (B.2) or (B.4). (As the calculation of standard deviation is cumbersome, see reference (58, p.211) for example for calculation of standard deviation of pressure or velocity measurement.) In order to use equation (B.4) it is necessary that the variables be distributed normally which may not be true at all times, and all variables should have the same odd number (20 to 1 or 30 to 1 etc.). However the procedures for calculation of uncertainty from equations (B.2) and (B.4) are the same and the answers are close (ratio is approximately $\sqrt{2}$), but equation (B.2) predicts the larger uncertainty which is safer because we can never consider all of the contributed uncertainties.

In the following, we will analyze and calculate the uncertainty or error of each particular parameter under the conditions of the present work, and then use equation (B.2) to calculate the total uncertainties.

B.2 Preston Tube Error

B.2.1 Deviation from Calibration

In using a Preston tube to measure the wall shear stress two basic assumptions are accepted as follows:

1) The velocity distribution in the region close to the wall ($y^+ \leq 30$) obeys the universal velocity law ($u^+ = f(y^+)$), i.e. the distribution is independent of the kind of flow (internal or external) or the shape of the flow cross section (uniform or nonuniform around the periphery) in case of internal flow.

2) The different sources of the disturbances of Preston tube measurement have also a universal character in accordance with the first assumption. These sources of disturbances can cause differences between the real stagnant pressure and that registered by the Preston tube. These differences are due to viscosity effects from the non-rotational, non-symmetrical flow around the Preston tube (different velocity gradient in different direction around the Preston tube) or the presence of the wall and turbulent velocity fluctuations.

Several investigators, e.g. (26), (27) and (28) did extensive studies on the phenomena which effects the application of Preston tube in the measurement of stagnant pressure and wall shear stress. The conclusion is as follows:

a) Viscosity effect: The total pressure will be determined too high for very low velocity flows. The reason is

that when the Preston tube Reynolds number is not very large compared with unity, the viscous forces acting in the fluid may be of sufficient size compared with the inertia forces (viscous flow). Then, under this condition, the Bernoulli formula is not applicable. Investigators of reference (27) did a series of experiments on Pitot tube measurement limitation. Their results show that when the Pitot tube (probe) Reynolds number ($Re = \rho \bar{V}d/2\mu$, where d is the outside probe diameter) is higher than 1000, there is no effect of viscosity. Between $Re = 50$ and $Re = 1000$ there is little effect of viscosity (less than 1%). For values of Re less than 50 the pressure registered by the Preston tube is larger than the true stagnant stream pressure (27). In the present work the minimum Re is 58 (corresponding to subchannel Reynolds number equal to 4140) which is larger than 50.

b) Shear flow effect: By using a Preston tube in flow passing through a pipe, an error occurs due to the velocity gradient, i.e. due to the non-rotational or non-symmetric flow passing around the Preston tube. The error can be accounted for by relocating the geometric center of the Preston tube in the calculations by an effective location which is shifted towards the higher velocity region. This displacement is less than $(15 \pm 1)\%$ of the Preston (Pitot) tube diameter no matter how large the velocity gradient is (28).

c) Wall effect: When the Preston tube approaches the

wetted wall, an effect occurs opposite to the shear flow effect. This effect is solely dependent upon the wall distance expressed in terms of the Preston tube diameter. A circular Preston tube will measure a velocity which is 1.5% lower than if it was located directly at the wall (28).

d) Effect of turbulent velocity fluctuations: The Pitot tube will measure a higher velocity due to the velocity fluctuations (26).

$$\Delta u_{\text{turb}} = u \left[\frac{1}{\sqrt{1 - \frac{u'^2 + v'^2 + w'^2}{u^2}}} \right]$$

Reference (27) asserts that the universal velocity law formula ($u^+ = f(y^+)$) changes as the Reynolds number changes, and also the formula is a function of the local radius of curvature of the wetted perimeter. The final form of the universal velocity law given in that paper is

$$x^+ = y^+ + 2 \log \left\{ \frac{1}{\sqrt{2} C_T C_w} \left[\frac{1.1513 y^+}{K} + C_T + \right. \right. \\ \left. \left. + \frac{2.3026}{K} \log \left(\frac{C_s (1+T) (2 - C_s \cdot d/2a)}{2 [T + (1 - C_s \cdot d/2a)^2]} \right) \right] \right\} \quad (B.5)$$

where

$$C_T = \frac{u}{u + \Delta u_{\text{turb}}} = 1 - 0.065 \exp \left(- \sqrt{\ln y^+ - 2.3} \right) \quad (\text{B.6})$$

and where x^* and y^* are as defined in Chapter 3,

$$x^* = \log_{10} \left(\frac{\Delta P_d \rho d^2}{4\mu^2} \right), \quad y^* = \log_{10} \left(\frac{r_w \rho d^2}{4\mu^2} \right),$$

where

$T = \text{constant}$ (depends upon the channel geometry)

$d = \text{outside diameter of the Preston tube}$

$\Delta P_d = \text{the difference between stagnant pressure measured by the Preston tube and local flow static pressure}$

$C_s = 1.3$

$C_w = 1.015$

$a = \text{the profile length between the wetted wall and, the point of maximum velocity}$

and K is the ratio of r_m to the local radius of curvature of the wetted perimeter (r_1 or r_2 , see Figure (B.1)), r_m is the radial coordinate of the point which is located on the same azimuthal angle (θ) as the measurement point is and has the maximum velocity. $K < 1$ characterizes a concave surface whereas $K > 1$ characterizes a convex curvature of the wetted perimeter ($K = 0$ for pipe flow).

Equation (B.5) agrees well with the experimental results of Patel (see Chapter 3) under his particular conditions for the range of $2.9 < u^+ < 7.6$.

The Patel calibration was derived for flow passing through tube ($K=0$) while in the present work the flow is passing through a non-uniform cross section. In reference (26) for the case of $K=5$ and $r_1/r_2=0.07$, the deviation from $K=0$ was calculated using equation (B.5) and the results are as follows:

1) If the measurement doesn't match the condition $d \ll 2a$ where d and a are as they were defined in equation (B.5) upon which the calibration curve is based, then the additional term in equation (B.5) which describes the deviation of the velocity profile from the logarithmic behavior becomes more pronounced. As $d/2a$ becomes larger, we will have larger deviation of the real $x^*=f(y^*)$ from the calibration curve, i.e. from the curve $d/2a \rightarrow 0$.

2) Even in the case where the condition $d \ll 2a$ is satisfied, when x^* increases the deviation from calibration curve becomes more pronounced (if the flow cross section is not uniform around the periphery-like tube). This is the effect of boundary layer thickness. Figure (B.2) shows the ratio of $\tau_{d=0}/\tau_{d,K=0}$, can be considered as being solely a function of the wall distance parameter (y_d^+) for $K=\text{constant}$. However, at $y_d^+ \leq 30$, the velocity distribution should be the same for all geometries, Figure (B.3) shows the ratio of $\tau_w/\tau_{d=0}$ which can be regarded as being independent of the Reynolds number for given values of $d/2a$ and K .

The total error related to deviation from calibration

curve can be determined by multiplication of the individual deviations, i.e.

$$\frac{\tau_w}{\tau_{d,K=0}} = \left(\frac{\tau_w}{\tau_{d=0}} \right) \left(\frac{\tau_{d=0}}{\tau_{d,K=0}} \right) \quad (B.7)$$

As Figure (B.2) shows, up to $K=1.5$ the condition of $y_d^+ \leq 30$ is close enough to the calibration criteria. In this work, $K=1.1$, $d/2a=0.03455$ and y_d^+ at its maximum value (corresponding to $Re=36170$ and $\tau_w(30)=16.09 \times 10^{-2} \text{ lb}_f/\text{ft}^2$) is $y_d^+=29.3$ (less than 30). These numbers provide a deviation less than 0.6% ($\tau_w/\tau_{d,K=0}$) which in comparison with the calibration uncertainty of 1.5% is negligible.

The conclusion of the former analysis is that using a Preston tube with outside diameter $d=0.014$ inches under the present conditions gives results (τ_w) within the accuracy of the Patel calibration curve and other errors parameters.

B.2.2 Preston Tube Alignment Uncertainty

The Preston tube was fixed parallel to the rod axis within 0.1 degrees. Then the uncertainty due to the alignment is

$$\frac{\Delta(\Delta P_d)}{\Delta(P_d)} = \cos^2 \phi = \pm 0.99997 \quad ,$$

where ϕ is the angle between the Preston tube axis and rod axis. As we see this uncertainty is very small and we ignore it.

B.3 Correlation Error

As mentioned in Chapter 3, three different formula are given by reference (15) for calculation of the wall shear stress. Each one of these formulae includes a certain amount of uncertainty and fits a particular Preston tube Reynolds number range. In the range $5.6 < x^* < 7.6$

$$x^* = 2 \log_{10} (1.95y^* + 4.1) + y^* \quad . \quad (B.8)$$

In the above-mentioned range, this curve fits the experimental calibration within $\pm 1\%$ of τ_w .

In the Preston tube Reynolds number range $1.5 < y^* < 3.5$ corresponding to $2.7 < x^* < 5.6$, the calibration is represented by the empirical relation

$$y^* = 0.8287 - 0.1381x^* + 0.1437x^{*2} - 0.006x^{*3} \quad (B.9)$$

to within $\pm 1.5\%$ of τ_w .

In the region where the Preston tube Reynolds number is $y^* < 1.5$, corresponding to $0 < x^* < 2.9$ we have

$$y^* = \frac{1}{2}x^* + 0.037 \quad (B.10)$$

to within $\pm 1\%$ of τ_w .

The results of Eqs. (B.9) & (B.10) are shown in Figure (B.4). In the Reynolds number range of this work, only relations (B.9) and (B.10) are needed.

B.4 Density Error

The density of the air flow was calculated using the following formula (29, p. F9):

$$\rho = 1.2929 (273.13/T) \left[(B - 0.3783e) / 760 \right] \quad (B.11)$$

where

B=absolute pressure of air (mm Hg)

e=absolute pressure of air moisture (mm Hg)

T=absolute dry temperature ($^{\circ}$ K)

ρ =density of moist air (gram/lit)

The parameter e could be found by the following relation:

$$\omega = \frac{18}{28.95} \times \frac{e}{760} \quad (B.12)$$

where ω is relative humidity (lb_m of moisture per lb_m of dry air). The parameter ω was found by knowing dry and wet bulb temperatures and using the psychometric chart (30).

For formula (B.11) we have $\rho = \rho(p, T)$

$$\Delta \rho = \frac{\partial \rho}{\partial T} \Delta T + \frac{\partial \rho}{\partial p} \Delta p + \frac{\partial \rho}{\partial e} \Delta e, \quad (B.13a)$$

where $\Delta \rho$, ΔT , Δp and Δe are density, temperature, pressure and moisture pressure uncertainty respectively.

The error due to deviation in humidity measurement is very low because if, for example, we have 1% error due to temperature determination then we have $\Delta \omega = 1.05 (\text{grain}) = 0.00015$

$\frac{\text{lb}_m}{\text{lb}_m \text{ dry air}}$ (31) and so the effect of Δe on ρ will be very

small. Therefore we ignore Δe and write

$$\Delta p = \frac{\partial p}{\partial T} \Delta T + \frac{\partial p}{\partial p} \Delta p \quad (B.13b)$$

For the first term on the right-hand side we have

$$\frac{\partial p}{\partial T} \Delta T = -1.2929 \times 273.13 [(B-0.3763e)/760] \frac{\Delta T}{T^2} \quad (B.14)$$

The temperature reading error was $\Delta T = \pm 0.5^\circ \text{C}$. In the case of using the compressor as an air source the calculation turned out ($Re = 36000$) to be

$$\frac{\partial p}{\partial T} \Delta T = 357.7 \frac{\Delta T}{T^2} = 1.252 \times 10^{-4} \Delta T \frac{\text{lb}}{\text{ft}^3}$$

$$\frac{\partial p}{\partial p} \Delta p = \frac{1.2929 \times 273.13}{Tx760} \Delta p \quad (B.15)$$

At high pressures, we measured the static pressure with the Hellicoid gage. The error included in the pressure gage was $\pm \frac{1}{4} \times \frac{\text{scale}}{100}$ and the gage subdivision was 0.1 psi.

Therefore in this case:

$$\Delta p = \pm 0.05 \pm 0.075 = 0.125 \text{ psi.} \quad (B.16)$$

There could be another error due to the pressure tap hole dimension as follows.

B.5 Pressure Tap Hole Error

The static pressure error for a tap hole with significant

hole-diameter/pipe-diameter ratio can be written (32):

$$\frac{\Delta p}{\tau_w} = f \left[\frac{d}{v} \sqrt{\frac{\tau_w}{\rho}}, \frac{l}{d}, \frac{D}{d}, \frac{D_p}{d} \right] \quad (B.17)$$

where D is the diameter of connection to the manometer, D_p is the pipe diameter, l is the length of the tap hole and d is the tap hole diameter.

The static pressure error for a finite hole size can be written

$$\frac{\Delta p}{\tau_w} = f \left[\frac{d}{v} \sqrt{\frac{\tau_w}{\rho}} \right] \quad (B.18)$$

The investigators of references (32) and (33) have done a series of systematic experiments to determine this pressure error. Figure (B.5) shows the pressure error for $l/d > 1.5$. Using Figure (B.5) and numerical results of $Re=36170$, it turns out for the present work that $\Delta p/p \approx 6 \times 10^{-4}$, which is about zero. But when the tap hole is different than what it is shown to be in Figure (B.5), e.g. one with a housing immediately behind the tap mouth, then the error will be considerable (like what usually is designed).

B.6 Barocel Pressure Sensors and Electronic Equipment Error

The Barocel and relevant electronic equipment are described in Chapter 3. The overall system accuracy depends upon the combination of the sensor, the Electronic Manometer

and the voltmeter. According to the instruction manual for the Barocel and Manometer used in the present work, the related error can be calculated from the following relation:

$$\pm(0.25\% \text{ of reading} + 0.02\% \text{ of operating scale}).$$

The voltmeter error is its last digit number or half of that depending on the case. The time required for the Barocel pressure difference to climb to 63% of the actual pressure difference between the Preston tube and static pressure reading (Δp_d) is 3.3 seconds (34) and for each data reading we waited from 4 to 9 minutes depending upon the case. Therefore, we do not have the effect of pressure transition in our data reading.

B.7 Overall Error in Measurement of τ_w

In the previous sections we have studied each individual uncertainty which could exist in the measurement of τ_w . Here we will see the overall error due to the measurement of the wall shear stresses τ_w . Consider the condition of Reynolds number greater than 9000. In this range we used equation (B.9) to calculate the wall shear stress:

$$\Delta y^* = (-0.1381 + 0.1437 \cdot 2 \cdot x^* - 0.006 \cdot 3 \cdot x^{*2}) \Delta x^* \quad (B.19)$$

or

$$\Delta y^* = K(x^*) \cdot \Delta x^* \quad , \quad (B.20)$$

where Δx^* and Δy^* are uncertainty included in x^* and y^* respectively and

$$K(x^*) = -0.1381 + 0.1437 \cdot 2x^* - 0.006 \cdot 3x^{*2} \quad (B.21)$$

$$\Delta x^* = 0.4343 \Delta \left[\ln \left(\frac{\tau_w d^2}{4\mu^2} \right) \right] \quad (B.22)$$

$$\Delta y^* = 0.4343 \Delta \left[\ln \left(\frac{\tau_w d^2}{4\mu^2} \right) \right] \quad (B.23)$$

Using equation (B.6) to calculate the uncertainty we obtain

$$\frac{\Delta x^*}{0.4343} = \frac{\Delta(\Delta p_d)}{\Delta p_d} + \frac{\Delta \rho}{\rho} \pm 2\Delta[\ln \mu] \quad (B.24)$$

$$\frac{\Delta y^*}{0.4343} = \frac{\Delta(\Delta p_d)}{\Delta p_d} + \frac{\Delta \rho}{\rho} \pm 2\Delta[\ln \mu] \quad (B.25)$$

Then equation (B.19) turns out to be

$$\frac{\Delta \tau_w}{\tau_w} \pm \frac{\Delta \rho}{\rho} \pm 2\Delta[\ln \mu] = K(x^*) \left[\frac{\Delta(\Delta p_d)}{\Delta p_d} \pm \frac{\Delta \rho}{\rho} \pm 2\Delta[\ln \mu] \right] \quad (B.26)$$

The simplified form is:

$$\begin{aligned} \frac{\Delta \tau_w}{\tau_w} = K(x^*) \frac{\Delta(\Delta p_d)}{\Delta p_d} + [1-K(x^*)] \frac{\Delta \rho}{\rho} + 2[1-K(x^*)] \\ \times [\Delta(\ln \mu)] \quad (B.27) \end{aligned}$$

The viscosity of the air is a linear function of temperature in logarithmic scale. Using data from referenc (35) we came

out with:

$$\Delta(\ln \mu) = 15.666 \times 10^{-4} \Delta T \quad (B.28)$$

$$\left[\frac{\text{lb}_f \text{sec}}{\text{ft}^2} \right] \quad [^\circ\text{C}]$$

where $\Delta T = \pm 0.5^\circ\text{C}$.

One of the main uncertainty of Δp_d is due to the visual reading of the integrator voltmeter. The accuracy of the voltmeter is negligibly small.

Although an integrating circuit with a certain time constant (10 or 100 seconds) was used, because of pressure difference fluctuation ($\Delta p'_d$), the reading of the integrator voltmeter was still not a constant. The fluctuation was sometimes the last digit number in two directions (+ or -) and sometimes in only one direction, dependent on the case.

In the case of $Re=36170$ at $\theta=0$ we have:

$$\tau_w(0) = 12.2 \times 10^{-2} \text{ lb}_f/\text{ft}^2 \quad ,$$

$$\Delta p_d = 11.868 \text{ lb}_f/\text{ft}^2 \quad ,$$

$$x^*(0) = 4.8$$

$$y^*(0) = 2.8135 \quad ,$$

$$K(x^*) = 0.8267 \quad ,$$

$$\Delta(\Delta P_d) = \frac{\left[\frac{1b_f}{ft^2} \right]}{100} \times (\text{reading}) + \frac{\left[\frac{1b_f}{ft^2} \right]}{100} \times \overset{\text{scale}}{\downarrow} 1 \times 2.7845 +$$

Barocel scale
(mm Hg)

$$+ 0.001 \times 10 \times 2.7845$$

Voltmeter error

$$\Delta(\Delta P_d) = \frac{0.25}{100} \times 11.848 + \frac{0.02}{100} \times 2.7845 +$$

$$+ 0.01 \times 2.7845 = 0.058$$

Using relations (B.15), (B.16), (B.13b) and data from Appendix A we get

$$\frac{\Delta \rho}{\rho} = 0.01 \quad \text{and}$$

$$\frac{\Delta \tau_w}{\tau_w} = 0.827 \frac{\Delta(\Delta P_d)}{\Delta P_d} + 0.173 \frac{\Delta \rho}{\rho} + 0.2715 \times 10^{-3}$$

Then the overall uncertainty due to measurement is

$$\frac{\Delta \tau_w}{\tau_w} = 0.6\%$$

Formula (B.9) can determine τ_w within $\pm 1.5\%$. Using equation (B.2) we obtain

$$\Sigma \frac{\Delta \tau_w}{\tau_w} = 1.5\% + 0.6\% = 2.1\%$$

But if we use equation (B.4) we will obtain

$$\Sigma \frac{\Delta \tau_w}{\tau_w} = \sqrt{\left(\frac{0.6}{100}\right)^2 + \left(\frac{1.5}{100}\right)^2} = 1.62\%$$

which the ratio of two uncertainties is approximately $\sqrt{2}$.

In the case of a very low Reynolds number (less than 5000) equation (B.10) governs. Then the term $K(x^*)$ in equation (B.27) is $K(x^*)=0.5$. Using equation (B.27) we obtain

$$\frac{\Delta \tau_w}{\tau_w} = 0.5 \times \frac{\Delta(\Delta p_d)}{\Delta p_d} + 0.5 \frac{\Delta \rho}{\rho} + 15.666 \times 10^{-4} \times 0.5.$$

For other Reynolds numbers the procedure for calculation of the τ_w uncertainty is the same as above. As we see, in the calculation of measurement uncertainty the error due to the integrator voltmeter clearly dominates the other uncertainty. This is due to the Δp_d fluctuations. In the calculation of total uncertainty, the uncertainty of the Patel calibration correlation clearly dominates the measurement uncertainty. Therefore the need of a more accurate calibration correlation is apparent.

Table (5.3) shows the τ_w uncertainty corresponding to each Reynolds number. These uncertainties are calculated using equation (B.2). As we see, the percent of error at $\theta=0$ is less than at $\theta=30$ degrees, but these errors are very

close and therefore the error is about uniform for all peripheral positions.

B.8 Flow Rate Uncertainty

The flow rate was calculated using semi-empirical formula from reference (20), i.e. equation (B.29):

$$w = 0.4219 D_2^2 K Y \sqrt{P_1 / T_1 G y \Delta P_u} \quad (B.29)$$

where

w = mass flow rate lb_m/sec

D_2 = orifice diameter, inches

K = flow coefficient, dimensionless (see Table B.1)

y = expansion factor, dimensionless (approximately 1 for air)

P_1 = static pressure before orifice, inches H_g absolute

T_1 = temperature before orifice, $^{\circ}R$

G = specific gravity of gas (dry air = 1.00)

Y = supercompressibility factor, dimensionless (see Figure B.6)

ΔP_u = pressure drop across orifice, inches H_g

The empirical expression for Y is :

$$Y = 1 - \left[0.41 + 0.35 \left(\frac{D_2}{D_1} \right)^4 \right] \left[\frac{\Delta P}{P_1} \times \frac{1}{k} \right] \quad , \quad (B.30)$$

where $k = C_p / C_v = \frac{\text{specific heat at constant pressure of air}}{\text{specific heat at constant volume of air}} \approx 1.4$

D_1 = pipe inside diameter (inches).

The parameter G for wet air can be calculated from the following equation:

$$G_{\text{wet air}} = \frac{1+w}{1+1.608w} \quad , \quad (B.31)$$

where w = specific humidity of the wet air, $\frac{\text{lb}_m \text{ water vapor}}{\text{lb}_m \text{ dry gas}}$.

The values of k are tabulated in Table (B.1) and Y can be calculated from equation (B.30). These values can be reliable within $\pm 0.5\%$ when the following conditions are fulfilled:

$$2 \text{ in.} < D_2 < 24 \text{ in.}$$

$$D_2 \geq 3/8 \text{ in.}$$

$$0.15 < D_2/D_1 < 0.75$$

$$\Delta p_u/p_1 \leq 0.2 \text{ .}$$

where all parameters are as defined before. Otherwise the "K" and "Y" uncertainty will be about $\pm 1\%$.

When these conditions are satisfied, the flow rate can be calculated within $\pm 0.5\%$ uncertainty.

Under the conditions of the present work in all cases the above conditions are satisfied except at Reynolds number = 18890. In following we try to evaluate the flow rate uncertainty corresponding to this Reynolds number.

The diameter of the orifice hole has been machined very accurately (10^{-4} inches) so we ignore its uncertainty.

Then using equations (B.2) and (B.29) we obtain

$$\begin{aligned}
 \frac{w}{0.4219D_2^2} &= Y \sqrt{\frac{P_1}{T_1} Gy \Delta p} \cdot \Delta K + K \sqrt{\frac{P_1}{T_1} Gy \Delta p} \Delta Y + \\
 &+ \frac{1}{2} KY \sqrt{\frac{Gy \Delta p}{T_1}} \Delta p_1 - \frac{1}{2} KY \sqrt{\frac{P_1}{T_1} Gy \Delta p} \Delta T_1 + \\
 &+ \frac{1}{2} KY \sqrt{\frac{P_1 Y \Delta p}{T_1 G}} \Delta G + \frac{1}{2} KY \sqrt{\frac{P_1 G \Delta p}{T_1 Y}} \Delta Y + \\
 &+ \frac{1}{2} KY \sqrt{\frac{P_1 Gy}{T_1 \Delta p}} \Delta(\Delta p) \quad . \quad (B.33)
 \end{aligned}$$

The uncertainty of G can be calculated using uncertainty of w. The uncertainty of w can be estimated using reference (31). For wet temperature reading the error is $\Delta T_{\text{wet}} = \pm 0.5$ F then we have

$$\Delta w = 1.05 \text{ grain} - 0.00015 \frac{\text{lb}_m \text{H}_2\text{O}}{\text{lb}_m \text{dry air}}$$

$$\Delta G = 8.98 \times 10^{-5} \quad .$$

Using data from $Re=18890$, the results of the other errors are:

$$\Delta K = \pm (0.01)K \quad \Delta K = \pm 0.006$$

$$\Delta Y = \pm (0.01)Y \quad \Delta Y = \pm 0.009$$

$$\Delta y = \pm 0.0002$$

$$\Delta P_1 = \pm 0.02 \text{ in } H_g$$

$$\Delta (\Delta p) = \pm 0.02 \text{ in } H_g$$

$$\Delta T = \pm 0.9^\circ F$$

Inserting these uncertainties into equation (B.33) we get

$$\Delta w = 0.011$$

and

$$\frac{\Delta w}{w} = 1.9\%$$

B.9 Reynolds Number Error

Reynolds number is defined:

$$Re = \frac{\rho \bar{V} D_h}{\mu} \quad (B.34)$$

where

$\bar{V}$ = average fluid velocity across the channel area

D_h = subchannel hydraulic diameter

Then we have

$$\bar{V} = \frac{w}{\rho A} = \frac{w_s}{\rho A_s}$$

$$D_h = \frac{4A_s}{P_w}$$

$$Re = \frac{4w_s}{P_w \mu}$$

where

A =channel flow area

w_s =subchannel flow rate ($0 < \theta < 30$)

A_s =subchannel flow area

p_w = subchannel wetted perimeter

From test section dimensions we have:

$$\frac{w_s}{w} = \frac{A_s}{A} = \frac{0.049217}{0.595} = 0.031 \quad .$$

Therefore

$$Re = 0.124 \frac{w}{\mu p_w} \quad . \quad (B.35)$$

Then the Reynolds number uncertainty is:

$$\begin{aligned} Re &= 0.124 \left[\frac{1}{\mu p_w} \Delta w + \frac{w}{\mu p_w^2} (\Delta p_w) + \frac{w}{\mu^2 p_w} \Delta \mu \right] = \\ &= 0.124 \left[\frac{1}{\mu p_w} \Delta w + \frac{w}{\mu p_w^2} (\Delta p_w) + \frac{w}{\mu p_w} \Delta (\ln \mu) \right] \quad . \quad (B.36) \end{aligned}$$

Dividing equation (B.36) by equation (B.35) we get:

$$\frac{\Delta Re}{Re} = \left[\frac{\Delta w}{w} + \frac{\Delta p_w}{p_w} + \Delta (\ln \mu) \right] \quad (B.37)$$

where Δp_w is due to the test section construction and we have $\Delta p_w = 0.001$ in., $p_w = 0.3927$ in. The term $\Delta (\ln \mu)$ can be calculated from equation (B.28). Then for all cases except

Re=18890 we have

$$\frac{\Delta Re}{Re} = \frac{0.5}{100} + \frac{0.001}{0.393} + 15.666 \times 10^{-4} \times 0.5$$

$$\frac{\Delta Re}{Re} = 0.008 \approx 0.8\%$$

And for the case of Re=18890 the result of the calculation is:

$$\frac{\Delta Re}{Re} = 2.2\%$$

It is worth mentioning that in the calculation of the flow rate error, corresponding to Re=18890, we have used pessimistically the upper limit of the uncertainty of each particular parameter. For instance we have considered the uncertainty related to K and Y to be within 11% while under present conditions it is most probable that their actual uncertainty is less than this amount. Therefore the 2.2% uncertainty in the Reynolds number corresponding to Re=18890 is an upper estimate of the possible uncertainty.

TABLE (B.1)

Values of Flow Coefficient K for 2" Pipe
(taken from reference (20))

$Re = \frac{W}{\pi D_2 \mu}$ (D_2/D_1)	25000	35000	50000	75000	100000
0.2	0.6026	0.6011	0.6000	0.5991	0.5987
0.25	0.6033	0.6017	0.6005	0.5997	0.5992
0.3	0.6062	0.6055	0.6032	0.6023	0.6018
0.675	0.7070	0.7000	0.6949	0.6908	0.6888
0.7	0.7227	0.7156	0.7092	0.7047	0.7025

FIGURE (B.1)

Identification of
Figures (B.2) and (B.3)
Parameters

FIGURE (B.2) Error of the Wall Shear
Stress Measurement due to the Geometry
Effects (after reference (26))

FIGURE (B.3) Error in the Wall Shear
Stress Measurement Due to the Finite

FIGURE (B.4)

Calibration of Preston Tube (after reference (15))

FIGURE (B.5) Dimensionless Measurement Pressure Error for l/d ratio > 1.5 (taken from reference (32)).

FIGURE (B.6) EXPANSION FACTORS FOR AIR

$k = 1.4$; Computed from $Y = 1 - \left[0.41 + 0.35 \left(\frac{D_2}{D_1} \right)^4 \right] \left[\frac{\Delta p}{p_1} / k \right]$; $p_2 = p_1 - \Delta p_u$

(after reference (20))

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