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GEOMETRIC GHOSTS AND UNITARITY

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GEOMETRIC GHOSTS AND UNITARITY

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ABSTRACT

We review the geometrical identification of the renormalization ghosts and the resulting derivation of Unitarity equations (BRST) for various gauges: Yang-Mills, Kalb-Ramond and Soft-Group-Manifold.

THE CONVENTIONAL TREATMENT

The (geometric) Yang-Mills Lagrangian $L_{INV} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a$ has no inverse Fourier-transform needed to write a propagator. It thus had to be supplemented by a gauge-fixing term $L_{FIX} = -\frac{1}{2} C^a C_a$ (with $C^a = \partial_\mu A_\mu^a$ for example) or more formally, the linear $L_{FIX} = -\sigma_a \Sigma^a$, with σ_a a (boson) scalar Lagrange multiplier, ensuring the appearance of $\delta\sigma(\Sigma)$ in the functional integration. To ensure Unitarity (i.e. cancellation of contributions due to redundant components of A or its F) Feynman introduced ghosts¹. The ghost Lagrangian $L_{GH} = \bar{X} \hat{m} X^a$ involves scalar ghost X^a and antighost $\bar{X}_a$ fields, $\hat{m}$ is given by the gauge variations of Σ^a (or C^a).

$$\delta A_\mu^a = D_\mu \epsilon \rightarrow \delta \Sigma^a = \hat{m} \epsilon^a \quad (1)$$

X^a and $\bar{X}_a$ have to be assigned Fermi statistics, to produce a minus sign guaranteeing cancellations between their closed loops and the redundant contributions. The quantum Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a - \sigma_a \Sigma^a + \bar{X}_a \hat{m} X^a \quad (2)$$

obeys Slavnov-Taylor invariance and provides generalized Ward-Takahashi identities. 't Hooft and Veltman then completed the proof of renormalizability, adding regularization. The conditions for unitarity have since been reduced to invariance under BRST (discrete) supertransformations². Let Λ be a (constant) odd element in a Grassmann manifold, and take (δ is the gauge infinitesimal variation of (1)),

$$\epsilon^a = X^a \Lambda, \Lambda^2 = 0, \{\Lambda, X\} = \{\Lambda, \bar{X}\} = 0; s = -\frac{\partial}{\partial \Lambda} \delta \quad (3)$$

$$\therefore s A_\mu^a = D_\mu X^a; s X^a = \frac{1}{2} f_{bc}^a X^b X^c = \frac{1}{2} [X, X]^a; s \Sigma^a = \hat{m} X^a \quad (4)$$

$$\text{Add the formal prescriptions } s \bar{X}_a = \sigma_a, s \sigma_a = 0 \quad (5)$$

$\therefore s L_{INV} = 0; s L_{FIX} = -\sigma_a \hat{m} X^a; s L_{GH} = \sigma_a \hat{m} X^a - \bar{X}_a s^2 \Sigma^a$; so that provided $s^2 A_\mu^a = 0$ (to make $s^2 \Sigma^a$ vanish), which also implies $s^2 X^a = 0$ (using Jacobi's identity or algebraic closure of the δ variations), one has $s \bar{X}_a = 0$. Note that in the more conventional quadratic $L_{FIX}, s C^a = \hat{m} X^a, s \bar{X}_a = C_a$, so that $s^2 \bar{X}_a = \hat{m} X^a \neq 0$. The transformation s , with s^2 vanishing on A_μ^a, X^a and (in the linear treatment only) on $\bar{X}_a$, has the features of an exterior differential.

THE GEOMETRICAL IDENTIFICATION

The geometrical identification of the ghosts, as vertical components of the connection P , a principal fiber bundle (P, M, G_0) has been recently demonstrated³. Using x^μ as co-ordinates on the base manifold M and $\omega^i (i = 1..n)$ on the group fiber G , a section I may be

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given by $\omega^1 = 0$, lifting the x^μ onto it. The dot ($\bullet$) represents right-multiplication $P \times G \rightarrow P$ so that $(p.g) \cdot g^{-1} = p \cdot (gg^{-1})$, $\forall p \in P$, $\forall g, g^{-1} \in G$. To produce that action, the dot maps the n -dimensional Lie algebra A_G of G onto the differential operators of P_* , the tangent manifold. The (quasi) inverse map (since P_* has $4 + n$ dimensions) is given (through contraction $dz^M \cdot \frac{\partial}{\partial z^N} \approx \delta^M_N$) by the connection ω^a . Let $z^M(x^\mu, \alpha^i)$ represent local co-ordinates on P . On Σ ,

$$\omega^a = \Xi^a_i(\alpha, x) d\alpha^i + A^a_\mu(\alpha, x) dx^\mu \quad (6)$$

We remind the reader that although he may be used to A^a_μ being only x^μ dependent, any gauge transformation will introduce α^i dependence, through the homogeneous term $e^{i\alpha^1 \lambda^1} i A^a_\mu(x) e^{-i\alpha^1 \lambda^1}$ (λ is a basis A_G). For Σ a global (trivial) horizontal section, $\Xi^a = \Xi^a_i d\alpha^i$ is the left-invariant Cartan 1-form. More generally, $\Xi^a = X^a$, the ghost field. To prove it, one writes the Cartan-Maurer structural equation on P .

$$F^a = \frac{1}{2} F^a_{MN} dz^M dz^N = \left(\frac{\partial}{\partial z^M} \omega^a_N - \frac{\partial}{\partial z^N} \omega^a_M + \frac{1}{2} f^a_{bc} \omega^b_M \omega^c_N \right) dz^M dz^N \quad (7)$$

has nothing but horizontal components

$F^a = \frac{1}{2} F^a_{\mu\nu} dx^\mu dx^\nu$. Denoting by d the full differential over z^M (more generally the exterior derivative, i.e. a curl when acting on a one-form, such as X^a) and by ℓ and $\bar{d}$ the α^i and x^μ components respectively, we have

$$\ell \delta = d\alpha^i \frac{\partial}{\partial \alpha^i} \delta, \quad d\delta = dx^\mu \frac{\partial}{\partial x^\mu} \delta, \quad \bar{d}\delta = dz^M \frac{\partial}{\partial z^M} \delta = \ell \delta + d\delta \quad (8)$$

$$d\alpha^i F^a_{i\mu} = 0 = \ell A^a_\mu - D_\mu X^a, \quad d\alpha^i [i d\alpha^j] \frac{1}{2} F^a_{ij} = 0 = \ell X^a - \frac{1}{2} [X, X]^a$$

i.e. the first two BRST equations (4), provided

$$\Xi^a \equiv X^a, \quad \ell \equiv s \quad (9)$$

Indeed, $\bar{d}^2 = d^2 = s^2 = sd + ds = 0$ from cohomology.

We now have to verify whether indeed the fermi statistics of X^a derive indeed from its being a 1-form: in non exterior-calculus words, a differential, anticommuting because of the antisymmetrization of differentials in a measure. Perturbation physics is described by the generating functional (G is the group volume)

$$e^{i\Gamma} = \int_G \mathcal{D}(\phi) e^{iS_{\text{INV}}(\phi)}, \quad \text{with } \phi = A^a_\mu \lambda^a dx^\mu \quad (10)$$

with $S_{\text{INV}}(\phi) = \int L_{\text{INV}} d^4x$, the Yang-Mills action. We fix the gauge by inserting $1 = \int \delta(\Sigma) d\Sigma$; however, $d\Sigma = d(i\partial_i \Sigma) = s\Sigma$ with no ∂_μ contribution through our choice of co-ordinates for Σ . Using the representation $\delta(\Sigma) = \int d\theta e^{i\theta \Sigma}$ and the Berezin integral $s\Sigma = \int dX e^{iX s\Sigma}$ (with $\int d\theta \delta(\theta) \equiv -i \frac{\partial}{\partial \theta} \delta(\theta) \big|_{\theta=0}$)

$$e^{i\Gamma} = \int \mathcal{D}(\phi, X, X, s) e^{iL} \quad (11)$$

where we have replaced the group volume G by an integral over X , the measure on the fiber. The functional integral is indeed geometrical, and the fermi statistics of $X, \bar{X}$ derive from their nature as differentials or 1-forms.

Ojima has recently suggested a geometric derivation for (5). Complexifying the fiber G , one writes for (8) $\bar{d} = s + r + d$, where $r = d\alpha^i \frac{\partial}{\partial \alpha^i}$ and $rs + sr = 0$, $r^2 = 0$. Note that since spin and

statistics do not co-relate, the r equations may violate CPT and differ from the s ; so does $\bar{X}$ differ from X in its BRST variations. The resulting new (rr) , (rs) equations are

$$r\bar{X} = \frac{1}{2}[\bar{X}, \bar{X}] ; \quad rX + s\bar{X} + [X, \bar{X}] = 0 \quad (12)$$

Fixing $s\bar{X}_a = \sigma_a$, the second equation reads $rX = -\sigma - [X, \bar{X}]$. Applying s to the (rr) equation, we have $r\sigma = -[\sigma, \bar{X}]$. We now apply s to both sides of the (rs) equation. The left hand side is $-rsX = -\frac{1}{2}r[X, X] = +[\sigma, X] + [[X, \bar{X}], X]$ where we have used the value rX again. For the right hand side, we have $-s\sigma - \frac{1}{2}[[X, X], \bar{X}] + [X, \sigma]$. The triple brackets cancel on both sides by Jacobi's identity, and we find $s\sigma = s^2 \bar{X} = 0$. Note that $L_{FIX} + L_{GH} = -s(X\bar{X})$.

Since $sL = 0$, and L is a 4-form over x^μ so that $dL = 0$ by saturation, we have geometric closure of L over P ,

$$dL = 0 \quad (13)$$

SUPERGROUPS AND SPONTANEOUS BREAKDOWN

In ref.⁴ we have shown that for G a supergroup, $H \subset G$ its even subgroup (indices $a, b, \dots \in H$; $i, j, \dots \in G/H$)

$$\left. \begin{aligned} \omega^a &= X^a + dx^\mu A_\mu^a \\ \omega^i &= \eta^i + dx^\mu \xi_\mu^i \end{aligned} \right\} \quad (14)$$

Due to the odd nature of G/H , the η^i are scalar boson fields, corresponding to a Goldstone-Higgs set. The ξ_μ^i is a vector-ghost set. This picture was used⁴ to reproduce chiral spontaneously-broken $SU(3)_L \otimes SU(3)_R$, applying the supergroup $Q(3)$. It may also provide the interpretation of $SU(2/1)$ as a unified weak-electromagnetic theory (see our paper in session QFDIII of these Proceedings).

THE KALB-RAMOND FIELD

The usefulness of this geometric formalism was displayed in recent work on the $B_{\mu\nu}^u$ antisymmetric tensor gauge field⁴. Both the original Abelian model⁴ and the more recent non-Abelian⁵ have been shown to represent scalar fields, the latter a non-linear σ -model. The Lagrangian is here

$$L = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma} B_{\mu\nu}^a F_{\rho\sigma}(A+H) - \frac{1}{2}m^2 H^a H_a - \frac{1}{2}F_{\mu\nu}^i(A) F^{\mu\nu}_i(A) \quad (15)$$

$$B^a = \frac{1}{2} B_{\mu\nu}^a dx^\mu \wedge dx^\nu ; \quad H^a = H_a^\mu dx^\mu ; \quad A^i = A_\mu^i dx^\mu , \quad A^m = 0 \quad (16)$$

a denotes the adjoint representation of G ; i the subgroup G' and m the quotient G/G' .

The first attempts to supply ghosts and BRST equations was done with no geometric guidance and failed. Applying geometry, Thierry-Mieg produced straight-forward results⁶. For instance, the full B "connection" is

$$B = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu + \frac{1}{2} \tilde{B}_{ij} d\alpha^i \wedge d\alpha^j + \tilde{B}_{\mu i} dx^\mu \wedge d\alpha^i \quad (17)$$

and we see that we have as ghosts a scalar boson ϕ and a vector fermion ξ_μ as in (14),

$$\phi = \frac{1}{2} \tilde{B}_{ij} d\alpha^i \wedge d\alpha^j ; \quad \xi_\mu = \tilde{B}_{\mu i} d\alpha^i \quad (18)$$

and the BRST equations represent the vanishing of all components of $\tilde{B}_{A+H}$ other than the fully horizontal⁶ (in $dx^\mu \wedge dx^\nu \wedge dx^\rho$).

THE SOFT GROUP MANIFOLD

To treat non-internal gauges (such as gravity and supergravity) we have introduced⁷ the Soft Group Manifold (SGM). The SGM becomes a principal bundle upon application of the equations of motion (spontaneous fibration), with a subgroup FCG as fiber⁸. We thus get

pseudo-closure $d_{(F)} L = 0$.

More recently, we have checked⁹ that the geometric identification of ghosts and of the BRST operator s holds over G (the SGM). We have used the algebra of Lie derivatives L_ϵ which holds off-mass-shell. However, the conventional quantum Lagrangian is not invariant under this algebra $L_\epsilon L = \epsilon_j dL + d(\epsilon_j L)$. The second term is a divergence and does not matter, but $dL \neq 0$ as long as L is not reduced by fibration to space-time only. Invariance has generally been obtained by adding a quartic ghost term in L , cancelling its vertical components¹⁰.

The geometric derivation also fixes the "natural" commutation relations among the various ghosts and fields¹¹.

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