MASTER

**An Investigation of
High Energy Runaway Electron Confinement
in the Oak Ridge Tokamak**

Stewart Jay Zweben

OAK RIDGE NATIONAL LABORATORY

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AN INVESTIGATION OF HIGH ENERGY RUNAWAY ELECTRON
CONFINEMENT IN THE OAK RIDGE TOKAMAK

A Thesis

Presented to the Faculty of the Graduate School
of Cornell University
in Partial Fulfillment for the Degree of
Doctor of Philosophy

by

Stewart Jay Zweben

Oak Ridge Associated Universities Graduate Participant

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AN INVESTIGATION OF HIGH ENERGY RUNAWAY ELECTRON CONFINEMENT
IN THE OAK RIDGE TOKAMAK

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Cornell University 1977

Abstract

High energy runaway electrons in the Oak Ridge tokamak ORMAK have been investigated through measurement of the bremsstrahlung produced when these electrons leave the discharge and strike the limiting aperture of the torus. The experimental results have been interpreted in terms of a classical single-particle model appropriate for collisionless particles in a tokamak, and it has been found that most of the confinement properties of high energy runaways in ORMAK can be understood on this basis. An experiment designed to directly test this model has disclosed an anomalous transport which has been described by a runaway diffusion coefficient $D \approx 10^2 - 10^4 \text{ cm}^2/\text{sec}$ appropriate for runaways near the outside of the plasma. A discussion of the possible mechanisms for this anomalous transport is given.

The work described in this thesis has extended the investigation of runaway electrons into the high energy (2–10 MeV) range. This was made possible through the use of an x-ray pulse height analysis system with appropriate pileup rejection circuitry and by a careful attention to the unfolding of the observed scintillator spectra. Although this method has some intrinsic limitations with respect to runaway energy analysis and confinement studies, it has succeeded in demonstrating that these electrons can accelerate freely and be nearly perfectly confined over the timescales typical of present tokamak devices.

In Chapter 1 we give a general description of ORMAK and of the theory of runaway production applied to the tokamak environment. The motivation for runaway studies is described along with a review of past and present experimental investigations in the field.

In Chapter 2 the classical theory of collisionless particle orbits in tokamaks is reviewed especially with reference to the interpretation of our experimental results. We list some of the non-ideal effects which might contribute to anomalous transport of the high energy runaways.

Chapter 3 and Appendixes A and B describe the hard x-ray diagnostic system including the data acquisition and analysis procedures. We also describe briefly other runaway diagnostics which have in some instances supplemented our information about runaways in ORMAK.

In Chapter 4 we present the results on high energy runaway confinement in normal ORMAK discharges and we interpret these results in terms of the classical drift orbit model of Chapter 2. It is shown that the main features of the observed confinement can be well understood through the simple classical model, but that in some circumstances the results are not so easily described on this basis alone.

The last two chapters represent our attempt to define the nature of the anomalies in the confinement of these high energy electrons. The inconclusiveness of the results should be understood in the context of the present uncertainties about tokamak transport in general.

Experiments aimed at studying runaway diffusion are described in Chapter 5. The "plasma shift experiment" clearly shows an anomalous runaway transport near the outside of the plasma which has been

modeled in terms of a diffusion coefficient of 10^2-10^4 cm²/sec. We find that our ability to reevaluate the data of Chapter 4 in the light of this result is limited by uncertainty concerning the extent of anomalous runaway transport in the plasma interior. This uncertainty is related to our lack of knowledge of the radial profile of the runaway creation rate, and to the fact that our detection system is capable of observing runaways only when they leave the discharge (see Appendix C).

In Chapter 6 we examine some mechanisms which have been observed to cause anomalous runaway transport in ORMAK; for example, the effect of MHD instability is observed to result in a correlation between runaway loss and magnetic fluctuations (see also Appendix D). A review of these mechanisms indicates that there are several possible causes for the anomalous diffusion described in Chapter 5.

INTRODUCTION

The study of plasma confinement is one of the most important and most difficult areas in the controlled fusion program. A device such as the tokamak has excellent confinement properties according to simple theory, yet in practice there exist "anomalous" loss processes which can dominate certain aspects of the observed behavior. Identification of these anomalous mechanisms proceeds through improved measurements and through refinements to existing theory, and often there are several theories which can account for the fragmentary experimental information.

This investigation of high energy runaways may be viewed as a subset of the continuing study of plasma confinement in tokamaks. The runaway problem is simplified by the fact that these particles should have relatively little interaction with the background plasma, yet it is complicated by the fact that the experimental information on runaways has been rather limited. Thus the study of runaway confinement is at the point where an anomalous transport has been shown to exist, but the detailed evaluation of the causes for this transport has just begun.

There have been three stages in the development of this work. In the first stage the runaway detection system was built and an unusually high runaway energy was observed, indicating that the confinement of these particles was better than had been expected. In the second stage the simplest confinement theory was applied to the data and a reasonably consistent interpretation was found. The last stage involved efforts to directly test this simple model and to understand the anomalies which were shown to exist. Although the work in this

last stage has disclosed some interesting new phenomena, the complexity of the physical situation is such that a complete understanding of high energy runaway confinement remains to be achieved.

In this thesis we first present an introduction to tokamaks and runaway electrons (Chapter 1), and then review the simple drift orbit model of runaway confinement (Chapter 2). After a discussion of the experimental methods (Chapter 3), we present runaway confinement data for normal discharges and give an interpretation of this data in terms of the simple model (Chapter 4). A test of this interpretation (Chapter 5) shows that there exists an anomalous behavior in some circumstances, the mechanism for which is presently not clear (Chapter 6). The overall conclusion is that the confinement of high energy runaways is to a first approximation given by the simple drift orbit model, but that some of the observed effects must be attributed to the influence of various anomalous transport mechanisms.

1. REVIEW OF TOKAMAKS AND RUNAWAY ELECTRONS

A tokamak is a toroidal plasma confinement system which has in recent years been the most promising approach to the achievement of controlled thermonuclear fusion. The success of the tokamak lies in the relative ease with which interesting high temperature plasma regimes can be achieved, and in the anticipation that the present results can be extrapolated to a reactor by the end of the century. Yet in order to proceed with the large and expensive machines involved in such extrapolations it has become important to understand in detail the complex physical phenomena which are occurring in present devices.

1.1 TOKAMAKS

The early development of the tokamak in the period from the mid-1950's to about 1968 was carried on entirely in the Soviet Union; hence, the generic name "tokamak" means roughly "toroidal (TO) magnetic (MAK) chamber (KA)." The work in this country on tokamaks began in 1968 when groups at Princeton and Oak Ridge decided to build the ST and ORMAK devices, respectively, and since then the number of major devices world-wide has increased rapidly to about twenty as of 1977. The largest and most sophisticated machines are currently operating at Princeton (PLT) and at the Kurchatov Institute in Moscow (T-10).

The field of tokamak research has been extensively reviewed by Artsimovich in 1972⁽¹⁾ and by Furth in 1975,⁽²⁾ and a convenient summary of machine parameters, theory and technological problems and prospects has been published by ERDA.⁽³⁾ The most recent results are available through the proceedings of the International Conference at

Berchtesgaden,⁽⁴⁾ and there have also been several recent reactor studies such as those by groups at Oak Ridge⁽⁵⁾ and Wisconsin.⁽⁶⁾ In this section a brief review is presented of the design, theory, and of the results for tokamaks in general and for ORMAK in particular. In the next two sections the topics of electron runaway and of runaways in tokamaks will be reviewed in turn.

Tokamak Design

A schematic drawing of the conventional tokamak configuration is shown in Fig. 1.1. The dominant features of the design are a strong toroidal magnetic field B_T and a large toroidal plasma current I . The toroidal magnetic field is basically necessary for achieving overall plasma stability, whereas the particle confinement is determined mainly by the poloidal field B_p produced by the plasma current (see Chapter 2). The large iron core shown in the figure is used in generating the plasma current through a simple transformer action. A third set of coils is needed to produce a small vertical magnetic field B_v which is necessary to control the plasma equilibrium. Thus the combination of these three magnetic fields results in a helical magnetic structure which can be centered in the chamber through the action of the radial $I \times B_v$ force.

A relation between B_T and I is set by the "safety factor," $q = B_T a / (B_p R) = 5 B_T (g) \cdot a^2 (cm) / [R (cm) \cdot I (amp)]$, where a and R are the minor and major radii of the torus, respectively; thus typically $q \lesssim 5^{(1)}$ for $a = 20$ cm and $R = 80$ cm implies that for a toroidal field of 20 kG the maximum plasma current is about 100 kA. The magnitude of this

current is perhaps the most important single parameter in tokamak design, such that improvements in plasma parameters have been achieved by raising I through increased B_T (i.e., Alcator at MIT with $B_T = 80$ kG) or by an increase in size (i.e., PLT and T-10).⁽³⁾ The plasma current not only determines the particle confinement through the poloidal field, but also has had the role of heating the plasma through ohmic dissipation. Since most tokamaks operate with a one-turn loop voltage of about 2 V, a typical plasma current of 100 kA can supply ≈ 200 kW of ohmic heating, which is generally enough to raise the plasma to a temperature of about 500 eV.

Among the secondary features of tokamak design are the requirements imposed by the vacuum system, the need for a large array of plasma diagnostics, and (in present and future devices) the requirements of a supplementary plasma heating system. The base pressure in the toroidal chamber must be in the 10^{-8} torr range in order to insure a low level of the impurities (i.e., oxygen) which can severely limit plasma performance,⁽³⁾ and it has been the experience that this requirement conflicts with the large number of ports needed for adequate diagnostic access. Supplementary heating schemes such as neutral particle injection or wave heating also depend upon good access to the plasma, as well as requiring large (≥ 1 MW) power supplies and control systems.

Therefore, even though the basic structure of the tokamak is relatively simple, the detailed design and construction of present machines is rather expensive and time consuming. For example, the PLT at Princeton was designed and built over a period of about five years

at a cost of over \$20 million, and a next-generation device [i.e., the Tokamak Fusion Test Reactor (TFTR), also at Princeton] is expected to cost an order of magnitude more than this.

Tokamak Theory

The theoretical study of tokamaks is generally divided into the areas of equilibrium, stability, and transport. The study of equilibrium⁽⁷⁾ aims at determining the magnetic field structure which is necessary in order to balance the plasma pressure $p = nkT$, where n is the plasma density and T its temperature. After an equilibrium has been found, the theory then attempts to define the range of parameter space over which stability can be expected, both in the macroscopic magnetohydrodynamic (MHD) limit⁽⁸⁾ and on the microscopic (kinetic theory) level.⁽⁹⁾ It has long been recognized that there is no state of a tokamak which is completely stable,⁽²⁾ thus the work in plasma transport has evolved the distinction between "neoclassical" transport calculated on the basis of an ideal (stable) plasma model,⁽¹⁰⁾ and "anomalous" transport due to the effects of MHD or kinetic instabilities.⁽⁹⁾ The basic constraints defined by tokamak theory can be summarized in a simplified form as follows:

Equilibrium:	$B_V/B_p \propto (\beta_p + \ell_i/2)$	Balance of outward pressure by inward $I \times B_V$ force.
Stability:	$q = B_T/B_p \ a/R_0 \gtrsim 3$	Practical stability for kink modes.
Transport:	$D_{\perp} \propto \nu_{ei} \rho_p^2$	Radial diffusion coefficient.

Where the following symbols are commonly used in tokamak studies,

- β_p = ratio of plasma pressure to poloidal field pressure = $\frac{nkT}{B_p^2/8\pi}$
- D_1 = radial diffusion coefficient for electrons or ions
- L_i = internal inductance of current channel
- ν_{ei} = electron-ion collision frequency
- ρ_p = Larmor radius of a particle evaluated in the poloidal field

Tokamak Results

In Table 1.1 we list some of the principal results achieved or expected for three generations of tokamak devices. The T-3 and ORMAK are representative of the first group of experiments built before ≈ 1970 . PLT and T-10 are typical of present devices, and TFTR is expected to be the first machine in which the power generated by fusion reactions is comparable to the power required to maintain the plasma. A figure of merit which is commonly used to measure the overall quality of the plasma with respect to fusion burning is $n \tau_E$, where τ_E is defined as the energy confinement time $\tau_E = nkT/P_{in}$, where P_{in} is the total input power due to ohmic and supplementary heating. A tokamak fusion reactor will have to satisfy the Lawson criterion $n\tau_E \gtrsim 2 \times 10^{14} \text{ cm}^{-3} \text{ sec}$ at a plasma temperature of $\gtrsim 5 \text{ kev}$. (3)

Table 1.1 Some Tokamak Results

Device	Year	Minor Radius cm	Maximum Current kA	Peak Temp. (ions)eV	Average Density cm ⁻³	Average n _T ^E cm ⁻³ sec
T-3(USSR) ⁽¹¹⁾	1968	15	120	500	5×10^{13}	$\geq 10^{11}$
ORMAK ⁽¹²⁾	1974	23	120	300	3×10^{13}	$\geq 10^{11}$
ORMAK ⁽⁴⁾	1976	23	230	1800	6×10^{13}	$\approx 5 \times 10^{11}$
PLT ⁽⁴⁾	1976-	45	600	1000	3×10^{13}	$\geq 10^{12}$
T-10(USSR) ⁽⁴⁾	1976-	39	500	800	8×10^{13}	$\approx 5 \times 10^{12}$
TFTR ⁽¹³⁾	c.1981	85	2500	5000	5×10^{13}	$\geq 10^{13}$

Perhaps more important than these specific results is the level of understanding which has been achieved as a result of the world-wide efforts on tokamak theory and experiment. A qualitative view of the situation with respect to the three areas of plasma equilibrium, stability, and transport is given below:

Table 1.2 Tokamak Knowledge

Area	Theoretical Understanding	Experimental Information	Agreement?
Equilibrium	Good	Good	Yes
Stability	Fair to Good	Fair	Probably
Transport	Unsettled	Fair	Partial

This table points out that plasma transport is the most uncertain and most complicated aspect of the tokamak problem, and it is also one of the most important areas since it is the main determinant of the density and energy confinement time factors in the Lawson criterion.

The present understanding can be summarized by saying that the ions behave according to neoclassical predictions, but the electron confinement is anomalous by a factor 10-100.⁽³⁾ The work described in this thesis has in a small way interacted with the unsolved problem of this anomaly in the electron confinement.

ORMAK

The Oak Ridge tokamak, ORMAK, began operation in 1971 and was terminated in December 1976 as the new ISX tokamak was being installed. The main activities of the ORMAK group have been in the areas of plasma confinement and neutral beam heating, as described in the recent paper given at the Berchtesgaden conference.⁽⁴⁾ The principal results involve an extensive series of plasma scaling experiments and the achievement of record ion temperatures in the 1500 eV range through intense neutral beam injection. Figure 1.2 shows ORMAK as it appeared in 1975 with its full complement of diagnostics.

Some typical parameters for an ORMAK discharge are shown in Fig. 1.3. The initial current rise is determined by the discharge of a large capacitor through the ohmic heating transformer, and after about 20 msec a steady state is reached in which the current is kept constant by a feedback-controlled battery bank. The addition of neutral beams or a cold gas feed during the steady state has been used to increase the plasma temperature and density, respectively.

For a fixed plasma current (usually determined by the toroidal field strength for $q = 5$) the character of the discharge is mainly determined by the average plasma density $\bar{n}_e$, where $.5 \times 10^{13} \leq \bar{n}_e \leq 5 \times 10^{13} \text{ cm}^{-3}$. The high and low density regimes are somewhat different

with respect to the runaway behavior as described in Chapter 4. A detailed description of the ORMAK device can be found in the Annual Reports of the Fusion Energy Division from 1974–1976.⁽¹⁴⁾

1.2 RUNAWAY ELECTRONS

If an electric field is applied to a plasma the bulk of the electrons are simply heated by ohmic dissipation, but it turns out that a fraction of the electron population can gain energy continuously from the field and so achieve energies much above the thermal level. This is made possible by the fact that the effective frictional force arising from small angle collisions with background electrons decreases with increasing electron energy⁽¹⁵⁾

$$F_{\text{coll}} = m\nu_e v = \frac{4\pi n e^4 \ln \Lambda}{m^2 v^2} \quad (1.2.1)$$

where v is the electron velocity, ν_e is the frequency for the accumulation of 90° scattering, n is the background electron density, and $\ln \Lambda$ is the usual Coulomb logarithm ($\ln \Lambda \approx 10$ – 20). Thus there is an electron velocity v_c above which the force due to the applied electric field is greater than the dynamical friction, $eE > F_{\text{coll}}$, such that for $v > v_c$ electrons will continuously gain energy, where

$$v_c = \left(\frac{4\pi e^3 n \ln \Lambda}{mE} \right)^{1/2}. \quad (1.2.2)$$

The degree to which the electrons in a plasma will run away is generally measured in terms of the critical electric field E_c at which thermal electrons with $v_{\text{th}} = kT_e/m$ will become runaways

$$E_c = \frac{4\pi e^3 n \ln \Lambda}{m v_{\text{th}}^2} \approx 4 \times 10^{-2} \left(\frac{n}{10^{13}} \right) \left(\frac{10^3}{T_e} \right) \text{volts/cm} \quad (1.2.3)$$

For $E/E_c \gtrsim 1$ essentially all electrons will run away in a collision time, whereas for $E/E_c \ll 1$ only an exponentially small fraction of the Maxwellian electron distribution has a $v > v_c$ needed for runaway. The main theoretical interest in this area has therefore been the calculation of the runaway rate as a function of E/E_c .

The theory of runaway production for $E/E_c \ll 1$ is a problem in kinetic theory which has been gradually refined since the work of Dreicer in 1960.⁽¹⁶⁾ Dreicer's original treatment divided velocity space into two regions separated by the critical velocity v_c , and it was assumed that the runaway rate was given by the diffusion of electrons from the low velocity thermal region into the runaway-depleted high velocity region. His results (as shown in Fig. 1.4) overestimated the runaway rate by over an order of magnitude due to his assumption of a Maxwellian velocity distribution below v_c . This error was noticed by Gurevich⁽¹⁷⁾ and Lebedev⁽¹⁸⁾ who treated the forward peaking of the electron distribution near v_c more carefully, and by Kruskal and Bernstein⁽¹⁹⁾ who found it necessary to divide velocity space into five distinct regions. Recently Kulsrud, et al.⁽²⁰⁾ solved a linearized Fokker-Planck equation using a numerical code which allowed the determination of a multiplicative constant missing from the Kruskal analytic theory. Finally, the influence of impurity ions and of the fully relativistic form of the collision cross section has been treated by Cohen⁽²¹⁾ and by Connor and Hastie,⁽¹⁵⁾ so that an accurate expression for the runaway rate S_r (fraction of electrons running away per collision time) can be written:⁽¹⁵⁾

$$S_r = C_r n v(v_{th}) \left(\frac{E}{E_c} \right)^{-h(\alpha, Z)} \exp \left\{ - \frac{\lambda(\alpha)}{4} \frac{E_c}{E} - \left(\frac{2E_c}{E} \right)^{1/2} \gamma(\alpha, Z) \right\} \quad (1.2.4)$$

where $C_r \approx 1$ and

$$\lambda(\alpha) = 8\alpha(\alpha - 1/2 - \sqrt{\alpha(\alpha-1)})$$

$$\gamma(\alpha, Z) = \sqrt{(1+Z)\alpha^2/8(\alpha-1)} \left[\frac{\pi}{2} - \sin^{-1} \left(1 - \frac{2}{\alpha} \right) \right]$$

$$h(\alpha, Z) = 1/16(\alpha-1) \left\{ \alpha(Z+1) - Z + 7 + 2 \sqrt{\alpha/\alpha-1} (1+Z)(\alpha-2) \right\}$$

and $\alpha = (E/E_c)(mc^2/KI_e)$, $Z = \sum Z_i^2 n_i / \sum Z_i n_i$ = effective ion charge

The main feature of this result is the extreme sensitivity of the runaway rate to small changes in the parameter E/E_c (see Fig. 1.4), such that an increase in temperature results in many more electrons near v_c and a higher runaway rate, while an increase in density enhances the dynamical friction and so decreases the runaway rate [see Eq. (1.2.3)]. The presence of impurity ions ($Z > 1$) increases the scattering cross section and so also reduces the production rate, as does the leveling-off of the relativistic collision cross section for $v_c \lesssim c$. This sensitivity of the runaway production rate on E/E_c makes it rather difficult to calculate precisely the runaway population given the uncertainties involved in the experimental determination of T_e , n_e , and Z (see Chapter 4).

Experiments

High energy electrons have been seen in many plasma devices since the beginning of the controlled fusion program. In all cases the most conspicuous sign of their appearance has been the hard x-ray bremsstrahlung emission produced when these electrons strike the walls

of the containment device, and the difficulties associated with the interpretation of runaway behavior through this x-ray emission persist through today.

Although high energy electrons have been observed in linear high current pulsed discharges,⁽²²⁾ most of the early studies of runaways were done on toroidal devices such as the Perhapsatron⁽²³⁾ in the US (1954), the TA 2000⁽²⁴⁾ in France (1958), and the T-1⁽²⁵⁾ toroidal Z-pinch in Germany (1961). In each of these experiments the x-ray emission was observed to occur in bursts during the early phases of these short (< 1 msec) high voltage discharges, and the measured runaway energy was in the 50 keV range. In the German device an attempt was made to compare the observations with the results of runaway production theory, and it was concluded that after the first few microseconds there was some instability mechanism which either suppressed the production of runaways or else prevented them from gaining an energy above about 1 keV.

The most sophisticated set of runaway experiments in the pre-tokamak era was performed on the B-1 and B-3 stellarators at Princeton in the late 1950's.⁽²⁶⁻²⁸⁾ Both x-ray pulse height analysis and microwave emission were used to detect runaways in the energy range below ≈ 1 MeV, and an attempt was made to understand the runaway particle confinement on the basis of single-particle orbit theory. Freefall acceleration of the runaways was seen up to .5 MeV and a general agreement with the orbit theory was shown by the location of runaway impact at the wall, but the effect of instabilities was also seen in the pattern of runaway loss during current steps and in a

collective loss mechanism observed in some cases.

Thus the study of runaway electrons has from its beginning aimed at an understanding of the processes of production, confinement, and loss of these particles, and it can be stated that in none of these areas has a complete and consistent picture emerged to fit the results of any experiment. The reason for this is not hard to find — it is that the plasma itself has not been completely understood. The same situation exists in present tokamak experiments, as described in the next section.

1.3 RUNAWAY ELECTRONS IN TOKAMAKS

All tokamaks produce runaways to some extent, and there have been many detailed studies of their behavior in present devices. The main reason for this interest is that runaways act as a natural probe of the complicated magnetic structure of the device; for example, the existence of 10 MeV electrons in ORMAK implies that particles can be confined for over 10^6 toroidal transits of the machine. Another reason for the interest in runaways is that they have been known to cause damage to the inner walls of the machine [i.e., TFR⁽²⁹⁾] and under some circumstances the runaway population is so large as to dominate the character of the discharge.⁽³⁰⁾ Calculations of runaway production for the future generation of large tokamaks also indicate that substantial numbers may be produced during the high voltage, low density gas breakdown phase,⁽³¹⁾ and since runaways do not contribute to fusion burning they may have to be suppressed by some means.

Early Experiments

An outline of the status of runaway experiments in tokamaks at the time this thesis work was begun (Summer, 1974) is presented in Table

1.3. There were three main results at this point:

1. Runaways could attain energies of $\lesssim 1$ MeV.
2. There was in some cases an anomalous loss mechanism for these runaways, especially in the presence of plasma instabilities.
3. Runaway discharges could be created in which nearly all the current was carried by the runaways.

Of these five studies given in the table, perhaps the most comprehensive was that on the TM-3 device.⁽³⁵⁻³⁶⁾ They used a plasma bremsstrahlung measurement to try to find the runaway energy spectrum below $\approx .5$ MeV, and were the first to study an instability mechanism associated with runaways at low plasma density. Runaway studies on TM-3 are still continuing, as discussed below.

Another noteworthy achievement was the ST experiment in which the plasma bremsstrahlung measurement was used to determine the runaway production rate and runaway confinement time for $E \lesssim 1$ MeV. A radial scan of the x-radiation indicated that the runaway population in this energy range in the steady state was peaked on axis, as would be predicted from production theory, and the inferred confinement time for these runaways was 10-20 msec.⁽³⁹⁾

The LT-3 experiments in Australia should also be noted for their early discussion of runaway diffusion. Their probe measurements of low energy runaway confinement have been the most detailed in this field to date (see also Table 1.4).

Table 1.3 Early Experimental Work on Runaways

DEVICE	DIAGNOSTICS	RESULTS
LT-1, LT-3 ⁽³²⁻³⁴⁾ (Canberra, 1971-present)	<ul style="list-style-type: none"> • NaI detector with movable target 	<ul style="list-style-type: none"> • Rapid runaway loss during instabilities for energies $< 100 \text{ keV}$⁽²²⁾ • Low energy runaway drift surfaces lie closely to magnetic surfaces • Runaway confinement at early times determines breakdown
TM-3 ^(35,36) (Kurchatov, 1972-1973)	<ul style="list-style-type: none"> • NaI pulse analysis of plasma bremsstrahlung • Hard x-ray flux from walls • Bolometer at wall • RF and microwave emission • diamagnetic loop 	<ul style="list-style-type: none"> • Plasma bremsstrahlung gave $T_e \leq 40 \text{ keV}$ • Instability at low density showing periodic runaway loss and RF emission with large energy transfer to walls • Possible positive slope in distribution function near 100 keV
ST ⁽³⁷⁻³⁹⁾ (Princeton, 1972-1973)	<ul style="list-style-type: none"> • Plasma bremsstrahlung with NaI 	<ul style="list-style-type: none"> • High energy ($< .5 \text{ MeV}$) x-ray spectra peaked on axis • Estimate $3 \times 10^{10} \text{ cm}^{-3}$ for 100 keV tail • Estimate of runaway production rate lower than theoretical predictions by $\approx \times 10$ • Estimate of runaway confinement time 10-20 ms for $F \leq 1 \text{ MeV}$
T-6 ⁽⁴⁰⁾ (Kurchatov, 1973)	<ul style="list-style-type: none"> • Hard x-ray flux • Diamagnetic loop 	<ul style="list-style-type: none"> • Runaway discharge regime has current carried by electrons 10-500 keV with cold background plasma • Instability shows current steps • Inhomogeneity in B_T reduces runaway confinement
ORMAK ⁽⁴¹⁻⁴⁴⁾ (Oak Ridge, 1974)	<ul style="list-style-type: none"> • Hard x-ray flux 	<ul style="list-style-type: none"> • Field errors may affect runaway confinement at early times in the discharge • Runaway discharge regime modeled with production and equilibrium theory • Correlation of runaways with MHD activity

Table 1.4 Recent Experimental Work

DIAGNOSTIC	DEVICE	RESULTS
Limiter bremsstrahlung (tangential observation with energy analysis)	ORMAK ⁽⁴⁷⁻⁵⁰⁾ (1974-1976)	<ul style="list-style-type: none"> • Energy up to 10 MeV • Approximately 100 A of high energy run-aways in normal discharges • Nearly freefall acceleration for $E > 2$ MeV • Confinement explained in terms of classical single-particle model • Small anomalous diffusion and occasional influence of instabilities (Chapters 5 and 6)
	PLT ⁽⁵¹⁾ (1976)	<ul style="list-style-type: none"> • Energy up to ≥ 8 MeV • Half-freefall acceleration
Limiter bremsstrahlung (uncollimated x-ray flux)	Pulsator ⁽⁵²⁾ (1975)	<ul style="list-style-type: none"> • Correlation of limiter x-rays with pre-disruptive MHD activity • Indirect energy estimate ≤ 8 MeV
Plasma bremsstrahlung	Pulsator ^(53,54) (1975-1976)	<ul style="list-style-type: none"> • Energy $\leq .5$ MeV • X-ray emission peaked at center of discharge • Confinement time estimated after gas puff $\approx$ energy confinement time (10 msec) • Runaways loss correlated with MHD activity during the current rise (wall bremsstrahlung measurement)
	TM-3 ^(55,56) (1975)	<ul style="list-style-type: none"> • $E < .3$ MeV (runaway temperature ≈ 30 keV) • Study of impurities radiation caused by runaways gives $I_{\text{runaway}} \approx \frac{1}{5} - \frac{1}{10} I_{\text{total}}$
Probe measurements with bremsstrahlung flux detection	LT-3 ⁽⁵⁷⁻⁵⁹⁾ (1975-1976)	<ul style="list-style-type: none"> • Energy ≤ 1 MeV • Rapid runaway loss before disruptive instabilities and during MHD activity early in the discharge • Diffusion coefficient $10^4 - 10^5$ cm²/sec for runaways in stable discharges, with higher energy runaways less well confined
Photonuclear activation of limiter by runaway bremsstrahlung	TFR ⁽⁶⁰⁾ (1976)	<ul style="list-style-type: none"> • High energy runaway impact concentrated at outer limiter • $E \leq 18$ MeV (≈ 30 A high energy runaways)
	PLT ⁽⁵¹⁾ (1976)	<ul style="list-style-type: none"> • Activation at outer limiter ≈ 1 kA of runaways with $E = 6-13$ MeV • Indication of steady state runaway population in long (1 sec) discharges
Synchrotron (micro- wave) radiation	Cleo, TFR ⁽⁶¹⁾	<ul style="list-style-type: none"> • High level of nonthermal emission (especially $\approx 3 \omega_{ce}$) due to runaways
	ATC ⁽⁶²⁾	<ul style="list-style-type: none"> • Steps in nonthermal radiation apparently caused by sudden pitch-angle scattering of runaways (ATC) • Decrease in synchrotron radiation with onset of $m = 2$ mode (ATC)

The first Oak Ridge runaway work was a theoretical study of strong runaway discharges which were unintentionally produced during early ORMAK operation. D. A. Spong's modelling studies of this regime are described at length in his thesis.⁽⁴⁴⁾ This work formed the basis for some recent experimental studies of this regime in ORMAK under more controlled and better diagnosed circumstances, as mentioned in the next section.

Also studied on ORMAK was a correlation observed by A. C. England between hard x-ray bursts and MHD activity in the normal discharge regime. A theoretical analysis⁽⁴³⁾ indicated that this could be explained by magnetic island structures in the discharge. Further results in this area are described in Sect. 6.2 of this thesis.

The original direction of the ORMAK runaway electron program was thus in part directed toward experimentation with the strong runaway regime in which most of the discharge current is carried by runaways, and a survey of this field had been prepared⁽⁴⁵⁾ by Dr. H. Knoepfel before coming to Oak Ridge to direct the runaway program. (The results of our work in this area have been published,⁽³⁰⁾ but are not included in this thesis.) The main area of interest has been the study of high energy runaway confinement in normal ORMAK discharges, as summarized below.

Recent Experimental Work

There has been a growing world-wide interest in runaway studies as witnessed by the recent International Conference on Synchrotron Radiation and Runaway Electrons in Maryland.⁽⁴⁶⁾ Although the extent of our knowledge has grown, the subject is apparently far from the

achievement of a unified understanding in either theory or experiment. The experimental situation is outlined in Table 1.4; the organization of this table makes it clear that the various experimental diagnostics have not yet converged to an unambiguous description of the runaway phenomenon. This is in part due to intrinsic differences between the small and large tokamaks, and partly due to the inability of any single diagnostic to measure the whole runaway distribution function.

The basic situation as described in Table 1.4 may be summarized as follows:

1) High energy runaways ($E \gtrsim 8$ MeV) have been observed by the tangential bremsstrahlung measurement and by the photonuclear activation method. The existence of this high energy component implies an unusually good confinement for these particles. In ORMAK, for example, the existence of 10 MeV electrons implies a confinement time of at least 50 ms (necessary for energy gain in the toroidal electric field), while the plasma electron energy confinement is only $\lesssim 5$ ms for these discharges.

2) Low energy runaways ($\lesssim 1$ MeV) have been observed by the plasma bremsstrahlung and probe measurements to show an apparent diffusion which is comparable to or faster than the thermal electron diffusion rate ($\tau_R < 10$ msec).

3) Runaways can dominate the microwave emission spectrum of a tokamak, but it has not yet been possible to deduce from this spectrum much about the distribution function of the runaways.

In addition to these generalizations about the runaway population in normal, stable discharges, we must add the following points:

4) Strong runaway discharges have occurred in nearly every tokamak; they have a distinctive set of characteristics related to the fact that the discharge current is carried by runaways and not by thermal electron drift.^(30,40)

5) The mechanism of liner damage in TFR due to runaway bombardment has been extensively studied⁽⁶³⁾ and been shown to be due to ~ 50 keV electrons which become trapped in the toroidal field coil ripples. These electrons are associated with an instability of the runaway electron population, apparently the same as that which has been extensively documented in TM-3.^(35,55) This instability is presently one of the most interesting areas of runaway studies, but since it has been only rarely seen in ORMAK (see Sect. 6.4) and since it primarily involved runaways with $E < 1$ MeV, we have not emphasized it in our work.

6) The Alcator device has demonstrated a "slide-away" regime which occurs at low density where a runaway discharge might be expected.⁽⁶⁴⁾ This regime is characterized by a lack of high energy runaways in spite of a large $E/E_c \gtrsim .1$, and by increased RF emission and anomalous ion heating in the plasma. A theoretical explanation of this regime in terms of an instability of the electron distribution function has been proposed,⁽⁶⁵⁾ but it is not yet clear whether this phenomenon is related to the runaway instability of the TM-3 TFR type.

In summary, there is a large amount of experimental information about runaway electrons in tokamaks, but the relationships among the observations on various machines are not presently clear. In this thesis we present observations on the high energy runaway electrons

in ORMAK, and describe a model which satisfactorily fits the bulk of these results. We then show that under some circumstances this model needs to be refined to include some anomalous diffusion and enhanced transport due to instabilities. In this way we are attempting to extend our understanding to include the effects which may be dominating the confinement of the lower energy runaways.

CHAPTER 2

DRIFT ORBIT THEORY

This chapter describes the theory of runaway dynamics in a tokamak as a basis for the interpretation of experimental results in Chapters 4-6. In the first section the "collisionlessness" of high energy runaways with respect to Coulomb scattering is established. The second section reviews the work of Spong⁽⁴⁴⁾ on the orbits of collisionless high energy electrons in an ideal tokamak field structure, and in Sect. 2.3 some further development of this work is reported. The application of this simple drift orbit model of confinement to the ORMAK runaway experiments is described in Sect. 2.4, and finally in Sect. 2.5 some non-ideal effects are reviewed which can modify this simple model.

2.1 COLLISIONALITY OF HIGH ENERGY RUNAWAYS IN TOKAMAKS

As a high energy electron with kinetic energy $E = (\gamma - 1) mc^2$ passes through a plasma it gives up energy primarily through small angle collisions with the thermal electrons. An expression for the rate of energy loss is^(66,67)

$$\frac{dE}{dt} \approx \frac{4\pi n_e e^4}{mv} \ln \Lambda \quad (2.1.1)$$

where $\ln \Lambda$ is the Coulomb logarithm (which is a slowly varying function of the electron energy). As $v \rightarrow c$ the rate of energy loss becomes nearly constant, since it is determined by the velocity with which the high energy electron passes by the slow-moving plasma electrons.⁽¹⁵⁾

For ORMAK $n_e \approx 2 \times 10^{13}$ electrons/cm³ and $\ln \Lambda \approx 15$, so that the energy loss rate is roughly 6 MeV/sec. This should be compared to the

rate of energy gain in the tokamak toroidal electric field:

$$\frac{dE}{dt} = eEv_{\phi} = eVf \quad (2.1.2)$$

where V is the one turn loop voltage and f is the toroidal transit frequency. Assuming $v_{\phi} \approx c$ gives $dE/dt \approx 60 \text{ V MeV/sec}$, i.e., for normal ORMAK steady state conditions $v \geq 2$ volts and so $dE/dt = 120 \text{ MeV/sec}$. Thus the rate of energy loss due to Coulomb scattering for any relativistic electron is negligible compared to the rate of energy gain for normal ORMAK plasma conditions.

There are some other energy loss mechanisms which are generally negligible for the case of runaway electrons in ORMAK. For example, the bremsstrahlung loss due to collisions with plasma ions can be written⁽⁶⁶⁾

$$\frac{dE_{\text{rad}}}{dt} \approx \frac{16}{3} \frac{N_i Z_i^2 e^2}{\hbar c} \left(\frac{e^4}{m^2 c^4} \right) \gamma m c^2 c \ln \gamma \quad (2.1.3)$$

For $n_i Z_i^2 = 7n_e = 10^{14} \text{ cm}^{-3}$ which is typical of ORMAK, the rate of energy loss for a 10 MeV electron due to bremsstrahlung is therefore less than 1 MeV/sec. Similarly the synchrotron radiation due to the electron's curvature in the toroidal direction is⁽⁶⁶⁾

$$\delta E (\text{MeV/revolution}) = 8.85 \times 10^{-2} \frac{[E(\text{BeV})]^4}{R(\text{m})} \quad (2.1.4)$$

According to this formula a 10 MeV electron in ORMAK ($R = 1 \text{ m}$) has $\delta E < 10^{-3} \text{ eV/rev} \approx .05 \text{ MeV/sec}$; thus, for this process also the energy loss per revolution is much less than the energy gain through the applied electric field. The synchrotron loss due to the gyromotion in the toroidal field is also generally negligible, as shown

below. A detailed evaluation of these and other loss mechanisms can be found in Ref. 67.

The conclusion of this analysis is that runaway electrons with an energy in the range 2–10 MeV should gain energy at very nearly the "freefall" rate given by Eq. (2.1.8). It should be noted clearly, however, that this conclusion is not valid if the critical velocity for runaway generation v_c is near the speed of light (for ORMAK $v_c < .1 c$), or if the electrons have a large pitch angle with respect to the toroidal magnetic field (see below). It may also be invalid in the presence of plasma instabilities which could anomalously slow down or scatter runaways through the action of small-scale electric or magnetic fields, as discussed in Sect. 2.5.

The other aspect of runaway collisionality is the evolution of the pitch angle ξ defined as the angle between the runaway velocity and the toroidal magnetic field. This pitch angle thus defines the ratio between perpendicular and parallel momenta $\xi = p_\perp / p_\parallel$, and can be important for determining the runaway orbit (see Sect. 2.2), the rate of energy gain in the applied electric field through $dE/dt = v_\phi \cos \xi$, and also the rate of synchrotron radiation due to the particle gyromotion. It is fairly obvious that the small energy loss rate due to Coulomb collisions in the energy range 2–10 MeV implies that the pitch angle of these electrons is also small, but it is interesting to determine just how small it is.

Pitch angle scattering of runaways in present tokamaks is mainly due to the few high Z impurity ions, since the scattering angle for each collision is proportional to Z_i^2 . A simple estimate of the mean

square pitch angle due to multiple scattering from ions where $Z_{\text{eff}} = \sum n_i Z_i^2 / n_i Z_i$ can be derived [from Ref. (66)]:

$$\langle \xi^2 \rangle = 8\pi n_e Z_{\text{eff}} L \left(\frac{e^2}{\gamma m c^2} \right)^2 \ln \Lambda \quad (2.1.5)$$

where n_e is the electron density and L is the path length of relativistic electron travel in the plasma. For example, if $Z_{\text{eff}} = 7$ and $n_e = 2 \times 10^{13} \text{ cm}^{-3}$, we find that a 10 MeV electron will accumulate a mean pitch angle of 10° in 50 ms if no energy change is taken into account. In fact, however, the electric field tends to reduce the pitch angle since it increases the parallel momentum without affecting the perpendicular momentum, such that for relativistic electrons freely accelerating in a constant electric field E

$$\frac{d\xi}{dt} = \frac{d}{dt} \left(\frac{p_\perp}{p_\parallel} \right) = -\xi \frac{d p_\parallel / dt}{p_\parallel} = -\frac{eE}{\gamma m c} \xi \quad (2.1.6)$$

Thus the effects of Coulomb scattering from Eq. (2.1.5) and the momentum change from Eq. (2.1.6) can be combined to form an equation for $\bar{\xi} \equiv \langle \xi^2 \rangle^{1/2}$:

$$\frac{d\bar{\xi}}{dt} = -\frac{A\bar{\xi}}{\gamma} + \frac{B}{\gamma^2 \bar{\xi}} \quad (2.1.7)$$

where $A = eE/mc$ and $B = 4\pi n_e Z_{\text{eff}} c (e^2/mc^2)^2 \ln \Lambda$. This equation can be rewritten using the relativistic approximation $\gamma = At$ in the form $d(\bar{\xi}^2 t^2)/dt = 2B/A^2$, which has the solution:

$$\bar{\xi}^2 = \frac{2B}{A\gamma} + \frac{\gamma_0^2}{\gamma^2} \left(\bar{\xi}_0^2 - \frac{2B}{A\gamma_0} \right) \quad (2.1.8)$$

where the initial condition is specified by some $\bar{\xi}_0$ at γ_0 .

This expression can be used to find the evolution of the pitch angle of high energy runaways in ORMAK; for example, if $\gamma \gg \gamma_0$, then $\bar{\xi} \propto \gamma^{-1/2}$, and assuming $n_e = 2 \times 10^{13} \text{ cm}^{-3}$, $Z_{\text{eff}} = 7$, and $E = 2 \text{ V/turn}$,

$$\bar{\xi} \approx .7 \gamma^{-1/2} \text{ (rad)} \quad (2.1.9)$$

Therefore the approximation $\xi \approx 0$ is fairly good for runaways with energy above 5 MeV.

This single particle analysis can be compared with the sophisticated calculations of Connor and Hastie,⁽¹⁵⁾ in which they find using a Fokker-Planck equation:

$$f \propto \frac{1}{q_{\parallel}} \exp \left(- \frac{(\alpha+1)}{2(1+Z_{\text{eff}})} \frac{q_{\perp}^2}{q_{\parallel}} \right) \quad (2.1.10)$$

where f is the runaway distribution function for $v \gg v_c$, $q_{\parallel}$ and $q_{\perp}$ are the momenta normalized to mc , $\alpha \equiv (E/E_c)(mc^2/kT)$, and Z_{eff} is the effective ionic charge. This formulation gives a gaussian-like distribution in pitch angle of width $\langle \xi^2 \rangle \sim 2(1+Z)/[(\alpha+1)q_{\parallel}]$, or typically $\bar{\xi} \equiv \langle \xi^2 \rangle^{1/2} \approx .6 \gamma^{-1/2}$ for the ORMAK parameters given above.

This estimate of the pitch angle can be used to calculate the rate of synchrotron radiation due to the gyromotion of high energy runaways. The total power radiated by an electron with a gyroradius ρ and a momentum $p_{\perp} = \gamma_{\perp} \beta_{\perp} mc$ perpendicular to the toroidal field is⁽⁶⁶⁾

$$P = \frac{2}{3} \frac{e^2 c}{\rho^2} \beta_{\perp}^4 \gamma_{\perp}^4 \quad (2.1.11)$$

A simple transformation to a reference frame in which the electron has a large momentum parallel to the field gives

$$P = \frac{2}{3} \frac{e^2}{c} \Omega_0^2 (\beta_{\text{tot}}^2 \gamma_{\text{tot}}^2 \sin^2 \xi) \quad (2.1.12)$$

where now the total momentum of the electron is given by $P_{\text{tot}} = \gamma_{\text{tot}} \beta_{\text{tot}} mc \approx \gamma_{\parallel} \beta_{\parallel} mc$, and $\Omega_0 = eB_T/mc$. For $B_T = 20$ kG and $\xi = .7 \gamma^{-1/2}$, Eq. (2.1.12) gives a rate of energy loss

$$P \approx .2 \gamma \text{ (MeV/sec)} \quad (2.1.13)$$

Thus this process again involves a negligible loss for high energy runaways in ORMAK.

In summary, it has been shown in this section that runaway electrons with energy about 5–10 MeV should be accelerated nearly freely in the tokamak electric field, and also that they should have small pitch angles throughout this energy range (assuming the absence of collective interactions). This conclusion motivates the following discussion of a "collisionless" runaway orbit model. We find that much of the high energy runaway data can be understood on this basis, but in some cases there may be non-Coulomb scattering mechanisms (such as those described in the last section of this chapter) which could modify this simple picture.

2.2 COLLISIONLESS PARTICLE ORBITS

In this section we describe the dynamics of collisionless particles in the tokamak field structure, and show that the particle trajectories lie on drift surfaces which can be confined within the torus. The structure of these orbits is well known,^(1,2,3) and the application to high energy runaways has been described extensively in Spong's thesis.⁽⁴⁴⁾

The collisionless particle — we will call it a runaway although the same theory applies to any particle for times short compared to the collision time — can be described in two ways. One derivation of its orbit⁽¹⁾ uses guiding center theory, which is appropriate as long as the length and time scales of the gyromotion in the toroidal field are short compared to the variations in the field itself. The other method takes advantage of the symmetry of the tokamak with respect to toroidal rotation and uses the constancy of the canonical toroidal angular momentum P_ϕ . These two derivations produce essentially the same result,^(44,68) although since neither is exact there may be some inconsistency on a finer level.⁽⁶⁹⁾

The following simplified derivation of the runaway orbit is based on the guiding center model as described by Artsimovich.⁽¹⁾ This model has somewhat more intuitive appeal than the conservation argument, and is also more useful in handling non-ideal magnetic field structures as described, for example, in Sect. 6.2. The assumptions of guiding center theory do hold well for the ideal tokamak field structure since the gyroradius for a runaway of $\lesssim 1$ cm is much smaller than the typical scale length for magnetic field changes ($\gtrsim 1$ m), and the cyclotron period is much shorter than any other time scale in this idealized tokamak model.

As usual, in guiding center theory,⁽⁷⁰⁾ the runaway tends to follow the magnetic field line $\vec{B}$ but has an additional drift velocity v_D perpendicular to $\vec{B}$ given in this case by the curvature and ∇B drifts:

$$v_D = \frac{1}{\gamma m^2 R_0 \Omega_{T0}} (P_\parallel^2 + P_\perp^2/2) \quad (2.2.1)$$

where the relativistic momenta parallel and perpendicular to $\vec{B}$ are $P_{\parallel}$ and $P_{\perp}$, and the cyclotron frequency at the major radius R_0 is $\Omega_{T0} = eB_{T0}/mc$ (note that the $1/R$ dependence of B_T cancels the variable major radius R). Since the magnetic field is nearly in the toroidal direction for $q \approx 5$ (see Sect. 1.1), this drift is predominantly in the vertical direction and has a magnitude $v_D \approx 10^8$ cm/sec for a typical runaway in ORMAK. Without the influence of the poloidal field the runaway would drift vertically and be lost in less than a microsecond. With the poloidal field, however, the runaway guiding center has an additional velocity component v_H given by its helical motion following the local magnetic field, $v_H = v_{\parallel} B_P/B_T$, so that the resulting guiding center equations can be written (see Fig. 2.1):

$$\begin{aligned}\frac{dx}{dt} &= - v_H \cos \theta \\ \frac{dr}{dt} &= + v_D \cos \theta\end{aligned}\tag{2.2.2}$$

Here r and θ are the usual cylindrical coordinates and x is measured outward along the major radius in the equatorial plane, as shown at the bottom of Fig. 2.1.

For small runaway pitch angles (see Sect. 2.1) we can assume that $v_{\parallel}$ is nearly constant around the torus, and so the trajectory can be described by the equation $dr/dx = - v_D/v_H$. This can be checked to be the equation for a shifted circle

$$(x - (v_D/v_H)r_0)^2 + y^2 = r_0^2\tag{2.2.3}$$

where r_0 is the minor radius of the shifted orbit. The outward displacement d_y can therefore be written

$$\frac{d_Y}{r_0} = \frac{v_D}{v_H} = \frac{v_{\parallel}^2 / R_0 \Omega_{T0}}{v_{\parallel} B_p / B_T} = \frac{\gamma m v_{\parallel} c}{e B_p R} \quad (2.2.4)$$

Here we have used the fact that $B_T = B_{T0}/R$, and have left the dependence on the poloidal field B_p unspecified for the moment. In this form it can be seen that the amount of the outward displacement depends linearly on the runaway parallel momentum and also upon the magnitude of the poloidal field $B_p(r)$ determined by the plasma current profile.

If $d_Y \ll r_0$ we can assume further that the poloidal field is constant along the orbit, so that for a flat current profile we have [in toroidal geometry]⁽⁴⁴⁾ $B_p \approx (2I r_0 / c r_L^2) R_0 / R$, where R_0 is the distance from the orbit to the center of the current distribution with total plasma current I ($I_{\text{runaway}} \ll I_{\text{plasma}}$), and r_L is the chamber (limiter) radius. The result is then:

$$\frac{d_Y}{r_L} \approx \frac{\gamma m v_{\parallel} c^2 r_L}{2e I R_0} = \frac{r_L}{2R_0} \frac{I_A}{I} \quad (2.2.5)$$

where $I_A = \gamma m v c^2 / e = 17\beta\gamma [\text{kA}]$ is called the Alfvén current.

This equation displays the general form of the drift orbit confinement model; namely that collisionless particles with small pitch angles have approximately circular orbits, the centers of which are displaced outward from the center of the plasma current distribution such that the amount of this displacement is directly proportional to the runaway energy (through I_A for $\gamma \gg 1$), and inversely proportional to the total plasma current I . The confinement of the runaway is therefore determined by whether or not the outer edge of the drift orbit intersects the material limiter at a radius r_L (see

Fig. 2.1). It should be noted explicitly that the minor radius r_0 of the drift orbit is not determined by the preceding analysis, so that for a given runaway energy and plasma current there are many possible values for r_0 corresponding to different confined orbits. In the case defined by Eq. (2.2.5), for example, the orbit displacement is independent of r_0 , but this is not true for an arbitrary current profile (as discussed in Sect. 2.4).

It is interesting to note that the outward orbit displacement for a relativistic runaway is larger than that for a thermal electron by a factor $\approx \gamma c/v_{th}$, or about two orders of magnitude. Thus the displacement effect can be significant for the confinement of high energy runaways at the same current for which it is insignificant for thermal electron confinement.

Before closing this section it would be worthwhile to mention the effects due to the breakdown of the assumptions used to derive Eq. (2.2.5); namely, the small pitch angle assumption which was used to find v_H , and the small displacement assumption which was used to calculate B_p . In the first case, it is well known that for a large enough pitch angle the drift orbit changes dramatically into what is commonly known as a "banana" or trapped particle orbit⁽¹⁾ in which the particle "mirrors" in the $1/R$ toroidal field due to its magnetic moment. It has been shown by Spong,⁽⁴⁴⁾ however, that for runaways with $\xi \leq 30^\circ$ the orbit is not significantly affected (see Fig. 2.2). Therefore on the basis of the scattering calculation of Sect. 2.1, and also by the fact that a trapped runaway would not continue to gain energy at the rate defined by Eq. (2.1.2), it can be safely assumed that high energy

runaways are indeed untrapped and can be described well by the analysis given above. This may not be the case for low energy runaways, however, as has been shown in the TFR experiments.⁽⁶³⁾

The other assumption used in deriving Eq. (2.1.5), namely $d_Y \ll r_0$, was made just so that the essential form of the result could be obtained without an explicit integration over the actual path of the runaway orbit. The results of a numerical calculation of some orbits using the exact $B_p(r)$ for a typical ORMAK current profile are shown in Fig. 2.2. Here it can be seen that the orbits for $d_Y \lesssim r_0$ are in general not circular, although it turns out that for a flat current profile the drift orbits are in fact given closely by Eqs. 2.2.3–2.2.5. For more details the reader is referred to Spong's work.^(44,68)

2.3 SYSTEMATIC EFFECTS ON THE ORBIT MINOR RADIUS

It has been shown in the last section that for the case of a flat current profile the runaway drift orbits are all shifted outward from the center of the plasma current distribution by an amount d_Y determined by Eq. 2.2.5. It was also noted that the minor radius r_0 was not determined by the analysis, so that for a given runaway energy and plasma current there are many possible orbits determined by the combinations of r_0 and I_A/I . Since the confinement of a particular runaway is determined by both of these factors, it is of interest to evaluate the evolution in time of the minor radius of a runaway which is accelerated from rest to some final kinetic energy $E = (\gamma-1)mc^2$ in a plasma in which the current may be varying.

Spong has derived from the variation of the toroidal canonical angular momentum in an external electric field an expression for the final form of a runaway orbit accelerated from rest for a flat current profile⁽⁴⁴⁾

$$(R - R_0 - \frac{r_L}{R_0} \rho_P)^2 + z^2 = r^2(t) \quad (2.3.1)$$

where

$$r^2(t) = r_0^2 - \frac{c^2 r_L^2}{2\pi R_0 I} \int_0^t V dt + 2r_L \rho_P + \frac{r_L^2}{R_0^2} \rho_P^2 \quad (2.3.2)$$

and

$$\rho_P \equiv \frac{v_\phi}{eB(r_L)/\gamma mc}$$

Here the orbit displacement for this case of a flat current profile is given by the term $r_L \rho_P / R_0$ on the left of Eq. (2.3.1) which can be checked to be identical with the displacement given by Eq. (2.2.5). The minor radius, however, does not appear to be constant since it depends upon the runaway energy through the freefall acceleration $E(t) \propto \int_0^t V dt$, where V is the applied loop voltage.

An evaluation of the right hand side originally⁽⁴⁴⁾ seemed to indicate that the minor radius of an accelerated runaway orbit did in fact expand as the energy increased. A subsequent analysis⁽⁶⁸⁾ which included properly the higher order terms in r_L/R_0 showed, however, that most of this effect was spurious and that in fact the change in minor radius expected due to the change in runaway energy was negligible for all cases of interest. Thus it appears that the minor radius of the runaway orbit retains the radius of the magnetic flux on which it was born, at least in the case in which the plasma current is kept constant.

The effect of a changing plasma current can be incorporated in the derivation of Eq. (2.3.2), and we find that the right side of this equation is unchanged except for the first term which becomes $r_0^2(I/I_0)$, where I_0 is the initial plasma current and I is the plasma current after some time. The result is that the orbit minor radius depends upon the plasma current according to

$$r^2(t) = r_0^2 I/I_0 \quad , \quad (2.3.3)$$

where we have assumed the exact cancellation of the last three terms on the right of Eq. (2.3.2).⁽⁶⁸⁾ This means, for example, that if a runaway was created at $I_0 = 10$ kA and was confined until $I = 90$ kA, the minor radius during this time should have decreased by a factor of three. This is an important consideration with respect to runaway confinement, but since it has apparently not been noticed previously, we have been cautious in applying it to the data of Chapter 4. Such a "shrinkage" of the runaway minor radius during the current rise phase of a discharge would greatly enhance the confinement of runaways born at early times at the outside of large tokamaks, and this concentration of runaways near the minor axis could affect the subsequent breakdown and energy balance of the discharge. The existence of such a shrinkage effect has not been directly demonstrated experimentally, however, as discussed in Sect. 4.1.

2.4 APPLICATION OF DRIFT ORBIT THEORY TO RUNAWAY EXPERIMENTS

We have seen in Sect. 2.2 that all collisionless untrapped particles in an ideal tokamak field structure with a flat current profile are displaced from the flux surfaces by:

$$\frac{d_Y}{r_L} \approx \frac{r_L}{2R_0} \frac{I_A}{I} \quad (2.4.1)$$

For runaways this displacement is always outward and can be significant for their confinement. For example, a 10 MeV electron in a 100 kA ORMAK discharge ($r_L = 23$ cm, $R_0 = 80$ cm) has $d_Y/r_L \approx .5$, according to this equation.

Since our measurements are made on the runaways only when they leave the discharge and strike the outside limiting aperture, it is important to calculate this marginal confinement condition. In order that the runaway orbit just intersect the outer limiter (point D in Fig. 2.1) we must have:

$$d_Y + r_0 + d_p = r_L \quad (2.4.2)$$

where r_0 is the minor radius of the drift orbit, d_p is the shift of the flux surfaces away from the center of the chamber, and r_L is again the radius of the limiter, which in ORMAK is a tungsten ring indented about 2 cm from the chamber wall at a toroidal location shown in Fig. 1.2 (see also Fig. A.6).

This relation sets an upper limit on the runaway energy which can be contained in the discharge for a given r_0 . Conversely, if we can measure the energy of the runaway as it intersects the outer limiter (which we do) then knowing the current I and the plasma shift d_p we can calculate d_Y and thus infer the runaway orbit minor radius r_0 . Knowledge of the minor radius is useful in checking the predictions of runaway production theory and in evaluating the confinement of runaways in the tokamak discharge.

It is important to note that the minor radius r_0 calculated in this way from the limiter intersection condition may not necessarily equal the minor radius of that runaway during its lifetime in the plasma. This is because r_0 is not constrained by the independent variables of runaway energy and plasma current, so that a detailed integration of the motion is necessary to relate the final minor radius of the runaway to the radius of the magnetic flux surface on which it was born. For example, the argument in Sect. 2.3 showed that the magnitude of the plasma current could affect r_0 , and in the next section we describe several non-ideal field structures which could affect the minor radius in a random manner. Thus our ability to infer from the limiter measurements of the origin and history of the runaway orbits is limited in general by the uncertainty about the extent of such possible variations in r_0 (as discussed in Sect. 5.2). On the other hand, the discussion of Sect. 2.3 has indicated that in an ideal tokamak field structure (at constant current) the minor radius does in fact remain constant during the runaway acceleration process, so that in this case it would be possible to infer a good deal about the overall runaway behavior from the limiter measurement alone.

Assuming for the time being this ideal situation in which the minor radius of a drift orbit remains constant in time, we have plotted in Fig. 2.3 the outer limiter intersection condition for runaway orbits for various current profiles. Using this graph the runaway orbit minor radius can be directly found from the measured value of current and runaway energy at intersection with the outer limiter:

$I_A/I \approx 34E \text{ (MeV)}/I(\text{kA})$. If the plasma current distribution is shifted

from the axis of the chamber by d_p , the inferred r_0 value should be corrected according to Eq. (2.4.2).

The curve labelled "filament" refers to an extreme case in which all the plasma current I is localized within the runaway orbit. The amount of orbit displacement for a given runaway energy and I is smaller in this case than for the flat current profile since the corresponding $B_p(r)$ is larger [see Eq. (2.2.4)]. This case has been calculated using the method of conservation of canonical angular momentum.⁽⁴⁹⁾ The curve labelled "type-B" refers to a current profile which approximates the low density ORMAK discharge⁽¹⁴⁾ which was analyzed by Speng.⁽⁴⁴⁾ This result will be used in the analysis of the data of Chapters 4-6. Note that for $r_0 \geq .5 r_L$ the orbit displacement is roughly 70% that calculated for the flat current case, and that for $r_0 < r_L$ the displacement is sensitive to the precise manner in which the toroidal correction is applied to the model. The result of Ref. 44 has been corrected to include the $1/R$ dependence of the poloidal field (see figure caption 2.3).

The picture of high energy runaway orbits in a tokamak should now be clear, at least in the case of a constant plasma current. A runaway can be created at any minor radius r_0 at any time t_0 according to the particular conditions for runaway production. It should gain energy at nearly the freefall rate and gradually increase its outward displacement until it hits the outer limiter. Runaways born at a given t_0 should move outward in a nested set, since d_Y is nearly independent of r_0 , such that the first runaways to strike the outer limiter would be those with the largest r_0 , and in time successively

smaller r_0 orbits with increasingly higher energy should be seen at the outer limiter.

In an ideal situation in which the orbit minor radii remained constant in time, a good deal of information could therefore be obtained from the measurement of runaway number and energy at the outer limiter. From the measured energy we infer r_0 , and from the number of electrons at this energy the local density of this energy component at r_0 can be found through:

$$S_R(E) = 4\pi^2 r_0 R_0 n(r_0, E) \frac{d(d_Y)}{dt} \quad (2.4.3)$$

where $S_R(E)$ is the measured number of electrons per unit time striking the limiter with some energy E , and $n(r_0, E)$ is then the local density of runaways with this energy.

If the runaway flux to the limiter $S_R(E, t)$ were measured and if the assumption of freefall acceleration was made, the production rate of runaways as a function of both r and t could be obtained since the energy could then be related to the birthtime through the measured loop voltage. We could in fact determine the whole runaway distribution function $f(E, r, t)$ under the same assumptions.

Unfortunately, there are several practical shortcomings in such a program. The main problem is that $S_R(E)$, the distribution function of runaways striking the outer limiter, cannot be readily determined below $E \lesssim E_{\max}/2$, as discussed in Chapter 3. Another problem is the possible existence of non-ideal field structures which could cause an unpredictable change in r_0 or in the orbit shape, as described in the next section and in Chapters 5 and 6. A third problem is that only a

fraction of the runaway orbits are shifted out to the limiter during the steady state part of the discharge, while the rest are lost during the current decay phase as described in Chapter 4. The lack of knowledge of the current profile and plasma position during the current profile and plasma position during the current decay increases the uncertainty with respect to the innermost runaway orbits.

Thus the complete picture of runaway confinement cannot in practice be obtained from the combination of drift orbit theory and limiter measurements, but at the least these methods have allowed a significant improvement in our knowledge and have provided a framework on which further refinements can build. In the next section we review some of the areas in which such refinements may be needed in order to understand runaway confinement in the real tokamak environment.

2.5 NON-IDEAL EFFECTS ON RUNAWAY DRIFT ORBITS

In order to interpret properly the measurements of runaways striking the outer limiter, we must carefully consider whether the simple drift orbit model described in Sects. 2.2 and 2.3 does adequately represent the real runaway orbits in ORMAK. The basic problem is that non-ideal magnetic or electric fields in the tokamak might alter the particle trajectories significantly.

There are several indications that such effects can be significant. Certainly the neoclassical transport theory for thermal electrons, which is based on a collisional version of the drift orbit model, is known to give electron heat conduction coefficients which are apparently an order of magnitude smaller than those observed.⁽¹⁴⁾

The anomaly is most likely due to the presence of electrostatic waves associated with density gradients, although even after over 10 years of intensive study the anomalous loss mechanisms are still not clear.⁽²⁾

The runaway component has also been shown to be affected by anomalous transport in some circumstances (see Sect. 1.3) where by "anomalous" we mean that which is not expected on the basis of the classical drift orbit model described in Sect. 2.2. In fact, it is often postulated that low energy runaways undergo the same anomalous transport as do the thermal electrons, or at least have a similar confinement time.^(39,53,59) In ORMAK there are also indications that the high energy runaways can show some anomalous transport in a quiet discharge (Chapter 5) and in the presence of instabilities (Chapter 6). On the other hand, it is clear that drift orbit model is a necessary and usually a sufficient condition for an understanding of high energy runaway behavior in the normal ORMAK discharge.

Before presenting the experimental results which can be used to examine the validity of this last statement, it will be useful to list some of these non-ideal effects which can require modifications of the simple drift orbit model. It must be said that a quantitative evaluation of all these effects has not been made at this point, and that a comparison with experimental results is available only in a few cases. The list serves, however, to define the rough range of applicability of the theory and the sorts of uncertainties involved in its application to the experimental results of Chapter 4.

Table 2.1 Some Non-Ideal Effects

- A. Plasma Instabilities
 - 1. Major disruption
 - 2. Internal disruption
 - 3. Magnetic islands
 - 4. Drift waves
- B. Static Magnetic Field Perturbations
 - 1. Field errors and stray fields
 - 2. Vertical field ripple
 - 3. Toroidal field ripple
- C. Runaway Collective Effects
 - 1. Observed "RF spike" instability
 - 2. Theories of the instability of the runaway tail

At this point we can briefly review these possible effects on runaway confinement, leaving a detailed discussion of the relevant experimental observations to Chapter 6.

A.1. Major disruption: This is a classical limitation to tokamak operation at high density and high current, the mechanism of which is presently not understood. Its symptoms are a sudden loss of confinement for both thermal and runaway particles, probably associated with a rearrangement of the flux surfaces. This phenomenon is easily noticed and was avoided during most of the runaway work described in this thesis.

A.2. Internal disruption: This is a relaxation oscillation in temperature which occurs near the center of the discharge in most

tokamak operating regimes. It probably involves a slight rearrangement of the current profile, but in most cases does not produce any noticeable effect on the runaway flux to the outer limiter.

A.3. Magnetic islands: These are a class of helical current perturbations which can exist in the steady state of normal tokamak discharges. The ubiquitous "Mirnov oscillations" (≈ 10 KHz $\hat{B}_p$ pickup at the chamber wall) are now generally identified with rotating magnetic island structures near the outside of the plasma. Magnetic islands cause a dramatic change in the particle drift surfaces (as described in Sect. 6.2) and can therefore significantly affect runaway confinement, although in most ORMAK discharges the amplitude of the oscillations is small and their effect seems negligible.

A.4. Drift waves: This is a generic term for a class of electrostatic waves which are basically associated with density and temperature gradients near the middle or outside of the discharge. They are thought to cause part of the anomalous electron heat conductivity, and may also affect runaways through a random $\vec{E} \times \vec{B}$ radial diffusion.⁽⁵⁴⁾ The wavelengths are typically $\lesssim 1$ cm, and so might also produce non-adiabatic effects on the runaway orbit (see Appendix E). There have only been a few measurements on these waves on tokamaks, and none on ORMAK.

B.1. Field errors and stray fields: The magnetic field symmetry of a real tokamak is spoiled by many things, including local field errors due to coil leads and stray fields which can penetrate the plasma especially at the gaps in the conducting shell.⁽¹⁾ One could imagine that a field line might be diverted so that it eventually

intersects the chamber wall, thus allowing runaways (which have a large velocity along the field line) to be lost more rapidly than thermal electrons. However, since the poloidal confining field is so much larger than these error fields [except at early times in the discharge⁽⁴¹⁾], it is likely that the flux surfaces in steady state would be only slightly distorted by such effects.

B.2. Vertical field ripple: The vertical field needed to keep the plasma in equilibrium is not uniform, especially at the outside nearest the coils which produce this field. A study of this effect showed⁽⁷¹⁾ that there could be an ≈ 2 cm ripple in the vertical field near the liner in ORMAK. This might produce some subtle enhancement of the radial motion of runaways near the limiter caused by a distortion of the local flux surfaces.

B.3. Toroidal field ripple: The finite number of toroidal field coils also produces ripple in B_T , especially near the outside of the chamber. Although this inhomogeneity has been seen to affect runaways in an extreme case in which one toroidal field coil was deactivated,⁽⁴⁰⁾ the field ripple in ORMAK is extremely small ($\lesssim 1\%$ at the outside edge) and so should have a negligible effect on runaway drift orbits. This was checked using a guiding center code developed at Oak Ridge⁽⁷²⁾ which showed that even for a 10% ripple at the edge of ORMAK the runaway (untrapped) orbit showed only a 2.5 cm periodic radial excursion near the outer limiter.

Another possible effect of the toroidal field ripple could be a resonance between the periodicity of the ripple (≈ 10 cm in ORMAK) and the period of the gyroorbit of the runaway (≈ 10 cm at $E = 10$ MeV and

$B_T = 20$ kG). Such an interaction is known to affect a relativistic electron's pitch angle in linear geometry,⁽⁷³⁾ but apparently no detailed study has been made of the problem in tokamak geometry.

C.1. "RF spike" instability: This is an interesting phenomenon that apparently is due to an instability of the runaway distribution function. It has been studied on TM-3,^(36,55) TFR,⁽⁶³⁾ and recently in ORMAK (see Sect. 6.4). It points out that at least in some circumstances the single-particle model for runaway dynamics may need to be modified to include collective effects. Unless otherwise mentioned the data reported in this thesis was taken during times when the symptoms of this instability were not observed.

C.2. Theories of the runaway tail: There have recently been several theories on the instability of the runaway tail⁽⁷⁴⁾ which aim to understand the "RF spike" instability and also the steps observed in runaway synchrotron emission.⁽⁶²⁾ The basic idea is that the anisotropy in the runaway distribution function has free energy which can drive plasma waves which tend to relax the anisotropy, and that these waves can cause enhanced pitch angle scattering of the runaways and possibly also an increase in the effective friction due to transfer of runaway energy to plasma waves. At present there is some controversy about the relationship between the various theories and the experimental results, partly because the theories are still in an idealized state (i.e., requiring assumptions about runaway production and loss rates). In general, however, the main attention is on the nonrelativistic epithermal runaways and not on the high energy runaways which are the subject of this thesis. Thus although this area

remains one of key interest for future runaway studies in tokamaks, we have taken the attitude here that only when the classical single-particle model fails to explain the high energy data, is it necessary to invoke such anomalous processes.

In summary, the non-ideal effects which we have listed can in theory have two basic manifestations with respect to runaway confinement. The more obvious one is a deviation of the guiding center from the simple drift orbit model prediction. This can occur in a gross way in the major disruption, or in a subtler but well defined way such as in the cases of magnetic island or toroidal field ripple. There is also the possibility that drift waves or stray fields can change the minor radius of the drift orbit in an unpredictable way. The effect of a variable minor radius will be discussed in Sect. 5.2.

The other main manifestation of non-ideal effects would be a non-adiabatic change in the runaway's magnetic moment (pitch angle) or an anomalous friction affecting the runaway energy. These would in general be caused by small-scale perturbations such as the drift waves, toroidal field ripple, or high frequency waves excited by runaway instabilities. In general we have at present only indirect and ambiguous experimental information on these microscopic effects.

3. EXPERIMENTAL METHOD

3.1 POSSIBLE DIAGNOSTICS FOR RUNAWAY ELECTRONS

Before describing the hard x-ray detection system which was used for our work it would be useful to review the other methods of runaway detection which have been developed. A table which lists the recent experimental work has been given in Chapter 1, so we concentrate here on the merits and shortcomings of the different methods.

Perhaps the most straightforward and classical detection method is plasma bremsstrahlung, in which the spectrum hard x-rays produced by runaway-ion collisions in the plasma is measured. Although this has been attempted with some success,^(39,53,56) there are several problems which limit its usefulness and which in fact have prevented its use on ORMAK.

The primary problem is the low cross section for this process, which makes it necessary to take great care to reduce the hard x-ray background due to limiter or wall radiation. In the Pulsator and TM-3 experiments, in which the detector was aimed normal to the toroidal field into a hole in the far wall, the rates were $\lesssim 50$ counts/shot⁽⁵⁴⁾ and "several hundred shots/spectrum,"⁽⁵⁶⁾ respectively.

In ORMAK this problem was exacerbated by the relatively large background of x-rays due to the high energy runaway components, by the small solid angle available to view the plasma, and by the lack of a "black background" at the liner far wall. Attempts to measure plasma bremsstrahlung both tangentially and normally to the toroidal field thus failed to observe anything but the limiter and wall background.

There are also several difficulties intrinsic to this measurement. Firstly, it is a chordal measurement which requires unfolding for interpretation of radial profiles. Secondly, it requires a knowledge of $\langle Z \rangle$ for an interpretation of the absolute intensity.^(39,56) Most significant, however, is that the choice of a viewing direction restricts the energy range of runaways to be measured, since the thin-target bremsstrahlung emission is always peaked in the forward direction in a cone of width $1/\gamma$. For example, viewing 90° to the electron direction essentially restricts the sensitivity to $E \lesssim 2$ MeV, and when viewing at 0° one becomes sensitive to the highest energy components only (see Appendix A). The 90° scheme which has been used in most of the experiments reported so far also requires an assumption about the runaway pitch angle, which (see Sect. 2.1) is uncertain for $E \lesssim 2$ MeV.

Nevertheless this plasma bremsstrahlung measurement is apparently the best diagnostic for the low energy (< 500 keV) runaway population. The recent work on TM-3 with runaway-excited impurity lines⁽⁵⁶⁾ should be mentioned as an important extension of the usefulness of this method.

Another method which has been used specifically to study runaway confinement is the probe technique introduced by the LT-3 group.^(57-59,74) Here a pair of small metal wires is inserted radially into the discharge; the innermost acts as a runaway absorber and the outer one is observed with a hard x-ray detector. The flux of runaways striking the outer probe is taken to be a measure of the runaway radial diffusion rate (see Sect. 5.1). It was consistently observed that the radial diffusion speed was very large, especially during the unstable phases of the LT-3 discharge, such that $D \sim 10^4 - 10^5$ cm²/sec (which corresponds to a runaway

confinement time of $\lesssim 1$ msec).

Although this is presently the most direct method for measuring runaway diffusion, there are several problems in its interpretation and practicality. Firstly, the use of probes is restricted to relatively small and cold plasmas, since even a 1 cm diameter probe will melt during low density discharges in larger machines such as ORMAK (see Fig. 5.1). Secondly, the effect of the probe itself on the runaways is open to question; for example, it is possible that runaway scatter from the surface of the inner probe and thus reach the outer probe in a manner unrelated to runaway diffusion (see Sect. 5.1). Thirdly, the runaway transport measured is basically characteristic of the region of plasma between the inner and outer probes, a region which may again be significantly influenced by the presence of the inner probe.

Nonetheless the probe experiments have shown that runaways are at times subject to rather violent transport, and that in these cases it is more appropriate to model their behavior in terms of a diffusion model rather than the drift orbit model of Chapter 2. We shall adopt in part this picture of runaway dynamics in our discussion in Chapter 5.

The other major diagnostic technique for runaways is the measurement of synchrotron radiation.^(27,61,62) This is generally microwave or far infrared radiation 20–500 GHz which is due to the gyromotion of the runaways at ω_{ce} (and not due to the toroidal motion as in synchrotrons). Measurements have been made on many tokamaks and a review of the state of the experiments and theory has recently been made by the Maryland group.^(75–77)

In general the difficulty with this method is in the interpretation of the observed frequency spectra. A runaway can radiate a broad spectrum up to $\sim 5 \omega_{ce}$ depending on its energy and pitch angle, and the frequency ω_{ce} is itself dependent on the runaway energy and on the local toroidal field ($\omega_{ce} = (eB_T/\gamma mc)R_0/R$). Furthermore the emission spectrum can be strongly modified by absorption by the plasma. (77,78) With appropriate theoretical modelling, however, this method may become the most elegant and practical way to observe the runaways and their interaction with the plasma.

3.2 THE HARD X-RAY DETECTION SYSTEM

Introduction

For the experiments described in this thesis the high energy runaways are generally detected by the bremsstrahlung they produce when they strike the outer edge of the tungsten limiter (see Figs. 3.1 and A.6). The main advantage of this method is in its experimental simplicity and in the absence of background effects (since the outer limiter is the strongest x-ray source in the machine). There is also an important practical advantage in that the detector does not have to be interfaced directly with the plasma confinement chamber, since the high energy x-rays can easily penetrate the material surrounding the machine. The main disadvantages of this method are that the thick target bremsstrahlung spectra do not allow an unfolding of runaway components below about half the maximum runaway energy, and the single-position nature of the measurement makes the interpretation of the radial profile of runaway confinement difficult.

The experimental arrangement basically consists of a well-collimated and shielded x-ray detector and its associated electronics and data acquisition system. In the next few pages we describe the operation of the detectors and the interpretation of the observed spectra, and in Sect. 3.3 we discuss the data analysis procedures used in obtaining the results described in Chapters 4-6.

X-Ray Detection

The basic element in the hard x-ray detection system is a large (5" x 5" or 3" x 3") NaI (Tl*) scintillation detector which is operated in the single pulse mode. These detectors were purchased from Harshaw with a reliable "sealed-unit" design, so that mounting and stray light problems were eliminated.

Of the various hard x-ray detectors available, the NaI has been the most widely used and documented in the energy range $\lesssim 10$ MeV.⁽⁷⁹⁾ Liquid and plastic scintillators have the advantage of a faster decay time ($\lesssim 10$ ns compared with the ~ 250 ns of NaI), but would have to be impractically large to have an efficiency comparable to the heavy NaI crystal. Even with the large 5" NaI detector the photopeak efficiency at 10 MeV is $\lesssim 20\%$, so that in interpreting the pulse spectrum the scintillator response function must be taken into account (see Figs. 3.2 and 3.3).

The energy resolution of these large NaI crystals is $\sim 8\%$ FWHM for the standard ^{137}Cs line at 662 keV, so with resolution $\propto 1/\sqrt{E}$ (see Appendix B) we have better than .5 MeV resolution at 10 MeV. Fortunately no better energy resolution than this is needed for studying the smooth bremsstrahlung spectra, but for study of high Z impurity lines

at $E < 100$ keV a germanium detector or bent-crystal spectrometer would be preferable.

The high energy bremsstrahlung photons are peaked in the forward (electron) direction and come almost entirely from the thick target interaction at the outer (tungsten) limiter and not from the thin target volume bremsstrahlung (see Appendix A). To reduce the background of scattered x-rays a large lead collimator was used which provided a spacial resolution of $\lesssim 6$ cm at the outer limiter. The design and positioning of the collimator are shown in Figs. 3.1 and 3.4. The important elements of the collimator are described in the figure caption; although the design criterion can become quite sophisticated,⁽⁸⁰⁾ we found it sufficient to use the rule "two inches of lead equals x10 attenuation" for the construction of the apertures and shielding. The apertures were defined by a total of ≈ 12 inches of lead.

It is important to note that there is a good deal of matter between the x-ray source at the outer limiter and the detector. Typically we have 25 cm aluminum, 5 cm copper, and $\approx .6$ cm tungsten in the tangential detection line due to the ORMAK shell and coil frames, coils, and to the limiter itself. In order to reduce the count rate to manageable levels we also chose to insert $1/2''$ – $2\ 1/2''$ of lead into the collimator as a "filter" and absorber. All this material typically reduces the high energy x-ray flux at the detector by a factor of ≥ 100 , and so of course it must be taken into account when attempting to unfold the pulse spectrum or determine the absolute number of runaways, as described in detail in Appendix A.

This material also serves as a source of Compton scattered x-rays which can broaden the apparent source and also affect the low energy end of the direct radiation spectrum. These effects have been estimated and shown to be negligible for our purposes; for example, the horizontal scan described in the next chapter showed a clear x-ray source at the outer limiter with $\leq 1\%$ background elsewhere.

The only spurious source of gamma radiation in the MeV range (besides the negligible cosmic ray component) is that due to thermal neutron capture in lead (≤ 7.4 MeV), for example, or in the iodine in the crystal (≤ 6.7 MeV). In fact, during strong neutron producing discharges (deuterium injection into deuterium) the hard x-ray detection system was overloaded with these neutron-induced gamma rays. In the usual case of hydrogen discharges, however, the main neutron production comes from photonuclear reactions in the limiter [tungsten threshold ≈ 7 MeV⁽⁵¹⁾], which ought to produce a negligible background for our system. This was checked by noting that the hard x-ray count rate was reduced by $\approx \times 3$ when a 1" lead absorber was placed in the collimator aperture (this absorber would not be effective for reducing the thermal neutron background at the crystal) and by the fact that there was a clear source at the outer limiter.

Interpretation

In the remainder of this section we will review some of the considerations necessary for the proper use and interpretation of this hard x-ray system. A detailed discussion of the two main points can be found in the Appendixes A ("Interpretation of Scintillation Spectra") and B ("Pileup Reduction in NaI"). Some further information about the

crystal and collimator can also be found in a memo "Hard X-Ray Detection System for ORMAK," ORNL/TM-5022.⁽⁸¹⁾

Although the design and operation of the system is fairly straightforward — aim the collimator at the limiter and turn on the high voltage to the crystal — it soon becomes obvious that there is a trade-off between the desire to achieve high count rates (good statistics) and the tendency of the pulses to "pile-up" on top of one another. This is a significant consideration for our system since we would like to accumulate spectra over timescales ≈ 10 msec. This generally requires summing over a number of similar discharges, so that the available count rate is inversely proportional to the number of shots required to accumulate a useful spectrum.

An example of a discharge showing a clear pileup effect is shown in Fig. 3.5. Each dot is the stretched output ($4 \mu\text{s}$ wide) of the amplified NaI pulse; the height of the pulses is proportional to the amount of energy the x-ray deposits in the crystal, as calibrated with a ^{22}Na source. The most interesting feature of these spectra is the maximum pulse height, which corresponds to the maximum runaway energy, and it is clear that this maximum energy appears to increase suddenly at times during this discharge (i.e., points A and B). Without a pileup inspection process it would be difficult to decide whether this is a real increase in energy or just the result of an increased count-rate leading to pileup. This is especially uncertain during more normal discharges in which the countrate is a smooth function of time.

Our resolution of this problem, as discussed in Appendix B, uses a fast-shaped form of each of the rather slow NaI output pulses to decide

whether that pulse is near enough to other pulses to become piled up. The pileup inspection process produces an "inhibit" signal which gates out the piled up pair; the inhibit signal is also recorded on an oscilloscope to indicate when pileup is becoming serious.

Although this pileup rejection system is nearly standard for scintillation spectroscopy,⁽⁸²⁾ we have explored the limits to which it can be useful for our application. The result is that the pileup resolving time is limited to $\approx 100\text{--}200$ ns by the random fluctuations in photoelectron emission at the crystal during the leading edge of each light pulse. This resolving time, although about an order of magnitude better than that without pileup inspection, does limit the count rate to $\approx 5 \times 10^4$ counts/sec for $\lesssim 1\%$ pileup. The details are discussed in Appendix B.

When it is clear that pileup effects have been eliminated, the next problem is that of interpreting the pulse spectrum in terms of the runaway electron distribution function at the limiter. In order to do this we have to consider the transformation of the electron spectrum into a thick target bremsstrahlung spectrum, the passage of this x-ray spectrum through the various absorbing materials, and finally the particular response of the NaI crystal to this remaining x-ray spectrum. The complexity of these transformations, plus the limited statistics available for sample pulse spectra, has forced us to adopt the indirect method of unfolding, whereby we match the observed pulse spectra with theoretically computed monoenergetic electron spectra instead of directly determining the electron distribution from the data.

The major difficulty with this procedure lies in the fact that the expected number of pulses per electron increases roughly as E^{3-4} in the MeV electron range, and only a small fraction of these scintillator pulses correspond to nearly the full electron energy (see Fig. 3.6). This is due to the forward peaking of the 0° thick target bremsstrahlung spectrum and to the relatively low photo peak efficiency of NaI in this energy range, as discussed in detail in Appendix A.

The result is that in most cases only the electrons near maximum energy contribute to the observed pulse spectra, and so the unfolding of energy components below $E_{\text{max}}/2$ becomes extremely difficult. An example of a spectrum matched with computed 10 MeV and 5 MeV mono-energetic components is shown in Fig. 3.7. In this case, which is typical rather than optimal, we see that basically only the maximum electron energy can be determined from the data.

Therefore the hard x-ray results in the following chapters refer only to this maximum energy component unless otherwise specified. In practice this maximum is usually defined to ± 1 MeV by the uncertainties in fitting the upper end of the spectrum. Even with this limitation many interesting results can be obtained through this procedure, as described in the next chapter.

Another point of concern with respect to the interpretation of these pulse spectra is the possibility that the x-rays may be "bunched" in time such that the pileup inspector would respond to the sum of bunched pulses with < 100 ns/bunch and thus allow the badly piled up sum of these pulses to be recorded as a single high energy pulse. This might occur, for example, if the runaways were to hit the limiter in

discrete groups defined by the transit time of the torus (≤ 20 ns), or if a number of low energy x-rays from a single electron (produced over its slowing down time of < 1 ns) were able to make it to the detector.

The latter possibility is extremely unlikely considering the small probability that two x-rays from a single electron would be detected. For example, with our typical solid angle and attenuation there is an expected rate of $< 10^{-9}$ counts/electron at our detector for all x-rays produced by high energy runaways (see Fig. 3.6). This is in part due to the absorption of nearly all low energy ($< .5$ MeV) x-rays by the absorbing material in the machine and collimator. Typically we detect $\approx 10^3$ counts for $\approx 10^{13}$ high energy runaways hitting the limiter (and many fewer counts come from the lower energy components) so that only one x-ray per electron is ever detected.

The other possibility of a collective runaway loss mechanism is somewhat less unlikely. In fact, an early experiment apparently saw such a loss mechanism for runaways which, in the absence of random pulse pileup, produced an x-ray pulse height larger than that expected from freefall acceleration of the runaways.⁽²⁶⁾ A coincidence measurement indicated that this was due to pileup of bremsstrahlung due to many electrons hitting the limiter in < 100 ns bunches.

Although in ORMAK we have not observed such anomalously large pulse heights in the absence of random pulse pileup, we checked this possibility in two ways. The more straightforward way was to observe that the maximum energy did not change significantly as the thickness of lead absorber was varied, as might be expected to happen, for instance, if many x-rays with $E < .5$ MeV were piling up within 100 ns

to produce a single high energy count. Another test was made to check the spacing in time of x-rays using a fast plastic scintillator (1" x 1", 2 ns time constant). A scope was repeatedly triggered on random x-ray pulses, and the time interval up to 200 ns after the triggering pulse was observed. No unexpected bunching of x-rays in the time interval $\approx$ 20–200 ns could be observed, except during the sudden runaway dumps caused by disruptions (see Sect. 6.1).

These observations, plus the very systematic and reproducible patterns of maximum energy vs. time described in the next chapter, convinced us that no rapid collective loss was occurring which would influence the interpretation of maximum energy as described above.

Before closing this section on the hard x-ray diagnostic it would be worthwhile to mention the integrating uncollimated hard x-ray intensity monitors which were always used in conjunction with the pulse system. These were small NaI or ionization detectors operated in the current mode (integration time $\lesssim$ 50 μ s) which responded to the x-ray flux escaping from the machine at various places around the room (see Fig. 3.4). They were thus sensitive to $\geq$.2 MeV x-rays which could pass through the aluminum shell and walls, but of course gave no energy information on higher energy components. They were useful to pick out fast dumps of runaways which the pulse system, with its limited count rate, could not accurately follow (see Fig. 3.5). Some phenomena such as the disruption and the MHD correlations could only be observed with the integrating detectors, as described in Chapter 6. These detectors were also useful to show that in most cases the runaway loss was a smooth process, i.e., that high count rate (pileup producing) x-ray losses were not occurring.

3.3 DATA ANALYSIS

The hard x-ray pulse data from one or two of our collimated detectors was routinely displayed on oscilloscopes and also collected in a multichannel analyzer linked to the ORMAK computer system. We have also made extensive use of the other diagnostic systems available on this machine, particularly the Thomson scattering (for electron temperature and density profiles), the 2 mm microwave interferometer (for line averaged density), the $\dot{B}$ pickup loops (for plasma position and MHD activity), and the PIN diode array (for internal mode correlations).

The hard x-ray pulse spectra were at first recorded solely with polaroid film by which it was possible (using a 4 μ s square wave version of the amplified NaI pulses) to count individual pulses with a sweep speed of 1–5 ms/cm. Although the counting procedure was tedious, it was certainly possible to accumulate useful spectra this way, especially for the case of early runaway loss described in Sect. 4.2.

The multichannel analyzer (HP5401A) was soon implemented to accumulate spectra during four time intervals and to send the data to the computer after each discharge. This was only marginally useful to study the energy evolution of the runaways, so a 16 group system was developed by Dave Swain which typically covered 80 ms of the discharge in consecutive 5 msec intervals. Even though this system was nearly indispensable for the later studies, we still found it useful to monitor the pulse train with oscilloscope photos to look for finer time structure and for possible intervals of pulse pileup.

The pulse spectra were compared to the calculated monoenergetic spectra produced from consideration of the bremsstrahlung and scintillation transformations, as discussed in Appendix A. The computer code for this calculation, originally written with the help of Tim Renk, was later extended by Dave Swain so that the data could be directly fitted by a computer/programmer interaction. An example of this procedure is shown in Fig. 3.8.

The procedure for fitting the observed spectra to the theoretical curves is evidently a delicate one, since the most significant information is contained at the upper end of the spectrum where there are fewest counts. The usual procedure involves a "stripping" process in which the upper end is fitted first and then its contribution to the lower end is subtracted out, allowing the next highest energy component to be fit, and so on. In adopting this procedure we generally found that a component which fit the upper 20% of the pulse height range could account for most of the lower energy counts as well. This should not be taken as an indication that there was only a single high energy runaway component, but rather that the higher energy components contribute pulses $\propto E^{3-4}$ to our spectra (see Fig. 3.6). This was confirmed by a version of the program which fit a flat distribution function to the data instead of a monoenergetic one, with relatively little change in the quality of fit as shown in Fig. 3.8.

It should be pointed out again that even a small fraction of piled up pulses near the upper end of the spectrum can significantly distort this unfolding procedure. Our criterion was generally that $\leq 1\%$ of the pulses in this region should be due to random pulse pileup (see

Appendix B); but even with this conservative figure, if there were a few pulses well above the apparent upper end of the spectrum they were not weighted too heavily in the energy determination.

Thus the two main constraints on the analysis leading to a "maximum" electron energy were that the upper 20% of the pulse height spectrum was fit by this component, and that the lower energy end of the observed spectrum had more counts than that expected from the background due to this maximum energy component. For a typical spectrum with a total of ≈ 500 counts this process generally resulted in an uncertainty of ± 1 MeV in the maximum energy component in the range 4–10 MeV.

In some cases it was considered worthwhile to attempt the unfolding of some lower energy components as well. For example, in Fig. 3.9 we show the spectrum observed during a series of strong runaway beam discharges in which it was important to find the beam distribution function.⁽³⁰⁾ Given the statistical uncertainty in the data it was felt that at best the unfolding procedure could be pushed to give such a semi-quantitative three point distribution.

In a few cases in which the maximum runaway was particularly clear from the upper end of the pulse spectrum, we also made use of the smooth envelope of the maximum pulse height as seen on the scope photo to estimate the energy evolution in time. This is particularly easy for maximum energies below about 6 MeV, since the fraction of pulses expected near the electron energy is relatively high (see again Fig. 3.6). If done with care, this procedure can smooth out some of the statistical errors introduced by the overbinning needed to

get good time resolution; an example of a pulse train for which this is possible is shown in Fig. 3.10.

Besides the maximum energy the other important property of these runaways is their absolute number. In principle this is given immediately by the absolute thick target bremsstrahlung spectrum, the known absorption of materials between the detector and the source, and the solid angle and efficiency of the detector. In fact, however, there are some other uncertainties which have also to be taken into account. First of all, although in theory all runaways should hit the outer limiter at the equatorial plane, in practice the ORMAK liner may have been sufficiently bumpy so that some electrons hit the liner instead of the limiter. That this was not a dominant factor can be seen in the fact that the limiter did show rather severe melting; had this energy been deposited on a bulge in the liner it would have put a hole in it. It is possible, however, that a diffuse spray of runaways may have hit the liner due to scattering of runaways from the edge of the limiter (see Sect. 6.1). Although such effects are difficult to estimate due to the uncertain geometry, we have guessed that the calculated flux of runaways should be multiplied by a source factor of up to $S = 10$ to account for runaways lost to the liner outside of our typical viewing diameter of $\lesssim 10$ cm at the outer limiter.

In addition to this geometrical factor it must be stressed that at a given time the detector is sensitive to only the runaway component near maximum energy, with lower energy components being overshadowed by the background of higher components, and with runaways of $E < 2$ MeV being essentially undetectable. In all likelihood the distribution

function of runaways is monotonically decreasing from thermal to the highest energy allowing a substantial contribution to the runaway population from the relatively low energy runaway tail. This component would be best measured by the plasma bremsstrahlung technique as described in Sect. 3.1. Therefore our estimates of high energy runaway populations are lower limits to the total superthermal electron component.

In view of these uncertainties, plus the additional factors relevant to the absolute bremsstrahlung spectra as discussed in Appendix A, it is clear that the absolute runaway flux near maximum energy can be determined only to within a factor of ~ 3 either way through our method. Relative fluxes can of course be determined with more accuracy as shown, for example, in Sect. 4.1.

4. EXPERIMENTAL RESULTS FOR NORMAL DISCHARGES AND THEIR INTERPRETATION IN TERMS OF THE DRIFT ORBIT MODEL

In the last two chapters we have shown that runaway electrons ought to be created and accelerated freely in normal ORMAK discharges, and that it is possible to study runaway confinement by observing the hard x-ray bremsstrahlung produced when the runaways strike the metal limiter. In this chapter we describe the basic patterns of runaway confinement observed in ORMAK, and we show to what extent these results can be understood on the basis of the runaway drift orbit model of Chapter 2.

This chapter is organized as follows: in Sect. 4.1 the dependence of the hard x-ray pattern on the plasma density is noted, and the evolution of runaway energy vs time is described in detail for the two regimes of interest for this thesis. In Sect. 4.2 the drift orbit theory is applied to the data of Sect. 4.1, and in Sect. 4.3 some further experimental tests of the drift orbit model are discussed. Overall we find that the runaway confinement is surprisingly well described by this simple model, but that there are also some results which seem to require the consideration of non-ideal effects such as those discussed in Sect. 2.5. The exploration of these nonideal effects is continued in Chapters 5 and 6 through a discussion of runaway diffusion and the effect of instabilities on runaway confinement, respectively.

4.1 RUNAWAY PATTERNS IN ORMAK

There are four basic regimes of ORMAK operation defined by the experimentally controllable parameters I and $\bar{n}$, where I is the total discharge current and $\bar{n}$ is the line averaged plasma density. The

distinctive features of these regimes are summarized in Table 4.1 and in Fig. 4.1. It can be seen that for a fixed plasma current there is only a limited range of density over which a "normal" discharge can be obtained, such that for densities at the high end of this range the discharge becomes violently unstable to the "disruptive" instability, and for discharges at the low density end of the range the runaway electron population begins to dominate the character of the discharge. In the density range between these two limits there are two regimes of stable operation labelled "high density" (or "Type A") and "low density" (or "Type B").

Table 4.1 Discharge Regimes

Regime	$\bar{n}_e$ at 100 kA (q=5)	Stability	Temperature	Comments
Runaway	$\lesssim 0.5 \times 10^{13} \text{ cm}^{-3}$	Unstable	Cold	Appearance sensitive to impurity level
Type B	$\approx 0.5 - 1.5 \times 10^{13} \text{ cm}^{-3}$	Stable	Hot	Fairly flat current profile; low level MHD activity
Type A	$\approx 1.5 - 2.5 \times 10^{13} \text{ cm}^{-3}$	Stable	Hot	Peaked current profile; high level MHD activity
Disruptive	$\gtrsim 2.5 \times 10^{13} \text{ cm}^{-3}$	Unstable	Cold	Violent MHD activity; plasma hits wall

Hard x rays indicating the presence of runaway electrons have been observed in each of these discharge regimes. In Fig. 4.2 we see that the x-ray patterns for the disruptive and runaway regimes show the effect of instabilities which cause a sudden loss of runaways, and that the two normal discharge regimes can be distinguished by the time at which the runaways are lost. The higher density Type A regime shows relatively fewer runaways than does the lower density Type B regime, and the runaways in Type A are all lost before about 10 msec, whereas those in Type B discharges are mainly lost during the current decay at the end of the discharge.

We have chosen for the purposes of this thesis to concentrate on these two normal discharge regimes. The plasma behavior leading to the disruption is not well understood at present,⁽²⁾ and ORMAK was seldom run in this regime due to the low plasma temperature and irreproducible nature of the discharges. The runaway regime is, of course, more interesting for our purposes, but the behavior in this regime is clearly decoupled from that in the normal discharges, and our results in this area have been published elsewhere.⁽³⁰⁾

Turning now to a more detailed description of the runaway behavior in the normal discharge regimes, we show in Figs. 4.3 and 4.4 the patterns of maximum runaway energy vs time observed for the Type A and Type B discharges, respectively. This data was obtained through analysis of the bremsstrahlung spectra produced when the runaways left the discharge and collided with the outer limiter, as described in Chapter 3. Thus the energy referred to in the graphs is characteristic of the highest energy runaway component hitting the limiter at any particular time.

A simple but important experimental result can be obtained from these figures; namely, that the maximum energy is observed to increase at nearly the freefall rate for both these regimes. This behavior is consistent with our calculation of the collisionality of high energy runaways in Sect. 2.1.

For the high density Type A discharge we have drawn curves representing the freefall acceleration which would be expected for a runaway born at $t=0$, and it can be seen that these curves fit the data well for both the 90 kA and the 162 kA cases. It should be noted that the loop voltage is rather high at the beginning of the discharge (see Fig. 4.2), thus allowing runaways to gain nearly 10 MeV in less than 10 msec. An inductive correction to the loop voltage has been applied using $L = 0.5 \mu\text{h}$; this correction amounts to about 3 MeV at $I = 100 \text{ kA}$, and is somewhat dependent on the minor radius assumed for the runaway orbit, as discussed in the next section.

For the lower density Type B discharge the runaways are seen to gain energy more slowly in the lower loop voltage characteristic of the steady state of the discharge, but again the rate is nearly consistent with a freefall acceleration during the time interval $t \approx 40\text{-}60 \text{ msec}$. In this case the x-ray intensity was too low to give a value for the runaway energy before 40 msec; however, the freefall curve can be extrapolated backwards to give a birth time of $t \approx 10 \text{ msec}$ for this group of runaways (note that there is no inductive correction during this constant-current phase). The maximum energy remains nearly constant during the initial stages of the current decay while many runaways are being lost, but a group of lower energy electrons are clearly contained until the very end of the discharge.

The relative population of the Type A and Type B runaways varies with the plasma density as shown in Figs. 4.5 and 4.6. The number of high energy runaways in the low density region increases rapidly with decreasing density in a manner reminiscent of the production rate scaling $S \propto E/E_c \propto 1/n$ (see Sect. 1.2), whereas the population of Type A runaways seems to increase with density. The patterns of maximum energy vs time do not vary with plasma density, however, and the existence of an intermediate regime in which both Type A and Type B patterns appear in the same discharge confirms the distinct identity of these two groups of runaways.

It is interesting to compare these observations of runaway loss with a calculation of runaway production rates for an ORMAK discharge. In Fig. 4.7 we show such a calculation for one of the few runs in which temperature data at early times was available. Two features of this graph should be noticed. First, at early time (≈ 5 msec) runaway production is largest near the outside of the discharge (due in part to an assumed electric field skin effect); and second, the production rate peaks at the center of the discharge at $t \approx 10$ msec. It appears that the two features of the observed x-ray patterns (Figs. 4.3 and 4.4) can be related to these two peaks in the runaway production rate, but a quantitative comparison with the data of Figs. 4.5 and 4.6 is made very difficult by the extreme sensitivity of the production rate to E/E_c (see Fig. 1.4). For example, the $\pm 15\%$ error estimates on T_e and n_e at early times can alone result in an order of magnitude change in the runaway rate for $E/E_c \approx 0.4$, and the uncertainty in Z can contribute an equal error. Furthermore, it is quite possible that not all the runaways

which are created will accelerate to relativistic energies;⁽⁴⁹⁾ and it is not even certain that the theoretical calculations properly account for the measured population of lower energy runaways⁽³⁹⁾ (i.e., the theory leaves out collective effects). Due to all these uncertainties we have tried to avoid in this work any direct reliance of the predictions of production theory in our attempts to define the confinement of high energy runaways.

4.2 INTERPRETATION OF RESULTS THROUGH THE DRIFT ORBIT MODEL

We recall from Chapter 2 that high energy runaway orbits are shifted outward from the magnetic flux surfaces by $d_Y \propto E(\text{MeV})/I(\text{kA})$, and so from the measured runaway energy at the outer limiter the runaway orbit minor radius can be inferred. In this section we will see how well this basic interpretation fits the runaway data for the low and high density discharges.

A. High Density Case

For the group of runaways born near $t=0$ in high density Type A discharges (see Fig. 4.3), we have calculated in Table 4.2 the parameter I_A/I and (through Fig. 2.2 and Eq. 2.4.2) the corresponding minor radii r_0 for runaways lost before $t=10$ msec. The result is that for all these runaways $I_A/I \approx 3-4$; thus, computed minor radii are all about 15-17 cm if a Type B current profile is assumed ($d_Y \approx 8-10$ cm), or about 12-15 cm for a flat current profile ($d_Y \approx 10-14$ cm). Although the current profile was not measured during these runs, it is safe to assume that the actual profile was somewhere between these two standard shapes.⁽¹⁴⁾

The picture therefore emerges of runaways being created near the outside of the discharge at $t \approx 0$, and of these runaways being gradually swept into the outer limiter as they gain energy over the first 10 msec of the discharge. The case for the creation of runaways on outer orbits is strengthened by runaway production calculations which show favorable conditions for production there at early times (see Fig. 1.3), and by the freefall calculation which shows a runaway energy vs time which is well fit by $L \approx 0.5 \mu\text{h}$ appropriate for the outside of the discharge (rather than an $L = 0.8 \mu\text{h}$ appropriate for the inside).

Let us now see how this result is influenced by the possibility that the runaway orbit minor radius varies with a changing current according to $Ir^2 = \text{constant}$ (see Sect. 2.3). The general effect of this term should be to decrease the runaway radii during the increasing current over $0 < t < 10 \text{ msec}$, so that the minor radii at birth should be larger than those calculated at intersection in Table 4.2. The most dramatic effect should occur for those runaways lost near $I = 80 \text{ kA}$, since from the freefall curves of Fig. 4.3 it appears that these electrons cannot be born after about $I = 20 \text{ kA}$. In this case the minor radius should have contracted by at least a factor of two between the runaway birth and its loss at the limiter, so that the computed radius at birth should be $\approx 30 \text{ cm}$ instead of $\approx 15 \text{ cm}$. But the limiter radius is only 23 cm !

This poses a dilemma which remains unsolved. If the current-dependent term is accepted within the framework of the ideal drift orbit model, then it is barely possible (using the flat current profile) to interpret this pattern of early runaway loss as due to runaways born very near the limiter. The resulting picture of runaway confinement

Table 4.2 Type A Runaway Loss

Time (msec)	I (kA)	d_p^* (cm)	γ	I_A/I	Minor Radius r_0 (cm) [†]	
					Flat Profile	Type B Profile
<u>90 kA Case</u>						
2	31	-2	5.5	3.0	15	17.5
3	42.5	-2.8	7.8	3.15	15.5	18
4	49	-3.6	11	3.82	13.5	17.5
5	54	-4.5	13	4.1	13.5	18
6	61	-5	14.5	4.04	14	18.5
7	68	-4.0	16	4.0	13.5	17.5
8	76	-3.5	17.5	3.9	13.0	17
<u>162 kA Case</u>						
2.75	53	-1.0	11	3.5	12	15.5
3.25	60	-1.1	11	3.1	13.5	16
3.75	64	-1.2	12	3.2	13.5	16
4.25	69	-1.3	14	3.2	13.5	16
4.75	71.5	-1.4	17	4.0	11	15
5.25	76	-1.5	17	3.8	11.5	15

*Shift of the outermost flux surfaces as calculated from B pickup loops (minus = inward).

†Limiter radius = 23 cm.

involves a delicate balance between the contraction of r_0 with increasing I and the slight variation of I_A/I with time, the details of which are uncertain due to the errors involved in the measurement of the runaway energy and current profile. On the other hand, there may well be some non-ideal effects which anomalously increase the minor radius faster than the current-dependent term decreases it; in this case the birth radius cannot be determined without a detailed knowledge of the anomalous process (see Sect. 6.1).

B. Low Density Case

For low density Type B discharges we can divide the runaways observed after ≈ 30 msec into two groups - those lost during the steady state, and those lost during the current decay after the loop voltage has been turned off. In the first case the current is constant and the plasma current profile is well known ("Type B profile"), whereas in the second case the current is changing, the plasma position is varying, and the current profile is not well known.

We have shown in Fig. 4.4 the evolution of maximum runaway energy vs. time for a set of discharges in which both these runaway groups have been measured. But since the hard x-ray intensity is much greater during the current decay phase than during the steady state, we have generally adjusted the detection system to view only one of these regions at a time. In Figs. 4.8-4.10 are shown some additional data for these two groups measured separately.

A typical analysis for the steady state runaway energy loss is given in Table 4.3. The pattern of increasing runaway energy at the outer limiter can in this case be readily interpreted as the loss of successively

Table 4.3 Type B Runaway Loss During Steady State

Time (msec)	I (kA)	d_p (cm)	γ	I_A/I	r_0 (cm)
35	104	-2	11	1.8	20
40	104	-2	13.3	2.2	19.5
45	104	-2	15.5	2.5	19
50	104	-2	17.7	2.9	18
55	104	-2	19.9	3.25	17
60	104	-2	22.2	3.6	16
65	104	-2	24.3	4.0	15.5

smaller runaway orbits due to the increasing orbit displacement with energy ($d_Y \approx 5$ cm at 5 MeV, 10 cm at 12 MeV). Here the runaways with $r_0 \approx 15-20$ cm are smoothly shifted into the limiter during the time $t = 35-65$ msec, according to the drift orbit interpretation.

The analysis of the runaway group lost during the current decay presents additional uncertainties concerning the plasma position and current profile; nevertheless, we show in Table 4.4 the computation of minor radii for the case graphed in Fig. 4.10 assuming a Type B current profile throughout. We can see that the computed minor radii are generally decreasing in time, as if the increasing orbit displacement with decreasing current is smoothly driving the remaining runaways to the limiter.

We note incidentally that the current-dependent effect should increase the minor radii during the current decay; for example, those lost at 62.5 kA in Table 4.4 have a computed minor radius of 6.5 cm

Table 4.4 Type B Runaways Lost During Current Decay

Time (msec)	Current (kA)	d_p (cm)	γ	I_A/I	r_0 (cm)
77.5	100	-1	21	3.6	15
82.5	84	-1	21.5	4.4	13.5
87.5	73.5	-1	21.5	5.0	12
92.5	62.5	-1	21	5.7	10.5
97.5	50	-1.5	21	7.1	6
102.5	40	-2	17.5	7.4	≈ 5
107.5	31	-2	13.5	7.4	≈ 5
112.5	23	-2.5	11	8.1	≈ 5
117.5	16	-3	7	7.4	≈ 5

if they were born during the steady state at $I = 164$ kA. A further discussion of these small-radii components is given in the next section.

The behavior of the runaways in low density discharges can thus be summarized by saying that the drift orbit model can account at least qualitatively for the observed pattern of runaway loss. During the steady state of the discharge all runaways are gaining energy and shifting outward according to the orbit displacement effect, such that by about 65 msec all runaways with $r_0 \gtrsim 15$ cm have intersected the limiter. When the voltage is turned off the runaways stop gaining energy (except possibly for a small induction emf), but since the current begins to decay the remaining orbits continue to shift outward and eventually all of the runaways are lost as $I \rightarrow 0$. The computed radii indicate that runaways can be born over the whole volume of the low density discharge, and that

they are apparently confined until the classical orbit displacement effect brings them to the outer limiter. The plausibility of this explanation of runaway confinement in low density discharges has been a major factor in favor of the drift orbit model interpretation.

C. Additional Checks of the Model for the Low Density Case

In the remainder of this section we will discuss four checks of the drift orbit model which can be made using hard x-ray data obtained during the course of normal low density ORMAK operation. The first three of these checks show a consistency with the model, but the last begins to disclose an anomaly which is not readily explained by the simple orbit theory (see Table 4.5).

Table 4.5 Checks of the Model

Check	Consistent with Model?
1. Location of runaway impact at outer limiter	Yes
2. Improved runaway confinement with increased current	Yes
3. Maximum energy confined during current decay	Yes
4. Energy vs. time at outer limiter	Usually

1. The collimated tangential hard x-ray detector was scanned between the inner and outer limiters in the equatorial plane (see Fig. 4.11). The observed pulse rate was corrected for the difference in material along the various lines, and it was concluded that the runaway flux at the outer limiter was about a factor 100 more than the

flux to the inner limiter. The residual x rays coming from the inner limiter can in fact be accounted for by Compton scattering of x rays from the outer limiter on material in the line of sight of the detector.

This observation is consistent with the outward shifts expected on the basis of the orbit model, and in particular it confirms that the outward shift is larger than the inward plasma motion which occurs during current decay.

2. We have consistently observed a systematic dependence of the pattern of runaway loss on the magnitude of plasma current in the steady state; namely, that the runaways are confined longer in higher current discharges. This is shown in the x-ray intensity patterns of Fig. 4.12, and can also be noted in the difference between the pulse data of Fig. 4.4 (101 kA) and Fig. 4.10 (164 kA).

This result agrees qualitatively with the drift orbit model prediction that $d_Y \propto 1/I$ for a given runaway energy. Knowledge of this feature of runaway confinement has been useful in planning further runaway experiments; for example, experiments which require a long period of runaway loss during the steady state should be run at low current (cf. Sect. 5.2).

3. A rather more direct check of the model can be made by comparing the maximum runaway energy during current decay with that predicted on the basis of the orbit model (see Fig. 2.2). The highest possible runaway energy confined at a given current should correspond to runaways with $r_0 \cong 0$; for these we have $I_A/I \cong 7$ (independent of the current profile) or $E_{\max}(\text{MeV}) \cong 0.2 I (\text{kA})$. This condition has been graphed along with the data of Fig. 4.9, and a good agreement is found for $I \gtrsim 50 \text{ kA}$. This

indicates that the low energy components lost at the end of the discharge are most likely born near $r \cong 0$, and that those lost earlier must have had larger radii.

We should note that there are probably also some low energy components lost during the period from 80 to 100 msec in Fig. 4.10, but these electrons cannot be readily detected in the presence of higher energy components (see Sect. 3.2). Only after about 100 msec have the high energy components been shifted to the limiter, allowing the better confined low energy components to be seen. This feature of the hard x-ray pattern is therefore in good agreement with the drift orbit model.

4. We have noted previously that in both high and low density discharges the maximum runaway energy observed at the outer limiter has increased at nearly the freefall rate (see Figs. 4.3 and 4.4). An exception to this behavior has been shown in Fig. 4.8, where in case (b) the observed energy gain is only about 60% freefall during this steady state phase of the discharge.

There are two ways in which this behavior can be reconciled with the simple drift orbit theory, both of which seem rather unlikely. The most obvious way is to assume that individual runaways still accelerate freely, but that the ones with smaller minor radii are born later in time. For example, in Fig. 4.4(b) the first runaways lost at $E = 4$ MeV have computed minor radii of $r_0 = 20$ cm and are born at about 22 msec according to an extrapolation based on a 0.19 MeV/msec freefall energy gain; but those lost late in the discharge at about 9.5 MeV have computed minor radii of 15 cm and a freefall-extrapolated birthtime of about 45 msec.

Thus, this explanation requires that runaways not be born at $r = 15$ cm until about 45 msec, a condition which is unlikely since the plasma is in steady state after about 20 msec. The other possibility is that the runaways have such a large pitch angle that their toroidal velocity is reduced to 60% the speed of light, and we have seen in Sect. 2.1 that the required angle of over 50° is an unlikely state for high energy runaways.

There are two alternative explanations for this lack of freefall at the outer limiter which involve departures from the classical drift orbit model; namely, one can postulate either an anomalous friction or an anomalous diffusion for these runaways. The effect of diffusion seems more likely for these high energy particles, and will be discussed at length in Chapter 5.

4.3 FURTHER EXPERIMENTAL TESTS OF THE MODEL

In this section we describe two experiments which were intended to help clarify the runaway confinement observed in both the high and low density regimes. The first involves changing the plasma position in the high density case in order to test whether those runaways were actually born at the outside of the discharge, as had been indicated by the drift orbit analysis of Sect. 4.2. The second set of experiments was designed to explore in detail the evolution of runaway energy vs. time in low density discharges, in order to look for evidence of anomalous friction or diffusion such as that discussed at the end of the last section.

A. Plasma Shift Experiment at Early Times

We recall that the minor radii calculated at intersection with the limiter for runaways lost during the first 10 msec in high density discharges was in the range $r_0 = 12-18$ cm (see Table 4.2), and that if the current-dependent minor radius effect is taken into account the initial birth radii must be very near the limiter radius of 23 cm. For such large radius orbits the influence of the plasma position should be significant in determining the confinement of these runaways.

In Fig. 4.13 we have plotted the magnitude of the plasma shift observed at the beginning of some typical ORMAK discharges. We see that in these cases the high density discharges were shifted inward about 2 cm more than the low density discharges, and the inward shift is even more pronounced for the case documented in Fig. 4.3. This leads us to believe that the appearance of high energy runaways early in the discharge is correlated with an inward shift of the plasma which allows "room" for runaways born near the outside of the discharge to achieve relativistic energies. For example, if the plasma were perfectly centered ($d_p = 0$), a runaway born at $r = 18$ cm would hit the limiter after an energy gain corresponding to only $I_A/I \cong 2$; whereas if the plasma were shifted inward about 4 cm the runaways born at 18 cm could gain an energy corresponding to the observed $I_A/I \cong 3-4$ before being shifted to the outer limiter. This effect could, therefore, explain the appearance of many high energy runaways at the beginning of the (inward shifted) high density discharge, and the absence of these runaways in the (better centered) low density discharges.

In order to test this hypothesis we intentionally varied the initial position of the plasma to see what the effect was on the confinement of this group of early runaways. The plasma position at early times is determined by the extent to which the vertical field has penetrated the thick aluminum shell around the torus, so in this experiment we varied the timing of the vertical field (relative to the ohmic heating field) in order to vary the position. The resulting plasma position vs timing is shown in Fig. 4.14; the results show that the increased vertical field at $t = 0$ due to an early pulsing of the vertical field coils does result in an inward shift as expected from the equilibrium equation (see Chapter 1). It should be noted, however, that the position calculation is somewhat uncertain at early times since the plasma is not really in equilibrium.⁽⁸³⁾

The resulting effect on the hard x-ray intensity during the current rise phase is shown qualitatively in Fig. 4.15, where the x-ray signal in this case comes from an uncollimated integrating detector whose response is roughly proportional to both the number and energy of the runaways ($\propto nE^2$). In Fig. 4.16 the integral of this signal over the first 10 msec of the discharge is plotted against the magnitude of the vertical field timing. Unfortunately, pileup problems prevented a measurement of energy for the runaways in these runs.

These figures show clearly that the hard x-ray intensity increases for the discharges which were shifted inward, in qualitative agreement with the confinement hypothesis described above. For example, the early runaways begin to appear in low density discharges with large inward shifts, presumably because the runaways born on large radius orbits

then have room to gain more energy before being shifted to the limiter. The x-ray flux in high density discharges also increases with the inward shift.

As a further test of this model, we attempted to look for the low energy runaways which should be lost at early times in normal well-centered low density discharge. In Fig. 4.17 we show the results of a brief experiment which does show some x rays apparently due to runaways which have been shifted to the limiter before gaining high energy. Their presence again confirms in a qualitative manner the model of runaways being born on outer orbits at $t = 0$, whose confinement is determined by the balance between the inward plasma shift and the outward orbit displacement effect.

It must be pointed out, however, that there are some alternative interpretations which can be given to the data shown in Figs. 4.15-4.17. It is possible, for example, that the conditions for runaway production could vary systematically with plasma position such that the increase in x-ray intensity could be due to an increase in runaway production rather than an improvement in runaway confinement. The theoretical production rates are indeed very sensitive to the parameter E/E_c in this range (see Figs. 1.4 and 4.7); but with the very limited knowledge of the plasma parameters for these conditions it is presently impossible to evaluate this effect. Another potential explanation is that some anomalous process (such as MHD activity) is actually determining the confinement of these early runaways, so that the systematic variation of runaway flux with shift could be due to a change in the level of

this anomalous process. Although such non-ideal effects have certainly been seen (see Sect. 6.2), there is at present no way to evaluate their influence in this experiment.

B. Experiments on Runaway Acceleration

In this section we examine the effect of a variation in the applied voltage on the maximum runaway energy in low density discharges. One would expect on the basis of the classical calculation of runaway energy losses (see Sect. 2.1) that high energy runaways should continue to gain energy at the freefall rate until their orbit displacement brings them out to the limiter. The results presented below seems to indicate, however, that there may well be some anomalous frictional or diffusive effects which are in part determining the evolution of runaway energy in the low density ORMAK discharges.

In Fig. 4.18 we show the results of a set of measurements in which the applied voltage was limited to the capacitor bank discharge which normally drives the current rise over 0-10 msec. Here the magnitude of the initial voltage V_c was varied (the decay time being fixed at about 5 msec), and the maximum runaway energy lost during the subsequent current decay was measured in the usual way. Note that when V_c is varied the maximum current and plasma density also increase linearly with V_c .

There are several interesting and unexpected features of these results. We should first note that the runaways are lost only at the very end of the current decay, so that they are clearly different from those lost during the first 10 msec in high density discharges (i.e., in Fig. 4.3). On the other hand, the observed maximum energies are consistent with a freefall acceleration in the applied loop voltage only if we assume

that these runaways were born at about $t = 3$ msec. This is significantly earlier than the birthtime of $t \approx 10$ msec previously deduced for runaways lost during the steady state of low density discharges (see Section 4.1).

Another unexpected property of these runaways is the unusually low current at which they are lost; for example, the 5 MeV runaways are lost at $I \approx 20$ kA. The resulting value of $I_A/I \approx 10$ indicates that these runaways must have had very small minor radii at intersection with the limiter, according to the drift orbit model interpretation.

A third interesting feature of this experiment was that an estimate of the relative number of runaways vs. magnetic flux swing showed (see Fig. 4.18) that the number of runaways near maximum energy decreased with an increase in V_c , contrary to the naive expectation that a higher voltage would lead to a far higher runaway production rate through Eq. (1.2.4). This is most likely due to systematic changes in n_e and/or T_e which are also involved in the runaway production process and which were not measured in the present experiment.

Thus far we have concluded on the basis of the collisionless drift model that the runaways observed in these capacitor discharges were created near the center of the discharge at about $t = 3$ msec. One would, therefore, expect that this group of electrons should be well contained and freely accelerated in a discharge in which the battery bank voltage was added at $t = 10$ msec (in the usual way) to form a steady state discharge. This behavior was tested in a further experiment in which a battery voltage of up to 80 msec duration was added to the normal 0.18 v-s capacitor discharge of Fig. 4.18. The maximum runaway energy was again measured during the current decay and the results are plotted in Fig. 4.19.

What was found was that the maximum energy increased very slowly with increasing battery duration, such that the runaways measured in the current decay at the end of a 96 msec long discharge was only 9 ± 1 MeV. If the 5 MeV electrons in the capacitor discharge would have been contained and accelerated freely, they should have had nearly 20 MeV after this pulse! Furthermore, the rate of maximum energy increase with a changing battery duration appeared to be only about 1/3 freefall, as shown in the data of Fig. 4.20.

As in the case where only 60% freefall energy gain was observed during a steady state discharge (see Sect. 4.2C), there are several potential explanations for this phenomenon. The most plausible one which retains the assumptions of the drift orbit model is that the very high energy runaways do indeed exist at the end of the long discharges, but that there are too few of them to be detected in the presence of a larger population of runaways born later in the discharge (even in spite of the E^3 dependence of pulse rate on electron energy, Fig. 3.6). This is possible since the plasma density was intentionally increased along with the battery length in order to keep the x-ray level within the operating range of the pulse detection system (see Fig. 4.21). Thus, the group of runaways observed in the capacitor discharge at a density of about $0.5 \times 10^{13} \text{ cm}^{-3}$ may have been a negligible component of the total runaway population of an 86 msec discharge run at $n_e = 1.2 \times 10^{13} \text{ cm}^{-3}$. There is unfortunately no way to check this hypothesis since the precise variation of runaway production among these discharges is not known. This uncertainty with respect to runaway production has been a persistent

difficulty in our attempts to understand runaway confinement. (It is hard to know how well runaways are confined if we're not sure where they come from!)

Another possible explanation of the observed energy vs time behavior is simply to say that there is some anomalous frictional force which slows the runaway energy gain to about 1/3 freefall. Although one could imagine that some wave-particle interaction could cause this, there is presently no clear indication why the frictional effect should appear in this instance and not in the "freefall" cases discussed in Section 4.1.

A last possible mechanism for this behavior involves an anomalous diffusion which would allow the runaways born early in the discharge to be lost after a certain time in a manner independent of the orbit displacement effect. For example, if there were runaways born at $t \approx 3$ msec near the center of the discharge, then one would expect on the basis of the drift orbit model that they would be contained until they gained at least $\sqrt{2}15$ MeV. If, on the other hand, there were some process that brought them to the limiter faster than this, then their absence at the end of the long battery pulse could be explained on this basis without recourse to an anomalous friction or to a systematic dependence on plasma density.

The simplest model for such a diffusive process would be to assume that runaways could be characterized by a confinement time τ_R related to a diffusion coefficient through

$$\tau_R = \frac{a^2}{D}$$

where "a" is the minor radius of the machine. This would lead to a maximum runaway energy of about $E_{\max} \text{ (MeV)} \propto \int_0^{\tau R} V \, dt \approx \tau R \cdot 60 \bar{V} \text{ (volt)} \cdot \tau R \text{ (sec)}$ so that, for example, a maximum energy of about 10 MeV implies a diffusion coefficient $D \approx 5 \times 10^3 \text{ cm}^2/\text{sec}$. Obviously, a diffusion model for ORMAK must be more complicated than this, but at least it can be seen that such an effect could result in a maximum runaway energy (as observed at the outer limiter) which does not increase at the single-particle freefall rate.

4.4 CONCLUSIONS

In this chapter we have examined the patterns of high energy runaway confinement in normal ORMAK discharges and have interpreted these patterns in terms of the drift orbit model of Chapter 2. In most cases we have found that this model provides a consistent and reasonable explanation for the observed behavior, for example:

1. the maximum runaway energy vs. time at the outer limiter usually increased at the freefall rate (as expected from the classical energy loss estimates),
2. the confinement of runaways born near $t = 0$ can be explained by assuming that they are born on large radius orbits,
3. the smooth loss of runaways after 30 msec in low density discharges can be understood through the orbit displacement scaling $d_Y \propto E/I$, and
4. the maximum runaway energy vs. time observed during the current decay agrees with the limit imposed by the drift orbit model.

On the other hand, there were also several experimental results which seemed to indicate - but did not prove - the existence of some anomalous frictional or diffusive process, for example:

1. if the current-dependent minor radius effect is applied to the data for early runaway loss, it is (marginally) necessary to assume an anomalous outward transport mechanism,
2. the maximum runaway vs time was in one case observed to increase at only 60% freefall during the course of a steady state discharge, and
3. increasing the duration of the applied voltage pulse did not result in the expected increase in maximum runaway energy.

The basic difficulty in our attempts to compare theory and experiment has been involved with the coupled nature of the runaway production and confinement processes. From our limiter measurement we can deduce a birthtime and a minor radius through the collisionless drift orbit model, but there has been no direct way to verify this prediction through either runaway production calculations or a spatially resolved (internal) runaway population measurement. We have, therefore, proceeded to show that the predictions were generally self-consistent and reasonable with respect to what is known about runaway production.

CHAPTER 5

EXPERIMENTS ON RUNAWAY DIFFUSION IN NORMAL DISCHARGES

In the last chapter it has been shown that the collisionless drift orbit model can provide a satisfactory explanation for most of the observed properties of high energy runaway confinement in ORMAK. In this chapter we continue the investigation of runaway confinement by describing two experiments which were designed specifically to look for an anomalous runaway diffusion. We find that the experimental results of this chapter do indicate an anomalous runaway transport near the outside of the discharge, and this transport is modelled by including a diffusive term in the drift orbit theory. The resulting diffusion coefficient of $D = 10^2 - 10^4 \text{ cm}^2/\text{sec}$ indicates the presence of non-ideal effects such as those discussed in Sect. 2.5.

In Sect. 5.1 we review the literature on runaway diffusion experiments, and in Sect. 5.2 we describe how such a diffusion could influence runaway confinement in ORMAK. In the next section we discuss an experiment designed to measure runaway diffusion in the shadow of a "point limiter," and the shortcomings of this type of experiment are noted. An improved method of measuring runaway diffusion is discussed in Sect. 5.4, where we show that the drift orbit model can be directly checked by measuring the runaway flux to the outer limiter after an inward plasma shift. This "plasma shift experiment" has been used to estimate the runaway diffusion coefficient near the plasma edge. In Sect. 5.5 we summarize the experimental information of runaway diffusion in ORMAK, and review the influence of this effect on the overall pattern of high energy runaway confinement. The discussion of some

mechanisms which may be responsible for runaway diffusion is presented in Chapter 6.

5.1 REVIEW OF RUNAWAY DIFFUSION EXPERIMENTS (EXCLUDING ORMAK)

By runaway diffusion we mean a radial motion of the runaway orbit which cannot be accounted for by the simple collisionless drift orbit model of Sect. 2.2. Such a motion is most likely due to the influence of non-ideal magnetic or electric fields present in a tokamak discharge, such as those described briefly in Sect. 2.5, but we should recall that this subject of "anomalous" transport is far from being closed. The experiments on runaway diffusion are presently at the stage in which the measurements can only be compared in a qualitative way with the existing information on these non-ideal effects.

The most extensive study of runaway diffusion has been carried out over the past five years by the LT-3 group^(32-34,57-59) [see Tables 1.3 and 1.4]. Their measurements have shown a very rapid loss of relatively low energy (< 1 MeV) runaways, especially during MHD unstable phases of their short ($\lesssim 2$ msec) discharges. The runaway loss has been attributed to "the free motion of runaways along ergodic magnetic surfaces"⁽⁵⁹⁾ with a diffusion step length indicative of the overall disruption of the magnetic surfaces.⁽⁵⁸⁾ A diffusion model has been applied to quantify the observed transport, and diffusion coefficients as high as 10^4 – 10^5 cm²/sec have been deduced. There is some evidence that the diffusion coefficient increases toward the outside of the discharge,⁽⁵⁸⁾ and that the diffusion is faster for higher energy (non-relativistic) electrons,⁽⁵⁹⁾ but it should be noted that this transport is measured in the shadow of a small probe inserted into the

discharge and may not be characteristic of the undisturbed plasma (see Sect. 3.1). Although the collisionless orbit displacement is not very small for these runaways ($d_Y/r_L \lesssim .3$), the erratic runaway loss and the large calculated diffusion rates have allowed them to neglect this term in their confinement model.

The other main experiments on runaway transport have used a plasma bremsstrahlung measurement to find the radial distribution of low energy ($\lesssim 1$ MeV) runaways in the ST⁽³⁹⁾ and Pulsator⁽⁵³⁾ discharges. A comparison of the observed runaway populations with the expected runaway production rates has allowed an estimate of the overall confinement time for these runaways, and the resulting $\tau_R \approx 5\text{--}20$ msec has been noted to be nearly the same as the confinement time for the thermal electron population. Presumably the same anomalous transport mechanism could be affecting both the thermal and runaway components, although no detailed theoretical model for this process has been proposed by these groups.

On the ATC tokamak a synchrotron radiation measurement was used to monitor the runaway electron population, and an example of unusually rapid runaway loss during enhanced MHD ($m = 2$) activity has been reported.⁽⁶²⁾ An estimate of the diffusion rate associated with this process⁽⁷⁶⁾ gave a value of 4×10^4 cm²/sec, most likely associated with low energy (< 1 MeV) runaways.

The only information on high energy runaway diffusion comes from a recent experiment on PLT⁽⁵¹⁾ in which a limiter bremsstrahlung measurement (similar to ours) was used to show that the maximum runaway energy observed at the outer limiter increased at only about 1/2

the freefall rate during a long (0.5 sec) discharge. This was taken to indicate a non-ideal confinement similar to that seen for low energy runaways in the ST, but no further analysis has been presented.

It should also be mentioned that there is an instability associated with runaway electrons which has been observed and studied extensively on the TM-3⁽⁵⁵⁾ and TFR⁽⁶³⁾ tokamaks. This instability can apparently cause a rapid pitch angle scattering of low energy runaways, such that they may become trapped in the local mirrors of the toroidal field coils and be lost to the walls through the rapid $\vec{B} \times \nabla \vec{B}$ drift. Although such an instability may produce waves which cause high energy runaway diffusion, there has been no evidence that the high energy runaways are lost through such a trapping mechanism.

From this brief survey it can be seen that there is rather little information available on high energy runaway diffusion in tokamaks. The main reason for this is that these runaways have only been measured at the limiter, so without knowing where they came from in the plasma, it has been difficult to tell whether their confinement is classical or anomalous.

5.2 INFLUENCE OF RUNAWAY DIFFUSION IN ORMAK

In this section we will describe in a general sense the influence of diffusion on high energy confinement in ORMAK. To do this we assume that there is some unspecified mechanism which allows the minor radius r_0 of the runaway drift orbit to change randomly by an amount δ per toroidal transit of the machine, while the center of the drift orbit is displaced outward by an amount d_y as usual (see Sect. 2.2). This is reasonable since the conservation of toroidal angular

momentum (for an axisymmetric system) determines d_Y , but it does not constrain the minor radius. We further assume in this simplified treatment that the mechanism which causes r_0 to vary does not cause the runaway energy or pitch angle to change. Of course such assumptions should be verified by a detailed analysis of the diffusion mechanism, but in the absence of a unique physical mechanism for diffusion (see Chapter 6), we can proceed in a heuristic way with this simple and rather general model.

An average runaway diffusion coefficient $\bar{D}$ can be defined in terms of δ , such that

$$\bar{D} \equiv \delta^2 / \tau_{\text{transit}} \approx \frac{\delta_c^2}{2\pi R_0} \quad (5.2.1)$$

Here $\bar{D}$ is considered to be independent of r and runaway energy E , at least for the runaway energy range above ≈ 2 MeV. How would such a diffusion affect our interpretation of the results described in Chapter 4?

The simplest measure of the relative importance of the classical orbit displacement and anomalous diffusive effects is given by the ratio of the radial particle velocities associated with these two processes. We recall that the classical model of runaway confinement (with d_p and I constant) allows only an outward motion of the center of the runaway orbit given by (see Sect. 2.2):

$$v_Y \equiv \frac{d(d_Y)}{dt} \approx \frac{d}{dt} \left[\frac{r_L^2}{2R_0} \frac{I_A}{I} \right] \propto \frac{dE(\text{MeV})}{dt} \quad (5.2.2)$$

A random variation in r_0 , however, will also result in a mean radial "velocity" for a particular runaway orbit given roughly by:

$$v_D \approx \frac{\bar{D}}{\Delta s} \quad (5.2.3)$$

where Δs is the radial distance over which the runaway motion is measured. Of course this v_D is only a convenient measure of $\bar{D}$, such that the ratio of these two quantities is a rough indication of the significance of the diffusive effect

$$\Sigma \equiv \frac{v_D}{v_\gamma} = \frac{\bar{D}}{v_\gamma \Delta s} \quad (5.2.4)$$

For $\Sigma \ll 1$ diffusion is negligible, but for $\Sigma \geq 1$ diffusion begins to be significant for runaway confinement.

In Table 5.1 we have summarized the available information on runaway diffusion in terms of this parameter Σ , and we have also indicated the expected influence of various possible diffusion coefficients for ORMAK. It is clear that the diffusive effect dominates in the other experiments, but that these effects only become important in ORMAK for $\bar{D} \geq 10^2 \text{ cm}^2/\text{sec}$.

A lower limit to high energy runaway diffusion in ORMAK can be set by an estimate of the classical (collisional) diffusion for these untrapped particles. We find that for a 5 MeV runaway under the usual ORMAK conditions

$$\bar{D}_{\text{class}} \approx \rho_{\text{tor}}^2 v_{ei} \approx (1 \text{ cm})^2 (1 \text{ sec}^{-1}) = 1 \text{ cm}^2/\text{sec} \quad (5.2.5)$$

A diffusion at this level would be completely negligible for the ORMAK experiments.

An upper limit on $\bar{D}$ can also be roughly set by the confinement time which is required in order that runaways can gain $\approx 10 \text{ MeV}$ in the applied loop voltage. For a typical low density discharge we have

Table 5.1 Magnitude of Diffusive Effects

Device	$\bar{D}$ cm ² /sec	$\Sigma \equiv v_D/v_\gamma^*$	Comments
ORMAK	10^2	$1(\Delta s = 1 \text{ cm})$	Diffusion important only near edge.
		$.05(\Delta s = 20 \text{ cm})$	
	10^3	$.5(\Delta s = 20 \text{ cm})$	Diffusion dominant at edge, significant for center.
	10^4	$5(\Delta s = 20 \text{ cm})$	Diffusion would limit maximum energy to ≈ 10 MeV.
ST(39) Pulsator(53)	10^4	≈ 10	Inferred for runaways < 1 MeV from $\tau_{\text{diff}} \approx 10$ msec for $\Delta s = 10$ cm.
LT-3(57-59)	$> 10^4$	> 10	Inferred for runaways lost in sudden bursts, $E < 1$ MeV, $\Delta s \approx 10$ cm.
PLT(51)	$\approx 10^4$	≈ 10	From maximum energy ≈ 10 MeV observed in long (.5 sec) discharges, assuming $\Delta s = 40$ cm.

* Using $v_\gamma \approx 100$ cm/sec for freefalling relativistic electrons in ORMAK ($V = 2$ volts), and scaling $v_\gamma \propto r_L^2/R_0^2 I$ for other machines at this voltage.

$V = 2$ loop volts, so that runaways lost at the end of the discharge must have been confined for at least 50 msec (see Eq. 2.1.2). If we make the "worst" possible assumption that these runaways came from the center of the discharge, then

$$\bar{D} \approx (23 \text{ cm})^2 / 50 \text{ msec} \approx 10^4 \text{ cm}^2/\text{sec} \quad (5.2.6)$$

A diffusion coefficient of this magnitude will obviously be significant for high energy runaway confinement.

In between these two extremes the relative importance of diffusion depends upon the radial distance over which the runaway motion is measured; for example, a $\bar{D} = 10^2 \text{ cm}^2/\text{sec}$ would be significant only over distances on the order of 1 cm, whereas a $\bar{D} = 10^3 \text{ cm}^2/\text{sec}$ would carry runaways from the plasma center to the limiter nearly as fast as the classical orbit displacement effect. Of course, at this level the radial dependence of $\bar{D}$ becomes crucial in evaluating the observed transport, and we shall see that this is the principal difficulty in our attempt to reevaluate the results of Chapter 4 in the light of the diffusion experiments described in the next sections.

5.3 POINT LIMITER EXPERIMENT

Our first attempt to measure high energy runaway diffusion was through the use of a "point limiter," which was a 1 cm diameter tungsten rod inserted radially into the plasma in the equatorial plane 90° downstream (for electrons) from the normal limiter (see Fig. 4.10). The method was similar to the probe measurements of the LT-3 group,⁽⁵⁹⁾ although our experiment was much more limited in scope.

In principle this technique allows the radial velocity of runaways at the edge of the plasma to be found by measuring the distance over which the runaways move past the tip of the point limiter. Let us consider the case in which a point limiter of diameter d_{PL} bounds the plasma at a radius r_{PL} . A runaway near the tip of the point limiter has a probability $d_{PL}/2\pi r_{PL}$ per toroidal transit of intercepting the point limiter, assuming that the runaway is not on a rational magnetic surface. If there is an average outward radial velocity v_r for these runaways, then they would be expected to move a distance d_d past the point limiter tip, where

$$v_r \approx d_d \left(\frac{d_{PL}}{2\pi r_{PL}} \right) \left(\frac{c}{2\pi R_0} \right) \quad (5.3.1)$$

Here we have assumed that the toroidal transit frequency is given by $v_{||} = c$, and that all runaways hitting the point limiter will be absorbed.

Thus a measurement of d_d can be used to compute a v_r ; for example, the classical drift orbit displacement of a freefalling runaway at constant current results in an outward velocity $v_Y \approx 100$ cm/sec, which should for $d_{PL} = 1$ cm and $r_{PL} = 20$ cm result in a $d_d = 10^{-4}$ cm. If d_d is observed to be larger than this in ORMAK, then we can compute a "diffusive" v_D and a rough diffusion coefficient

$$D \approx v_D d_d \quad (5.3.2)$$

Of course, such an outward movement of runaways gives an indication of the radial transport only at the edge of the plasma or in the shadow of the point limiter.

This method was implemented by viewing the outer limiter as usual during a series of discharges in which the point limiter was inserted at various radial positions in the plasma. One would expect that as the point limiter was gradually moved inside the normal limiter radius of $r_L = 23$ cm that the runaway flux to the normal limiter would gradually be reduced, so that d_d could be found from a plot of hard x-ray flux (to the normal limiter) vs position of the point limiter. Such a plot is shown in Fig. 5.1. We note that these normal low density discharges were not significantly changed by the presence of the point limiter inside r_L , and that the x-ray signal plotted here was mainly due to runaways lost during the current decay phase (as usual).

From this plot we can infer the rather surprising result that $d_d \approx 2$ cm under these conditions. This was apparently confirmed by an examination of the damage pattern observed on the electron-impact side of the point limiter (see photo in Fig. 5.1). The observed melting is almost certainly due to runaway bombardment, and it indicates that some runaways can move at least 1.5 cm past the point limiter tip. (This damage was most likely sustained during those discharges in which the point limiter was inserted more than 2 cm past the normal limiter, so that all runaways were eventually lost at the point limiter itself.) There was unfortunately no measurement of hard x-ray flux from the point limiter during the series documented in Fig. 5.1, although a detector aimed at the inner edge of the normal limiter showed no variation of the small x-ray flux there as the point limiter position was varied, in agreement with the large orbit displacements expected for these runaways.

A runaway radial velocity and diffusion coefficient can be computed from Eqs. (5.3.1) and (5.3.2) with $d_d = 1$ cm and $r_{PL} = 20$ cm, and we find $v_D \approx 5 \times 10^5$ cm/sec and $D \approx 5 \times 10^5$ cm²/sec! It is unlikely that such a rapid radial transport could be characteristic of the runaway population inside the plasma, since at this rate runaways would be transported from the center to the limiter in a time $t = r_L^2/D = 1$ msec, whereas we have seen that it takes about 50 msec for runaways to gain the characteristic energy of 10 MeV observed in these experiments.

We will consider two possible interpretations of this result:

- 1) the observed transport is characteristic only of the region at the edge of the plasma (i.e., near the tip of the point limiter); and 2) the observed transport is not characteristic of the plasma at all, but is due to a violent scattering of runaways from the point limiter itself.

Interpretation 1) cannot be ruled out by experiments of this sort, since the radial velocity is only measured in the shadow of the point limiter. There are indeed some non-ideal plasma perturbations which are expected to be strongest at the plasma edge, such as those associated with high m -mode MHD activity or toroidal and vertical field coil ripple (see Sect. 2.5). One can therefore imagine an extreme case in which the anomalous transport was confined to the plasma boundary so as to form a "virtual limiter" for runaways. In such a case the transport would be classical until this layer was reached, at which point the runaways would pick up a large v_D characteristic of the perturbations in the boundary layer. Such an explanation allows a reconciliation of the large v_D observed in this experiment and the classical

type of confinement discussed in Chapter 4, but although this explanation is qualitatively consistent with the measurement of D vs r performed by the LT-3 group,⁽⁵⁸⁾ there has been no direct way to test this effect in ORMAK.

Interpretation 2) should also be considered before attempting to understand the observed runaway transport in terms of non-ideal plasma conditions. Let us assume that an electron has a chance of being pitch-angle scattered from the curved surface of the (tungsten) point limiter, or of escaping through the flat edge facing the plasma. Such an electron may acquire a large gyroradius, such that on a subsequent toroidal transit it may be absorbed (or further scattered) at a point up to a gyroradius away from the initial impact. Since the gyroradius of a 10 MeV electron is roughly 1 cm for a 30° pitch angle, it would require only about one scattering per electron to explain the appearance of runaways up to 1 cm past the tip of the point limiter.

It is obviously difficult to make a quantitative evaluation of this effect, but we can at least show that it is not necessarily negligible. An electron and photon transport code⁽⁸⁴⁾ has been run to calculate the reflection of 10 MeV electrons incident at 30° to the surface of a .6 cm planar tungsten target. The reflected electrons have an energy spectrum roughly flat in the interval 1–9 MeV, and it is found that over 30% of the incident electrons are reflected between 0° and 20° from the target surface. Thus if the average angle of incidence of 10 MeV runaways onto the curved surface of the point limiter was 30° , then roughly 30% of these electrons would be scattered back into the plasma with a pitch angle of over 30° . Thus on

subsequent toroidal passes we could expect that a significant fraction of these electrons would be deposited up to 1 cm past the tip of the point limiter. [The magnitude of this effect could be checked by varying the atomic number of the point limiter material, since electron reflection is substantially reduced for low-Z materials.⁽⁸⁵⁾]

We conclude from this experiment that there may be a substantial runaway diffusion near the edge of the plasma, but it is clear that the results give little information about the processes which might be involved in such an anomalous runaway transport. It should also be noted that such probe experiments are not really suitable for runaway studies in large tokamaks, since the probes rapidly melt from the runaway bombardment and the evaporated atoms probably contaminate the plasma.

5.4 PLASMA SHIFT EXPERIMENT

In this section we describe an experiment which directly measured the transport of high energy runaways across a known distance near the outside of the discharge, and we show that the results are consistent with a runaway diffusion coefficient of 10^2 – 10^4 cm²/sec appropriate for this region of the plasma. In the first part of this section the experimental method and results are discussed, and in the second part the results are interpreted in terms of a generalized confinement model used to estimate the diffusion coefficient.

Experiment

The idea of this experiment was to suddenly shift the plasma position inward during the steady state of a normal low density discharge, and to then hold it steady at the inward-shifted position.

One would expect on the basis of the diffusionless drift orbit model that the runaway orbits should all be pulled inward along with the plasma current distribution, such that the runaway flux to the outer limiter ought to decrease sharply immediately after the inward shift. Since the runaways will continue to gain energy, we would further expect that after some time the runaways on the outermost orbits (which were just about to intersect the limiter before the shift) should eventually move back out to the limiter. A hard x-ray detector aimed at the outer limiter should therefore see a "gap" in the runaway flux beginning with the inward shift and ending at the time when the net inward plasma shift Δs is overcome by the additional d_Y displacement due to the runaway energy gain. A schematic description of this process is given in Fig. 5.2.

In order to calculate the expected time gap in the x-ray flux we use the expression for the classical orbit displacement (from Fig. 2.2)

$$d_Y(\text{cm}) \approx 2.4 I_A/I \quad (\text{for ORMAK}) \quad (5.4.1)$$

Thus the time necessary for a relativistic runaway to move outward Δs near the outside of the discharge is given by

$$T_{\text{gap}} = \frac{\Delta s}{\frac{d(d_Y)}{dt}} = \frac{\Delta s(\text{cm})I(\text{kA})}{82 \frac{dE(\text{MeV})}{dt}} \quad (5.4.2)$$

where we have assumed that the current I is held constant during this time. In the experiment described below we have $I = 74$ kA and a free-fall runaway energy gain given by $dE/dt \approx 120$ MeV/sec, so that for a typical inward shift of $\Delta s = 2$ cm we would expect an x-ray time gap of $T_{\text{gap}} \approx 17$ msec.

The pattern of hard x-ray flux after the plasma shift can therefore be used to check the validity of the drift orbit model. If some runaways are found at the limiter during T_{gap} , or if there is no gap at all, then there must be some mechanism which anomalously transports the runaways when they are within a distance Δs from the limiter.

Experimental results were obtained for about 85 normal low density discharges in which a variety of plasma shifts were performed. We first discuss the most extensive series (53 discharges) for which the results are particularly interesting. In this case the x-ray gaps were clearly and reproducibly observed after the inward shift, as shown for example in the oscilloscope photos of Fig. 5.3.

We recall that the scintillator pulse patterns recorded by the collimated tangential detector (such as those of Fig. 5.3) contain information on both the flux and the energy of runaways striking the outer limiter (see Sect. 3.3). Therefore in order to have a measure of the gap in the runaway flux at the outer limiter, we must first check whether the pulse height spectra are of similar form with or without the plasma shift. In Fig. 5.4 we show some examples of spectra obtained during this series of experiments. It can be seen that at a given time the shapes of the spectra are similar with or without the plasma shift, and from Fig. 5.5 we see that in both cases the maximum runaway energy increases in time at nearly the freefall rate. We can thus make the simplifying assumption that the high energy runaway flux to the limiter at a given time is proportional to the total scintillator count rate, independent of whether or not there was an inward plasma shift.

The analysis of the gap in the runaway flux after the inward shift proceeds by normalizing the scintillator countrate observed with a plasma shift to that observed without a shift. In Fig. 5.6 we show an example of the raw countrate data and the normalized flux obtained for the case of a 3.5 cm shift. This normalization presumably removes any systematic variation in the runaway flux to the limiter which is not associated with the shifting process, and indeed there was such a systematic variation observed consistently between bins 4 and 5 of the multichannel analyzer scan of these discharges. This structure apparently occurred with and without the plasma shift, as can be most clearly seen in the traces obtained from an uncollimated detector shown at the bottom of Fig. 5.6, and is itself described further in Chapter 6. It is clear from the oscilloscope traces (i.e., Fig. 5.3) that the beginning of the observed x-ray gaps are related to the inward shift and not to this unanticipated structure in the unnormalized x-ray flux.

In Fig. 5.7 we have plotted the normalized x-ray flux patterns for this series of 53 discharges in which seven different plasma shifts were performed. These patterns show a relatively clear gap for the larger inward shifts, but in all cases there is evidently a measurable runaway flux to the limiter throughout the whole duration of T_{gap} . The gaps are an indication of runaway confinement as expected on the basis of the classical model of Eq. (5.4.2), whereas the gradual leakage of runaways during T_{gap} indicates the presence of an anomalous diffusion.

In Table 5.2 we list the various trials along with an estimate of the experimental time gap deduced from the graphs of Fig. 5.7. The experimental time gap has been defined as the duration between the beginning of the inward shift and the time when the normalized flux returns to 50% of the value it had before the shift. [This procedure minimized the irrelevant variation in the absolute value of the x-ray flux, which may change from shot-to-shot due to a slight variation in the conditions for runaway production, for example (see Fig. 5.7).]

Table 5.2 Plasma Shift Trials

Trial #	Amount of Shift Δs (cm)	Time of Shift(msec)	# of Shots	Experimental T_{gap} (msec)	Theory T_{gap} (msec)
0	0	-	4	-	-
1	1	71	5	2.5 ± 2.5	$5 \pm 5^*$
2	1.7	76	5	10 ± 5	14 ± 2
3	1.8	66	10	15 ± 5	13 ± 2
4	2.4	71	5	11 ± 5	18 ± 2
5	2.6	61	5	15 ± 5	17 ± 2
6	3.5	66	19	26 ± 5	22 ± 2
7	3.6	61	4	20 ± 5	22 ± 2

* Error bars based on an assumed uncertainty of $\pm .2$ cm in Δs .

We have also listed in this table the theoretical value for the x-ray gap as calculated on the basis of the diffusionless drift orbit model. In these calculations we assume that the runaway energy gain is given by freefall acceleration in the measured loop voltage (see Fig. 5.2). The actual plasma position vs time [as calculated from B_p

pickup loops in the usual way⁽⁸³⁾] is used to determine T_{gap} as the duration between the beginning of the shift and the time when the additional d_Y displacement equals the net inward shift (see Fig. 5.8).

The comparison between experiment and theory is shown in Fig.

5.9. We conclude that there is at least a semi-quantitative agreement with the drift orbit model predictions for this series of experiments, despite the fact that a certain amount of anomalous diffusion blurs the clarity of the experimental gaps.

Calculation of the Diffusion Coefficient

It has been pointed out that the appearance of a non-zero x-ray flux during T_{gap} can be interpreted as an indication of runaway diffusion in the region within Δs from the limiter. This process can be modelled through an equation of the form

$$\frac{\partial n(r,t)}{\partial t} = D \frac{\partial^2 n(r,t)}{\partial x^2} + V \frac{\partial n(r,t)}{\partial x} \quad (5.4.3)$$

where n is the local runaway density and V is the convective velocity of the runaway population as a whole. Here the first term on the right represents the anomalous transport due to some D (assumed to be independent of space and time), and V is the classical outward velocity of the runaway orbits due to the orbit displacement effect and the motion of the plasma as a whole. In this one-dimensional treatment the position r is considered to be the location of the outermost point of the runaway orbit.

The simplest model of the plasma shift experiment is:

$$\begin{aligned}
 (1) \quad n(r,0) &= \begin{cases} 0 & (0 < r < \Delta s) \\ 1 & (\Delta s < r < \infty) \end{cases} \\
 (2) \quad n(0,t) &= 0 \\
 (3) \quad V &= \Delta s / T_{\text{gap}}
 \end{aligned}
 \tag{5.4.4}$$

where the first condition represents the runaway spacial distribution just after an inward plasma shift Δs , the second condition models the outer limiter as a perfectly absorbing boundary at $r = 0$, and the third condition is a simplified expression for the classical outward orbit velocity. The flat distribution of runaway population is motivated by the fairly constant runaway loss during steady state as shown, for example, in Figs. 4.4 and 4.8. The diffusion equation (5.4.3) has been solved numerically for various D (see Appendix C), and the relative runaway flux to the limiter $(D \partial n / \partial x)_{x=0}$ vs time can be compared with the experimental flux patterns. Such a comparison for the 3.5 and 3.6 cm trials is shown in Fig. 5.10.

The comparison indicates fairly clearly that the observed patterns of runaway loss can be explained by a diffusion coefficient of order $D \approx 10^2 \text{ cm}^2/\text{sec}$. It is clear that $D \sim 10 \text{ cm}^2/\text{sec}$ produces too sharp a gap to fit the data, and it can also be seen that the gap is destroyed by the diffusive term when $D = 10^3 \text{ cm}^2/\text{sec}$, in agreement with the considerations of Sect. 5.2 ($\Sigma \approx 2$).

A slight refinement of the model has been attempted by including the inward plasma motion in the term V (see caption on Fig. 5.10), and this is seen to explain in a qualitative way the gradual drop in x-ray flux over the first ≈ 5 msec after the start of the shift (when Δs is still small). Of course there are many other possible refinements

to the model, such as varying $D(r,t)$ or $n(r,0)$, but we consider the experimental uncertainties to be too large to justify such attempts. These uncertainties include (besides the shot-to-shot variation shown in Fig. 5.10) the assumptions of freefall acceleration, the use of the total x-ray countrate to estimate the runaway flux, and the assumption that the normalization process removes the effect of the systematic structure in the x-ray patterns at $t = 76$ msec.

As a final point in the analysis of this series, we note that the addition of a diffusion coefficient into the theory reduces somewhat the expected gap in the x-ray flux, and we have plotted in Fig. 5.9 the revised theoretical time gaps for two possible D 's. The comparison with the data again shows that a $D \approx 10^2 \text{ cm}^2/\text{sec}$ is consistent with the data, but that $D = 10^3 \text{ cm}^2/\text{sec}$ is too large to be consistent with the observed x-ray gaps.

We note incidentally that the scintillator pulse spectra of Fig. 5.4 show a perceptible increase in the relative number of counts below ≈ 2 MeV for the 3.5 cm shift case, compared to the discharges without a shift. This may be interpreted qualitatively as an indication that low energy (≤ 2 MeV) runaways diffuse faster than high energy runaways, although as we have seen (Fig. 3.6) the tangential detection system cannot accurately determine the runaway population in these low energy components. An enhanced diffusion of low energy runaways can also account qualitatively for the absence of a gap in the signal from the uncollimated detector (see Fig. 5.6). The results documented in Figs. 5.5–5.10 therefore refer mainly to the highest energy runaways only, as we recall was also the case for the results of Chapter 4.

We now discuss some additional trials of the plasma shift experiment in which there was generally no clear gap observed after the inward shift. Three different series were run with a total of about 30 shifted discharges, and in only one series did there appear in about four out of ten cases a perceptible x-ray gap. In all other cases there was either no change in the x-ray pattern after the shift, or else there was actually an increase in the x-ray flux just after the shift. These discharges were similar with regard to their gross discharge parameter (i.e., I , n_e , V) to those documented above; however, some internal instabilities apparently triggered by the plasma shift were observed in some of these cases.

An example of a discharge in which the x-ray flux increased after an inward shift is shown in Fig. 5.11. This x-ray behavior coincides with an increase in the level of instability in the discharge, as can be seen by the increased amplitude of oscillation seen by the $\dot{B}_p$ pickup loops ($m \geq 2$) and the PIN diodes ($m = 1$). Presumably the inward shift in this discharge triggered the growth of these modes, and it is most likely that these modes then increased substantially the radial transport of the runaways. A lower limit to the diffusion coefficient can be set by the condition that the diffusion be large enough to destroy the gap expected from the inward plasma velocity; roughly $D > V_{in} \cdot \Delta s \approx 10^3 \text{ cm}^2/\text{sec}$. It can also be seen from Fig. 5.11 that the x-ray flux increases most dramatically during the enhanced $m \geq 2$ activity during the first 10 msec after the start of the shift, but that there was still a significant x-ray flux to the limiter even after 30 msec of strong $m = 1$ activity. This may indicate that the

runaways near the outside of the discharge may be lost during the $m = 2$ activity at $r > .5 r_L$, but that the internal $m = 1$ cannot so easily transport runaways to the limiter (see Chapter 6).

Among this group of 30 discharges there were also some cases in which shifts of up to 6 cm were produced without signs of increased instability, in which there was no significant difference between shifted and unshifted x-ray patterns. Again this implies a large $D > 3 \times 10^3 \text{ cm}^2/\text{sec}$ in order that the outward diffusion outweigh the inward plasma shift. Here an upper limit on the overall diffusion coefficient can be set by the fact that most of the high energy runaways are still lost during the current decay beginning about 30 msec after the shift; thus $\bar{D} < (23 \text{ cm})^2/30 \text{ msec} \approx 2 \times 10^4 \text{ cm}^2/\text{sec}$. It is certainly possible, however, that this fast diffusion is appropriate only for the outer edge of the discharge.

5.5 SUMMARY OF RUNAWAY DIFFUSION EFFECTS

The two experiments described in this chapter have indicated that there can be a significant anomalous transport of high energy runaways near the outside of normal low density ORMAK discharges. The clearest indication of such a transport came from the plasma shift experiment, in which the pattern of runaway flux to the limiter after an inward plasma shift was interpreted in terms of a runaway diffusion coefficient in the range 10^2 – $10^4 \text{ cm}^2/\text{sec}$. In this section we point out that a general picture of runaway confinement is difficult to achieve through our limiter measurement without an independent knowledge of the runaway creation process.

In Table 5.3 we list some results of Chapter 4 in a form which displays the possible influence of runaway diffusion in each case. On the right in this table are some bounds on the average $\bar{D}$ obtained from simple order-of-magnitude estimates based on the observed confinement property. For example, the observation that most of the 10 MeV runaways are lost during the current decay phase at the end of the discharge implies that the confinement time for these particles is at least 200 msec (see note 3); therefore if these runaways were all born near $r = 0$, then an upper limit for the diffusion coefficient would be $\bar{D} = 5 \times 10^3 \text{ cm}^2/\text{sec}$. On the other hand, if we allow the possibility that these runaways are born on outer orbits ($r > 0$), then we have seen in Sect. 4.2 that the observed confinement can be explained in terms of the diffusionless drift orbit model alone. Thus it is clear that the measurement of runaway loss to the outer limiter cannot provide a complete understanding of runaway confinement without an independent knowledge of the radial profile of the runaway creation rate.

We can show this interdependence of runaway production and confinement more explicitly through a consideration of the experimental result of Chapter 4 in which the maximum runaway energy at the outer limiter increased at less than the expected freefall rate (Table 5.3, point 5). We recall that one possible explanation of this phenomenon involved the diffusive loss of early-born runaways, such that the maximum runaway energy would be expected to saturate due to an anomalously short confinement time τ_R related to a runaway diffusion process.

Table 5.3 Influence of Diffusion on the Results of Chapter 4

Results Explicable Through Drift Orbit Model	Influence of Diffusion?
1) Freefall energy gain to 10 MeV	$\bar{D} < 10^4$
2) Most of runaways lost during current decay	$\bar{D} \lesssim 5 \times 10^3$
3) Low energy runaways lost at the end of current decay with $E_{\max} \propto I$	$\bar{D} \lesssim 10^4$ for $E \lesssim 5$ MeV
4) Confinement $\propto 1/I$ in steady state	$\bar{D} \propto 1/I(?)$
Apparent Failures of Drift Orbit Model	
5) Lack of freefall gain at outer limiter	$\bar{D} > 10^3(?)$
6) Unusually poor confinement for run- aways lost during current rise (only if $Ir^2 = \text{const}$)	$\bar{D} \approx 10^5(?)$
7) Unusually good confinement with capacitor only	$\bar{D}$ small

Notes to Table 5.3

1) The confinement time for 10 MeV runaways lost during the steady state of low density discharges is $\tau_r \gtrsim 80$ msec from freefall acceleration; thus the diffusion rate $\bar{D}$ (averaged over r) is at most $\bar{D} = (23 \text{ cm})^2 / 80 \text{ msec} \approx 10^4 \text{ cm}^2/\text{sec}$.

2) The fact that most of the highest energy runaways are lost only during the current decay phase (see Fig. 4.4) indicates somewhat better confinement than supposed in 1); for example, if only 10% of the 10 MeV runaways are lost during the time required to gain 10 MeV (see Fig. 4.9), then we would expect that roughly two more e-folding

times would be necessary for runaways to diffuse completely from the center of the plasma ($\tau_R \approx 3(80) \text{ msec} \Rightarrow \bar{D} < 5 \times 10^3 \text{ cm}^2/\text{sec}$).

3) The confinement of low energy $E \leq 5 \text{ MeV}$ runaways for up to 60 msec after the termination of the applied loop voltage (at which time runaway creation presumably stops) indicates a $\bar{D} < 10^4 \text{ cm}^2/\text{sec}$ for these runaways (note that the presence of runaways at the inferred radius of $r \approx 0$ is not inconsistent with a diffusive model).

4) If a diffusive mechanism were dominating runaway confinement, we would require $\bar{D} \propto 1/I$ to explain the result shown in Fig. 4.11.

5) Diffusion begins to become significant over the whole plasma volume when $\Sigma \geq 1$ (see Table 5.1).

6) Depending upon the (unknown) initial birth radius, the runaways lost during the current rise phase may have $\bar{D} (23 \text{ cm})^2/5 \text{ msec} = 10^5 \text{ cm}^2/\text{sec}$.

7) Confinement of runaways until $I_A/I \lesssim 10$ indicates a very small diffusion for the group lost after the capacitor discharges ($r \approx 0$ after 40 msec; see Fig. 4.17).

The experimental situation can be modelled by considering the transport of runaways born at some radius r and some time t to be given by a general function $P(r_L - r, t' - t)$, where P is the probability that a runaway born at (r, t) is lost to the limiter at r_L at time t' . For example, if the transport were completely specified by the diffusionless drift orbit model, then P would be a δ -function defined by the limiter intersection condition [Eq. (2.4.2)], or if the transport was purely diffusive, P would be a smooth curve peaked near some time given by $t' - t \approx (r_L - r)^2/\bar{D}$. The evolution of "maximum"

runaway energy $\bar{E}$ vs time observed at the outer limiter can therefore be written

$$\bar{E}(t') = \frac{\int_{r=0}^{r=r_L} \int_0^{t'} S(r,t) P(r_L-r, t'-t) E(t'-t) W(E) r^2 dr dt}{\int_{r=0}^{r=r_L} \int_0^{t'} S(r,t) P(r_L-r, t'-t) W(E) r^2 dr dt} \quad (5.5.1)$$

where $S(r,t)$ is the runaway creation rate, $E(t'-t)$ is the energy of an individual runaway born at t and lost at t' (given by freefall in the applied voltage, for example), and $W(E)$ is a weighting function for runaways striking the limiter at the same time ($W(E) \propto E^3$ for the tangential bremsstrahlung determination of "maximum" energy (see Fig. 3.6)).

Thus we can see explicitly that a measurement of $\bar{E}(t')$ can be used to determine the runaway transport P only when the source function S is known, and that even given $S(r,t)$ the unfolding problem may be non-trivial. Some numerical examples of the dependence of $\bar{E}(t')$ on various P and S are discussed further in Appendix C, where we show that an observation such as "maximum energy increased at 60% freefall (Sect. 4.2)" can be interpreted in terms of various combinations of source and transport functions.

Our knowledge of runaway diffusive effects may therefore be summarized by saying that an anomalous transport has been shown to exist near the outside of the discharge, but aside from the upper limits for D given by the large energies attained by these fast electrons, the extent of runaway diffusion in the plasma interior remains uncertain. This uncertainty is directly related to the fact that our limiter measurements cannot unambiguously define the location

of runaway creation in the discharge. This difficulty should not be too surprising, since even a full radial profile of some plasma parameter (i.e., electron temperature) is barely sufficient to define the associated transport processes.⁽¹⁴⁾ In the absence of clear evidence for runaway diffusion in the plasma interior, we conclude that the drift orbit model interpretation of Chapter 4 is satisfactory as a first approximation to the overall pattern of high energy runaway confinement in ORMAK.

CHAPTER 6

MECHANISMS FOR ANOMALOUS RUNAWAY TRANSPORT IN ORMAK

In the previous chapter we have seen that there can be a significant anomalous transport of high energy runaways near the outside of a normal discharge, and it was also shown that the existence of such a transport in the plasma interior cannot be ruled out by our limiter x-ray measurements. In this chapter we discuss several possible mechanisms for such non-collisional runaway transport, all of which involve the presence of some non-ideal field structure in the discharge (see Sect. 2.5), and indications for all of which have been observed in ORMAK.

The most obvious case of anomalous runaway transport is associated with the disruptive instability (Sect. 6.1) during which the runaways are lost due to a violent rearrangement of the magnetic structure of the plasma. Less violent plasma instabilities such as the $m = 2$ kink-tearing mode (Sect. 6.2) and the internal disruption (Sect. 6.3) have also been observed to affect runaway confinement in some cases. We have further observed in low density discharges an unusual instability which seems to affect only the runaway population, as discussed in Sect. 6.4. The anomalous effect of neutral gas puffing is discussed in Sect. 6.5, and the influence of neutral beam injection on runaway confinement is mentioned briefly in Sect. 6.6.

In the last section of this chapter we summarize these results and review the state of our knowledge concerning anomalous runaway transport. We find that the mechanism for the runaway diffusion observed in Chapter 5 cannot at present be unambiguously identified.

The results of this chapter do, however, provide an outline upon which future investigations of anomalous runaway transport can be based.

6.1 DISRUPTIVE INSTABILITY

The symptoms of a disruptive instability in ORMAK are very clear-cut and nearly identical to disruptions observed in other tokamaks. In Fig. 6.1 we show the behavior of the plasma current, loop voltage, plasma density, and hard x-ray flux during a disruptive discharge in ORMAK. The characteristic negative voltage spike and positive current spike are accompanied by a sudden drop in plasma density and temperature. Observations on the ATC tokamak⁽⁸⁶⁾ have shown that during the disruption the plasma expands in minor radius and contracts in major radius until it hits the wall of the vacuum vessel. The onset of this instability represents one of the main limits to present tokamak operation, such that most of ORMAK's operation was confined within a non-disruptive parameter space determined by a density limit⁽⁸⁷⁾ and safety factor $q(a) \gtrsim 3$ (see Sects. 1.1 and 4.1).

The influence of this mode on runaway confinement can be clearly seen in the hard x-ray signal of Fig. 6.1 in which the first three disruptions are accompanied by large runaway dumps to the limiter (or liner). The absence of hard x-rays after the third disruption indicates that essentially all the runaways were lost from the discharge before the usual runaway dump during the current decay. The short duration and the irreproducibility of the main x-ray spikes ($\lesssim 1$ msec) has made it difficult to use the pulse detection system to determine the runaway energy (see Appendix B); thus we cannot tell whether the runaways lost after the first disruption were created early in the

discharge or just preceding subsequent disruptions.

In Fig. 6.2 we show an example of a discharge in which the disruption was preceded by a correlation between the runaway loss and MHD activity as monitored by a $\dot{B}_p$ pickup loop. A similar correlation was observed by the Pulsator group during their detailed study of the disruption,⁽⁴⁾ although in their case the correlation lasted for only a few cycles and the runaways were all lost before the negative voltage spike. This correlation between runaway confinement and MHD activity most likely indicates that the runaways are being perturbed by a distortion in the magnetic structure of the plasma, such that the large runaway dump terminating the correlation would be associated with a complete destruction of the magnetic surfaces.

In order to understand this runaway behavior it would be useful to have an understanding for the disruption itself, but unfortunately no clear explanation for this phenomenon presently exists.^(2,3) The consensus seems to be that it involves the nonlinear development of the $m = 2$ mode and its interaction with the internal $m = 1$ mode (i.e., Refs. 52, 89, 95). Some details of the MHD mode structure have been determined from soft x-ray sensitive PIN diode arrays on ORMAK,⁽¹⁴⁾ but unfortunately there is not enough data on the behavior of runaways in these cases to attempt a systematic explanation of the mechanism for the anomalous runaway transport.

It is interesting to note that disruptive-type events can also be associated with the loss of runaways during the current rise phase of high density ORMAK discharges (see Sect. 4.2). In Fig. 6.3 we show some examples of this phenomenon. In Fig. 6.3(a) is a case in which

runaways created near $t = 0$ are fairly well contained until the first negative voltage spike, but in (b) and (c) it can be seen that there are x-ray bursts corresponding to as many as seven successive disruptive events. The hard x-ray pulse system indicates that the maximum runaway energy increases at nearly the freefall rate in these discharges, much as it did in the more stable cases documented in Fig. 4.3, although pileup problems have made a quantitative evaluation difficult.

This behavior appears to be consistent with a model proposed by Fussman⁽⁵⁴⁾ of the Pulsator group [and Mirnov of the TM-3 group⁽⁸⁷⁾] in which the runaway dumps are associated with the development of high m -mode activity at the boundary of the plasma column. They have noticed that the appearance of the negative voltage spikes is well correlated with integral $q(a)$ values such that the development of a mode with $m \approx q(a)$ may be expected to affect the confinement of runaways near that rational surface. A test of this correlation for the ORMAK data shows a good agreement with $q(a) = 6-9$ in case (b), and a fairly good agreement with $q(a) = 6-10$ in case (b). It thus appears that an outer layer of runaway orbits may be affected by these disruptions, while the inner runaways may continue to accelerate until either their orbit displacement or a subsequent disruption brings them out to the limiter. Of course a quantitative test of this model is not possible without an independent knowledge of the runaway creation processes, as discussed at the end of Chapter 5.

Our conclusion concerning these examples of anomalous runaway transport during disruptions is that in all cases the disturbance of

the magnetic field structure can strongly deteriorate runaway confinement, but the details of the transport process remain obscure. It appears, however, that the effect of high m -mode disruptions may be localized near the outside of the plasma, while the normal $m = 2$ type disruption can readily dump all the runaways in the discharge.

6.2 MHD ACTIVITY ($m = 2$)

A common type of tokamak instability is the MHD kink or tearing mode, but unlike the disruptive instability these modes can exist in a steady state discharge without a severe penalty in plasma confinement. Their overall structure is understood theoretically⁽³⁾ although the observed saturation of the mode amplitude is a nonlinear problem that is just being resolved.⁽⁸⁹⁾ In this section we first describe the observations which suggest that runaways can be affected by this mode, and then we present a model of the $m = 2$ magnetic island which can explain some of these observations.

Experiments

There have been many experimental indications that MHD activity can affect runaway confinement. The clearest of these are the ORMAK observations of a periodic runaway loss at the same frequency as the $\dot{B}$ pickup loop signals,⁽⁴³⁾ and the recent ATC observation of decreased synchrotron emission from runaways after an onset of the $m = 2$ mode.⁽⁶²⁾ There may also be an influence of the $m = 2$ amplitude on the bulk plasma energy confinement time such as could explain the anomalous electron energy loss,⁽¹⁴⁾ but this connection is speculative at present.

The MHD behavior of ORMAK has been studied extensively using magnetic field ($\dot{B}$) pickup loops and a soft x-ray PIN diode array.⁽¹⁴⁾ Typical $\dot{B}$ signals as shown in Fig. 6.4 measure the fluctuating poloidal field amplitude $\Delta A/A \sim 10^{-2}$ – 10^{-3} at the liner, where the fluctuation at 5–15 KHz is probably due to a toroidal rotation of the mode structure. The amplitude of these oscillations is usually higher in the high density than in the low density regime, and during the steady state [with $q(a) \approx 5$] the dominant $m = 2$ or 3 mode is saturated for most of the discharge. The $m = 2$ growth rate peaks at $q(r) = 2$ so that for a typical ORMAK current profile we have $r \approx 14$ cm for this mode. Note that during the current rise phase the $\dot{B}$ signal shows a large amplitude corresponding to the rather erratic growth of high m -number modes at the outside of the plasma, and that the MHD activity usually decreases strongly after the loop voltage is turned off.

In the normal low density ORMAK discharges such as described in Sect. 4.3 there is no obvious correlation between MHD activity and runaway confinement. The hard x-ray flux shows a smooth increase through the steady state when the MHD amplitude is constant, and there is a large runaway loss during the current decay when there is little or no MHD signal.

There are some cases in which a clear correlation is manifest, however, as shown in Fig. 6.5. Here we see a dramatic phasing of hard x-ray flux and MHD activity as monitored by the $\dot{B}$ and PIN signals. This behavior was most commonly seen during the low level runaway loss associated with "gas puffing" experiments (see Sect. 6.4), but it is possible that a very small hard x-ray modulation exists in more normal circumstances (although several attempts to find this proved negative).

An extreme case of the effect of MHD is shown in Fig. 6.7 in which a sudden increase in MHD amplitude resulted in a nearly complete loss of runaway confinement in $\lesssim 10$ msec. Although such behavior was very rare in ORMAK it does appear to be very similar to an observation of the ATC synchrotron group.^(49,62)

A final observation concerns the correlation between MHD and runaway loss observed during the pre-disruptive phase (see Fig. 6.2). In this case the $m = 2$ mode is most likely growing slowly until the disruption, at which time the magnetic surfaces become severely disturbed through some unknown process. In this case the existence of a phasing of the MHD and runaway loss for over 100 cycles indicates that the effect on runaway transport is relatively weak until the disruption.

We conclude from these observations that the $m = 2$ mode can produce a modulation of the runaway loss process in phase with the rotation frequency of the mode. We have also seen that a strong MHD oscillation can anomalously remove all runaways from the discharge in a time $\lesssim 10$ msec without the appearance of a major disruption.

Magnetic Island Model

We will now give a brief discussion of these results in terms of the perturbation of runaway drift surfaces due to a $m = 2$ magnetic island structure in the discharge. Our interpretation is similar to that of the Pulsator group⁽⁵²⁾ but we present a new result concerning the influence of the orbit displacement effect on the formation of island structures in the runaway drift orbit. Although there is qualitative agreement between this model and our data, we must point out that the lack of runaway energy resolution in this case prevents an

attempt at a detailed comparison between the observed $m = 2$ mode structure and the measured runaway confinement.

A magnetic island is a helical current perturbation which results in flux surfaces such as shown in Fig. 6.7. The basic physical effect is a pinching of the perturbed currents on a rational surface, but such an "island" in the magnetic flux is possible only when a finite resistivity is admitted into the MHD equations allowing a change in the magnetic topology. It has been estimated that resistive timescales in tokamaks are of the proper magnitude to account for the observed mode structure,⁽⁸⁹⁾ although the direct experimental evidence for such structures is rather limited at present.⁽⁹⁰⁾

To calculate the influence of such islands on runaway confinement we have followed the work of Chrisman, Clarke, and Rome⁽⁴³⁾ and have used a guiding center computer code to map out the trajectory of a runaway orbit in the presence of a typical $m = 2$ magnetic island structure. We were especially interested in the drift orbits for high energy runaways, since these particles no longer follow field lines and so they might be expected to lose the island structure when the orbit displacement is large.

The following set of simplified guiding center equations was used:

$$\left\{ \begin{array}{ll} \frac{dx}{dt} = v_{\parallel} \frac{B_x}{B} & \text{(horizontal velocity)} \\ \frac{dy}{dt} = v_{\parallel} \frac{B_y}{B} - v_d & \text{(vertical velocity)} \\ \frac{dz}{dt} = v_{\parallel} & \text{(toroidal velocity)} \end{array} \right. \quad (6.2.1)$$

where we simulate the toroidal geometry by adding a vertical drift v_d [Eq. (2.2.1)] and a toroidal field $B = B_0/R$ to a cylindrical model. We have also assumed a runaway pitch angle $\xi = 0$ for this application, although the code does produce the proper banana orbits for trapped particles when $\xi \gtrsim 45^\circ$. These equations are solved numerically using a guiding center code as described in Appendix D.

Instead of solving for the perturbed fields due to the islands by using an assumed current perturbation (as was done in Ref. 43), we have used a perturbed flux function given by Bruce Waddell for the saturated state of an $m = 2$ island.⁽⁹¹⁾ [see Appendix D]. The resulting flux surfaces (i.e., Fig. 6.7) should resemble the actual magnetic structure during the steady state of an ORMAK discharge.

Since these flux surfaces are each the locus of a $\vec{B}$ field line we can see how runaway orbits can be significantly perturbed by these island structures. The next thing we would like to know is how high energy runaways with significant orbit displacement $d_y/r_L \propto E/I$ are affected by the perturbed field structure. A typical result is shown in Fig. 6.18.

It turns out that as the orbit displacement increases, the influence of a (stationary) magnetic island perturbation gradually decreases until the drift orbit is hardly affected at all, as if there was a lack of resonance between the particle orbit and the field perturbation. The displacement has to be at least as large as the island width before the island structure in the runaway orbit disappears, as indicated by Fig. 6.9.

This shows that it is possible to have a magnetic island localized inside the discharge and yet have the runaway drift surfaces maintain an island structure as they are touching the limiter. Therefore we would expect that as the runaways gain energy and their orbit displacement increases their drift surfaces will gradually be scraped off at the outer limiter with a modulated intensity in phase with the rotational motion of the mode. Presumably this continues until the island itself grows enough to touch the limiter causing a major disruption.⁽⁸⁰⁾ This is a qualitative explanation for the periodic runaway loss as shown in Figs. 6.5 and 6.6.

It should be noted that the experimental data shows that there must be some mechanism which allows the periodic runaway loss to appear over ≈ 100 cycles since if the drift orbits and island structure were stationary it would take only about one toroidal rotation for the runaways on drift surfaces touching the limiter to be lost. The orbit displacement effect is in fact slow enough to allow a gradual loss such as that of Fig. 6.5 but it apparently cannot explain the complete loss of runaways over ≤ 10 msec as shown in Fig. 6.6. In this case it is hard to explain how all the runaways of the discharge could be directly affected by the island at the $q = 2$ surface; it seems as though the island enhances the anomalous diffusion of the runaways as well as perturbing their drift orbit structure. In fact we must have roughly $D \sim (20 \text{ cm})^2 / 10 \text{ msec} \approx 10^5 \text{ cm}^2/\text{sec}$ to account for this rapid runaway loss. One possible mechanism for this would be the formation of a large scale ergodic field structure due to the interaction of the $m = 2$ mode with the $m = 1$ toroidal field or plasma shift as suggested

by Finn.⁽⁸⁶⁾ This would allow a field line to wander from within the plasma to the wall without necessarily becoming localized at the main island. Unfortunately we do not have enough experimental information to test this hypothesis.

Our conclusion is that the observed correlation between MHD activity and runaway loss can be qualitatively understood in terms of the formation of island structures in the runaway drift orbit, but that in the case of strong MHD activity there may be additional loss mechanisms indirectly produced by the instability. We should note, incidentally, that the high density regime is characterized by a large MHD amplitude and an absence of high energy runaways in steady state, but for this case it is likely that this does not reflect a large runaway loss rate, but rather that the runaway production rate is greatly reduced by the high plasma density.

6.3 INTERNAL DISRUPTION

The internal disruption or "sawtooth oscillation" was discovered when a soft x-ray sensitive PIN diode was used to scan radial chords of the ST plasma.⁽⁹²⁾ A relaxation oscillation in electron temperature was observed such that a peaking of T_e inside $q(r) = 1$ was accompanied by a drop in T_e outside $q(r) = 1$, and vice versa. The study of this mode on ORMAK has shown that it is present in nearly all discharges but that it has no effect on the gross energy confinement since it is localized near the center of the discharge [usually $q = 1$ at $r \approx 2-5$ cm].⁽¹⁴⁾

In nearly all of our observations of hard x-rays in ORMAK we have observed no modulation of the hard x-ray flux to the limiter correlated

with these internal oscillations. The exceptions are shown in Fig. 6.12. where in case (a) there is clearly an enhanced loss of runaways probably associated with sawtoothing (although no PIN signals are available) and in (b) there is a clear correlation between sawtoothing and the hard x-ray flux. [Similar behavior has been observed in TFR.⁽⁹³⁾] In (a) the discharge was run at an unusually low $q(a) = 3$, and in case (b) there was an intense neutral beam heating. For both of these situations it is expected that the $q = 1$ surface was at a particularly large radius and so would more readily cause some perturbation of runaways hitting the limiter.

It can be seen in the figure that the modulations of the hard x-ray flux has the phasing of the outer PINs . This may be understandable in terms of the drift orbit model as follows: since the current profile is presumably oscillating with the temperature profile, its sudden flattening should increase somewhat the orbit displacements of runaways near the limiter. The slow peaking of the temperature profile should gradually pull runaway orbits away from the limiter reducing the x-ray flux ("A" in drawing) until another disruption flattens the profile again ("B").

Even though the sawtoothing in most instances does not affect the runaway flux to the limiter, it probably does have a strong influence on runaway diffusion especially in the region $q \lesssim 1$. We recall that in one of the shift experiments (Sect. 5.3) the inward shift resulted in a large amplitude internal disruption which apparently caused an increase in the runaway diffusion near the outside of the plasma (Fig. 5.10). Although in this case a causal relationship

is not proven, there is an indication from theoretical calculations of the $m = 1$ tearing mode⁽⁹⁴⁾ that the internal disruption involves a severe distortion of the flux surfaces in the plasma interior which would naturally affect runaway orbits. It is also possible that a strong $m = 1$ mode in the presence of an $m = 2$ can create large-scale ergodic regions allowing runaways to travel directly from the inside to the limiter.⁽⁹⁵⁾ There is unfortunately not enough experimental data on this interaction to attempt an evaluation of these possibilities, however.

6.4 AN "RF SPIKE" INSTABILITY

In this section we describe an instability which occurred near the end of some normal low current ORMAK discharges such as those of the plasma shift experiment described in Sect. 5.3. The most characteristic symptom of this instability in ORMAK was the appearance of periodic bursts of large amplitude RF in the range 700–1400 MHz ("RF spikes"), although there were also clear effects on the hard x-ray and microwave diagnostics as described below. The general features seem to be similar to those observed for the runaway instability described by the TM-3⁽⁵⁵⁾ and TFR⁽⁶³⁾ groups, and it seems likely that this behavior may be related to an instability of the runaway tail which has been the subject of some recent theoretical work.⁽⁷⁴⁾ In order to help the unraveling of the various theoretical possibilities we will document in some detail the recent observations of its appearance in ORMAK.⁽⁹⁶⁾ Although the presence of this instability does affect runaway confinement it has not been observed in

the discharged documented in Chapter 4 and only where noted in those of Chapter 5.

A typical observation of this instability is shown in Fig. 6.11. Here it occurs just before the end of the steady state of this low density, low current discharge when we notice a sudden drop in the hard x-ray flux, the sudden onset of the RF spikes, and a sudden increase in the microwave emission in the 6 mm range (33-40 GHz) and at 140 GHz. All three symptoms occur within 1 msec of each other at a time when the voltage, current, and density are constant. There is also no sign of the instability on the radiometer (measuring particle and energy flux to the liner), the H_β light monitor, or in the plasma position indicator, indicating that either the instability is "internal" or else does not affect the bulk thermal population as a whole. The period of the RF spikes does not correlate with either the $\dot{B}$ loop or sawtooth frequency.

In Table 6.1 we list all known occurrences of the instability in ORMAK. It has been seen only relatively recently since in the past the discharges have usually been terminated before 80 msec, and also because the RF loop signal had not been monitored routinely. It is interesting that the instability can occur during the current decay phase (loop voltage = 0) as well as during steady state, indicating that it is not driven by the external energy supplied by the ohmic heating system (but there may be an internal voltage after $V_L = 0$).

When this phenomenon occurs it has a remarkable shot-to-shot reproducibility. For example, in the series 16603-16621, when the instability began during the current decay at the end of the discharge the

Table 6.1 Observations of RF Spike Instability

Shot No.	Voltage Length (ms)	Time of Instability (ms)	B_{Tor} (kg)	Steady State (30 ms)			Comments
				I (kA)	$\bar{n}_e (\times 10^{13})$	Volt	
16603-621	115, 125	133 $\pm$ 1	14.7	74	1.0-1.1	2.0-2.4	Current pulse $\approx$ 30-50 ms. Instability at $I \sim 25$ kA, $V_e = 0$; very small x-ray dump at end; clear RF spikes and 6 mm radiation; no correlation of RF spikes and x-rays.
16550-560	115	106	14.8	78	0.5-0.6	2.0-2.6	Good 140 GHz data; clear RF plus 6 mm plus HX data.
15876-902	80-110	80-100	12-18	73	0.5	2.0-2.4	Not very clear except 897-902 (have RF, 6 mm, HX very clear for 897-902); 900 and 902 @ 80 ms; 901 @ 100 ms.
15709-742	70	96-110	10-18.5	103-120	0.8-1.5	2-3	Very periodic RF (see no MHD correlation); all instabilities start during current decay; 6 mm microwave goes <u>down</u> for 18 kG cases.
14319-414	110	80	12	74	0.5	1.8	Characteristic HX signature (neg. step) consistent during "shift expt;" series of $\approx$ 100 shots (no RF, microwave available).

No sign of the instability during 15830-845, 16172-181 (i.e., at 150 and 120 kA, respectively, n_e 1.3×10^{13}), or during the neutral beam experiment 16257-309. Typically for $I = 75$ kA, $T_e = 400$ eV, $T_{e0} = 700$ eV, and the temperature probably decays with the current at the end of the shot.

RF spikes started at 133 ± 1 msec in 14 out of 14 shots monitored. There is no obvious plasma condition which can be correlated with the start of the instability, however.

The RF spikes are shown in more detail in Fig. 6.12. They have a period of .05–.3 msec or longer and persist from the onset of the instability until nearly the end of the current decay (i.e., well over 100 oscillations). They can appear in groups of 2 or 3 (Fig. 6.12a) or singly with a very regular period (Fig. 6.12b), but in all cases appear to be very fast (< 30 μ sec wide as shown in Fig. 6.12c). In some cases the spikes are correlated with a fluctuation in the hard x-ray amplitude, but their frequency has been seen to be ≈ 3 times the MHD ($\dot{B}$) frequency in the absence of sawtooth oscillations. The spike structure does not appear on either the 6 mm or 2 mm microwave signals, but it is possible that if its width was < 10 μ s it would not be detected by these systems.

A few RF spikes are generally seen during the first few milliseconds of every discharge, and some spikes were also seen at a time when the voltage was suddenly increased during the steady state of a discharge (16603–16621). This may indicate that there is some relationship to the runaway distribution function which should be changing at these times (see also the microwave signal in Fig. 4.15).

The microwave and hard x-ray signatures were not always as shown in Fig. 6.11. For example, in shots 15709–15720 and 15899–15902 the 6 mm emission went down sharply (instead of up) at the onset of the instability, as shown in Fig. 6.13; these were the only shots at the high toroidal field of 18 kG. In these shots (and some others with

the increased 6 mm radiation) there also appeared a single fast (< 1 msec) spike in the hard x-ray flux at the onset of the instability. In other cases the x-ray flux was seen to peak about 20 msec before the instability and to either increase, decrease, or remain nearly constant at the time of the first RF spikes. We should also note that the flux of high energy x-rays (≈ 5 MeV) detected by the pulse system at the outer limiter also decreased when the uncollimated hard x-rays decreased, as documented in the 14300 series of Sect. 5.2.

The experimental results may be summarized as follows:

1. The instability correlates with runaway electrons in some way causing changes in hard x-ray and 140 GHz runaway radiation. The runaway population was not necessarily very large in all cases of the instability (i.e., it was relatively small in the 16600 run).
2. The instability does not produce a gross perturbation of the plasma (I , V , n_e , plasma position, visible radiation, MHD, and radio-meter are not affected by its onset).
3. There is no obvious clue as to its triggering mechanism, but we have observed it only after ~ 80 ms in relatively low current, low density discharges. One faint clue may be the 140 GHz pattern, which seems to level off just before onset of the instability.
4. It occurs both during the steady state (loop voltage = 2 V) and during the current decay at the end of the shot (loop voltage = 0 V).

In the light of the results from TM-3 and TFR and also from the Maryland group⁽⁷⁶⁾ we might speculate that these symptoms are caused by an instability of the runaway distribution function which results

in a transfer of runaway energy from the parallel into the perpendicular direction. This would naturally result in the observed increase in cyclotron harmonic radiation ($140 \text{ GHz} \gtrsim 3 f_{ce}$), but not necessarily in radiation around f_{pe} or f_{ce} ⁽⁶³⁾ ($f_{pe} \approx 20 \text{ GHz}$) since this radiation is strongly affected by the plasma. A periodic oscillation in the runaway distribution function and RF emission has been predicted by some of the theoretical work,⁽⁷⁴⁾ and the lack of gross macroscopic effects suggests that the instability involves a relatively small group of particles such as runaways. This type of behavior is obviously significant for the confinement of high energy runaways, although it is clear that the overall runaway confinement is not seriously affected by its presence since the hard x-ray flux pattern is basically unchanged by its presence. The unusual drop in the flux at the time of the instability might be explained by a decrease in orbit displacement with an increase in pitch angle, since the vertical drift velocity is proportional to $p_{\parallel}^2 + p_{\perp}^2/2$ [see Eq. (2.2.1)]. In an extreme case some runaways may enter large banana orbits, which could explain the x-ray spike at the onset of the instability and also the cases in which the hard x-ray flux was modulated with the RF frequency.

Although this instability will probably not be present in future high density devices (if the runaway population is low!), we believe that this feature may be useful for plasma diagnosis in present devices if a theoretical understanding of its appearance can be achieved. It also represents one of the more interesting physics problems in current tokamak research.

6.5 EFFECT OF GAS PUFFING

Gas puffing is simply the addition of cold neutral gas during the discharge in an attempt to raise the plasma density. We have observed some interesting runaway behavior during this process which may be related to an instability of the plasma or of the runaway electron tail.

In Fig. 6.14 we show an example of our consistent observation that the gas puff reduces the total hard x-ray flux seen at the end of the discharge. In case (a) the puff begins at 20 msec and so might be expected to reduce the subsequent runaway production through the increase in density, but in case (b) the puff is started nearly at the end of the voltage pulse and so would not be expected to affect the total runaway production very much. In both cases the hard x-ray signal shows bursts as if an instability was causing sudden losses of runaway confinement.

This effect of gas puffing on the runaway population has been seen on several occasions and some attempt was made to understand the apparent reduction in the runaway population. In Fig. 6.15 we show some data for similar discharges with a varying amount of gas puffing. These discharges were nearly identical up until about 35 msec when the density began to rise due to the cold gas feed ($I = 174$ kA, $V = 3.5$, $B_T = 25$ kG, $n_e @ 30 \text{ msec} \approx 1.6\text{--}1.9 \times 10^{13} \text{ cm}^{-3}$).

It can be seen that the larger gas puffs affected the runaways more strongly; i.e., with a small density change the hard x-ray flux retains the normal low density pattern, but with the large puffs the runaways begin to be lost in bursts during the density rise. These

bursts coincide with increases in MHD activity which we can tentatively identify as disruptions (see Sect. 6.1). Indeed, for the largest puff we can see that a density limit has been reached at which the plasma is definitely disruptive. It is interesting to note that in the intermediate cases there are still some high energy runaways which remain confined after these bursts.

The most curious aspect of this data is that the total x-ray flux appears to be significantly reduced by the puffing. This may be accounted for in several ways: The number of runaways may be reduced by the lack of production after the gas puff, the average runaway energy may be less since they were lost before the full flux swing (total x-ray intensity $I \propto E^2$), the runaways may be lost to the liner and not the limiter during the disruptive-type bursts ($I \propto Z$ and the liner is $Z = 26$, limiter $Z = 74$), or else there may be some interaction inside the plasma itself which depletes either the runaway population or its average energy. The latter possibility could be related to an unstable form of the runaway distribution function after puffing, since increased density will greatly reduce the production rate⁽⁵⁴⁾ and so tend to form a bump-on-tail runaway distribution.

Our conclusion is that gas puffing is an effective method for reducing the population of runaways in the discharge, but that the exact mechanism for this reduction is not yet clear. This is not too surprising since the mechanism of density increase during gas puffing is not understood either — some anomalous particle transport is apparently necessary to explain the diffusion of the thermal electrons into the center of the discharge on a timescale much faster than would

be expected on the basis of neoclassical theory. A possible clue to the mechanism of runaway loss may be noted in that the most clear-cut examples of MHD/hard x-ray correlation were noticed during the steady state of a discharge in which gas puffing had been used (see Sect. 6.2), so that perhaps the enhanced particle transport is associated with increased MHD activity.

6.6 EFFECT OF NEUTRAL BEAM INJECTION

In most cases the addition of neutral beam injection into a steady state discharge did not affect the runaway confinement at all. An exception to this was during counterinjection when disruptive-type x-ray bursts were seen which may be associated with the deleterious effect on plasma stability which has been associated with this type of injection.⁽¹⁴⁾ Unless otherwise specified all the runaway data described in this thesis was taken without injection.

We may note here that the plasma heating associated with neutral beams has been successfully explained by a classical single particle model similar to that described in Chapter 2.⁽⁹⁷⁾ Although the high energy injected ions are not collisionless (they slow down and heat the plasma in ≈ 10 msec), there may be an analogy with the confinement of high energy runaways which could help define the extent of any anomalous particle transport processes. Certainly the injection experiments have shown a basic confirmation of neoclassical orbit theory, and this may be directly relevant to electrons with similar values of momentum (a 30 keV ion has the same momentum as a 5 MeV electron).

6.7 SUMMARY OF ANOMALOUS TRANSPORT MECHANISMS

In this chapter we have discussed in a qualitative manner several mechanisms which have been seen to affect high energy runaway confinement in ORMAK. In Table 6.2 we list these along with some other effects which may be responsible for runaway transport but which have not been directly measured in ORMAK. It can be seen from the last column that there are quite a few possible mechanisms that could explain the anomalous transport observed in the plasma shift experiment of Chapter 5.

In order to clarify the runaway transport problem it seems to be necessary to perform further controlled experiments in which each of these possible mechanisms is isolated. For example, there have been experiments on small tokamaks at MIT⁽⁹⁸⁾ and in Japan⁽⁹⁹⁾ which have observed systematic losses of runaways associated with non-axisymmetric perturbations of the applied vertical field, and in the T-6 tokamak⁽⁴⁰⁾ the runaway confinement deteriorated when the toroidal field ripple at one point around the torus was increased. Yet even in these cases the precise effect on the runaway orbits has not been determined. Such experiments and the associated theoretical studies are an appropriate subject for future research.

Table 6.2 Summary of Anomalous Transport Mechanisms

Mechanism	Physical Process	Effect on Runaway Confinement	Present in Shift Expt.?
Disruptive instability	Destruction of magnetic surfaces, mechanism unknown	Runaways follow magnetic field lines to wall	No
MHD ($m = 2$)	Formation of magnetic islands	Distortion of runaway drift surfaces, modulation of loss to limiter	Yes
Internal disruption	$m = 1$ instability near $q = 1$ surface	Internal effect unknown, occasional modulation of runaway loss to limiter	Weakly
RF spikes	Instability of runaway tail?	Pitch angle scattering of runaways in instability generated waves (?)	Partially
Gas puff	Unknown mechanism which allows enhanced thermal electron transport	Either increases runaway losses or anomalous friction	No
Neutral beam counter-injection	Enhanced losses of counter-injected beam on unconfined orbits (?)	Disruptive type runaway loss	No
Drift waves	Trapped electron mode (?)	Possible $E \times B$ drift or pitch angle scattering (see App. E)	Not measured in ORMAK
Field ripple	Modulation of magnetic surfaces due to finite toroidal or vertical field coils	Possible fluting of runaway drift surfaces, pitch angle scattering (?)	Probably negligible
Error fields	Destruction of magnetic surfaces near edge, formation of magnetic islands (?)	Possible loss of runaways along unconfined field lines	Not measured

SUMMARY AND CONCLUSIONS

This work has demonstrated that runaway electrons in ORMAK can be accelerated freely and confined until they reach energies of ≥ 10 MeV. It has been shown that the patterns of runaway loss to the limiter can generally be explained by a collisionless single-particle orbit model, but that an anomalous transport can exist at least near the outside of the discharge. This transport has been modelled by a runaway diffusion coefficient $D \approx 10^2 - 10^4 \text{ cm}^2/\text{sec}$, and an attempt has been made to identify the mechanism of this anomaly.

In Chapter 2 the collisionless drift orbit model of runaway confinement has been described. It was shown that high energy runaways ought to accelerate freely in the applied electric field, and that their pitch angles should be small. The application of this model to our limiter measurement was discussed, along with a brief review of the mechanisms which may cause anomalous runaway transport.

In Chapter 3 it has been shown how high energy runaways can be measured through the thick target bremsstrahlung produced when these electrons strike the limiter of the torus. The ORMAK hard x-ray diagnostic system was described along with the procedures used for data acquisition and analysis. The two fundamental limitations to this system were explained in detail in Appendix A (Interpretation of Scintillator Spectra) and Appendix B (Pileup Reduction in NaI).

In Chapter 4 the data on high energy runaway loss patterns was presented along with an interpretation based on the drift orbit model of Chapter 2. The runaway confinement in low density discharges could be explained by the outward displacement of runaway orbits with

increasing energy (during the steady state) or with decreasing current (during the current decay). The confinement in high density discharges could be explained in terms of the outward displacement of runaways born near the outside of the discharge, but in this case the influence of MHD activity on runaway confinement could not be discounted. Several additional experiments aimed at clarifying the confinement process in these two cases were described.

In Chapter 5 the subject of high energy runaway diffusion was introduced along with two experiments which have indicated the presence of such diffusion. The point limiter experiment has shown that runaways can have an anomalously large radial velocity near the edge of the discharge, but the interpretation of this result is in doubt due to the possibility of runaway scattering from the point limiter itself. The plasma shift experiment has shown that the transport of runaways to the limiter after an inward plasma shift can be modelled by adding a diffusion coefficient $D \approx 10^2 - 10^4 \text{ cm}^2/\text{sec}$ to the drift orbit model of runaway confinement (see also Appendix C). The lower limit in this range has been observed in the most extensive run and represents a negligible correction to the overall transport of runaways. The upper limit has in some cases been associated with MHD instability in the discharge, and represents a significant modification to the drift orbit model. It was shown that a detailed evaluation of runaway confinement in the plasma interior cannot be made with our limiter detection system, although an upper limit on the average diffusion coefficient of $\lesssim 10^4 \text{ cm}^2/\text{sec}$ can be deduced.

In Chapter 6 some mechanisms for anomalous runaway diffusion were discussed. Phenomenon which have been seen to affect runaway confinement in ORMAK include MHD activity, cold gas puffing, and a new instability apparently associated with the runaway population itself. The presence of other effects such as drift waves and various static magnetic perturbations cannot be ruled out as a cause of the runaway diffusion observed in the experiments of Chapter 5. Details of the expected influence of $m = 2$ magnetic islands and of drift waves have been discussed in Appendixes D and E, respectively.

The general implication of this work is that the magnetic structure of ORMAK can be extremely efficient in confining these collisionless runaways, since over 10^6 toroidal transits of the machine are required to gain the measured energy of ≈ 10 MeV. On the other hand, the observed deviations from the collisionless orbit model have again shown that anomalous transport processes can be a significant factor in the confinement properties of tokamaks. The success of the tokamak fusion reactor will depend to a large extent on the identification and control of the instabilities responsible for this anomalous transport.

One practical implication of this work involves a set of prescriptions which can be given to avoid the appearance of high energy runaways in tokamaks. An increase in plasma density has been shown to decrease the runaway population roughly in accordance with the classical runaway production theory (Fig. 4.5); however, the creation of runaways during the breakdown phase is not inhibited by this means (Fig. 4.6). Runaways created during breakdown can be removed from

the discharge by an outward shift of the plasma column at early times (Sect. 4.3), while runaways which exist during the steady state can be eliminated by a puff of cold neutral gas (Sect. 6.5). It should also be mentioned that a strong increase in runaway production with increasing impurity level has been demonstrated during the course of this work. (30)

POSSIBILITIES OF FURTHER RESEARCH

There are many possible extensions of the work described in this thesis, both in the experimental and theoretical areas. The main experimental challenge is to measure the runaway distribution function inside the plasma in order to deduce the detailed nature of the runaway transport, and possibly to correlate a measured runaway diffusion coefficient with some other experimental indications of anomalous transport. On the theoretical side, an extensive program of calculation is needed in order to understand how the various plasma instabilities can affect the confinement of the high energy runaway population, so that the measured anomalous transport can eventually be related to a specific physical mechanism.

Some of the more promising and tractable avenues for future research are listed below:

Experimental

1. A tangential plasma bremsstrahlung measurement can possibly be used to determine the high energy end of the runaway distribution function inside the plasma (see Appendix A.8). The forward peaking of the x-ray emission should reduce this chordal measurement to nearly a point measurement, but one has to be very careful to avoid the background due to limiter radiation. An estimate of the x-ray flux based on Fig. A.8 has indicated an available count rate of $\geq 10^5$ x-rays/sec for typical low density ORMAK conditions. A radial profile of runaway energy and population vs time would be very useful toward understanding the runaway production and transport mechanisms.

2. The introduction of known magnetic field errors or a variation of the toroidal field ripple can be used to test the significance of these factors in determining high energy runaway confinement. A deterioration of runaway confinement with an increasing non-axisymmetric vertical field has been shown, for example, on an experiment on the Versator tokamak at MIT.⁽⁹⁸⁾

3. A series of scaling experiments would be useful if a large amount of machine time were available. One could check the scalings of maximum runaway energy vs. τ_E , n , I , etc. in order to further check the drift orbit model and to gain information about the possible diffusive mechanisms.

4. The role of runaways in limiter damage has become of some importance recently, when it was noticed that high-Z impurities can be transferred from the limiter to the liner to the plasma. A study of the runaway-limiter interaction may become of some practical significance for future tokamaks.

5. The low energy part of the runaway distribution has not yet been measured at the same time as the high energy part. A 90° plasma bremsstrahlung detector plus a 0° limiter (or plasma) bremsstrahlung measurement should be able to cover most of the runaway distribution function.

Theoretical

1. The influence of more than one magnetic island may lead to interesting effects on runaway confinement. For example, is it possible that the orbit displacement effect can allow a runaway to "jump" from one island to another? The recent theory of "magnetic

flutter" by Callen⁽¹²⁶⁾ may be a particularly interesting branch of this problem.

2. The influence of short wavelength electrostatic or electromagnetic waves or field ripples on the motion of high energy runaways may show some interesting resonance effects when the time for the runaway to cross the perturbation is nearly the gyroperiod. One would expect an enhanced pitch angle scattering or energy change which could be important in explaining the observed anomalies in the confinement of high energy runaways in ORMAK.

APPENDIX A

INTERPRETATION OF SCINTILLATOR SPECTRA

1. INTRODUCTION

In this appendix we give the details of the computation of the expected pulse spectra for monoenergetic electrons as shown in Fig. 3.6. The procedure is basically straightforward: for each electron energy we find the absolute 0° bremsstrahlung emission for electrons normally incident on a thick tungsten target, calculate the attenuation of this x-ray spectrum through the known materials between the source and detector, and finally transform the x-ray spectrum into the scintillator pulse height spectrum through the known response matrix for our 5" x 5" NaI crystal.

As shown in more detail in subsequent sections, this can be symbolically expressed as:

$$P(k_m) = \int_0^{k_{\max}} g(k-k_m) s(k, E) e^{-\sum_i d_i \mu_i(k)} dk \quad (A.1)$$

where $P(k_m)$ is the pulse height spectrum as given in Fig. 3.6, k_m is the pulse height, $g(k-k_m)$ is the response function of the scintillator transforming a photon k into a pulse height k_m , $s(k, E)$ is the bremsstrahlung spectrum for one electron with kinetic energy E , and the exponential term gives the x-ray attenuation through a thickness d of material i with attenuation cross section $\mu(k)$. These functions (see Sects. 2-4) were tabulated in matrix form and incorporated in a computer program which produced pulse height spectra given a set of

material thicknesses d_i and an electron energy between 2 and 12 MeV (in integral steps).

The use of these spectra as described in Sect. 3.3 is dominated by two facts of life about the thick target bremsstrahlung spectrum due to runaways. First, the higher energy components so dominate the pulse spectrum that unfolding of components below $E_{\text{max}}/2$ is basically impossible (assuming a nearly flat distribution function); and second, the absolute number of high energy components can be found only to within about a factor of 10 due to uncertainties in the source function (Sect. 3.3), the unknown pitch angle distribution, and the non-ideal target effects. The last two effects will be discussed below.

2. 0° BREMSSTRAHLUNG SPECTRA

A useful approximation for our experiment is that the runaways are normally incident on a plane thick target of tungsten, and that the x-rays are observed at 0° to the incident electron direction. There is a rather extensive literature on the bremsstrahlung calculation for this case.⁽¹⁰⁰⁻¹⁰⁵⁾

It is important to realize that thin target bremsstrahlung is strongly forward peaked in the MeV energy range⁽⁶⁶⁾ with a mean half-angle of $1/\gamma$, where $E = (\gamma-1)m_0c^2$ is the kinetic energy of the electron. For thick targets the electron is deflected by multiple scattering as it is slowed down, but nevertheless the high energy end of the x-ray spectrum is still peaked by nearly this much (see Chapter 6). The usual relation⁽¹⁰⁰⁾ for total x-ray yield $I \propto T^2$ is thus modified by this peaking so that for 0° observation we have nearly $I \propto T^4$.

The most extensive experimental data for thick target bremsstrahlung in this energy range is from a LINAC experiment by O'Dell, et al.,⁽¹⁰²⁾ who measured absolute 0° spectra for nine electron energies between 5.3 and 20.9 MeV. A composite Au-W target was used with a combined thickness equivalent to .038 cm W. Only photon energies above 3 MeV were measured.

Dickinson and Lent⁽¹⁰¹⁾ calculated forward bremsstrahlung for electron energies 1, 2, 4, and 10 MeV, both for stopping targets and for optimally thick targets (see below). They also show that shape agreement with O'Dell is reasonably good; however, their theoretical values are consistently 35–40% larger than O'Dell.

Calculations by Berger and Seltzer⁽¹⁰⁵⁾ and Ferdinande⁽¹⁰⁴⁾ show better agreement with O'Dell, but the published spectra include only 5.3 and 10 MeV electrons onto the O'Dell target. The improved agreement is partly due to better tailoring to the experimental arrangement.

We have chosen to use the "optimal" thickness* values of Dickinson and Lent, basically because the O'Dell target thickness is considerably less than optimal for ≈ 10 MeV electrons in tungsten.⁽¹⁰²⁾ Since the target in ORMAK is .25" tungsten, we assume that all photon production occurs inside the optimal thickness ($< .1$ cm), and simply correct for the remaining target attenuation. Interpolation is aided

*For targets thicker than optimal, the photon attenuation begins to outweigh any additional bremsstrahlung production. For targets thinner than optimal, the total intensity is quickly reduced and the spectral shape changes. Optimal thickness for $Z = 74$, $E = 10$ MeV is $\leq .1$ cm. The relative spectrum changes by $\leq 5\%$ between optimal thickness and $\approx 3x$ optimal thickness for 10 MeV electrons.

by an empirical formula in Dickinson and Lent giving total intensity as a function of energy at optimal thickness.

The results are plotted in Fig. A.1. These curves are used to produce photon spectra divided up into 1/2 MeV wide bins centered on multiples of 1/2 MeV. It is these spectra that we use as the function $s(k,T)$ in Eq. (A.1). The shapes of the spectra are in good agreement with the results of a series of runs using Berger's and Seltzer's Monte Carlo code as described in Sect. 6, although we would expect that the approximations inherent in these calculations result in an uncertainty $\leq 30\%$ in the absolute bremsstrahlung intensity [estimated error in Monte Carlo code is $\pm 10\%$ ⁽¹⁰⁵⁾].

3. ATTENUATION

This is the simplest part of the transformation process. We simply use the total attenuation coefficients given in Refs. (106) and (107) for the measured material thicknesses in the machine (25 cm Al, 5 cm Cu for the 0° line, 11 cm Al and 5 cm Cu for the 18° line), the estimated limiter attenuation ($\approx 1/4"$ W as shown in Fig. A.2), and the known lead filter thickness (usually 1/2" to 2-1/2"). Total attenuation for 25 cm Al, 5 cm Cu, and 2" Pb is ≈ 300 for 5 MeV x-rays.

The basic effect of this attenuation is to eliminate the x-rays below $\approx .5$ MeV, since in particular the attenuation of photons in lead is sharply increasing below this energy (and nearly flat from 1-10 MeV). This is very useful since these low energy photons are not used in finding the electron distribution function and would otherwise reduce the number of useful high energy counts given a fixed count rate (see Appendix B).

We note that the uncertainties in the amount of aluminum and copper in the machine along these lines is roughly 20%, and that Compton scattering from off-line materials into the detector is small due to the small solid angle of our system. The attenuation coefficients are good to a few percent, so that a $\pm 20\%$ uncertainty in the Al and Cu thicknesses results in total uncertainty of about a factor of two in the absolute bremsstrahlung intensity.

4. SCINTILLATOR RESPONSE

The final step in the computation of the expected pulse spectra for monoenergetic electrons is the scintillator response function $g(k, k_m)$ of Eq. (A.1). This function takes into account the fact that only a small fraction of the incident photons of energy k result in a pulse proportional to this energy, since most of them eventually produce some x-rays which leave the crystal.

We have used the experimental results of Jarczyk, et al.⁽¹⁰⁸⁾ [as shown in Fig. 3.2] to construct a scintillator response matrix as shown in Fig. A.3 [a calculated scintillator response function is also available⁽¹⁰⁹⁾]. This matrix represents the response of our 5" x 5" NaI crystal, and has been incorporated into the computer program which calculates monoenergetic pulse spectra. The output of this program for typical values of material attenuation has been given in Fig. 3.6.

Occasionally we used a 3" x 3" NaI crystal in addition to the 5" x 5" crystal, and we observed no systematic difference between the two spectra. The principal difference between their expected responses is a relatively lower photopeak efficiency for the 3", but

this is somewhat compensated for by the relatively larger 1-electron escape peak .5 MeV below the center of the photopeak.⁽¹⁰⁸⁾ This .5 MeV energy difference should not be significant for our analysis of pulses in the energy range 5–10 MeV.

5. FITTING SPECTRA

We have discussed in Sect. 3.3 some of the difficulties involved in unfolding the observed pulse height spectra in terms of the computed monoenergetic components. The main point is that the total number of counts/electron increases like T^{3-4} in the electron energy range 2–10 MeV, so that it is nearly impossible to distinguish the contribution of runaway components below half the maximum energy. In most cases we have only used the spectra for determining the maximum electron energy, as discussed in Chapter 3.

It is also important for the fitting process to be sure that pulse pileup does not distort the high energy end of the pulse distribution. A safe level of $\leq 1\%$ pileup has limited our count rate to $\leq 5 \times 10^4$ counts/sec (see Appendix B), which allows the accumulation of only about 250 counts per shot over a typical ORMAK timescale of 5 msec during which the maximum energy can be considered constant. Practical considerations have allowed only 5–10 identical discharges to accumulate data for each of our experiments, so that the statistical certainty in our pulse spectra is still rather large (see Fig. 3.7). It is possible that with a greatly improved data base the unfolding procedure could be applied to yield the runaway distribution function in the range 2–10 MeV, but the accuracy of such procedures

will depend also on the non-ideal effects discussed in the next two chapters.

6. OFF-AXIS BREMSSTRAHLUNG

Based on the calculations of runaway pitch angle of Sect. 2.1 we have generally assumed that the 0° thick target bremsstrahlung spectra can be used for interpretation of our results. In this section we discuss the sensitivity of these spectra to a change in the incident electron's angle with respect to target, or to a change of the detector's viewing angle for normal incidence. The latter point is significant since occasionally we used a detector at an 18° angle with respect to the usual tangential detector. Although there has been some analytic theory on this problem^(103,110) we have used a comprehensive electron-photon computer code ETRAN written by Berger and Seltzer⁽⁸⁴⁾ to find the spectra for the energy range of our interest.

In Fig. A.4 we show bremsstrahlung spectra as viewed at various angles for the case of 10 MeV electrons incident normally on a plane tungsten target. It can be seen that the photon intensity is highest for a viewing angle of 0° and falls off rapidly for angles above $\approx 1/\gamma$. This effect is shown more explicitly in Fig. A.5 and is related to the relativistic forward peaking of the bremsstrahlung emission. This peaking ought to be $1/\gamma$ for thin targets, but apparently persists for the high energy photons in thick target emission.

Figure A.5 shows that there is $\lesssim$ order of magnitude decrease in expected intensity for viewing $\approx 10^\circ$ off-axis in this case, but that the shape of the spectrum is not affected much. For viewing angles

above about 30° the high energy end of the photon spectrum begins to be depleted and the overall intensity is reduced further. The analytic results of Lent and Dickinson which were used to construct the pulse spectra of Fig. 3.6 are also shown; they agree well in shape with the code but are somewhat below the 0° – 1° results. The conclusion is that for the purpose of energy analysis the analytic results can be used for viewing angles (with respect to the target) of up to about 20° , but that for an estimate of the absolute electron population the appropriate off-axis intensities should be used. This has been confirmed by experiment in that there was no systematic difference between the spectral shapes observed between the 0° and 18° detector angles.

The complementary variable is of course the angle of electron incidence onto the target. In Fig. A.6 we show that for non-normal incidence (here 75° with respect to the target plane) the x-ray spectra are shifted off 0° ; in fact, most of the photons appear at 0° with respect to the incident electron's direction, independent of the target position. This sensitivity could be used to include the effect of a non-zero runaway pitch angle, but unfortunately there is presently no independent measurement of the pitch angle distribution of runaways incident onto the target.

We have compromised with this uncertainty by using the spectra of Lent and Dickinson in the interpretation of our data. Their results correspond to an average pitch angle of $\approx 3^\circ$ according to the ETRAN code, and this is within the range of the expected pitch angle as discussed in Sect. 2.1. What actually happens is that for a fixed

viewing angle the system is most sensitive to electrons headed in the direction of the detector independent of the target orientation, so we may say that our estimates of absolute electron intensity refer only to this subset of the runaway population hitting the target. There is obviously room for improvement upon this procedure, but for our purposes it has been sufficient to have this order of magnitude estimate of the absolute runaway population.

7. NON-IDEAL TARGET EFFECTS

For computation of the expected scintillator response we have assumed a plane tungsten target of 1/4" thickness. The actual target is shown in Fig. A.7. Each of the tungsten links are 1/8" in thickness, and this photo shows the damage due to runaway impact at the outside of the limiter (this is the original ORMAK limiter which was removed before we began our experiments — the present one is similar).

The most serious non-ideal target effect would be the edge effect, i.e., that some runaways will hit the limiter so near its edge that they can escape back out into the plasma. We note that although the mean range of 10 MeV electrons in tungsten is $.3 \text{ cm}^{(105)}$ the high energy photons which we measure should be produced in the early stages of the electron slowing-down and so the target can be considered thick for electrons passing through $\geq 1 \text{ mm}$ of the limiter. For runaways passing through less material the spectrum should approach the thin-target values, but again for the high energy end of the photon spectrum the shape of the spectrum should not be affected much.

The other aspect of an edge effect is that the scattered electrons will acquire some pitch angle after collision with the limiter. This

may cause enhanced transport of these electrons into the shadow of the limiter due to their finite Larmor radius or to the formation of trapped particle orbits.⁽⁴⁴⁾ This may cause an extended damage pattern such as observed in the point limiter experiment (Sect. 6.1) and to a lesser extent in the normal limiter as shown in Fig. A.7. The increased average pitch angle of runaways finally hitting the limiter may in turn affect the bremsstrahlung spectra as discussed in the last section.

We note that the construction of the limiter may allow some of the bremsstrahlung to pass through more than 1/4" of tungsten, so that the expected absolute intensity would need to be increased by $\approx \times 2$. But since the dominant x-ray absorber is the lead in the collimator, the expected shape of the spectra should not be much changed.

A further evaluation of these effects would need a quantitative evaluation of the electron scattering processes for this geometry. Although we have not attempted to do this, we include in Fig. A.8 a table showing the surprisingly large backscattered electron emission from a plane tungsten target (from the Berger and Seltzer code). This result, along with some experimental work on electron backscattering,⁽⁸⁵⁾ suggests that $\geq 20\%$ of the electrons incident onto the limiter may be inelastically scattered back into the plasma. Most of these electrons would have only a fraction of the incident electron energy, however.

8. THIN TARGET BREMSSTRAHLUNG SPECTRA

We include for the sake of completeness some thin target bremsstrahlung spectra for hydrogen taken from Logan [Fig. A.9].⁽¹¹¹⁾ It

is interesting to note the extreme insensitivity of the 90° plasma bremsstrahlung measurement to electrons with $E > 2$ MeV; also we can see that for 0° bremsstrahlung only the highest relativistic components will be detected. Possibly a system using 2 or 3 different viewing angles simultaneously may be able to resolve the whole runaway distribution function. This figure can also be used to show that 0° thin target bremsstrahlung is generally negligible compared to the tungsten thick target yield.

APPENDIX B

PILEUP REDUCTION IN NaI

1. INTRODUCTION

For our x-ray pulse height analysis system it is important to maximize the count rate capability in order to minimize the number of shots needed to accumulate a useful spectrum. It was found that the count rate limit is set by the requirement that the pulse spectrum be undistorted by pulse pileup, but that the usual count rate limit for NaI detectors can be improved by about a factor of five with the pileup rejection system described below.

Pulse pileup occurs when the time interval between two pulses is shorter than the integration time of the linear amplifier system, that is, the time over which the information from a single pulse is normally accumulated. For example, since in a NaI (Tl*) crystal the scintillation light decays with a principal time constant of 250 ns, the pulses are usually processed with an amplifier shaping time constant of at least .5 μ s in order to catch all the light from each scintillation. This results in a pulse about 2 μ s wide. If we require that not more than 1% of the pulses have time intervals less than 2 μ s in order to avoid pileup, the count rate limit would be given roughly by $NT = .01$, where N is the count rate (in counts/sec) and T is the pileup resolving time, defined as that time interval below which two pulses are considered to cause pileup. For $T = 2 \mu$ s we have $N \approx 5 \times 10^3$ counts/sec which would allow accumulation of only about 50 counts in a typical ORMAK time scale of 10 ms. To improve this count rate we could either

attempt to reduce the pileup resolving time, or else accept a larger fraction of piled-up pulses. In the bulk of this appendix we describe a circuit which reduces the pileup resolving time from $\approx 2 \mu\text{s}$ to .1 to .2 μs , so let us first consider what would be the maximum acceptable fraction of piled-up pulses for our system.

2. PILEUP DISTORTION

The most important consideration with respect to pileup distortion is that the high energy (large pulse height) end of the spectrum should not be disturbed, since the important measurement of maximum electron energy depends on finding the maximum pulse height. We can calculate the amount of pileup distortion vs. count rate for a simplified form of our undistorted pulse height spectrum.

The undistorted NaI pulse height spectrum produced by runaways in our experiment can be approximated by (see Appendix A):

$$\begin{cases} S(E) = dN/dE = (2N/E_0^2) (E_0 - E) & (0 < E < E_0) \\ S(E) = 0 & E > E_0 \end{cases} \quad (\text{B.1})$$

where E is the pulse height and E_0 is some maximum pulse height related to the maximum electron energy, and we have normalized to a total count rate N . For our case, where the fraction of piled-up pulses is small, we need only consider the distortion due to the summing of two pulses and this can be written^(82,112)

$$D(E) = S(E) + NT \int_0^E P(x) S(E - x) dx \quad (\text{B.2})$$

where $D(E)$ is the distorted spectrum, T is the pulse width, and $P(x)$ is the probability of finding a pulse height x already in the system at

the time when the next pulse peaks at a height of $E - x$.

The probability $P(x)$ depends upon the pulse shape. For example, if the pulses are rectangular $P(x) = S(x)/N$; or if the pulse has a long negative tail (undershoot), $P(x) > 0$ for $x > 0$. For our case the pulse can be approximated by a triangular shape so that $P(x) = S(x)/2N$; that is, the average "baseline shift" due to the first pulse is half its peak height. Thus,

$$D(e) \approx S(E) + T/2 \int_0^{E_0} S(x) S(E - x) dx \quad (B.3)$$

A convenient measure of the extent of pileup for our case is the ratio of the second term to the first term (on the right side of this equation) integrated over the pulse height interval, $.8E_0 < E < 2E_0$. This is the fraction (f) of piled-up pulses to "real" pulses near the maximum energy. After an elementary integration we find:

$$f \approx 2NT \quad (B.4)$$

This is only a factor two larger than the naive estimate of the number of overlapping pulses $f = NT$.

The maximum allowable value of f depends upon the accuracy which we need to find E_0 . Our choice has usually been an extremely safe $f = .01$, such that for a normal spectrum with ≈ 100 counts near maximum energy we expect only one count due to pileup. In practice the count rate varies uncontrollably, so that our system generally is considered safe from pileup distortion for $f \lesssim .05$, with increasing caution being used in the interpretation of spectra past this point.

3. PILEUP REDUCTION SYSTEMS

There is an extensive literature on the problems of high count rate pulse systems, especially from the field of nuclear spectroscopy.⁽¹¹³⁻¹²⁰⁾ The case of NaI detectors is particularly difficult due to the long decay constant and relatively low number of electrons in the primary signal, but there have been two successful approaches toward maximizing the count rate without pileup:

- A. Attempts to reduce the width of the pulses by shaping them with a suitably terminated delay line (s), so that when a pulse is added to its reflection it returns to zero in a time shorter than the scintillator decay time.⁽¹¹⁶⁾
- B. Attempts to reject piled-up pulses by gating out those pulses which have time intervals less than about 2 μ s. The time interval inspection process is done using a fast, nonlinear form of the original pulse.^(117,119,120)

The system type (A) is suitable for NaI since the pulse risetime (determined mainly by the PM tube) is ≤ 20 ns, so that the "clipped" pulse can in principal be shortened to roughly double this length. A fundamental limit with this system is that the energy resolution naturally degrades as the clipped pulse is shortened below the scintillator decay constant [i.e., by a factor of 2 for a 50 ns pulse length in NaI⁽¹¹⁶⁾]. Another limitation is that the fluctuations in the source signal cannot be cancelled out by the clipping process, and the result is a minimum detectable signal above the noise which limits the dynamic range of the spectrum. Two other practical factors ultimately decided our choice of system (B). Firstly, the clipping system does not

directly indicate when and if pulse pileup does become serious as may happen, for example, during a high count rate burst from ORMAK. Secondly, it relies on the reproducibility of the leading edge shape of the PM pulses, which is usually good, but may become distorted during the transient magnetic and radiation fields in the ORMAK environment. In conclusion, we chose to operate a version of system (B) to be safe about our linearity and to have warning of possible pileup situations.

The pileup rejection method of systems (B) has been described for a bremsstrahlung measurement⁽¹²⁰⁾ and for a nuclear spectroscopy application.⁽¹¹⁹⁾ The latter system was used for investigating a narrow region of the spectrum in the presence of a large count rate of lower energy pulses. Thus it was possible to achieve a pileup resolving time of $\lesssim 50$ ns with a small dynamic range. The system for bremsstrahlung study, similar to ours in that we may be interested in the whole spectrum, was operated with a pileup resolving time of 250 ns.

A detailed evaluation of the limitations of these pileup rejection systems is given in Sect. 5; first we describe the principles of their operation.

4. DESCRIPTION OF THE SYSTEM

A block diagram and a timing diagram for the pulse analysis system is shown in Fig. B.1. All the electronic modules are standard ORTEC or Tennelec NIM units. The linear output pulses are stored in up to 16 segments of the MCA memory for consecutive time intervals, and are also displayed on oscilloscopes along with other signals of

interest.

The operation is basically simple. All pulses are routed into two different branches, the slow linear branch for pulse height analysis and the fast pileup inspection branch for determining which pulses are to be rejected. The linear branch uses a standard $.5 \mu\text{s}$ shaping amplifier (with baseline restoration and pole/zero cancellation), a $2 \mu\text{s}$ delay so that the pileup inspection can be completed before the linear pulse goes to the MCA, and a gated pulse stretcher which performs tail-pileup rejection as described below. The pileup branch uses a "timing filter amplifier" to produce a fast pulse (which is essentially the shaped derivative of the PM pulse), a 100 MHz discriminator to reject low amplitude noise on the fast pulse, and a "pileup-inspector" which examines a $2 \mu\text{s}$ interval after each detected pulse. If a second pulse is detected within this inspection interval the pileup inspector "inhibits" or gates out the linear branch for $5 \mu\text{s}$, which (with the $2 \mu\text{s}$ delay) should eliminate the piled-up pulse pair.

There are two sorts of pileup which are rejected by this circuit; often these are called peak pileup and tail pileup.⁽¹¹⁵⁾ Tail pileup means that the pulse separation is larger than the linear pulse's risetime ($\approx 1 \mu\text{s}$) and peak pileup refers to the more difficult case when the separation is less than the pulse risetime (see Fig. B2).

Tail pileup is rejected automatically by the linear branch. The linear amplifier has pole-zero cancellation and baseline restoration so that an accumulation of low amplitude, long decay time undershoot cannot shift the baseline at high count rates. In addition, the d.c.

coupled "linear gate stretcher" starts its output at the first peak of any pulse pair and will not refire until the input has fallen below its discriminator level (usually .2 MeV). The system has been tested with a γ source and shows negligible peak broadening and $< 1\%$ peak shift at 5×10^4 cts/sec.

Peak pileup rejection cannot be accomplished by the linear branch since the resulting pulse does not have a double-humped structure. The pileup inspection branch uses a fast version of the pulse, but still cannot resolve two pulses whose separation is less than the width of the fast pulse, since the 100 MHz discriminator will not refire until its threshold level is recrossed (see Fig. B2). The whole problem reduces to the question: how short can we make the width of the fast pulse? In the next section we show that there is a trade-off between the pileup resolving time and the available dynamic range, and that the limiting factor is the fluctuation level on the PM pulse due to the relatively small number of photoelectrons per scintillation in NaI.

5. LIMITS TO PILEUP REJECTION IN NaI

The pileup inspection branch can resolve two pulses only if their time interval is large enough so that the 100 MHz discriminator threshold is recrossed after the first pulse. For our case, where we are interested in the shape of the whole spectrum, we would like to set the discriminator level to $\approx 1/20$ th, the maximum expected energy. Thus the pileup resolving time is essentially the full width of the fast pulse (see Fig. B2).

As the width of the fast pulse is reduced, the signal becomes increasingly "noisy," as shown in Fig. B3. The pileup rejection system begins to fail when a single pulse has a chance of being rejected because the 100 MHz discriminator mistakenly triggers on a fluctuation in the tail of that single pulse. We call this condition a "spurious inhibit," since it would leak to rejection of the single pulse as if it were a piled-up pulse pair. For a given width of the fast pulse, we find that as the discriminator level is lowered the spurious inhibits begin to dominate the pileup rejection circuit.

In Fig. B4, we plot the ratio of inhibits to total counts for various settings of fast pulse width and discriminator level, using the pulses from a Cs^{137} source at 4.4×10^3 cts/sec. The smaller the width of the fast pulses, the larger the discriminator level necessary to avoid onset of spurious inhibits. The requirement that the discriminator level be 5% of the maximum expected energy, plus a requirement that $\lesssim 1\%$ of the counts be lost as spurious inhibits, has fixed the minimum fast pulse width to be in the range of 100–200 ns. For pulses with smaller widths the fluctuations shown in Fig. B3 begin to trigger the pileup circuits.

What is the origin of these fluctuations? We have checked the dependence of signal/noise on the high voltage level and on the γ -ray energy. The s/n is independent of voltage and roughly proportional to $\sqrt{\text{energy}}$ in the range 20 keV $\rightarrow$ 1.2 MeV. We also observe that the fluctuations are largest at high signal levels and negligible between pulses. This suggests that they are due to the limited photocathode statistics. An estimate based on a typical photo cathode current for

NaI of 2 electrons/keV⁽⁸⁰⁾ gives a photopeak resolution reasonably consistent with the observed energy resolution of 8% for Cs¹³⁷, assuming an intrinsic resolution of about 4%.⁽⁷⁹⁾ We conclude that the fluctuations in Fig. B3 are due simply to the statistics of electron arrival at the photocathode over the scintillation decay time, since the total number of photoelectrons per pulse is only ~ 1300 .

A calculation of the expected effect of this sort of fluctuation level on the pileup resolving time can be made and compared with the experimental results of Fig. B4. We assume that two scintillator pulses are separated by a time t (see Fig. B5) and calculate the minimum amplitude of the second pulse which can be distinguished from the fluctuations on the tail of the first pulse. Each pulse is assumed to have an exponential decay constant τ and we integrate the signal only over an interval T corresponding to the width of a fast-shaped pulse. If we assume that the number of electrons in the second pulse must be less than 2.5 times the square root of the number of electrons in the tail of the first pulse during T , in order that there be $< 1\%$ "spurious inhibits," we find that $N_2 > 2.5 N_1^{1/2}$ implies:

$$\frac{N_2}{N_0} \geq \frac{2.5}{(N_{\text{tot}})^{1/2}} \frac{[e^{-t/\tau} - e^{-(t+T)/\tau}]^{1/2}}{(1 - e^{-T/\tau})} \quad (\text{B.5})$$

We can compare this ratio to the observed value at which we found spurious inhibits to become significant for the Cs¹³⁷ pulses. For a 662 keV pulse we would expect $N_{\text{tot}} = 2500$; for $T = 100$ ns and $t \approx 50 - 100$ ns we find $N_2/N_0 = .11$. That is, we could expect that the discriminator level must be set to less than 11% of the peak energy in order to limit the number of spurious inhibits to $< 1\%$. This agrees

reasonably with the 105 ns case of Fig. B5, where 1% spurious inhibits occurred at an $\approx 7\%$ discriminator level. We conclude that the observed occurrence of spurious inhibits is consistent with this explanation, and we infer from this that the fluctuation level for 10 MeV pulses should be relatively less than observed for the Cs^{137} pulses. Thus the use of the 662 keV pulses for evaluating pileup provides a conservative solution to the pileup problem for the x-ray spectra of the experiment.

6. CONCLUSION

Our pileup rejection system with NaI can reduce the pileup resolving time from the usual $2\ \mu\text{s}$ to $\approx .1 \rightarrow .2\ \mu\text{s}$ while still retaining a $\approx 10:1$ dynamic range (determined by the discriminator level $\approx .5\ \text{MeV}$). With our requirement of $\leq 1\%$ pileup we have operated this system successfully at $5\text{--}10 \times 10^4$ cts/sec. An example of the operation using a Cs^{137} source at 5×10^4 cts/sec is shown in Fig. B6. Pileup is reduced approximately a factor of 5 with this rejection system, corresponding to a reduction in the effective pileup resolving time from $\approx .60\ \mu\text{s}$ (for significant pileup in the linear branch alone) to $\approx .12\ \mu\text{s}$ with the rejection system operating.

The limit of our ability to reject pileup appears to be the statistical fluctuations in the PM signal due to the rather small number of photoelectrons/keV characteristic of large NaI detectors. These fluctuations may mistakenly trigger the pileup rejection circuit resulting in a spurious inhibit; this process would tend to reject large amplitude, unpiled-up pulses which are vital to our spectrum analysis.

In practice we have used a safe setting of our fast shaped pulse which assures absence of spurious inhibits for 10 MeV pulses, resulting in a pileup resolving time of $\lesssim .2 \mu\text{s}$ for all pulses in the range 1–10 MeV. The signals indicating pileup rejection are displayed along with the scintillator pulses in order to indicate the presence of possible pileup during high count rate bursts.

APPENDIX C

RUNAWAY DIFFUSION MODELS

In this section we will first describe the diffusion model used in Sect. 5.3 to simulate the plasma shift experiment, and then we will discuss the more general problem of interpreting the flux of runaways to the limiter in terms of the possible runaway diffusion in the plasma. Some attempt will be made to understand the observed lack of freefall energy gain (Sects. 4.2 and 4.5) in terms of a runaway diffusion coefficient.

We recall that the shift experiment of Sect. 5.2 was designed so that the runaways were made to move across a known distance ΔS corresponding to the magnitude of an inward plasma shift. In Fig. 5.9 we have compared the observed runaway flux back to the outer limiter vs time with some theoretical curves representing the addition of various diffusion coefficients to the drift orbit model. We show here how those curves were obtained.

A simple diffusion model has been described by the LT-3 group for various circumstances appropriate for their probe measurements.⁽⁵⁷⁻⁵⁹⁾ Based on their idea of a runaway spatial diffusion coefficient D we have solved the 1-dimensional diffusion equation $\partial n / \partial t = D \frac{\partial^2 n}{\partial x^2} + V \frac{\partial n}{\partial x}$ appropriate for the boundary conditions of the shift experiment. We are interested in solving for the runaway particle flux vs. time appearing at the outer limiter after the inward plasma shift, so the experimental situation has been modelled by assuming:

(a) an initial condition in which the runaway spatial distribution is a step function shifted ΔS away from an absorbing boundary

(outer limiter) at which the runaway density is zero [see Fig. C.1(a)];

(b) the diffusion coefficient is independent of time, space, and runaway energy inside the plasma;

(c) no runaway creation occurs during the diffusion process;

(d) all runaways have a constant drift velocity toward the boundary corresponding to the orbit displacement process due to free-fall acceleration.

We note that the solution without the drift velocity assumption

(d) for $\Delta S = 0$ can be obtained analytically through a Fourier series:

$$n(x,t) = \sum_{m \geq 1} A_m(t) \sin\left(\frac{\pi m x}{L}\right)$$

where

(C.1)

$$A_m(t) = \frac{4}{\pi m} e^{-(\pi^2 m^2 D/L^2)t}$$

solves the diffusion equation for a periodic boundary at $\pm L$ in one dimension. The general form of the flux vs. time at the boundary shows a peak at $t = 0$ and a gradual depletion of the runaways over $\pm L$ in a time $t \approx D/L^2$. In order to include the outward movement, however, we were forced to solve the diffusion equation numerically.

In Fig. C.1 we show some results from the numerical calculation used to obtain the flux vs. time curves for Fig. 5.9. The 1-D diffusion equation was solved on a 150 x 150 space-time grid by a backward difference scheme⁽¹²¹⁾ in order to avoid numerical instability:

$$n(x,t+1) - n(x,t) = D[n(x+1,t+1) - 2n(x,t+1) + n(x-1,t+1)]$$

$$n(x,t+1) \rightarrow n(x-1,t+1)$$

For $D = 0$ the distribution moves with a constant velocity from slab 30 to slab 1 (absorbing boundary) producing a flux vs. time that looks like a step in function [Fig. C.1(a)]. The time gap which here is 30 time steps corresponds to the net effect of the drift orbit displacement and a small plasma movement during the time after the inward plasma shift, such that the 3.5 cm shift case of Fig. 5.9 has an expected time gap of 30 msec ($D = 0$).

For $D > 0$ some particles arrive at the boundary before 30 time steps, and obviously this effect increases with D . We note that the effect of the absorbing boundary is to increase the flux relative to the case without the boundary since the local gradient of runaway density is enhanced. Thus the time required for the flux of runaways to reach half its asymptotic value is gradually reduced for large D . This corresponds to the "closing" of the time gap as pictured in Fig. 5.8, and would not be present if the diffusion was perfectly symmetrical about the moving edge of the runaway distribution.

This model could evidently be made more realistic by the inclusion of the inward plasma motion, a spatially dependent diffusion coefficient, or a more realistic runaway density profile. For our purposes, however, it has been considered reasonable to extract from the data only an estimate of the runaway diffusion coefficient $D \approx 100 \text{ cm}^2/\text{sec}$, as described in Sect. 5.4.

In the rest of this appendix we discuss the problem outlined in Sect. 5.5 concerning the interpretation of limiter x-ray measurements in terms of a runaway diffusion coefficient in the plasma. This is relevant in an attempt to explain the observed lack of freefall

after a certain average runaway confinement time $\bar{\tau}_R$. We described there how the experimental information concerning the runaway energy evolution at the limiter was insufficient to define a diffusion coefficient without an independent knowledge of the radial profile of the runaway creation rate. Here we will show some computations of this effect.

One can construct a simple model of the experimental situation as follows. We assume a runaway creation rate $S(r,t)$ and a generalized transport process such that a runaway created at (r,t) is lost to the outer limiter with a probability $P(t'-t,r)$. This probability would be determined by a combination of orbit displacement and diffusive effects. Assuming a weighting of electrons $W(E)$ according to E^3 in the determination of maximum energy (see Appendix A), such that with freefall for individual electrons $E = \dot{E}(t'-t)$, we find an approximate expression for the "maximum" energy vs. time at the outer limiter:

$$\bar{E}(t') = \frac{\int_{r=0}^{r_L} \int_0^{t'} S(r,t) P(t'-t,r) E W(E) r^2 dr dt}{\int S P W r^2 dr dt} \quad (C.2)$$

where all of the physics related to runaway confinement is in the function $P(t'-t,r)$.

This expression shows explicitly that in order to determine from the limiter measurement the transport properties in P one needs to know independently where the runaways come from, or vice versa. We now show some examples of this transformation. Unfortunately we have not found a tractable analytic solution for the interesting choices of $S(x,t)$ and $P(t'-t,r)$, so we have used a simple computer model to display the qualitative features of the results. In this model we

specify $S(x,t)$ in 20 space and 50 time slabs (1-D model) and average over the number of electrons hitting the limiter at every time. We have factored the source function $S(x,t)$ into $S(x)T(t)$ for convenience.

In Fig. C.2(a) we show the result for a transport model which simulates the pure drift orbit limit, that is, $P(t'-t, x) = \delta(t-t'-x)$. Thus all electrons born at (x,t) will reach the limiter at the same time proportional to the distance x from the limiter. For a uniform space-time distribution $S(x) = 1$, $T(t) = 1$, we see that the energy vs. time increases at exactly the freefall rate until the electrons from $x = 20$ arrive at the limiter with the highest possible energy of $\bar{E} = 20$ units, but the steady state solution for $t \geq 20$ has $\bar{E} \approx 16$ due to the averaging over all energy components $(\sum_{1}^{20} E(E^3)) / \sum_{1}^{20} E^3 \approx 16)$. The steady state in this situation corresponds to the case (not observed in the experiment) in which the runaways born at the center of the discharge ($x = 20$ here) have gained enough energy to reach the limiter.

We also show the results for $T(t) = t$ and $T(t) = 1/t$. In the former case the average energy in steady state is a bit lower than with $T(t) = 1$, since lower energy components near $x = 0$ are contributing more heavily, and in the latter case the final energy is a bit higher due to the reduced production near $x = 0$ as time goes on.

In Fig. C.2(b) we show what happens for a spatial source function $S(x) = H(10)$, meaning that $S(x) = 1$ for $x < 10$, $S(x) = 0$ for $x > 10$. The pattern is similar to that for $S(x) = 1$ except that the steady state energies are a factor of two lower than for the case $S(x) = 1$ for $D \times 20$. One can imagine that other source functions produce

varying final energies, but that generally the energy vs. time is nearly freefall until steady state has been reached. This would seem to be characteristic of the pure drift orbit model.

On the other hand, in Fig. C.3 we show that a diffusive-type model given by $P(t'-t, s) = c(t'-t)^2 \exp [-K(t'-t)/x^2]$, where $\int_0^\infty P dt' = 1$, gives a gradually changing maximum energy vs. time until steady state has been reached. Again the final energy is dependent upon the choice of $S(x)$ and $T(t)$, but we can see that here the slope of the curve is also quite variable. In Fig. C.4 we show the effect of a varying diffusion rate on the average energy vs. time, showing the expected inverse relationship between diffusion rate and maximum energy.

Our conclusion from this simple model is that it is impossible to deduce solely from the measurement of average energy vs. time at the outer limiter the corresponding diffusion coefficient, since for a given diffusion the result is equally sensitive to the details of the runaway creation rate $S(r, t)$. In order to isolate $D(r, t)$ we would need at least an independent measure of $S(r, t)$, and even then the unfolding problem may not be well determined. A determination of $S(r, t)$ can in principle be possible from runaway production theory (see Fig. 1.4), but we have seen (Sect. 4.2) that in practice these rates are presently too uncertain and unreliable to be used as a basis for the analysis of runaway confinement.

APPENDIX D

RUNAWAY ORBITS WITH MAGNETIC ISLANDS

In this appendix we outline the method used to derive the drift orbits for runaways in the presence of $m = 2$ magnetic island perturbations in the plasma. The notation is that of B. Waddell⁽⁸⁹⁾ who has derived the perturbed fields appropriate to this problem. The method is to insert the perturbed magnetic fields due to the islands into a guiding center computer code, and the results have been described in Sect. 6.2 (Fig. 6.11).

The unperturbed tokamak magnetic field can be modeled by a current profile (in cylindrical geometry):

$$J_{z0} = \frac{-2[1 + (\hat{r}_s/\hat{r}_0)^2]}{1 + (\hat{r}/\hat{r}_0)^2} \quad (D.1)$$

where $\hat{r}$ is the minor radius, $\hat{r}_0$ is the effective radius of the current channel, and $\hat{r}_s$ is the radius of the $q = 2$ singular surface (all radii normalized to the limiter radius r_L). On top of this unperturbed current are toroidal current perturbations due to the $m = 2$ island which tend to flatten the overall current profile around r_s , as shown schematically in Fig. D.1. This flattening is due to the large parallel transport which averages plasma parameters along a flux surface.

Following Waddell, the magnetic fields can be defined in terms of a helical flux function ψ such that:

$$\left\{ \begin{array}{l} B_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \\ B_\theta = -\frac{\partial \psi}{\partial r} - \frac{kr}{m} B_z \\ B_z = \text{constant} \end{array} \right. \quad (\text{D.2})$$

where standard cylindrical coordinates (r, θ) are used, with $k = n/R$ corresponding to the periodic boundary condition used for simulating a torus of major radius R . Normalizing

$$\hat{J}_z = \frac{m\mu_0 J_z}{kB_z} \quad \text{and} \quad \hat{\psi} = \frac{\psi}{r_L^2 \frac{k}{m} B_z},$$

we have

$$\hat{J}_z = -\nabla_\perp^2 \hat{\psi} - 2 \quad (\text{D.3})$$

where

$$\nabla_\perp^2 \psi = \frac{1}{r} \frac{\partial}{\partial r} \left(\hat{r} \frac{\partial \hat{\psi}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \hat{\psi}}{\partial r^2}.$$

Now $\hat{\psi} = \hat{\psi}_0 + \hat{\psi}_1 \cos(m\theta + n\phi)$ represents the total flux due to the unperturbed current $\hat{J}_{z0}$ and a perturbed current $\hat{J}_{z1}$ due to $m = 2, n = 1$ island, such that any choice of $\hat{\psi}_1$ automatically satisfies $\nabla \cdot \hat{\mathbf{B}} = 0$ [see Eq. (D.1)]. The unperturbed flux corresponding to $\hat{J}_{z0}$ is:

$$\begin{aligned} \hat{\psi}_0(\hat{r}) &= c_2 \ln [1 + (\hat{r}/\hat{r}_0)^2] + \frac{1}{2} \hat{r}^2 + c_1 \\ c_2 &\equiv \frac{1}{2} \hat{r}_0^2 [1 + (\hat{r}_s/\hat{r}_0)^2] \\ c_1 &= -c_2 \ln [1 + \hat{r}_0^{-2}] + \frac{1}{2} \end{aligned} \quad (\text{D.4})$$

which is equivalent to a safety factor:

$$q(\hat{r}) = \frac{m}{n} \frac{[1 + (\hat{r}/\hat{r}_0)^2]}{[1 + (\hat{r}_s/\hat{r}_0)^2]}, \quad (D.5)$$

and the perturbed flux has been found from Waddell's resistive MHD computer code to be given nearly by:

$$\hat{\psi}_1 = C_3 \hat{r}^2 \exp\left(-\frac{\hat{r}^2}{w^2}\right) (1 - \hat{r}) \cos(2\theta + \phi) \quad (D.6)$$

Here w is the $m = 2$ island (in units of r_L), ϕ is the "toroidal" angle $\phi = z/2\pi R$, and C_3 is a constant related to w giving the magnitude of the perturbed flux

$$C_3 = \frac{w}{16f(\hat{r}_s, w)} \left(\frac{(\hat{r}_s/\hat{r}_0)^2}{1 + (\hat{r}_s/\hat{r}_0)^2} \right) \quad (D.7)$$

where $f(\hat{r}_s, w)$ is the right-hand side of Eq. (D.6) evaluated at $\hat{r} = \hat{r}_s$ (without the factor C_3). Using these forms for $\hat{\psi}_0$ and $\hat{\psi}_1$ we then calculate the magnetic fields needed in the guiding center code.

The total magnetic field components can be written:

$$\begin{aligned} B_x &= \frac{B_z}{2R} \left\{ \left[\frac{2r}{q(r)} + B_{\theta 1} \right] \left(\frac{-y}{r} \right) + B_{r1} \left(\frac{x}{r} \right) \right\} \\ B_y &= \frac{B_z}{2R} \left\{ \left[\frac{2r}{q(r)} + B_{\theta 1} \right] \left(\frac{x}{r} \right) + B_{r1} \left(\frac{y}{r} \right) \right\} \end{aligned} \quad (D.8)$$

where the terms with $1/q(r)$ represent the unperturbed flux given by Eq. (D.5), and the perturbed fields B_{r1} and $B_{\theta 1}$ are found through Eq. (D.2) and (D.6):

$$\begin{aligned}
B_{r1} &= \frac{1}{r} \frac{\partial \hat{\psi}_1}{\partial \theta} = C_3 (1 - \hat{r}) \hat{r} \exp(-\hat{r}^2/w^2) 2 \sin(2\theta + \phi) \\
B_{\theta 1} &= - \frac{\partial \hat{\psi}_1}{\partial r} = C_3 \left\{ \hat{r}^2 (1 - \hat{r}) \left(-2 \frac{\hat{r}}{w^2}\right) + 2\hat{r} - 3\hat{r}^2 \right\} \cdot \exp(-\hat{r}^2/w^2) \cdot \cos(2\theta + \phi)
\end{aligned} \tag{D.9}$$

The resulting poloidal flux function such as shown in Fig. 6.9 have the typical island structure indicating that the field lines have been significantly perturbed by this perturbed current on the rational surface $m/n = 2/1$. The orbits for runaways with a negligible outward displacement ($E \rightarrow 0$) follow the field lines and therefore map out these island structures in their drift surfaces. For $E > 0$ the drift surfaces are displaced outward and distorted, as described in Sect. 6.2.

We might mention here that the guiding center code [originally written by Dave Swain⁽¹²¹⁾] uses an extremely accurate differential equation solving routine ODE, so that many hundreds of toroidal transits can be calculated without significant error accumulation. It should also be noticed that our treatment of this problem has been simplified by assuming only a constant vertical drift of the guiding center; that is, we neglect the higher order drifts (e.g., $B_p \times \nabla B$) and the perturbation in the drift velocity due to the fields of the islands. We suppose these drifts to be of negligible importance, so that the effect of the island comes only through the tendency of runaways to follow the total $\vec{B}$ field, the drift velocity being given by the curvature drift in the unperturbed field.

APPENDIX E

EFFECT OF DRIFT WAVES ON RUNAWAY CONFINEMENT

It has been mentioned in Sects. 2.5 and 6.7 that drift waves are a possible mechanism for anomalous high energy runaway transport in tokamaks. These waves are caused by density and temperature gradients,⁽⁷⁰⁾ and have been observed in the ATC^(123,124) and TFR⁽¹²⁵⁾ tokamaks. They involve an electrostatic potential $e\phi/kT \lesssim .02$ associated with (mainly) poloidal wave vectors $k_{\perp} \rho_i \lesssim 1$ (i.e., $\lambda_{\perp} \sim .1-1$ cm) and frequencies ~ 100 kHz. Small magnetic perturbations accompanying these waves may also produce fluctuating magnetic island structures which themselves may affect runaway transport.⁽¹²⁶⁾

There are three distinct physical mechanisms by which these waves can interact runaways: 1) $E_p \times B_T$ radial drifts; 2) pitch angle scattering of runaway by the perturbed electric field; and 3) the magnetic "flutter" associated with these waves.

1) As proposed by Fussman,⁽⁵⁴⁾ runaways should drift radially when passing through the poloidal electric field. Using⁽¹²⁷⁾:

$$v_r \text{ (cm/sec)} = 10^8 \frac{E_p \text{ (V/cm)}}{B_T \text{ (gauss)}}$$

we find that a relativistic electron should have a radial displacement $\sim 10^{-5}$ cm over a voltage drop given by $e\phi \sim 50$ eV. If the runaway had a random radial step size per transit, the resulting diffusion coefficient would be a negligible $D \approx (10^{-5})^2 / 2 \times 10^{-9} \text{ sec} \lesssim 10^{-1} \text{ cm}^2/\text{sec}$. It is possible, however, that high energy runaways passing obliquely through the poloidal structure of these waves may see as many

as $r_L k_\perp / q \lesssim 200$ of these waves during one toroidal transit, resulting in a $D \lesssim 10 \text{ cm}^2/\text{sec}$. Since the wave frequency is much less than the runaway transit frequency, it is presently not clear to what extent the radial steps due to this drift can actually be considered to be uncorrelated.

2) The runaway gyrofrequency goes down with increasing electron energy, such that a 10 MeV runaway would execute a gyroorbit over a toroidal distance of $L = 2\pi c/\Omega \approx 10 \text{ cm}$ (@ $B_T = 20 \text{ kG}$, see also Sect. 2.5). If the runaway passes through an electric (or magnetic) field with a wavelength shorter than this, it is possible that a pitch angle scattering may occur. For example, if a runaway with gyrolength $L = c/\Omega/2\pi$ passes through a localized ($\ll L$) perpendicular electric field perturbation with a total voltage drop $e\phi$, then it is easy to show that the runaway pitch angle changes by an amount

$$\Delta\theta = \frac{\Delta p_\perp}{p_\perp} \sim \frac{e\phi}{\gamma m_0 c^2}$$

Thus a random walk with this angular step size can result in a slow pitch angle scattering, i.e., if $e\phi = 100 \text{ eV}$, $\Delta\theta \sim 10^{-4} \text{ rad}$ for a 1 MeV electron, so that the sum of $\approx 10^8$ such scatterings would be necessary to scatter $\approx 1 \text{ rad}$. This angle would be sufficient to form trapped runaway orbits. Also, if the perturbation wavelength was near the gyroorbit length, a resonant scattering may occur on a much more rapid timescale. More detailed measurements of the toroidal structure of drift waves would be needed in order to evaluate this effect on high energy runaways in tokamaks.

3) The magnetic flutter accompanying drift waves seems to be more likely to affect runaway confinement than these two electrostatic interactions, since runaways are strongly influenced by radial magnetic fields (see Sect. 6.2). Callen⁽¹²⁶⁾ has estimated a diffusion coefficient $D \sim 10^4 \text{ cm}^2/\text{sec}$ for low energy runaways (using contemporary tokamak parameters); thus this mechanism appears to be a strong candidate for explaining the anomalous runaway diffusion discussed in Chapter 5.

It is interesting to note that the effect of electrostatic waves on runaway confinement was considered by Artsimovich in 1962:⁽¹²⁸⁾

"At the same time, it must be considered improbable that there is any correlation between the x-rays and the development of those instabilities which may lead to anomalous diffusion but do not break down the structure of the self-magnetic field. Instabilities of this type, which figure very widely in recent theoretical research, will lead to the appearance of electric fields of varying size and direction in different sections of the plasma column. Because of their great speed, the accelerated electrons are not influenced by such fields. A fast electron, during its motion, integrates fluctuating electric fields over a very long distance in a very short time, and therefore it does not undergo any perceptible drift under the action of this field."

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Fig. 1.1 Basic Tokamak Apparatus- A toroidal plasma confined in a helical field created by the superposition of a strong, externally generated toroidal field and the poloidal field generated by the plasma current (figure from ref.3).

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Fig. 1.2 Ormak- Photo shows Ormak c. 1975 with its full complement of diagnostics and neutral beam injectors. The torus is located inside the large vacuum vessel. Top view shows the positioning of the major diagnostics including the hard x-ray detectors; side view shows the location of the toroidal, vertical, and Ohmic heating coils.

Fig. 1.2 (continued)

Fig. 1.3 Ormak Parameters- Typical Ormak parameters from ref. 48 (Lausanne Conf.). At the left is the time evolution of a 120 kA discharge. At top right is the scaling of temperature with discharge current, and at the lower right is the variation in current profile with I .

Fig. 1.4 Runaway Production Rates- Runaway production rates are a sensitive function of E/E_c and Z_{eff} . These figures (from ref.21) show calculated rates for various theories: $Z_{eff}=1$ (left) and $Z_{eff}>1$ (right)

Fig. 2.1 Runaway Orbits in a Tokamak- A runaway drift surface AB is shifted outward from the center of the current distribution by an amount d_γ . The current distribution itself may be shifted an amount d_p from the center of the chamber. At bottom are the drift components used to derive eq.2.2.2.

Fig. 2.2 Runaway Drift Orbits- Top: orbit trajectories projected onto a minor cross-section for relativistic electrons of energies starting at 1/2 Mev and going up in 1/2 Mev increments. Bottom: orbits for 3 and 5 Mev electrons with pitch angles of 0°, 20°, 40°, 60°, and 80° (toroidal current = 100 kA). From Spong's thesis (ref. 44).

Fig. 2.3 Outer Limiter Intersection Condition- The minor radius of a runaway at intersection with the outer limiter can be obtained from the measured value of I_A/I . If there is a plasma shift the radius must be corrected according to eq. 2.4.2. Dashed curve shows Spong's original calculation⁽⁴⁴⁾ for Type B⁽⁴⁴⁾; solid line shows new calculation with $B_p \propto 1/R$.

Fig. 3.1 Hard X-ray Detector Positioning- The tangential detectors can be aimed at the outer limiter on lines A (0°) or C (18°), or at the inner limiter on line B. An uncollimated detector is located at D.

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Fig. 3.2 Properties of Various Scintillators- Photopeak efficiencies and absorbtion fractions are shown for various sized NaI crystals and a germanium crystal. In all cases diameter=height, except for the .1 cm. NaI which is 1/2" diameter.

Fig. 3.3 NaI Scintillator Response Curves- The shape of the scintillator response function depends upon the incident x-ray energy. These normalized curves for a 5"x4" crystal were used in the construction of the scintillator response matrix (Fig.A.2). This figure is from ref. 108.

Fig. 3.4 Collimator Design- Apertures (1) and (3) define the acceptance angle of this collimator. The x-ray intensity is adjustable with a variable absorber (2). The NaI crystal (4) is shielded from backscattered x-rays by (5). All shaded areas are lead.

Fig. 3.5 Example of Pulse Pileup- The pileup effect is shown in this strong runaway discharge³⁰ in which sudden dumps of runaways appear to produce high energy pulses.

Fig. 3.6 Computed Pulse Spectra for Monoenergetic Electrons- Scintillation spectra for monoenergetic electrons are computed assuming normal incidence on tungsten, with attenuation due to 25 cm Al, 5 cm Cu, and 2.5" Pb, as seen by our 5"x5" detector. Insert (a) shows for this case the total number of scintillator counts vs. E ; insert (b) shows same for only 5 cm Al attenuation.

Fig. 3.7 Fit of Observed Spectra I - A typical scintillator pulse height spectrum matched with theoretical curves. The histogram represents data obtained during an interval summed over four similar discharges (90-95 msec in Fig. 4.10). The solid curve is the expected spectrum for 0° emission from 2×10^{11} 10 MeV electrons normally incident on tungsten. The broken curve includes an additional component of 4×10^{11} electrons at 5 MeV.

Fig. 3.8. Fit of Observed Spectra II - Computed monoenergetic electron pulse spectra for 5, 6, and 7 MeV electrons are shown (*) fit to hard x-ray data (solid line). The maximum runaway energy is determined to be $E = 6 \pm 1 \text{ MeV}$. A flat distribution function with a maximum energy of 6 MeV is shown at the lower right.

Fig. 3.9 Three Point Fit - Scintillator pulse height spectrum obtained from runaway dominated discharges.(30) The histogram is fitted by thick target bremsstrahlung produced by a group of 7, 5, and 3 MeV electrons with relative numbers of 1:3:16.

Fig. 3.10 Example of Smooth Envelope of Maximum Energy vs. Time- In some cases the maximum runaway energy can be found by forming the envelope of maximum scintillator pulse height. This is particularly useful in high density discharges such as this in which runaways are lost over ≤ 10 msec (from 90 kA case of Fig. 4.3).

Fig. 4.1 Discharge Regimes in ORMAK - Normal operation of ORMAK is limited at high densities by the disruptive instability and at low densities by the x-ray production due to a large runaway electron component (acceptable x-ray level ≈ 100 mr per shot at the tank wall). The points plotted near the runaway limit refer to the "knee" of the x-ray intensity vs. density curves such as shown in Fig. 4.5.

Fig. 4.2 X-ray Patterns for ORMAK Operation Regimes - The x-ray patterns produced in various operation regimes (see Table 4.1) can be easily monitored using an uncollimated NaI detector located about 10 m from the machine, as shown in Fig. 3.4. Such patterns measure the x-ray intensity due to runaways above about .5 MeV in energy (intensity $\propto n E^2$). Both the high and low density patterns can often be seen in the same discharge. Note that the runaway pattern is from a discharge in which the voltage pulse was limited to 10 msec duration.⁽³⁰⁾

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Fig. 4.3 Maximum Runaway Energy vs. Time for High Density Discharges - Runaways lost during the first 10 msec in low density discharges gain energy in a manner consistent with freefall acceleration from near $t = 0$. The two curves are calculated on the basis of freefall acceleration in the applied loop voltage, including an inductive correction with $L = .5 \mu\text{H}$. The evolution of the plasma current and shift are shown for the 90 kA case.

Fig. 4.4 Maximum Runaway Energy vs. Time for Low Density Discharges -
 The maximum runaway is observed to increase at nearly the freefall rate during the steady state of a low density discharge. The experimental points can be extrapolated backward to predict a birthtime of about 10 msec for this group of runaways. The intensity curve gives the relative number of maximum energy runaways lost vs. time, calculated through the pulse rate using energy correction factors such as those shown in Fig. 3.6.

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Fig. 4.5 Number of Runaways vs. Density for Low Density Discharges -
 The relative number of maximum energy runaways increases sharply with decreasing density in this regime. Here the maximum energy of runaways lost during the current decay was roughly constant for different densities, so that the relative runaway population is shown to be proportional to the number of pulses counted by the tangential detector. The three symbols refer to three different experimental runs at $I = 100$ kA, and the curve shows the runaway production scaling⁽²¹⁾ with density according to Fig. 1.2, assuming the temperature and $\langle z \rangle$ to be constant. The runaway current is calculated on the basis of the absolute bremsstrahlung intensity for the 10 MeV component.

Fig. 4.6 Number of Runaways vs. Density for High Density Discharges -
 The number of counts per shot recorded by the tangential detector increases with density for the high density regime. This is most likely due to a small systematic increase in E/E_c , but due to the extreme sensitivity of runaway production to variations of this parameter (see Fig. 1.2), and due to the uncertainties in the experimental parameters at early times, a quantitative interpretation of this behavior is not possible.

Fig. 4.7 Runaway Rates for ORMAK - Runaway rates for a pure hydrogen plasma calculated vs. time and radius from measured density and temperature profiles in ORMAK. The electric field is calculated from the loop voltage, using $L = 0.5 \mu\text{H}$ for the outside ($r \approx 15 \text{ cm}$) and $L = 0.9 \mu\text{H}$ for the inside ($r \approx 0$). Note that the steady state rate is more commonly higher on axis due to a peaked temperature profile.

Fig. 4.8 Maximum Runaway Energy vs. Time During the Steady State of Low Density Discharges - The maximum runaway energy is usually observed to increase at nearly the freefall rate during the steady state phase of low density discharges. The pulse spectra used to show this for curve (a) are given in Fig. 4.8. In curve (b), however, only 60% freefall is observed. The relative number of runaways lost vs. time for the components of curve (a) are shown at the bottom of the figure. Data was accumulated over ≥ 5 discharges in each case.

Fig. 4.9 Pulse Spectra During Steady State Runaway Loss - The scintillator pulse spectra shown here have been used to find the maximum runaway energies labelled A, B, C in Fig. 4.8.

Fig. 4.10 Maximum Runaway Energy vs. Time During the Current Decay in Low Density Discharges - The maximum runaway energy during the current decay (after the loop voltage ends) first shows the loss of a 10 MeV component, but below about 50 kA the maximum confined energy falls in time in a manner similar to the drift orbit model prediction (dashed curve $E(\text{MeV}) = .2 I(\text{kA})$). There is a particularly large component of runaways below about 5 MeV in this high current, high temperature discharge, as shown at the bottom of the figure. Data was accumulated over four discharges such as the one shown in the photo.

Fig. 4.11 Location of Runaway Impact in ORMAK - A strong x-ray source is observed at the outer limiter during a horizontal scan of the limiter region. The number of counts recorded by the tangential detector during this series of identical discharges has been corrected for the varying material attenuation in the detector line. Melting due to runaway impact has also been observed in the equatorial plane at the outer edge of the limiter.

Fig. 4.12 Comparison of Runaway Loss Patterns for High and Low Current Discharges - Runaways appear to be better confined in high current discharges, in qualitative agreement with the drift orbit model prediction $d\gamma \propto 1/I$. Here it can be seen that most of the runaways are confined until the current falls below about 50 kA in both discharges. The x-ray signal here comes from an uncollimated integrating detector.

Fig. 4.13 Plasma Position at Early Times - There is generally a systematic difference in the plasma position between high and low density discharges. The increased inward shift for low density discharges can be used to explain the enhanced confinement of high energy runaways early in the discharge.

Fig. 4.14 Plasma Shift vs. Vertical Field Timing - The plasma position early times can be varied by varying the timing of the vertical field relative to the ohmic heating field. The plasma position is calculated using the outputs of B_p pickup loops on the inside and outside of the torus. The absolute values of the positions are uncertain due to the lack of equilibrium at this stage of the discharge.

Fig. 4.15 Patterns of X-ray Intensity with Varying Vertical Field Timings - The x-ray intensity as monitored by the uncollimated detector is seen to increase strongly for inward-shifted discharges. The intensity in the bottom figure is off-scale in this drawing. The oscillations seen in the top trace are discussed in Sect. 6.2, and the microwave emission seen in the bottom trace is mentioned in Sect. 6.4.

Fig. 4.16 X-ray Intensity vs. Vertical Field Timing - The x-ray emission integrated over the first 10 msec of the discharge increases with an increased shift. This behavior is consistent with the improved runaway confinement expected with an inward shift, but the runaway production rates may also vary in a way which contributes to the observed effect.

Fig. 4.17 Measurement of Low Energy X-rays Using a 90° Detector - A small NaI detector was mounted so as to view low energy x-rays scattered from the inner wall of the machine. The pattern of pulses shown at the bottom of the figure indicates that there is a group of low energy runaways lost at very early times in low density discharges, even when these electrons are not detectable in the tangential detector (middle). There also appears to be a distinct group born at about 10 msec, which cannot be seen in the tangential detector until they have gained about 4 MeV.

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Fig. 4.18 Runaway Energy vs. Magnetic Flux Swing for Capacitor Discharges - The maximum runaway energy was observed to increase as the magnetic flux swing was increased in these short discharges. The population of runaways near maximum energy was seen to decrease with increasing voltage V_c , but the other parameters involved in runaway production (i.e., average density, as shown) also varied with V_c . There was no systematic effect due to the magnitude of the preionization pulse on the observed x-ray intensity. The energy points for this plot were based on single shots.

Fig. 4.19 Maximum Runaway Energy vs. Battery Duration - The maximum runaway energy (as measured in the current decay) increased with longer battery pulses, but not as fast as would be expected for a freefall acceleration of a fixed group of runaways. The plasma density was systematically increased with pulse length in order to keep the x-ray pulse rate at manageable levels.

Fig. 4.20 Pulse Spectra for Battery Duration Experiment - These pulse spectra were used to determine the maximum runaway energies shown in Fig. 4.18. Only the pulses above about 5 MeV were used in this case to find the maximum energy.

Fig. 4.21 Variation of X-ray Intensity with Plasma Density and Pulse Length - The x-ray intensity increases as the pulse length increases, partly because of the additional runaway energy gain (see Fig. 3.6) and partly because new runaways may be created. This behavior may be used to explain the pattern of runaway energy vs. battery duration observed in Fig. 4.18.

Fig. 5.1 Point Limiter Experiment - The graph shows the relative x-ray signal at the normal (outer) limiter for various insertions of the 1 cm diameter point limiter. This result indicates that runaways can be transported radially up to about 2 cm past the tip of the point limiter. The x-ray signal plotted is proportional to the total number of counts observed by the collimated tangential detector, most of which occur during the current decay phase of the discharge. The inset shows the damage to the point limiter due to runaway bombardment.

Fig. 5.2 Basic Concept of the Plasma Shift Experiment - Actual behavior of the plasma position, loop voltage, and current are shown for a 3.5 cm inward plasma shift. If there were no diffusion of runaways, we expect that the x-ray flux should drop to zero at the beginning of the shift, and return to its former value after a time T_{gap} determined by the runaway energy gain and actual plasma shift vs. time. This is shown schematically at the bottom of the figure.

Fig. 5.3 Effect of Plasma Shift on Scintillator Pulse Patterns - Photos show typical patterns of x-rays observed at the outer limiter with no inward shift (top), 1.7 cm inward shift (middle), and 3.5 cm inward shift (bottom). Arrows indicate gaps expected on the basis of diffusionless drift orbit theory. The existence of pulses within these expected gaps indicates the presence of an anomalous runaway diffusion. The 0° detector viewed an area 8 cm in this experiment.

Fig. 5.4 Scintillator Pulse Spectra With and Without a Plasma Shift - The pulse spectra at two times after a 3.5 cm inward shift are nearly identical to those obtained at the same times without a plasma shift. This allows the relative scintillator countrates to be used as a measure of relative runaway flux to the limiter. The enhancement in the pulse spectra for the shifted discharges below about 2 MeV may indicate that the low energy runaways diffuse faster than the maximum energy components. Arrows indicate maximum energy.

Fig. 5.5 Runaway Energy Gain During the Plasma Shift Experiment - The maximum runaway energy vs. time deduced from the upper end of the scintillator spectra is seen to increase at nearly the freefall rate during these experiments, whether or not the plasma is shifted inward. The freefall slope shown is calculated for a constant loop voltage of 1.9 V (as occurs without a plasma shift); the expected energy gain with a shift is about 10% larger over 66-96 msec due to the variation in the loop voltage (see Figs. 5.2 and 5.8). Bottom shows pulse spectra for 3.5 cm shift case.

Fig. 5.6 Countrate With and Without a Plasma Shift - The scintillator countrate vs. time without a plasma shift shows a reproducible structure at $t = 76$ msec. The plasma shift produces a drop in the x-ray flux which occurs well before the occurrence of this structure. An uncollimated detector of relatively soft x-rays (bottom) shows that the structure occurs both with and without a shift, motivating the normalization process. The uncertainties shown in countrate data are shot-to-shot deviations for 4 shots (unshifted) and 19 shots (3.5 cm shift).

Fig. 5.7 Normalized Flux vs. Time Patterns - Various trials of the shift experiment (see Table 5.2) resulted in these normalized flux vs. time patterns. The x-ray gap can be clearly seen for the larger shifts only, the smaller shifts being more strongly influenced by diffusion. The experimental values for T_{gap} are shown by the arrows.

Fig. 5.7 (continued)

Fig. 5.8 Calculation of T_{gap} from Drift Orbit Model - This example shows how the actual plasma position vs. time and the measured loop voltage are used to calculate the T_{gap} expected from drift orbit theory. The runaway energy gain in MeV is calculated through the freefall assumption [Eq. (2.1.2)], and T_{gap} is defined in this case by $\Delta s \approx 1.1 \Delta E$ [see Eq. 5.4.1 and 5.4.2]. The uncertainty in this value is on the order of ± 2 msec for an uncertainty in Δs of $\pm .2$ cm.

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Fig. 5.9 Comparison of T_{gap} From Theory and Experiment - The experimental gaps obtained from Fig. 5.7 can be seen to agree fairly well with those predicted on the basis of the drift orbit model. The solid line shows a rough scaling of the theoretical predictions (see Table 5.2), and the dashed curves show the effect of a diffusive transport tending to close the gaps. We conclude that the experiment is consistent with the drift orbit model supplemented by a diffusion on the order of $10^2 \text{ cm}^2/\text{sec}$.

Fig. 5.10 Estimation of the Diffusion Coefficient From the Flux vs. Time Pattern - The simplified diffusion model described in the text has been used to determine an order-of-magnitude value $D = 10^2 \text{ cm}^2/\text{sec}$ for these trials of the plasma shift experiment. A diffusion coefficient of order $10 \text{ cm}^2/\text{sec}$ would have produced a much sharper x-ray gap than that observed, while a $D = 10^3 \text{ cm}^2/\text{sec}$ would have resulted in the near disappearance of the gap. The actual plasma position vs. time is more closely modelled by including a constant inward plasma velocity over the first 10 msec (see Fig. 5.2); this results in an increased flux during this time, as shown for the $300 \text{ cm}^2/\text{sec}$ case for the 3.5 cm shift. Note that the experimental T_{gap} has been used in the modelling of the diffusive process.

Fig. 5.10 (continued)

Fig. 5.11 Example of Enhanced Diffusion Associated with MHD Activity -
 In this discharge the inward shift (arrow) apparently triggered an increase in the MHD activity, as can be seen in the soft x-ray sensitive PIN diode signals (14) ($m = 1$ sawtoothing) and in the external B_D pickup loop signal ($m \geq 2$). The normalized x-ray flux pattern in this case showed an increase in runaway flux to the limiter, especially during the first 10 msec after the shift when the $m \geq 2$ activity peaks. The enhanced MHD level most likely results in an increase in the runaway diffusion rate.

Fig. 6.1 Disruptive Instability- Current, voltage, plasma density and hard x-ray traces are shown for a discharge in which there are four disruptive instabilities (at 22,32,47, and 60 msec). Time resolution for these traces ~ 0.5 msec.

Fig. 6.2 Predisruptive MHD Activity- In these discharges a correlation can be seen between MHD activity as monitored by a $\dot{B}_p$ pickup loop and runaway loss as monitored by an uncollimated hard x-ray detector. The large x-ray dumps at the end of this correlation are coincident with the disruptive instability.

Fig. 6.3 Disruptions During Current Rise - Negative voltage spikes similar to those seen in Fig. 6.1 can occur during the runaway loss at the beginning of high density discharges. Hard x-ray dumps are correlated with these spikes. Integral $q(a)$ values are seen to be nearly correlated with the spikes, indicating the presence of high m -mode MHD activity.

Fig. 6.4 Typical MHD Behavior in Steady State- The MHD instability level is generally higher in high density discharges, as shown in these traces of $\dot{B}_\theta$ and PIN diode signals (courtesy of J. Dunlap).

Fig. 6.5 MHD and Hard X-ray Correlation- Top photo shows correlated MHD loop signal and hard x-ray emission. Note that the asymmetric signal is common to both traces. Bottom shows correlation between these signals and the 6 cm PIN diode signal (courtesy of J. Dunlap).

Fig. 6.6 Strong Runaway Loss During MHD Activity- In this shot a sudden increase in the MHD level apparently caused the loss of all runaways from the discharge within 10 msec. The bottom photo shows that this runaway loss is in phase with the $\dot{B}_p$ loop signal.

Fig. 6.7 Magnetic Island Structures- The location of the $q=2$ surface and the width of the island determine the flux surfaces in this model. Low energy runaways follow these flux surfaces (each dot is the intersection of a magnetic field line with a fixed toroidal cut).

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Fig. 6.8 Runaway Drift Surfaces in an $m=2$ Island- High energy runaway drift surfaces are shifted outward from the magnetic flux surfaces, but they retain an island structure for $d_\gamma \approx .5r_L$ in this case (Fig.6.7 top).

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Fig. 6.9 Effect of Orbit Displacement on Runaway Drift Surfaces in Magnetic Islands- The width of the largest island in the runaway drift surface W_R is plotted against the orbit displacement ($\propto$ runaway energy). Case (a) corresponds to Fig. 6.8, case (b) to the island at the bottom of Fig. 6.7. The distortion of the drift surfaces especially for $d\gamma \approx r_s$ is not well shown by this plot.

HARD X-RAY

CURRENT

PINS

+10 cm

+12 cm

+14 cm

HARD X-RAY

+14 cm

HARD X-RAY

Fig. 6.10 Effect of Internal Disruption on Runaway Loss- In rare cases a correlation was observed between runaway loss and internal disruptions. Case (a) was for a discharge in which $q(a)=2.5$; case (b) was during strong neutral beam heating.

Fig. 6.11 "RF Spike Instability- At time A before the end of this low density, low current discharge the RF began to show spikes. Note that the timescales are slightly different for these three scope traces (RF and microwave signals courtesy of D. Swain and J. Wilgen, respectively).

Fig. 6.12 Details of RF Spikes- The RF spikes have a width less than $30\mu\text{sec}$ and are often regularly spaced. A correlation with hard x-ray flux was sometimes (but not always) observed.

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Fig. 6.13 Reversal of Microwave Emission- The 6 mm microwave emission increased at the onset of the instability for $B_T = 14.6 \text{ kG}$, but decreased at $B_T = 17.9 \text{ kG}$.

Fig. 6.14 Effect of Gas Puff on Hard X-ray Pattern- A sudden increase in plasma density due to a controlled injection of cold gas resulted in a strong disturbance of the runaway loss pattern. The x-ray spikes are correlated with disruptive-type MHD activity during this discharge.

Fig. 6.15 Hard X-ray Flux vs. Puff Strength - Larger density increases associated with larger puffs caused a progressively stronger reduction in the hard x-ray flux at the end of the discharge.

Fig. A.1 Calculated Thick Target Bremsstrahlung Spectra - Points show calculation for 0° bremsstrahlung on an "optimal thickness" tungsten from Ref. 101. Solid lines were obtained from an interpolation formula in that reference (see also Ref. 81).

Scintillator Response Matrix																											
Photon Energy (MeV)	Pulse Height (MeV)																										Total Eff.
	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	
1.0	184	655																									0.932
1.5	128	144	539	5																							0.880
2.0	94	110	171	429	6																						0.856
2.5	36	71	94	172	385	8																					0.838
3.0	57	57	60	89	178	341	12																				0.824
3.5	49	49	49	54	89	180	310	15																			0.819
4.0	42	42	42	43	51	90	180	280	18																		0.810
4.5	39	39	39	39	40	47	85	179	255	21																	0.803
5.0	37	37	37	37	37	37	44	92	172	229	24																0.800
5.5	35	35	35	35	35	35	35	43	92	165	207	28															0.800
6.0	34	34	34	34	34	34	34	34	43	94	158	188	32														0.800
6.5	32	32	32	32	32	32	32	32	33	45	96	151	170	35													0.800
7.0	30	30	30	30	30	30	30	30	30	35	48	100	144	154	37												0.800
7.5	28	28	28	28	28	28	28	28	28	28	37	51	101	136	139	38											0.800
8.0	26	26	26	26	26	26	26	26	26	26	28	38	55	100	133	130	39										0.802
8.5	25	25	25	25	25	25	25	25	25	25	25	28	40	58	99	129	122	40									0.803
9.0	24	24	24	24	24	24	24	24	24	24	24	24	28	40	60	98	127	116	40								0.807
9.5	23	23	23	23	23	23	23	23	23	23	23	23	23	28	42	62	98	125	111	41							0.814
10.0	22	22	22	22	22	22	22	22	22	22	22	22	22	22	28	43	62	95	121	103	41	11					0.820
10.5	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	29	42	62	92	117	98	41	12				0.824
11.0	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	29	41	62	90	114	93	40	13			0.828
11.5	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	29	42	62	88	110	89	39	14		0.831
12.0	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	29	42	61	86	107	85	39	15	0.835

Fig. A.2 Scintillation Response Matrix - This matrix was calculated based on the experimental NaI response functions from Ref. 108. A correction to the total absorption fraction was applied to correct their 5" x 4" data for our 5" x 5" crystal (see also Ref. 81).

Fig. A.3. Off-Axis Bremsstrahlung - The ETRAN code was run to check the 0° bremsstrahlung spectra calculated in Fig. A.1. The shape of the A.1 spectra is in good agreement with ETRAN for $\theta \approx 0^\circ$ -10° for this case of 10 MeV electrons on a .6 cm tungsten slab. The data from ref. 101 was for a .3 cm W target.

Fig. A.4 Forward Peaking of 0° Bremsstrahlung - The photon spectra for 10 MeV electrons normally incident on a .6 cm tungsten slab (from ETRAN) show peaking $\sim 1/\gamma$ in the forward direction. The high energy photons are somewhat more peaked than the low energy photon.

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Fig. A.5 Non-Normal Incidence Spectra - Non-normal incidence of 10 MeV electrons on a .6 cm tungsten (from ETRAN) slab produces photon spectra peaked in the incident electron direction.

Fig. A.6 Photo of ORMAK Limiter - The original ORMAK tungsten limiter shows melting at the outer edge in the equatorial plane, probably sustained during strong runaway discharges. (This limiter was removed prior to 1974; the present limiter sustained similar damage.) We have assumed that x rays are attenuated by two of these 1/8" plates.

ANGULAR DISTRIBUTIONS OF REFLECTED ELECTRONS
(NUMBER/SR, NORMALIZED TO ONE INCIDENT PARTICLE)

THETA (DEG) Z/R= 1.8619			THETA (DEG) Z/R= 1.8619	
0.0	- 15.0000	1.87E-02	0-5	9.20E-2
15.0000	- 30.0000	3.03E-02	5-10	8.39E-2
30.0000	- 45.0000	2.00E-02	10-15	8.60E-2
45.0000	- 60.0000	2.07E-02	15-20	7.89E-2
60.0000	- 75.0000	7.92E-03	20-25	6.67E-2
75.0000	- 90.0000	8.61E-03	25-30	7.19E-2
			30-35	7.06E-2
			35-40	6.17E-2
			40-45	6.86E-2
			45-50	6.09E-2
			50-55	7.31E-2
			55-60	6.10E-2
			60-65	5.06E-2
			65-70	4.30E-2
			70-75	4.06E-2
			75-80	3.33E-2
			80-85	2.39E-2
			85-90	2.05E-2

ENERGY SPECTRA OF REFLECTED ELECTRONS
(NUMBER/MEV, NORMALIZED TO ONE INCIDENT PARTICLE)

10.0000	- 9.5000	0.0	10.0-9.5	0
9.5000	- 9.0000	0.0	9.5-9.0	2.32E-2
9.0000	- 8.5000	4.00E-03	9.0-8.5	4.04E-2
8.5000	- 8.0000	2.00E-03	8.5-8.0	4.40E-2
8.0000	- 7.5000	0.0	8.0-7.5	4.24E-2
7.5000	- 7.0000	0.0	7.5-7.0	4.52E-2
7.0000	- 6.5000	1.20E-02	7.0-6.5	3.68E-2
6.5000	- 6.0000	1.40E-02	6.5-6.0	4.00E-2
6.0000	- 5.5000	1.20E-02	6.0-5.5	4.08E-2
5.5000	- 5.0000	1.60E-02	5.5-5.0	4.28E-2
5.0000	- 4.5000	8.00E-03	5.0-4.5	4.60E-2
4.5000	- 4.0000	1.60E-02	4.5-4.0	3.64E-2
4.0000	- 3.5000	1.20E-02	4.0-3.5	4.36E-2
3.5000	- 3.0000	1.80E-02	3.5-3.0	3.40E-2
3.0000	- 2.5000	1.40E-02	3.0-2.5	3.20E-2
2.5000	- 2.0000	2.40E-02	2.5-2.0	3.52E-2
2.0000	- 1.5000	2.20E-02	2.0-1.5	3.12E-2
1.5000	- 1.0000	1.80E-02	1.5-1.0	3.52E-2

Fig. A.7 Backscattered Electron Distribution- About 20% of normally incident 10 Mev electrons on tungsten are backscattered (left), while a larger number are reflected when the angle of incidence is 30° to the plane of the target (right). Results from ETRAN.

Fig. A.8 Thin Target Bremsstrahlung Spectra - Spectra from Ref. 111 on hydrogen. At a viewing angle of 90° to electron direction, mostly low energy electrons are seen (≈ 1 MeV), whereas at 0° mostly the high energy electrons are seen due to forward peaking (Fig. A.4). (r_0 is the classical electron radius)

Fig. A.8 (continued)

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Fig. B.2 Pulse Pileup and Pileup Rejection - Top shows weak pileup effect for two well separated pulses (tail pileup); bottom shows strong pileup due to two nearly coincident pulses (peak pileup). The fast shaped 454 pulses must recross the 436 discriminator level in order to trigger pileup rejection.

Fig. B.3 Noise on Scintillator Pulses - Fluctuations in the 454 signals increase with reduced shaping constant (top), probably due to the statistics of photocathode current. These fluctuation levels limit the width of the 454 pulse to ≈ 100 ns (bottom).

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Fig. B.4 Spurious Inhibits - The limit to pileup rejection is given by the point at which the fluctuations on the tail of the first pulse equal the magnitude of the second pulse (top). In a test of the pileup rejection circuit with G^{137} , it was observed that the lower the 454 shaping constants, the more often the system triggered on fluctuations.

Fig. B.5 Example of Pileup Rejection - This test of pileup rejection at $\sim 5 \times 10^4 \text{ sec}^{-1}$ shows the effect of (a) no rejection, (b) 100 ns 454 shaping, (c) 50 ns, and (d) 20 ns. The double sum peak at 1324 KeV is due to two nearly coincident photopeak pulses.

Fig. C.1 Examples of Runaway Diffusion Model- Patterns of runaway spacial density and flux to the absorbing boundary (slab 0) for (a) $D=0$, (b) $D=1$, and (c) $D=10$, where D is in units of slabs and time steps.

Fig. C.1(b)

Fig. C.1(c)

Fig. C.2 Orbit Displacement Simulation Model - Calculation of $\langle E^3 \rangle$ for runaways transported to limiter according to an orbit displacement model (constant outward particle velocity): (a) Initial spatial distribution of $S(x)$ constant $0 < x < 20$ slabs. (b) Initial spatial distribution constant $0 < x < 10$, 0 for $10 < x < 20$ slabs. Runaway production rate $T(t)$ at all slabs is constant (left), or varying in time (right).

Fig. C.3 Diffusive Simulation Model - Same as C.3 but for diffusive transport model. The "maximum" energy vs time is seen to depend upon the functions $S(x)$ and $T(t)$ for a given diffusion coefficient.

Fig. C.4 Varying Diffusion Coefficient in Diffusive Model - The "maximum" energy vs time and the maximum energy for $t \rightarrow \infty$ vary as expected with D. Higher D results in a short confinement time and a lower maximum energy.

Fig. D.1 Current Perturbation for $m=2$ Island - The effect of an $m=2$ island is to flatten the current distribution near the $q=2$ surface.

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